Exercise 6.2.1

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Given the Poisson's equation in one dimension:

$$-\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} = f(x)$$

With boundary conditions:

$$u(0) = 0, \frac{\mathrm{d}u}{\mathrm{d}x}(1) = 0$$

Only homogeneous boundary conditions and operator $L = \frac{d^2}{dx^2}$, so the corresponding minimisation problem is given by:

$$J[u] = \int_0^1 \frac{1}{2} u L u - u f dx = \int_0^1 \frac{1}{2} (\frac{du}{dx})^2 - u f dx$$