CS 188 Spring 2022 Introduction to Artificial Intelligence — Written HW 5

Due: Friday 03/04/2022 at 10:59pm (submit via Gradescope).

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: It is recommended that your submission be a PDF that matches this template. You may also fill out this template digitally (e.g. using a tablet). **However, if you do not use this template, you will still need to write down the below four fields on the first page of your submission.**

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Collaborators	

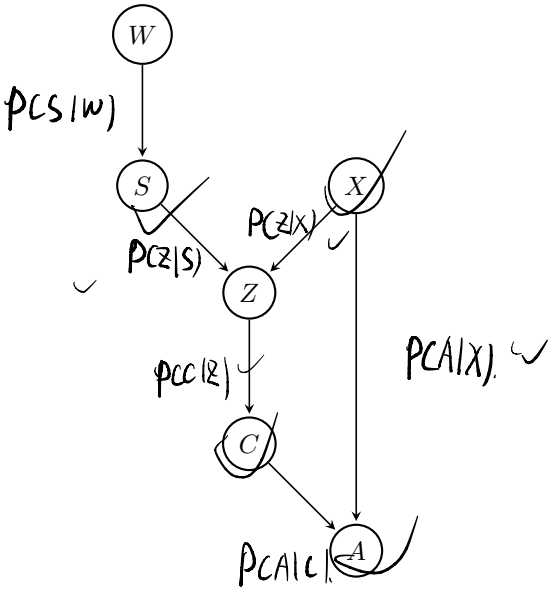
For staff use only:

Q1. Quadcopter: Spectator	/30
Total	/30

Q1. [30 pts] Quadcopter: Spectator

Flying a quadcopter can be modeled using a Bayes Net with the following variables:

- W (weather) $\in$ {clear, cloudy, rainy}
- S (signal strength) $\in$ {strong, medium, weak}
- X (true position) $= (x, y, z, \theta)$ where x, y, z **each** can take on values $\in \{0, 1, 2, 3, 4\}$ and θ can take on values $\in \{0°, 90°, 180°, 270°\}$ $5 \times 5 \times 5 \times 4$
- Z (reading of the position) $= (x, y, z, \theta)$ where x, y, z **each** can take on values $\in \{0, 1, 2, 3, 4\}$ and θ can take on values $\in \{0°, 90°, 180°, 270°\}$
- C (control from the pilot) $\in$ {forward, backward, rotate left, rotate right, ascend, descend} (6 controls in total)
- A (smart alarm to warn pilot if that control could cause a collision) $\in$ {bad, good}



(a) Representation

(i) [3 pts] What is N_x, where N_x is the domain size of the variable X? Please explain your answer.

Answer: $N_x =$ 4×5^3

Explanation: x, y, z can be $\{0, 1, 2, 3, 4\}$, which own five possible choices each. and θ can be $\{0, 90°, 180°, 270°\}$ which own four possible choices

(ii) [4 pts] Please list **all** of the Conditional Probability Tables that are needed in order to represent the Bayes Net above. Note that there are 6 of them.

W	P(W)

X	P(X)

S	W	P(S|W)

Z	S	X	P(Z|S,X)

C	Z	P(C|Z)

A	C	X	P(A|C)

$P(Z \mid S, W)$

(iii) [3 pts] What is the size of the Conditional Probability Table for Z? You may use N_x in your answer.

$3N_X \times N_X = 3N_X^2$

Now, assume that we look at this setup from the perspective of Spencer – a spectator who can observe A and W. Spencer observes A=bad and W=clear, and he now wants to infer the signal strength. In BN terminology, he wants to calculate $P(S|A = \text{bad}, W = \text{clear})$.

(b) [5 pts] Inference by Enumeration

If Spencer chooses to solve for this quantity using inference by enumeration, what are the different probability terms that need to be multiplied together in the summation?

$P(S, Z, C, X, A=\text{bad}, W=\text{clear})$

hidden: S, Z, C, X evidence: A, W

there are 6 item need to multiply

(c) [15 pts] Inference by Variable Elimination

Spencer chooses to solve for this quantity by performing variable elimination in the order of $Z - X - C$. Answer the following prompts to work your way through this procedure.

(1a) First, we need to eliminate Z. Which factors (from the 6 CPTs above) are involved?

$P(Z \mid S, X)$ $P(C \mid Z)$

(1b) Describe how you eliminate the variable of interest by multiplication of those factors. What conditional probability **factor** results from this step?

$f_1(C \mid X, S) = \sum_Z P(Z \mid S, X) \cdot P(C \mid Z)$

(2a) Second, we need to eliminate X. Which factors are involved?

$f_1(C \mid X, S)$, $P(X)$ $P(A \mid X, C)$

(2b) Describe how you eliminate the variable of interest by multiplication of those factors. What conditional probability **factor** results from this step?

$$f_2(C, A|S) = \sum_X f_1(C|X,S)\, P(A|X,C)\, P(X)$$

(3a) Third, we need to eliminate C. Which factor/s are involved?

$$f_2(C, A|S)$$

(3b) Describe how you eliminate the variable of interest by multiplication of those factors. What conditional probability **factor** results from this step?

$$f_3(A|S) = \sum_C f(C, A|S)$$

(4) List the 3 conditional probability factors that you calculated as a result of the 3 elimination steps above, along with their domain sizes. You may use N_x in your answer. Which factor is the biggest? Is this bigger or smaller than the biggest factor from the "inference by enumeration" approach?

$f_1(C|X,S)$: $6 \times N_X \times 3 = 18N_X$

$f_2(C, A|S)$: $6 \times 2 \times 3 = 36$

$f_3(A|S)$: $2 \times 3 = 6$

$f(C|X,S)$ is the biggest this is small than inference by enumeration

(5) List the **1** unused conditional probability factor from the 3 that you calculated above, and also list the **2** resulting conditional probability factors from the 6 original CPTs.

$P(C|W)$

$P(A|S)$, $P(W)$

(6) Finally, let's solve for the original quantity of interest: $P(S|A = \text{bad}, W = \text{clear})$. After writing the equations to show how to use the factors from (5) in order to solve for $f(S|A = bad, W = clear)$, don't forget to write how to turn that into a probability $P(S|A = bad, W = clear)$.
Hint: use the definition of conditional probability, and use the 3 resulting factors that you listed in the previous question.

$$P(S|A=bad, W=clear) = \frac{P(S|W) \cdot P(W) \cdot f_3(A=bad|S)}{P(W=clear) \cdot \sum_S f_3(A=bad|S)}$$