CS 188 Spring 2022 | Introduction to Artificial Intelligence | **Written HW 3**

Due: Wednesday, February 16 at 10:59pm (submit via Gradescope).

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	Qingjing
Last name	Zhang
SID	3037581096
Collaborators	None.

For staff use only:

Q1.	Propositional Logic	/40
	Total	/40

Q1. [40 pts] Propositional Logic

(a) Consider a propositional language with 4 symbols: A, B, C, D. For each of the following sentences, mark how many models satisfy the sentence out of the 16 possible models.

(i) [3 pts] $\alpha_1 = A \vee B$: $\boxed{12}$

4×3

(ii) [3 pts] $\alpha_2 = (A \wedge B) \Rightarrow C$: $\boxed{14}$

T → F

false case A B C D.

T T F × 2

16 − 2 = 14

(iii) [4 pts] $\alpha_3 = (A \wedge B) \vee (\neg C \vee D)$: $\boxed{13}$

or

$\alpha_3 = F$

$(A \wedge B) = F \Rightarrow 3$

$(\neg C \vee D) = F \Rightarrow 1$

A

(b) Suppose there are three chairs in a row, labeled L(eft), M(iddle), R(ight) and three persons A, B, and C. Everyone has to sit down but, unfortunately,

- A doesn't want to sit next to B
- A doesn't want to sit in the left chair
- C doesn't want to sit to the (adjacent) right of B

We will formulate these constraints in propositional logic using only variable $X_{p,c}$ to mean that person p sits in chair c. **Please express the constraints in CNF.**

(i) [2 pts] A doesn't want to sit next to B.

$\neg [(X_{AL} \wedge X_{BM}) \vee (X_{AM} \wedge (X_{BL} \vee X_{BR})) \vee (X_{AR} \wedge X_{BM})]$

$\neg (X_{AM} \vee X_{BM})$

$= \neg X_{AM} \wedge \neg X_{BM}$

(ii) [1 pt] A doesn't want to sit in the left chair.

$\neg (X_{AL} \wedge ((X_{BM} \wedge X_{CR}) \vee (X_{BR} \wedge X_{CM})))$

$= \neg X_{AL} \vee ((\neg X_{BM} \vee \neg X_{CR}) \wedge (\neg X_{BR} \vee \neg X_{CM}))$

$= (\neg X_{AL} \vee \neg X_{BM} \vee \neg X_{CR}) \wedge (\neg X_{AL} \vee \neg X_{BR} \vee \neg X_{CM})$

(iii) [1 pt] C doesn't want to sit to the (adjacent) right of B.

$\neg ((X_{BL} \wedge X_{CM}) \vee (X_{BM} \wedge X_{CR}))$

$= (\neg X_{BL} \vee \neg X_{CM}) \wedge (\neg X_{BM} \vee \neg X_{CR})$

(iv) [4 pts] Besides the constraints from the first 3 parts, are there any other constraints needed to solve the problem? If yes, express them in propositional logic, including only the minimum number of constraints needed to solve the problem.

$$(X_{AL} \wedge \neg X_{BL} \wedge \neg X_{CL}) \vee (\neg X_{AL} \wedge X_{BL} \wedge \neg X_{CL}) \vee (\neg X_{AL} \wedge \neg X_{BL} \wedge X_{CL})$$

$$\vee (X_{AM} \wedge \neg X_{BM} \wedge \neg X_{CM}) \vee (\neg X_{AM} \wedge X_{BM} \wedge \neg X_{CM}) \vee (\neg X_{AM} \wedge \neg X_{BM} \wedge X_{CM})$$

$$\vee (X_{AR} \wedge \neg X_{BR} \wedge \neg X_{CR}) \vee (\neg X_{AR} \wedge X_{BR} \wedge \neg X_{CR}) \vee (\neg X_{AR} \wedge \neg X_{BR} \wedge X_{CR})$$

(v) [2 pts] Lastly, can we satisfy these constraints? (A yes/no answer with some justification is sufficient for the problem. You do not need a formal proof.)

NO.

Sine A doesn't want to sit next to B.
then, A&, B should be in L or R.
with A not want to sit in the left chair,
A should be R. However, B sit right of A, so no case holds

$A \wedge A$

(c) Logical Inference

Given

$\neg(A \wedge (A \Rightarrow B) \wedge (A \Rightarrow C) \wedge (B \wedge C \Rightarrow D)) \vee D$

$$KB = (A, A \Rightarrow B, A \Rightarrow C, B \wedge C \Rightarrow D)$$

(i) [4 pts] Show the steps in a forward chaining algorithm for proving $KB \models D$. Make sure to list the agenda, inferred, and count at the beginning and at each iteration of the algorithm.

init: Agenda = [A]
Inferred = [A = F, B = F, C = F, D = F]
Count = [0. 1. 1. 2]

S1: pop A, then inferred[A] = true.
then we can decrement count of A => B and A => C and add B & C to agenda
Agenda = [B, C]
Inferred = [A = T, B = F, C = F, D = F]
Count = [0. 0. 0. 2]

S2 pop B. inferred[B] = true,
then we can decrement count of B ∧ C => D
Agenda = [C]
Inferred = [A = T, B = T, C = F, D = F]
Count = [0. 0. 0. 1]

S3: pop C. inferred[C] = true,
then we can decrement count of B ∧ C => D and add D to agenda
Agenda = [D]
Inferred = [A = T, B = T, C = T, D = F]
Count = [0. 0. 0. 0]

S4. pop D and find that D = D.
so return true.
Agenda = []
Inferred = [A = T, B = T, C = T, D = F]
Count = [0. 0. 0. 0]

(ii) [2 pts] How do we prove $KB \models D$ using a SAT solver? (Hint: The solution is simple and takes just one line.)

Solve $KB \wedge \neg D$ via SAT solver. if we get false, then $KB \models D$.

clause $\wedge$ clause

$\neg$ $\quad$ only $\vee$

(iii) [4 pts] Write out the necessary clauses in CNF representation of the sentence required for the previous part.

$(A \wedge (A \Rightarrow B) \wedge (A \Rightarrow C) \wedge (B \wedge C \Rightarrow D)) \wedge \neg D$

$= A \wedge (\neg A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee \neg C \vee D) \wedge \neg D$

(iv) [10 pts] Show the steps in the operation of DPLL, assuming a fixed variable ordering (A, B, C, D) and a fixed value ordering (true before false). Remember to apply early termination, pure literals (repeatedly), and unit clauses (repeatedly), keeping track of which clauses have already been satisfied in the process. Make sure to list the intermediate steps for the model, symbols, and clauses at the beginning and at each iteration of the algorithm.

Init: clause: [A, $\neg A \vee B$, $\neg A \vee C$, $\neg B \vee \neg C \vee D$, $\neg D$]
symbol: [A, B, C, D]
model: [ ]

S1 Find a pure symbol A and set A as True
clause: [A, $\neg A \vee B$, $\neg A \vee C$, $\neg B \vee \neg C \vee D$, $\neg D$]
symbol: [B, C, D]
model: [A = T]

S2. Find unit clause $\neg A \vee B$, and set B = true.
clause: [A, $\neg A \vee B$, $\neg A \vee C$, $\neg B \vee \neg C \vee D$, $\neg D$]
symbol: [C, D]
model: [A = T, B = T]

S3. Find unit clause $\neg A \vee C$, and set C = true.
clause: [A, $\neg A \vee B$, $\neg A \vee C$, $\neg B \vee \neg C \vee D$, $\neg D$]
symbol: [D]
model: [A = T, B = T, C = T]

S4. Find unit clause $\neg B \vee \neg C \vee D$, and set D = true.
clause: [A, $\neg A \vee B$, $\neg A \vee C$, $\neg B \vee \neg C \vee D$, $\neg D$]
symbol: [ ]
model: [A = T, B = T, C = T, D = T]

S4 $\neg D$ become false, then return false.