HW1 Numerical Analysis 22Fall

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20220922

Abstract

The solution for 1.8.1 theoretical problems.

I.

- **What is the width of the interval at the nth step?**

Solution: The width of the interval at the nth step is $\frac{1}{2^n}$.

- **What is the maximum possible distance between the root r and the midpoint of the interval?**

Solution: The maximum possible distance is 1.

II.

Solution: Let x_0 be the root of $f(x)$, at the nth step,

$$\frac{b_0 - a_0}{2^n}/x_0 < \frac{b_0 - a_0}{2^n}/a_0 < \epsilon$$

Then we have,

$$\begin{aligned} \frac{b_0 - a_0}{a_0 \epsilon} &< 2^n \\ log(b_0 - a_0) - log(\epsilon) - log(a_0) &< n \\ log(b_0 - a_0) - log(\epsilon) - log(a_0) - 1 &\leq n \end{aligned}$$

III.

We have $f'(x) = 12x^2 - 4x$. Then we can easily establish a frame,

.	x	f(x)	f'(x)
0	-1	-3	-16
1	-0.8125	-0.4658	11.1719
2	-0.7708	-0.0201	10.2128
3	-0.7688	0.0003	10.1678
4	-0.76883	$-1.95\text{E} - 5$	10.1685
Result	-0.768828	.	.

IV.

$$e_n = f(x_n) - 0 = f(x_n)$$

According to Taylor expansion of order 1, we have,

$$f(x) = f(x_n) + (x - x_n)f'(x) + O(x - x_n)$$

Thus,

$$e_{n+1} = f(x_n + 1) = f(x_n) + (x_{n+1} - x_n)f'(x_n) + O(x - x_n)$$

$$e_{n+1} = f(x_n) - \frac{f(x_n)}{f(x_0)}f'(x_n) + O(x - x_n)$$

$$e_{n+1} = f(x_n) - \frac{f'(x_n)}{f(x_0)}f(x_n) + O(x - x_n)$$

$$e_{n+1} = (1 - \frac{f'(x_n)}{f(x_0)})e_n$$

$$e_{n+1} = Ce_n^s$$

where $C = (1 - \frac{f'(x_n)}{f(x_0)}), s = 1$

V.

Within $(-\frac{\pi}{2}, \frac{\pi}{2})$, the iteration $x_{n+1} = tan^{-1}(x_n)$ will converge.
Since $tan^{-1}(x) < x, x_{n+1} < x_n$.Since x_n is a monotonically decreasing sequence and it's bounded, it converges.Since $tan^{-1}(0) = 0$, it comes to a stable point.
Thus it converges to 0.

VI.

We can easily derive the iteration formula: $x_{n+1} = \frac{1}{p+x_n}$ and $x_1 = \frac{1}{p}$.
Thus, we can solve the equation:

$$x = \frac{1}{x}$$

$$x_{1,2} = \frac{-p \pm \sqrt{p^2 + 4}}{2}$$

Since $x_n > x_{n+1}$ and $x_1 > 0$, we only reserve the positive root of the equation above.
Since x_n is a monotonically decreasing sequence and it's bounded, by Monotonic Theorem, it converges to $\frac{-p+\sqrt{p^2+4}}{2}$

VII.

Since $a < 0$, $log(a)$ doesn't hold.But We still have $\frac{b_0-a_0}{2^n}/x_0 < \epsilon$.
Thus,

$$n \geq log(b_0 - a_0) - log(\epsilon) - 1$$