

PROPERTIES AND APPLICATIONS OF THE 2D FOURIER TRANSFORM

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The outputs presented in the succeeding pages are created using MATLAB. Moreover, the codes are uploaded in <u>Github</u>.

ROTATION PROPERTY OF THE FOURIER TRANSFORM

CANVAS WEAVE MODELING
AND REMOVAL

CONVOLUTION THEOREM
REDUX

FINGERPRINTS: RIDGE ENHANCEMENT

LUNAR LANDING SCANNED PICTURES: LINE REMOVAL

ROTATION PROPERTY OF THE FOURIER TRANSFORM

Objectives:

 Demonstrate the rotation property of the Fourier Transform





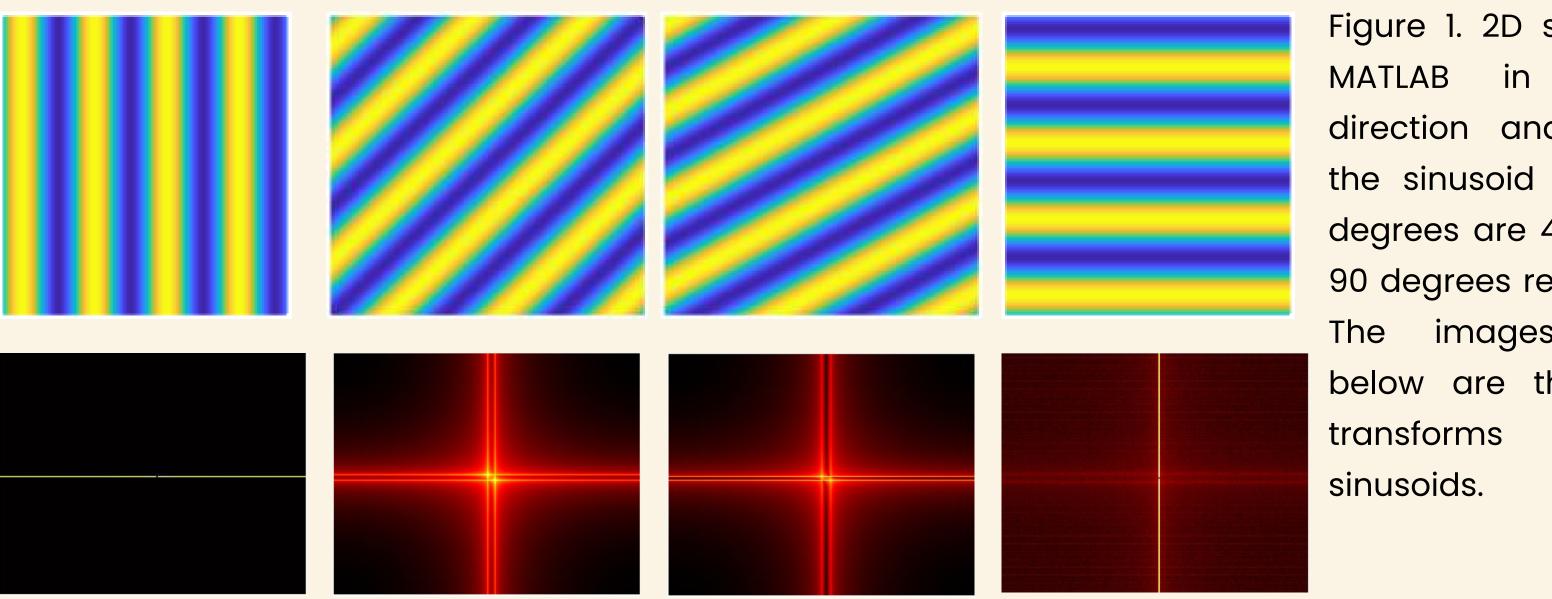
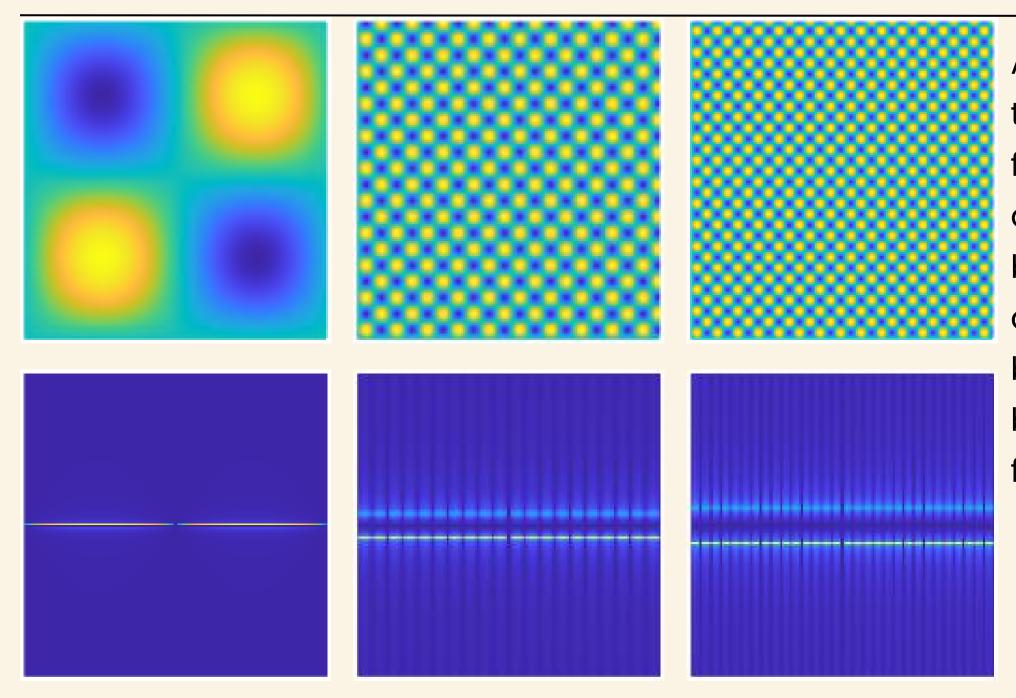


Figure 1. 2D sinusoid in in the xdirection and rotating the sinusoid such that degrees are 45, 30, and 90 degrees respectively. images directly below are the Fourier transforms of the

The results above show that the Fourier transform rotates along with the rotation of the sinusoids. This demonstrates the rotation property of the Fourier transform, which is a consequence of the fact that rotating an image in the spatial domain corresponds to a phase shift in the frequency domain [1].



As the frequency of the sinusoids increases, the Fourier transform has more high-frequency components, resulting in additional peaks. Moreover, the distance between these peaks becomes narrower due to the increased frequency. This behavior is a result of the relationship between the spatial domain and the frequency domain in the Fourier transform.

Figure 2. 2D sinusoid created through varying frequencies. The frequencies are increasing from left to right. The images directly below are the Fourier transforms of the sinusoids.



CANVAS WEAVE MODELING AND REMOVAL

Objectives:

 Remove the canvas weave to properly investigate the brush strokes of a painter.

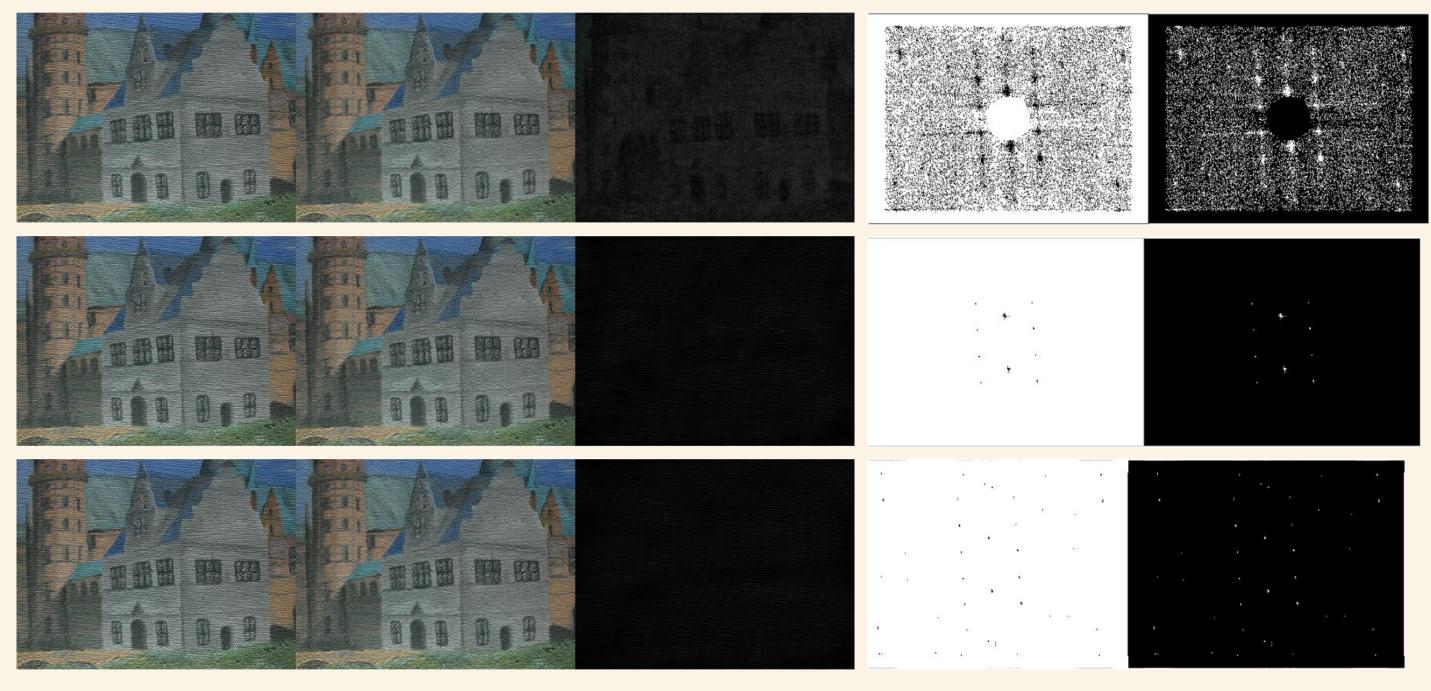


Figure 3. The first column is the copy of the original Daria picture. The second column images are the result of masking the Fourier transform in column 4 and the 3rd column are created by the masking of the last column.

CANVAS WEAVE MODELING AND REMOVAL

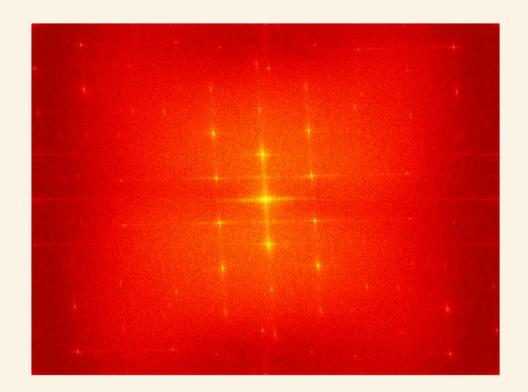


Figure 4. FFT of the painting.

Based on the Fourier transform of the painting, I came up with three ways to mask its peaks. The first mask used adaptive thresholding with "ForegroundPolarity" set to "bright". The second mask utilized thresholding where I set the values less than or equal to 0.75 to 0 and the rest are 1. Finally, the third mask is created using adaptive thresholding using "Sensitivity" set to zero to produce an image with fewer foreground objects. Overall, I think that all methods produced the desired results.



Figure 5. From left to right: original image, image produced by adaptive foreground polarity, thresholding, and adaptive sensitivity.



Figure 6. Shown from left to right are the original image, produced image, weave pattern, mask used for producing the results, and mask used for extracting the patterns.

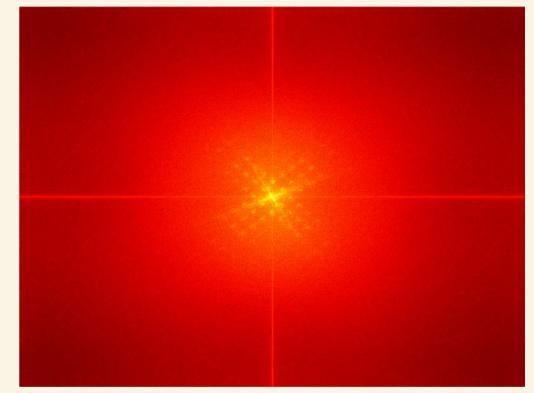


Figure 7. FFT of the ecobag.

Masking the peaks can either remove or reduce the patterns present in an image. To verify if the pattern removal was successful, the remaining patterns can be viewed in a separate image. In the results shown above, the pattern was only partially removed, indicating that a more appropriate masking technique should be used.

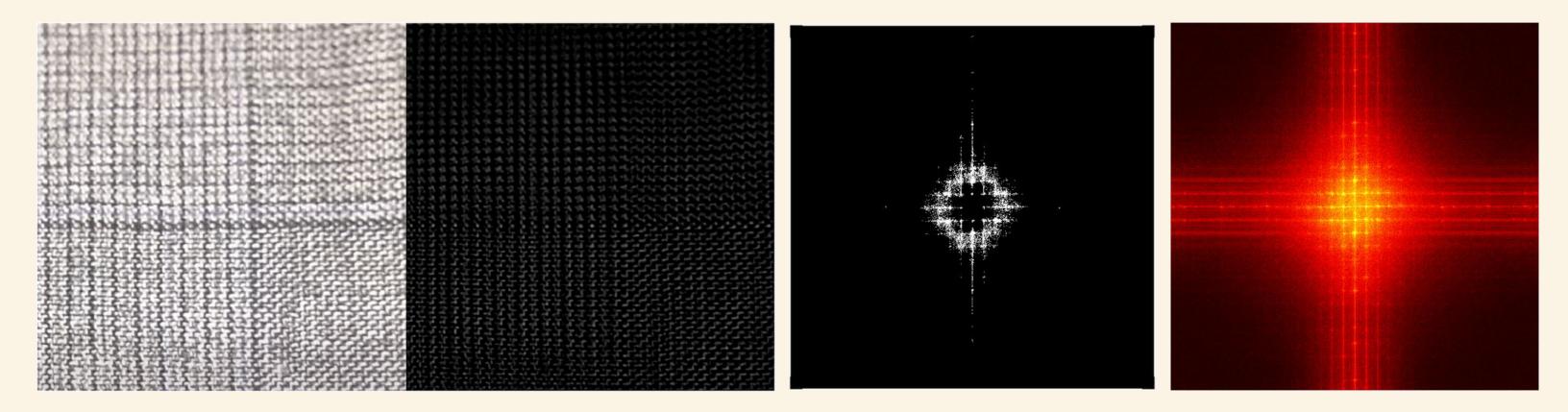


Figure 7. Shown from left to right are the image used, weave pattern extracted, mask used to extract the pattern and the Fourier transform of the image used.

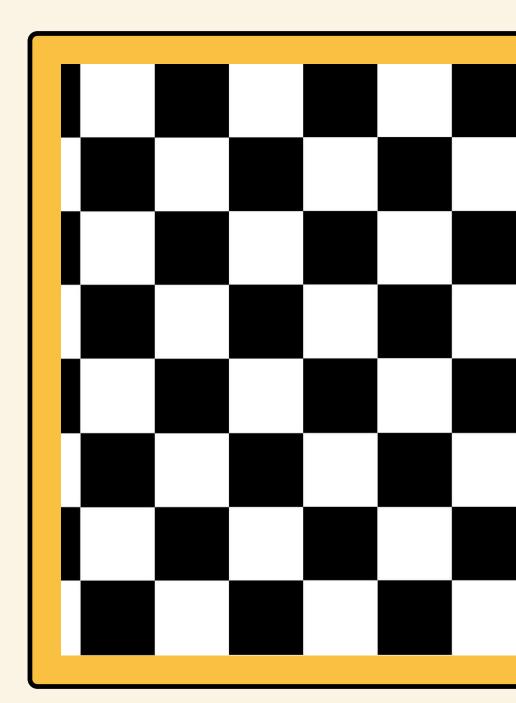
Here, our objective was to highlight the weaving pattern in a fabric. To achieve this, I set the peaks to I and the rest to 0, creating a mask that displays only the peaks. As a result, we can observe the weaving pattern more clearly in the fabric.

CONVOLUTION THEOREM REDUX

Objectives:

 Demonstrate convolution theorem redux





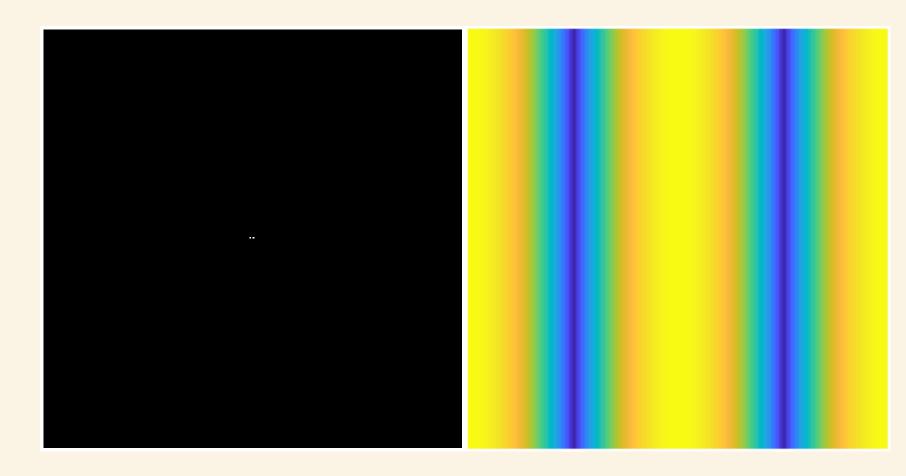


Figure 8. The 2 dots created and its Fourier Transform

Convolution in Image Space



Multiplication in Frequency

Convolution theorem states that the Fourier transform of the convolution of two functions is the product of their individual Fourier transforms [2]. In the first part, we generated an array with symmetric small dots along the x and y axis, along with its corresponding Fourier transform. As we progress through the slides, we observe that compressing a function in the spatial domain results in broadening its frequency spectrum, and vice versa, which is known as the scale property of the Fourier transform. Additionally, for image convolution, two different packages were used: one from MATLAB and another based on provided code. Both produced identical images, as shown in the results.

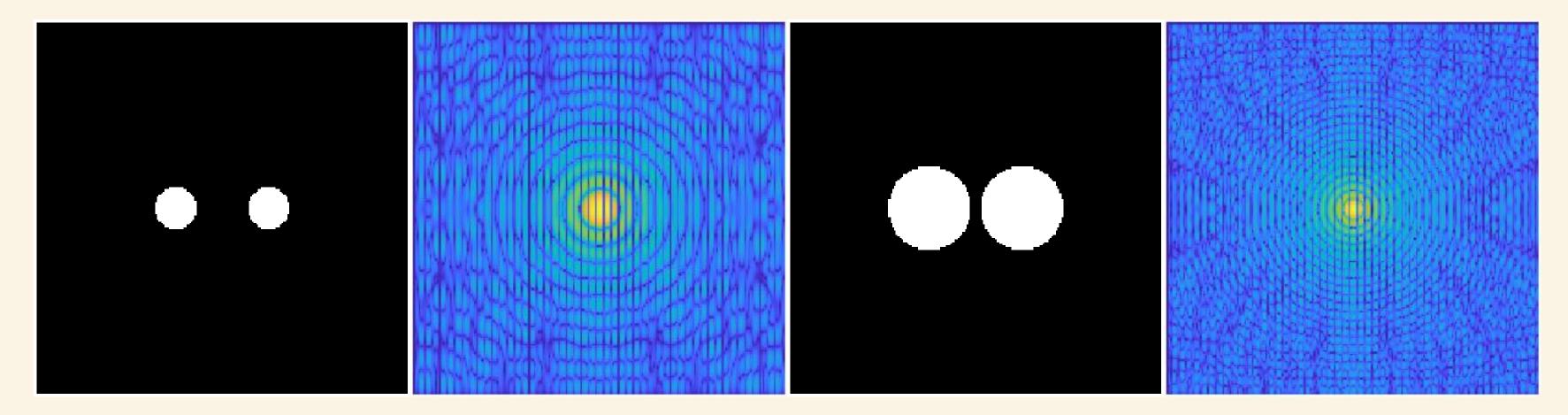


Figure 9. Comparsion of the Fourier Transform of the circles as the circle enlarges.

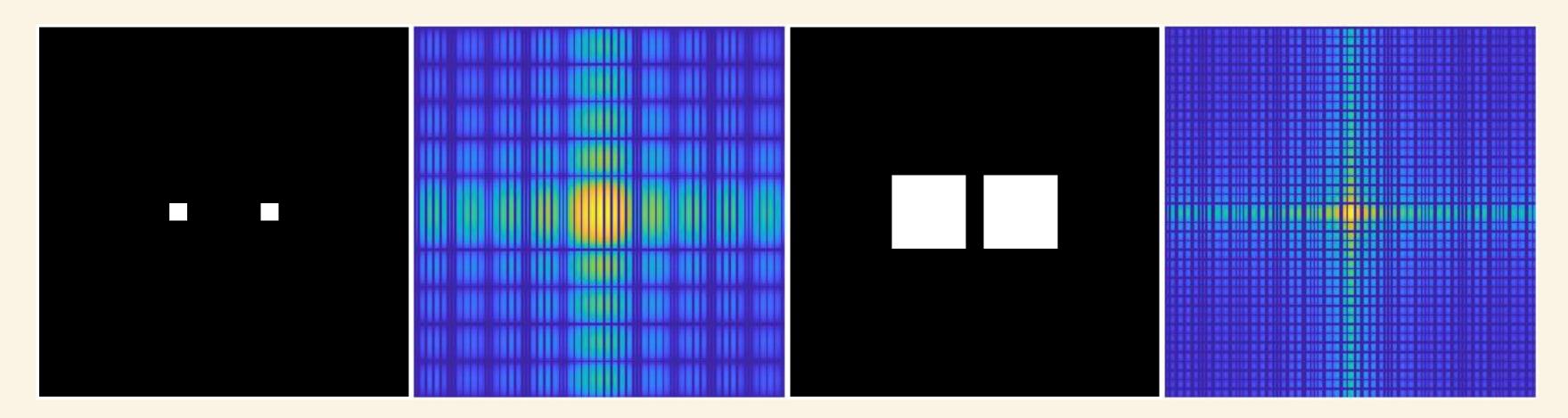


Figure 10. Comparsion of the Fourier Transform of the squares as the squares enlarges.

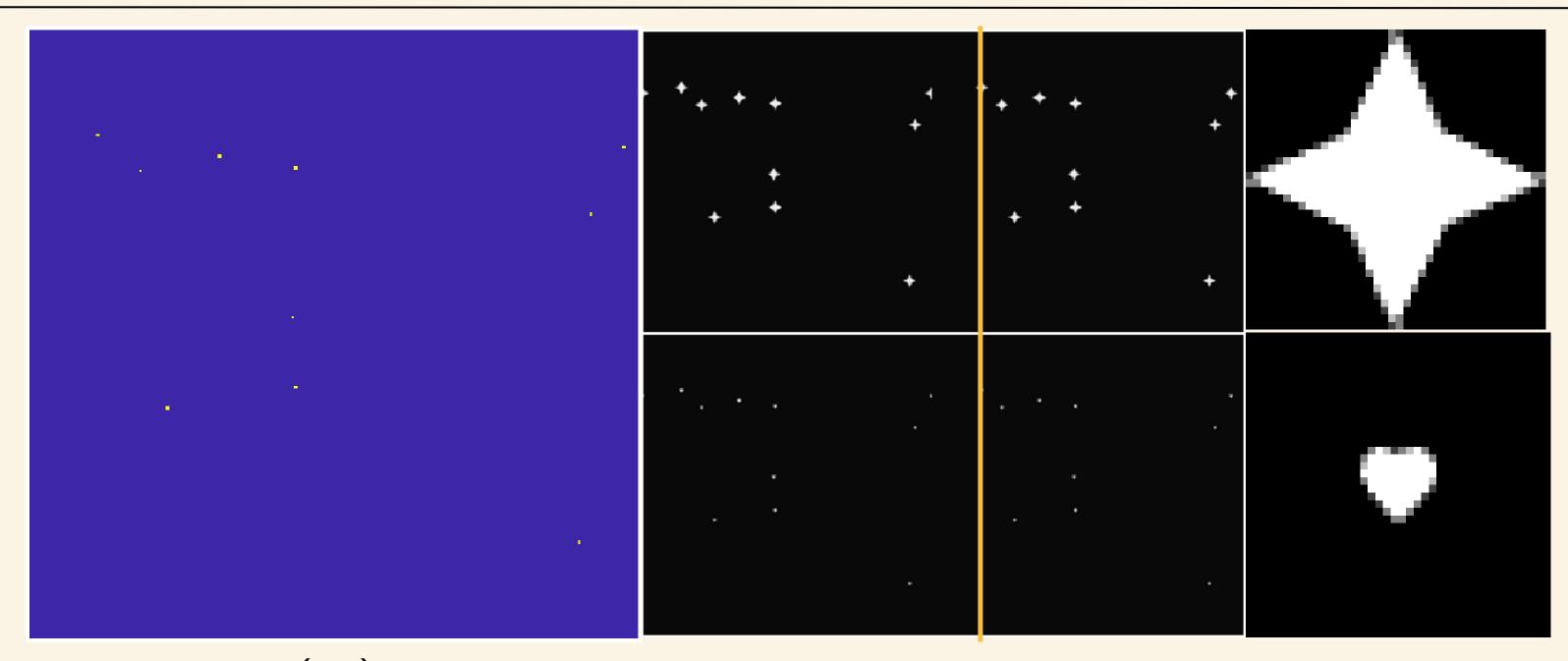


Figure 11. Random 1's (left) and the convoluted outputs using the convolution patterns from the right. The convoluted images are created using two different methods: packagen and own-made.

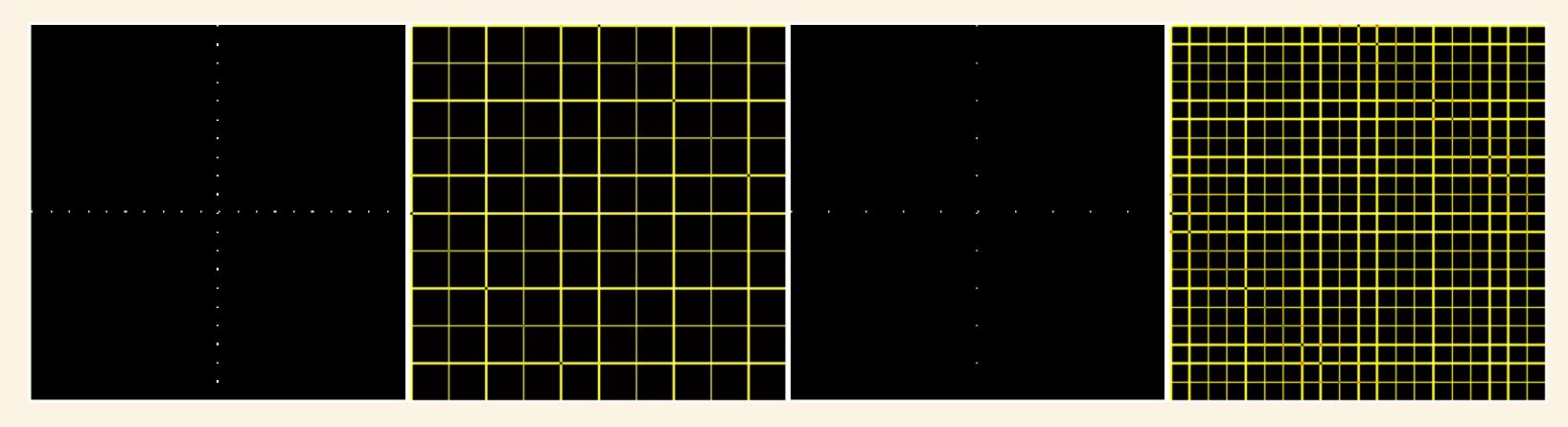


Figure 12. Comparison between two equally spaced 1's with different spacings and their corresponding Fourier Transforms.

FINGERPRINTS: RIDGE ENHANCEMENT





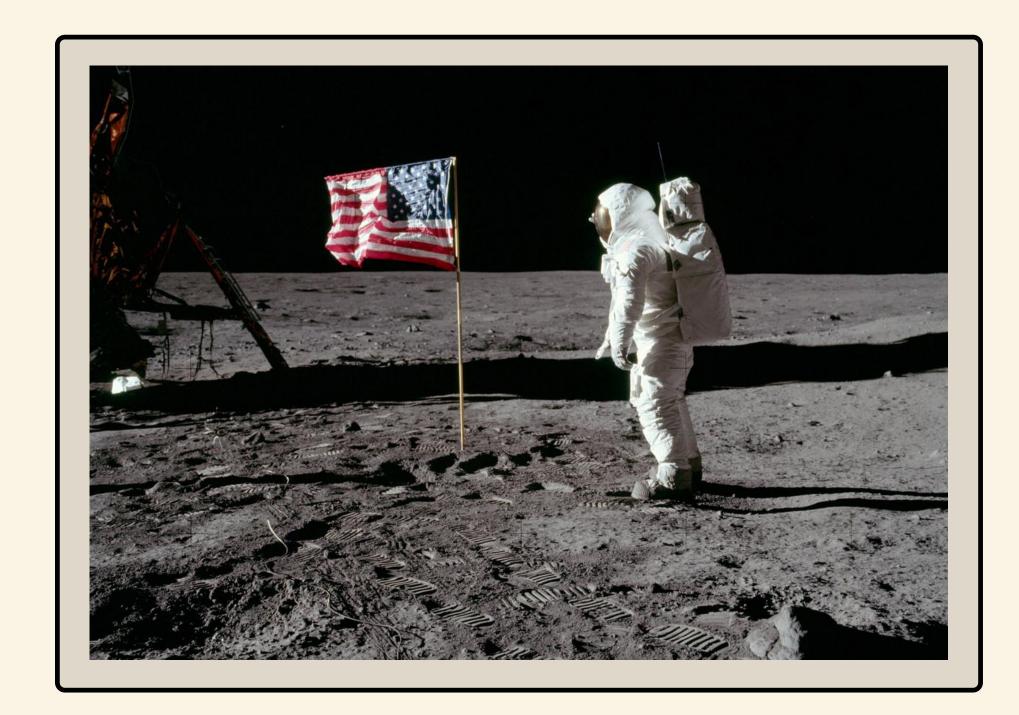
Objectives:

Enhance the ridges of a provided fingerprint



Figure 13. From left to right: FFT of the fingerprint, original fingerprint used, enhanced fingerprint, pattern extracted from the image.

Based on the obtained results, it is evident that the fingerprint image is enhanced through the implementation of necessary steps. However, there is still potential for further improvements. Therefore, it is recommended to create a more optimal mask to achieve better enhancement results.

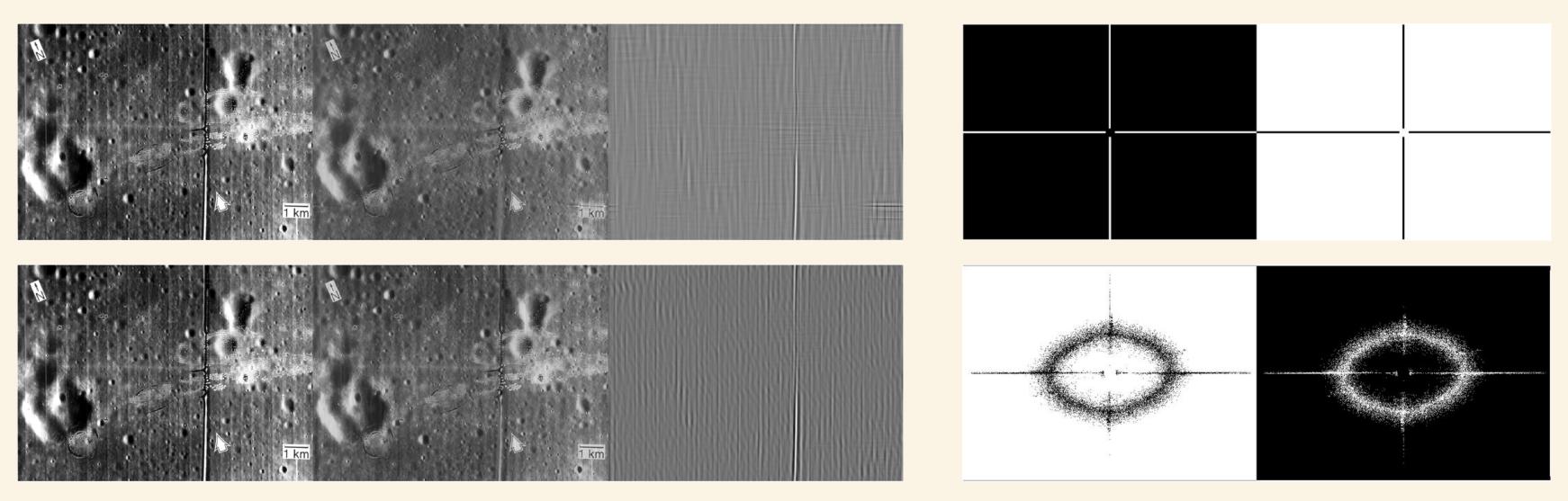


LUNAR LANDING SCANNED PICTURES: LINE REMOVAL

Objectives:

 Remove the vertical lines in the image by filtering in the Fourier Domain.

LUNAR LANDING SCANNED PICTURES: LINE REMOVAL



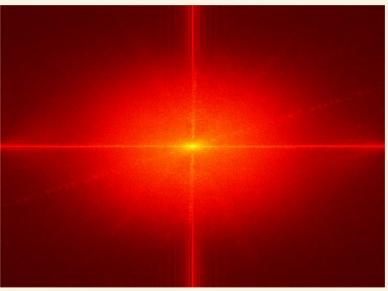


Figure 14. Outputs of the lunar landing line removal. In the first row, a manual masking was used and on the second row, I used thresholding to cover the peaks. The image below shows the Fourier transform of the original image.



Figure 15. From left to right: Original picture, manual masking, masking through adaptive thresholding

It is evident that the manual masking method resulted in fewer lines compared to the adaptive thresholding technique, albeit with reduced contrast between the images. Conversely, the adaptive thresholding approach demonstrated more contrast between the images, yet the lines were not as clearly removed as with the manual method. The masking method would have been better if we utilized the array done with equal dot spacing.

REFLECTION

RATING: 90 / 100

I have submitted my outputs days late and with that, I deserve the appropriate deductions. Supposedly, my score should be 80 but I still think that I deserve an extra 10 points for going above and beyond. To be honest, I really enjoyed doing the activity so I didn't want to submit the output created half-heartedly. Despite it, I still think that I have a lot to learn since I seem to forgot the underlying principles beyond what is being done.

Nevertheless, I want to thank my VIP labmates for helping me whenever I encounter a misunderstanding on the instructions and even the code. I am also grateful for the advancement of technology for paving the way for chatGPT to help me whenever I get stuck on my proper grammar usage. Being sleep-deprived for months has taken a toll in me and I am grateful for everyone for their patience and support.



- [1] Rotation Property of Discrete Fourier Transform for an image. (n.d). Mathworks. https://www.easybib.com/guides/citation-guides/citation-basics/how-to-cite-website-no-author/
- [2] Zisserman A. (2014). Lecture 2: 2D Fourier transforms and applications [PowerPoint slides]. https://www.robots.ox.ac.uk/~az/lectures/ia/lect2.pdf