Massively Parallel Finite Element Methods

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With the advent of massively parallel computer architecture, traditional serial algorithms used in the process of formulating, solving, and post-processing Finite Elements need to be updated to reflect current and future changes to computer architecture. From development of sparse parallel linear solvers to domain decomposition techniques to parallel mesh generation, various techniques will be explored and evaluated

Introduction

Using the finite element method, various physical phenomena in classical physics can be represented within a body/continuum. This continuum can be solid, liquid, gaseous, or anything in between. The only requirement of this method is that the physical phenomena is able to be represented/approximated with a discrete number of points using differential equations and occurs spatially x-y-z space.

One of the biggest problems with trying to solve phenomena such as turbulence is that the theoretical grid sizes and memory requirements needed to get an accurate representation of the domain over time and space (x, y, z, t) can be prohibitively expensive in terms of RAM, Bandwidth, and FLOPS with todays current machines. One possible technique to lessen the impact of this is to use data compression techniques to store vector fields of the temporal derivative in space along with a coarser vector field. Because recent trends in supercomputing suggest that RAM and Bandwidth will become more expensive in relation to FLOPS in the future, algorithms which utilize far less RAM and network bandwidth at the expense of an increase FLOPS with play to the strength of current and future trends in technology. As an example, consider a 2-D complex fluid velocity field $\vec{V}$. Pretend that we are using a GPU to process this field. To make the best use of the GPU, first we must consider in what areas it excels at and at which areas it does not.

Strengths of the GPU:

- Massive Parallel Computation (1000 cores)

  High Theoretical FLOPS (Teraflops)

  Very High Energy Efficiency per Flop

- Graphics

  Dedicated Hardware on the GPU

  Well Established Graphics APIs for compression

- Bandwidth

  High Global Memory Bandwidth

  Programmable Cache

Weaknesses of the GPU:

- Thread Divergence
- Low Bandwidth between the GPU Memory and CPU Memory (5GB/s)
- Low individual clock speed (1Ghz)

Because the GPU was originally designed for graphics computation, it has many well-established graphics APIS as well as dedicated hardware for graphics purposes. If we were to represent the gradient $\nabla\vec{V}$ of a 2d velocity using a compression technique, we could perform integration and decoding on this compressed field to reproduce the velocity field on the GPU reducing CPU and GPU bandwidth at the expense of numerical artifacts being introduced into the velocity field. While this is certainly not desirable, this is partially mitigated by integration which helps to reduce artificial noise introduced by the compression algorithm. Moreover, if we performed additional image filtering on the uncompressed velocity field, we could further reduce artificial noise at the expense of more FLOPS. Moreover, because of the limited global memory on the GPU compared to the CPU, we could store a compressed version of the field that would be a higher in fidelity than an uncompressed version using the same amount of global memory. By uncompressing the data, we would be using FLOPS on the high fidelity decoded field but sending compressed versions to reduce strain on precious system bandwidth.

In Finite element analysis, one of the most important the differential equations into a linear system of equations given by Equation 1

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{1}$$

where A is a sparse square matrix of size $n \times n$ and x is a vector of length n For sparse matrices there exists many methods for solving linear systems [Matrix Computations, 2013].

- Iterative Methods
- Direct Methods

Subsection 1

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Table 1: *Random table*

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First name	Last Name	Grade
John	Doe	7.5
Richard	Miles	2

Subsection 2

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