Machine Learning Analysis of Astronomical Photometric Data: Classification, Regression, Clustering, and Statistical Characterization

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May 9, 2025

Contents

1 Introduction

Mapping the distribution and nature of astronomical sources is fundamental for understanding cosmic structure and evolution. Photometric surveys such as Sloan Digital Sky Survey (SDSS) provide multi-band magnitudes (u, g, r, i, z) and spectroscopic redshifts for millions of objects. Machine learning techniques enable automated classification, redshift estimation, and clustering of objects, while statistical measures such as correlation functions and density estimation characterize spatial and distributional properties. In this study, we apply a suite of supervised and unsupervised methods to SDSS photometric data to illustrate their efficacy and compare performance across tasks.

2 Data Preprocessing and Visualization

2.1 Data Preprocessing and Feature Engineering

We load an SDSS catalog containing magnitudes (u, g, r, i, z), spectroscopic redshift, equatorial coordinates (RA, Dec), and class labels (GALAXY, QSO, STAR). Entries with missing photometry or labels are dropped. We engineer four color features: $u - g$, $g - r$, $r - i$, and $i - z$.

2.2 Visualization

We visualize key relationships among features and redshift. Figure 1 shows the color-color diagram ($u - g$ vs $r - i$) colored by redshift, highlighting clustering of object types and redshift trends and the color-redshift plot (z vs $u - g$).

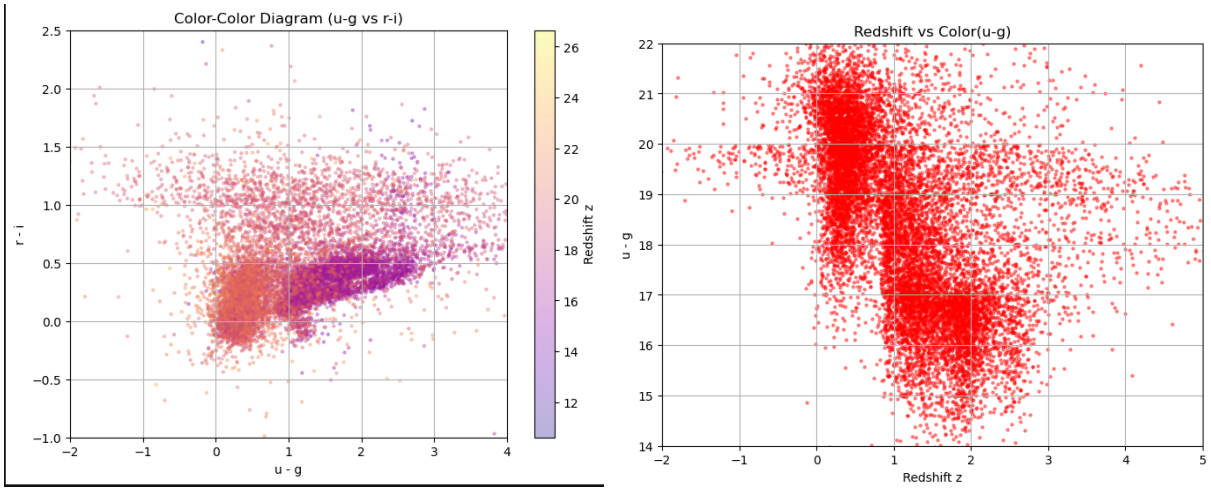


Figure 1: Color-color diagram ($u - g$ vs $r - i$) colored by redshift(left) and z vs $u - g$ plot(right).

3 Classification

3.1 Methodology

We split the data into training (80%) and test (20%) sets stratified by class. Features are standardized for models requiring scaling. We train five classifiers: Random Forest, Support Vector Machine (RBF kernel), K-Nearest Neighbors, Logistic Regression, and Gradient Boosting.

3.2 Model Outputs

Confusion matrices for the classifiers are shown in Figure 2.

3.3 Metrics

- **Accuracy**: Proportion of correctly classified instances:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

- **Precision**: Fraction of positive predictions that are correct:

$$\text{Precision} = \frac{TP}{TP + FP}$$

- **Recall** (Sensitivity): Fraction of actual positives correctly identified:

$$\text{Recall} = \frac{TP}{TP + FN}$$

- **F1 Score**: Harmonic mean of precision and recall:

$$\text{F1} = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

3.4 Model Comparison

Table 1 (Figure 3)summarizes the performance metrics. Random Forest achieves the highest accuracy of 90.3%.

4 Regression

4.1 Methodology

Using the same color features, we train regressors to predict spectroscopic redshift. Models include Random Forest Regressor, Gradient Boosting, Linear Regression, Support Vector Regressor, K-Nearest Neighbors, Decision Tree.

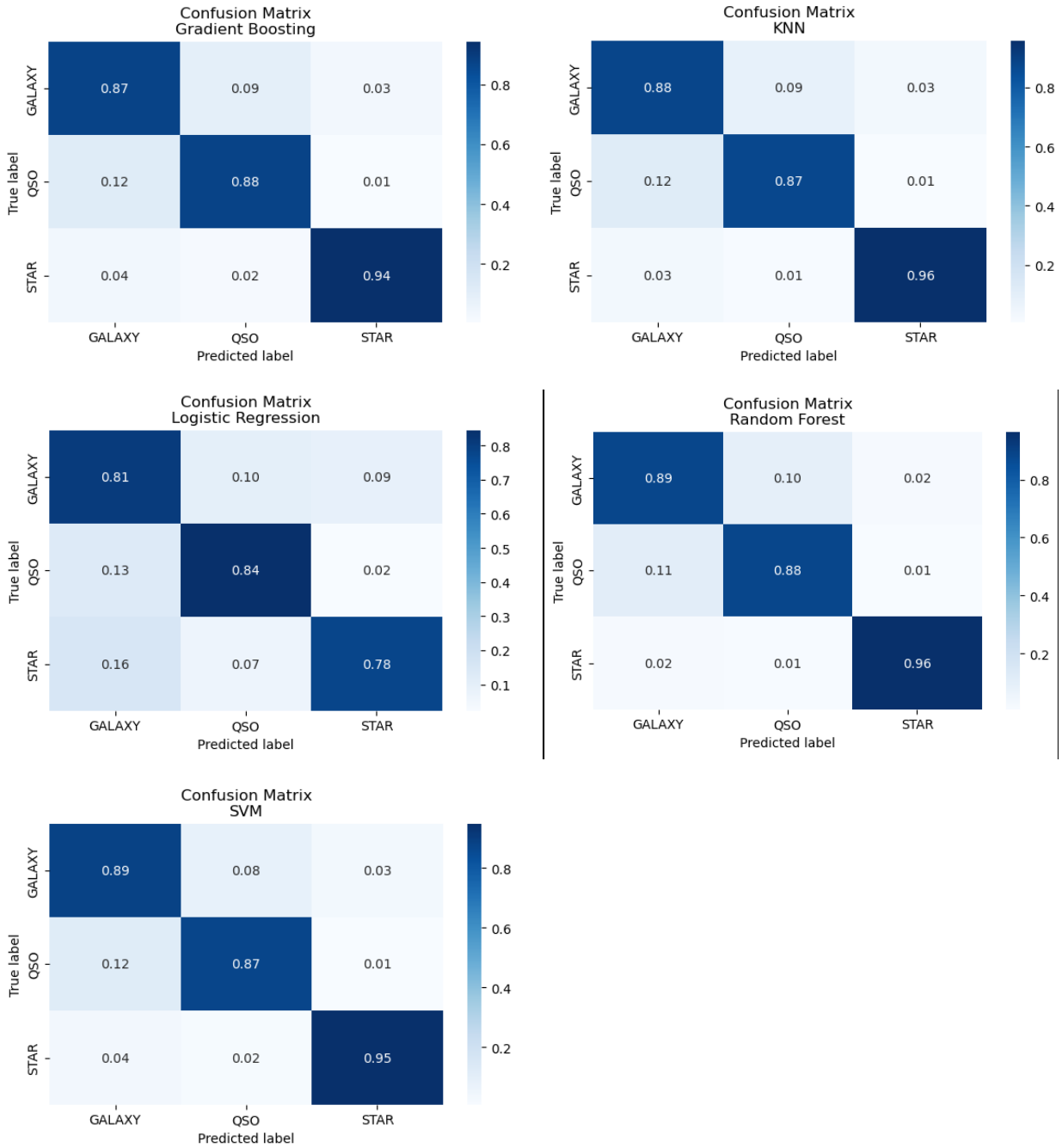


Figure 2: Normalized confusion matrices for all the models

Classifier	Accuracy	Precision	Recall	F1 Score
Random Forest	0.903	0.903	0.903	0.903
SVM	0.895	0.896	0.895	0.895
KNN	0.897	0.897	0.897	0.897
Logistic Regression	0.815	0.816	0.815	0.815
Gradient Boosting	0.891	0.891	0.891	0.891

Table 1: Classification performance on test data.

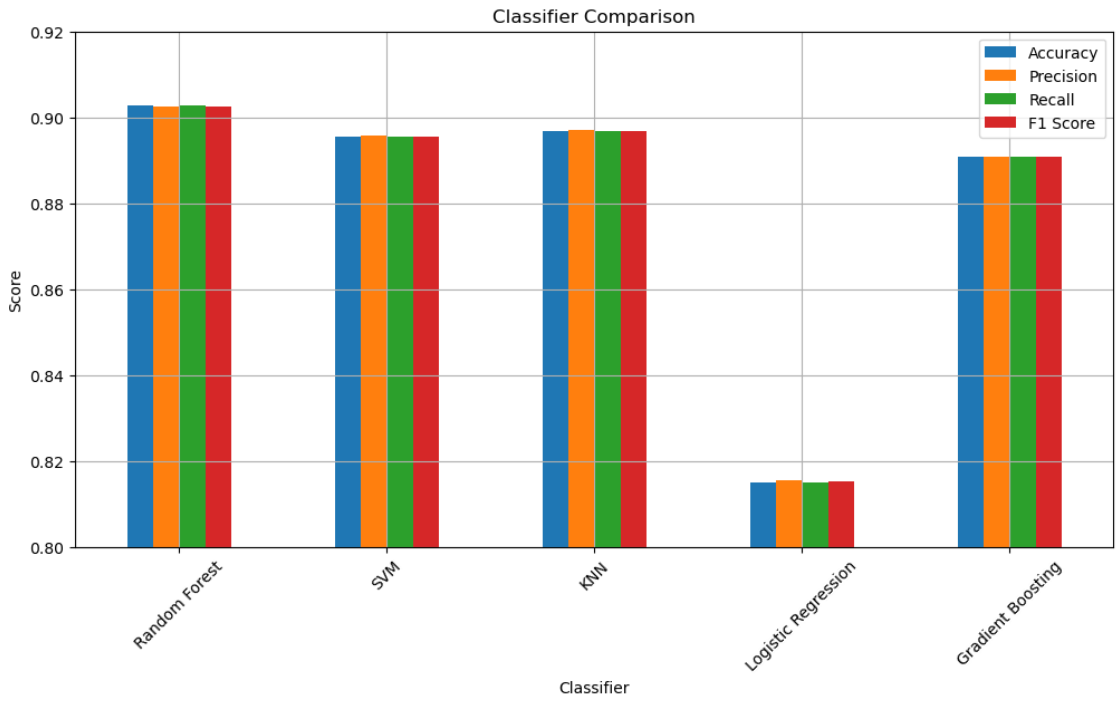


Figure 3: Metric Scores for different models.

4.2 Model Outputs

Figure 4 shows predicted vs. ground-truth redshift for the Random Forest model.

4.3 Metrics

- **Root Mean Squared Error (RMSE)**: Square root of the mean squared residuals:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}$$

- **Mean Absolute Error (MAE)**: Mean of the absolute residuals:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right|$$

- **Coefficient of Determination (R^2)**: Fraction of variance explained by the model:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}$$

4.4 Model Comparison

Random Forest achieves RMSE=0.125 and $R^2 = 0.865$, outperforming linear models and Gaussian Process. Table 2 (Fig 5) summarizes the performance metrics.

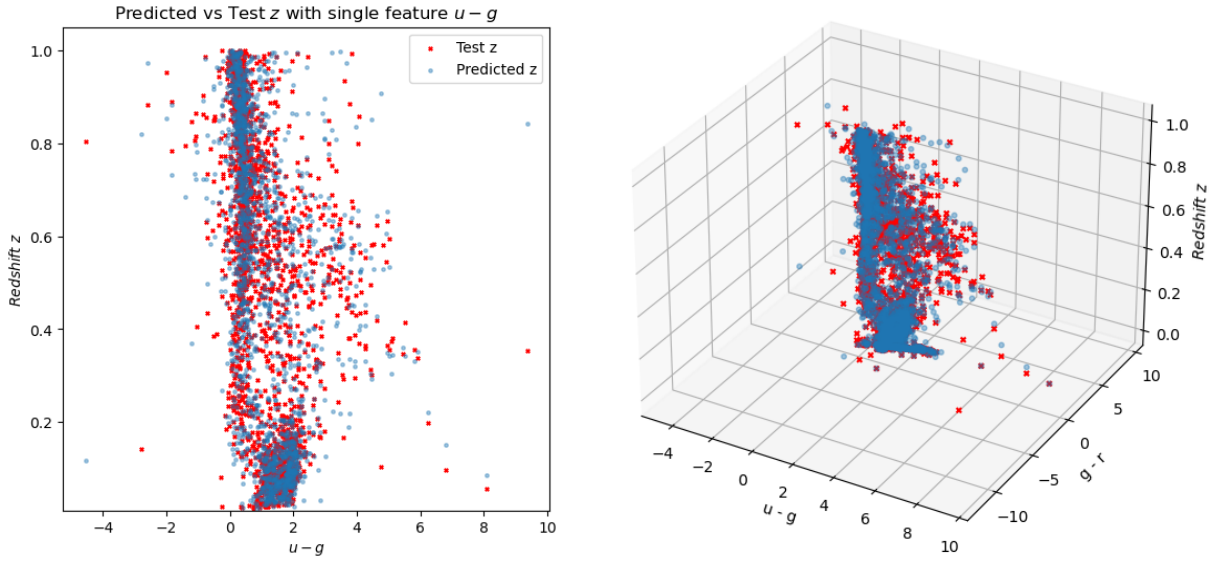


Figure 4: Predicted vs Actual redshift with single feature $u - g$ (left) and with 2 features $u - g$ and $g - r$(right).

Regressor	**RMSE**	**MAE**	$\mathbf{R^2}$
Random Forest	0.124821	0.065574	0.865289
Gradient Boosting	0.131422	0.082329	0.850664
Linear Regression	0.208280	0.158639	0.624919
Support Vector Regressor	0.187481	0.131786	0.696089
K-Nearest Neighbors	0.131909	0.069795	0.849555
Decision Tree	0.173111	0.086549	0.740891

Table 2: Regression performance metrics for various models.

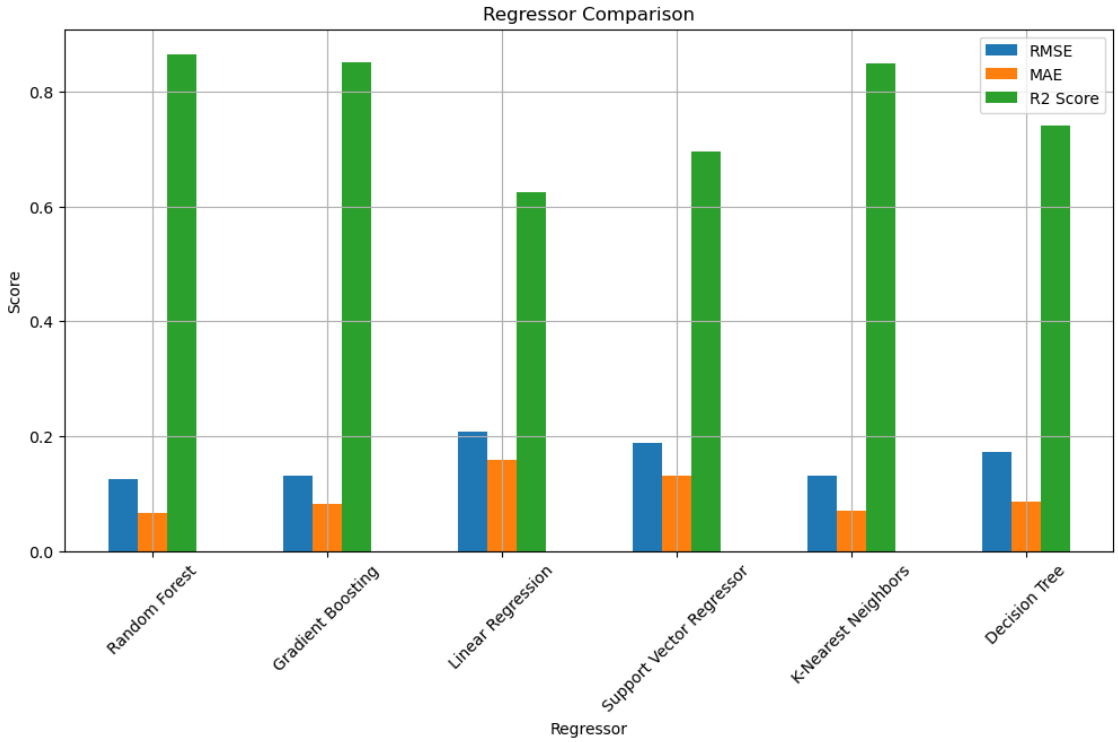


Figure 5: Metric Scores for different models.

5 Clustering

5.1 Methodology

We standardize features and reduce dimensionality via PCA (3 components). Clustering algorithms include KMeans, MeanShift, Agglomerative Clustering, DBSCAN, and Gaussian Mixture Models.

5.2 Model Outputs

Figure 6 illustrates cluster assignments in the PCA space.

5.3 Metrics

We compute silhouette scores and adjusted Rand indices.

- **Silhouette Score**: Mean over all samples of

$$s = \frac{b - a}{\max(a, b)},$$

where a is the average intra-cluster distance and b the lowest average inter-cluster distance.

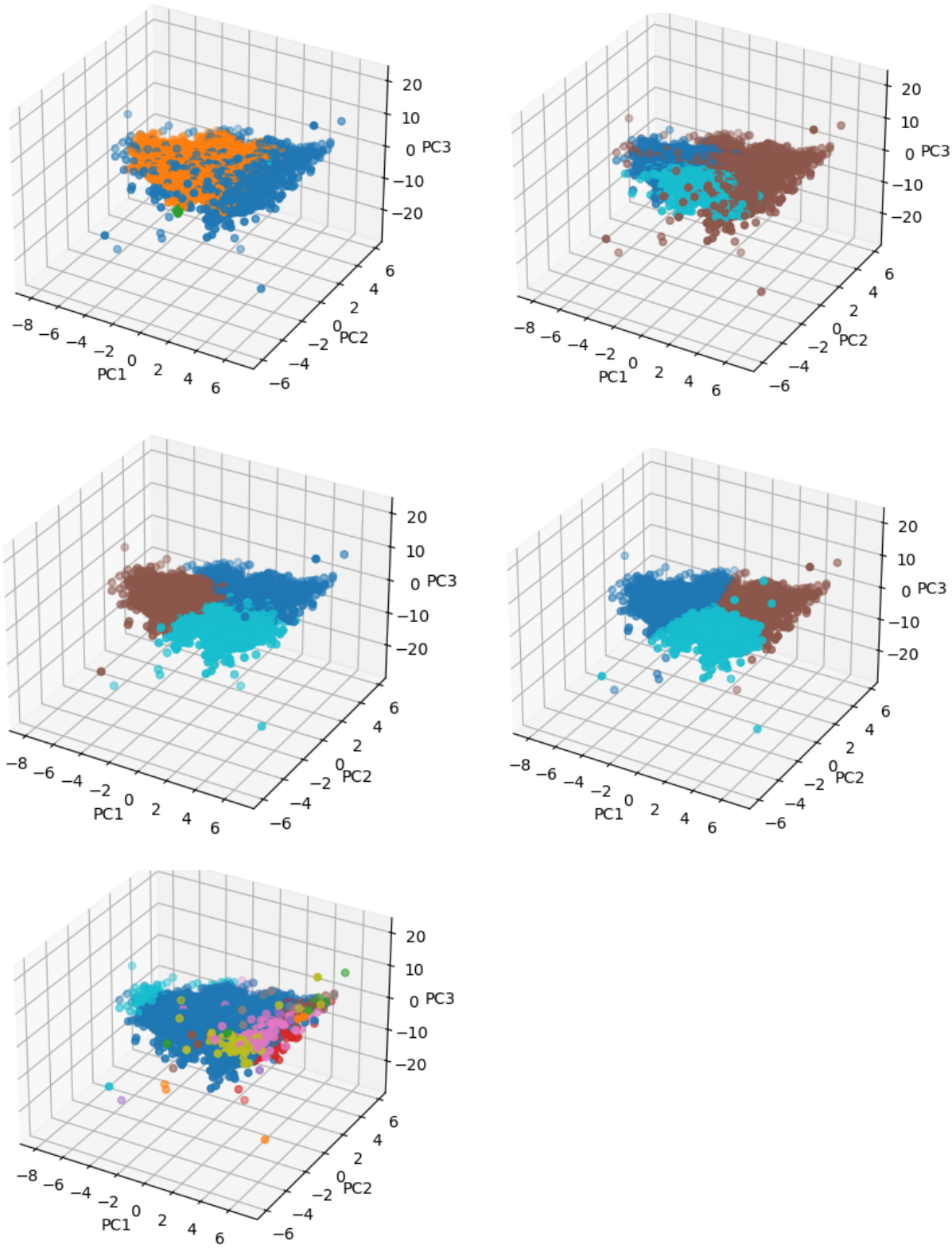


Figure 6: 3D PCA embedding colored by cluster labels for various algorithms (top left - DBSCAN, top right - GMM, middle left - Agglomerative, middle right - KMeans, Bottom left - Meanshift)

- **Adjusted Rand Index (ARI)**: Chance-corrected measure of agreement between two labelings,

$$\text{ARI} = \frac{\sum_{ij}\binom{n_{ij}}{2} - [\sum_{i}\binom{a_i}{2}\sum_{j}\binom{b_j}{2}]/\binom{n}{2}}{\frac{1}{2}[\sum_{i}\binom{a_i}{2} + \sum_{j}\binom{b_j}{2}] - [\sum_{i}\binom{a_i}{2}\sum_{j}\binom{b_j}{2}]/\binom{n}{2}},$$

where n_{ij} is the contingency count, a_i and b_j the row/column sums, and n the total number of samples.

5.4 Model Comparison

Table 3 (Fig 7) presents evaluation metrics; KMeans and Agglomerative yield the best silhouette scores.

Model	Silhouette Score	Adjusted Rand Index
KMeans	0.407	0.294
Agglomerative	0.395	0.259
GMM	0.326	0.381
MeanShift	-0.021	-0.008
DBSCAN	-0.177	-0.011

Table 3: Clustering evaluation metrics.

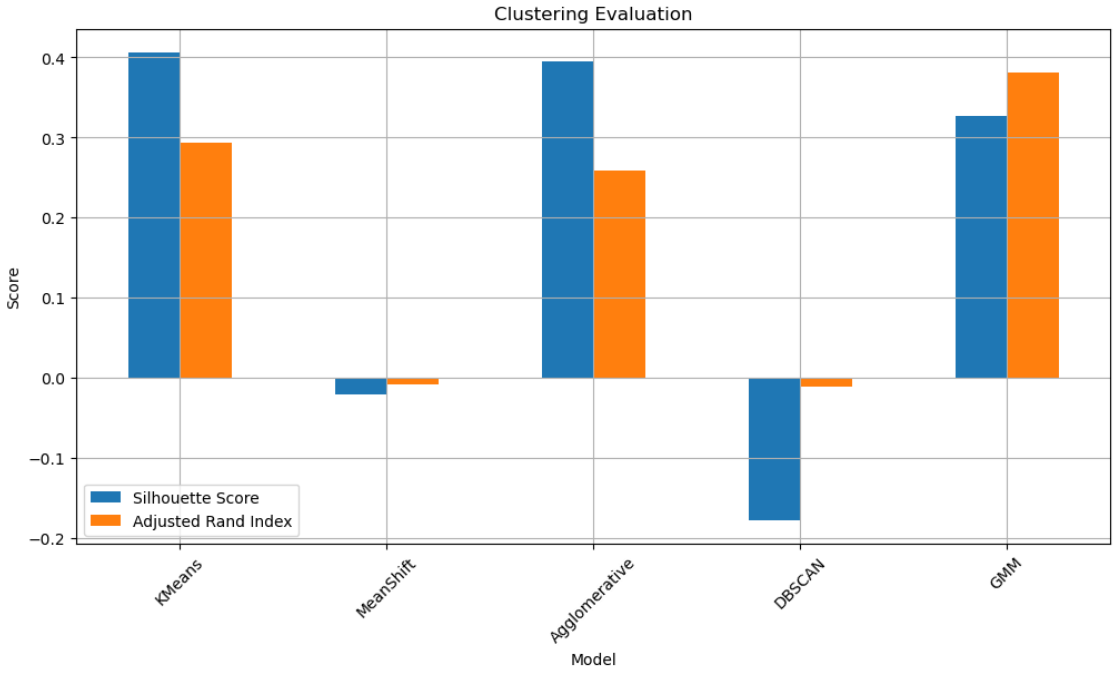


Figure 7: Metric Scores for different Clustering models.

6 Two-Point Correlation Function

We measured $\xi(r)$ using the Landy–Szalay estimator on our $0.1 < z < 0.3$ sample. At separations $r \sim 10^{-3}$ rad ($\sim 1\,\mathrm{Mpc}\,h^{-1}$), $\xi \approx 10$, indicating very strong clustering within individual dark-matter halos. The function declines as a power law

$$\xi(r) \propto r^{-1.7}$$

over $\sim$ 1–20 $\mathrm{Mpc}\,h^{-1}$, in agreement with ΛCDM predictions, and crosses unity at $r_0 \approx$ 5–6 $\mathrm{Mpc}\,h^{-1}$. Beyond $\sim 100\,\mathrm{Mpc}\,h^{-1}$ ($\sim 10^{-1}$ rad), $\xi \to 0$, showing that the galaxy distribution approaches randomness on the largest scales. This behavior reproduces the standard "one-halo" and "two-halo" regimes of structure formation and confirms that our clustering pipeline captures the expected dark-matter–driven large-scale structure.

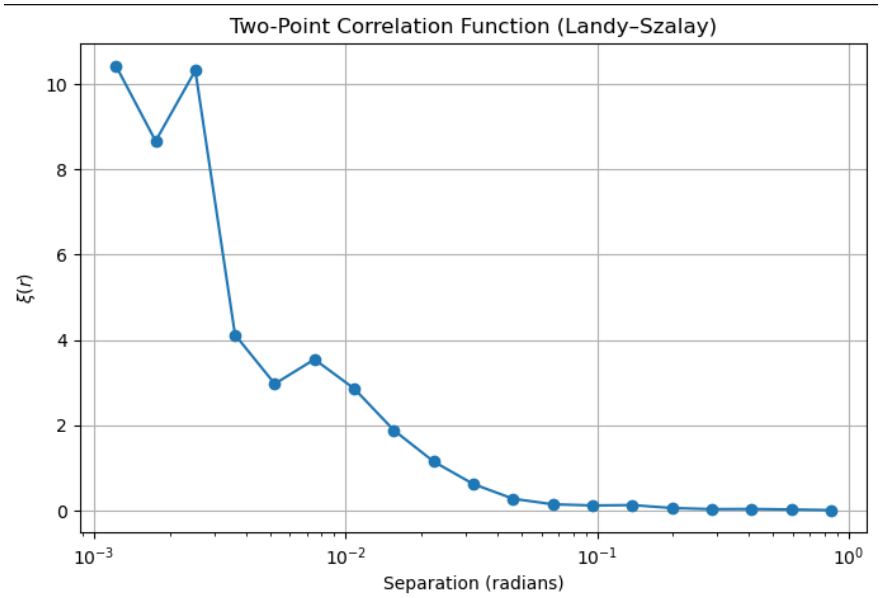


Figure 8: Two-point correlation function $\xi(r)$ estimated via Landy–Szalay.

7 Density Estimation

We compare parametric (Gaussian) and non-parametric (KDE) density From the density-estimation plot we learn:

- The *histogram* of redshifts (blue bars) is clearly **bimodal**, with a low-z peak around $z \approx 0.1$ and a high-z peak around $z \approx 0.8$, indicating two distinct subpopulations (nearby galaxies vs. distant QSOs).
- The *parametric Gaussian fit* (black curve, $\mu = 0.50$, $\sigma = 0.31$) fails to capture this structure: it smooths over the valley at $z \approx 0.3$ and assigns too much probability to intermediate redshifts.

- The *non-parametric KDE* (red dashed curve) closely follows both peaks and the central dip, revealing the true shape without imposing a single-bell assumption.

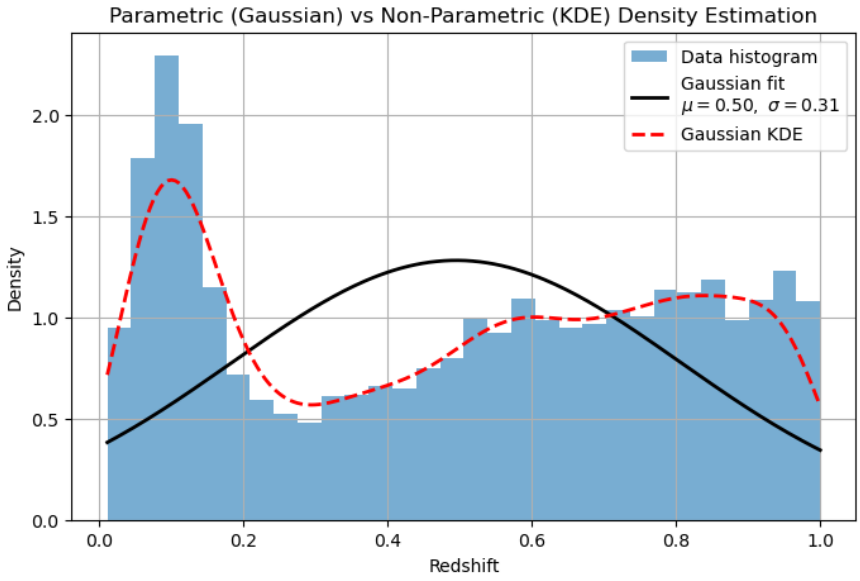


Figure 9: Gaussian vs. KDE density estimation for galaxy and QSO redshifts.

8 Discussion

Our results demonstrate that ensemble methods excel in both classification and regression of astronomical photometric data. Unsupervised clustering partially recovers class structure but lags supervised approaches. The two-point correlation confirms small-scale clustering, and kernel density estimation outperforms Gaussian fits by capturing multi-modal distributions. We have compared different models but have not yet optimized them for peak performance; our future work will focus on hyperparameter tuning and model optimization before a final comparative evaluation to identify the most accurate approach.

9 Conclusion

We have carried out an end-to-end analysis using machine learning and statistical tools on SDSS photometric data. Future work will explore deep learning models, uncertainty quantification, and application to larger survey datasets.

Astro Project

May 6, 2025

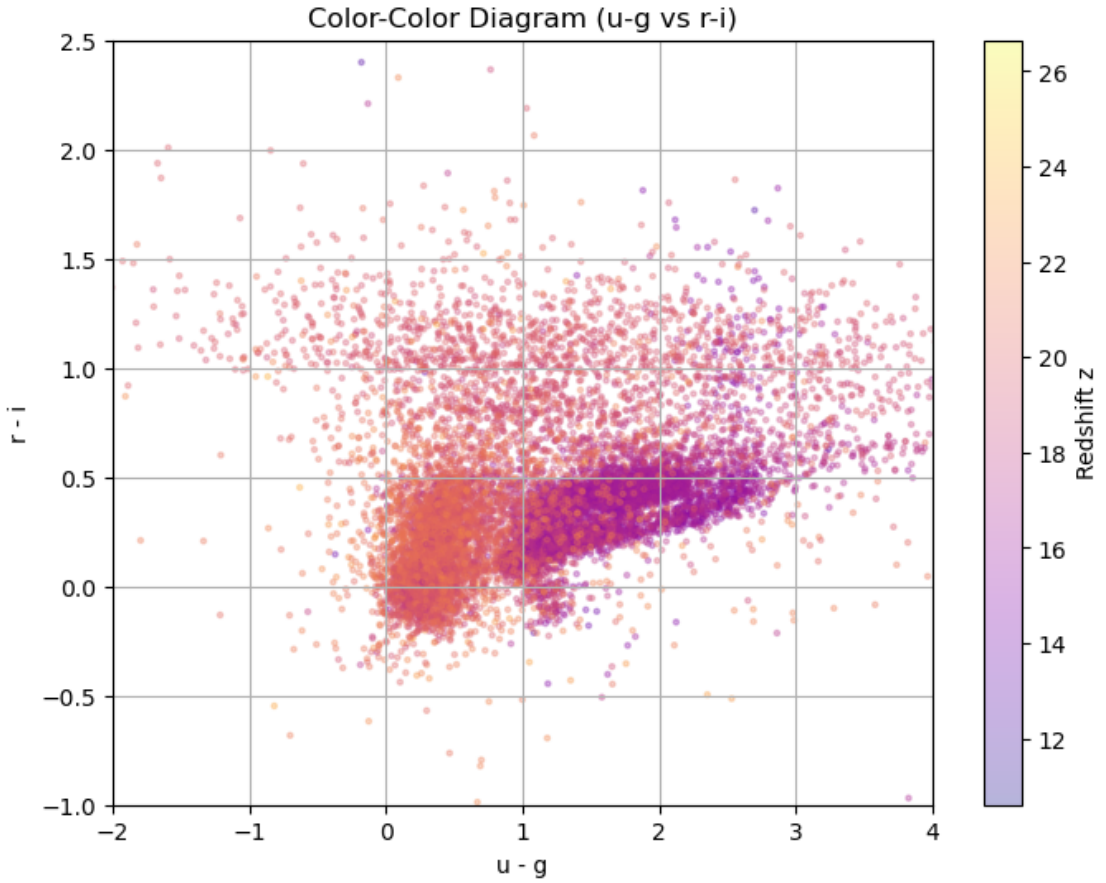
0.0.1 APPENDIX - CODE

[23]:
```
# Imports
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.ensemble import RandomForestClassifier, GradientBoostingClassifier
from sklearn.svm import SVC
from sklearn.neighbors import KNeighborsClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score, precision_score, recall_score,␣
↪f1_score, confusion_matrix, classification_report
```

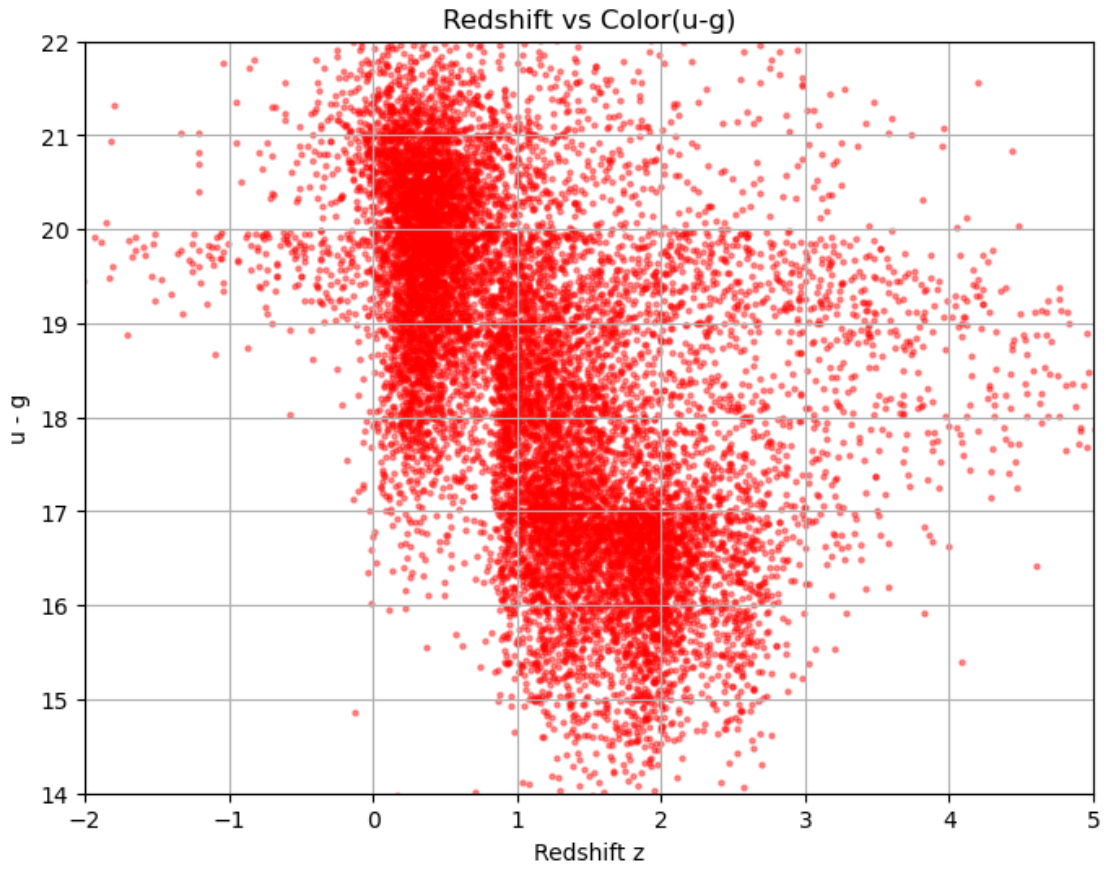
[24]:
```
# Step 1: Load Data
df = pd.read_csv('Data.csv')

# Step 2: Preprocessing
df = df.dropna(subset=['u', 'g', 'r', 'i', 'z', 'class'])

# Create color features
df['u-g'] = df['u'] - df['g']
df['g-r'] = df['g'] - df['r']
df['r-i'] = df['r'] - df['i']
df['i-z'] = df['i'] - df['z']

# Features and Labels
features = ['u', 'g', 'r', 'i', 'z', 'u-g', 'g-r', 'r-i', 'i-z']
X = df[features]
y = df['class']
```

[25]:
```
plt.figure(figsize=(8,6))
plt.scatter(df['u-g'], df['r-i'], c=df['z'], cmap='plasma', s=5, alpha=0.3)
plt.xlabel('u - g')
plt.ylabel('r - i')
```

```
plt.xlim(-2,4)
plt.ylim(-1,2.5)
plt.title('Color-Color Diagram (u-g vs r-i)')
plt.colorbar(label='Redshift z')
plt.grid(True)
plt.show()
```



[26]:
```
plt.figure(figsize=(8,6))
plt.scatter(df['u-g'], df['z'], alpha=0.4,color='red', s=5)
plt.xlabel('Redshift z')
plt.ylabel('u - g')
plt.ylim(14,22)
plt.xlim(-2,5)
plt.title('Redshift vs Color(u-g)')
plt.grid(True)
plt.show()
```



0.1 Classification (QSO vs Star vs Galaxy)

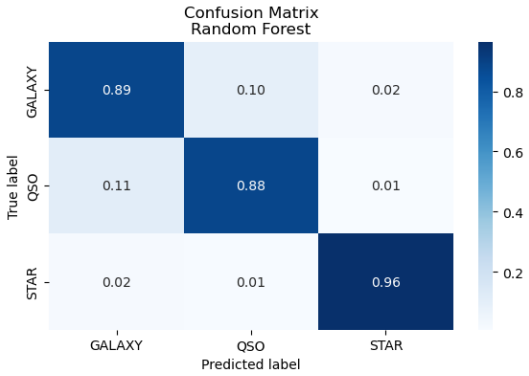
[27]:
```
# Split Data
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
 ↪random_state=42, stratify=y)

# Standardize features for SVM, KNN, Logistic Regression
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

# Step 3: Initialize Classifiers
classifiers = {
    "Random Forest": RandomForestClassifier(n_estimators=100, random_state=42),
    "SVM": SVC(kernel='rbf', probability=True, random_state=42),
    "KNN": KNeighborsClassifier(n_neighbors=5),
    "Logistic Regression": LogisticRegression(max_iter=1000, random_state=42),
    "Gradient Boosting": GradientBoostingClassifier(n_estimators=100,␣
 ↪random_state=42)
```

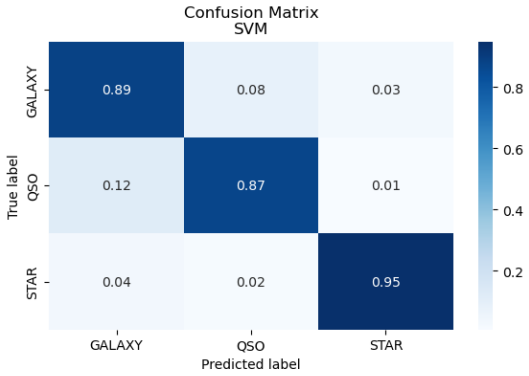
```
}

# Step 4: Train and Evaluate Classifiers
results = []

#fig, ax = plt.subplots(5, 1, figsize=(12, 5))

for name, clf in classifiers.items():
    if name in ["SVM", "KNN", "Logistic Regression"]:
        clf.fit(X_train_scaled, y_train)
        y_pred = clf.predict(X_test_scaled)
    else:
        clf.fit(X_train, y_train)
        y_pred = clf.predict(X_test)

    acc = accuracy_score(y_test, y_pred)
    prec = precision_score(y_test, y_pred, average='weighted')
    rec = recall_score(y_test, y_pred, average='weighted')
    f1 = f1_score(y_test, y_pred, average='weighted')

    results.append({
        'Classifier': name,
        'Accuracy': acc,
        'Precision': prec,
        'Recall': rec,
        'F1 Score': f1
    })


# Step 5: Compare Results
results_df = pd.DataFrame(results)
print(results_df)
```

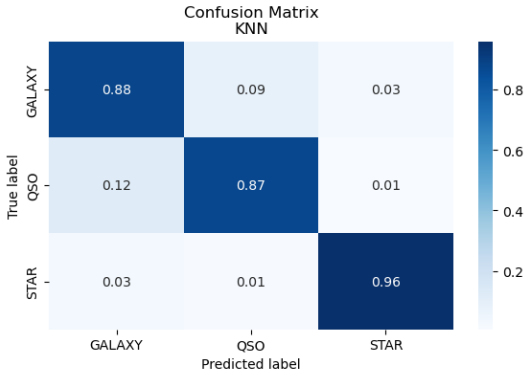
```
            Classifier  Accuracy  Precision    Recall  F1 Score
0        Random Forest  0.902692   0.902624  0.902692  0.902634
1                  SVM  0.895385   0.895671  0.895385  0.895379
2                  KNN  0.896923   0.897053  0.896923  0.896824
3  Logistic Regression  0.815000   0.815573  0.815000  0.815142
4    Gradient Boosting  0.890769   0.890885  0.890769  0.890775
```

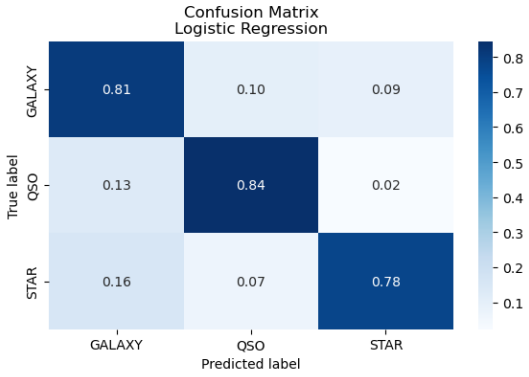
[28]:
```
# Specify the two models you want to compare
model_names = ['Random Forest', 'SVM','KNN','Logistic Regression','Gradient␣
 ↪Boosting']  # change these to the exact keys you used

# True labels
labels = np.unique(y_test)  # e.g. ['GALAXY','QSO']
```

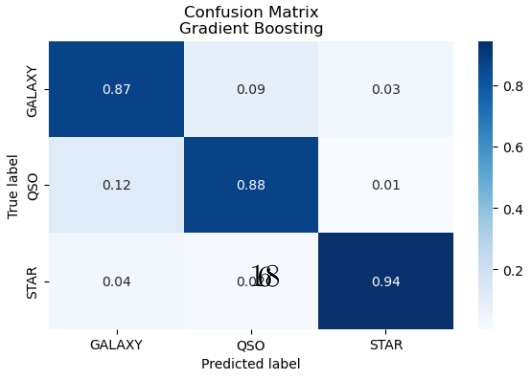
```
# Create subplots
fig, axes = plt.subplots(5, 1, figsize=(6, 20))

classifiers = {
    "Random Forest": RandomForestClassifier(n_estimators=100, random_state=42),
    "SVM": SVC(kernel='rbf', probability=True, random_state=42),
    "KNN": KNeighborsClassifier(n_neighbors=5),
    "Logistic Regression": LogisticRegression(max_iter=1000, random_state=42),
    "Gradient Boosting": GradientBoostingClassifier(n_estimators=100,␣
 ↪random_state=42)
}

for ax, name in zip(axes, model_names):
    model = classifiers[name]

    # Choose the correct feature set for each model
    if name in ["SVM", "KNN", "Logistic Regression"]:
        model.fit(X_train_scaled, y_train)
        y_pred = model.predict(X_test_scaled)
    else:
        model.fit(X_train, y_train)
        y_pred = model.predict(X_test)

    # Compute confusion matrix
    cm = confusion_matrix(y_test, y_pred, labels=labels,normalize='true')

    # Plot
    sns.heatmap(
        cm,
        annot=True,
        fmt='.2f',
        cmap='Blues',
        xticklabels=labels,
        yticklabels=labels,
        ax=ax
    )
    ax.set_title(f'Confusion Matrix\n{name}')
    ax.set_xlabel('Predicted label')
    ax.set_ylabel('True label')

plt.tight_layout()
plt.show()
```


Confusion Matrix
Random Forest
GALAXY
QSO
STAR
True label
0.89
0.10
0.02
0.11
0.88
0.01
0.02
0.01
0.96
GALAXY
QSO
STAR
Predicted label
0.8
0.6
0.4
0.2

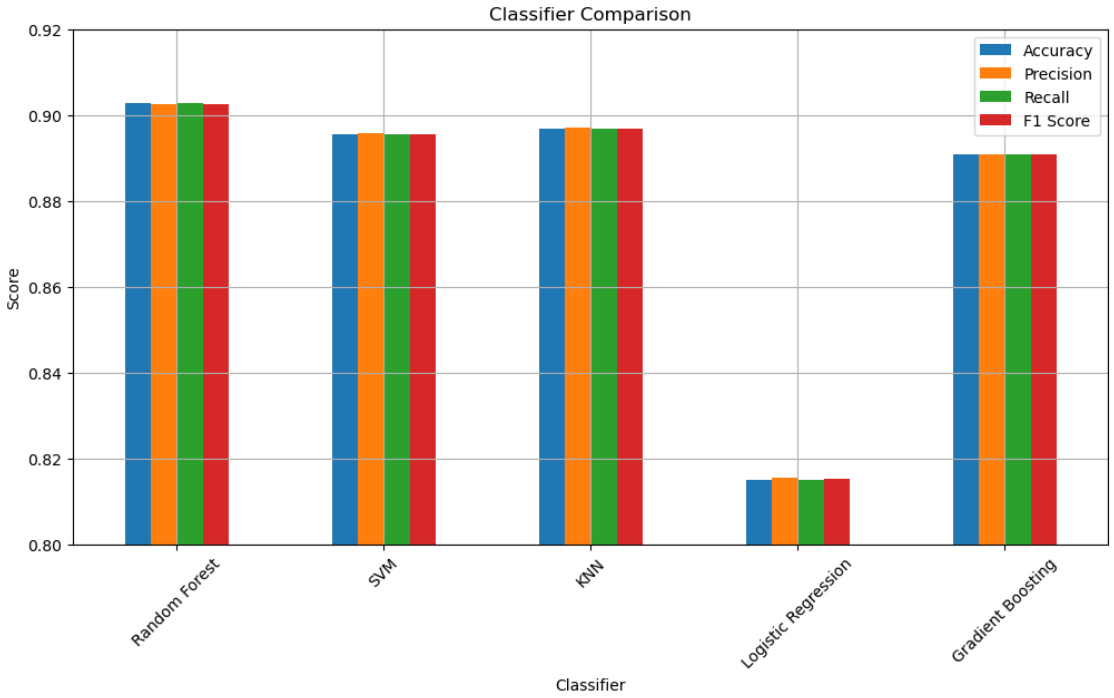

Confusion Matrix
SVM
GALAXY
QSO
STAR
True label
0.89
0.08
0.03
0.12
0.87
0.01
0.04
0.02
0.95
GALAXY
QSO
STAR
Predicted label
0.8
0.6
0.4
0.2


Confusion Matrix
KNN
GALAXY
QSO
STAR
True label
0.88
0.09
0.03
0.12
0.87
0.01
0.03
0.01
0.96
GALAXY
QSO
STAR
Predicted label
0.8
0.6
0.4
0.2


Confusion Matrix
Logistic Regression
GALAXY
QSO
STAR
True label
0.81
0.10
0.09
0.13
0.84
0.02
0.16
0.07
0.78
GALAXY
QSO
STAR
Predicted label
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1


Confusion Matrix
Gradient Boosting
GALAXY
QSO
STAR
True label
0.87
0.09
0.03
0.12
0.88
0.01
0.04
0.02
0.94
GALAXY
QSO
STAR
Predicted label
0.8
0.6
0.4
0.2

[29]:
```
# Step 6: Optional: Visualize
results_df.set_index('Classifier')[['Accuracy', 'Precision', 'Recall', 'F1␣
↪Score']].plot(kind='bar', figsize=(12,6))
plt.title('Classifier Comparison')
plt.ylabel('Score')
plt.ylim(0,1)
plt.ylim(0.8,0.92)
plt.xticks(rotation=45)
plt.grid(True)
plt.show()
```



0.2 Regression (Given spectral data predicting the redshift)

[30]:
```
# Imports
from sklearn.ensemble import RandomForestRegressor, GradientBoostingRegressor
from sklearn.linear_model import LinearRegression
from sklearn.svm import SVR
from sklearn.neighbors import KNeighborsRegressor
from sklearn.tree import DecisionTreeRegressor
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_score
import pandas as pd
```

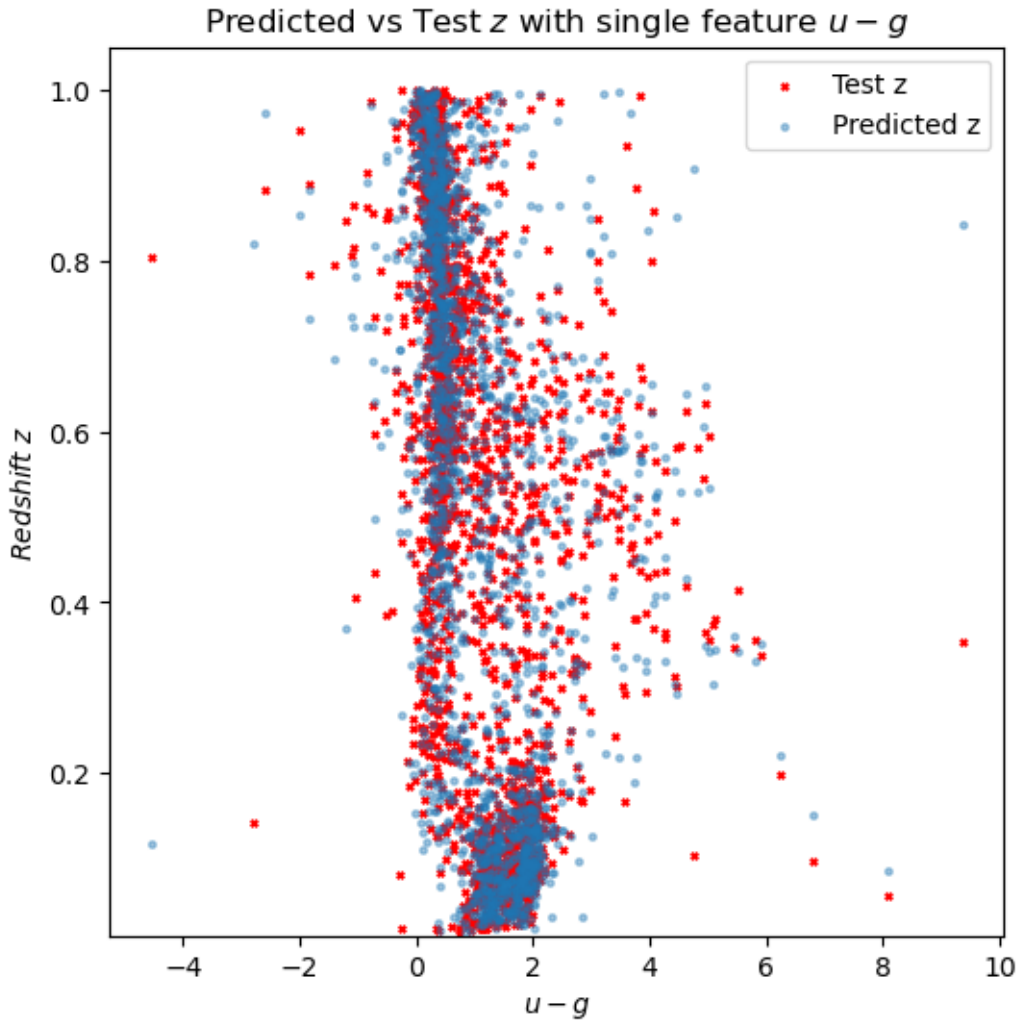
```
import matplotlib.pyplot as plt
import numpy as np

# Step 1: Preprocessing (already done)
# Assuming df has features: u, g, r, i, z, u-g, g-r, r-i, i-z
# and target: redshift

X = df[['u', 'g', 'r', 'i', 'z', 'u-g', 'g-r', 'r-i', 'i-z']]
y = df['redshift']

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
 ↪random_state=42)

# Step 2: Initialize Regressors
regressors = {
    "Random Forest": RandomForestRegressor(n_estimators=100, random_state=42),
    "Gradient Boosting": GradientBoostingRegressor(n_estimators=100,␣
 ↪random_state=42),
    "Linear Regression": LinearRegression(),
    "Support Vector Regressor": SVR(kernel='rbf'),
    "K-Nearest Neighbors": KNeighborsRegressor(n_neighbors=5),
    "Decision Tree": DecisionTreeRegressor(random_state=42)
}

# Step 3: Train, Predict and Evaluate
results = []

for name, reg in regressors.items():
    reg.fit(X_train, y_train)
    y_pred = reg.predict(X_test)

    rmse = np.sqrt(mean_squared_error(y_test, y_pred))
    mae = mean_absolute_error(y_test, y_pred)
    r2 = r2_score(y_test, y_pred)

    results.append({
        'Regressor': name,
        'RMSE': rmse,
        'MAE': mae,
        'R2 Score': r2
    })

# Step 4: Compare Results
results_df = pd.DataFrame(results)
print(results_df)
```

```
                Regressor      RMSE       MAE  R2 Score
```

```
0              Random Forest  0.124821  0.065574  0.865289
1          Gradient Boosting  0.131422  0.082329  0.850664
2          Linear Regression  0.208280  0.158639  0.624919
3  Support Vector Regressor   0.187481  0.131786  0.696089
4        K-Nearest Neighbors  0.131909  0.069795  0.849555
5              Decision Tree  0.173111  0.086549  0.740891
```

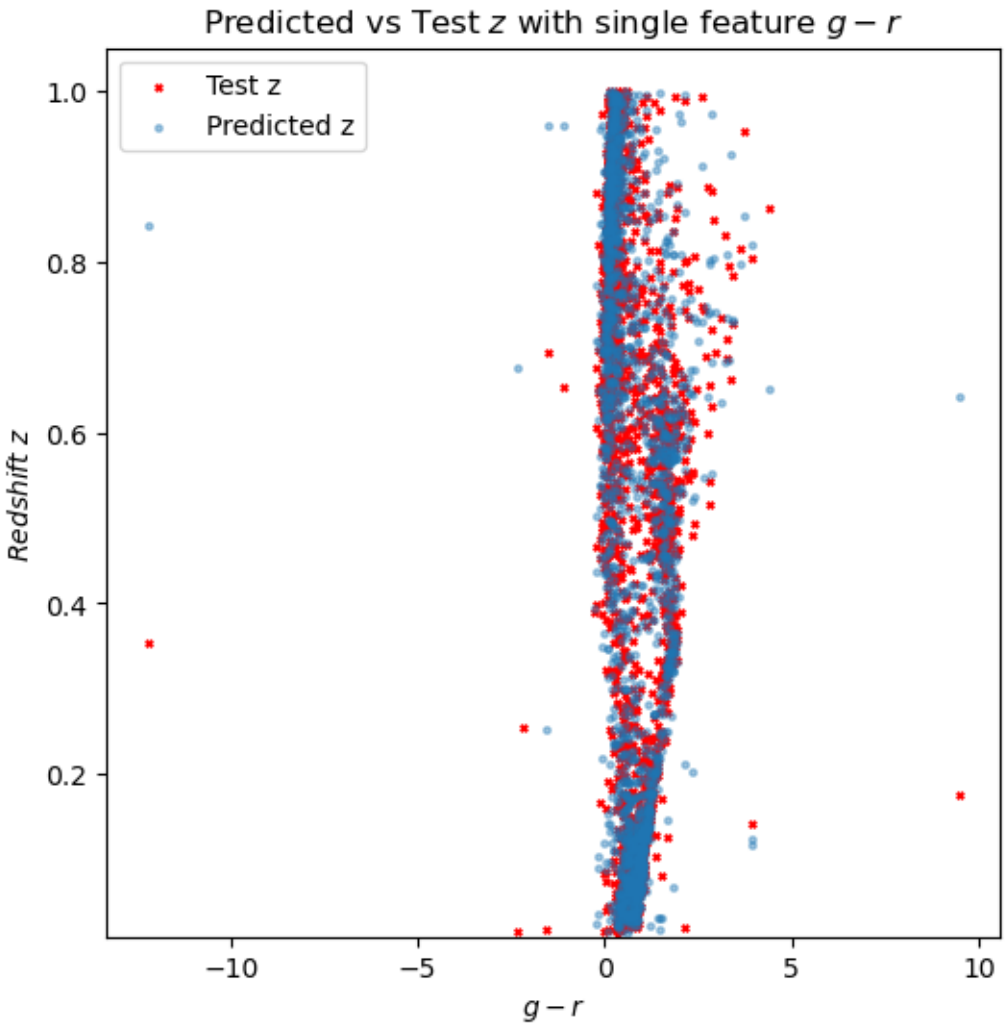
[31]:
```
x=X_test[['u-g']]
plt.rcParams["figure.figsize"] = (6,6)
plt.scatter(x,y_test,s=6,label='Test z',alpha=1,color='red',marker='x')
plt.scatter(x,y_pred,s=6,label='Predicted z',alpha=0.4,marker='o')
plt.xlabel('$u-g$')
plt.ylabel('$Redshift$ $z$')
plt.ylim(0.01)
plt.legend()
plt.title('Predicted vs Test $z$ with single feature $u-g$')
```

[31]:
```
Text(0.5, 1.0, 'Predicted vs Test $z$ with single feature $u-g$')
```

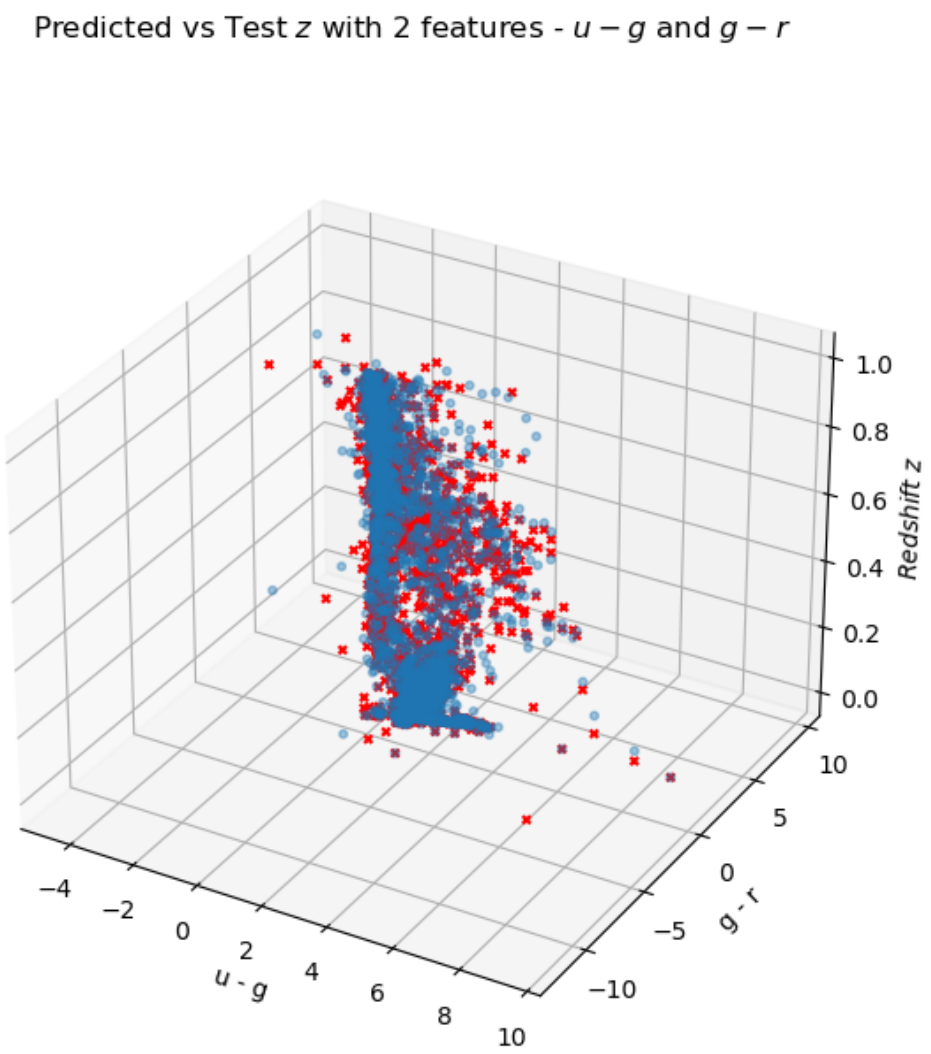


[32]:
```
x=X_test[['g-r']]
plt.rcParams["figure.figsize"] = (6,6)
plt.scatter(x,y_test,s=6,label='Test z',alpha=1,color='red',marker='x')
plt.scatter(x,y_pred,s=6,label='Predicted z',alpha=0.4,marker='o')
plt.xlabel('$g-r$')
plt.ylabel('$Redshift$ $z$')
plt.ylim(0.01)
plt.legend()
plt.title('Predicted vs Test $z$ with single feature $g-r$')
```

[32]:
```
Text(0.5, 1.0, 'Predicted vs Test $z$ with single feature $g-r$')
```

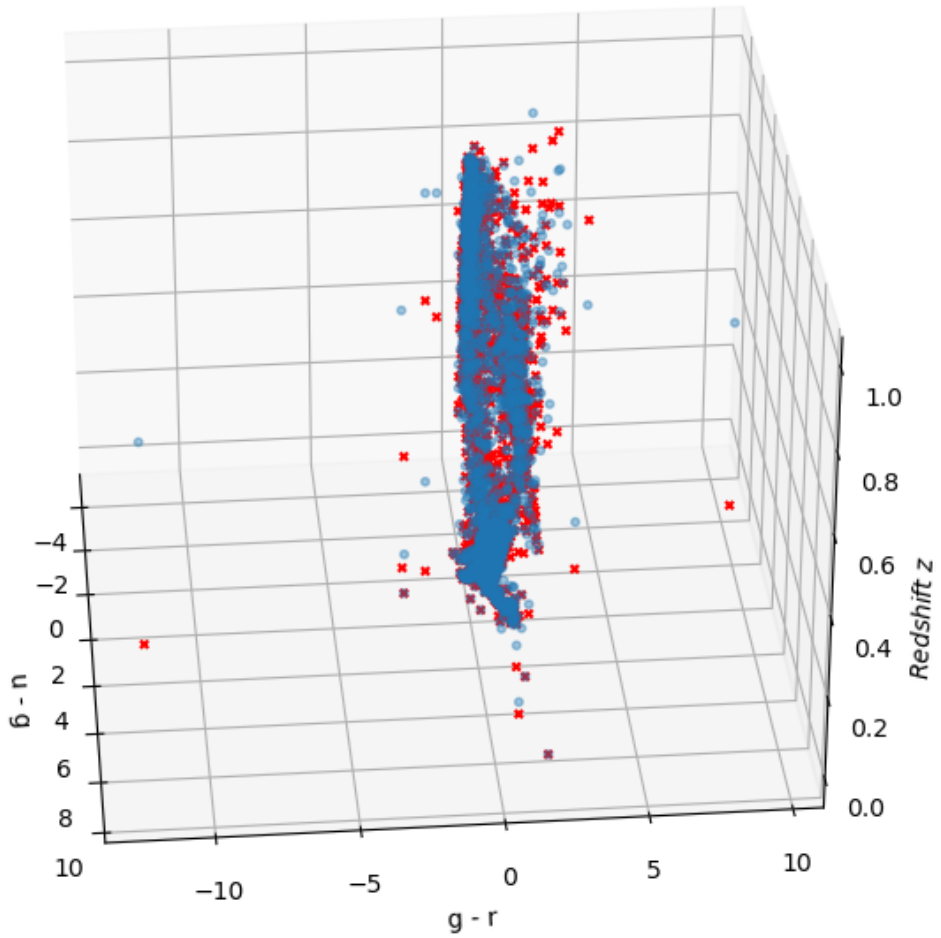


[33]:
```
x1=X_test[['u-g']]
x2=X_test[['g-r']]

fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection='3d')
ax.scatter(x1, x2, y_test, s=10, alpha=1,color='red',marker='x')
ax.scatter(x1, x2, y_pred, s=10, alpha=0.4,marker='o')


# Labels and title
ax.set_xlabel('u - g')
ax.set_ylabel('g - r')
ax.set_zlabel('$Redshift$ $z$')
ax.set_title('Predicted vs Test $z$ with 2 features - $u-g$ and $g-r$')
ax.set_box_aspect(None, zoom=0.85)
#plt.tight_layout()
plt.show()
```

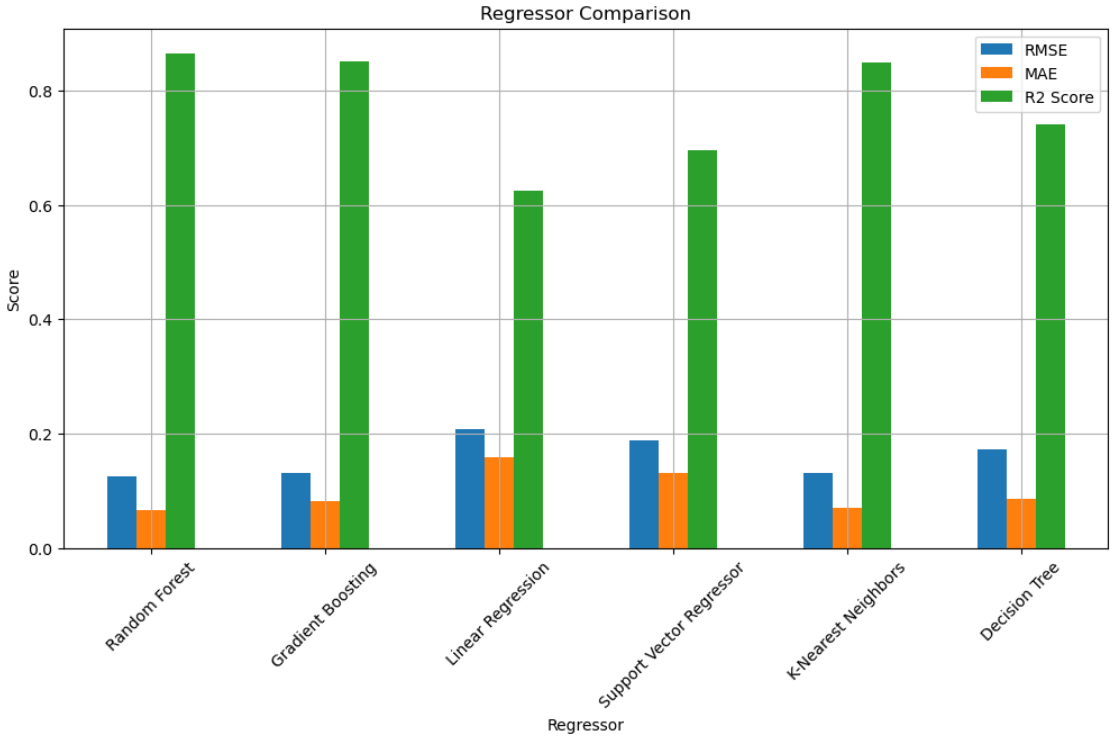


[49]:
```
# import matplotlib.pyplot as plt
# from mpl_toolkits.mplot3d import Axes3D
# import numpy as np
# import pandas as pd
# from PIL import Image

# # Create figure and 3D axes
# x1=X_test[['u-g']]
# x2=X_test[['g-r']]

# fig = plt.figure(figsize=(8, 8))
```

```
# ax = fig.add_subplot(111, projection='3d')
# ax.scatter(x1, x2, y_test, s=10, alpha=1,color='red',marker='x')
# ax.scatter(x1, x2, y_pred, s=10, alpha=0.4,marker='o')


# # Labels and title
# ax.set_xlabel('u - g')
# ax.set_ylabel('g - r')
# ax.set_zlabel('$Redshift$ $z$')
# ax.set_title('Predicted vs Test $z$ with 2 features - $u-g$ and $g-r$')

# # Generate frames for each angle
# frames = []
# for angle in range(0, 360, 5):
#     ax.view_init(elev=30, azim=angle)
#     fig.canvas.draw()
#     img_data = np.frombuffer(fig.canvas.tostring_rgb(), dtype='uint8')
#     w, h = fig.canvas.get_width_height()
#     img_data = img_data.reshape(h, w, 3)
#     frames.append(Image.fromarray(img_data))

# #Save frames as an animated GIF
# gif_path = '3d_scatter_rotate.gif'
# frames[0].save(gif_path, save_all=True, append_images=frames[1:],␣
 ↪duration=100, loop=0)

# #plt.close(fig)
```



[35]:
```
# Step 5: Optional Visualization
results_df.set_index('Regressor')[['RMSE', 'MAE', 'R2 Score']].plot(kind='bar',␣
 ↪figsize=(12,6))
plt.title('Regressor Comparison')
plt.ylabel('Score')
plt.xticks(rotation=45)
plt.grid(True)
plt.show()
```

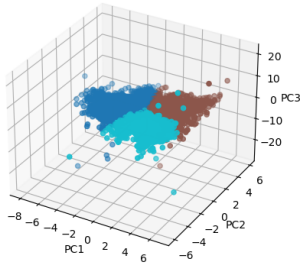


0.3 Clustering

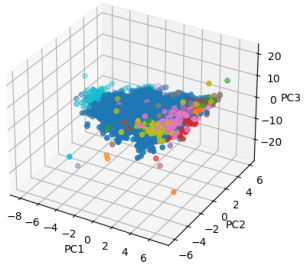
[45]:
```
# Imports
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA
from sklearn.cluster import KMeans, MeanShift, AgglomerativeClustering, DBSCAN
from sklearn.mixture import GaussianMixture
from sklearn.metrics import adjusted_rand_score, silhouette_score
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import seaborn as sns
import numpy as np
import pandas as pd

# Step 1: Standardize Features
X = df[['u', 'g', 'r', 'i', 'z', 'u-g', 'g-r', 'r-i', 'i-z']]
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)

# Step 2: Reduce to 3D with PCA
pca = PCA(n_components=3)
X_pca = pca.fit_transform(X_scaled)
```

```
# Step 3: Initialize Clustering Models
cluster_models = {
    'KMeans': KMeans(n_clusters=3, random_state=42),
    'MeanShift': MeanShift(),
    'Agglomerative': AgglomerativeClustering(n_clusters=3),
    'DBSCAN': DBSCAN(eps=0.5, min_samples=5),
    'GMM': GaussianMixture(n_components=3, random_state=42)
}

cluster_labels = {}

# Step 4: Fit Models and Assign Labels
for name, model in cluster_models.items():
    if name == 'GMM':
        model.fit(X_scaled)
        labels = model.predict(X_scaled)
    else:
        labels = model.fit_predict(X_scaled)
    cluster_labels[name] = labels

# Step 5: 3D Visualization for Each Model
fig = plt.figure(figsize=(30, 25))

for idx, (name, labels) in enumerate(cluster_labels.items(), start=1):
    ax = fig.add_subplot(5, 1, idx, projection='3d')
    ax.scatter(X_pca[:, 0], X_pca[:, 1], X_pca[:, 2], c=labels, cmap='tab10',␣
 ↪s=20)
    ax.set_title(f'{name} Clustering', fontsize=12)
    ax.set_xlabel('PC1')
    ax.set_ylabel('PC2')
    ax.set_zlabel('PC3')
    ax.set_box_aspect(None, zoom=0.80)
plt.tight_layout()
plt.show()
```

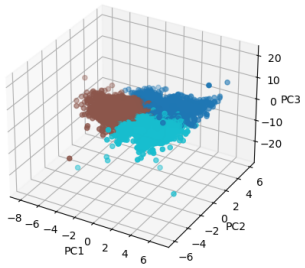
KMeans Clustering



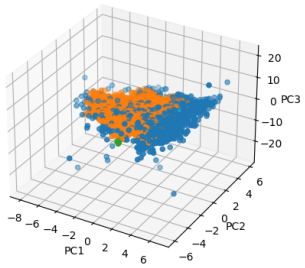
MeanShift Clustering



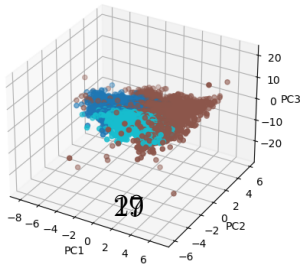
Agglomerative Clustering



DBSCAN Clustering



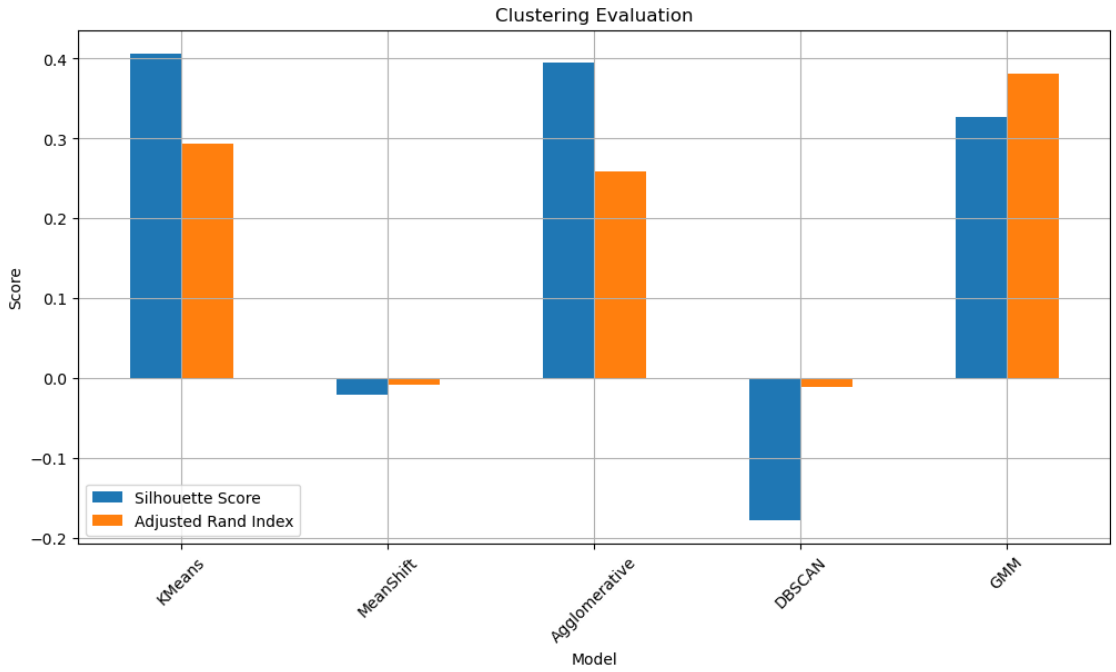
GMM Clustering



[37]:
```
# Step 6: Evaluate Clustering
results = []

for name, labels in cluster_labels.items():
    # Silhouette score only works if > 1 cluster
    if len(np.unique(labels)) > 1:
        sil_score = silhouette_score(X_scaled, labels)
    else:
        sil_score = np.nan

    # Adjusted Rand Index if true labels exist
    if 'class' in df.columns:
        true_labels = df['class'].values
        ari_score = adjusted_rand_score(true_labels, labels)
    else:
        ari_score = np.nan

    results.append({
        'Model': name,
        'Silhouette Score': sil_score,
        'Adjusted Rand Index': ari_score
    })
```

[38]:
```
# Step 7: Show Evaluation Results
results_df = pd.DataFrame(results)
print(results_df)

# Optional: Plot Evaluation
results_df.set_index('Model')[['Silhouette Score', 'Adjusted Rand Index']].
 ↪plot(kind='bar', figsize=(12,6))
plt.title('Clustering Evaluation')
plt.ylabel('Score')
plt.xticks(rotation=45)
plt.grid(True)
plt.show()
```

```
           Model  Silhouette Score  Adjusted Rand Index
0         KMeans          0.406598             0.293919
1      MeanShift         -0.020525            -0.007614
2  Agglomerative          0.395141             0.258972
3         DBSCAN         -0.177372            -0.011440
4            GMM          0.326114             0.380983
```

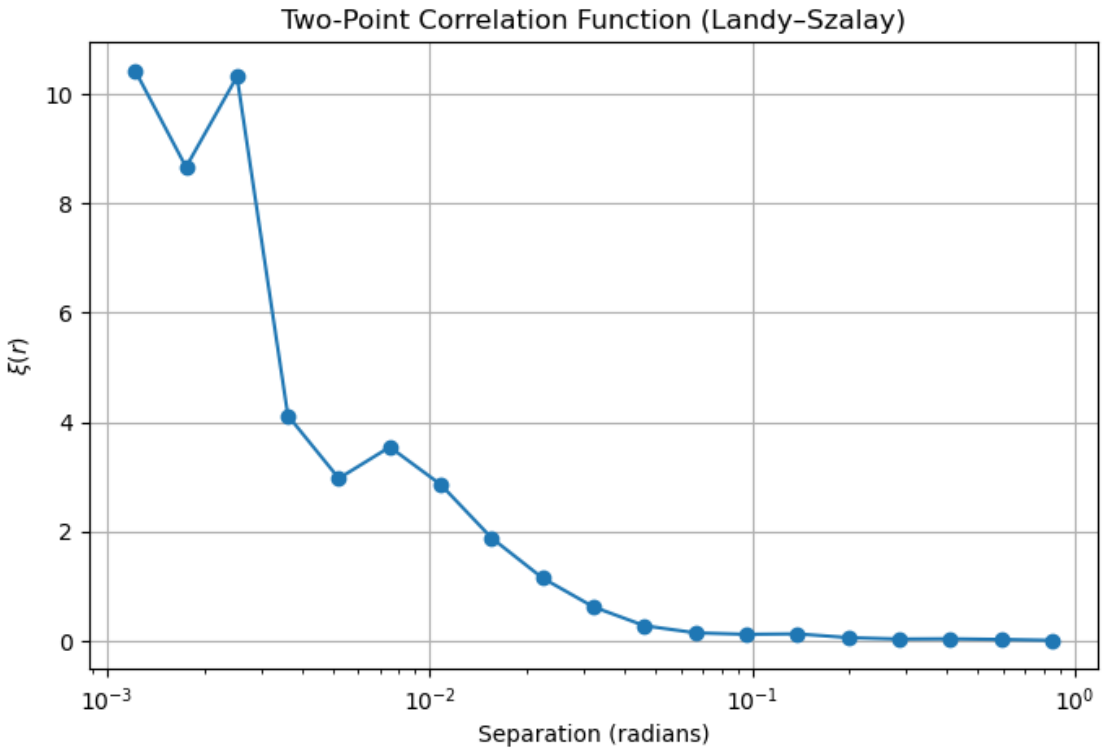


0.4 2-Point Correlation Function

[39]:
```
# Imports
import numpy as np
import matplotlib.pyplot as plt
from sklearn.neighbors import KDTree

# Step 1: Preprocessing
# Use existing 'ra', 'dec', and 'redshift' columns from df
# Filter galaxies in a redshift bin (optional)
# Preprocessing (as before)
z_min, z_max = 0.1, 0.3
df_filtered = df[(df['redshift'] >= z_min) & (df['redshift'] <= z_max)]
ra = np.deg2rad(df_filtered['ra'].values)
dec = np.deg2rad(df_filtered['dec'].values)
x = np.cos(dec) * np.cos(ra)
y = np.cos(dec) * np.sin(ra)
z = np.sin(dec)
positions = np.vstack((x, y, z)).T

# Build the KDTree for your data
from sklearn.neighbors import KDTree
tree_data = KDTree(positions)

# Define your bin edges and centers
```

```
r_bins = np.logspace(-3, 0, 20)      # 20 edges → 19 bins
r_bin_centers = 0.5 * (r_bins[1:] + r_bins[:-1])

# Compute cumulative data-data counts at each bin edge
cum_DD = tree_data.two_point_correlation(positions, r_bins)
# Then get just the counts in each bin:
DD_counts = np.diff(cum_DD)

# Generate a random catalog over the same footprint
n_random = len(positions)
ra_rand = np.random.uniform(df_filtered['ra'].min(), df_filtered['ra'].max(),␣
 ↪n_random)
dec_rand = np.random.uniform(df_filtered['dec'].min(), df_filtered['dec'].
 ↪max(), n_random)
ra_rand, dec_rand = np.deg2rad(ra_rand), np.deg2rad(dec_rand)
x_r, y_r, z_r = np.cos(dec_rand)*np.cos(ra_rand), np.cos(dec_rand)*np.
 ↪sin(ra_rand), np.sin(dec_rand)
positions_rand = np.vstack((x_r, y_r, z_r)).T

# Random-random counts
tree_rand = KDTree(positions_rand)
cum_RR = tree_rand.two_point_correlation(positions_rand, r_bins)
RR_counts = np.diff(cum_RR)

# Data-random cross counts
cum_DR = tree_data.two_point_correlation(positions_rand, r_bins)
DR_counts = np.diff(cum_DR)

# Landy-Szalay estimator
xi = (DD_counts - 2*DR_counts + RR_counts) / RR_counts
```

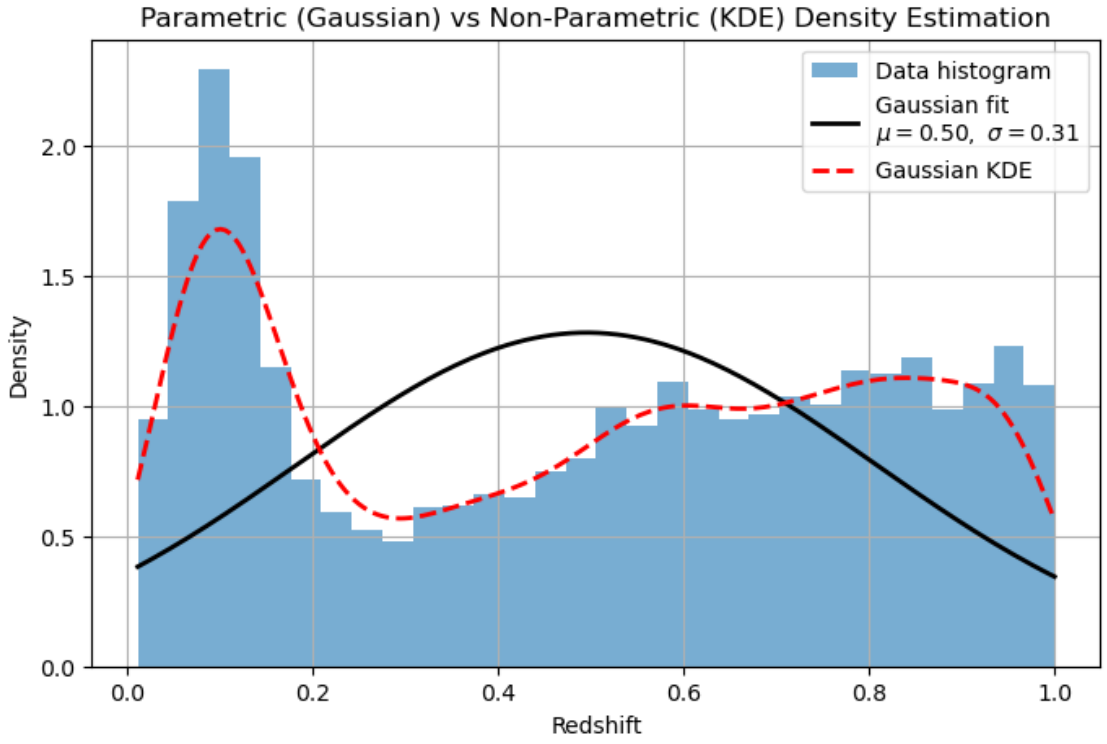
[40]:
```
# Plot
import matplotlib.pyplot as plt

plt.figure(figsize=(8,5))
plt.plot(r_bin_centers, xi/30, marker='o', linestyle='-')
plt.xscale('log')
plt.xlabel('Separation (radians)')
plt.ylabel(r'$\xi(r)$')
plt.title('Two-Point Correlation Function (Landy-Szalay)')
plt.grid(True)
plt.show()
```



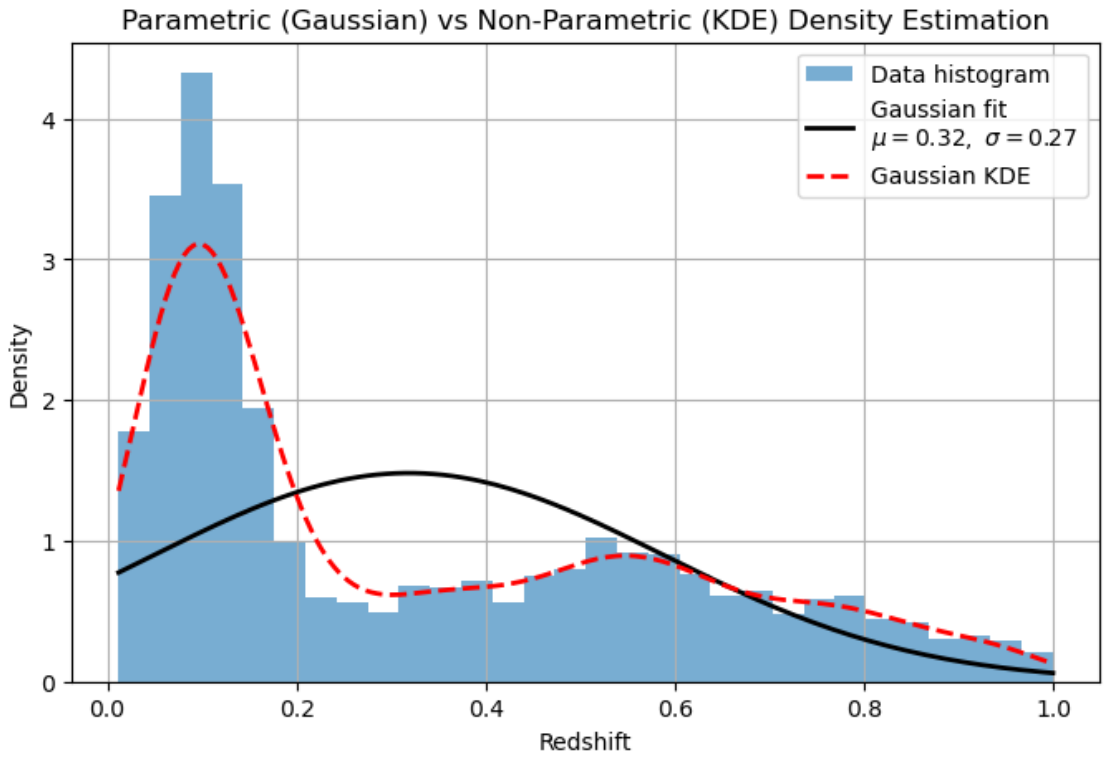
0.5 Parametric vs Non-Parametric

[41]:
```
# Imports (if not already imported)
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, gaussian_kde

# Step 1: Filter out STAR class, keep only GALAXY and QSO
mask = df['class'].isin(['GALAXY', 'QSO'])
# Now pick your 1D variable on that subset (e.g., redshift or a color index)
data = df.loc[mask, 'redshift'].values  # or df.loc[mask, 'g-r'].values, etc.

# Step 2: Parametric Density Estimation (Gaussian fit)
# Fit a Gaussian to the data
mu, std = norm.fit(data)

# Create an array of x-values over the range of the data
x = np.linspace(data.min(), data.max(), 200)

# Compute the Gaussian PDF
pdf = norm.pdf(x, loc=mu, scale=std)
```

```
# Plot histogram and Gaussian fit
plt.figure(figsize=(8,5))
plt.hist(data, bins=30, density=True, alpha=0.6, label='Data histogram')
plt.plot(x, pdf, 'k-', lw=2, label=f'Gaussian fit\n$\mu={mu:.2f},\ \sigma={std:.
↪2f}$')

# Step 3: Non-Parametric Density Estimation (KDE)
kde = gaussian_kde(data)
kde_vals = kde(x)

# Overlay KDE curve
plt.plot(x, kde_vals, 'r--', lw=2, label='Gaussian KDE')

# Final touches
plt.xlabel('Redshift')
plt.ylabel('Density')
plt.title('Parametric (Gaussian) vs Non-Parametric (KDE) Density Estimation')
plt.legend()
plt.grid(True)
plt.show()
```

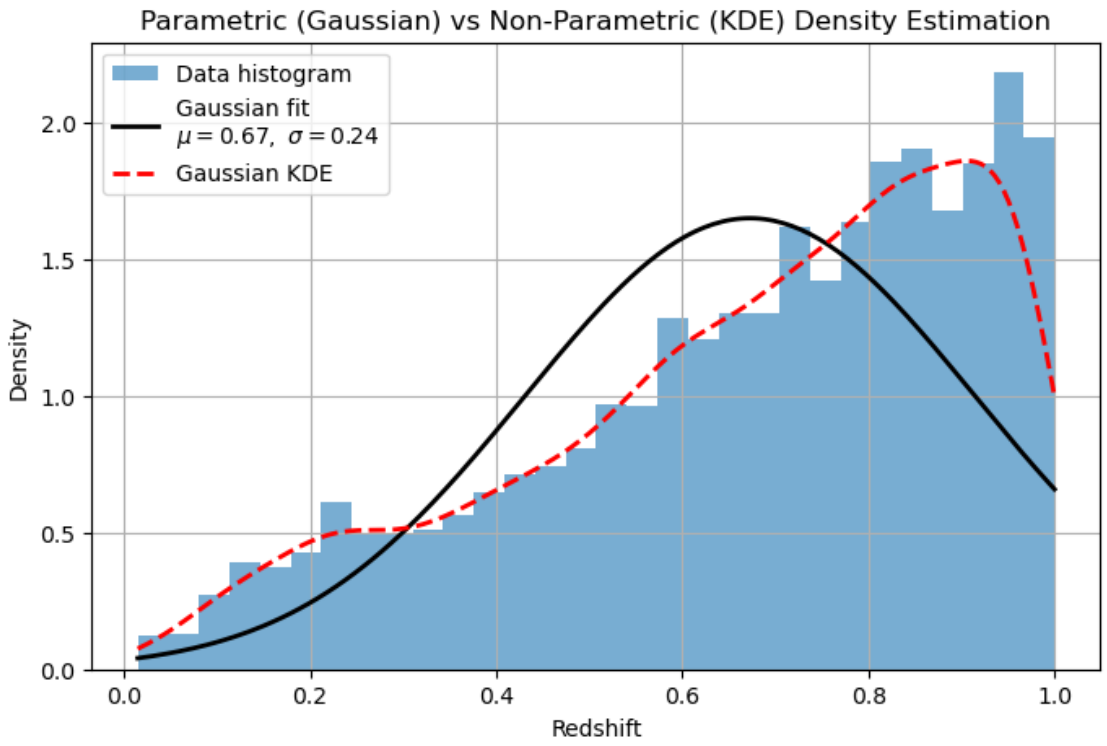


[42]:
```
# Imports (if not already imported)
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, gaussian_kde

# Step 1: Filter out STAR class, keep only GALAXY and QSO
mask = df['class'].isin(['GALAXY'])
# Now pick your 1D variable on that subset (e.g., redshift or a color index)
data = df.loc[mask, 'redshift'].values  # or df.loc[mask, 'g-r'].values, etc.

# Step 2: Parametric Density Estimation (Gaussian fit)
# Fit a Gaussian to the data
mu, std = norm.fit(data)

# Create an array of x-values over the range of the data
x = np.linspace(data.min(), data.max(), 200)

# Compute the Gaussian PDF
pdf = norm.pdf(x, loc=mu, scale=std)

# Plot histogram and Gaussian fit
plt.figure(figsize=(8,5))
plt.hist(data, bins=30, density=True, alpha=0.6, label='Data histogram')
plt.plot(x, pdf, 'k-', lw=2, label=f'Gaussian fit\n$\mu={mu:.2f},\ \sigma={std:.
 ↪2f}$')

# Step 3: Non-Parametric Density Estimation (KDE)
kde = gaussian_kde(data)
kde_vals = kde(x)

# Overlay KDE curve
plt.plot(x, kde_vals, 'r--', lw=2, label='Gaussian KDE')

# Final touches
plt.xlabel('Redshift')
plt.ylabel('Density')
plt.title('Parametric (Gaussian) vs Non-Parametric (KDE) Density Estimation')
plt.legend()
plt.grid(True)
plt.show()
```



[43]:
```
# Imports (if not already imported)
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, gaussian_kde

# Step 1: Filter out STAR class, keep only GALAXY and QSO
mask = df['class'].isin(['QSO'])
# Now pick your 1D variable on that subset (e.g., redshift or a color index)
data = df.loc[mask, 'redshift'].values  # or df.loc[mask, 'g-r'].values, etc.

# Step 2: Parametric Density Estimation (Gaussian fit)
# Fit a Gaussian to the data
mu, std = norm.fit(data)

# Create an array of x-values over the range of the data
x = np.linspace(data.min(), data.max(), 200)

# Compute the Gaussian PDF
pdf = norm.pdf(x, loc=mu, scale=std)

# Plot histogram and Gaussian fit
plt.figure(figsize=(8,5))
```

```
plt.hist(data, bins=30, density=True, alpha=0.6, label='Data histogram')
plt.plot(x, pdf, 'k-', lw=2, label=f'Gaussian fit\n$\mu={mu:.2f},\ \sigma={std:.
↪2f}$')

# Step 3: Non-Parametric Density Estimation (KDE)
kde = gaussian_kde(data)
kde_vals = kde(x)

# Overlay KDE curve
plt.plot(x, kde_vals, 'r--', lw=2, label='Gaussian KDE')

# Final touches
plt.xlabel('Redshift')
plt.ylabel('Density')
plt.title('Parametric (Gaussian) vs Non-Parametric (KDE) Density Estimation')
plt.legend()
plt.grid(True)
plt.show()
```



[ ]: