

Exercise 6.2.2

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Given the Poisson's equation in one dimension:

$$-\frac{d^2u}{dx^2} = f(x)$$

With corresponding minimisation problem:

$$J[u] = \int_0^1 \frac{1}{2} \left(\frac{du}{dx} \right)^2 - u f dx$$

After piecewise approximation the above becomes:

$$J[u_a] = \sum_0^n \frac{1}{2} \left(\frac{du_{a,n}}{dx} \right)^2 - u_{a,n} f$$

As $u_{a,0} = 0$ and $u_n - u_{n-1} = 0$ (second boundary condition $\frac{du}{dx}(1) = 0$) the summation it should remain finite.