

Specification of Red JonPRL Signatures

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1 Grammar

The grammar of Red JonPRL signatures is presented in Figure 1. Note that an optional production of sort s is formatted $\langle s \rangle$ in the rules.

| | | | |
|------------|-------|---|--------------------------|
| $sigexp$ | $::=$ | $\langle \cdot \rangle$ | empty signature |
| | | $sigexp. sigdec$ | signature extension |
| $sigdec$ | $::=$ | Def $opid \langle [params] \rangle \langle (args) \rangle : sortid = [term]$ | operator definition |
| | | Tac $opid \langle [params] \rangle \langle (args) \rangle = [term]$ | tactic definition |
| | | Thm $opid \langle [params] \rangle \langle (args) \rangle : [term] \text{ by } [term]$ | theorem declaration |
| $params$ | $::=$ | $\langle \cdot \rangle$ | empty parameter list |
| | | $params, symbind$ | parameter list extension |
| $args$ | $::=$ | $\langle \cdot \rangle$ | empty argument list |
| | | $args, metabind$ | argument list extension |
| $symbind$ | $::=$ | $symid : sortid$ | symbol binding |
| $metabind$ | $::=$ | $metaid : valence$ | metavariable binding |
| $valence$ | $::=$ | $\langle \langle \{ sortlist \} \rangle \langle [sortlist] \rangle . \rangle sortid$ | valence |
| $sortlist$ | $::=$ | $\langle \cdot \rangle$ | empty sort list |
| | | $sortlist, sortid$ | sort list extension |

Figure 1: Grammar of signature expressions. The identifier sorts $opid$, $sortid$, $symid$ and $metaid$ can be assumed to be arbitrary strings; the sort $term$ is left uninterpreted.

2 Static Semantics

The static semantics for Red JonPRL signatures begins with a specification of the class of *semantic* objects that will serve as the meanings for the *syntactic* objects defined in Section 1. We assume an ambient abstract binding tree signature such that at least the following facts hold:

$$\frac{\overline{tac \ sort} \quad \overline{thm \ sort} \quad \overline{exp \ sort} \quad \overline{opid \ sort}}{\Upsilon \Vdash \text{prove} : (. \text{exp}, . \text{tac}) \text{ thm}}$$

Then, our semantic objects are defined as in Figure 2.

| | | | |
|--------------------------|-------|-------------------------------------|--|
| a, b | \in | Sym | |
| \mathbf{m}, \mathbf{n} | \in | Metavar | |
| σ, τ | \in | Sort | $\triangleq \{ \tau \mid \tau \text{ sort} \}$ |
| v | \in | Valence | $\triangleq \{ v \mid v \text{ valence} \}$ |
| ϑ | \in | Opid | $\triangleq \text{Sym}$ |
| Υ | \in | Params | $\triangleq \text{Sym} \rightarrow \text{Sort}$ |
| Θ | \in | Args | $\triangleq \text{Metavar} \rightarrow \text{Valence}$ |
| M, N | \in | $\text{Tm}(\Theta, \Upsilon, \tau)$ | $\triangleq \{ M \mid \Theta \triangleright \Upsilon \parallel \cdot \vdash M : \tau \}$ |
| D | \in | Decl | $\triangleq \coprod_{\Upsilon, \Theta, \tau} \text{Tm}(\Theta, \Upsilon, \tau)$ |
| Σ | \in | Sig | $\triangleq \text{Opid} \rightarrow \text{Decl}$ |

Figure 2: Specification of the semantic objects.

A *natural semantics* hinges on the elaboration judgment $E \vdash A \Longrightarrow A'$, which means that the syntactic object A elaborates to the semantic object A' in the environment E . Let the $\Upsilon_\Sigma \in \text{Params}$ be defined as follows:

$$\Upsilon_\Sigma(\vartheta) \triangleq \begin{cases} \text{opid} & \text{if } \vartheta \in \text{dom}(\Sigma) \\ \perp & \text{otherwise} \end{cases}$$

Symbol Bindings

$$\boxed{\Sigma \vdash \text{symbolbind} \Longrightarrow (a, \tau)}$$

$$\frac{\Sigma \vdash \text{symid} \Longrightarrow a \quad \Sigma \vdash \text{sortid} \Longrightarrow \tau}{\Sigma \vdash \text{symid} : \text{sortid} \Longrightarrow (a, \tau)} \quad (1)$$

Metavariable Bindings

$$\boxed{\Sigma \vdash \text{metabind} \Longrightarrow (\mathbf{m}, v)}$$

$$\frac{\Sigma \vdash \text{metaid} \Longrightarrow \mathbf{m} \quad \Sigma \vdash \text{metaid} \Longrightarrow v}{\Sigma \vdash \text{metaid} : \text{metaid} \Longrightarrow (\mathbf{m}, v)} \quad (2)$$

Parameters

$$\boxed{\Sigma \vdash \text{params} \Longrightarrow \Upsilon}$$

$$\overline{\Sigma \vdash \langle \cdot \rangle \Longrightarrow \{ \}} \quad (3)$$

$$\frac{\Sigma \vdash \text{params} \Longrightarrow \Upsilon \quad \Sigma \vdash \text{symbolbind} \Longrightarrow (a, \tau)}{\Sigma \vdash \text{params}, \text{symbolbind} \Longrightarrow \Upsilon \cup a \mapsto \tau} \quad (4)$$

Arguments

$$\boxed{\Sigma \vdash \text{args} \Longrightarrow \Theta}$$

$$\overline{\Sigma \vdash \langle \cdot \rangle \Longrightarrow \{ \}} \quad (5)$$

$$\frac{\Sigma \vdash \text{args} \Longrightarrow \Theta \quad \Sigma \vdash \text{metabind} \Longrightarrow (\mathbf{m}, v)}{\Sigma \vdash \text{args}, \text{metabind} \Longrightarrow \Theta \cup \mathbf{m} \mapsto v} \quad (6)$$

Declarations

$$\boxed{\Sigma \vdash \text{sigdec} \Rightarrow (\vartheta, D)}$$

$$\frac{\begin{array}{lll} \Sigma \vdash \text{params} \Rightarrow \Upsilon & \Sigma \vdash \text{sortid} \Rightarrow \tau & \Sigma \vdash \text{opid} \Rightarrow \vartheta \\ \Sigma \vdash \text{args} \Rightarrow \Theta & \Sigma \vdash \text{term} \Rightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \tau \end{array}}{\Sigma \vdash \text{Def opid} \langle [\text{params}] \rangle \langle (\text{args}) \rangle : \text{sortid} = [\text{term}] \Rightarrow (\vartheta, \langle \Upsilon, \Theta, \tau, M \rangle)} \quad (7)$$

$$\frac{\begin{array}{ll} \Sigma \vdash \text{params} \Rightarrow \Upsilon & \Sigma \vdash \text{opid} \Rightarrow \vartheta \\ \Sigma \vdash \text{args} \Rightarrow \Theta & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \text{tac} \\ \Sigma \vdash \text{term} \Rightarrow M & \end{array}}{\Sigma \vdash \text{Tac opid} \langle [\text{params}] \rangle \langle (\text{args}) \rangle = [\text{term}] \Rightarrow (\vartheta, \langle \Upsilon, \Theta, \text{tac}, M \rangle)} \quad (8)$$

$$\frac{\begin{array}{llll} \Sigma \vdash \text{params} \Rightarrow \Upsilon & \Sigma \vdash \text{term}_1 \Rightarrow P & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash P : \text{exp} & \Sigma \vdash \text{opid} \Rightarrow \vartheta \\ \Sigma \vdash \text{args} \Rightarrow \Theta & \Sigma \vdash \text{term}_2 \Rightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \text{tac} & \end{array}}{\Sigma \vdash \text{Thm opid} \langle [\text{params}] \rangle \langle (\text{args}) \rangle : [\text{term}_1] \text{ by } [\text{term}_2] \Rightarrow (\vartheta, \langle \Upsilon, \Theta, \text{thm}, \text{prove}(P; M) \rangle)} \quad (9)$$

Signatures

$$\boxed{\vdash \text{sigexp} \Rightarrow \Sigma}$$

$$\overline{\vdash \langle \cdot \rangle \Rightarrow \{ \}} \quad (10)$$

$$\frac{\vdash \text{sigexp} \Rightarrow \Sigma \quad \Sigma \vdash \text{sigdec} \Rightarrow (\vartheta, D)}{\vdash \text{sigexp. sigdec} \Rightarrow \Sigma \cup \vartheta \mapsto D} \quad (11)$$