Specification of Red JonPRL Signatures

Jon Sterling

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1 Grammar

The grammar of Red JonPRL signatures is presented in Figure 1. Note that an optional production of sort s is formatted $\langle s \rangle$ in the rules.

sigexp	::=	$\langle \cdot \rangle$	empty signature
		sigexp. *sigdec*	signature extension
sigdec	::=	`Def` *opid* $\langle$[*params*]$\rangle$ $\langle$(*args*)$\rangle$: *sortid* = [*term*]	operator definition
		`Tac` *opid* $\langle$[*params*]$\rangle$ $\langle$(*args*)$\rangle$ = [*term*]	tactic definition
		`Thm` *opid* $\langle$[*params*]$\rangle$ $\langle$(*args*)$\rangle$: [*term*] `by` [*term*]	theorem declaration
params	::=	$\langle \cdot \rangle$	empty parameter list
		params, *symbind*	parameter list extension
args	::=	$\langle \cdot \rangle$	empty argument list
		args, *metabind*	argument list extension
symbind	::=	*symid* : *sortid*	symbol binding
metabind	::=	*metaid* : *valence*	metavariable binding
valence	::=	$\langle\langle${*sortlist*}$\rangle$ $\langle$[*sortlist*]$\rangle$.$\rangle$ *sortid*	valence
sortlist	::=	$\langle \cdot \rangle$	empty sort list
		sortlist, *sortid*	sort list extension

Figure 1: Grammar of signature expressions. The identifier sorts *opid*, *sortid*, *symid* and *metaid* can be assumed to be arbitrary strings; the sort *term* is left uninterpreted.

2 Static Semantics

The static semantics for Red JonPRL signatures begins with a specification of the class of *semantic* objects that will serve as the meanings for the *syntactic* objects defined in Section 1. We assume an ambient abstract binding tree signature such that at least the following facts hold:

$$\frac{}{\texttt{tac}\ \mathit{sort}} \qquad \frac{}{\texttt{thm}\ \mathit{sort}} \qquad \frac{}{\texttt{exp}\ \mathit{sort}} \qquad \frac{}{\texttt{opid}\ \mathit{sort}}$$

$$\frac{}{\Upsilon \Vdash \texttt{prove} : (.\,\texttt{exp}, .\,\texttt{tac})\,\texttt{thm}}$$

Then, our semantic objects are defined as in Figure 2.

$$
\begin{array}{rcll}
a, b & \in & \text{Sym} & \\
\mathfrak{m}, \mathfrak{n} & \in & \text{Metavar} & \\
\sigma, \tau & \in & \text{Sort} & \triangleq \{\, \tau \mid \tau \ \mathit{sort} \,\} \\
v & \in & \text{Valence} & \triangleq \{\, v \mid v \ \mathit{valence} \,\} \\
\vartheta & \in & \text{Opid} & \triangleq \text{Sym} \\
\Upsilon & \in & \text{Params} & \triangleq \text{Sym} \rightharpoonup \text{Sort} \\
\Theta & \in & \text{Args} & \triangleq \text{Metavar} \rightharpoonup \text{Valence} \\
M, N & \in & \text{Tm}(\Theta, \Upsilon, \tau) & \triangleq \{\, M \mid \Theta \triangleright \Upsilon \parallel \cdot \vdash M : \tau \,\} \\
D & \in & \text{Decl} & \triangleq \coprod_{\Upsilon, \Theta, \tau} \text{Tm}(\Theta, \Upsilon, \tau) \\
\Sigma & \in & \text{Sig} & \triangleq \text{Opid} \rightharpoonup \text{Decl}
\end{array}
$$

Figure 2: Specification of the semantic objects.

A *natural semantics* hinges on the elaboration judgment $E \vdash A \Longrightarrow A'$, which means that the syntactic object A elaborates to the semantic object A' in the environment E. Let the $\Upsilon_\Sigma \in \text{Params}$ be defined as follows:

$$
\Upsilon_\Sigma(\vartheta) \triangleq \begin{cases} \texttt{opid} & \mathit{if} \quad \vartheta \in \mathbf{dom}(\Sigma) \\ \bot & \mathit{otherwise} \end{cases}
$$

Symbol Bindings

$$\boxed{\Sigma \vdash \mathit{symbind} \Longrightarrow (a, \tau)}$$

$$
\frac{\Sigma \vdash \mathit{symid} \Longrightarrow a \quad \Sigma \vdash \mathit{sortid} \Longrightarrow \tau}{\Sigma \vdash \mathit{symid} : \mathit{sortid} \Longrightarrow (a, \tau)} \tag{1}
$$

Metavariable Bindings

$$\boxed{\Sigma \vdash \mathit{metabind} \Longrightarrow (\mathfrak{m}, v)}$$

$$
\frac{\Sigma \vdash \mathit{metaid} \Longrightarrow \mathfrak{m} \quad \Sigma \vdash \mathit{metaid} \Longrightarrow v}{\Sigma \vdash \mathit{metaid} : \mathit{metaid} \Longrightarrow (\mathfrak{m}, v)} \tag{2}
$$

Parameters

$$\boxed{\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon}$$

$$
\frac{}{\Sigma \vdash \langle\cdot\rangle \Longrightarrow \{\}} \tag{3}
$$

$$
\frac{\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon \quad \Sigma \vdash \mathit{symbind} \Longrightarrow (a, \tau)}{\Sigma \vdash \mathit{params}, \mathit{symbind} \Longrightarrow \Upsilon \cup a \mapsto \tau} \tag{4}
$$

Arguments

$$\boxed{\Sigma \vdash \mathit{args} \Longrightarrow \Theta}$$

$$
\frac{}{\Sigma \vdash \langle\cdot\rangle \Longrightarrow \{\}} \tag{5}
$$

$$
\frac{\Sigma \vdash \mathit{args} \Longrightarrow \Theta \quad \Sigma \vdash \mathit{metabind} \Longrightarrow (\mathfrak{m}, v)}{\Sigma \vdash \mathit{args}, \mathit{metabind} \Longrightarrow \Theta \cup \mathfrak{m} \mapsto v} \tag{6}
$$

Operator Identifiers

$$\boxed{\Sigma \vdash \mathit{opid} \Longrightarrow \vartheta}$$

$$
\frac{\vartheta \notin \mathbf{dom}(\Sigma)}{\Sigma \vdash \mathit{opid} \Longrightarrow \vartheta} \tag{7}
$$

Declarations

$\boxed{\Sigma \vdash \mathit{sigdec} \Longrightarrow (\vartheta, D)}$

$$\frac{\begin{array}{lll}\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon & \Sigma \vdash \mathit{sortid} \Longrightarrow \tau & \Sigma \vdash \mathit{opid} \Longrightarrow \vartheta \\ \Sigma \vdash \mathit{args} \Longrightarrow \Theta & \Sigma \vdash \mathit{term} \Longrightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \tau\end{array}}{\Sigma \vdash \texttt{Def}\ \mathit{opid}\ \langle [\mathit{params}] \rangle\ \langle (\mathit{args}) \rangle : \mathit{sortid} = [\mathit{term}] \Longrightarrow (\vartheta, \langle \Upsilon, \Theta, \tau, M \rangle)} \tag{8}$$

$$\frac{\begin{array}{ll}\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon & \Sigma \vdash \mathit{opid} \Longrightarrow \vartheta \\ \Sigma \vdash \mathit{args} \Longrightarrow \Theta & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \texttt{tac} \\ \Sigma \vdash \mathit{term} \Longrightarrow M & \end{array}}{\Sigma \vdash \texttt{Tac}\ \mathit{opid}\ \langle [\mathit{params}] \rangle\ \langle (\mathit{args}) \rangle = [\mathit{term}] \Longrightarrow (\vartheta, \langle \Upsilon, \Theta, \texttt{tac}, M \rangle)} \tag{9}$$

$$\frac{\begin{array}{lll}\Sigma \vdash \mathit{params} \Longrightarrow \Upsilon & \Sigma \vdash \mathit{term}_1 \Longrightarrow P & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash P : \texttt{exp} \\ \Sigma \vdash \mathit{args} \Longrightarrow \Theta & \Sigma \vdash \mathit{term}_2 \Longrightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \texttt{tac}\end{array} \quad \Sigma \vdash \mathit{opid} \Longrightarrow \vartheta}{\Sigma \vdash \texttt{Thm}\ \mathit{opid}\ \langle [\mathit{params}] \rangle\ \langle (\mathit{args}) \rangle : [\mathit{term}_1]\ \texttt{by}\ [\mathit{term}_2] \Longrightarrow (\vartheta, \langle \Upsilon, \Theta, \texttt{thm}, \texttt{prove}(P; M) \rangle)} \tag{10}$$

Signatures

$\boxed{\vdash \mathit{sigexp} \Longrightarrow \Sigma}$

$$\frac{}{\vdash \langle \cdot \rangle \Longrightarrow \{\}} \tag{11}$$

$$\frac{\vdash \mathit{sigexp} \Longrightarrow \Sigma \quad \Sigma \vdash \mathit{sigdec} \Longrightarrow (\vartheta, D)}{\vdash \mathit{sigexp}.\ \mathit{sigdec} \Longrightarrow \Sigma \cup \vartheta \mapsto D} \tag{12}$$