Section 12.2 Vectors in Three Dimension

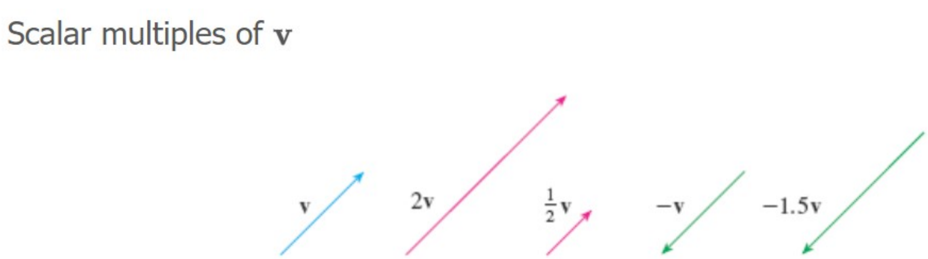
A three dimensional vector is an ordered triple $\overrightarrow{a} = \langle a_1, a_2, a_3\rangle$. The numbers a_1, a_2 and a_3 are called the **components** of the vector $\overrightarrow{a}$. If a vector starts at the point $A(a_1, a_2, a_3)$ and ends at the point $B(b_1, b_2, b_3)$, then the vector with representation $\overrightarrow{AB}$ is given by $\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3\rangle$.

Note: There are several conventions for vector notation, for example $\overrightarrow{a}$, $\hat{a}$, or $\mathbf{a}$ all are understood to represent the vector $\langle a_1, a_2, a_3\rangle$. The 'pointy brackets' distinguish the vector $\langle a_1, a_2, a_3\rangle$ from the point (a_1, a_2, a_3). Notation is important!

Example 1: Find the components of the vector with initial point $A(-2, 4, 1)$ and terminal point $B(0, 2, 5)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin (called the **position vector**).

The Algebra of Vectors: Suppose $\overrightarrow{a} = \langle a_1, a_2, a_3\rangle$ and $\overrightarrow{b} = \langle b_1, b_2, b_3\rangle$ are vectors and c is a scalar.

a.) Scalar Multiplication: $c\overrightarrow{a} = \langle ca_1, ca_2, ca_3\rangle$. (Changes magnitude if $c \neq \pm 1$ and direction if $c < 0$). Below illustrates various scalar multiples of a vector $\mathbf{v}$.



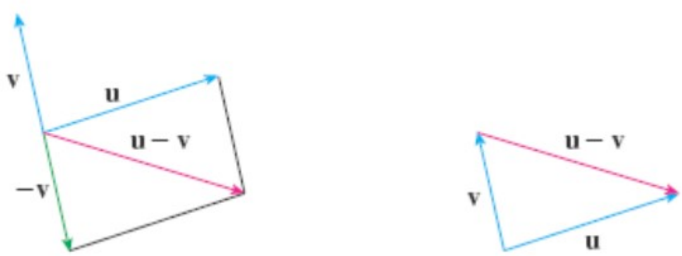
b.) Vector Length (magnitude): $|\overrightarrow{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$.

c.) Unit Vector: A unit vector in the direction of $\overrightarrow{a}$ is $\overrightarrow{u} = \dfrac{\overrightarrow{a}}{|\overrightarrow{a}|}$.

d.) Vector Sum: $\overrightarrow{a+b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

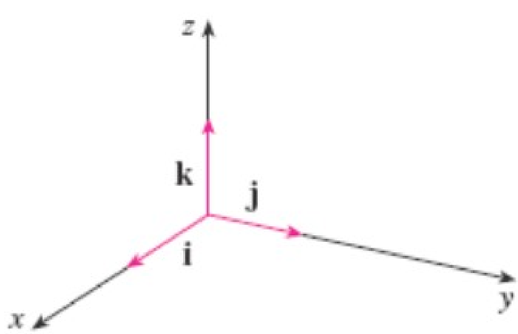
Below is an illustration of the sum of of two vectors, **u** and **v**, the second being the parallelogram rule.

e.) Vector Difference: $\overrightarrow{a-b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$. Below is an illustration of the difference of two vectors, **u** and **v**, in the order $\mathbf{u} - \mathbf{v}$.



(a) (b)

f.) We define $\overrightarrow{i} = \langle 1, 0, 0 \rangle$, $\overrightarrow{j} = \langle 0, 1, 0 \rangle$ and $\overrightarrow{k} = \langle 0, 0, 1 \rangle$, and $\langle a_1, a_2, a_3 \rangle = a_1 \overrightarrow{i} + a_2 \overrightarrow{j} + a_3 \overrightarrow{k}$.



Example 2: Given $\overrightarrow{a} = \langle 1, 2, -1 \rangle$ and $\overrightarrow{b} = \langle 0, 0, 4 \rangle$:

a.) Find $\overrightarrow{a+b}$ and illustrate with a sketch.

b.) Find $\overrightarrow{a-b}$ and illustrate with a sketch. Next, find a unit vector in the direction of $\overrightarrow{a-b}$.

c.) Find a vector in the direction of $-\mathbf{a} + 2\mathbf{b}$ with length 7.