

Cubical Abstract Machine

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M, A	$::= a \mid \text{coe}_{x.A}^{r \rightsquigarrow r'}(M) \mid \dots$	(terms)
K	$::= \text{coe}_{\square}^{r \rightsquigarrow r'}(M) \mid \dots$	(continuations)
C	$::= M \triangleleft \mathcal{E}$	(closures)
\mathcal{E}	$::= (\gamma, \psi)$	(environments)
γ	$::= \frac{a \hookrightarrow C}{a \hookrightarrow C}$	(object env.)
ψ	$::= \frac{x \hookrightarrow r}{x \hookrightarrow r}$	(dimension env.)
f	$::= (K, \mathcal{E}, \Psi)$	(frames)
π	$::= \cdot \mid f :: \pi$	(stacks)
\mathcal{C}	$::= \langle M \triangleleft \mathcal{E} \parallel \pi \rangle$	(states)

We will write $M[\mathcal{E}]$ for substituting the free dimensions and variables of a term M using an environment \mathcal{E} .

Stack frames and machine states A stack frame (K, \mathcal{E}, Ψ) represents a continuation whose whole binds Ψ dimensions. For a stack π , let $\vec{\Psi}$ be the aggregation of all the dimension bindings Ψ mentioned in π ; then the extension of a stack $(K, \mathcal{E}, \Phi) :: \pi$ is wellformed when \mathcal{E} is an environment for $\mathbf{FD}(K) \setminus \vec{\Psi}$; moreover, K is allowed to mention the dimensions in $\vec{\Psi}$.

A machine state $\langle M \triangleleft \mathcal{E} \parallel \pi \rangle$, where $\vec{\Psi}$ is the aggregation of dimension bindings in π , is wellformed when \mathcal{E} is an environment for $\mathbf{FD}(M) \setminus \vec{\Psi}$; moreover, M is allowed to mention the dimensions in $\vec{\Psi}$.

Selected transition rules

$$\langle a \triangleleft (\gamma, \psi) \parallel \pi \rangle \succ \langle \gamma(a) \triangleleft (\gamma, \psi) \parallel \pi \rangle \quad (1)$$

$$\langle M(N) \triangleleft \mathcal{E} \parallel \pi \rangle \succ \langle M \triangleleft \mathcal{E} \parallel (\square(N), \mathcal{E}, []) :: \pi \rangle \quad (2)$$

$$\langle \lambda a. M \triangleleft \mathcal{E} \parallel (\square(N), \mathcal{E}', []) :: \pi \rangle \succ \langle M \triangleleft [\mathcal{E}, a \hookrightarrow N \triangleleft \mathcal{E}] \parallel \pi \rangle \quad (3)$$

$$\langle \text{fst}(M) \triangleleft \mathcal{E} \parallel \pi \rangle \succ \langle M \triangleleft \mathcal{E} \parallel (\text{fst}(\square), \mathcal{E}, []) :: \pi \rangle \quad (4)$$

$$\langle \text{snd}(M) \triangleleft \mathcal{E} \parallel \pi \rangle \succ \langle M \triangleleft \mathcal{E} \parallel (\text{snd}(\square), \mathcal{E}, []) :: \pi \rangle \quad (5)$$

$$\langle \langle M, N \rangle \triangleleft \mathcal{E} \parallel (\text{fst}(\square), \mathcal{E}', []) :: \pi \rangle \succ \langle M \triangleleft \mathcal{E} \parallel \pi \rangle \quad (6)$$

$$\langle \langle M, N \rangle \triangleleft \mathcal{E} \parallel (\text{snd}(\square), \mathcal{E}', []) :: \pi \rangle \succ \langle N \triangleleft \mathcal{E} \parallel \pi \rangle \quad (7)$$

$$\langle \text{loop}_r \triangleleft (\gamma, \psi) \parallel \pi \rangle \succ \langle \text{base} \triangleleft (\gamma, \psi) \parallel \pi \rangle \textbf{ when } r\psi = \epsilon \quad (8)$$

$$\langle \text{coe}_{x.A}^{r \rightsquigarrow r'}(M) \triangleleft \mathcal{E} \parallel \pi \rangle \succ \langle A \triangleleft \mathcal{E} \parallel (\text{coe}_{\square}^{r \rightsquigarrow r'}(M), \mathcal{E}', [x]) :: \pi \rangle \quad (9)$$

$$\langle \text{bool} \triangleleft \mathcal{E} \parallel (\text{coe}_{\square}^{r \rightsquigarrow r'}(M), \mathcal{E}', [x]) :: \pi \rangle \succ \langle M \triangleleft \mathcal{E}' \parallel \pi \rangle \quad (10)$$

$$\langle (a : A) \rightarrow B \triangleleft \mathcal{E} \parallel (\text{coe}_{\square}^{r \rightsquigarrow r'}(M), \mathcal{E}', [x]) :: \pi \rangle \succ \langle \lambda b. \text{coe}_{x.B[\mathcal{E}, a \hookrightarrow \text{coe}_{x.A[\mathcal{E}}]^{r' \rightsquigarrow r}}(b) \triangleleft \mathcal{E}' \parallel \pi \rangle \quad (11)$$

The dynamics of coercion on a functional type are unfortunate: it is necessary to force the substitution \mathcal{E} on the types A and B , since we are “suspending” the coercion and placing it underneath a λ -abstraction. The same problem occurs for pair types. This suggests that we may ultimately need some kind of explicit substitution in our calculus.

$$\langle (a : A) \times B \triangleleft \mathcal{E} \parallel (\text{coe}_{\square}^{r \rightsquigarrow r'}(M), \mathcal{E}', [x]) :: \pi \rangle \succ \langle \langle \text{coe}_{x.A[\mathcal{E}]}^{r \rightsquigarrow r'}(\text{fst}(M)), \text{coe}_{x.B[\mathcal{E}, a \mapsto \text{coe}_{x.A[\mathcal{E}]}^{r \rightsquigarrow r'}(\text{fst}(M))}^{r \rightsquigarrow r'}(\text{snd}(M)) \rangle \triangleleft \mathcal{E}' \parallel \pi \rangle \quad (12)$$

Unloading the machine We can unload the machine at any time; this is useful if we are computing an open term and hit a variable.

$$\frac{}{\langle M \triangleleft \mathcal{E} \parallel \cdot \rangle \Longrightarrow M[\mathcal{E}]} \qquad \frac{\langle K[\Psi.M[\mathcal{E}]] \triangleleft \mathcal{E}' \parallel \pi \rangle \Longrightarrow N}{\langle M \triangleleft \mathcal{E} \parallel (K[\square], \mathcal{E}', \Psi) :: \pi \rangle \Longrightarrow N}$$