EXPERIMENTS IN MODERN AND APPLIED PHYSICS

Error Analysis Homework.

FALL 2017

Due: September 20'th

This homework set is designed to help in your self paced study of error analysis and statistics. Mastering the concepts covered here and in the suggested readings will be indispensable for analyzing data and writing lab reports. Before attempting to solve the problems you will find it useful to consult one of the suggested texts or on-line resources:

Taylor, "Introduction to Error Analysis"
Bevington "Data reduction and error analysis for phyiscal sciences"
A.C. Melissinos and J. Napolitano *"Experiments in Modern Physics"* Chapter 10.
https://www.khanacademy.org/math/statistics-probability

Useful definitions:.

- The uncertainty (or *absolute error*) , ΔQ, in a measured variable, Q, is the standard deviation σ of the parent Gaussian distribution function. It is calculated by using the formulas of error propagation.

For example, a length reported as (1.61 ± 0.03)m would indicate a mean measurement value of L = 1.61m and a standard deviation σ_L=0.03 m. The range $L \pm \sigma_L$ defines the "one-sigma" confidence level. This means that there is a 68.3% probability that the value of any measurement will fall in this range. The "two-sigma" confidence level is defined as the range $L \pm 2\sigma$. There is a 95.4% probability that a measurement will fall in this range.

If the accepted value of a variable, say the speed of light, lies within the one-sigma confidence level of the measured value in the laboratory we will regard the two to be "equal" or in agreement.

- The *relative error* in a measured quantity, Q, is defined as $\Delta Q/Q$.

Problem 1.

a. Which of the measured values in the table below are equal to the expected ones within one-sigma confidence level?

	measured mean value	$\pm\sigma$	$\pm 2\sigma$	Expected value
i	15.432	0.025	0.050	15.489
ii	1.23 x10^{-3}	1 x10^{-5}	2 x10^{-5}	1.22 x10^{-3}
iii	2.95 x10^{6}	0.03x10^{6}	0.06x10^{6}	2.85 x10^{6}

b. How would your answers in *a* change for the case of a two-sigma confidence level.

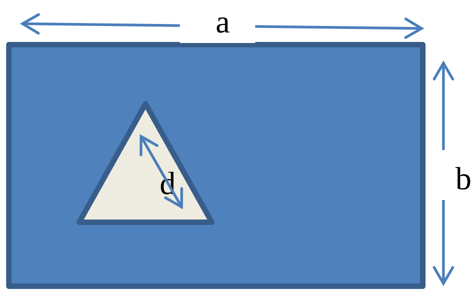
Problem 2.
A cart rolls down a slope, and the velocity is measured at two points separated by a distance ***s***. From these measurements, the acceleration ***a*** of the cart can be calculated using the formula:
$$v_2^2 = v_1^2 + 2as$$

The experiment was repeated 9 times with the following results:

v_1[cm/s]	92.6	90.0	91.5	93.1	93.3	94.0	92.1	92.7	93.0
v_2 [cm/s]	161.3	162.0	161.5	160.0	161.7	162.2	161.5	160.9	161.4
s [cm]	100.0	100.1	99.8	99.9	100.0	100.5	100.2	99.8	100.1

a. Calculate the mean, variance and standard deviation for each of the three measured quantities.
b. Calculate ***a***, and determine the absolute and the relative error.

Problem 3.



The measured values on the figure above are: $a = (2.25 \pm 0.03)cm$
$b = (1.30 \pm 0.03)cm$ $d = (0.55 \pm 0.03)cm$

Determine the area of the dark region and calculate its absolute and relative error. Give reason(s) for your choice of error propagation.

Problem 4.
You make five measurements of a quantity that has some random noise. Your measurements are {12,10,14,16,14} all in mm.
a) Calculate the mean value for your five measurements.
b) Calculate the standard deviation.

Problem 5
Neils Bohr showed that the energy of the quantum states of a Hydrogen atom are given by:
$$E_n = -\frac{1}{8}\frac{me^4}{\varepsilon_0^2 h^2}\frac{1}{n^2}$$
where *m* is the mass of the electron, *e* its charge, *h* is Planck's constant , ε_0 the permittivity of free space and *n= 1,2,3.. is* the principal quantum number. Suppose the *m*, *e* and *h* are

obtained experimentally and that the relative error in each of these measured quantities is 0.001, 0.002 and 0.0001 respectively.
What is the relative error in the energy of the third quantum state (n=3)?

Problem 6.

The conductivity, σ, of a metallic wire is derived by measuring the relationship between the voltage drop, V, and the applied current I:

$$V = \frac{1}{\sigma}\frac{L}{A} I \,.$$

Here L is the length of the wire and A is its cross section.
The results of such measurements and corresponding errors are summarized in the table below.

V (Volts)	I (mA)
1.00 ± 0.01	39.3 ± 2.0
1.30 ± 0.01	50.8 ± 2.0
1.70 ± 0.01	66.3 ± 2.5
2.00 ± 0.01	79.1 ± 2.5
2.25 ± 0.01	89.1 ± 2.4
2.5 ± 0.01	99.4 ± 23.2
2.75 ± 0.01	109 ± 4.1
3.00 ± 0.01	119 ± 2.0
3.25 ± 0.01	129 ± 5.5

a. Using Origin (or another program) make a scatter plot of V versus I for this data. Include error bars for both axes.
b. Using a fitting program make a linear fit of the data to the expected formula. Note that you will need to include a fit that takes into account the error on both the axes (Origin analysis can do this).
c. What is the value of the slope and its standard deviation?
d. Repeat parts b and c but without taking into account the error in the values of I and V. Comment on the difference in the results.
e. The slope you obtained from the fit gives $s = \frac{1}{\sigma}\frac{L}{A}$ and the corresponding standard deviation. Use these results and the measured value of L= (1.07 ± 0.03)cm and A= (0.01 ± 0.05)cm^2 to calculate the mean value of σ and its standard deviation.

Problem 7

Suppose a solution of concentration C_o and volume V_o is diluted by successive additions each of volume V, of solvent. What is the error in C_n, the concentration after n such additions? Assume the error in V is ΔV and that C_o and V_o are known precisely.

Problem 8

The following data was collected for an experiment measuring a free falling object :

t_i(sec) +/- 0.001sec	S_i (m) +/- 0.01m
0.000	0.3
0.079	0.4
0.132	0.5
0.174	0.6
0.212	0.7
0.244	0.8
0.271	0.9
0.301	1.0
0.325	1.1
0.349	1.2
0.373	1.3

Use a fitting program to fit the data to the free falling formula
$s = s_o + v_o t + 1/2gt^2$

and extract the values of the parameters s_o, v_o, g and their errors.