Mapping Scotland’s Sickening: Using Lexis Surface Visualisations to compare mortality patterns between populations

# Abstract

# Introduction

This paper will present hundreds of thousands of mortality rates, recorded at over one hundred different ages and over more than sixty years, covering populations in Scotland and those of our nearest neighbours, and selected other populations from across the globe. However it will contain no statistical models, no tables, and very few numbers. It will do this by presenting a series of maps of demographic data, known as Lexis surfaces, which show how something – mortality rates, in this case – varies by both age and by year. Conceptually, Lexis surfaces are to population data as spatial maps are to physical landscapes: in spatial maps, the height of a landscape is shown as a function of both its latitude and its longitude; the Lexis surfaces presented here show mortality as a function of both age and of year. Mortality hazards can be thought to vary continuously over both age and year, much as height varies continuously with both latitude and longitude. And just as it usually makes little sense to try to understand the height of physical landscapes only as a function of their latitude or longitude alone, so this paper will argue that important patterns and features in mortality landscapes are often missed when public health research aims to understand population health, and differences between population, by looking only at how mortality risks vary as a function of age or time alone.

## Thinking in slices: Life expectancy, age schedules, drift and cohorts

A life expectancy is a common way that many individual age-specific mortality risks are summarised as a function of year alone. Life expectancies are important summary statistics, and show that, on average, one population tends to live less long than another population in a given year. It is known, for example, that the general trend has been for life expectancies to have increased in richer countries for more than a century. (Leon, 2011) Plotting life expectancies against time for many populations can also show how both relative and absolute inequalities can increase despite outcomes improving for everyone, but at a faster rate for some populations than others. For example, life expectancy at birth, for countries submitting data to the Human Mortality Database (HDM) tended to increase faster for females than males for much of the Twentieth Century, leading to a gap in life expectancy by sex opening up until the 1970s, before starting to narrow again in more recent decades, in large part due to falling rates of cardiovascular disease death in men over 60 years of age. (Glei & Horiuchi, 2007; White et al., 2014) Similarly, much of Scotland’s partly deserved status as ‘Sick Man of Europe’ is due to its life expectancy trends improving more slowly than those of European neighbours, rather than falling in absolute terms. (McCartney, Walsh, Whyte, & Collins, 2012; Whyte & Ajetunmobi, 2012) Throughout the UK, and in England & Wales in particular, there has been an unprecedented stalling in life expectancies since 2011, leading public health researchers to suggest the effects of austerity are to blame, and for actuarial models to be revised substantially. (Fransham & Dorling, 2017; Seekings, 2017)

The risk of dying, known as the force of mortality, also varies in characteristic ways with age, according to a ‘schedule’ that tends to be simple and predictable over some ages, but more complicated and unpredictable at other ages. After around the age of 50, the risk of dying in the next year has tended to increase exponentially with each additional year of age, resulting in a schedule that appears as a straight line when the logarithm of mortality is plotted as a function of age, at least until very old age. Much of this increasing mortality hazard with age has been labelled ‘senescent’, the results of chronic and cumulative deterioration of bodily structure and function with age, of cells imperfectly copying themselves, each time with some errors, which eventually accumulate and become catastrophic for bodily functioning. Long because the biology of ageing was understood, however, the straight line schedule of log mortality against age was noted by insurers, academics, and actuaries, and today is usually referred to as the Gompertz curve, after the author of a publication written in 1825. (Gompertz, 1825) However, not all deaths are caused by old age; others are caused by accidents, violence, childbirth, and other things that ‘just happen’ to bodies whatever state of repair or disrepair they are in. To represent this hazard of death, a baseline hazard has been added at all ages, producing the next simplest age-mortality schedule, known as the Gompertz-Makeham model, named additionally after the author of a paper published in an actuarial journal in 1860. (Makeham, 1860) There is also a distinct and high hazard within the first year of life, and within this first year concentrated in the first months, weeks and days. Historically the risk of dying in infancy was similar to the risk of dying at very old age, producing empirical schedules of hazard against age that were high in the first year, low in childhood, then rising again with adulthood, and because of this shape referred to as ‘bathtub curves’. Adding yet more complexity to the age schedule is the observation that the ‘baseline risk’ does not appear constant at all ages, or the same between sexes. Instead, the mortality hazard appears to diverge between in the first few years of adulthood: jumping up sharply for males at the start of childhood, largely due to violent and accidental deaths, and historically for females between around the ages of 25 and 30 years, due to the risk of death in childbirth. These additional risks in early adulthood have been described as ‘the accident hump’, although these deaths are not all accidental. Relatively few models have attempted to parametrically fit the whole age schedule, with the most comprehensive and complicated models, incorporating up to eight parameters being proposed in 1980 (Heligman & Pollard, 1980), and a somewhat simpler five parameter model, first developed for modelling mortality risks in other animals, being proposed by Siler in 1979 (Siler, 1979), and applied to humans over subsequent years. (Gage & Dyke, 1986)

Age-specific mortality risks have been constantly changing over time, however, and so any attempts to predict both healthy and overall life expectancies for populations that are not already extinct needs to account for how these age-specific risks will evolve over coming years and decades. Perhaps the most widely used modelling framework for doing this was proposed by Lee & Carter in 1992, and involves making the assumption that the logarithm of age-specific mortality rates will tend to ‘drift’ downwards over time at a constant rate. (Lee & Carter, 1992) Methodologically, it has been noted that the Lee-Carter model is a form of principle component analysis (PCA) which summarises the age-year surface of values using only the first principle component, the ‘drift parameter’, (Girosi & King, 2007) and that there may be additional value in characterising populations using additional principle components. (Giordano, Russolillo, & Haberman, 2008) Conceptually, the Lee-Carter model is based around the Gompertz-Makeham model, assuming its log-linear shape will tend to evolve according to a constant tempo, the ‘drift parameter’. For this reason, Lee-Carter models tend to be less effective in estimating mortality rate changes at younger ages (before around age 50), but are often nevertheless relatively effective at estimating future life expectancies because the majority of deaths that occur in richer countries tend to be at these older ages.

An additional complication in thinking about mortality patterns relates to efforts to uniquely model and estimate age-period-cohort (APC) effects. All populations exist and age as cohorts, ageing one year per year, and some specific cohorts appear distinct in their mortality risks than cohorts from neighbouring birth years. For example, people born around the time of the ‘Spanish Flu’ of 1918 appear to have somewhat higher mortality risks at any given age than might be expected from trends observed in earlier and later cohorts, (Almond, 2006; J Minton, Vanderbloemen, & Dorling, 2013) and people born in England and Wales in the 1950s to have a somewhat lower mortality risk as they age than might be expected from broader trends. (Willets, 2003) There have been various attempts to uniquely partition away cohort effects from age effects and period effects in statistical models (for example (Yang, Fu, & Land, 2004; Yang, Schulhofer‐Wohl, Fu, & Land, 2008)), but doing so is logically impossible, because each of the three effects cannot be uniquely identified (Wilmoth, 2006), leading to effects to do so being branded ‘futile’. (Bell & Jones, 2014)

## Thinking in surfaces: Visualising and intuiting the geography of mortality using Lexis surfaces

This paper argues that a more intuitive, and often more productive, way of thinking about and exploring mortality is by visualising log mortality rates by both age and year at once. This can be done by producing level plots, or contour plots, in which year runs across the horizontal axis, age along the vertical axis, and the ‘cells’ at each age-year ‘coordinate’ are shaded, coloured, or otherwise marked graphically according to the corresponding age-year specific log mortality hazards. Conceptually Lexis surfaces are maps of age-time much as topographic maps are maps of latitude-longitude. In epidemiology, Lexis surfaces represented as shaded contour maps have been used to identify both mortality surfaces in selected European countries, (J Minton et al., 2013) but their use in demography dates back at least to the 1980s, (Vaupel, Gambill, & Yashin, 1987) and they are based on the Lexis diagram as discussed in 1875 (Lexis, 1875).

Figure 1a presents a blank Lexis surface, with some additional bands as annotation. The Lexis surface is arranged with equal projection along both axes, such that one year in time is as wide as one year in age as tall. Because of this cohorts run diagonally from bottom left to top right at 45 degree angles. The background of the Lexis surface marks out decades as vertical lines, decadal age groups as horizontal lines, and birth cohort decades as diagonal lines. Within this figure, a specific age band, decade band, and cohort band is highlighted with green, red, and blue colours respectively. In addition to thinking about patterns on Lexis surfaces in terms of bands of parallel horizontal (age), vertical (period) and diagonal (cohort) lines, it is also often useful to look for and identify specific regions within the surface that do not strictly run across the entirety of any of these three planes. These regions can sometimes be usefully interpreted as representing either age-period interaction effects, or age-cohort interaction effects, two examples of which are marked in the figure. Within the demographic time-frame of a century or longer, a common example of an age-period interaction is a war, during which the mortality risks of young adult males tend to rise sharply, before falling back more slowly. Examples of age-cohort interactions are perhaps less common, but potentially include any *additional* additional mortality risk experienced by 1918 birth cohorts as infants, i.e. the observation of an even higher mortality rate in the first year of life for this cohort even when the whole life-course log mortality multiplier associated with membership of this cohort is taken into account. Particularly high mortality and morbidity risks in the first year of life can be signals of one of the main mechanisms through which persistent life-course cohort effects emerge, namely through interuterine and perinatal exposure to deleterious environments at these earliest life-course stages, often referred to as a Barker Effect, (Barker, 2004) after research by Barker and colleagues demonstrating links between low birthweight and a range of early onset morbidities in adulthood. (Gluckman, Hanson, & Pinal, 2005; Hales & Barker, 1992) The importance of later stages in the life course, beyond infancy, for either establishing or accelerating mortality and morbidity hazards at later age, has also been recognised in life-course epidemiology, (Ben-Shlomo, 2002) and ongoing research into the associations between deprivation, gender and cohort membership in Scotland seems to highlight the importance of young adulthood as an important life-course stage, and the macroeconomic conditions established young adults are exposed to at this stage, in establishing persistently elevated hazards of suicides and drug-related deaths throughout the later life-course. (J Minton, 2017; Parkinson, Minton, Lewsey, Bouttell, & McCartney, 2017a, 2017b) Given the role of critical stages in life-course epidemiology, the value of identifying changes in mortality risk at younger age, as potential determinants of changes in older ages, therefore seems clear.

Figure 1b presents an impressionistic schematic of different patterns and trends that are often observed within Lexis surfaces of log mortality rates for rich world populations. Regions within this surface are coloured blue where trends in age-specific mortality tend to be improving, red where they tend to be worsening or relatively bad, and purple where the trends are more inconsistent over time and population. Arrows point upwards to indicate these improving trends, as on Lexis surfaces the corresponding ‘bands’ of mortality hazards appear to be moving upwards over time, and downwards where the mortality rates are high and/or worsening. For researchers concerned predominantly with life expectancy alone, the two most important age-year regions to note are at the bottom and top of the map, labelled ‘Falling Infant Mortality’ and ‘Lee-Carter Drift’. Mortality hazards within the first age of life have tended to fall, exponentially, since the early twentieth century in most European and North American countries; improvements in many other regions of the world have tended to have started later, but occurred even more quickly, over the middle and later parts of the twentieth century and beyond. Though the force of mortality in the first year is still much higher than over the next few years of age, it is a small fraction of levels that were typical and unchanged for the vast majority of recorded human history.

Because of this fall in infant mortality, the importance of Lee-Carter Drift as a primary determinant of future longevity has tended to rise. On the schematic of Figure 1b this is represented as applying mainly from around the age of 50 years, and until old age, but not the oldest ages. Below this age range, mortality trends tend to be more erratic, to differ more between populations and change over time. For male populations in particular, such trends may tend to be more responsive to relatively short term macroeconomic, cultural and social changes that occur within particular countries and regional population groups. For this reason, although changes in mortality rates at these younger adult ages may not contribute greatly to the sum total of mortality that occurs in any given year, they can still be important to explore, both as potential signals of period specific societal phenomena, such as violent conflicts and economic recessions, and also – via cohort effects which emerge from exposures to environmental conditions at later life-stage critical periods – as forerunners of lagged effects which may manifest as elevated mortality hazards in older age (‘scarring’). Finally, after the first year of age, mortality hazards within childhood tend to be very low, likely lower than at any previous time in human history, and to be continuing to fall; this makes the increase in mortality hazards for males, in particular, in the first few years of adulthood appear all the more abrupt and stark.

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| 1. Age, period and cohort bands, and age-period and age-cohort interactions | 1. Examples of key features often identified in Lexis surfaces of log mortality |

Figure Lexis surface illustrations

# Methods

The data and methods used in this paper are largely as described in a previous paper on quantifying ‘excess deaths’ in Scotland compared with neighbouring regions over time and at different ages. (Jon Minton et al., 2017) The earlier paper used one year by one year ‘Lexis squares’ of both exposures (adjusted population counts) and death counts for a range of countries, extracted from the Human Mortality Database (HMD), covering ages in single years up to 109 years old, and years from 1950 to the last available period. In the earlier paper, populations from individual countries were nested into narrower and broader geographic regions, allowing nested comparison of Scottish age-year specific mortality rates to be compared with both our nearest neighbours (the rest of the United Kingdom), and more distant neighbours (the rest of Western Europe). This earlier paper introduced a variation of the Lexis surface, the comparative level plot (CLP), which is essentially a Lexis surfaces of differences in mortality risks between two Lexis surfaces, each covering the same range of ages and years, but for two different populations, A and B. Though it can sometimes be more intuitive to present differences between populations on the Lexis surface as ratios (B/A), (Vanderbloemen, Dorling, & Minton, 2016) CLPs instead are coloured according to differences in log mortality (Log(B) – Log(A)). Log differences of zero are represented by white cells, cells where B has a higher mortality than A by red shades, and where A has a higher mortality than A by blue shades. The magnitude of the difference between B and A is therefore indicated by the degree of shade, with deeper reds and blues indicating greater differences, and lighter shades indicating smaller differences. An advantage of visualising log mortality differences is that mortality differences of A/B and B/A will therefore be of the same shade, with A/B appearing as blue as B/A appears red. (For example, 10 and 0.1 can be represented as 101 and 10-1 respectively, and it is the magnitude of these exponents, 1 and 1, which determine the cells’ shade, whereas the sign of the exponents, positive and negative, which determine the cells’ colour.)

This paper differs from the earlier paper, however, in three ways: firstly, the previous paper also calculated life-table estimates of the effect of excess Scottish mortality risks at different ages; this paper will focus only on describing and engaging with the Lexis surfaces themselves, and as by design more impressionistic rather than actuarial in intent. Secondly, this paper will focus as much on presenting Lexis surfaces of log mortality for individual populations, as much as the CLPs showing differences in log mortality between two populations; for brevity, Lexis surfaces of log mortality will be referred to as Shaded Level Plots (SLPs), and Lexis surfaces of differences between two populations as CLPs. Thirdly, the figures within this paper will present all individual age-year specific log mortality values largely ‘as-is’, without any smoothing of values to make neighbouring values more similar. Such smoothing of neighbouring values often becomes necessary when contour lines are used, as sudden discontinuities within the surfaces tend to a result in maps that are heavily overplotted with contours, obscuring underlying patterns. However, the SLP Lexis surfaces presented here use colour and shade alone, which removes the overplotting issues that result from using contour lines. Using colour and shade alone often creates a different interpretative risk, however, as illustrated in the famous Checkershadow Illusion, in which two equally dark grey cells either light if surrounded by darker cells, or dark if surrounded by lighter cells, highlighting the perceptual challenge of correctly decoding specific values from slight variations in cell shade. (Jonathan Minton, 2014) To reduce the risk of incorrectly

## Source of data

## Preparation of Data

### Unsmoothed

## Choice of Colour Schemes

### SLP

#### Paired Colour scheme

### CLP

#### Balanced Colour: Magnitude and Colour

## Scottish Disadvantage

### Age groups

#### Older Age

#### Younger Age

## Healthy ageing

### Compression of Mortality

### Compression of Morbidity

## Life course

### Life course stages

#### Infancy

#### Adulthood

#### Senescence

### Barker & Critical Period

## Different model types

### Life Expectancies

#### Conditional and Unconditional

#### Period or Cohort

#### Extrapolation if Cohort

### Life course structures

#### Gompertz

#### Gompertz-Makeham

#### Siler

#### Non-Parametric

#### Heligman Pollard

### Drift Models:

#### Lee-Carter

#### Quasi-Spatial (Girosi & King)

## Lexis surfaces

### Concept

### Origins

### Applications

### Variants

## Ways of Reasoning about data

### Reasoning: Inductive, Deductive, Abductive

### Case-based and variable-based

### Lexis surfaces as case-based approach

## Aim

### Use Lexis surfaces to apply case-based reasoning to populations, and population comparisons

### To understand accumulation of Scottish disadvantage over the life course

### To understand additional selected phenomena

# Results

## Lexis Plots of the British Isles

## Comparative Level Plot: Scotland and its Neighbours

## Lexis Plots of other populations

## Comparative Level Plots: Selected Other Populations

# Discussion

## Implications for Practice

### Conclusions regarding Scotland

### Conclusions regarding other countries

## Implications for Research

### Case-based reasoning

### Modelling approaches

### Lexis surfaces in an abductive research workflow

### Interactive Lexis surfaces

## Final summary