Jonathan Han Test 4 Part 1

[1] $= \{p \wedge {\sim}q \to r,\ p \vee q,\ q \to p,\ {\sim}r\}$

$= \{{\sim}(p \wedge {\sim}q) \vee r,\ p \vee q,\ {\sim}q \vee p,\ {\sim}r\}$

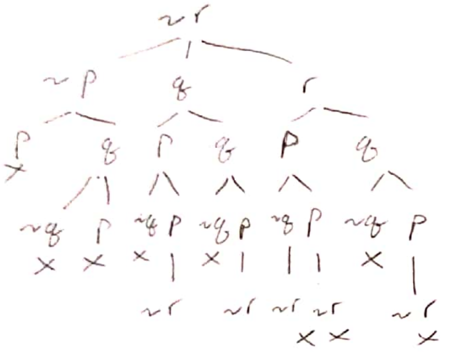
$= \{{\sim}p \vee q \vee r,\ p \vee q,\ {\sim}q \vee p,\ {\sim}r\}$

${\sim}p \vee q \vee r$

$p \vee q$

${\sim}q \vee p$

${\sim}r$



The set is consistent and thus the argument is invalid.

$q, p, {\sim}r = T$

p	q	${\sim}r$	$p \wedge {\sim}q \to r$	$p \vee q$	$q \to p$	r
T	T	T	T	T	T	F

[2] Part 1

$3x - 2 > 4$

$3x > 6$

$x > 2$

a) $x > 2,\ x \in \mathbb{Z}$

b) $x \leq 2,\ x \in \mathbb{Z}$

Part 2

a) Less than two of my library books are overdue.

b) Neither of my two friends misplaced their homework assignments.

c) Someone expected that to happen.

[3] Proof by PMI :

Let $P(n)$ be the predicate :

"$1+5+9+\cdots+(4n-3)=2n^2-n$"

Base case :

We must show that $P(1)$ holds, see that $P(1)=4(1)-3=1$ and $2(1)^2-1=1$.
Since both equations equal to 1, our base case holds and $P(1)$ is true.

Inductive step :

We must show that $P(k)\to P(k+1)$, $\forall k\in\mathbb{Z}^+$ is true.
Assume $P(k)$. Thus $P(k)=2k^2-k$.
We must show that the k+1 element satisfies $2(k+1)^2-(k+1)$.
Take a look at $P(k+1)=1+5+9+\cdots+(4k-3)+(4(k+1)-3)$

$$\begin{aligned}
&= 2k^2-k+4k+4-3\\
&= 2k^2+4k+4-k-3\\
&= 2k^2+4k+2+2-k-3\\
&= 2(k^2+2k+1)-k-1\\
&= 2(k+1)^2-(k+1)
\end{aligned}$$

See that $P(k+1)$ holds and the k+1 element satisfies $2(k+1)^2-(k+1)$.
Thus, by the principle of mathematical induction, the original predicate is true for all n in $\mathbb{Z}^+$.

[4] Proof by WOP

Let $P(n)$ be the predicate "$4 | 5^n - 1$ for every nonnegative integer n".
Now, suppose $P(n)$ is false.

Let X be a set such that $X = \{n \in \mathbb{Z}^+ \mid 4 \nmid 5^n - 1\}$. See that $X \neq \phi$ and $X \subseteq \mathbb{Z}^+$.

By the well ordering principle, X must contain a least element x^*. See that $x^* \neq 1$ since $5^1 - 1 = 4$ and $4 | 4$. Thus $P(1)$ is true. This means $x^* > 1$ and $x^* - 1 > 0$ and $x^* - 1 \in \mathbb{Z}^+$. Thus $P(x^* - 1)$ is true. This means $4 | 5^{x^*-1} - 1$ or $5^{x^*-1} - 1 = 4k$, for some $k \in \mathbb{Z}^+$

Take a look at
$$
\begin{aligned}
P(x^*) &= 5^{x^*} - 1 \\
&= 5(5^{x^*-1} - \tfrac{1}{5}) \\
&= 5(5^{x^*-1} - 1 + 1 - \tfrac{1}{5}) \\
&= 5(4k + 1 - \tfrac{1}{5}) \\
&= 20k + 5 - 1 \\
&= 20k + 4 \\
&= 4(5k + 1)
\end{aligned}
$$

See that $5k + 1$ is an integer since integers have closure under addition and multiplication. By the definition of divisibility, $4 | P(x^*)$. Thus, $X = \phi$ and this is a contradiction. And the original statement is true. ∎