CS 188 Spring 2022 Introduction to Artificial Intelligence — Written HW 4

Due: Wednesday, February 23 at 10:59pm (submit via Gradescope).

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	Qingjing
Last name	Zhang
SID	3037581026
Collaborators	

For staff use only:

Q1.	Probability Review	/30
	Total	/30

Q1. [30 pts] Probability Review

This question is meant to review part of the probability prerequisite. It might be helpful to look into resources under **General Resources** at `https://piazza.com/berkeley/spring2022/cs188/resources`.

Let A, B, C, D be four random variables.

(a) What is the smallest set of independence or conditional independence relationships we need to assume for the following scenarios?

(i) [1 pt] $P(A, B) = P(A|B)P(B)$

None

(ii) [1 pt] $P(A, B) = P(A)P(B)$

$\{A \perp\!\!\!\perp B\}$

(iii) [2 pts] $P(A, B, C) = P(A|B)P(B|C)P(C)$

$\{A \perp\!\!\!\perp C \mid B\}$

(iv) [3 pts] $P(A, B, C) = P(A)P(B|C)P(C)$

$\{A \perp\!\!\!\perp (B, C)\}$

(v) [3 pts] $P(A, B, C) = P(A)P(B)P(C)$

$\{A \perp\!\!\!\perp (B, C) \quad B \perp\!\!\!\perp C\}$

(b) Simplify the following expressions to one probability expression. Please show your work.

(i) [3 pts] $\frac{P(A,B)}{\sum_a P(a,B)}$

$= \frac{P(A,B)}{P(B)} = P(A|B)$

(ii) [3 pts] $\frac{P(A,B,C,D)}{\sum_a \sum_b P(a,b,C,D)}$

$= \frac{P(A,B,C,D)}{\sum_a \sum_b P(a,C,D)} = \frac{P(A,B,C,D)}{P(C,D)} = P(A,B|C,D)$

(iii) [4 pts] $\frac{P(A,C,D|B)}{P(C,D|B)}$

$= \frac{P(A,B,C,D) \,/\, P(B)}{P(B,C,D) \,/\, P(B)} = \frac{P(A,B,C,D)}{P(B,C,D)} = P(A|B,C,D)$

(iv) [4 pts] $\frac{P(A|B)}{\sum_c P(c|B)}$

$= \frac{P(A|B) \cdot P(B)}{\sum_c P(c|B) \cdot P(B)} = \frac{P(A,B)}{P(B)} = P(A|B)$

(v) [6 pts] $\frac{\sum_b P(A,b|C)P(D|A,b,C)}{P(A|B,C)}$, given $A \perp\!\!\!\perp B|C$

$= \frac{\sum_b P(A,b|C) \cdot P(C) \cdot P(D|A,b,C)}{P(A|B,C) \cdot P(C)} = \frac{P(A,C,D)}{P(B,C|B)} = \frac{P(A,C,D)}{P(A,C)} = P(D|A,C)$