The Definition of RedPRL, *the people's refinement logic*

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Contents

Chapter 1

Signatures

Decisively Smash The Formalist Clique!

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A *signature* is a collection of definitions, including terms, tactics and theorems.

1.1 Grammar

The grammar of RedPRL signatures is presented in Figure 1.1. Note that an optional production of sort s is formatted $\langle s \rangle$ in the rules.

$sigexp$	::=	$\langle \cdot \rangle$	empty signature
		$sigexp\ sigdec.$	signature extension
		$sigexp\ symdec.$	
$sigdec$	::=	`Def` $opid\langle[params]\rangle\langle(args)\rangle : sortid = [term]$	operator definition
		`Tac` $opid\langle[params]\rangle\langle(args)\rangle = [term]$	tactic definition
		`Thm` $opid\langle[params]\rangle\langle(args)\rangle : [term]$ `by` $[term]$	theorem declaration
$symdec$	::=	`Sym` $symbind$	symbol declaration
$params$	::=	$\langle \cdot \rangle$	empty parameter list
		$params, symbind$	parameter list extension
$args$	::=	$\langle \cdot \rangle$	empty argument list
		$args, metabind$	argument list extension
$symbind$	::=	$symid : sortid$	symbol binding
$metabind$	::=	$metaid : valence$	metavariable binding
$valence$	::=	$\langle\langle\{sortlist\}\rangle\langle[sortlist]\rangle.\rangle sortid$	valence
$sortlist$	::=	$\langle \cdot \rangle$	empty sort list
		$sortlist, sortid$	sort list extension

Figure 1.1: Grammar of signature expressions. The identifier sorts $opid$, $sortid$, $symid$ and $metaid$ can be assumed to be arbitrary strings; the sort $term$ is left uninterpreted.

1.2 Static Semantics

The static semantics for RedPRL signatures begins with a specification of the class of *semantic* objects that will serve as the meanings for the *syntactic* objects defined in Section 1.1. We assume an ambient abstract binding tree signature such that at least the following facts hold:

$$\frac{}{\texttt{tac}\ \mathit{sort}} \qquad \frac{}{\texttt{thm}\ \mathit{sort}} \qquad \frac{}{\texttt{exp}\ \mathit{sort}} \qquad \frac{}{\texttt{opid}\ \mathit{sort}}$$

$$\frac{}{\Upsilon \Vdash \texttt{prove} : (\texttt{.exp}, \texttt{.tac})\,\texttt{thm}}$$

Then, our semantic objects are defined as in Figure 1.2.

$$\begin{array}{rcll}
a, b & \in & \mathrm{Sym} & \\
\mathfrak{m}, \mathfrak{n} & \in & \mathrm{Metavar} & \\
\sigma, \tau & \in & \mathrm{Sort} & \triangleq \{\, \tau \mid \tau\ \mathit{sort} \,\} \\
v & \in & \mathrm{Valence} & \triangleq \{\, v \mid v\ \mathit{valence} \,\} \\
\vartheta & \in & \mathrm{Opid} & \triangleq \mathrm{Sym} \\
\Upsilon & \in & \mathrm{Params} & \triangleq \mathrm{Sym} \xrightarrow{\text{fin}} \mathrm{Sort} \\
\Theta & \in & \mathrm{Args} & \triangleq \mathrm{Metavar} \xrightarrow{\text{fin}} \mathrm{Valence} \\
M, N & \in & \mathrm{Tm}(\Theta, \Upsilon, \tau) & \triangleq \{\, M \mid \Theta \triangleright \Upsilon \parallel \cdot \vdash M : \tau \,\} \\
D & \in & \mathrm{Decl} & \triangleq \coprod_{\Upsilon,\Theta,\tau} \mathrm{Tm}(\Theta, \Upsilon, \tau) \\
\Sigma & \in & \mathrm{Sig} & \triangleq \left(\mathrm{Opid} \xrightarrow{\text{fin}} \mathrm{Decl}\right) \cap \left(\mathrm{Sym} \xrightarrow{\text{fin}} \mathrm{Sort}\right)
\end{array}$$

Figure 1.2: Specification of the semantic objects.

A *natural semantics* hinges on the elaboration judgment $E \vdash A \Longrightarrow A'$, which means that the syntactic object A elaborates to the semantic object A' in the environment E. Let the $\Upsilon_\Sigma \in \mathrm{Params}$ be defined as follows:

$$\Upsilon_\Sigma(u) \triangleq \begin{cases} \texttt{opid} & \textit{if} \quad u \equiv \vartheta \in \mathbf{dom}(\Sigma) \\ \tau & \textit{if} \quad \Sigma(u) \equiv \tau \\ \bot & \textit{otherwise} \end{cases}$$

Symbol Bindings

$$\boxed{\Sigma \vdash \mathit{symbind} \Longrightarrow (a, \tau)}$$

$$\frac{\Sigma \vdash \mathit{symid} \Longrightarrow a \quad \Sigma \vdash \mathit{sortid} \Longrightarrow \tau}{\Sigma \vdash \mathit{symid} : \mathit{sortid} \Longrightarrow (a, \tau)} \tag{1.1}$$

Metavariable Bindings

$$\boxed{\Sigma \vdash \mathit{metabind} \Longrightarrow (\mathfrak{m}, v)}$$

$$\frac{\Sigma \vdash \mathit{metaid} \Longrightarrow \mathfrak{m} \quad \Sigma \vdash \mathit{valence} \Longrightarrow v}{\Sigma \vdash \mathit{metaid} : \mathit{valence} \Longrightarrow (\mathfrak{m}, v)} \tag{1.2}$$

Parameters

$$\boxed{\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon}$$

$$\frac{}{\Sigma \vdash \langle\, \cdot \,\rangle \Longrightarrow \{\}} \tag{1.3}$$

$$\frac{\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon \quad \Sigma \vdash \mathit{symbind} \Longrightarrow (a, \tau)}{\Sigma \vdash \mathit{params}, \mathit{symbind} \Longrightarrow \Upsilon \cup a \mapsto \tau} \tag{1.4}$$

Arguments

$\boxed{\Sigma \vdash \mathit{args} \Longrightarrow \Theta}$

$$\frac{}{\Sigma \vdash \langle\,\cdot\,\rangle \Longrightarrow \{\}} \tag{1.5}$$

$$\frac{\Sigma \vdash \mathit{args} \Longrightarrow \Theta \quad \Sigma \vdash \mathit{metabind} \Longrightarrow (\mathtt{m}, v)}{\Sigma \vdash \mathit{args}, \mathit{metabind} \Longrightarrow \Theta \cup \mathtt{m} \mapsto v} \tag{1.6}$$

Symbols

$\boxed{\Sigma \vdash \mathit{symid} \Longrightarrow u}$

$$\frac{u \notin \mathbf{dom}(\Sigma)}{\Sigma \vdash \mathit{symid} \Longrightarrow u} \tag{1.7}$$

Operator Declarations

$\boxed{\Sigma \vdash \mathit{sigdec} \Longrightarrow (\vartheta, D)}$

$$\frac{\begin{array}{lll}\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon & \Sigma \vdash \mathit{sortid} \Longrightarrow \tau & \Sigma \vdash \mathit{opid} \Longrightarrow \vartheta \\ \Sigma \vdash \mathit{args} \Longrightarrow \Theta & \Sigma \vdash \mathit{term} \Longrightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \tau\end{array}}{\Sigma \vdash \mathtt{Def}\ \mathit{opid}\langle[\mathit{params}]\rangle\langle(\mathit{args})\rangle : \mathit{sortid} = [\mathit{term}] \Longrightarrow (\vartheta, \langle\Upsilon, \Theta, \tau, M\rangle)} \tag{1.8}$$

$$\frac{\begin{array}{ll}\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon & \Sigma \vdash \mathit{opid} \Longrightarrow \vartheta \\ \Sigma \vdash \mathit{args} \Longrightarrow \Theta & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \mathtt{tac} \\ \Sigma \vdash \mathit{term} \Longrightarrow M & \end{array}}{\Sigma \vdash \mathtt{Tac}\ \mathit{opid}\langle[\mathit{params}]\rangle\langle(\mathit{args})\rangle = [\mathit{term}] \Longrightarrow (\vartheta, \langle\Upsilon, \Theta, \mathtt{tac}, M\rangle)} \tag{1.9}$$

$$\frac{\begin{array}{llll}\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon & \Sigma \vdash \mathit{term}_1 \Longrightarrow P & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash P : \mathtt{exp} & \Sigma \vdash \mathit{opid} \Longrightarrow \vartheta \\ \Sigma \vdash \mathit{args} \Longrightarrow \Theta & \Sigma \vdash \mathit{term}_2 \Longrightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \mathtt{tac} & \end{array}}{\Sigma \vdash \mathtt{Thm}\ \mathit{opid}\langle[\mathit{params}]\rangle\langle(\mathit{args})\rangle : [\mathit{term}_1]\ \mathtt{by}\ [\mathit{term}_2] \Longrightarrow (\vartheta, \langle\Upsilon, \Theta, \mathtt{thm}, \mathtt{prove}(P; M)\rangle)} \tag{1.10}$$

Symbol Declarations

$\boxed{\Sigma \vdash \mathit{symdec} \Longrightarrow (u, \sigma)}$

$$\frac{\Sigma \vdash \mathit{symbind} \Longrightarrow (u, \sigma)}{\Sigma \vdash \mathtt{Sym}\ \mathit{symbind} \Longrightarrow (u, \sigma)} \tag{1.11}$$

Signatures

$\boxed{\vdash \mathit{sigexp} \Longrightarrow \Sigma}$

$$\frac{}{\vdash \langle\,\cdot\,\rangle \Longrightarrow \{\}} \tag{1.12}$$

$$\frac{\vdash \mathit{sigexp} \Longrightarrow \Sigma \quad \Sigma \vdash \mathit{sigdec} \Longrightarrow (\vartheta, D)}{\vdash \mathit{sigexp}\ \mathit{sigdec}. \Longrightarrow \Sigma \cup \vartheta \mapsto D} \qquad \frac{\vdash \mathit{sigexp} \Longrightarrow \Sigma \quad \Sigma \vdash \mathit{symdec} \Longrightarrow (u, \sigma)}{\vdash \mathit{sigexp}\ \mathit{symdec}. \Longrightarrow \Sigma \cup u \mapsto \sigma} \tag{1.13}$$

Chapter 2

Nominal LCF: a language for tactics

This chapter needs to be re-written in light of changes to the Nominal LCF formalism.

Bibliography