

The Definition of **Red**PRL,  
*the people's refinement logic*

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April 26, 2016

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# Chapter 1

## Signatures

*Decisively Smash The Formalist  
Clique!*

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Chairman Jon

A *signature* is a collection of definitions, including terms, tactics and theorems.

### 1.1 Grammar

The grammar of **Red**PRL signatures is presented in Figure 1.1. Note that an optional production of sort  $s$  is formatted  $\langle s \rangle$  in the rules.

$sigexp$	$::=$	$\langle \cdot \rangle$ $sigexp\ sigdec.$ $sigexp\ symdec.$	empty signature signature extension
$sigdec$	$::=$	$\text{Def } opid \langle [params] \rangle \langle (args) \rangle : sortid = [term]$ $\text{Tac } opid \langle [params] \rangle \langle (args) \rangle = [term]$ $\text{Thm } opid \langle [params] \rangle \langle (args) \rangle : [term] \text{ by } [term]$	operator definition tactic definition theorem declaration
$symdec$	$::=$	$\text{Sym } symbind$	symbol declaration
$params$	$::=$	$\langle \cdot \rangle$ $params, symbind$	empty parameter list parameter list extension
$args$	$::=$	$\langle \cdot \rangle$ $args, metabind$	empty argument list argument list extension
$symbind$	$::=$	$symid : sortid$	symbol binding
$metabind$	$::=$	$metaid : valence$	metavariable binding
$valence$	$::=$	$\langle \langle \{ sortlist \} \rangle \langle [sortlist] \rangle . \rangle sortid$	valence
$sortlist$	$::=$	$\langle \cdot \rangle$ $sortlist, sortid$	empty sort list sort list extension

Figure 1.1: Grammar of signature expressions. The identifier sorts  $opid$ ,  $sortid$ ,  $symid$  and  $metaid$  can be assumed to be arbitrary strings; the sort  $term$  is left uninterpreted.

## 1.2 Static Semantics

The static semantics for **RedPRL** signatures begins with a specification of the class of *semantic* objects that will serve as the meanings for the *syntactic* objects defined in Section 1.1. We assume an ambient abstract binding tree signature such that at least the following facts hold:

$$\frac{\overline{\text{tac sort}} \quad \overline{\text{thm sort}} \quad \overline{\text{exp sort}} \quad \overline{\text{opid sort}}}{\Upsilon \Vdash \text{prove} : (\text{exp}, \text{tac}) \text{ thm}}$$

Then, our semantic objects are defined as in Figure 1.2.

$$\begin{array}{llll} a, b & \in & \text{Sym} \\ \mathbf{m}, \mathbf{n} & \in & \text{Metavar} \\ \sigma, \tau & \in & \text{Sort} & \triangleq \{ \tau \mid \tau \text{ sort} \} \\ v & \in & \text{ProdValence} & \triangleq \{ v \mid v \text{ valence} \} \\ \vartheta & \in & \text{Opid} & \triangleq \text{Sym} \\ \Upsilon & \in & \text{Params} & \triangleq \text{Sym} \xrightarrow{\text{fin}} \text{Sort} \\ \Theta & \in & \text{Args} & \triangleq \text{Metavar} \xrightarrow{\text{fin}} \text{ProdValence} \\ M, N & \in & \text{Tm}(\Theta, \Upsilon, \tau) & \triangleq \{ M \mid \Theta \triangleright \Upsilon \parallel \cdot \vdash M : \tau \} \\ D & \in & \text{Decl} & \triangleq \coprod_{\Upsilon, \Theta, \tau} \text{Tm}(\Theta, \Upsilon, \tau) \\ \Sigma & \in & \text{Sig} & \triangleq \left( \text{Opid} \xrightarrow{\text{fin}} \text{Decl} \right) \cap \left( \text{Sym} \xrightarrow{\text{fin}} \text{Sort} \right) \end{array}$$

Figure 1.2: Specification of the semantic objects.

A *natural semantics* hinges on the elaboration judgment  $E \vdash A \Longrightarrow A'$ , which means that the syntactic object  $A$  elaborates to the semantic object  $A'$  in the environment  $E$ . Let the  $\Upsilon_\Sigma \in \text{Params}$  be defined as follows:

$$\Upsilon_\Sigma(u) \triangleq \begin{cases} \text{opid} & \text{if } u \equiv \vartheta \in \text{dom}(\Sigma) \\ \tau & \text{if } \Sigma(u) \equiv \tau \\ \perp & \text{otherwise} \end{cases}$$

### Symbol Bindings

$$\boxed{\Sigma \vdash \text{symbind} \Longrightarrow (a, \tau)}$$

$$\frac{\Sigma \vdash \text{symid} \Longrightarrow a \quad \Sigma \vdash \text{sortid} \Longrightarrow \tau}{\Sigma \vdash \text{symid} : \text{sortid} \Longrightarrow (a, \tau)} \quad (1.1)$$

### Metavariable Bindings

$$\boxed{\Sigma \vdash \text{metabind} \Longrightarrow (\mathbf{m}, v)}$$

$$\frac{\Sigma \vdash \text{metaid} \Longrightarrow \mathbf{m} \quad \Sigma \vdash \text{valence} \Longrightarrow v}{\Sigma \vdash \text{metaid} : \text{valence} \Longrightarrow (\mathbf{m}, v)} \quad (1.2)$$

### Parameters

$$\boxed{\Sigma \vdash \text{params} \Longrightarrow \Upsilon}$$

$$\overline{\Sigma \vdash \langle \cdot \rangle \Longrightarrow \{ \}} \quad (1.3)$$

$$\frac{\Sigma \vdash \text{params} \Longrightarrow \Upsilon \quad \Sigma \vdash \text{symbind} \Longrightarrow (a, \tau)}{\Sigma \vdash \text{params}, \text{symbind} \Longrightarrow \Upsilon \cup a \mapsto \tau} \quad (1.4)$$

## Arguments

$$\boxed{\Sigma \vdash \text{args} \Rightarrow \Theta}$$

$$\overline{\Sigma \vdash \langle \cdot \rangle \Rightarrow \{ \}} \quad (1.5)$$

$$\frac{\Sigma \vdash \text{args} \Rightarrow \Theta \quad \Sigma \vdash \text{metabind} \Rightarrow (\mathbf{m}, v)}{\Sigma \vdash \text{args}, \text{metabind} \Rightarrow \Theta \cup \mathbf{m} \mapsto v} \quad (1.6)$$

## Symbols

$$\boxed{\Sigma \vdash \text{symid} \Rightarrow u}$$

$$\frac{u \notin \text{dom}(\Sigma)}{\Sigma \vdash \text{symid} \Rightarrow u} \quad (1.7)$$

## Operator Declarations

$$\boxed{\Sigma \vdash \text{sigdec} \Rightarrow (\vartheta, D)}$$

$$\frac{\begin{array}{ccc} \Sigma \vdash \text{params} \Rightarrow \Upsilon & \Sigma \vdash \text{sortid} \Rightarrow \tau & \Sigma \vdash \text{opid} \Rightarrow \vartheta \\ \Sigma \vdash \text{args} \Rightarrow \Theta & \Sigma \vdash \text{term} \Rightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \tau \end{array}}{\Sigma \vdash \text{Def opid} \langle [\text{params}] \rangle \langle (\text{args}) \rangle : \text{sortid} = [\text{term}] \Rightarrow (\vartheta, \langle \Upsilon, \Theta, \tau, M \rangle)} \quad (1.8)$$

$$\frac{\begin{array}{ccc} \Sigma \vdash \text{params} \Rightarrow \Upsilon & & \Sigma \vdash \text{opid} \Rightarrow \vartheta \\ \Sigma \vdash \text{args} \Rightarrow \Theta & \Sigma \vdash \text{opid} \Rightarrow \vartheta & \\ \Sigma \vdash \text{term} \Rightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \text{tac} & \end{array}}{\Sigma \vdash \text{Tac opid} \langle [\text{params}] \rangle \langle (\text{args}) \rangle : [\text{term}] \Rightarrow (\vartheta, \langle \Upsilon, \Theta, \text{tac}, M \rangle)} \quad (1.9)$$

$$\frac{\begin{array}{ccc} \Sigma \vdash \text{params} \Rightarrow \Upsilon & \Sigma \vdash \text{term}_1 \Rightarrow P & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash P : \text{exp} \\ \Sigma \vdash \text{args} \Rightarrow \Theta & \Sigma \vdash \text{term}_2 \Rightarrow M & \Theta \triangleright \Upsilon_\Sigma \oplus \Upsilon \parallel \cdot \vdash M : \text{tac} \end{array} \quad \Sigma \vdash \text{opid} \Rightarrow \vartheta}{\Sigma \vdash \text{Thm opid} \langle [\text{params}] \rangle \langle (\text{args}) \rangle : [\text{term}_1] \text{ by } [\text{term}_2] \Rightarrow (\vartheta, \langle \Upsilon, \Theta, \text{thm}, \text{prove}(P; M) \rangle)} \quad (1.10)$$

## Symbol Declarations

$$\boxed{\Sigma \vdash \text{symdec} \Rightarrow (u, \sigma)}$$

$$\frac{\Sigma \vdash \text{symbind} \Rightarrow (u, \sigma)}{\Sigma \vdash \text{Sym symbind} \Rightarrow (u, \sigma)} \quad (1.11)$$

## Signatures

$$\boxed{\vdash \text{sigexp} \Rightarrow \Sigma}$$

$$\overline{\vdash \langle \cdot \rangle \Rightarrow \{ \}} \quad (1.12)$$

$$\frac{\vdash \text{sigexp} \Rightarrow \Sigma \quad \Sigma \vdash \text{sigdec} \Rightarrow (\vartheta, D)}{\vdash \text{sigexp sigdec.} \Rightarrow \Sigma \cup \vartheta \mapsto D} \quad \frac{\vdash \text{sigexp} \Rightarrow \Sigma \quad \Sigma \vdash \text{symdec} \Rightarrow (u, \sigma)}{\vdash \text{sigexp symdec.} \Rightarrow \Sigma \cup u \mapsto \sigma} \quad (1.13)$$

## Chapter 2

# Nominal LCF: a language for tactics

This chapter needs to be re-written in light of changes to the Nominal LCF formalism.

# Bibliography