Given:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \boldsymbol{\nabla} \cdot \boldsymbol{\nabla} u = f(x, y, z) \text{ in } \Omega$$

Boundary conditions:

$$u = g \text{ on } \Gamma$$

Not homogeneous so use $L = \boldsymbol{\nabla}^2$. This is clearly a linear and self adjoint operator so check positiveness.

$$\int_\Omega u L u \mathrm{d}\Omega = \int_\Omega \boldsymbol{\nabla}^2 u^2 \mathrm{d}\Omega \geq 0$$

$$J(u) = \int_\Omega \frac{1}{2}(u-g)(Lu + Lg)\mathrm{d}\Omega - \int_\Omega uf \mathrm{d}\Omega = \int_\Omega \frac{1}{2}(u-g)(\boldsymbol{\nabla}^2 u + \boldsymbol{\nabla}^2 g)\mathrm{d}\Omega - \int_\Omega \boldsymbol{\nabla}^2 u^2 \mathrm{d}\Omega$$