Exercise 5.8.4

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Given:

$$\frac{\mathrm{d}}{\mathrm{d}x}(p(x)\frac{\mathrm{d}u}{\mathrm{d}x}) = f$$

With boundary conditions:

$$u(0), u(1) + \frac{\mathrm{d}u}{\mathrm{d}x}(1) = a$$

Not homogeneous, so need to find a function which satisfies the boundary conditions. For example $w = -\frac{a}{\pi}\sin(x\pi)$ satisfies both boundary conditions. Now the corresponding minimisation problem is ($L = \frac{\mathrm{d}}{\mathrm{d}x}(p(x)\frac{\mathrm{d}}{\mathrm{d}x})$):

$$J(u) = \int_\Omega \frac{1}{2}(u-w)(Lu+Lw)\mathrm{d}\Omega - \int_\Omega uf\mathrm{d}\Omega = whatevernotgonnawriteitout$$