CS 188 Spring 2022 | Introduction to Artificial Intelligence | Written HW 4 Sol.

Solutions for HW 4 (Written)

Q1. [30 pts] Probability Review

This question is meant to review part of the probability prerequisite. It might be helpful to look into resources under **General Resources** at `https://piazza.com/berkeley/spring2022/cs188/resources`.

Let A, B, C, D be four random variables.

(a) What is the smallest set of independence or conditional independence relationships we need to assume for the following scenarios?

Note that independence is denoted as $\perp\!\!\!\perp$. In addition, $A \perp\!\!\!\perp C|B$ denotes that A is conditionally independent of C given B.

(i) [1 pt] $P(A, B) = P(A|B)P(B)$

None (always true)

(ii) [1 pt] $P(A, B) = P(A)P(B)$

$A \perp\!\!\!\perp B$

(iii) [2 pts] $P(A, B, C) = P(A|B)P(B|C)P(C)$

$A \perp\!\!\!\perp C|B$

(iv) [3 pts] $P(A, B, C) = P(A)P(B|C)P(C)$

$A \perp\!\!\!\perp B, C$

Wrong answer: $A \perp\!\!\!\perp B$ and $A \perp\!\!\!\perp C$

(v) [3 pts] $P(A, B, C) = P(A)P(B)P(C)$

$A \perp\!\!\!\perp B \perp\!\!\!\perp C$; A, B, C are mutually independent.

Wrong answers: $A \perp\!\!\!\perp B$ and $A \perp\!\!\!\perp C$ and $B \perp\!\!\!\perp C$; or A, B, C are pairwise independent

(b) Simplify the following expressions to one probability expression. Please show your work.

(i) [3 pts] $\frac{P(A,B)}{\sum_a P(a,B)}$

$= \frac{P(A,B)}{P(B)} = P(A|B)$

(ii) [3 pts] $\frac{P(A,B,C,D)}{\sum_a \sum_b P(a,b,C,D)}$

$= \frac{P(A,B,C,D)}{P(C,D)} = P(A, B|C, D)$

(iii) [4 pts] $\frac{P(A,C,D|B)}{P(C,D|B)}$

$= \frac{\frac{P(A,B,C,D)}{P(B)}}{\frac{P(B,C,D)}{P(B)}} = \frac{P(A,B,C,D)}{P(B,C,D)} = P(A|B, C, D)$

(iv) [4 pts] $\frac{P(A|B)}{\sum_c P(c|B)}$

Leaving the numerator unchanged,

$= \frac{P(A|B)}{\frac{\sum_c P(c,B)}{P(B)}} = \frac{P(A|B)}{\frac{P(B)}{P(B)}} = P(A|B)$

(v) [6 pts]

$\frac{\sum_b P(A,b|C)P(D|A,b,C)}{P(A|B,C)}$, given $A \perp\!\!\!\perp B|C$

Note that since $A \perp\!\!\!\perp B|C$, $P(A|B, C) = P(A|C)$ in the denominator

$= \frac{\sum_b P(A,b,D|C)}{P(A|C)} = \frac{P(A,D|C)}{P(A|C)} = \frac{\frac{P(A,C,D)}{P(C)}}{\frac{P(A,C)}{P(C)}} = \frac{P(A,C,D)}{P(A,C)} = P(D|A, C)$