```
# This is formatted as code
```

Bayesian NN

you can check the github link: https://github.com/JonOnEarth/Bayesian-Neural-network/blob/master/BayesianNN.ipynb

or colab link: https://colab.research.google.com/drive/1VuCl_E8Q-lADV7plnUvclUurosiC5KE1

Papers

- This implementation and code based on the paper: weight uncertainty of Neural network

Codes and contents are heavily inspired by

- https://github.com/cpark321/uncertainty-deep-learning
- https://joshfeldman.net/ml/2018/12/17/WeightUncertainty.html
- http://krasserm.github.io/2019/03/14/bayesian-neural-networks/

Other code or tutorials

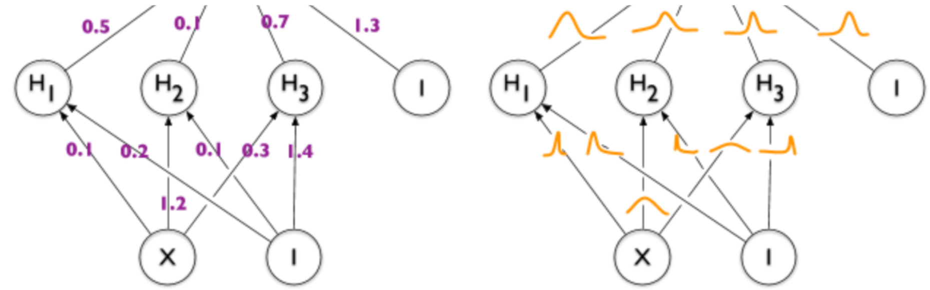
- https://github.com/krasserm/bayesian-machine-learning
- https://github.com/JavierAntoran/Bayesian-Neural-Networks
- https://github.com/nitarshan/bayes-by-backprop/blob/master/Weight%20Uncertainty%20in%20Neural%20Networks.ipynb
- https://paperswithcode.com/paper/weight-uncertainty-in-neural-networks
- https://github.com/anassinator/bnn

What is Bayesian Neural network

Bayesian neural networks differ from plain neural networks in that their weights are assigned a probability distribution instead of a single value or point estimate.

These probability distributions describe the uncertainty in weights and can be used to estimate uncertainty in predictions.

We will train a bayesian nerual network via variational inference learns the parameters of these distributios instead of the weights directly.



Left: each weight has a fixed value, as provided by classical backpropagation. Right: each weight is assigned a distribution, as provided by Bayes by Backprop

Probabilistic model

A neural network can be viewed as probabilistic model $p(y|x, w)$.

MLE

Given a training dataset $D = (x^{(i)}, y^{(i)})$ we can construct the likelihood function $p(\mathcal{D}|\mathbf{w}) = \prod_i p(y^{(i)}|\mathbf{x}^{(i)}, \mathbf{w})$ which is a fuction of parameters w. Maximizing the likelihood function gives the maximimum likelihood estimate (MLE) of w

$$\begin{aligned}\mathbf{w}^{\text{MLE}} &= \arg\max_{\mathbf{w}} \log P(\mathcal{D}|\mathbf{w}) \\ &= \arg\max_{\mathbf{w}} \sum_i \log P\left(\mathbf{y}_i|\mathbf{x}_i, \mathbf{w}\right)\end{aligned}$$

MAP

Multiplying the likelihood with a prior distribution p(w) is, by Bayes theorem, proportional to the posterior distribution $P(\mathbf{w}|\mathcal{D}) = P(\mathcal{D}|\mathbf{w})P(\mathbf{w})$. Maximizing $p(D|w)p(w)$ gives the maximum a posteriori (MAP) estimate of w. Computing the MAP estimate has a regularizing effect and can prevent overfitting. The optimization objectives here are the same as for MLE plus a regularization term coming from the log prior.

$$\begin{aligned}\mathbf{w}^{\text{MAP}} &= \arg\max_{\mathbf{w}} \log P(\mathbf{w}|\mathcal{D}) \\ &= \arg\max_{\mathbf{w}} \log P(\mathcal{D}|\mathbf{w}) + \log P(\mathbf{w})\end{aligned}$$

Variational inference

Unfortunately, an analytical solution for the posterior $p(w|d)$ in neural network is untractable. We therefore have to approximate the true posterior with a variational distribution $q(w|\theta)$ of known functional form whose parameters we want to estimate. This can be done by minimizing the Kullback-Leibler divergence between $q(w|\theta)$ and the true posterior $p(w|D)$ w.r.t. to θ.

$$\begin{aligned}\theta^\star &= \arg\min_{\theta} \text{KL}[q(\mathbf{w}|\theta)||P(\mathbf{w}|\mathcal{D})] \\ &= \arg\min_{\theta} \int q(\mathbf{w}|\theta) \log \frac{q(\mathbf{w}|\theta)}{P(\mathbf{w})P(\mathcal{D}|\mathbf{w})} \mathrm{d}\mathbf{w}\end{aligned}$$

$$= \arg\min_{\theta} \mathrm{KL}[q(\mathbf{w}|\theta)||P(\mathbf{w})] - \mathbb{E}_{q(\mathbf{w}|\theta)}[\log P(\mathcal{D}|\mathbf{w})]$$

This is known as the variational free energy. The first term is the Kullback-Leibler divergence between the variational distribution q(w|\theta) and the prior p(w) and is called the complexity cost. The second term is the expected value of the likelihood w.r.t. the variational distribution and is called the likelihood cost. By re-arranging the KL term, the cost function can also be written as

$$\mathcal{F}(\mathcal{D}, \boldsymbol{\theta}) = \mathbb{E}_{q(\mathbf{w}|\boldsymbol{\theta})} \log q(\mathbf{w}|\boldsymbol{\theta}) - \mathbb{E}_{q(\mathbf{w}|\boldsymbol{\theta})} \log p(\mathbf{w}) - \mathbb{E}_{q(\mathbf{w}|\boldsymbol{\theta})} \log p(\mathcal{D}|\mathbf{w})$$

We see that all three terms in equation are expectations w.r.t. the variational distribution $q(w|\theta)$. The cost function can therefore be approximated by drawing samples $w(i)$ from $q(w|\theta)$.

$$\mathcal{F}(\mathcal{D}, \boldsymbol{\theta}) \approx \frac{1}{N} \sum_{i=1}^{N} \left[\log q(\mathbf{w}^{(i)}|\boldsymbol{\theta}) - \log p(\mathbf{w}^{(i)}) - \log p(\mathcal{D}|\mathbf{w}^{(i)})\right] \tag{3}$$

In the following example, we'll use a Gaussian distribution for the variational posterior, parameterized by θ=(\mu,\sigma) where μ is the mean vector of the distribution and σ the standard deviation vector. The elements of σ are the elements of a diagonal covariance matrix which means that weights are assumed to be uncorrelated. Instead of parameterizing the neural network with weights w directly we parameterize it with μ and σ and therefore double the number of parameters compared to a plain neural network.

Network training

A training iteration consists of a forward-propagate and and backward-progagate. During a forward pass a single sample is drawn from the variational posterior distribution. It is used to evaluate the approximate cost function defined by equation 3. The first two terms of the cost function are data-independent and can be evaluated layer-wise, the last term is data-dependent and is evaluated at the end of the forward-pass. During a backward-pass, gradients of μ and σ are calculated via backpropagation so that their values can be updated by an optimizer.

Re-parameterization

Since a forward pass involves a stochastic sampling step we have to apply the so-called re-parameterization trick for backpropagation to work. The trick is to sample from a parameter-free distribution and then transform the sampled ϵ with a deterministic function $t(\mu, \sigma, \epsilon)$ for which a gradient can be defined.
Here, ϵ is drawn from a standard normal distribution i.e. $\epsilon \sim N(0, I)$ and function $t(\mu, \sigma, \epsilon) = \mu + \sigma \odot \epsilon$ shifts the sample by mean \mu and scales it with \sigma where $\odot$ is element-wise multiplication.

For numeric stability we will parameterize the network with ρ instead of σ directly and transform ρ with the softplus function to obtain $\sigma = log(1 + exp(\rho))$, so the σ is always non-negative.
Thus the transform from a sample of parameter-free noise and the variational posterior parameters that yields a posterior sample of the weights w is:

$$\mathbf{w} = t(\theta, \epsilon) = \mu + \log(1 + \exp(\rho)) \circ \epsilon$$

where $\circ$ is point-wise multiplication.

Back propagation (Gradient Descent)

From above, let ϵ be a random variable having a probability density given by $q(\epsilon)$ and let $w = t(\theta, \epsilon)$ where $t(\theta, \epsilon)$ is a deterministic function. Suppose further that the marginal probability density of w, $q(w|\theta)$, is such that $q(\epsilon)d\epsilon = q(w|\theta)dw$. Then for a function f with derivatives in w:

$$\frac{\partial}{\partial \theta}\mathbb{E}_{q(\mathbf{w}|\theta)}[f(\mathbf{w},\theta)] = \mathbb{E}_{q(\epsilon)}\left[\frac{\partial f(\mathbf{w},\theta)}{\partial \mathbf{w}}\frac{\partial \mathbf{w}}{\partial \theta} + \frac{\partial f(\mathbf{w},\theta)}{\partial \theta}\right]$$

Proof.

$$\begin{aligned}\frac{\partial}{\partial \theta}\mathbb{E}_{q(\mathbf{w}|\theta)}[f(\mathbf{w},\theta)] &= \frac{\partial}{\partial \theta}\int f(\mathbf{w},\theta)q(\mathbf{w}|\theta)\mathrm{d}\mathbf{w} \\ &= \frac{\partial}{\partial \theta}\int f(\mathbf{w},\theta)q(\epsilon)\mathrm{d}\epsilon \\ &= \mathbb{E}_{q(\epsilon)}\left[\frac{\partial f(\mathbf{w},\theta)}{\partial \mathbf{w}}\frac{\partial \mathbf{w}}{\partial \theta} + \frac{\partial f(\mathbf{w},\theta)}{\partial \theta}\right]\end{aligned}$$

Then combined, each step of optimisation proceeds as fllows:

1. Sample $\epsilon\ N(0, I)$
2. let $w = \mu + \log(1 + \exp(\rho)) \circ \epsilon$
3. Let $\theta = (\mu, \rho)$.
4. Let $f(w, \theta) = logq(w|\theta) - logP(w)P(D|w)$.
5. Calculate the gradient with respect to the mean
$$\Delta_\mu = \frac{\partial f(\mathbf{w},\theta)}{\partial \mathbf{w}} + \frac{\partial f(\mathbf{w},\theta)}{\partial \mu}$$
6. Calculate the gradient with respect to the standard deviation parameter ρ
$$\Delta_\rho = \frac{\partial f(\mathbf{w},\theta)}{\partial \mathbf{w}}\frac{\epsilon}{1 + \exp(-\rho)} + \frac{\partial f(\mathbf{w},\theta)}{\partial \rho}$$
7. Update the variational parameters:
$$\mu \leftarrow \mu - \alpha\Delta_\mu$$
$$\rho \leftarrow \rho - \alpha\Delta_\rho$$

Implementation example

Unlike the easy example or tutorials shown, only 1 dimension dataset was shown. Here I am use multiple dimension dataset to do a regression problems.

This dataset is from my research, which is indoor positioning dataset. The Received Signal strength(RSS value) as y out, and locations as input x. We are trying to using the position to predict the RSS filed.

[ ]

[ ]
```
import torch
print(torch.__version__)
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
from torch.distributions import Normal
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
import plotly.graph_objects as go
```

```
import plotly
print(plotly.__version__)
```

```
1.3.1
4.1.1
```

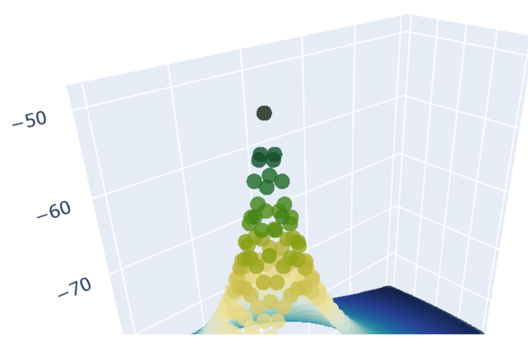
```
[ ] # load the dataset
    X = np.loadtxt('pos_train.csv', delimiter=',')
    y = np.loadtxt('rss_train.csv', delimiter=',')
    X_test = np.loadtxt('pos_rand.csv', delimiter=',')
    y_test = np.loadtxt('rss_rand.csv', delimiter=',')
    input_dim = X.shape[1]
    output_dim = y.shape[1]
```

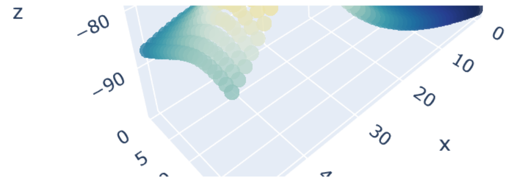
```
[ ] # numpy to tensor
    train_x = torch.from_numpy(X).float()
    train_y = torch.from_numpy(y).float()
    test_x = torch.from_numpy(X_test).float()
    test_y = torch.from_numpy(y_test).float()
```

```
[ ] train_x.type()
```

```
'torch.FloatTensor'
```

```
[ ] # The dataset filed visulization
    X_real = train_x.detach().numpy()
    y_real = train_y.detach().numpy()
    fig = go.Figure(data=[go.Scatter3d(x = X_real[:,0], y = X_real[:,1], z = y_real[:,1], mode='markers', marker=dict(
            size=6,
            color = y_real[:,1],                # set color to an array/list of desired values
            colorscale='delta',   # choose a colorscale
            opacity=0.8
        ))])
    fig.show()
```






```
[ ] # Tranfer the data into the batch data
    from torch.utils.data import TensorDataset
    train_data = TensorDataset(train_x, train_y)
    test_data = TensorDataset(test_x, test_y)
    batch_size_train = 32
    batch_size_test = 32
    train_loader = torch.utils.data.DataLoader(
        train_data, batch_size=batch_size_train, shuffle=True)
    test_loader = torch.utils.data.DataLoader(
        test_data, batch_size=batch_size_test, shuffle=True)
```

Standard_MLP

First, we are using the standard Neural network.

```
[ ] from mlp import *
```

```
[ ] # parameters for neural network
    layer_wid1 = [16,32,128,64,16,output_dim]
    nonlinearity1 = 'relu'
    stand_net=Net(input_dim, layer_wid1, nonlinearity1)
```

```
[ ] optimizer1 = optim.Adam(stand_net.parameters(),lr=0.001,weight_decay = 1e-5)
```

```
[ ] # training
    epochs1 = 2000
    for epoch in range(epochs1):  # loop over the dataset multiple times
      losses, mses = [], []
      for batch, (x_train,y_train) in enumerate(train_loader):
        optimizer1.zero_grad()
        # forward + backward + optimize
        y_pred = stand_net(x_train)
        loss = stand_net.criterion(y_pred, y_train)
        loss.backward()
        optimizer1.step()

        # output_train = bnet(x_train)
        # train_mse = F.mse_loss(y_train, output_train)
```

```
        # mses.append(train_mse.item())
        losses.append(loss.item())

      train_loss = np.mean(losses)
      # epoch_mse = np.mean(mses)
      if epoch % 100 == 0:
        print('epoch: {}/{}'.format(epoch+1,epochs1))
        print(f'epoch{epoch}, train_loss^mse:{train_loss}')

      test_losses, test_mses = [], []
      for i, (x_test, y_test) in enumerate(test_loader):
          # test_loss = bnet.sample_elbo(x_test, y_test, 5)
          test_pred = stand_net(x_test)
          test_mse = F.mse_loss(y_test, test_pred)
          # test_losses.append(test_loss.item())
          test_mses.append(test_mse.item())

      # test_loss = np.mean(test_losses)
      epoch_mse_test =  np.mean(test_mses)
      if epoch % 100 == 0:
        print('epoch: {}/{}'.format(epoch+1,epochs1))
        print(f'epoch_mse_test_loss:{epoch_mse_test}')
    print('Finished Training')
```

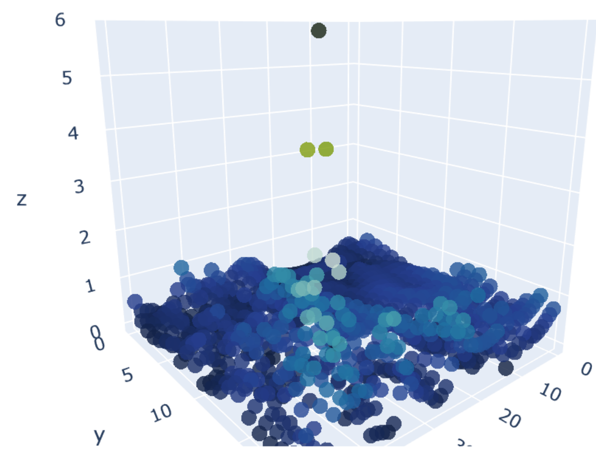
```
epoch: 1/2000
epoch0, train_loss^mse:6922.080437155331
epoch: 1/2000
epoch_mse_test_loss:6054.784469604492
epoch: 101/2000
epoch100, train_loss^mse:18.602494464201087
epoch: 101/2000
epoch_mse_test_loss:19.31205326318741
epoch: 201/2000
epoch200, train_loss^mse:5.412386817090652
epoch: 201/2000
epoch_mse_test_loss:5.773011438548565
epoch: 301/2000
epoch300, train_loss^mse:4.454936465796302
epoch: 301/2000
epoch_mse_test_loss:4.153872765600681
epoch: 401/2000
epoch400, train_loss^mse:2.565233076319975
epoch: 401/2000
epoch_mse_test_loss:2.547386534512043
epoch: 501/2000
epoch500, train_loss^mse:1.9160785885418163
epoch: 501/2000
epoch_mse_test_loss:2.207970429211855
epoch: 601/2000
epoch600, train_loss^mse:1.5793781876564026
epoch: 601/2000
epoch_mse_test_loss:1.9743670672178268
epoch: 701/2000
epoch700, train_loss^mse:1.521827732815462
epoch: 701/2000
epoch_mse_test_loss:1.8401612844318151
epoch: 801/2000
epoch800, train_loss^mse:1.3505019054693335
```

```
epoch: 801/2000
epoch_mse_test_loss:1.4345218446105719
epoch: 901/2000
epoch900, train_loss^mse:1.406448739416459
epoch: 901/2000
epoch_mse_test_loss:1.3287188243120909
epoch: 1001/2000
epoch1000, train_loss^mse:1.23554229035097
epoch: 1001/2000
epoch_mse_test_loss:1.5034247692674398
epoch: 1101/2000
epoch1100, train_loss^mse:1.1583131595569498
epoch: 1101/2000
epoch_mse_test_loss:1.2246176935732365
epoch: 1201/2000
epoch1200, train_loss^mse:1.1125480217092178
epoch: 1201/2000
epoch_mse_test_loss:1.2760322894901037
epoch: 1301/2000
epoch1300, train_loss^mse:1.1244444347479765
epoch: 1301/2000
epoch_mse_test_loss:1.1516575757414103
epoch: 1401/2000
epoch1400, train_loss^mse:1.1953153592698715
epoch: 1401/2000
epoch_mse_test_loss:1.2347836829721928
epoch: 1501/2000
epoch1500, train_loss^mse:0.8122498743674335
epoch: 1501/2000
epoch_mse_test_loss:0.76664924249053
epoch: 1601/2000
epoch1600, train_loss^mse:0.5885740623754614
epoch: 1601/2000
epoch_mse_test_loss:0.5427891630679369
epoch: 1701/2000
epoch1700, train_loss^mse:0.5023539723718867
epoch: 1701/2000
epoch_mse_test_loss:0.4833252774551511
epoch: 1801/2000
epoch1800, train_loss^mse:0.49618442356586456
epoch: 1801/2000
epoch_mse_test_loss:0.47791446559131145
epoch: 1901/2000
epoch1900, train_loss^mse:0.4483284284086788
epoch: 1901/2000
epoch_mse_test_loss:0.5049703205004334
Finished Training
```

[ ]

```
# Show the training dataset absolute prediction error.
Y = stand_net(train_x)
y_error = torch.abs(Y - train_y)
X_adam = train_x.detach().numpy()
y_error = y_error.detach().numpy()
fig = go.Figure(data=[go.Scatter3d(x=X_adam[:,0], y=X_adam[:,1], z=y_error[:,1], mode='markers', marker=dict(
        size=6,
        color=y_error[:,1],                # set color to an array/list of desired values
        colorscale='delta',   # choose a colorscale
        opacity=0.8
```

```
        ))])
    fig.show()
```



Bayesian Neural Networks

```
[ ] from bayesianNN import *
```

```
[ ] # Bayesian Neural network
    layer_wid2 = [1024,output_dim]
    nonlinearity2 =  'relu'
    bnet = MLP_BBB(input_dim,layer_wid2, nonlinearity2, noise_tol = 0.1, prior_var=1)
```

Parameters adjustment

Noise_tol is very important for the result. As noise tolerance is the covariance we set for the likelihood $P(D|w)$. if noise tolerance is big, that mean we tolerate the big error between real output and prediction. In opposite, samll noise tolerance mean we can't tolerate a big mistake for our prediction. That will give us some trade-off, too samll maybe overfitting, too big maybe underfitting.

```
[ ] optimizer2 = optim.Adam(bnet.parameters(), lr=.01, weight_decay = 1e-5)
```

```python
[ ] # bayesian neural network training
    epochs2 = 2000
    for epoch in range(epochs2):  # loop over the dataset multiple times
      losses, mses = [], []
      for batch, (x_train,y_train) in enumerate(train_loader):
        optimizer2.zero_grad()
        # forward + backward + optimize
        loss = bnet.sample_elbo(x_train, y_train, 10)
        loss.backward()
        optimizer2.step()

        output_train = bnet(x_train)
        train_mse = F.mse_loss(y_train, output_train)
        mses.append(train_mse.item())
        losses.append(loss.item())

      train_loss = np.mean(losses)
      epoch_mse = np.mean(mses)
      if epoch % 100 == 0:
        print('epoch: {}/{}'.format(epoch+1,epochs2))
        print(f'epoch{epoch}, train_loss:{train_loss}, epoch_mse:{epoch_mse}')

      test_losses, test_mses = [], []
      for i, (x_test, y_test) in enumerate(test_loader):
          test_loss = bnet.sample_elbo(x_test, y_test, 5)
          test_pred = bnet(x_test)
          test_mse = F.mse_loss(y_test, test_pred)
          test_losses.append(test_loss.item())
          test_mses.append(test_mse.item())

      test_loss = np.mean(test_losses)
      epoch_mse_test =  np.mean(test_mses)
      if epoch % 100 == 0:
        print('epoch: {}/{}'.format(epoch+1,epochs2))
        print(f'test_loss:{test_loss}, epoch_mse_test:{epoch_mse_test}')
    print('Finished Training')
```

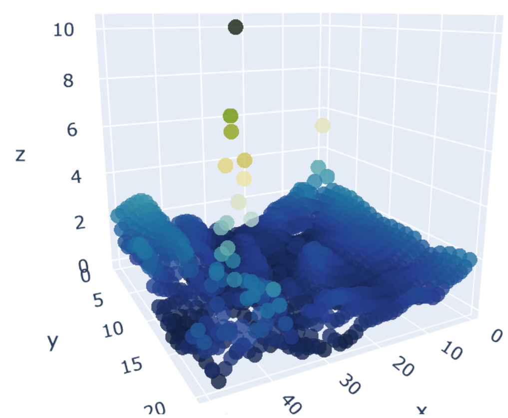
```
epoch: 1/2000
epoch0, train_loss:29230656.588235293, epoch_mse:2477.111166561351
epoch: 1/2000
test_loss:16656606.9765625, epoch_mse_test:1498.0246810913086
epoch: 101/2000
epoch100, train_loss:89718.65165441176, epoch_mse:6.268639115726247
epoch: 101/2000
test_loss:96636.04443359375, epoch_mse_test:7.122472286224365
epoch: 201/2000
epoch200, train_loss:51191.38671875, epoch_mse:3.1722595376126907
epoch: 201/2000
test_loss:54275.28497314453, epoch_mse_test:3.51962872967124
epoch: 301/2000
epoch300, train_loss:31372.83674172794, epoch_mse:1.5185675919055939
epoch: 301/2000
test_loss:30435.562072753906, epoch_mse_test:1.5843053609132767
epoch: 401/2000
epoch400, train_loss:23764.744399126837, epoch_mse:1.2661514440003563
epoch: 401/2000
```

```
test_loss:30822.393524169922, epoch_mse_test:2.05694667622447
epoch: 501/2000
epoch500, train_loss:24574.12126608456, epoch_mse:1.3643891601001514
epoch: 501/2000
test_loss:36488.59408569336, epoch_mse_test:2.6360652931034565
epoch: 601/2000
epoch600, train_loss:20151.86830767463, epoch_mse:0.9869749633704915
epoch: 601/2000
test_loss:20537.749877929688, epoch_mse_test:1.1518556345254183
epoch: 701/2000
epoch700, train_loss:17761.81752642463, epoch_mse:0.8294167755281224
epoch: 701/2000
test_loss:18011.608459472656, epoch_mse_test:0.9296817630529404
epoch: 801/2000
epoch800, train_loss:19863.51384420956, epoch_mse:0.9716104409273933
epoch: 801/2000
test_loss:30411.554443359375, epoch_mse_test:2.0092415623366833
epoch: 901/2000
epoch900, train_loss:17873.45490579044, epoch_mse:0.8148514120017781
epoch: 901/2000
test_loss:18185.988037109375, epoch_mse_test:0.9215692896395922
epoch: 1001/2000
epoch1000, train_loss:17703.774327895222, epoch_mse:0.802768712534624
epoch: 1001/2000
test_loss:21430.447387695312, epoch_mse_test:1.285710845142603
epoch: 1101/2000
epoch1100, train_loss:17614.014906939337, epoch_mse:0.761520687271567
epoch: 1101/2000
test_loss:21200.527099609375, epoch_mse_test:1.2140387576073408
epoch: 1201/2000
epoch1200, train_loss:17703.100786994484, epoch_mse:2.025664773934028
epoch: 1201/2000
test_loss:20157.801696777344, epoch_mse_test:1.0812946502119303
epoch: 1301/2000
epoch1300, train_loss:17079.563074448528, epoch_mse:0.73799989241011
epoch: 1301/2000
test_loss:20661.140502929688, epoch_mse_test:1.1014525163918734
epoch: 1401/2000
epoch1400, train_loss:17138.776166130516, epoch_mse:0.7249807128134895
epoch: 1401/2000
test_loss:16570.775482177734, epoch_mse_test:0.7874064017087221
epoch: 1501/2000
epoch1500, train_loss:16983.442354090075, epoch_mse:0.7520636390237248
epoch: 1501/2000
test_loss:15434.659790039062, epoch_mse_test:0.7075760643929243
epoch: 1601/2000
epoch1600, train_loss:20516.852338005516, epoch_mse:1.0334057737799252
epoch: 1601/2000
test_loss:29348.53924560547, epoch_mse_test:2.195502758026123
epoch: 1701/2000
epoch1700, train_loss:14786.926642922794, epoch_mse:0.5741768873789731
epoch: 1701/2000
test_loss:14725.407653808594, epoch_mse_test:0.6193345636129379
epoch: 1801/2000
epoch1800, train_loss:17145.681382123163, epoch_mse:0.6661794150576872
epoch: 1801/2000
test_loss:19266.58755493164, epoch_mse_test:1.013560488820076
epoch: 1901/2000
epoch1900, train_loss:16440.57666015625, epoch_mse:0.6811232856091332
epoch: 1901/2000
test_loss:25040.27716064453, epoch_mse_test:1.5921094790101051
```

```
Finished Training
```

```
[ ] # %debug
```

```
[ ] # The prediction error
    Y2 = bnet(train_x)
    y_error2 = torch.abs(Y2 - train_y)
    X_bnet = train_x.detach().numpy()
    y_error2 = y_error2.detach().numpy()
    fig = go.Figure(data=[go.Scatter3d(x=X_bnet[:,0], y=X_bnet[:,1], z=y_error2[:,1], mode='markers', marker=dict(
            size=6,
            color=y_error2[:,1],                # set color to an array/list of desired values
            colorscale='delta',   # choose a colorscale
            opacity=0.8
        ))])
    fig.show()
```



Result

Bayesian neural network(BNN) training result is similar to regular neural network training, but it takes much more time for BNN to train. The benefits for BNN to trian is that it can give us the distribution of the prediction as we discuss before. Let's see the result.

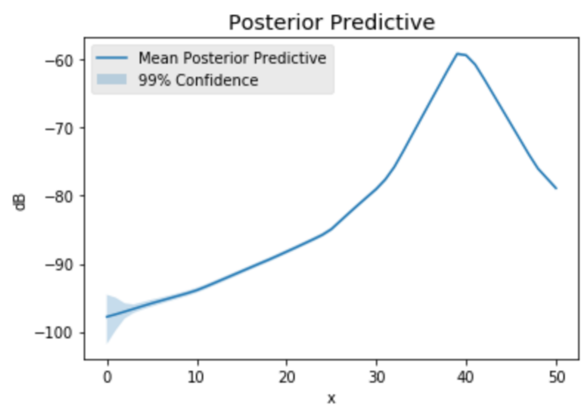
The trianing is a little bit of overfitting. So you can see sample 1000 data points, all prediction will locate around the mean value closely.

But at the edge of the dataset area, the confidence is very low than the other area of datasets. That mean few dataset at the area, we can't have a very confindent prediction. Unlike the regular neural network, they will only give the fix prediction value no matter the confindence.

[ ]

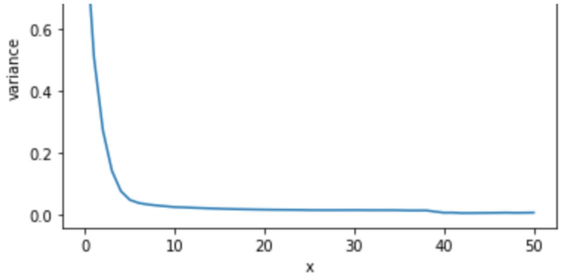
```
# samples is the number of "predictions" we make for 1 x-value
samples = 1000

y_samp = np.zeros((samples,train_y.shape[0],train_y.shape[1]))
for s in range(samples):
    y_tmp = bnet(train_x).detach().numpy()
    y_samp[s] = y_tmp#-train_y.detach().numpy()
trainx1 = train_x.numpy()[:,1]
index = np.where(trainx1 == 16)
trainx2 = train_x[:,0]
x_tmp = trainx2[index]
y_samp1 = y_samp[:,index,1].reshape(samples,-1)
plt.plot(x_tmp, np.mean(y_samp1, axis = 0), label='Mean Posterior Predictive')
plt.style.use('ggplot')
plt.fill_between(x_tmp.numpy().reshape(-1), np.percentile(y_samp1, 1, axis = 0), np.percentile(y_samp1, 99, axis = 0), alpha = 0.25, label='99% Confidence')
# plt.ylim(-100,-80)
plt.xlabel('x')
plt.ylabel('dB')
plt.legend()
plt.title('Posterior Predictive')
plt.show()
```



[ ]

```
# The variance of the prediction at each location of x where y is a fixed value
y_var = np.var(y_samp1, axis = 0)
plt.plot(x_tmp, y_var)
plt.xlabel('x')
plt.ylabel('variance')
plt.show()
```



variance
0.6
0.4
0.2
0.0
0
10
20
30
40
50
x

[ ]

[ ]