Numerical Analysis Project2

数学与应用数学 2002 王锦宸

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B

when n = 2,
the polynomial is:
$0.03846 + 0.19231(x + 5) - 0.03846(x + 5)x$
when n = 4,
the polynomial is:
$0.03846+0.03979(x+5)+0.06101(x+5)(x+2.5)-0.02653(x+5)(x+2.5)x+0.00531(x+5)(x+2.5)x(x-2.5)$
when n = 6,
the polynomial is:
$0.03846+0.02646(x+5)+0.02485(x+5)(x+3.33333)+0.01494(x+5)(x+3.33333)(x+1.66667)-0.01317(x+5)(x+3.33333)(x+1.66667)x+0.00420(x+5)(x+3.33333)(x+1.66667)x(x-1.66667)-0.00084(x+5)(x+3.33333)(x+1.66667)x(x-1.66667)(x-3.33333)$
when n = 8,
the polynomial is:
$0.03846+0.02234(x+5)+0.01396(x+5)(x+3.75)+0.01170(x+5)(x+3.75)(x+2.5)+0.00067(x+5)(x+3.75)(x+2.5)(x+1.25)-0.00490(x+5)(x+3.75)(x+2.5)(x+1.25)x+0.00244(x+5)(x+3.75)(x+2.5)(x+1.25)x(x-1.25)-0.00069(x+5)(x+3.75)(x+2.5)(x+1.25)x(x-1.25)(x-2.5)+0.00014(x+5)(x+3.75)(x+2.5)(x+1.25)x(x-1.25)(x-2.5)(x-3.75)$

The plot is drawn by ***GEOGEBRA*** as follows.

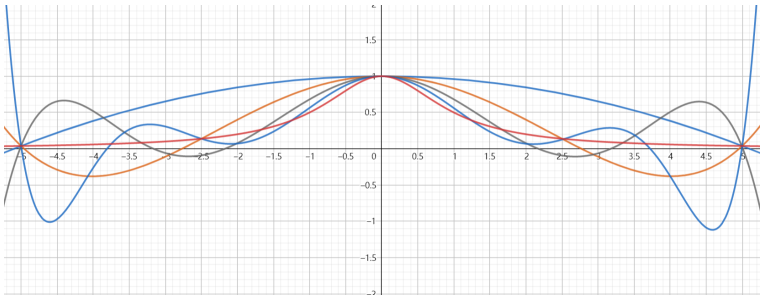

图 1: The Plot of Runge phenomenon

C

Since the polynomial is too long, we only display the plot. In this Problem, we choose the interpolating points to be the zeros of Chebyshev polynomials Tn,

$$x_k = cos\left(\frac{2k-1}{2n}\right) \qquad k = 1, \ldots, n$$

The plot is displayed as follows.

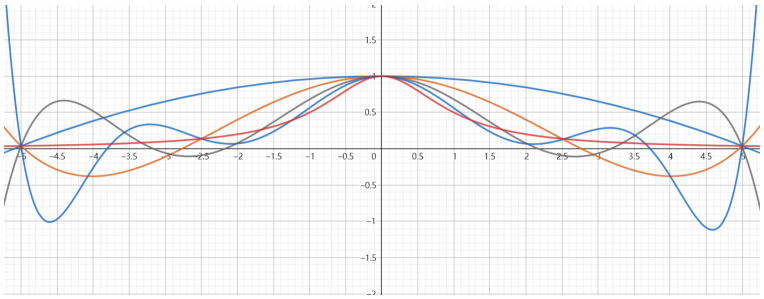

图 2: The Plot of Runge phenomenon

D

(a) f(10) = 742.5 and f'(10) = 48.38

(b) As is displayed in the plot below, we can easily find that car has exceeded the speed limit.

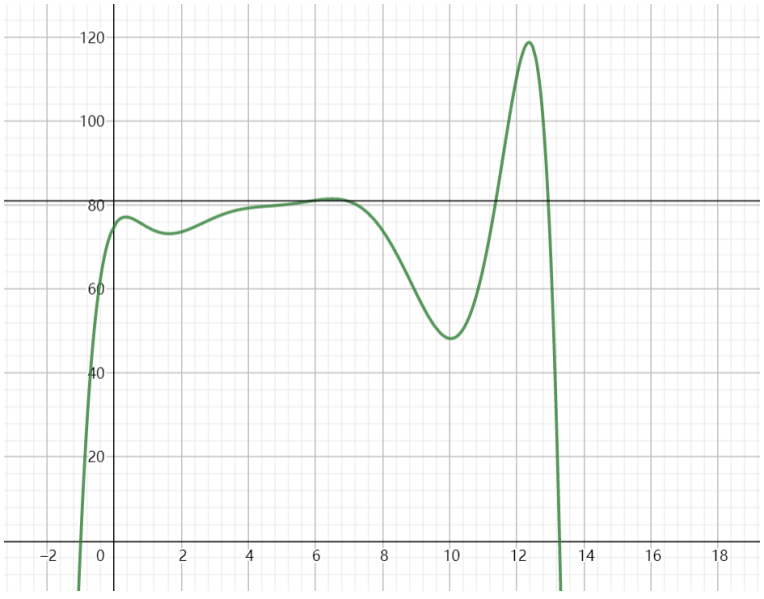

图 3: Speed Curve

E

(a)$p_1(x) = 6.67 + 1.77167 * x + 0.45783 * x(x-6) - 0.12478 * x(x-6)(x-10) + 0.01357 * x(x-6)(x-10)(x-13) - 0.00098 * x(x-6)(x-10)(x-13)(x-17) + 0.00004 * x(x-6)(x-10)(x-13)(x-17)(x-20)$

$p_2(x) = 6.67 + 1.57167 * x - 0.08717 * x(x-6) - 0.01527 * x(x-6)(x-10) + 0.00258 * x(x-6)(x-10)(x-13) - 0.00020 * x(x-6)(x-10)(x-13)(x-17) + 0.00001 * x(x-6)(x-10)(x-13)(x-17)(x-20)$

(b)According to the plot as follows, the larvae will be still alive in day43 and will even be immortal.

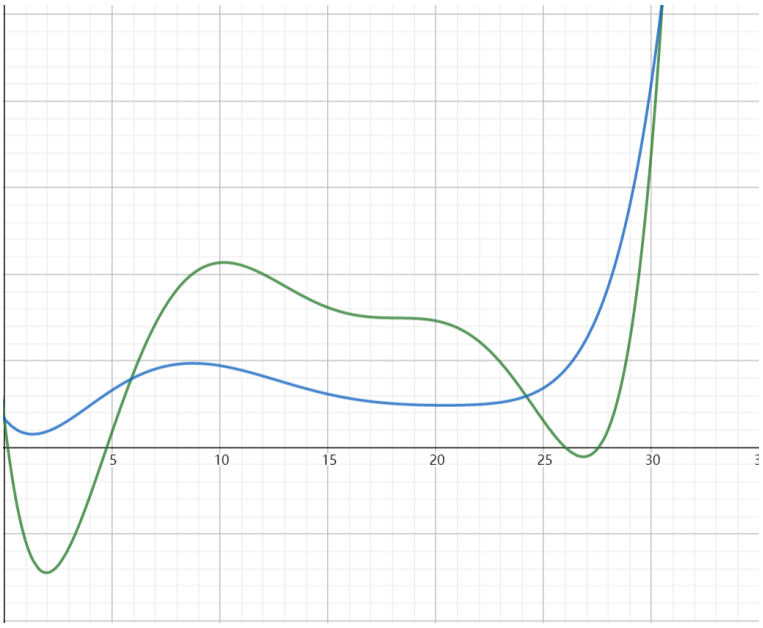

图 4: Larvae Curve