CS 188
Spring 2022

Introduction to
Artificial Intelligence

Written HW 7

Due: Wednesday 03/30/2022 at 10:59pm (submit via Gradescope).

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: It is recommended that your submission be a PDF that matches this template. You may also fill out this template digitally (e.g. using a tablet). **However, if you do not use this template, you will still need to write down the below four fields on the first page of your submission.**

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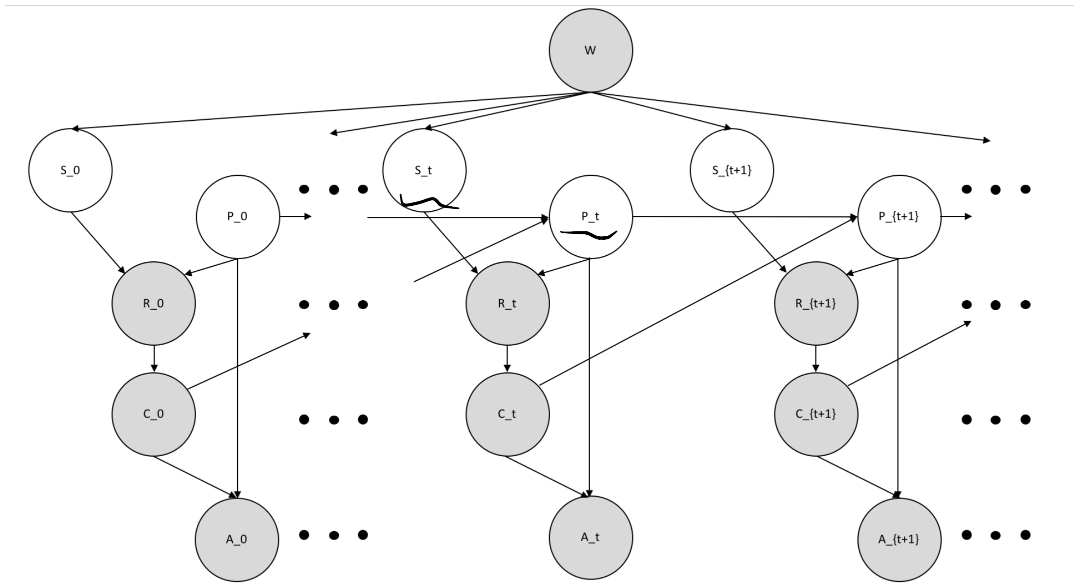
Collaborators	None.

For staff use only:

Q1.	Quadcopter: Pilot	/27
Q2.	MangoBot Human Detector	/8
	Total	/35

Q1. [27 pts] Quadcopter: Pilot

In this question, we look at the same setup as the previous homework, but we now look at it from the perspective of Paul, a quadcopter pilot who can **observe W (weather), R (reading of position), C (control from the pilot), and A (smart alarm warning)**. As before, suppose weather (W) does not change throughout the quadcopter's flight.



(a) Forward Algorithm: The real deal

(i) [2 pts] Now that the only hidden states are S_t and P_t, is this graph a well-behaving HMM (where $E_{t+1} \perp\!\!\!\perp E_t \mid X_{t+1}$ and $X_{t+1} \perp\!\!\!\perp E_t \mid X_t$, recall that X is the hidden variable and E is the evidence variable)? Please explain your reasoning.

Yes,

It is equivalent to prove.

$S_{t+1} \perp R_t, C_t, A_t$ Given S_t.
$P_{t+1} \perp R_t, C_t, A_t$ Given P_t.
$R_{t+1} \perp R_t$, Given S_{t+1} P_{t+1}
$C_{t+1} \perp C_t$, Given S_{t+1}, P_{t+1},
$A_{t+1} \perp A_t$, Given S_{t+1}, P_{t+1} they all holds.

(ii) [4 pts] What is the time-elapsed **prediction** formula from time-step t to time-step $t+1$? Be sure to include all hidden states and observed states, and show how to assemble the formula from the conditional probability tables corresponding to the graph. Denote $B(S_t, P_t) = P(S_t, P_t \mid W, R_{0:t}, C_{0:t}, A_{0:t})$. Find $B'(S_{t+1}, P_{t+1}) = P(S_{t+1}, P_{t+1} \mid W, R_{0:t}, C_{0:t}, A_{0:t})$. Hint: refer to the "Filtering algorithm" lecture slides.

- ○ $B'(S_{t+1}, P_{t+1}) = \max_{s_t} \sum_{p_t} P(S_{t+1} \mid W) * P(P_{t+1} \mid p_t, c_t) * B(S_t, P_t)$
- ○ $B'(S_{t+1}, P_{t+1}) = \sum_{s_t} \sum_{p_t} P(S_{t+1} \mid S_t) * P(P_{t+1} \mid p_t) * B(S_t, P_t)$
- ○ $B'(S_{t+1}, P_{t+1}) = \sum_{s_t} \sum_{p_t} P(R_{t+1} | S_{t+1}, P_{t+1}) P(S_{t+1} \mid S_t) * P(P_{t+1} \mid p_t) * B(S_t, P_t)$
- ○ $B'(S_{t+1}, P_{t+1}) = \sum_{s_t} \sum_{p_t} P(S_{t+1} \mid S_t) * P(P_{t+1} \mid p_t) P(P_{t+1} \mid c_t) * B(S_t, P_t)$
- ✓ $B'(S_{t+1}, P_{t+1}) = \sum_{s_t} \sum_{p_t} P(S_{t+1} \mid W) * P(P_{t+1} \mid p_t, c_t) * B(S_t, P_t)$
- ○ $B'(S_{t+1}, P_{t+1}) = \sum_{s_t} \max_{p_t} P(S_{t+1} \mid W) * P(P_{t+1} \mid p_t, c_t) * B(S_t, P_t)$

$P(S_{t+1}, P_{t+1} \mid W, R_{0:t}, C_{0:t}, A_{0:t})$.

$= P(S_t, P_t \mid W, R_{0:t}, C_{0:t}, A_{0:t}) \cdot P(S_{t+1} \mid S_t) \cdot P(P_{t+1} \mid P_t)$

(iii) [4 pts] How do we include the observation **update** at time-step $t+1$? Be sure to include all hidden states and observed states, and show how to assemble the update from the conditional probability tables corresponding to the graph. Denote $B'(S_{t+1}, P_{t+1}) = P(S_{t+1}, P_{t+1} \mid W, R_{0:t}, C_{0:t}, A_{0:t})$, find $B(S_{t+1}, P_{t+1})$

- ◯ $B(S_{t+1}, P_{t+1}) = P(R_{t+1} \mid S_{t+1}, P_{t+1}) * P(C_{t+1} \mid R_{t+1}) * P(A_{t+1} \mid C_{t+1}) * B'(S_{t+1}, P_{t+1})$
- ◯ $B(S_{t+1}, P_{t+1}) = P(R_{t+1} \mid S_{t+1}, P_{t+1}) * P(C_{t+1} \mid R_{t+1}) * B'(S_{t+1}, P_{t+1})$
- ◯ $B(S_{t+1}, P_{t+1}) = P(R_{t+1} \mid S_{t+1}, P_{t+1}) * P(C_{t+1} \mid R_{t+1}) * P(A_{t+1} \mid C_{t+1}, P_{t+1}) * B'(S_{t+1}, P_{t+1})$
- ✓ $B(S_{t+1}, P_{t+1}) \propto P(R_{t+1} \mid S_{t+1}, P_{t+1}) * P(C_{t+1} \mid R_{t+1}) * P(A_{t+1} \mid C_{t+1}, P_{t+1}) * B'(S_{t+1}, P_{t+1})$
- ◯ $B(S_{t+1}, P_{t+1}) \propto P(R_{t+1} \mid S_{t+1}, P_{t+1}) * P(C_{t+1} \mid R_{t+1}) * B'(S_{t+1}, P_{t+1})$?

(b) Consider a simpler scenario where we only track the 2D position (x, y) of the quadcopter. Paul, the pilot, wants to infer the quadcopter's true position P as accurately as possible.

- x, y **each** can take on values $\in \{0, 1, 2\}$.
- We have four controls: forward, backward, left, and right. 4
- Let variable E_R be Paul's estimate of the current position, and this variable depends on the reading R. The utility is based on the difference between the estimate of current position E_R and the actual position P: $U(P, E_R) = -||P - E_R||_x - ||P - E_R||_y$, in dollars.
- We consider only one time step. In that time step the **reading R is** (1, 0) and that the weather is cloudy.
- Under cloudy weather, the signal strength can take on 2 values with equal probability: weak and strong. The signal strengths correspond to the following errors in readings:
  - Weak: The reading R returns a random number (for each position element) sampled uniformly from the domain of possible positions.
  - Strong: The reading R is identical to the true position.

Answer the following questions:

(i) [2 pts] Among the hidden variables S and P, Which variable should intuitively have the greatest VPI? Explain your answer. You should not do any calculations for this part.

P

Given P, we can directly maximize the utility

Paul's coworker offers to tell him the signal strength (S) in exchange for some cash.

(ii) [3 pts] Suppose the signal strength is strong. Given the current reading R, what is the Maximum Expected Utility after knowing this information of S?

Since S is strong,

Reading R is identical to true position.

$U(P, E_R) = U(P, P) = 0$

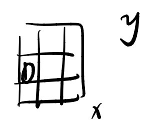
.

(iii) [3 pts] Suppose the signal strength is weak. Given the current reading R, what is the Maximum Expected Utility after knowing this information of S?

Since Signal strength is weak.

we take $E_R = (1, 1)$,

$MEV = -\frac{1}{9}(2+1+2+1+0+1+2+1+2) = -\frac{12}{9} = -\frac{4}{3}$



(iv) [3 pts] Considering the possibility of both signal strength, how much should Paul should pay to know this information of S?

$MEU(\phi) = -\frac{1}{9}(1+2+3+0+1+2+1+2+3) \times \frac{1}{2} = -\frac{5}{6}$

$VPI = MEU(S, S) = \frac{1}{2}(-\frac{4}{3}+0) - (-\frac{5}{6}) = \frac{1}{6}$

(c) **(i)** [3 pts] Suppose your coworker only tells you the signal strength with probability q, and with probability $1-q$, they don't tell you the signal strength even after payment. How much would you be willing to pay in this scenario? Your result should contain q.

$VPI = \max\{q(-\frac{2}{3}) + (1-q)(-\frac{5}{6}) - (-\frac{5}{6}), 0\}$

$= \max\{\frac{1}{6}q, 0\}$

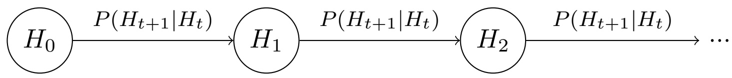
$= \frac{1}{6}q$

(ii) [3 pts] How much would you pay to know the true position (P)?

$VPI = \Delta U = 0 - (-\frac{5}{6}) = \frac{5}{6}$

Q2. [8 pts] MangoBot Human Detector

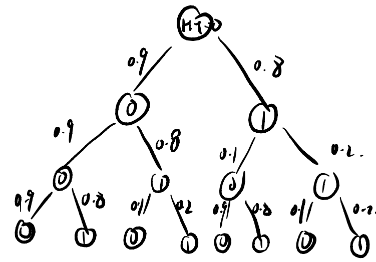
Your startup company MangoBot wants to build robots that delivers packages on the road. One core module of the robot's software is to detect whether a human is standing in front of it. We model the presence of humans with a Markov model:



where $H_t \in \{0, 1\}$ corresponds to a human being absent or present respectively. The initial distribution and the transition probabilities are given as follows:

H_0	$P(H_0)$
0	p
1	$1-p$

H_t	H_{t+1}	$P(H_{t+1}|H_t)$
0	0	0.9
0	1	0.1
1	0	0.8
1	1	0.2



(a) Express the following quantities in terms of p:

$0.8 p + 0.8(1-p)$

(i) [1 pt] $P(H_1 = 1) =$ $\sum_{H_0} P(H_1 = 1 \mid H_0) = 0.1p + 0.2(1-p) = 0.2 - 0.1p$

(ii) [1 pt] $\lim_{t\to\infty} P(H_t = 0) =$ $\sum_{H_{t-1}} P(H_t = 0 \mid H_{t-1}) = \lim_{t\to\infty} P(H_t = 0) = \lim_{t\to\infty} 0.\dot{8} - (0.1)^t p \approx 0.\dot{8}$

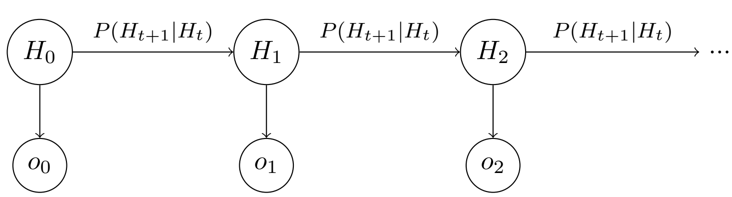
(b) The first-order Markov assumption in the model above can be inaccurate in real-world situations. Some potential ways to improve the model are listed below. For each option, determine whether it is possible to rewrite the process as a first-order Markov process, potentially with a different state representation.

(i) [1 pt] H_t depends not only on H_{t-1} but also on H_{t-2}. ✓ Yes ○ No

(ii) [1 pt] H_t depends not only on H_{t-1} but also on $H_{t-2}, H_{t-3}, \cdots, H_{t-k}$ for some fixed $k \geq 3$. ✓ Yes ○ No

(iii) [1 pt] H_t depends not only on H_{t-1} but also on $H_{t-2}, H_{t-3}, \cdots, H_1, H_0$. ✓ Yes ○ No

To make things simple, we stick to the original first-order Markov chain formulation. To make the detection more accurate, the company built a sensor that returns an observation O_t each time step as a noisy measurement of the unknown H_t. The new model is illustrated in the figure, and the relationship between H_t and O_t is provided in the table below.

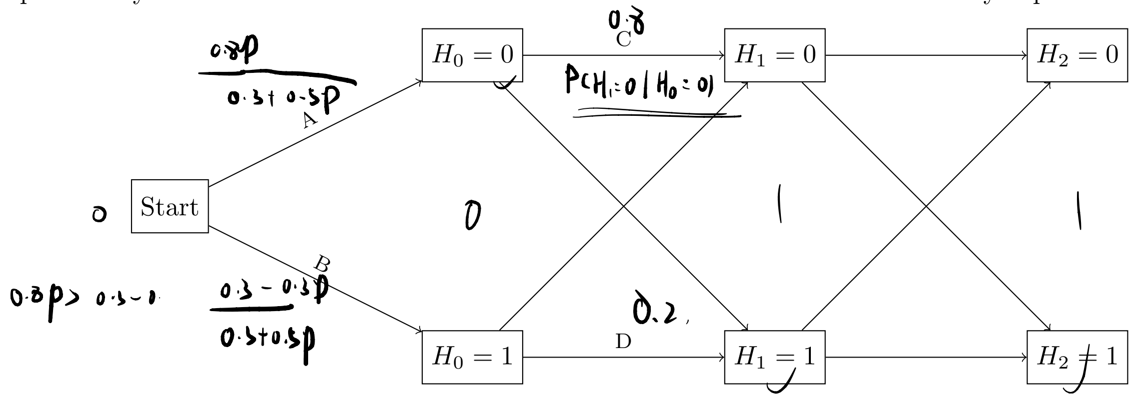


	H_t	O_t	$P(O_t|H_t)$
	0	0	0.8
p	0	1	0.2
	1	0	0.3
$1-p$	1	1	0.7

(c) Based on the observed sensor values $o_0, o_1, \cdots, o_t$, we now want the robot to find the most likely sequence $H_0, H_1, \cdots, H_t$ indicating the presence/absence of a human up to the current time.

$P(H_1 \mid H_0, O_0) = \frac{P(H_1, O_0 \mid H_0)}{P(H\ O_0)} =$ ——

(i) [1 pt] Suppose that $[o_0, o_1, o_2] = [0, 1, 1]$ are observed. The following "trellis diagram" shows the possible state transitions. Fill in the values for the arcs labeled A, B, C, and D with the product of the transition probability and the observation likelihood for the destination state. The values may depend on p.



(ii) [1 pt] There are two possible most likely state sequences, depending on the value of p. Complete the following (Write the sequence as "x,y,z" (without quotes), where x, y, z are either 0 or 1):
Hint: it might be helpful to complete the labelling of the trellis diagram above.

- When $p <$ $\boxed{\frac{3}{11}}$, the most likely sequence H_0, H_1, H_2 is $\boxed{0.1.1}$.
- Otherwise, the most likely sequence H_0, H_1, H_2 is $\boxed{1.1.1}$.

(d) [1 pt] True or False: For a fixed p value and observations $\{o_0, o_1, o_2\}$ in general, H_1^*, the most likely value for H_1, is always the same as the value of H_1 in the most likely sequence H_0, H_1, H_2. ○ True ✓ False