Exercise 5.8.6

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Find the minimisation problem corresponding to:

$$\frac{\mathrm{d}^4 u}{\mathrm{d}x^4} = f$$

With boundary conditions:

$$u(0) = \frac{\mathrm{d}u}{\mathrm{d}x}(0) = 0, \frac{\mathrm{d}u}{\mathrm{d}x}(1) = 0, \frac{\mathrm{d}^3 u}{\mathrm{d}x^3}(1) = 1$$

Operator is:

$$L = \frac{\mathrm{d}^4}{\mathrm{d}x^4}$$

This operator is clearly linear, self adjoint and positive. Not all boundary conditions are homogeneous so we have to find a function which satisfies the non homogeneous conditions.

$$w = \frac{1}{24}x^4$$

Satisfies the non homogeneous boundary condition. Use:

$$J(u) = \int_\Omega \frac{1}{2}(u - w)(Lu + Lw)\mathrm{d}\Omega - \int_\Omega uf\mathrm{d}\Omega$$

To find minimisation problem.