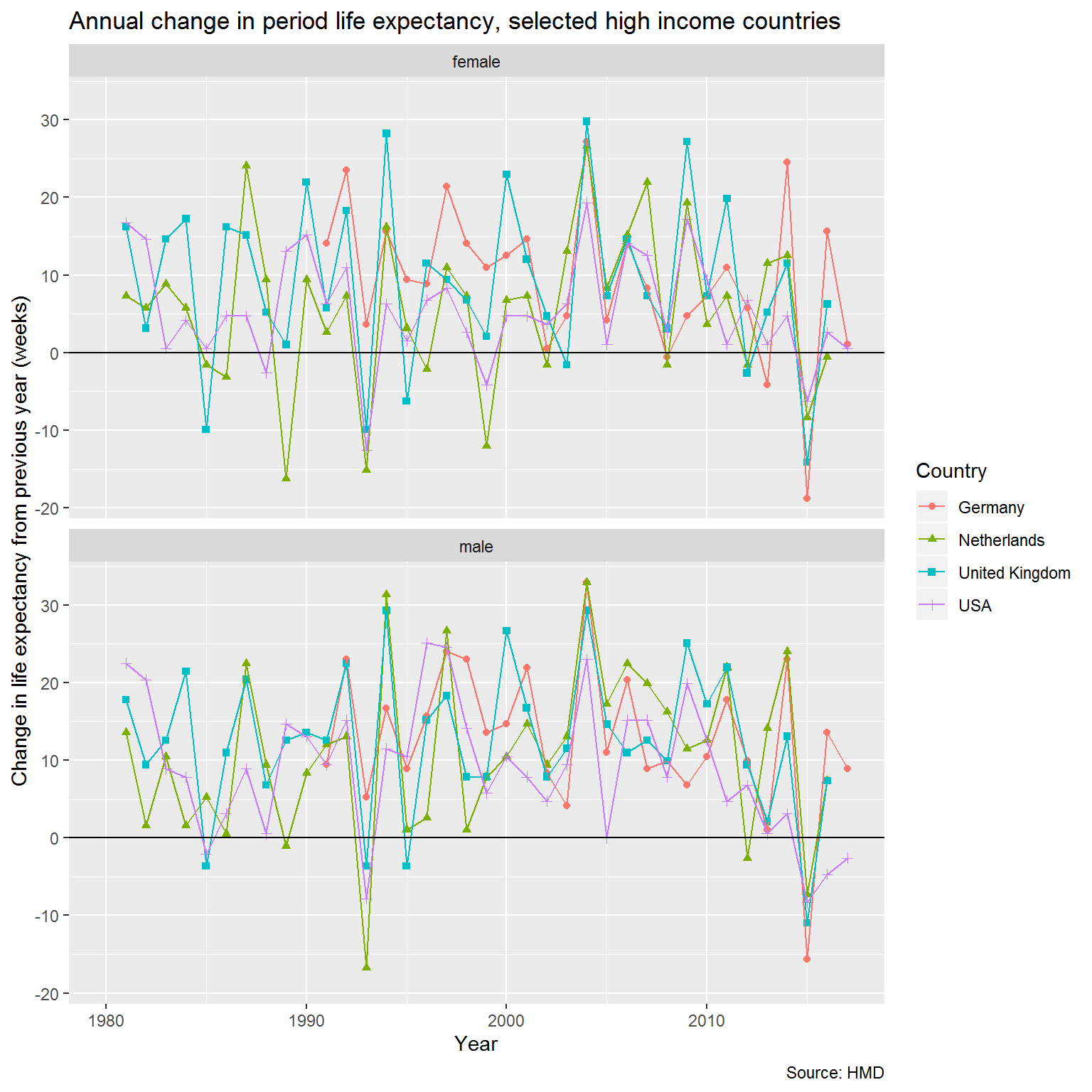


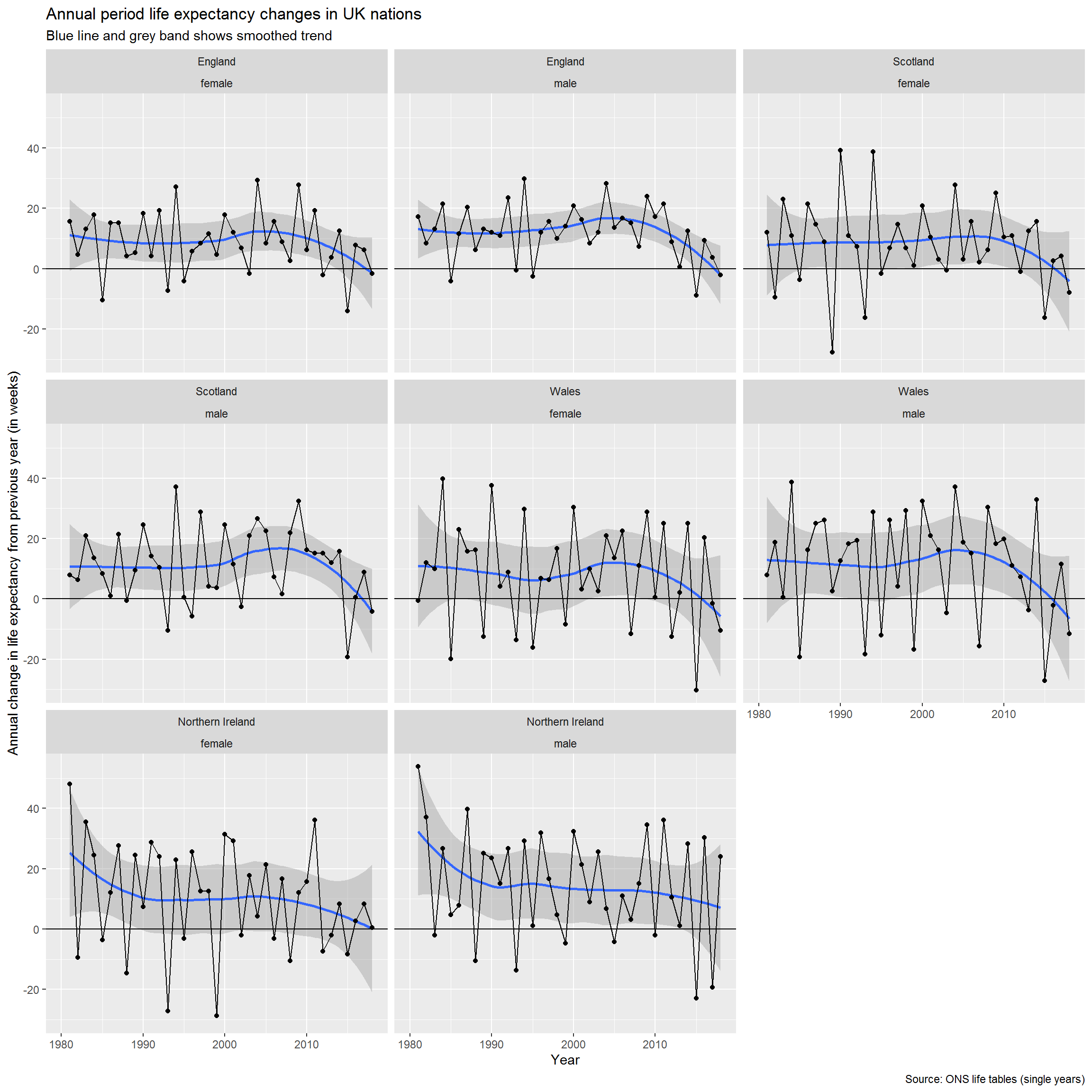
**1.2.1.3 Annual change in slow gainers since 1980**

There are some important differences, however, in how annual change rates have varied in the the slowest-improving countries. Figure X shows this for the five slowest-improving countries excluding Iceland, which due to its small population size shows much greater levels of annual variability than the other countries. From this figure it is apparent that the USA not only tended to show lower rates of improvement before 2010, but has also been exhibiting continuing and more persistent declines than the other countries, with three consecutive years of declining mortality for males in the last three available years, and only modest improvements for females.



| **sex** | **decade** | **HMD** | **ONS** |
| --- | --- | --- | --- |
| f | 80s | 0.168 | 0.168 |
| f | 90s | 0.168 | 0.170 |
| f | 00s | 0.244 | 0.241 |
| f | 10s | 0.091 | 0.080 |
| m | 80s | 0.229 | 0.230 |
| m | 90s | 0.229 | 0.232 |
| m | 00s | 0.319 | 0.313 |
| m | 10s | 0.164 | 0.131 |

Let’s now present this as a barplot

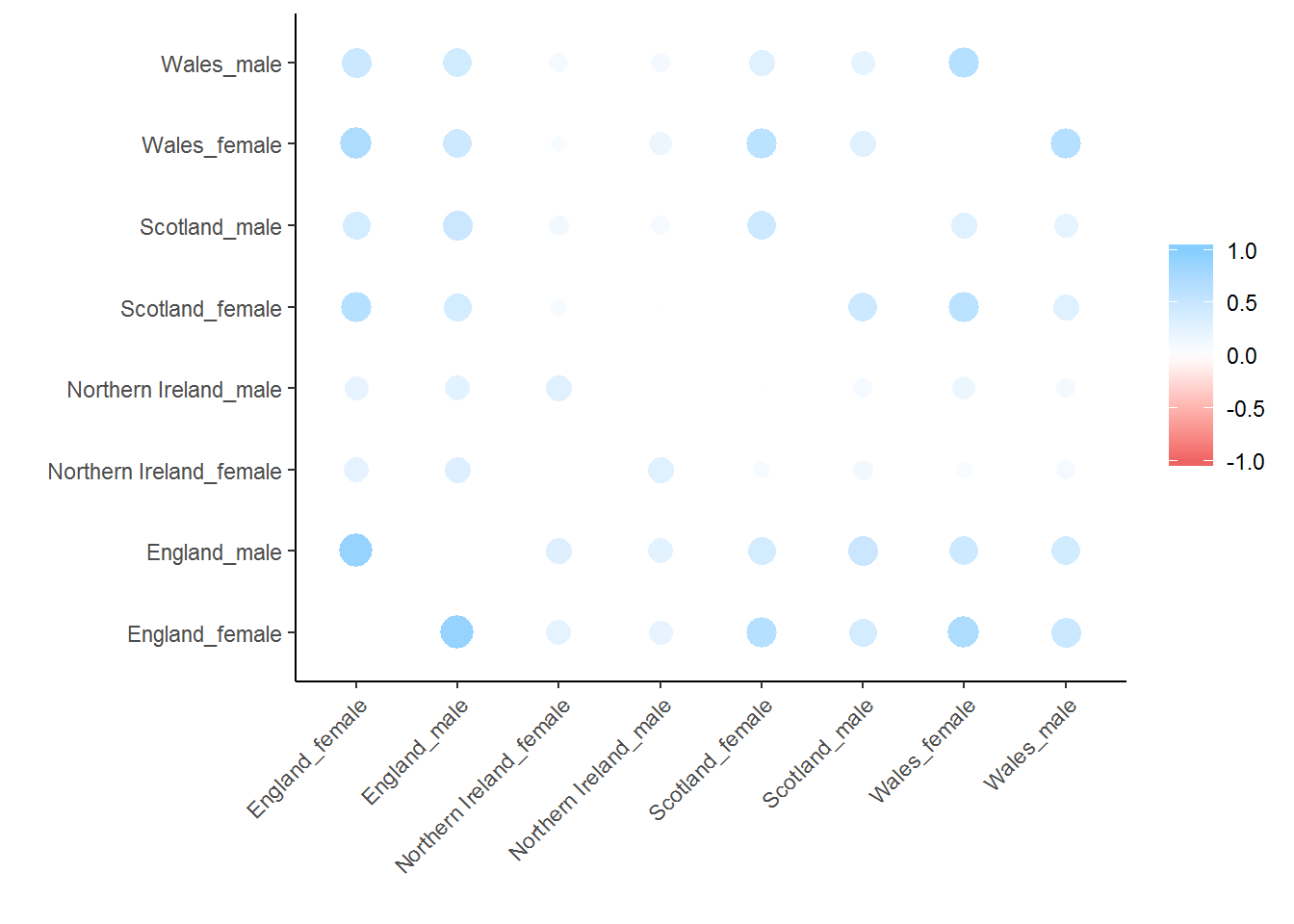
****

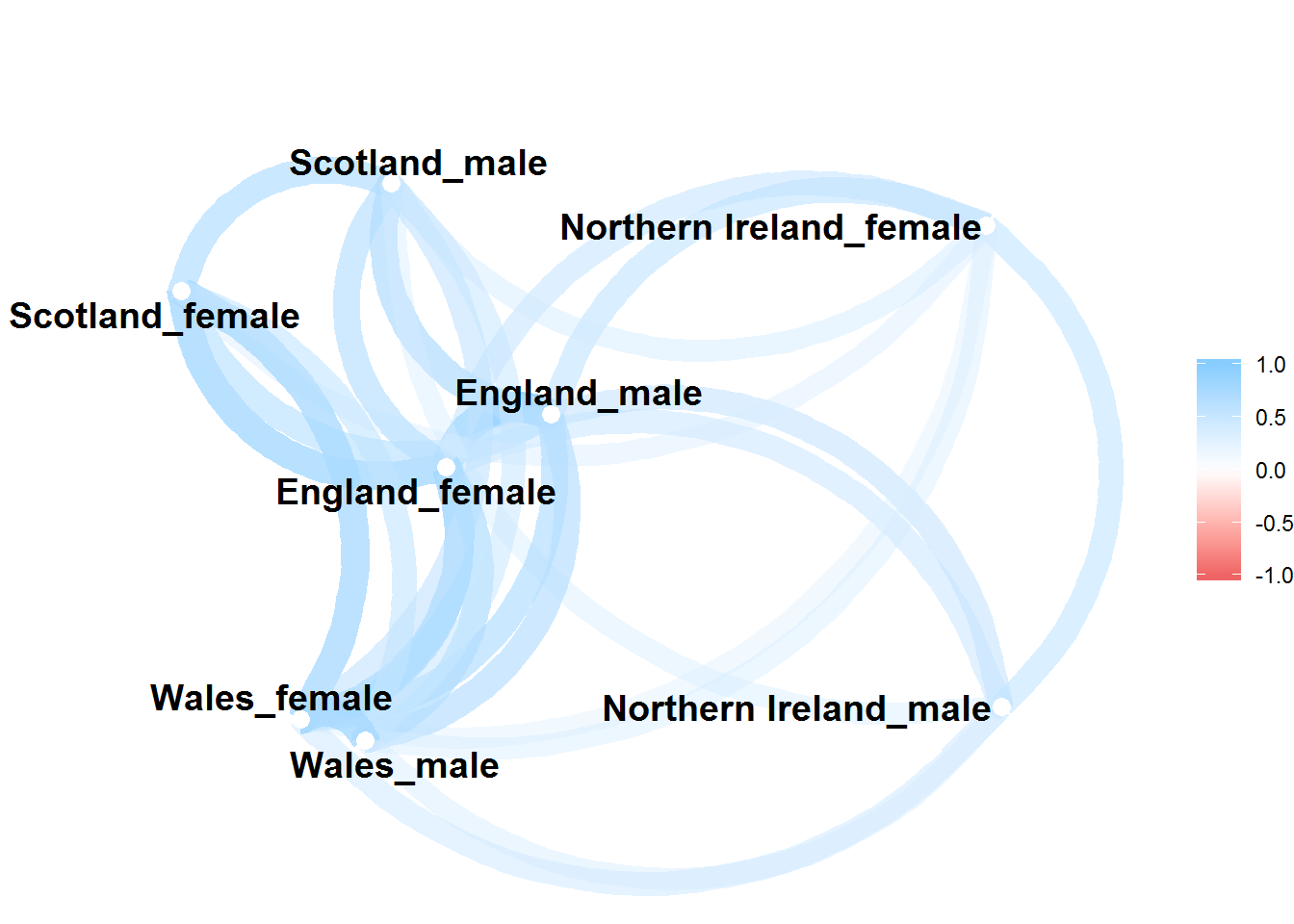
There are clear similarities between the trends in each of the UK nations, again with the exception of Northern Ireland. The trends even seem to correspond in terms of which years are ‘good years’ and which years are ‘bad years’ (i.e. they oscillate in phase with each other). To check this let’s look at the correlation between the trends.

|  |
| --- |
|  |

| **rowname**  <chr> | **England\_female**  <dbl> | **England\_male**  <dbl> | **Northern Ireland\_female**  <dbl> |  |
| --- | --- | --- | --- | --- |
| England\_female | *NA* | 0.8948164 | 0.5202739 |  |
| England\_male | 0.8948164 | *NA* | 0.5633628 |  |
| Northern Ireland\_female | 0.5202739 | 0.5633628 | *NA* |  |
| Northern Ireland\_male | 0.4978847 | 0.5355912 | 0.5540548 |  |
| Scotland\_female | 0.7748321 | 0.6215946 | 0.3588286 |  |
| Scotland\_male | 0.6337257 | 0.6817908 | 0.4110520 |  |
| Wales\_female | 0.8082490 | 0.6682959 | 0.3316528 |  |
| Wales\_male | 0.6785138 | 0.6435823 | 0.3780715 |  |

8 rows | 1-4 of 9 columns





Trends in males and females in England are highly correlated (r = 0.89). The correlation between male and female trends in Wales are also strong (r = 0.77), which is slightly below the correlation between females in England and Wales (r = 0.81). Correlations between males and females in Scotland are slightly weaker (r = 0.67), and the associations between sexes are weakest in Northern Ireland (r = 0.55).

The network plot places series that are more correlated with each other closer together, and less correlated series further from each other. This confirms that males’ and females’ trends are closely correlated to each other in England and Wales, somewhat less so in Scotland, and least in Northern Ireland, where trends between sexes are less correlated with each other than are the correlations between countries elsewhere in the UK.

This suggests that any general trends which apply throughout the UK will apply less strongly in Northern Ireland than elsewhere. This should be considered when looking at the results in the next section, which aims to identify if and when there have been breakpoints in the trends in UK nations.

Older projections that had to be uprated [to do]

**3.2 Visualise projections**

**3.2.1 Change in projections**

How do the projections (e0 at birth) change over time between countries?

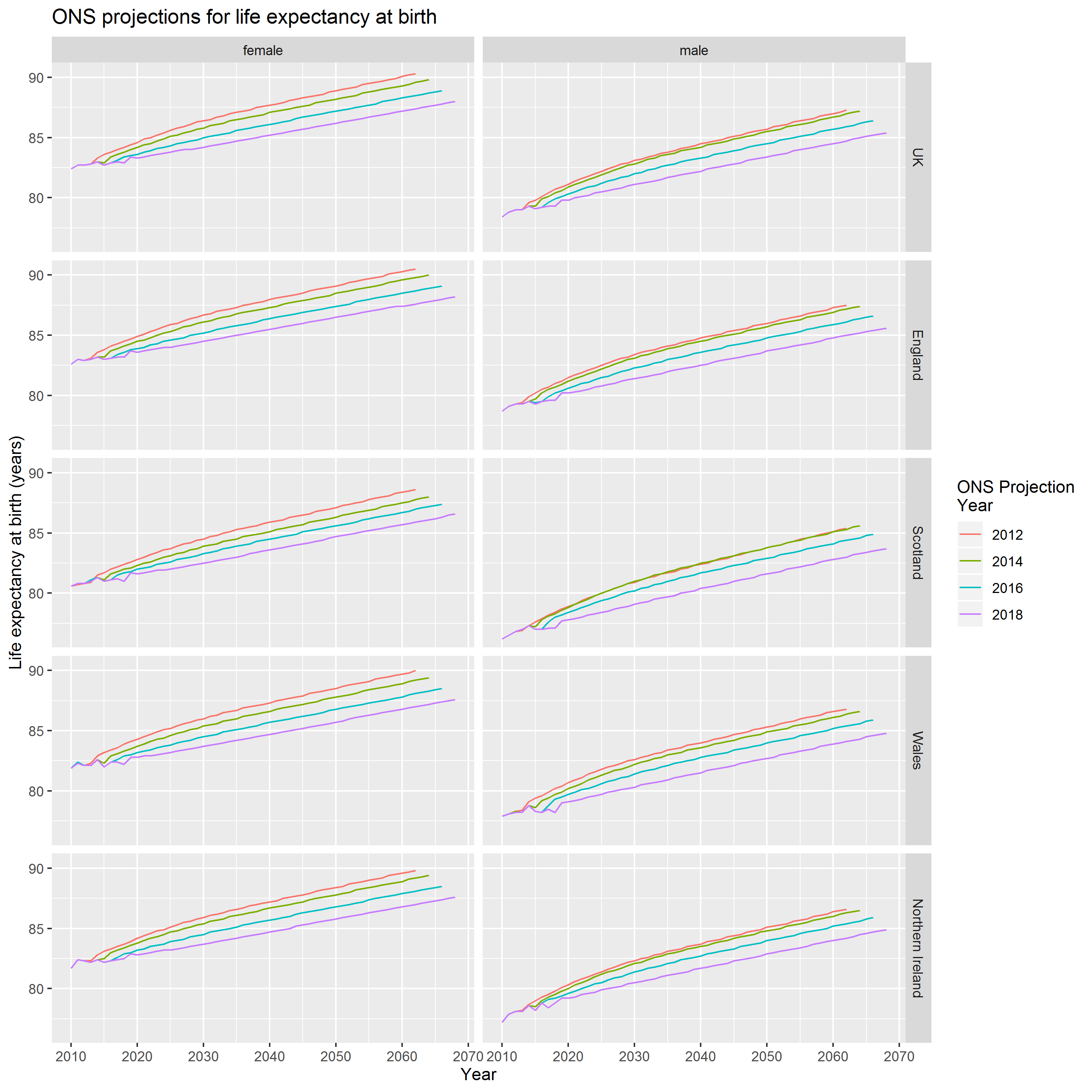
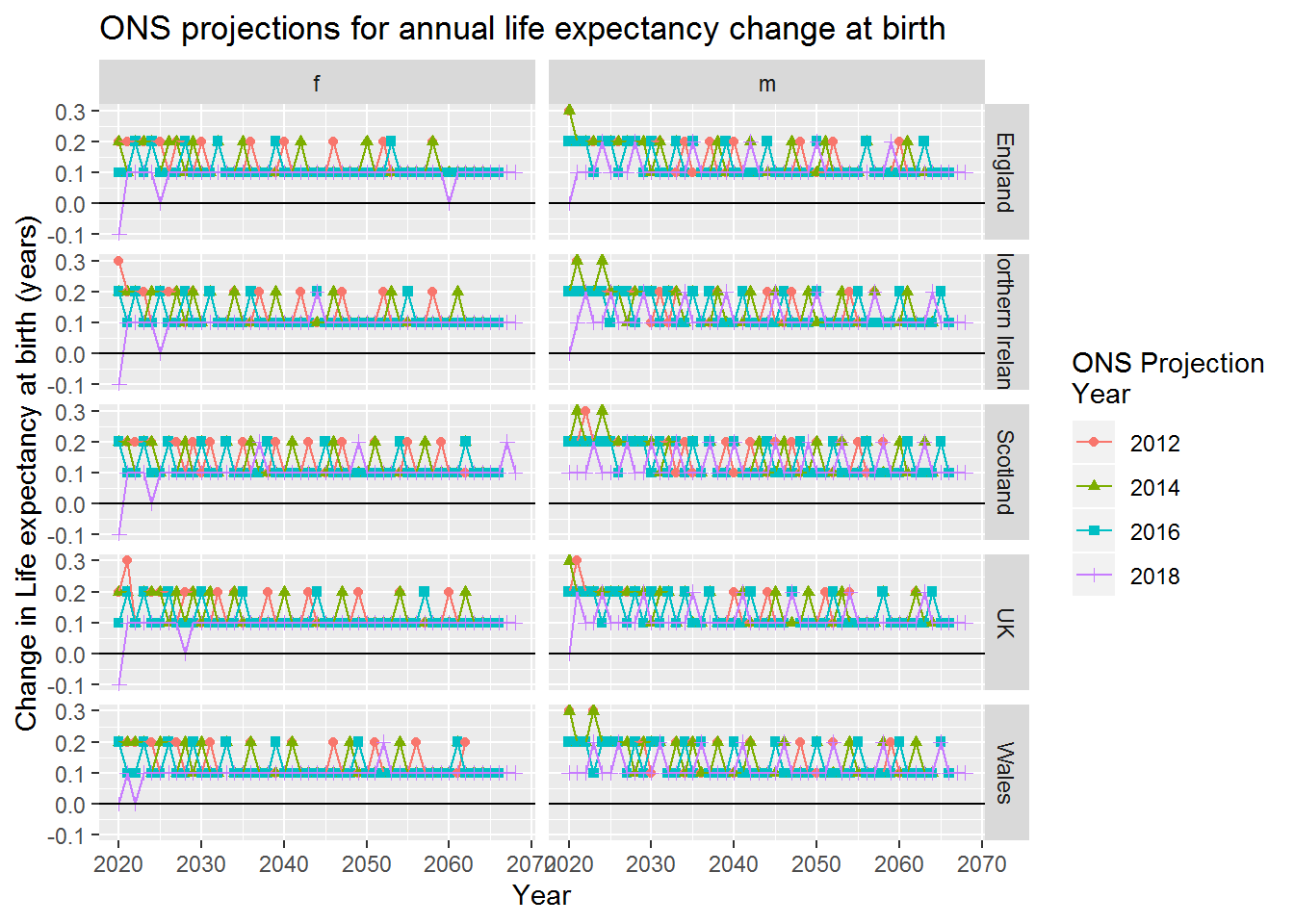


Figure 1 ONS life expectancy projections at birth, 2012 to 2018 revisions

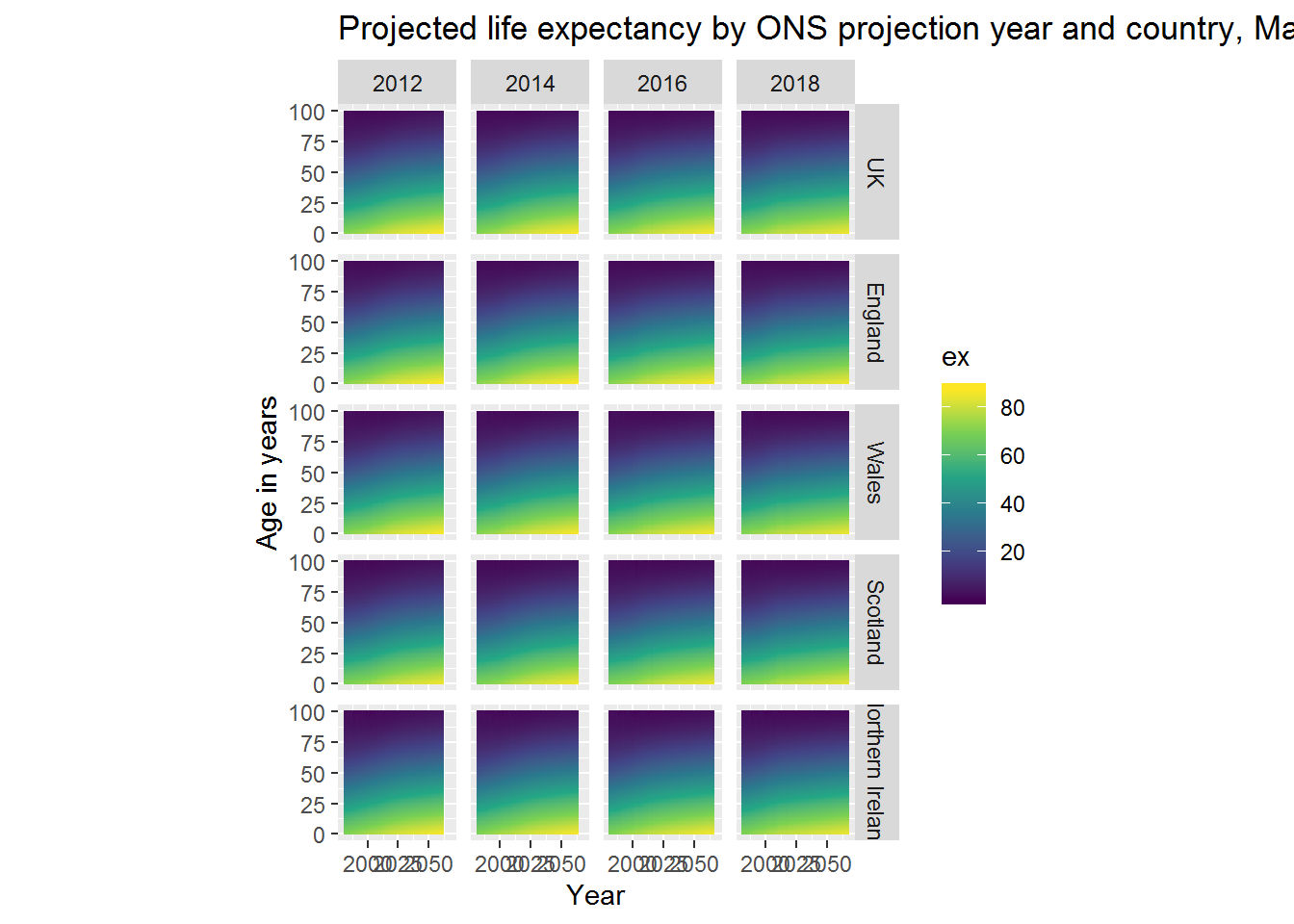
So, the projections involve adding varying numbers of increments of 0.1 life expectancy gain per year. Values within these intervals are created by changing the numbers of years of 0.2, 0.1 and 0.0 gain.

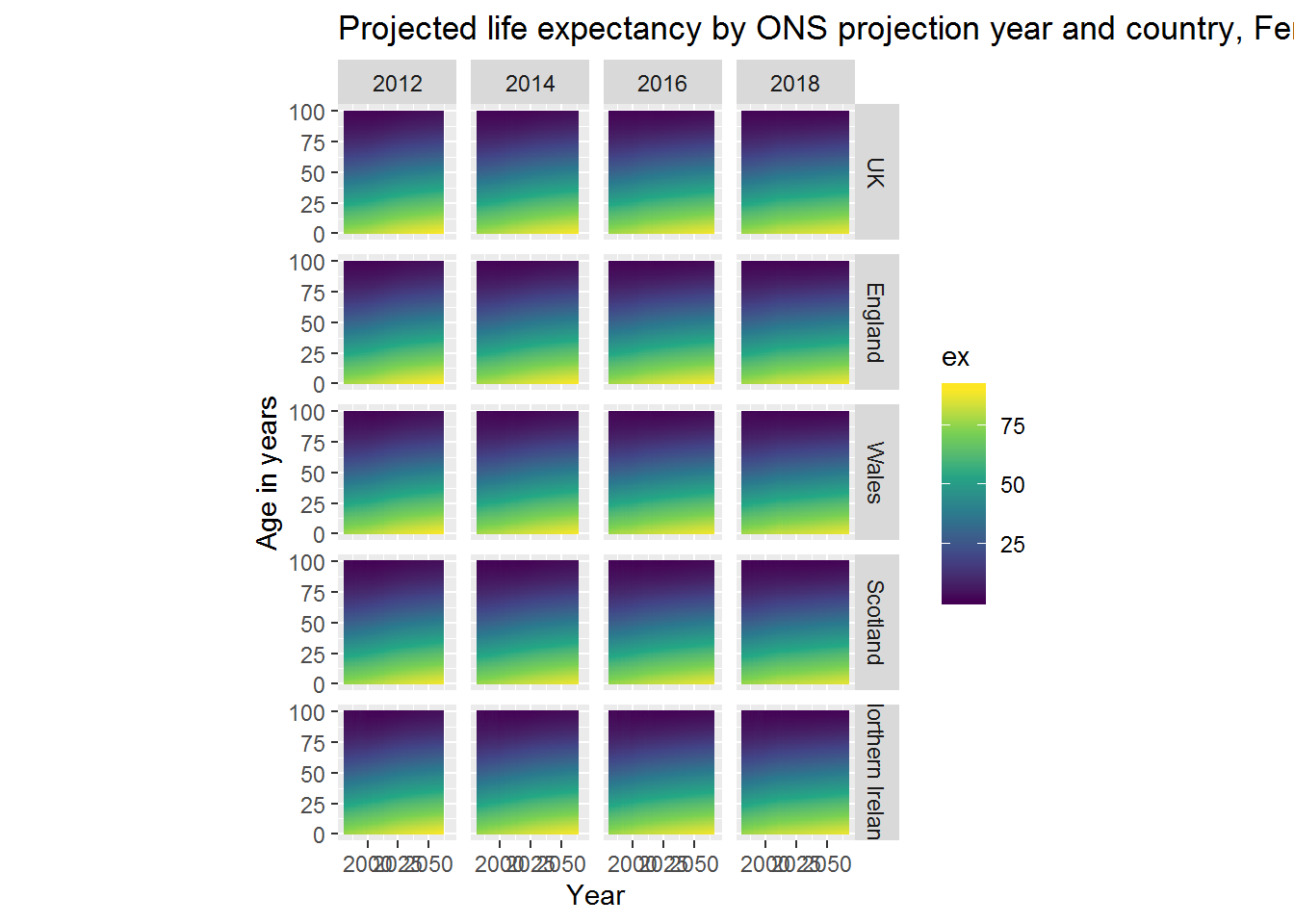
**3.2.2 Change in projections - annual increases**

For each of these projections, what’s the implied expected improvement level?

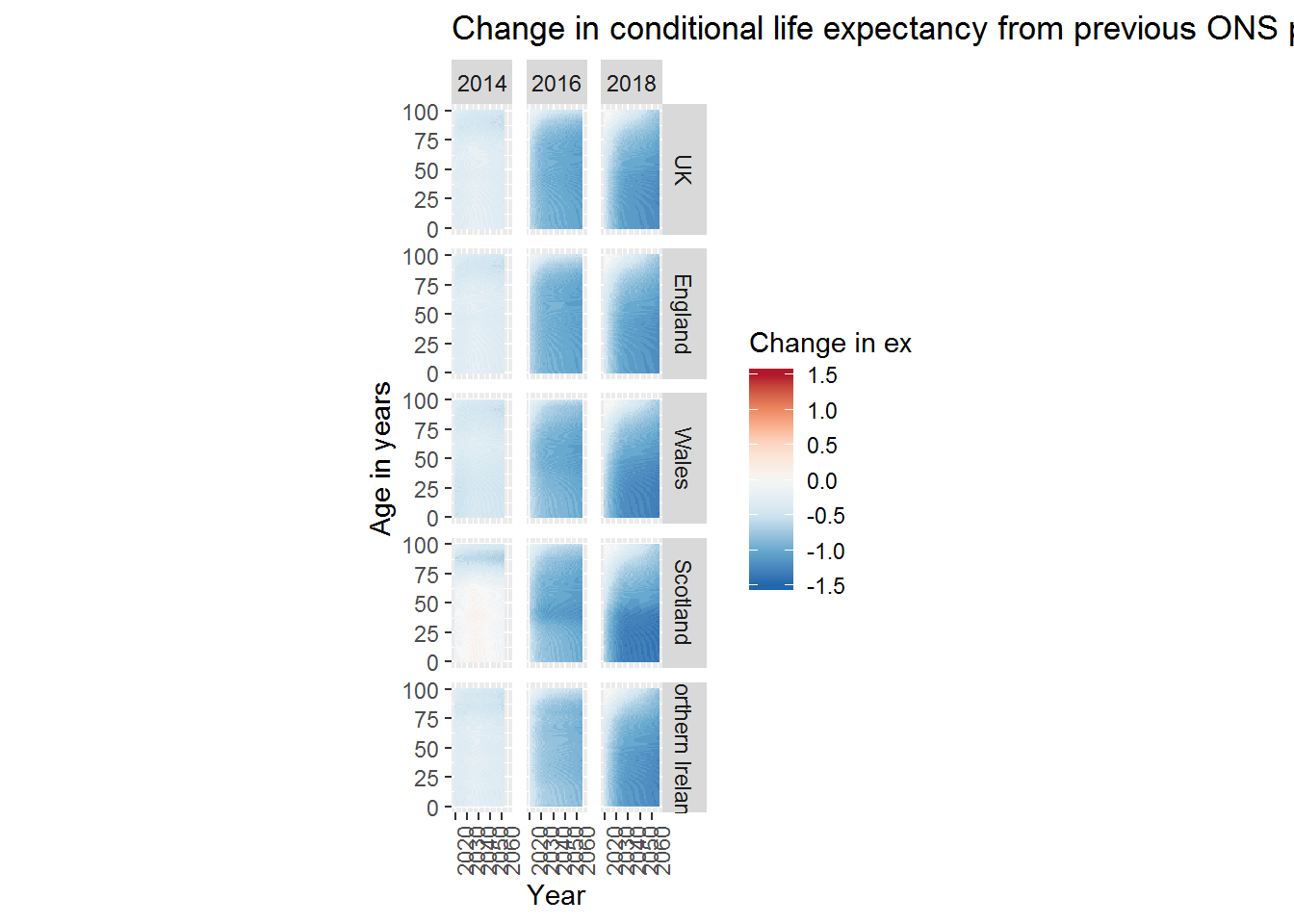


**3.2.3 Projections at all ages**





**3.2.4 Change in projections at all ages**



# R3.A Breakpoint analysis

This appendix shows results of running breakpoint analysis on UK and constituent nation annual life expectancy change data from the ONS single year lifetables. The R package segmented was used. (1,2) One breakpoint models were fit, and the effect of using different random number seeds in fitting these models explored.

## R3.1A Breakpoint estimates and standard errors

Figure R3.1A and table R3.1A show the point estimates for the breakpoints in trends. The errors bars show two standard errors around these point estimates. For each country except Northern Ireland, a breakpoint is identified around 2009-2010. The confidence intervals tend to be wider for females than for males, and (again with the exception of Northern Ireland) for smaller compared with larger countries.

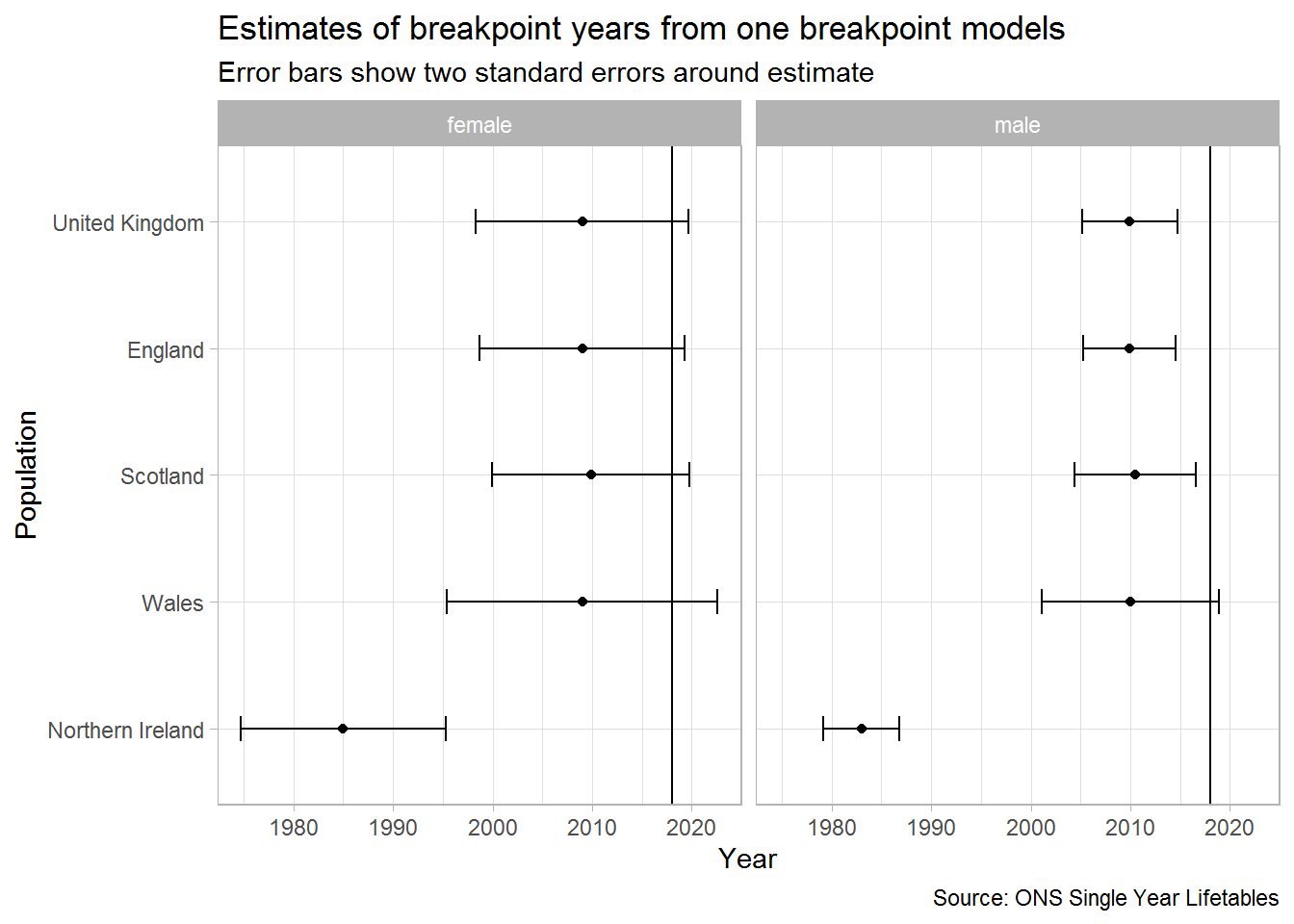


Figure R3.1A Breakpoint estimates and standard errors

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Population** | **Sex** | **Breakpoint** | **Standard Error** | **Lower CI** | **Upper CI** |
| England | female | 2009.00 | 5.15 | 1998.70 | 2019.30 |
| England | male | 2009.93 | 2.34 | 2005.26 | 2014.60 |
| Northern Ireland | female | 1985.00 | 5.13 | 1974.73 | 1995.27 |
| Northern Ireland | male | 1983.00 | 1.92 | 1979.16 | 1986.84 |
| Scotland | female | 2009.87 | 4.95 | 1999.97 | 2019.77 |
| Scotland | male | 2010.45 | 3.05 | 2004.36 | 2016.54 |
| Wales | female | 2009.00 | 6.80 | 1995.40 | 2022.60 |
| Wales | male | 2010.00 | 4.43 | 2001.14 | 2018.86 |
| United Kingdom | female | 2009.00 | 5.35 | 1998.30 | 2019.70 |
| United Kingdom | male | 2009.95 | 2.39 | 2005.18 | 2014.72 |

Table R3.1A Breakpoint analyses and standard errors

## R3.2A Effect of random number seed choice on breakpoint estimate

The results can be dependent on the choice of random number seed used by the algorithm to identify the breakpoint. With some random number seeds, the algorithm may not be able to identify a breakpoint at all. Figure R3.2A shows the differences in breakpoint estimates produced by passing the first five random number seeds that identify a breakpoint estimate. (Seeds 4, 5, and 7 do not consistently identify a breakpoint for all populations. With the exception of females in Wales, the majority of random number seeds lead to very similar breakpoint estimates.

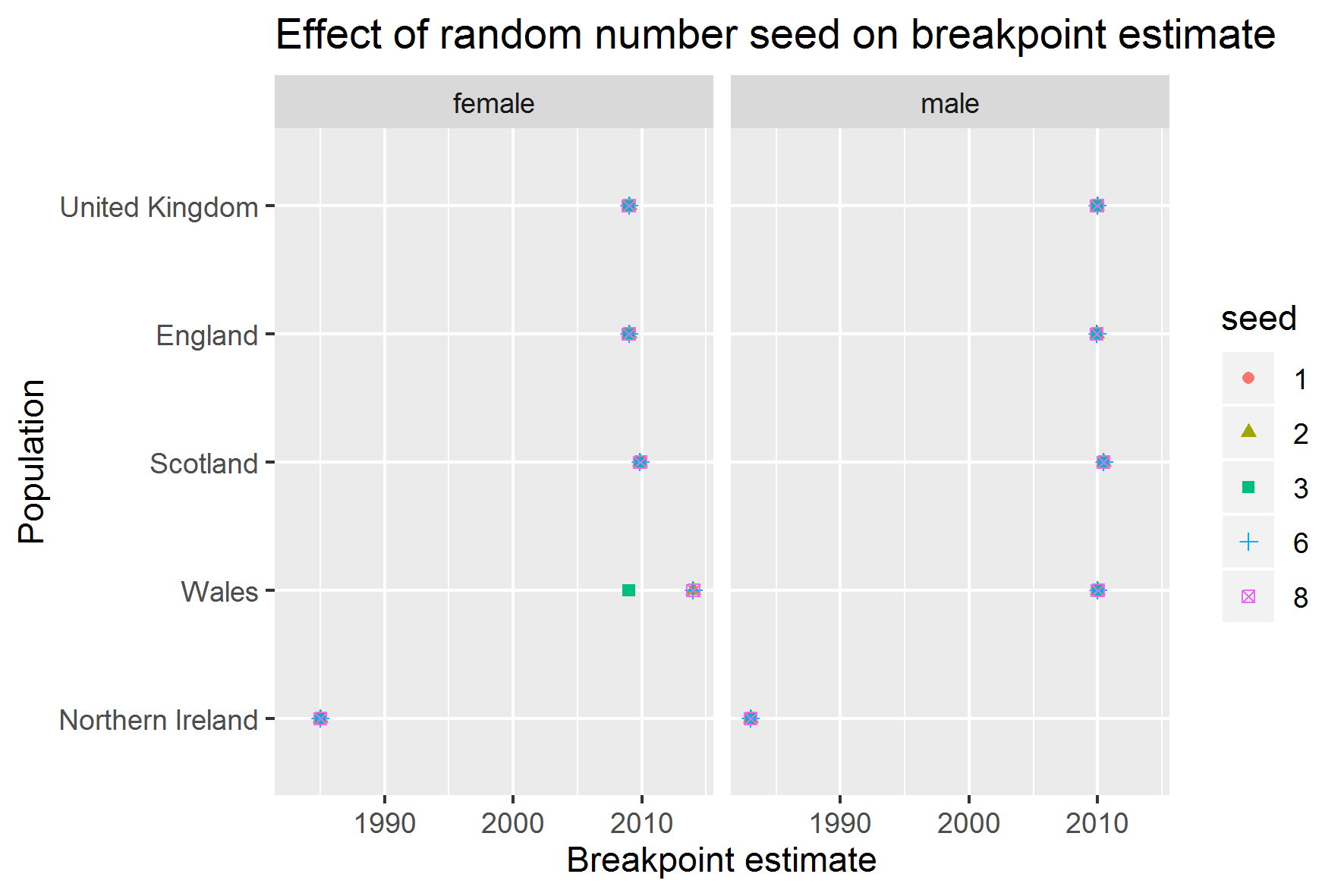


Figure R3.2A Effect of random number seed on breakpoint estimate

Bibliography

1. Muggeo VMR. Estimating regression models with unknown break-points. Stat Med. 2003 Oct 15;22(19):3055–3071.

2. Muggeo VMR. Testing with a nuisance parameter present only under the alternative: a score-based approach with application to segmented modelling. J Stat Comput Simul. 2016 Oct 12;86(15):3059–3067.

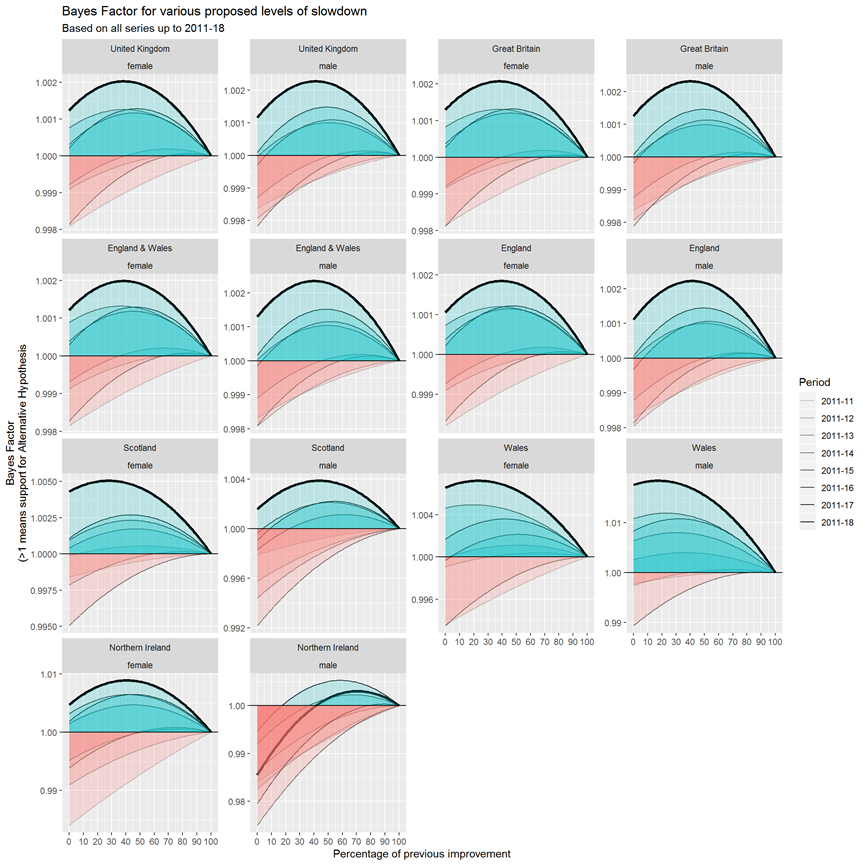
The before period used in the Bayes Factor calculations before is 1991 onwards. Let’s identify what the average improvement was over this period.

| **sex** | **mean\_ch\_e0** | **var\_ch\_e0** |
| --- | --- | --- |
| female | 0.191 | 0.039 |
| male | 0.276 | 0.026 |

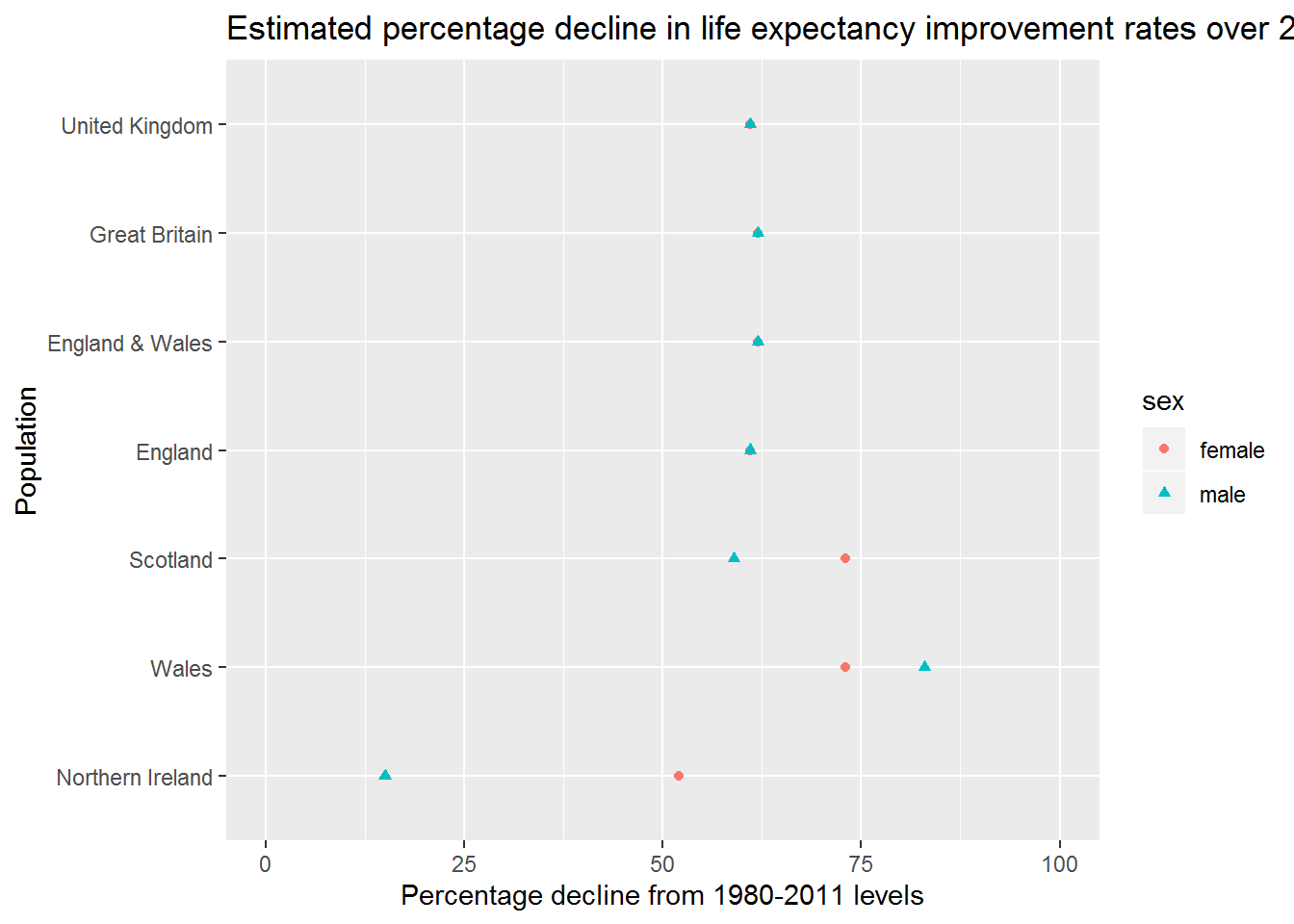
This checks out with what I’ve calculated before. (Phew!)

|  |
| --- |
|  |

| **population**  <chr> | **sex**  <chr> | **after\_end**  <int> | **perc**  <dbl> | **mu**  <dbl> | **ll**  <dbl> | **bayes\_factor**  <dbl> | **period**  <chr> |
| --- | --- | --- | --- | --- | --- | --- | --- |
| England | female | 2011 | 1.00 | 0.194500 | 0.7173006 | 1.0000000 | 2011-11 |
| England | female | 2011 | 0.99 | 0.192555 | 0.7172876 | 0.9999870 | 2011-11 |
| England | female | 2011 | 0.98 | 0.190610 | 0.7172745 | 0.9999739 | 2011-11 |
| England | female | 2011 | 0.97 | 0.188665 | 0.7172612 | 0.9999606 | 2011-11 |
| England | female | 2011 | 0.96 | 0.186720 | 0.7172478 | 0.9999472 | 2011-11 |
| England | female | 2011 | 0.95 | 0.184775 | 0.7172342 | 0.9999336 | 2011-11 |
| England | female | 2011 | 0.94 | 0.182830 | 0.7172205 | 0.9999199 | 2011-11 |
| England | female | 2011 | 0.93 | 0.180885 | 0.7172066 | 0.9999060 | 2011-11 |
| England | female | 2011 | 0.92 | 0.178940 | 0.7171926 | 0.9998920 | 2011-11 |
| England | female | 2011 | 0.91 | 0.176995 | 0.7171785 | 0.9998779 | 2011-11 |



For all populations except males in Northern Ireland, the addition of the 2018 single year life expectancy data led to sizeable increases in the empirical support for the belief that there has been a slowdown in life expectancy after 2010; this is seen by noting how much higher the bold line, which incorporates the 2018 data, is than the fainter lines representing cumulative data based on shorter series of observations. For most of these populations, the peak of the bold line is to the left of peaks based on earlier series, meaning not only did the 2018 observations increase the strength of evidence supporting belief in a slowdown in life expectancy improvements, but also suggested more severe magnitudes of slowdown than the series excluding this most recent observation had indicated. For the UK as a whole, the addition of the life expectancy data for 2018 suggested an overall slowdown of around 60% was most likely, compared with a most likely magnitude of slowdown of around 50% based on data up to 2017. For each of these populations, what does the Bayes Factor maximise at?



And as a table

|  |
| --- |
|  |

| **population**  <fctr> | **sex**  <chr> | **perc**  <dbl> | **bayes\_factor**  <dbl> |
| --- | --- | --- | --- |
| England | female | 61 | 1.002120 |
| England | male | 61 | 1.002958 |
| England & Wales | female | 62 | 1.002173 |
| England & Wales | male | 62 | 1.003081 |
| Great Britain | female | 62 | 1.002221 |
| Great Britain | male | 62 | 1.003173 |
| Northern Ireland | female | 52 | 1.004581 |
| Northern Ireland | male | 15 | 1.000435 |
| Scotland | female | 73 | 1.003834 |
| Scotland | male | 59 | 1.005845 |

**Technical appendix**

**Likelihood and Log Likelihood of the Normal Distribution**

For computational reasons it is more common to calculate the log likelihood of a function rather than the likelihood itself. Defining X={x1,x2,...,xn}X={x1,x2,...,xn} as a series of nn observations, the Log Likelihood of the Normal Distribution is as follows:

logL(μ,σ2|X={x1,x2,...,xn})=−n2log(2π)−nlog(σ)−12σ2∑i=1n(xi−μ)2logL(μ,σ2|X={x1,x2,...,xn})=−n2log(2π)−nlog(σ)−12σ2∑i=1n(xi−μ)2

This is implemented as a function in R as follows:

get\_ll <- **function**(x, mu, sig\_sq){

sig <- sqrt(sig\_sq)

n <- length(x)

- n \* log(sig) - (n/2) \* log(2 \* pi) - (1 / 2 \* sig\_sq) \* sum((x - mu)^2)

}

The Bayes Factor is defined as ratio of Likelihoods of two models. In the general case, if g(θ)g(θ) refers to a model with parameters θθ, and θnullθnull and θaltθalt to two different candidate parameters, then the Bayes Factor is

L(g(θalt)|X)L(g(θnull)|X)L(g(θalt)|X)L(g(θnull)|X)

Note that the alternative and null model specifications both contain a number of parameters in the Log likelihood that are identical. This includes n2log(2π)n2log(2π) and nlog(σ)nlog(σ) (because we are not concerned about testing proposed difference in the variance before and after). This means Bayes Factor could be calculated without including these parameters. However, they have been included for completeness.

An advantage of the Bayes Factor approach is that it is trivial to update it every year, taking only a minute or so to rerun with an additional year’s worth of data. This means that as soon as new data becomes available, it can be used to update our beliefs about long-term trends, and the extent of the deterioration from long-term trends if the accumulated recent data is considered representive of how long-term trends are likely to progress.

So, up to 2014, the ONS was projecting a slower improvement rates than 2011-12 alone would suggest. For the 2016 and 2018 projections, the rates were slightly higher than the Bayes Factor alone would suggest, especially for females. Now, the remaining analysis (possibly the only remaining analysis) is to express the UK’s recent improvement rates and ONS projections as a % of the mean improvement from 1980 to 2010.

Code

|  |
| --- |
|  |

| **sex**  <chr> | **mean\_ch\_e0**  <dbl> | **var\_ch\_e0**  <dbl> |
| --- | --- | --- |
| female | 0.1974194 | 0.03615978 |
| male | 0.2654839 | 0.02369226 |

So, by 2012 the ONS was projecting future improvement rates that were around 30% lower (1 - 0.137 / 0.197) than long-term average improvement rates for females, and around 44% lower than long-term trends (1 - 0.148 / 0.265) for males. By contrast the Bayes Factor approach alone would predict slowdowns of around 18% for females, and gains of around 4% for males.

By 2014 the ONS was projecting slowdowns of around 35% for females, and 44% for males. This contrasts with Bayes Factor estimates of around a

| **Year** | **BF\_female** | **BF\_male** | **ONS\_female** | **ONS\_male** |
| --- | --- | --- | --- | --- |
| 2011-12 | 0.816 | 1.040 | 0.694 | 0.557 |
| 2011-14 | 0.826 | 0.821 | 0.653 | 0.554 |
| 2011-16 | 0.446 | 0.520 | 0.583 | 0.505 |
| 2011-18 | 0.380 | 0.407 | 0.476 | 0.429 |