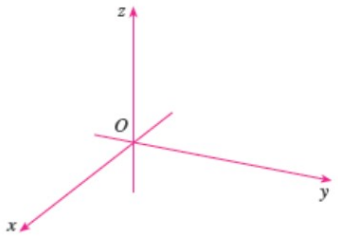
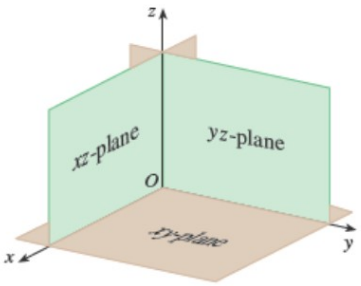
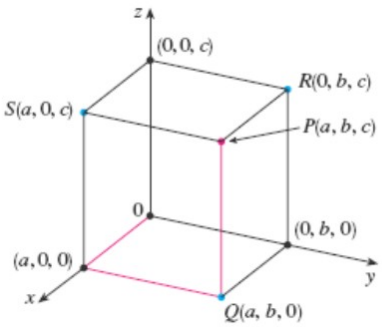
Section 12.1 Three-Dimensional Coordinate Systems

In three dimension, a point P is represented as an ordered triple $P(x, y, z)$. The orientation of the x, y, and z axes is shown below, and the three axes divide space into eight octants. The first octant, in the foreground, is determined by the positive x, y, and z axes.



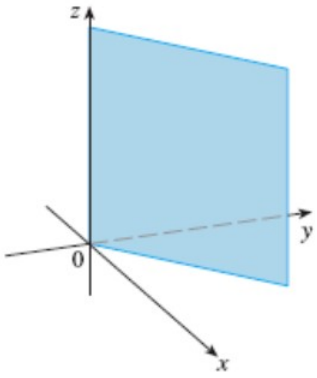


We can project a given point $P(a, b, c)$ onto the three coordinate planes.



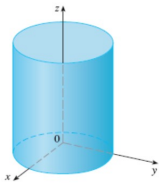
We know an equation of the form $y = mx + b$ determines a line in two dimension ($\mathbb{R}^2$). In three dimension ($\mathbb{R}^3$), $y = mx + b$ is a plane.

Planes: The equation of a plane is an equation of the form $ax + by + cz = d$. Below is the graph of the plane $y = x$ for $z \geq 0$. What is the projection of this plane onto the xy - plane?



Example 1: Sketch the graph of $2x + 4y + 6z = 12$ by finding the x, y, and z intercepts.

Cylinders: Recall the equation of a circle in $\mathbb{R}^2$ with center (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$. In $\mathbb{R}^3$, $(x-h)^2 + (y-k)^2 = r^2$ is a **cylinder**. The sketch below is the cylinder $x^2 + y^2 = r^2$.

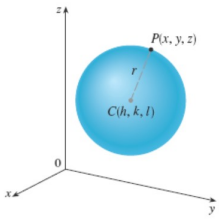


Example 2: Sketch the graph of

a.) $y^2 + z^2 = 4$, where $x \geq 0$

b.) $x^2 + z^2 = 9$, $-2 \leq y \leq 5$

Spheres: An equation of a **sphere** with center (h, k, l) and radius r is $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$.



Example 3: Find the center and radius of the sphere $x^2 + y^2 + z^2 = -8x + 6y - 17$.

Example 4: Find the equation of the sphere with radius 3 and center $(1, 4, 3)$.

What is the intersection of this sphere with the three coordinate planes?

a.) Intersection with xy - plane

b.) Intersection with xz - plane

c.) Intersection with yz - plane

Example 5: What is the equation of the sphere with center $(1, 2, 3)$ that touches the xy - plane? The yz - plane?

Distance and Midpoint: Consider the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

a.) The distance between the points P and Q is $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

b.) The midpoint of the line segment joining P and Q is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Example 6: What is the equation of the sphere if one of its diameters has endpoints $(2, 4, 3)$ and $(1, -6, 4)$?