CSC 225 Assignment 1

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Question 1

A)$\theta(log^2n)$

B)$\theta(nlogn)$

Question 2

A)

Function LargestMissing(A, n, k)
$B[] \leftarrow k$
for $i \leftarrow 0, 1, 2, \ldots, n$ **do**
  $B[A[i]] \leftarrow A[i]$
end for
for $i \leftarrow k, k-1, k-2, \ldots, 0$ **do**
  if $j == 0$ **then**
    $LM \leftarrow B[j]$
  end if
end for
return LM

This algorithm works because by creating a new array that is the size of the largest number we can put each number as we go through the original array in that numbers corresponding index in the new array. By doing this we get a new array of size k that is sorted and has 0 in all positions of missing numbers in the time of O(n). Therefore finding the largest missing number we just look through the new array backwards to find the first 0. This works because the array given will always have numbers from 1 to k and this will have O(k). This means that it will have an overall run time of O(n+k)

B)

Function LargestMissing(A, n, k)
Sort(A)
if A[n-1] != k **then**
  return k
end if
if A[0] != 1 **then**
  return 1
end if
for $i \leftarrow n-1, n-2, \ldots, 1$ **do**
  if A[i] != A[i-1] and (A[i]-1) != A[i-1] **then**
    $LM \leftarrow (A[i] - 1)$
  end if
end for
return LM

C) Any algorithm for LargestMissingNumber must be Ω(n) in worst case because it will always have to go through the list of length n at least one time to find the largest missing number.

Question 3

Consider the recusance 2T(n - 1) + 3n + 1 if n ¿= 1

While T(n) = 10 if n = 0

2T(n-1) + 3n + 1

2[2T(n-2) + 3(n-1) + 1] + 3n + 1

$=2^2$T(n-2) + (2)(3)(n-1) + 3n + 2 +1

2[2^2T(n-2) + (2)(3)(n-1) + 3n + 2 +1] +3n + 1

$=2^3$T(n-3) + (2^2)(3)(n-2) + (2)(3)(n - 1) + 3n + 2^2 + 2 +1

2^iT(n - i) + (2^{i-1})(3)(n - (i - 1)) + (2^{i-2})(3)(n - (i - 2)) + 3n + 2^{i-1} + 2^{i-2} + 2^{i-3}

= 2^iT(n - i) + $3\sum_{j=0}^{i-1} 2^j(n-j)$ + $\sum_{k=0}^{i-1} 2^k$

Need n = 0 for T(n) to be 10 Therefore i = n

= 2^nT(n - n) + $3\sum_{j=0}^{n-1} 2^j(n-j)$ + $\sum_{k=0}^{n-1} 2^k$

= $2^n(10)$ + $3\sum_{j=0}^{n-1} 2^j(n-j)$ + $\sum_{k=0}^{n-1} 2^k$