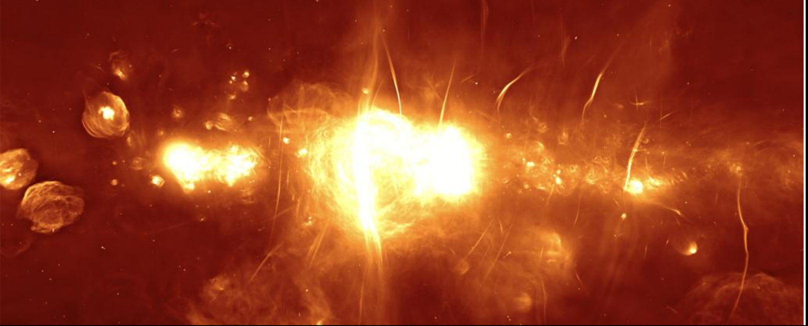
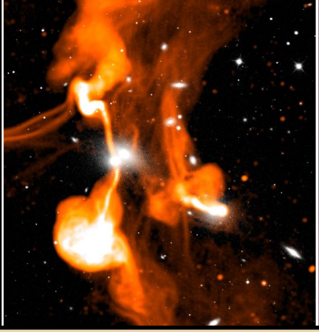
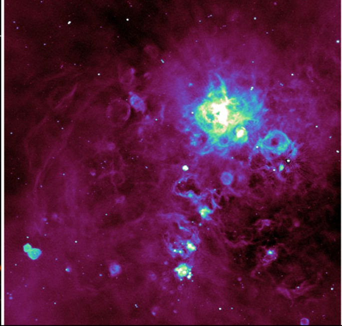


Interferometry for dummies

Jonah Wagenveld

Fundi tutorials, February 14, 2025



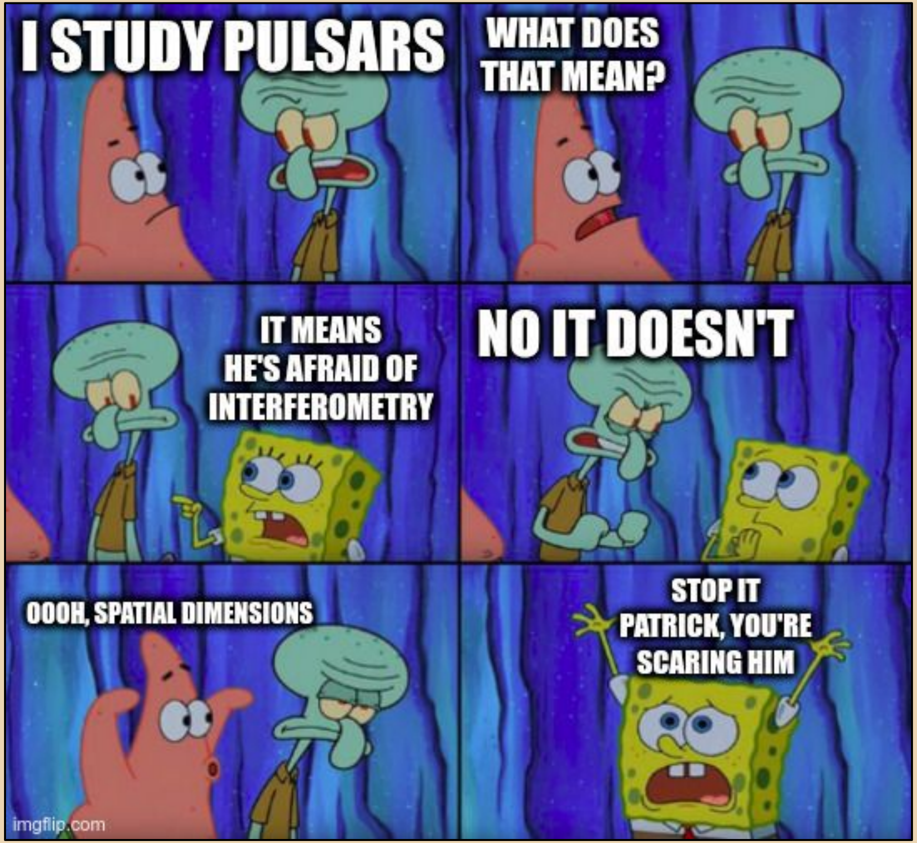
Outline

- Introduction
- Interferometry essentials
  - Fourier transforms (without the math)
- Two element interferometer
- N element interferometers
  - The UV plane
  - Westerbork
  - MeerKAT
- Synthesis imaging
- Basics of CLEAN

Some references

- Essential Radio Astronomy by Condon & Ransom
  - https://www.cv.nrao.edu/~sransom/web/xxx.html
- Synthesis imaging in radio astronomy
  - https://leo.phys.unm.edu/g̃btaylor/astr423/s98book.pdf
- CASA tutorials or documentation
  - https://casaguides.nrao.edu/index.php?title=VLA_CASA_Imaging-CASA6.5.2
  - https://casadocs.readthedocs.io/en/stable/notebooks/synthesis_imaging.html
- VLA workshops
  - 2019: https://science.nrao.edu/science/meetings/2019/vla-data-reduction
  - 2021: https://science.nrao.edu/science/meetings/2021/vla-data-reduction
- Simulating interferometric observations
  - https://github.com/crpurcell/friendlyVRI

Interferometry essentials



Interferometry essentials

Two things required to understand most of radio astronomy

- Some basic Fourier transforms
- The relation between physical scale and scale in the Fourier space

$$\theta = \frac{\lambda}{D}$$

Interferometry essentials

Two things required to understand most of radio astronomy

- Some basic Fourier transforms
- The relation between physical scale and scale in the Fourier space

$$\theta = \frac{\lambda}{D}$$

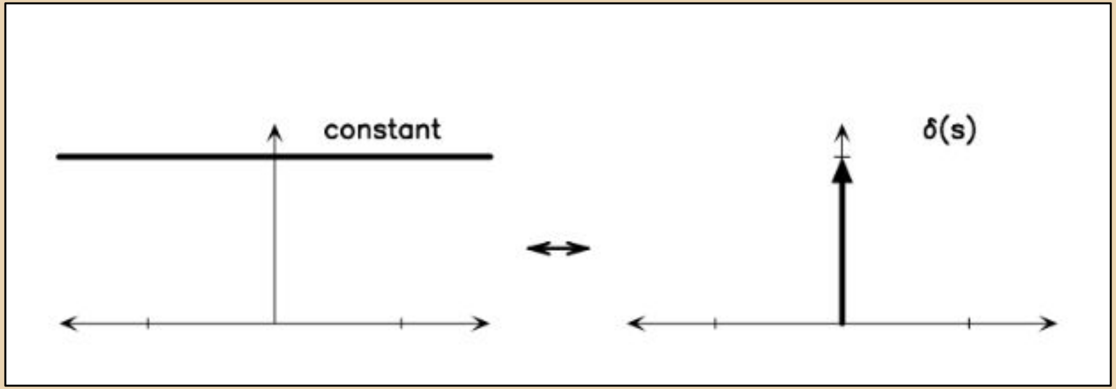
Some added concepts for interferometry

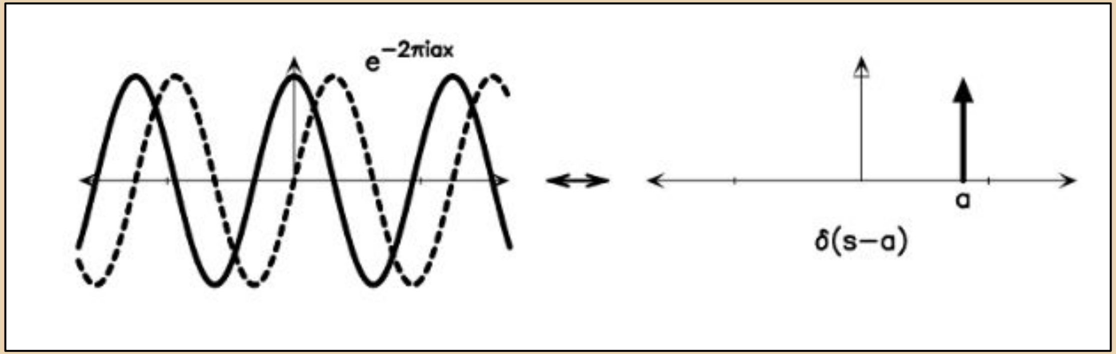
- The UV plane
- Image synthesis
- Deconvolution

Fourier transforms (without the math)

- Basically everything in radio astronomy involves Fourier transforms
  - Antenna responses
  - Interferometric aperture synthesis
  - Source morphologies
- However: most things are (geometrically) simple
- Knowing some Fourier transforms by heart will help you identify things

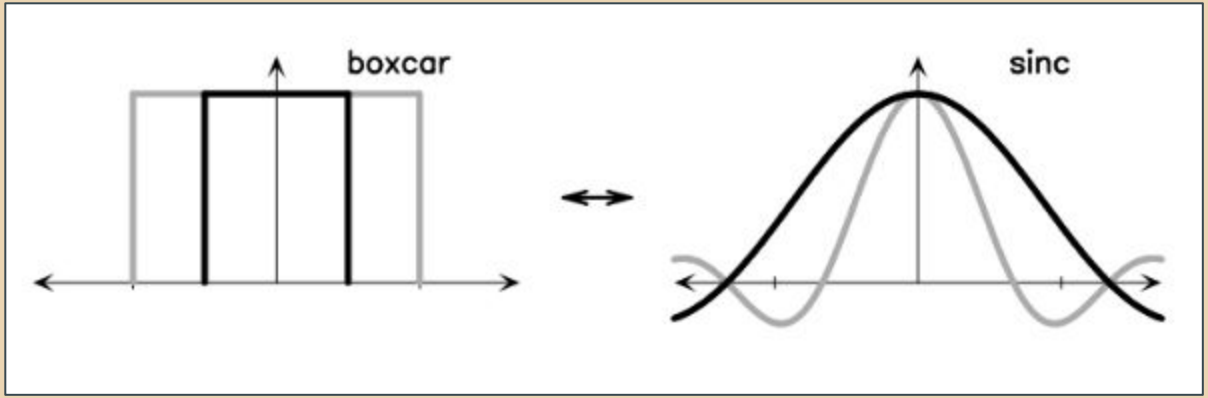
Fourier transforms (without the math)

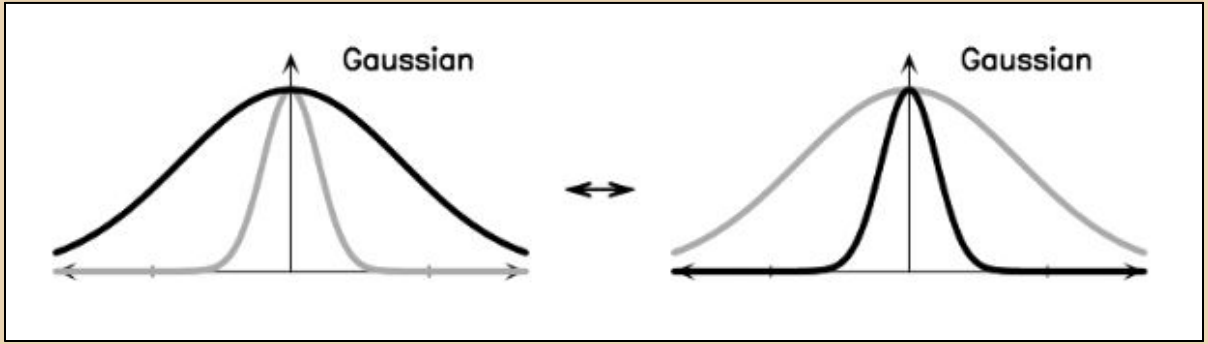




Credit: Condon & Ransom

Fourier transforms (without the math)





Credit: Condon & Ransom

Basic Fourier theorems (now a little math)

Addition theorem: Addition in real space is addition in Fourier space

$$f(x) + g(x) \leftrightarrow F(s) + G(s)$$

Similarity theorem: Big in real space is small in Fourier space (and vice versa)

$$f(ax) \leftrightarrow \frac{F(s/a)}{|a|}$$

Convolution theorem: Convolution in real space is multiplication in Fourier space

$$f(x) \otimes g(x) \leftrightarrow F(s) \cdot G(s)$$

Cross-correlation theorem: Cross-correlation in real space is multiplication in Fourier space with one of the signals complex conjugated

$$f(x) \star g(x) \leftrightarrow \bar{F}(s) \cdot G(s)$$

Single dish

- Single completely filled aperture
- Only one resolution defined by size of dish

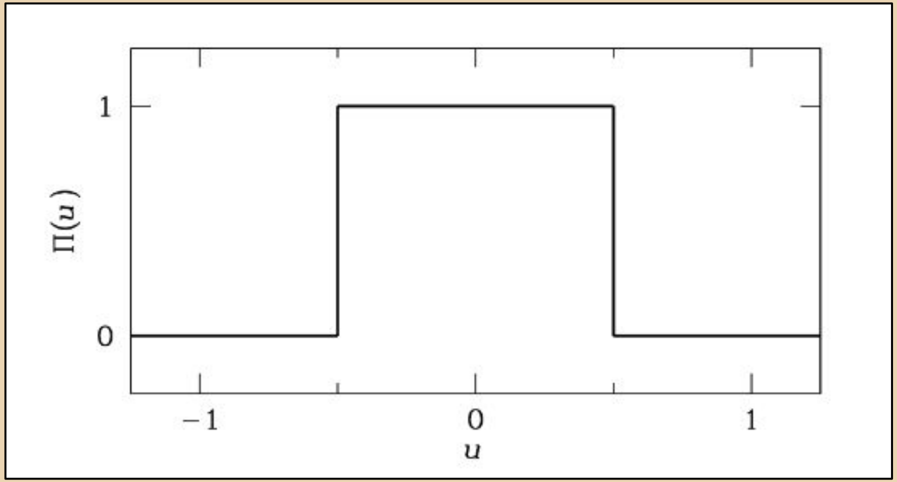
$$\theta = 1.22\frac{\lambda}{D}$$

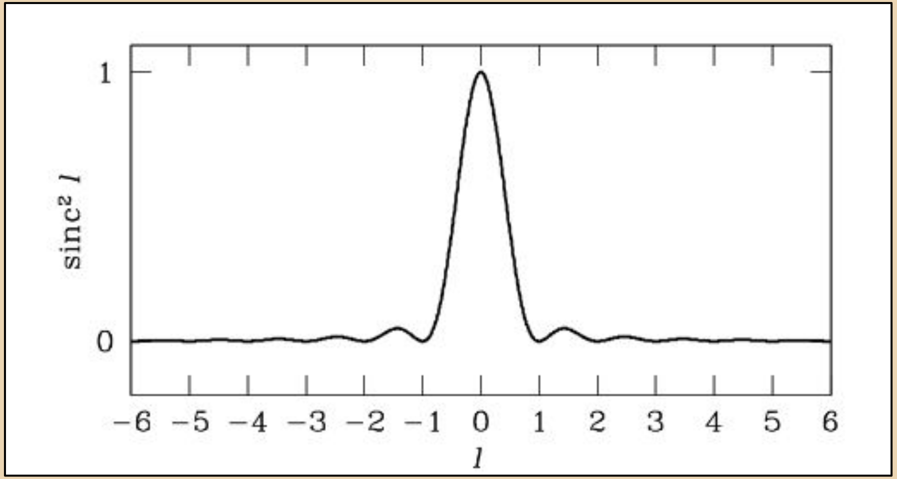
- Works well if no spatial resolution is required
  - Time domain things like pulsars
  - Frequency domain things like emission/absorption lines
- Image can be created by scanning across the sky
  - Limited resolution (even with very big dish)
  - Slow (and slower if your resolution is better)

Credit: Wikipedia

Single dish - Beam

Beam pattern of single dish → square of Fourier transform of illumination pattern

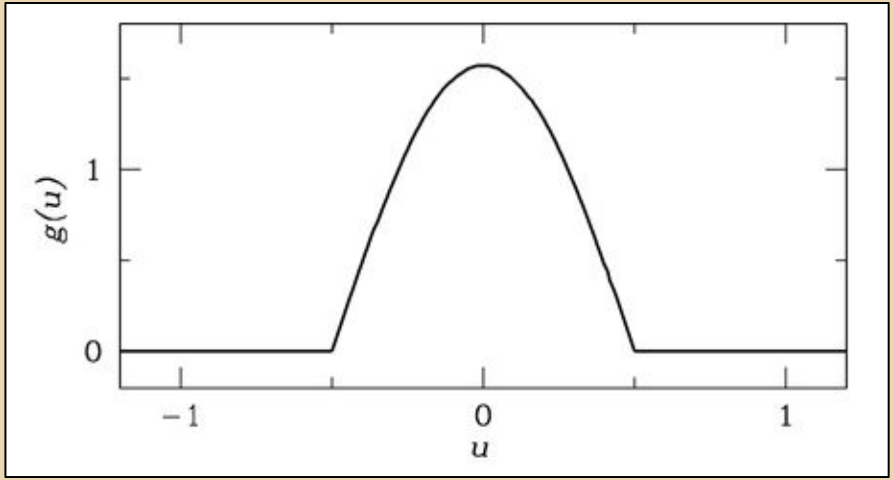


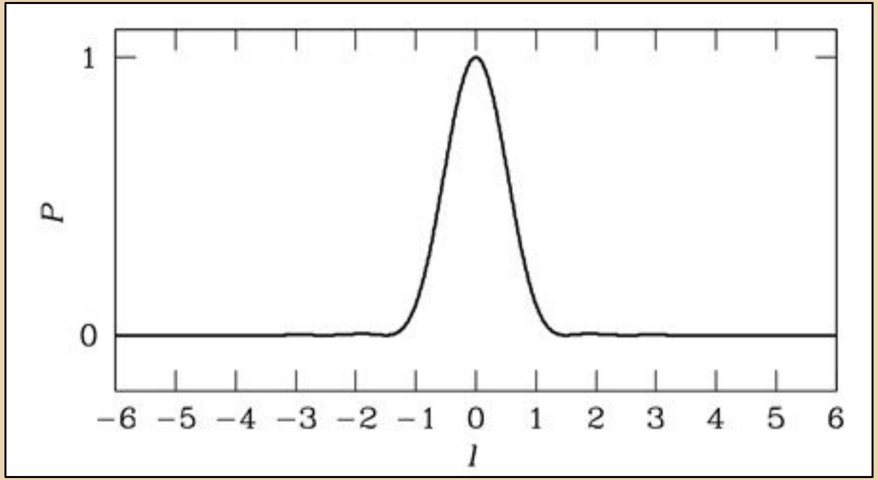


Credit: Condon & Ransom

Single dish - Beam

Taper illumination to control sidelobes

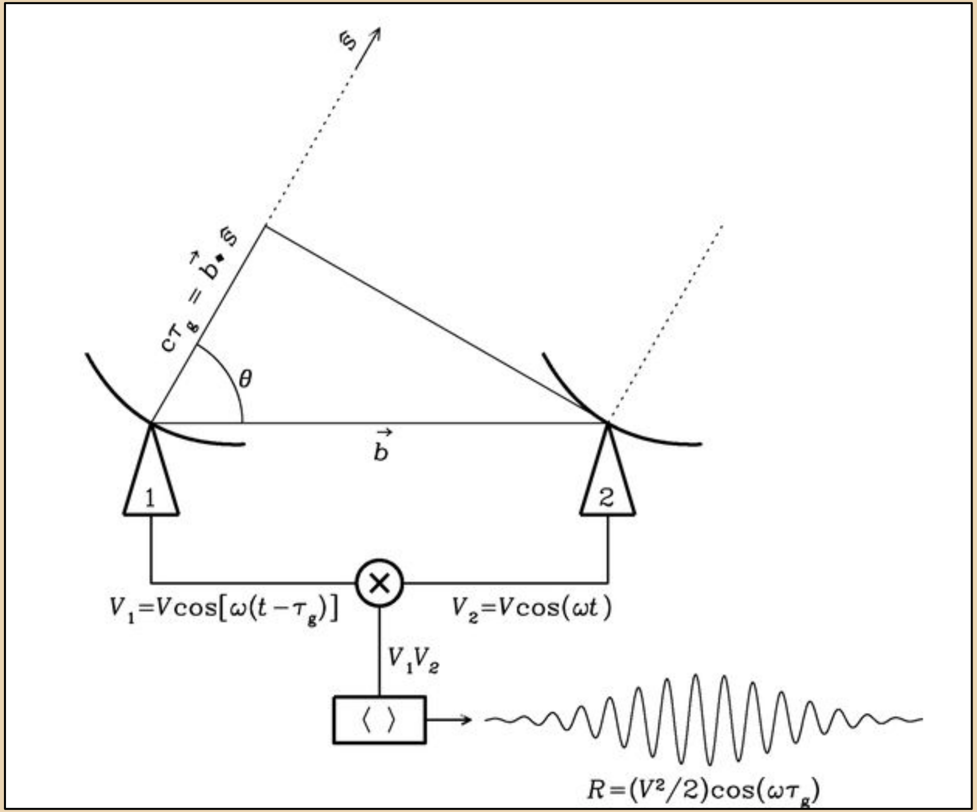




Credit: Condon & Ransom

Two element interferometer

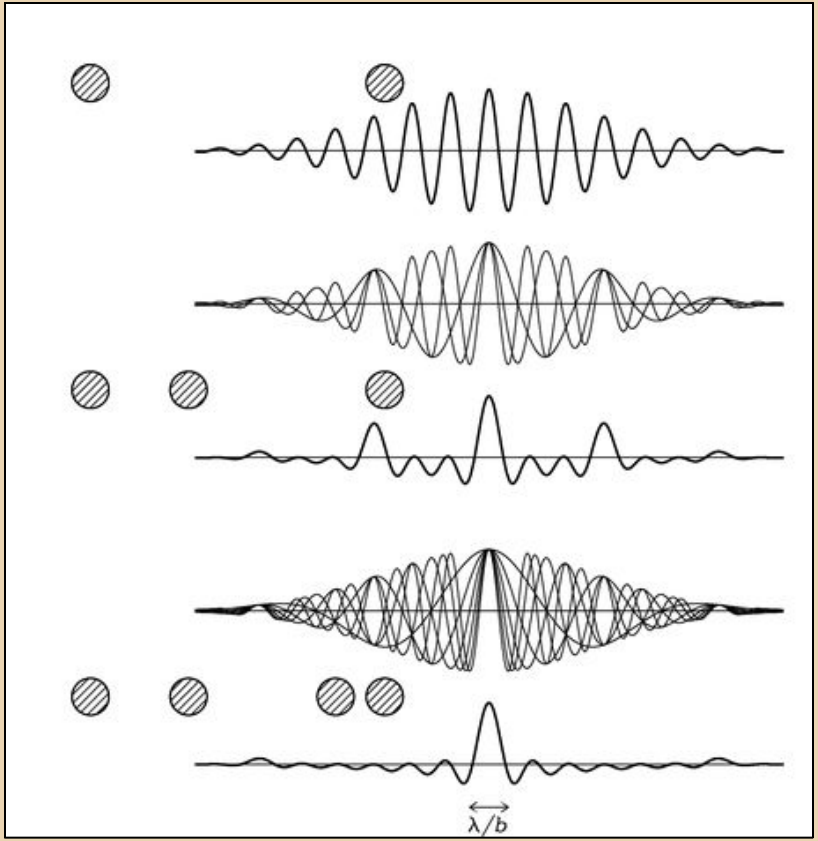
- Simplest interferometer
- Cross-correlates the signals coming from the sky
- Correlator multiplies and averages voltages from antennas, which eliminates all signals not common between antennas
- Resulting signal characterised by geometrical delay τ_g, determined by the **baseline** between the dishes, commonly referred to as **fringe**



Credit: Condon & Ransom

N-element interferometer

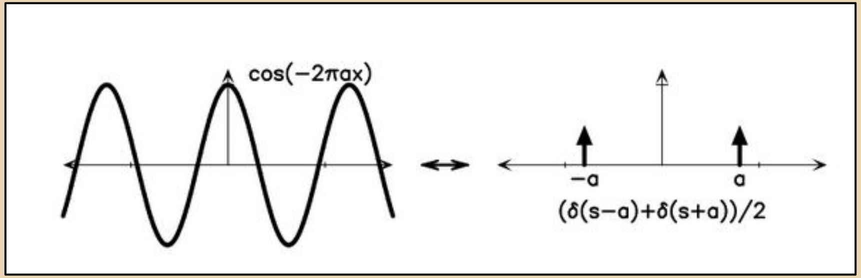
- Thing that we the antennas are pointing at is the **phase center**
- Adding more baselines constrains the spatial origin of the signal
- Basic principles stay the same, an interferometer with **N** elements can be characterised as **N(N-1)/2** two element interferometers
- **Synthesized beam** approaches a Gaussian with size $\theta = \lambda / B$ with **B** the longest baseline



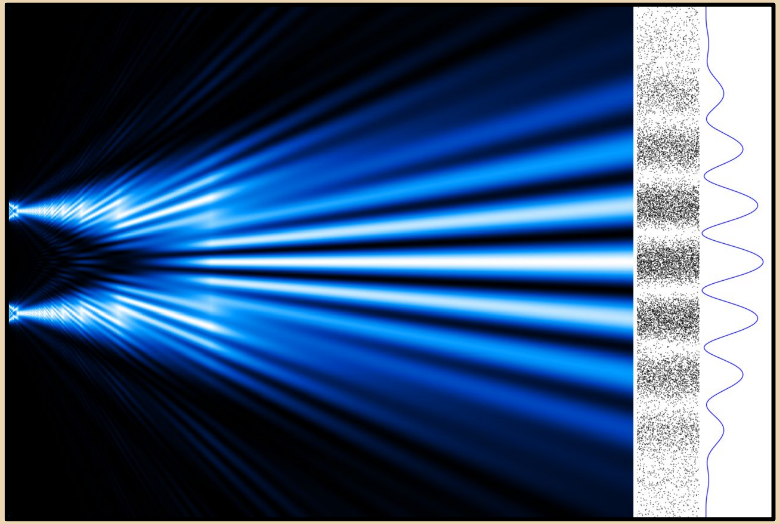
Credit: Condon & Ransom

Representing our baselines in Fourier space

- Each combination of dishes are essentially a **double slit**
- The resulting measured radiation pattern is their **Fourier transform**



Credit: Condon & Ransom

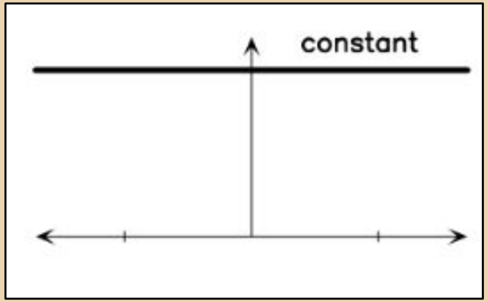


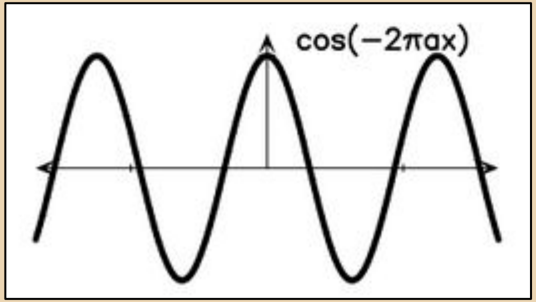
Credit: Wikipedia

Minimum and maximum angular scales

- Each baseline has an associated physical scale that it is sensitive to, according to $\theta = \lambda / B$
- The **minimum angular scale** visible to the telescope is determined by the longest baseline; this the **resolution**
- The **maximum angular scale** is determined by the shortest baseline, any physical structure larger than this will be **averaged out.**

$$\mathcal{V} = \int I(s) \exp(-2\pi i \frac{b \cdot s}{\lambda}) \mathrm{d}\Omega$$

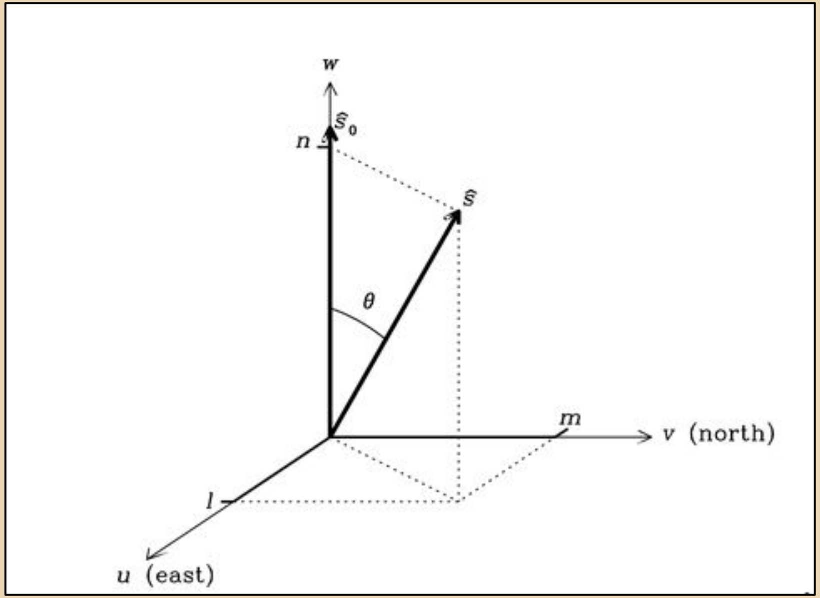




Credit: Condon & Ransom

Going 2D: The (u,v) plane

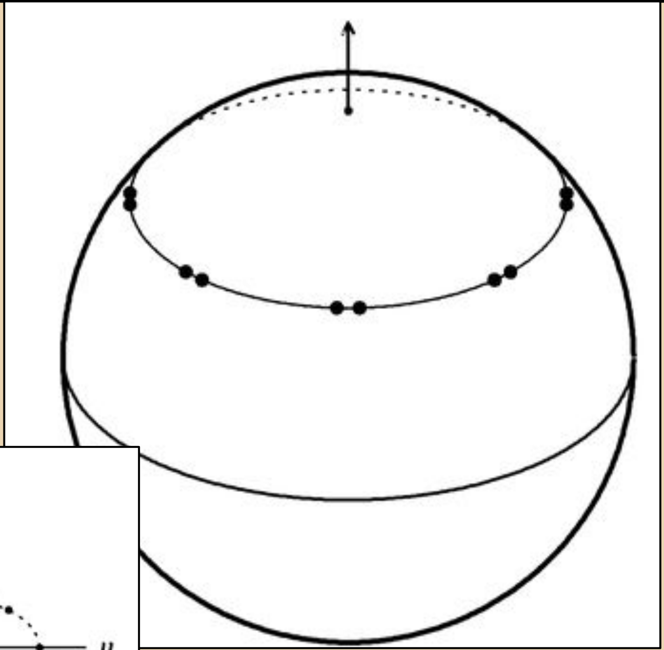
- Represent baselines as points in (u,v) space
- Each baseline adds two points
- Most conveniently described in units of λ
- The synthesized beam of the interferometer is then the Fourier transform of the (u,v) plane
- To match spatial dimensions, the complete coordinate system is **(u,v,w)**, with matching coordinates **(l,m,n)** describing the direction of the source

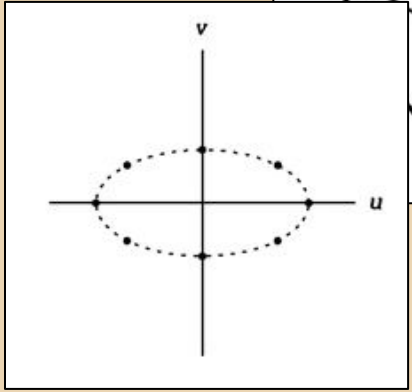


Credit: Condon & Ransom

Rotation aperture synthesis

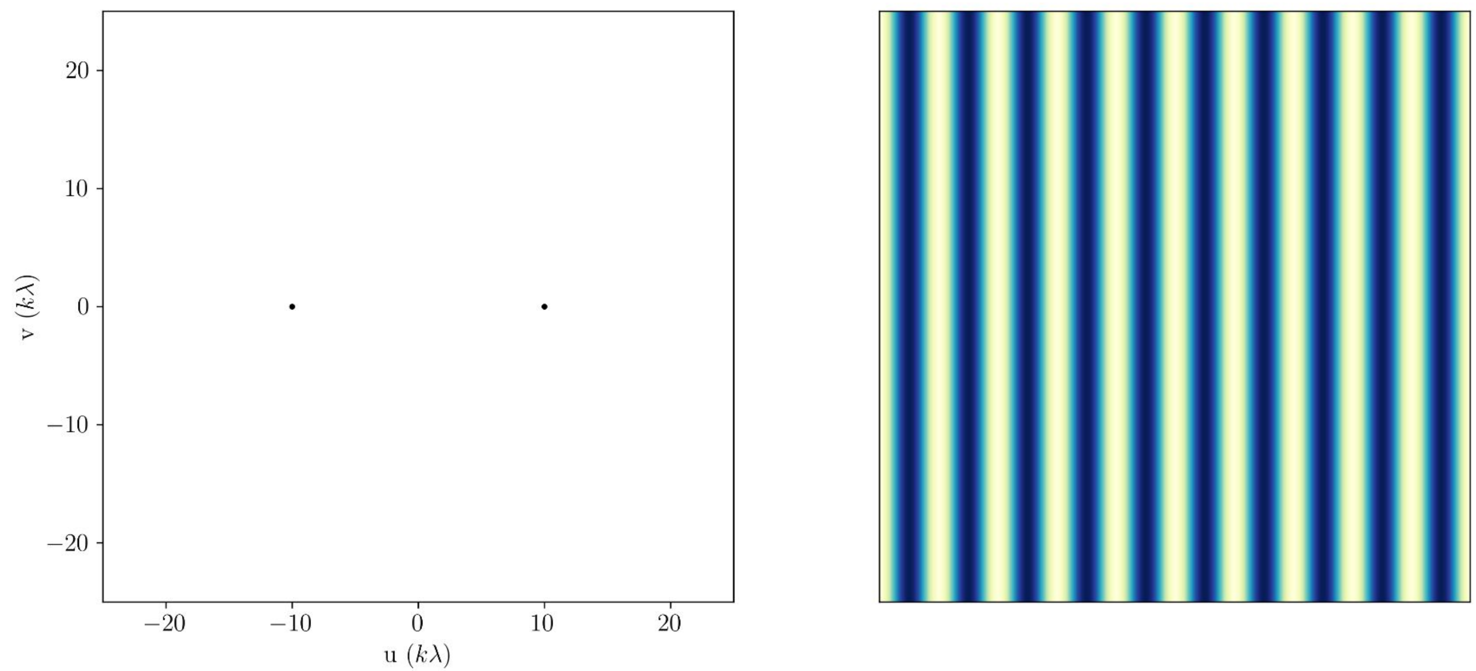
- **Q:** How to increase coverage of the (u,v) plane?
- **A:** Let the earth do it for you
- As the earth rotates, baselines trace out a path in (u,v) space
- A 12-hour integration will maximally fill the (u,v) plane (or less time depending on array configuration)



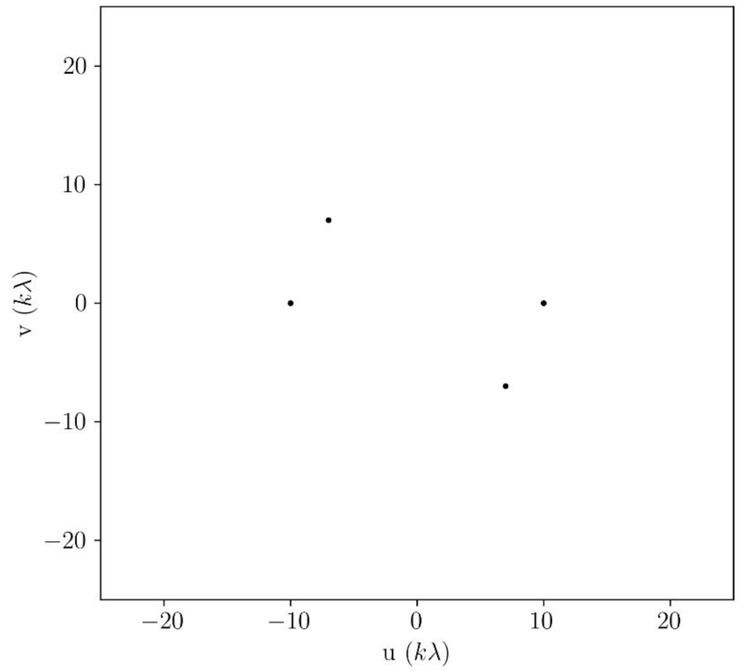


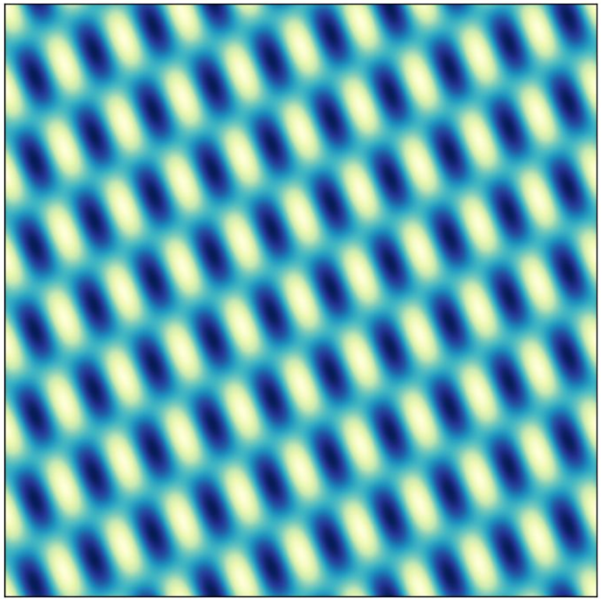
Credit: Condon & Ransom

Filling up the UV plane

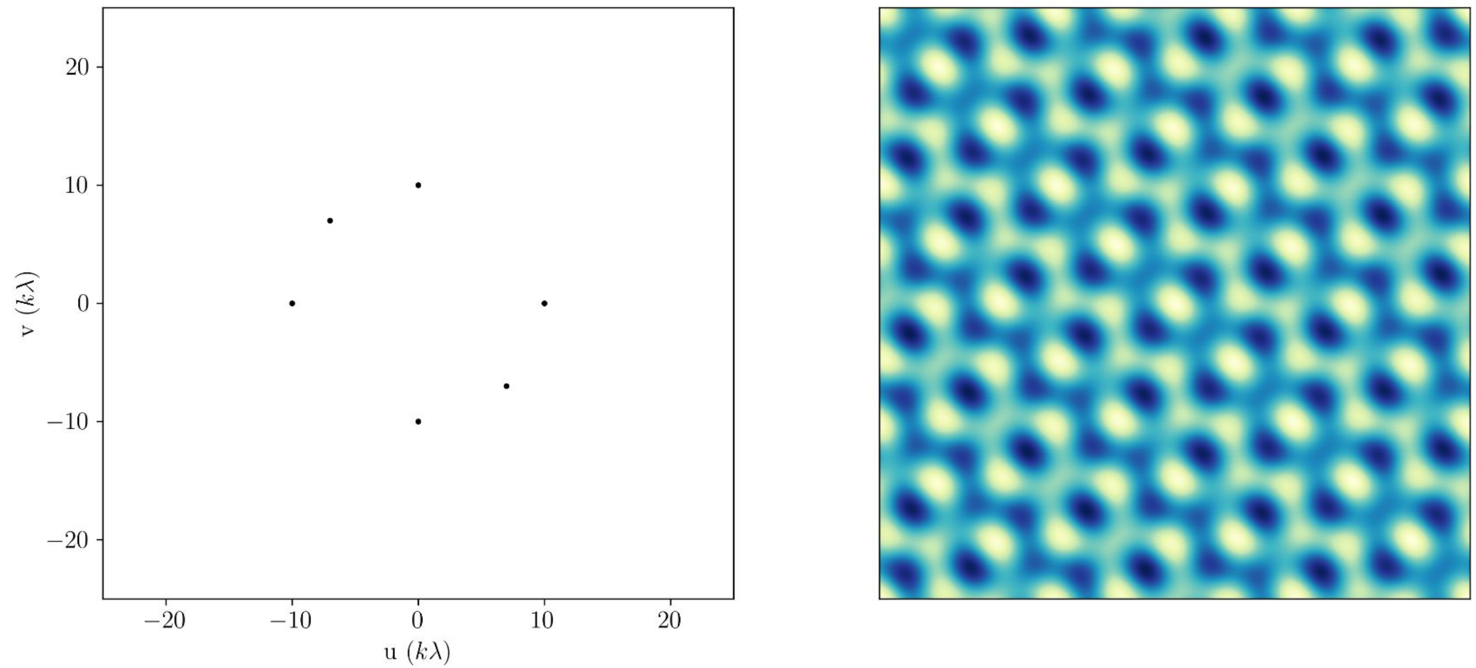


Filling up the UV plane

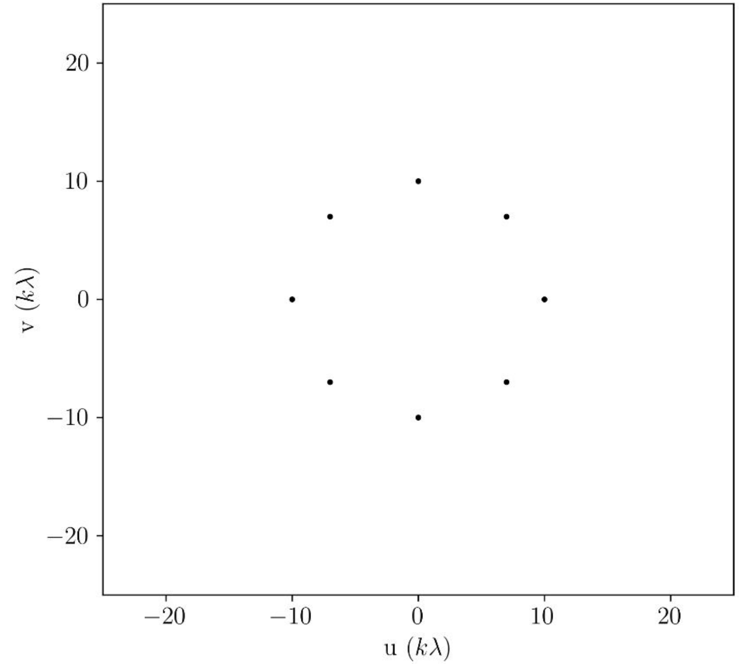


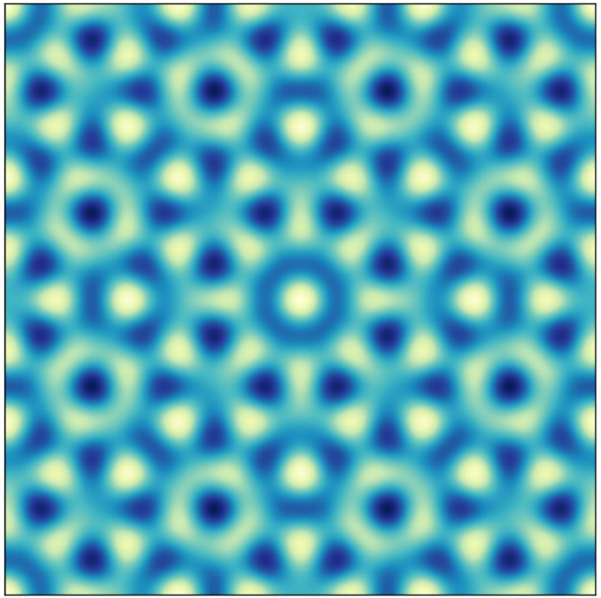


Filling up the UV plane

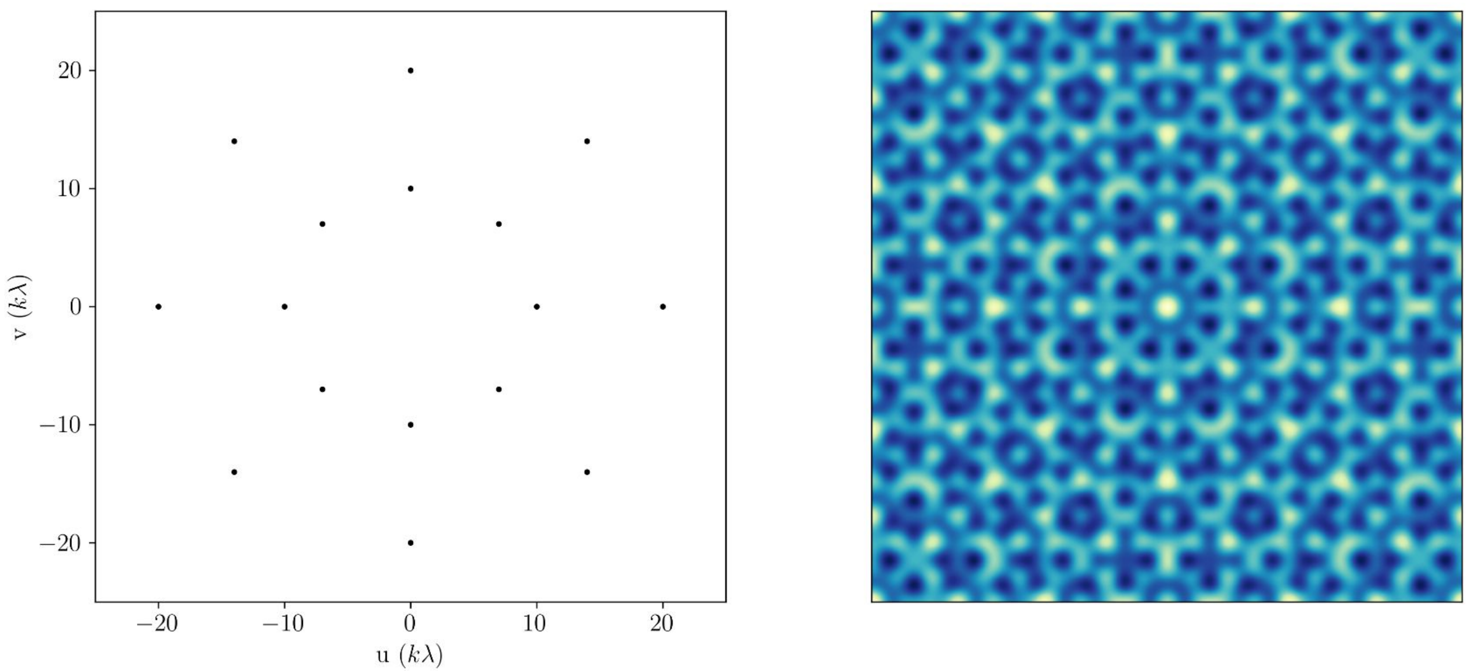


Filling up the UV plane





Filling up the UV plane



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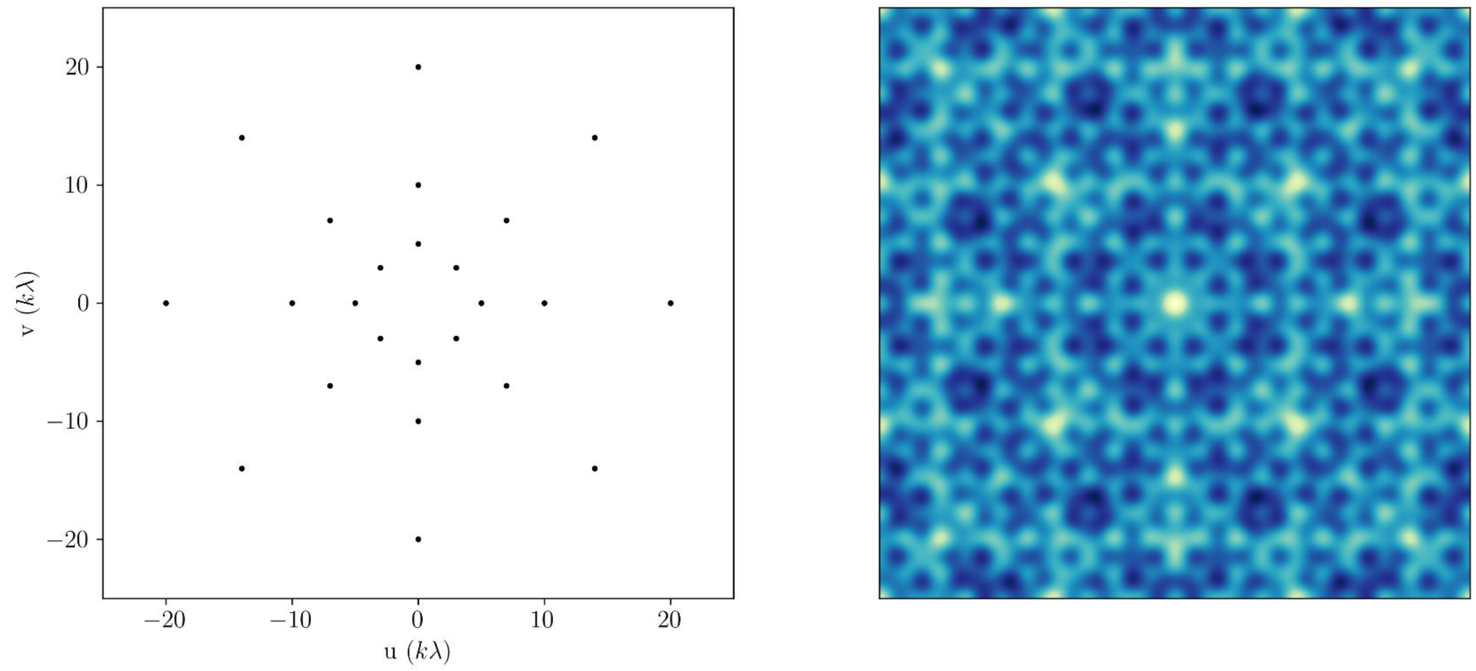


Visibilities

- The signal measured at a single point in (u,v,w) space is called a **visibility**

$$\mathcal{V}(u,v,w) = \int\int \frac{I_\nu(l,m)}{1-l^2-m^2}\exp[-2i\pi(ul+vm+wn)]\mathrm{d}l\mathrm{d}m$$

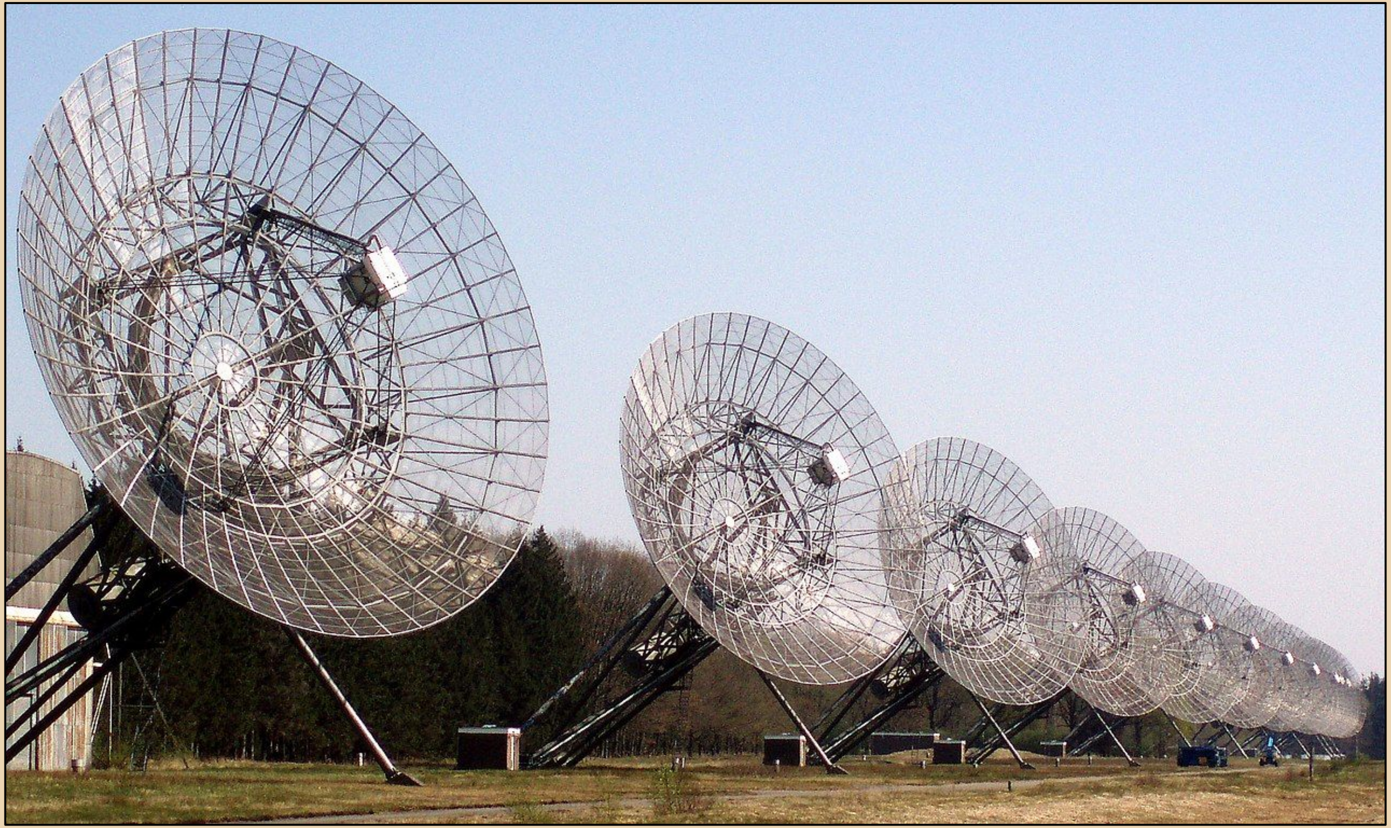
- This is not a 3D Fourier transform, so we don't like it
- To make it a Fourier transform, we have to neglect w, so assume our array is 2D

$$\mathcal{V}(u,v) = \int\int I_\nu(l,m)\exp[-2i\pi(ul+vm)]\mathrm{d}l\mathrm{d}m$$

- If we cannot neglect w, we use **w-projection** or **faceting**

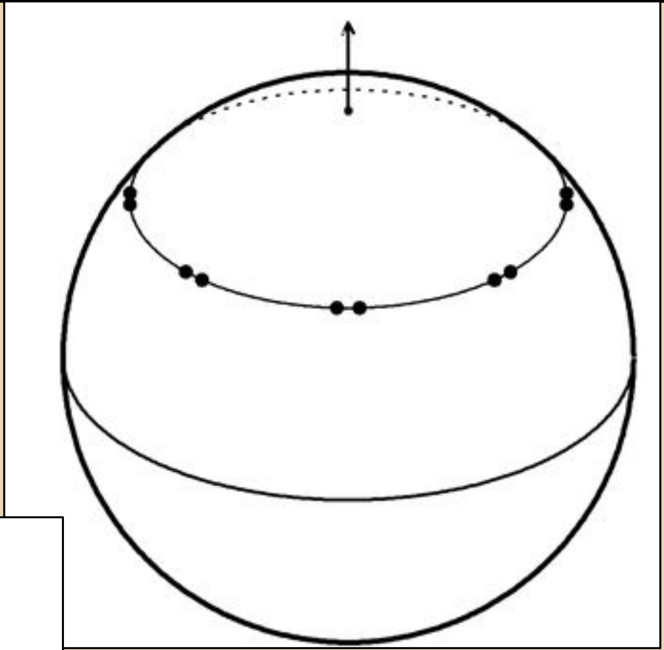
Westerbork

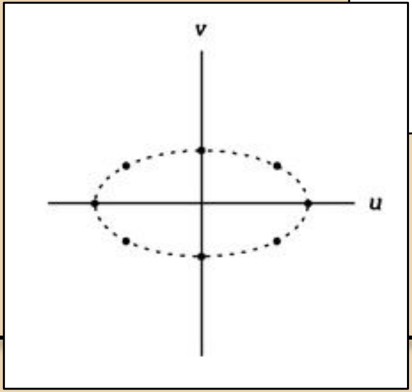
Credit: Wikipedia



Westerbork

- Fourteen dishes
  - D = 25 m
- Array is perfectly East-West aligned forming a line
- Antennas spread over 2.8 km
  - Min baseline: B = 144 m
  - Max baseline: B = 2.8 km
- Add earth rotation to create an ellipse in UV-plane
- Fourier transform is perfectly 2D



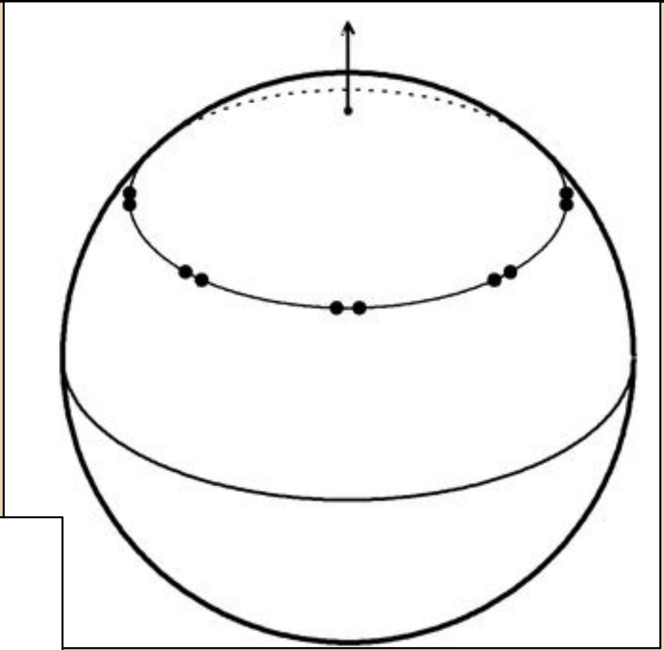


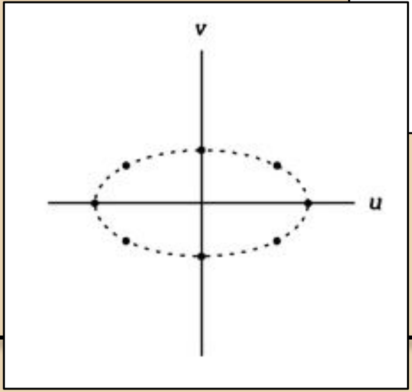
Credit: Condon & Ransom

Westerbork

$$\lambda = 21 \text{ cm}$$

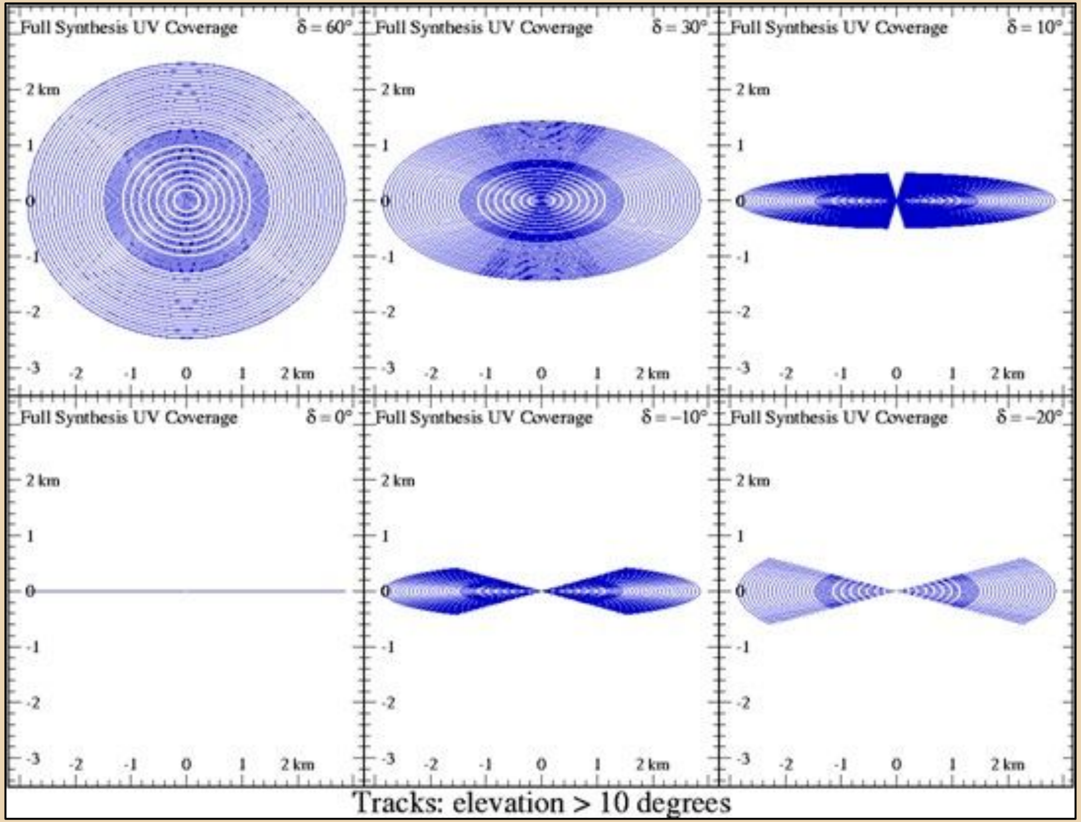
- Fourteen dishes
  - D = 25 m; **θ = 0.5°**
- Array is perfectly East-West aligned forming a line
- Antennas spread over 2.8 km
  - Min baseline: B = 144 m; **θ = 5'**
  - Max baseline: B = 2.8 km; **θ = 15"**
- Add earth rotation to create an ellipse in UV-plane
- Fourier transform is perfectly 2D





Credit: Condon & Ransom

Westerbork



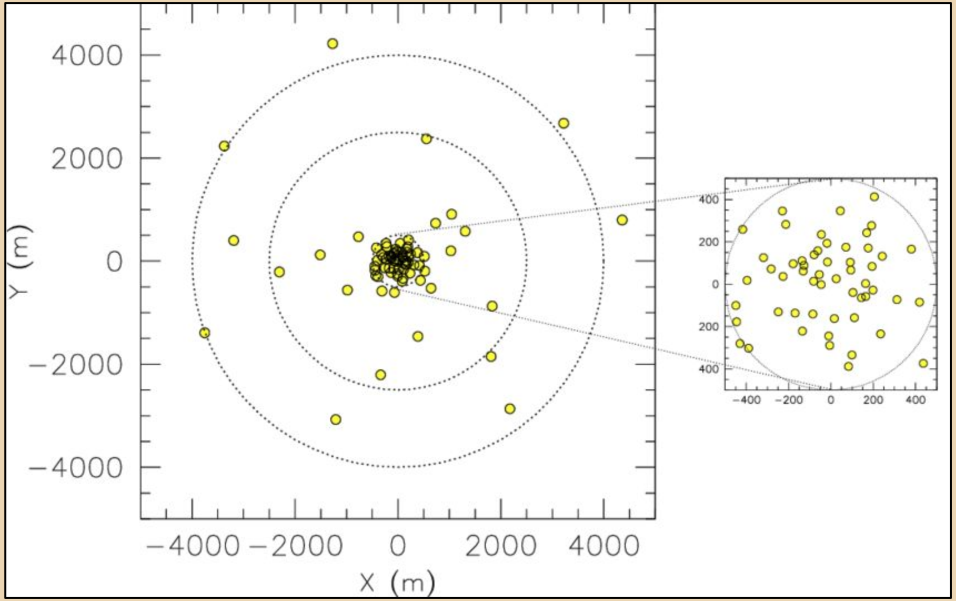
http://www.aoc.nrao.edu/events/synthesis/2006/lectures/TuesdayJune20/Cohen.pdf

MeerKAT

Credit: SARAO

MeerKAT

- First light in 2018
- 64 dishes
  - D = 13.5 m
- Many antennas in small core, with sparser longer baselines
  - Min baseline: B = 20 m
  - Max baseline: B = 8 km
- Sensitive to extended emission due to short baselines
- Semi-random spacings between telescopes to get better UV-coverage

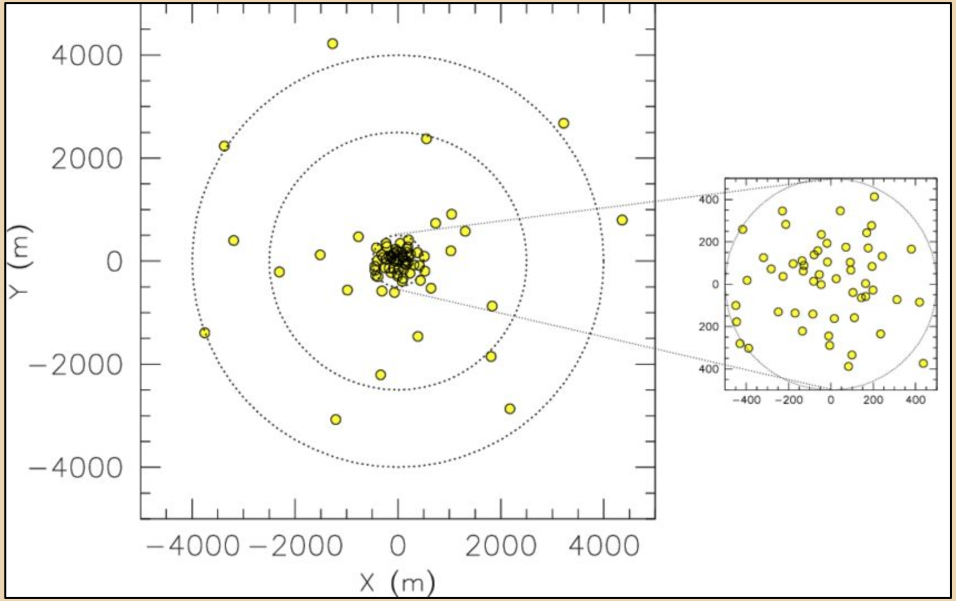


Credit: Booth et al. (2009)

MeerKAT

$\lambda = 21 \text{ cm}$

- First light in 2018
- 64 dishes
    - D = 13.5 m; **θ = 0.9°**
- Many antennas in small core, with sparser longer baselines
    - Min baseline: B = 20 m; **θ = 0.6°**
    - Max baseline: B = 8 km; **θ = 5.4”**
- Sensitive to extended emission due to short baselines
- Semi-random spacings between telescopes to get better UV-coverage

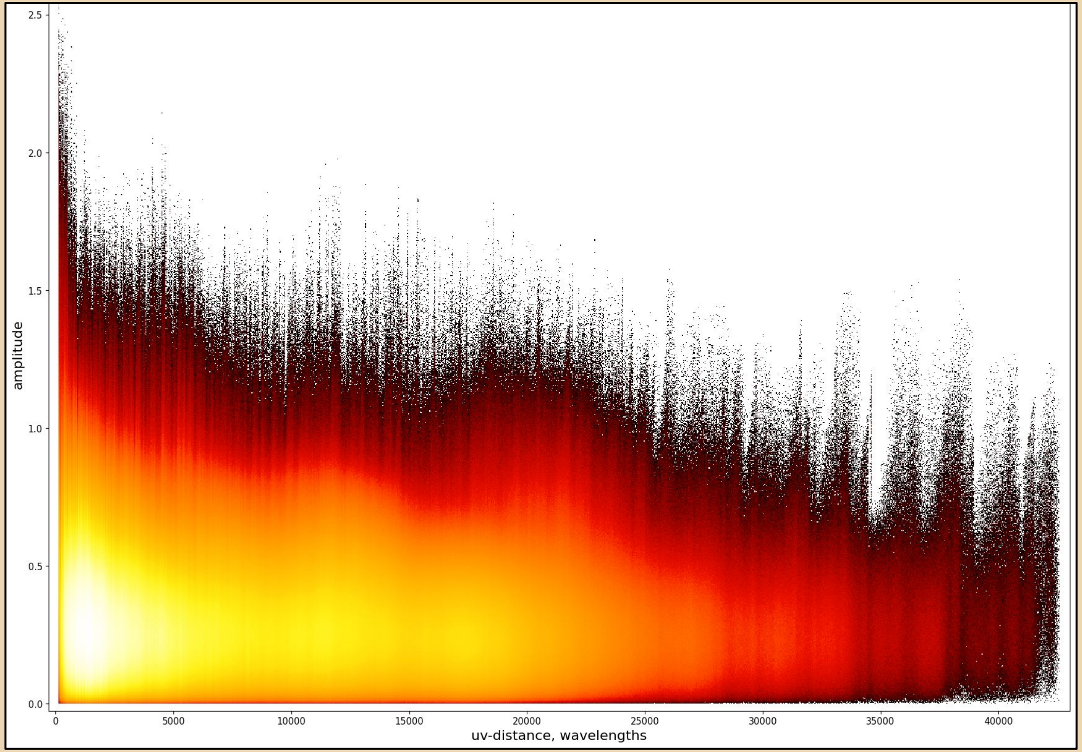


Credit: Booth et al. (2009)

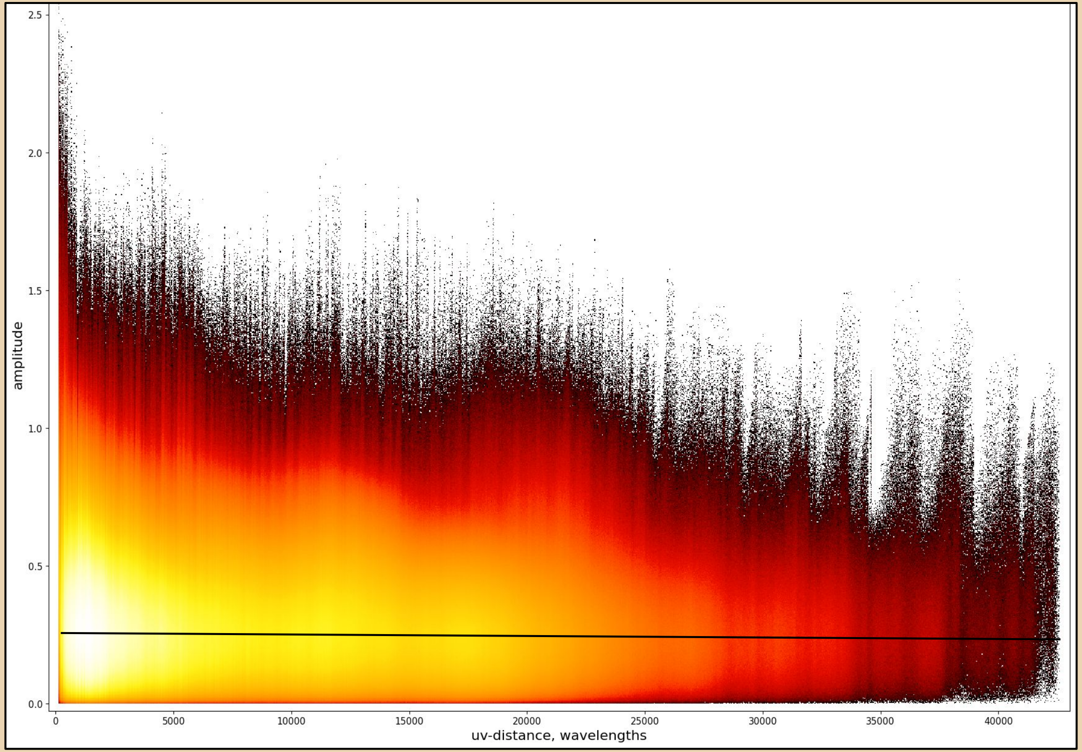
Dealing with interferometry data

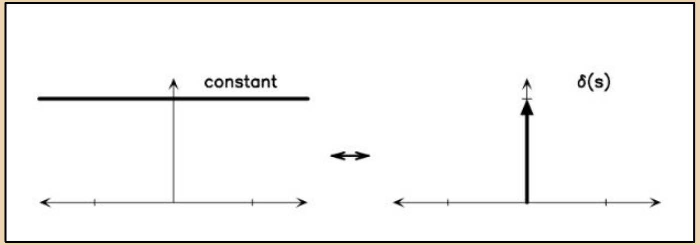
- Let's say we have visibilities from an interferometric observation
- We can make an image using Fourier transform straight away, but a good first inspection tool is the **UV-distance plot**
- Plot **visibilities** as a function of **baseline length**
- With a little knowledge of Fourier transforms, allows **deduction of source morphology and extent** without imaging, and was widely used in old radio astronomy papers

The UV-distance plot



The UV-distance plot





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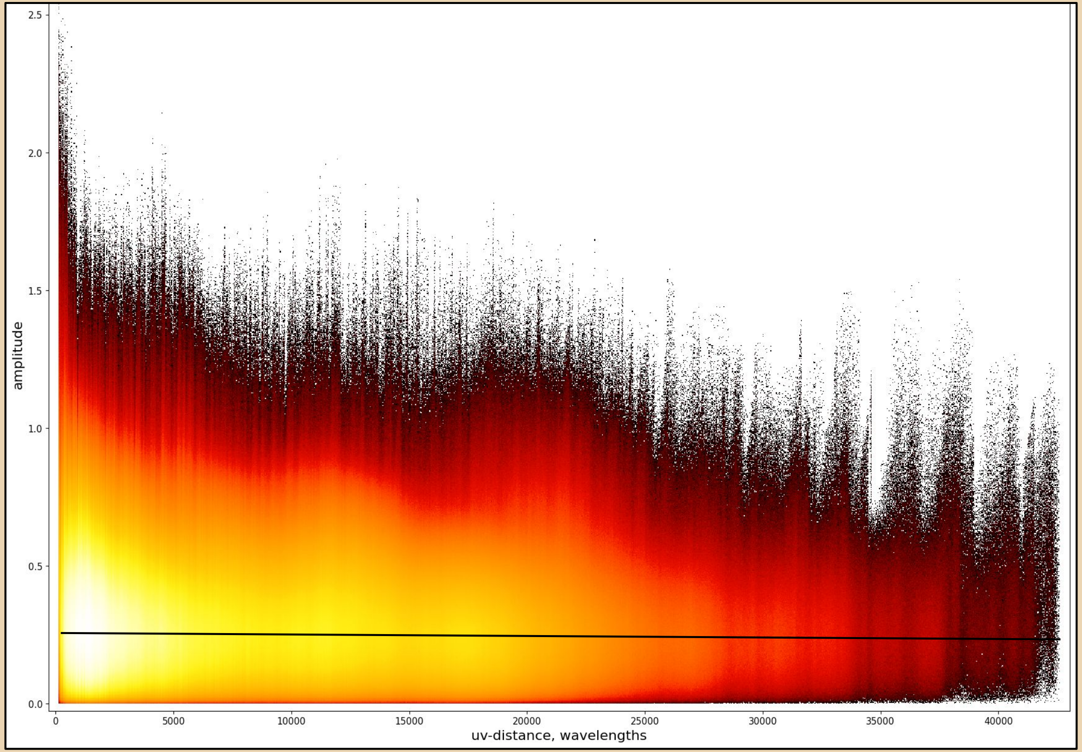

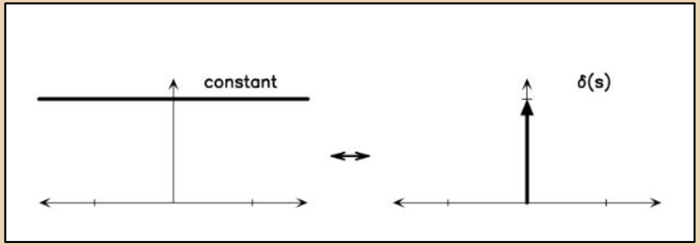

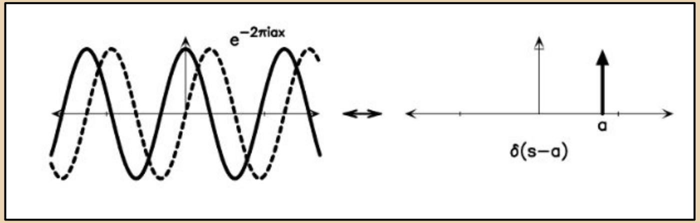

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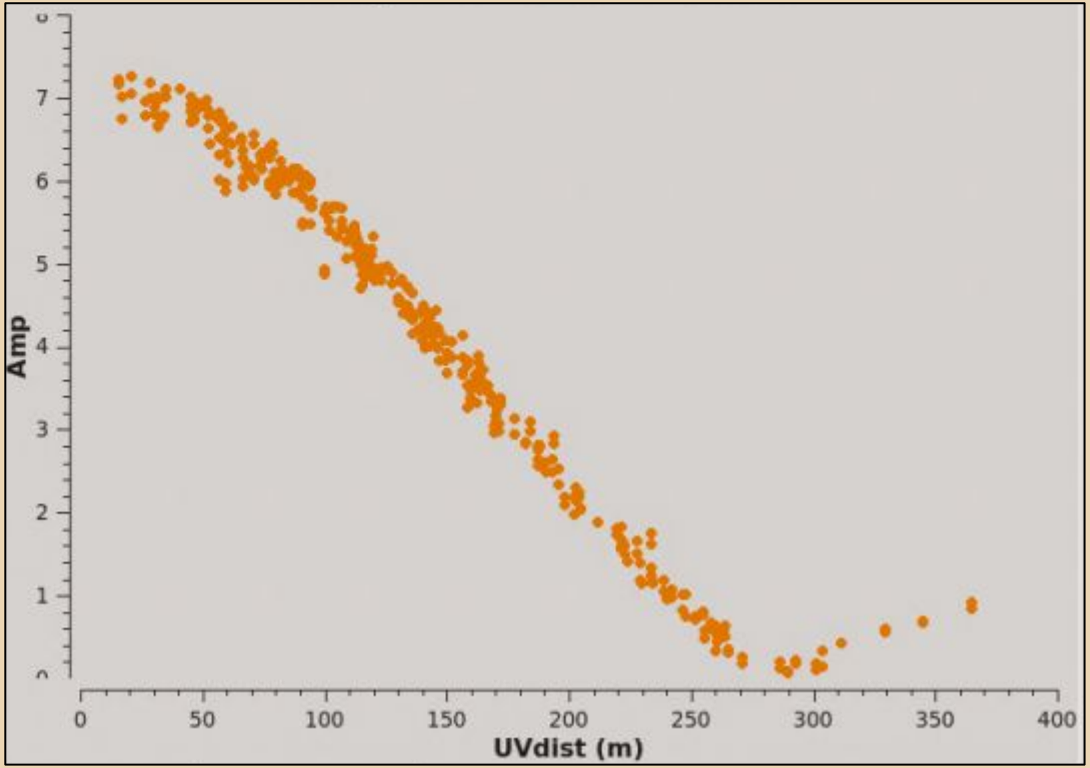

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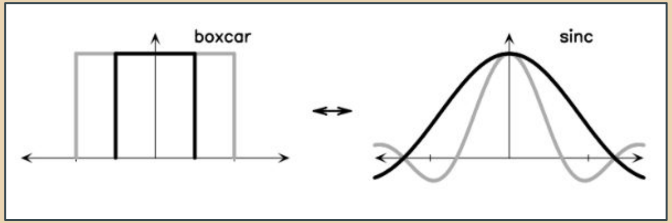
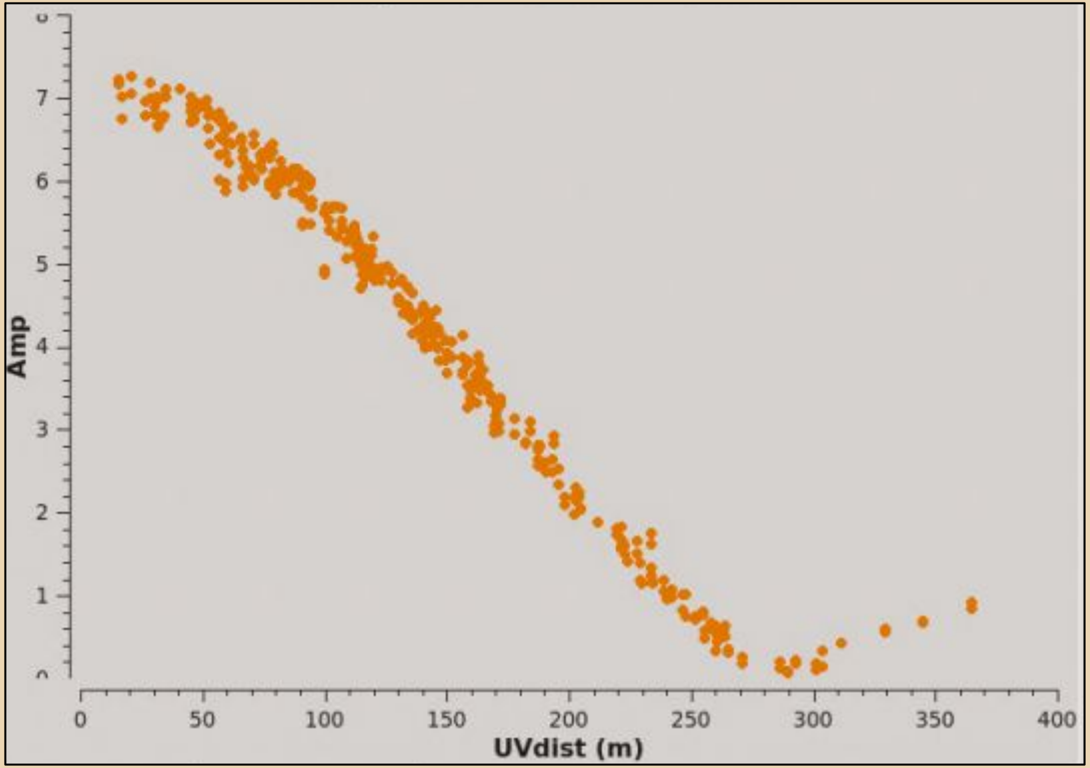


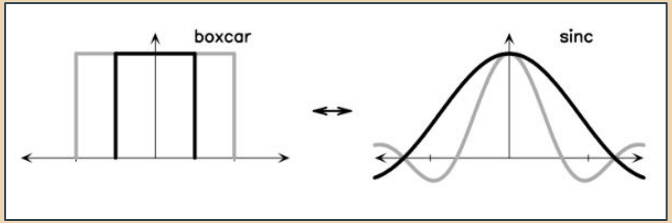


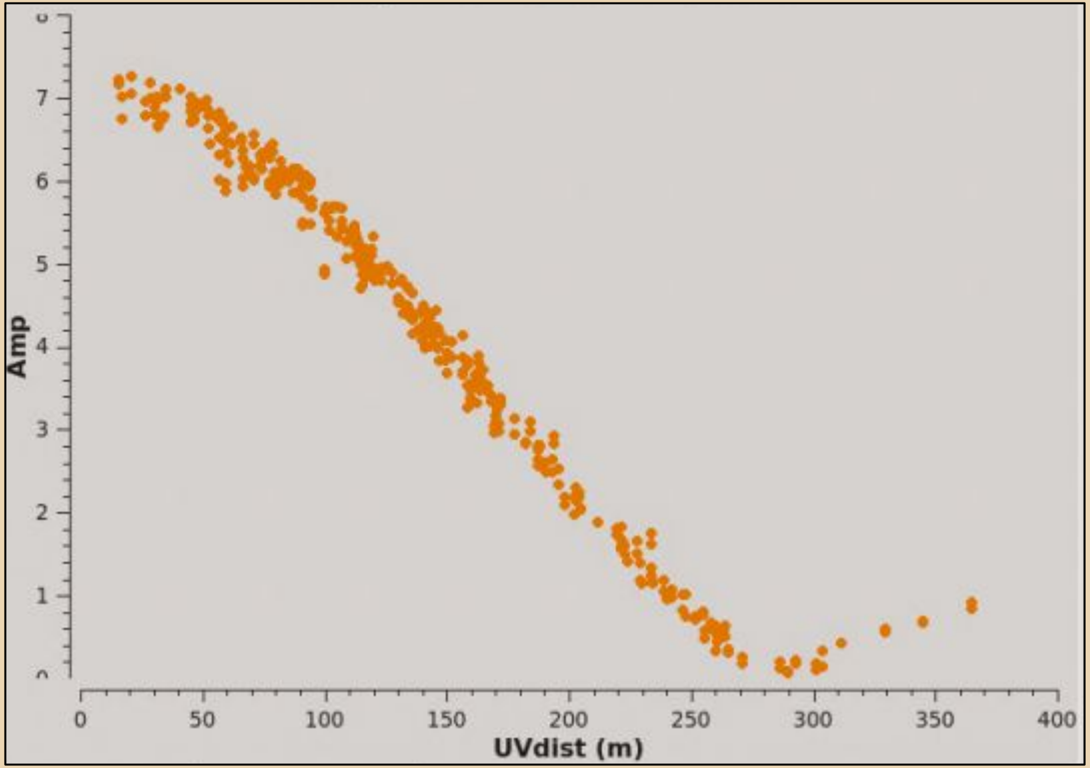
The UV-distance plot

ALMA observation

$\nu = 372$ GHz; $\lambda = 800$ μm

First null at: 280 m = 350 kλ





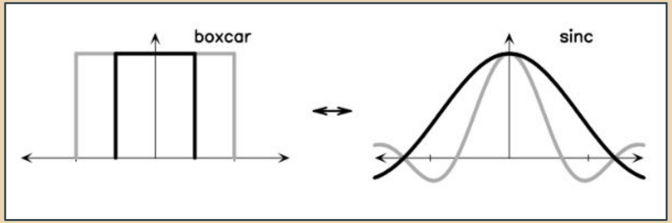
The UV-distance plot

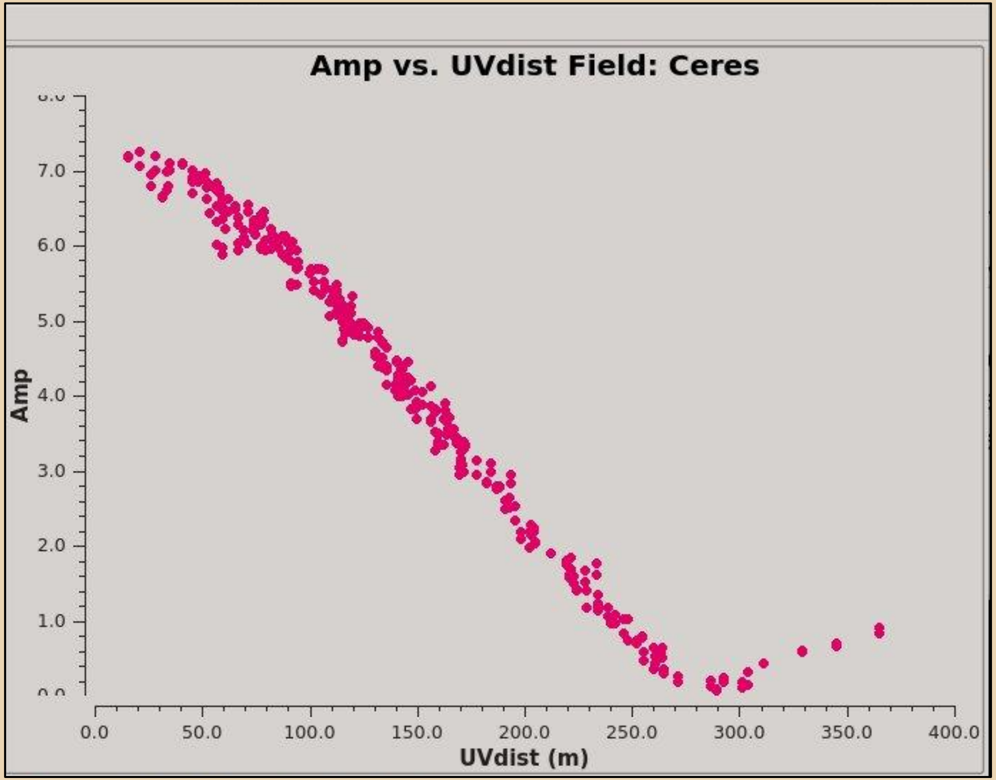
ALMA observation

$\nu = 372$ GHz; $\lambda = 800\ \mu$m

First null at: 280 m = 350 kλ

Size of object: $\boldsymbol{\theta = 0.6''}$





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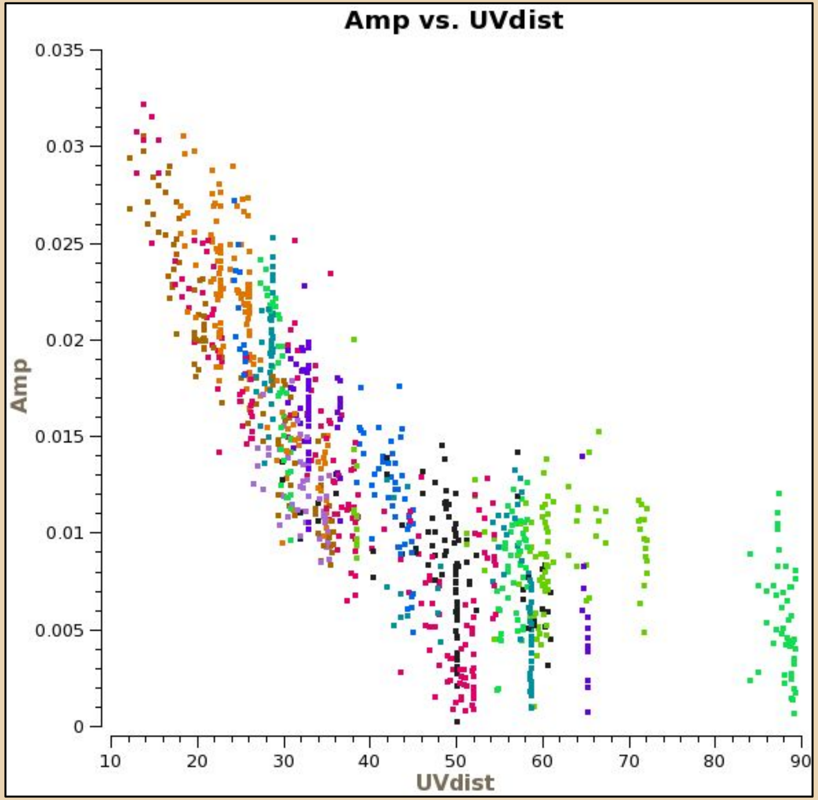

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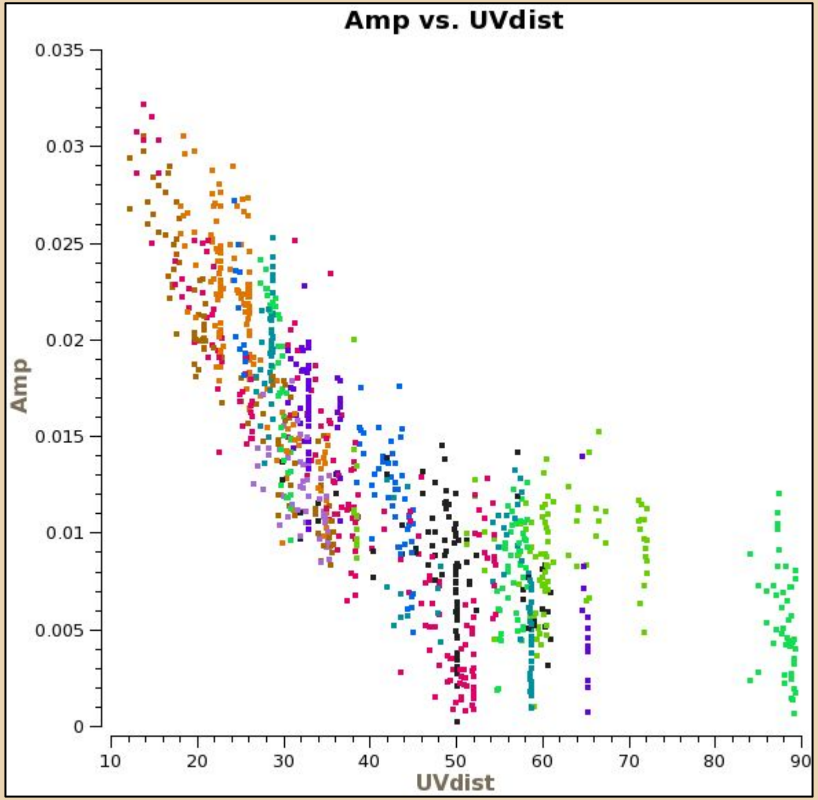
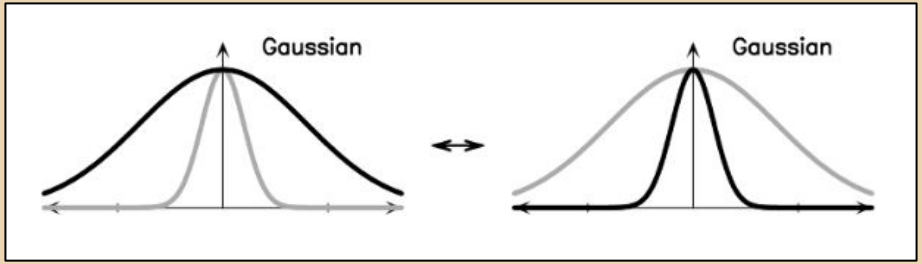
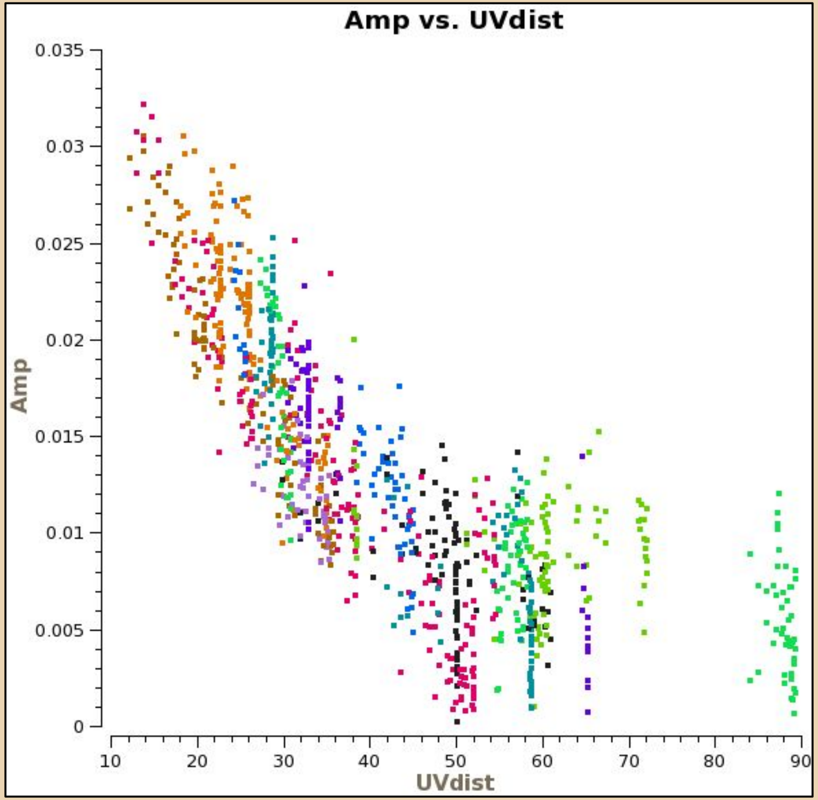
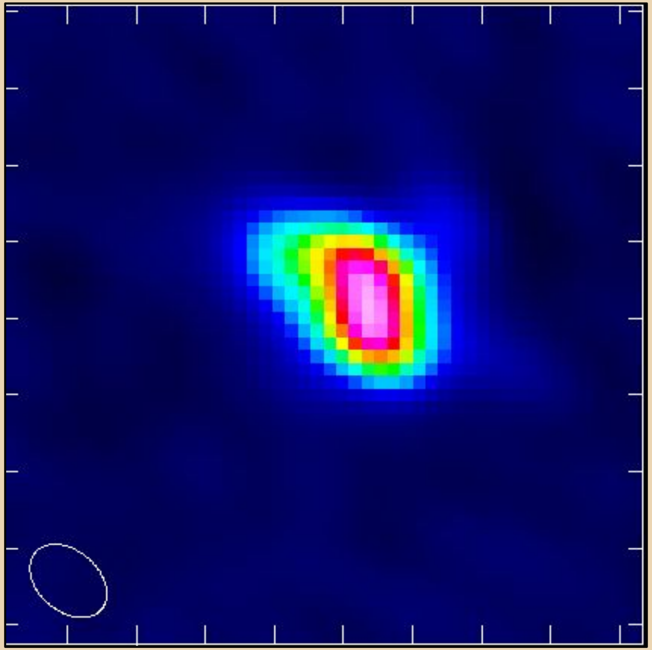




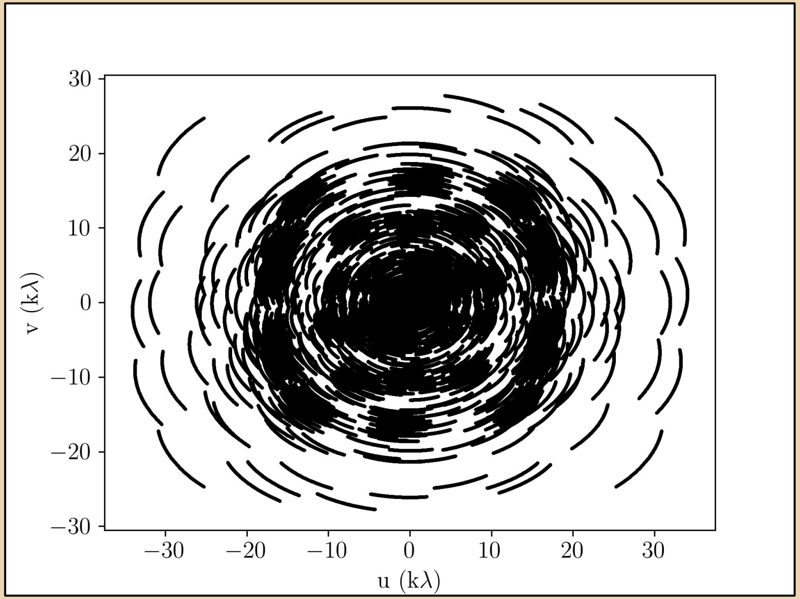
The UV-distance plot



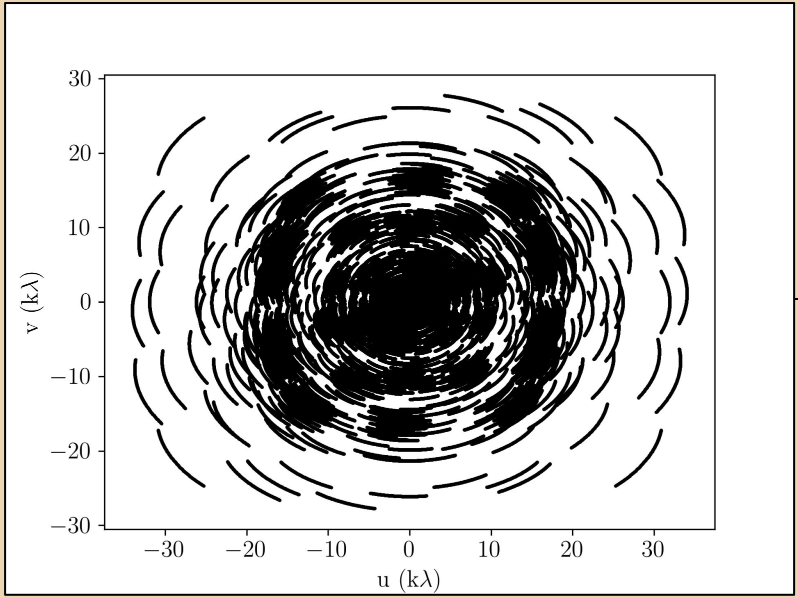


Making our image

- In order to compute the image (and synthesised beam), visibilities must be Fourier transformed, and therefore **gridded**
- Many pixels will contain multiple points, how to combine them together?
- **Weighting**
    - **Natural**: sum all visibilities in cell, emphasizes shorter baselines, worse resolution, better sensitivity
    - **Uniform**: correct cell for number of visibilities, emphasizes longer baselines, better resolution, worse sensitivity
    - **Briggs** : Anywhere between natural and uniform, based on **robust** parameter

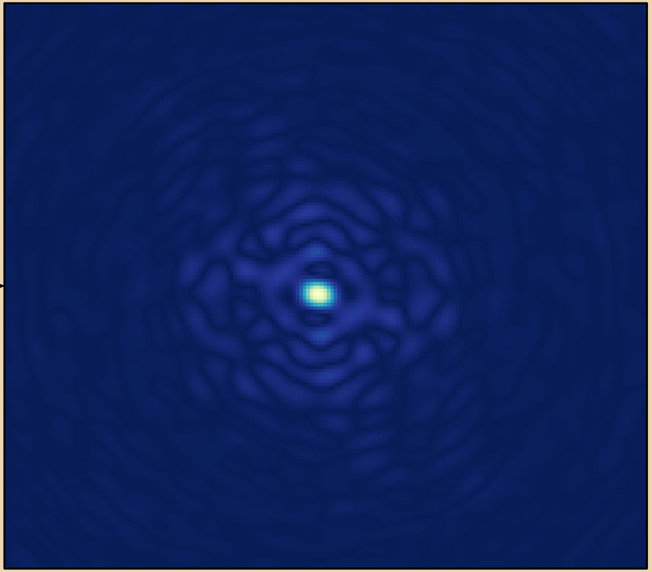


The synthesised beam



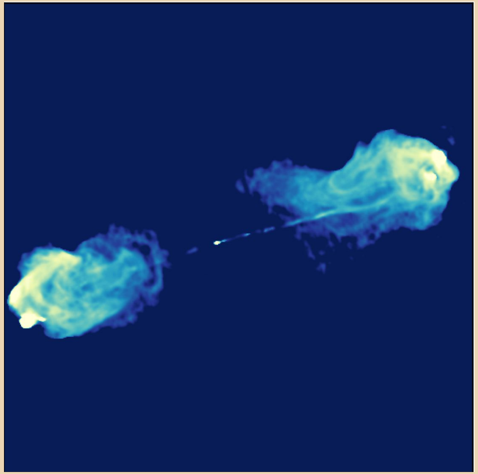
UV coverage

$\mathcal{F}$

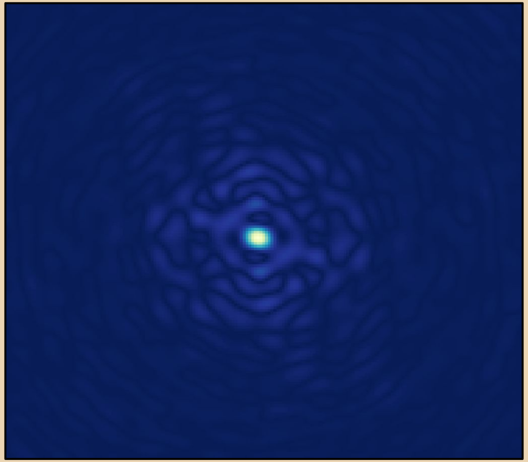


PSF/Synthesized beam/Dirty beam

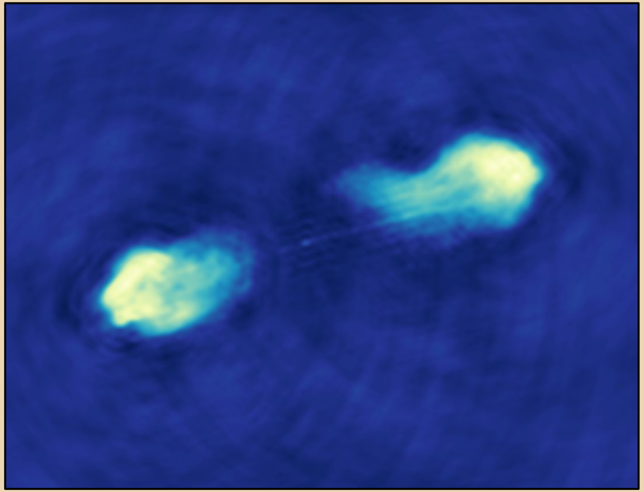
The final image



$\otimes$



$=$



Sky intensity

Dirty beam

Dirty image

Image deconvolution

- Fourier transform the observed visibilities $\mathcal{V}(u, v)$ to get dirty image $I_\nu^D(l, m)$
- We are essentially observing with missing information on various scales, which is represented by the shape of the dirty beam/synthesised beam/PSF
- If we would have no further knowledge of the sources in the sky, this is the best that we can do
- However: things in nature follow certain rules
  - Emission cannot be negative
  - Things should be smoothly varying
  - Many sources are unresolved (=smaller than the beam)
- Thus: we can build a **model** of the sky, with which we can **deconvolve** the image

Högbom CLEAN algorithm

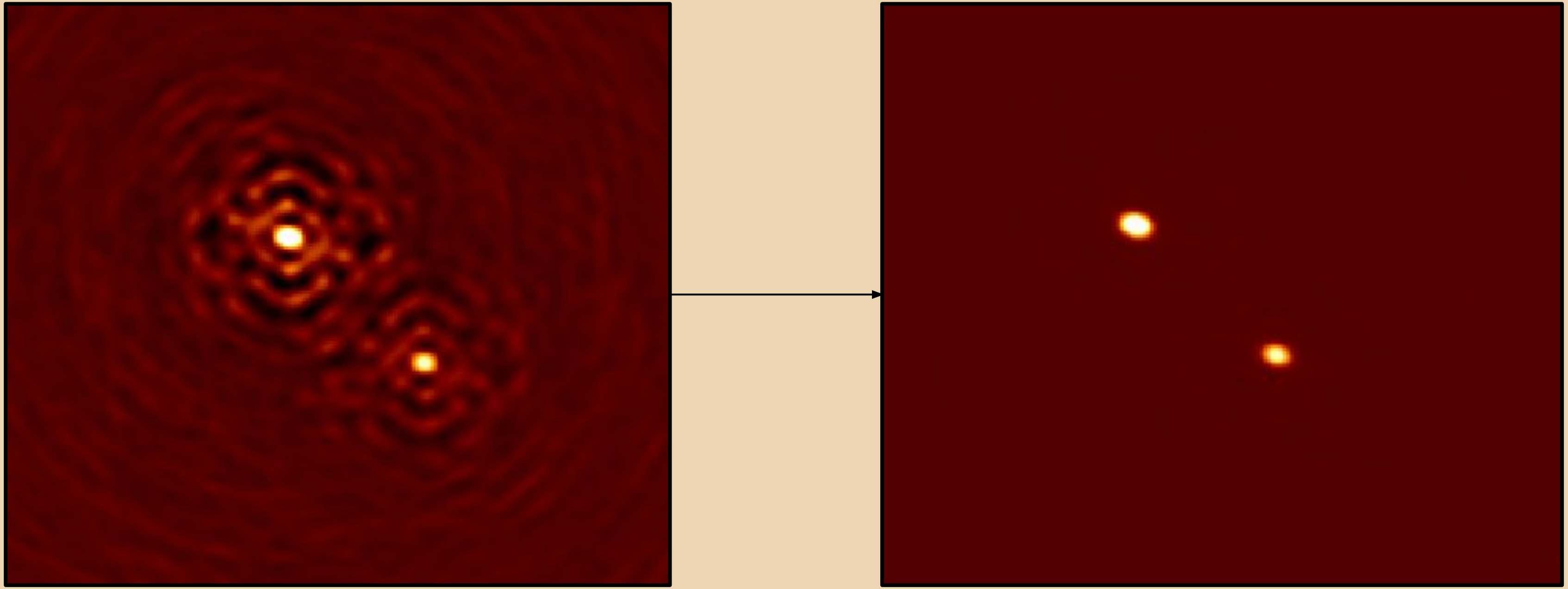
1. Find the brightest pixel in the dirty image
2. Subtract from the dirty image, at the position of the pixel, the dirty beam multiplied by the brightness of the pixel and a gain factor
3. Add the subtracted brightness, at the position of the pixel, to the point-source model image
4. Repeat 1-3 until either:
    a. The highest peak is below a user-specified threshold
    b. A user-specified number of iterations is reached
5. Convolve the point-source model with the CLEAN beam (usually an elliptical Gaussian fitted to the central lobe of the dirty beam) to create CLEAN image
6. Add the CLEAN image and the residual dirty image together

Usual extensions to CLEAN

- **Multi-frequency synthesis**: for larger bandwidth, model emission as a function of frequency by fitting Taylor polynomials
- **Multi-scale:** Model the emission at various scales, not just as a collection of point sources (useful for extended emission)
- **Faceting:** Divide the image into a number of facets with individual phase centers (to minimize w-term)
- **W-projection:** Use a discrete number of w-planes during gridding to account for w-term
- **A-projection:** Correct for variability of the primary beam over time and per antenna, for each baselines

CLEAN implementations

- AIPS (old), CASA (standard), WSclean (state of the art)



Do it yourself

- Go to: https://github.com/JonahDW/Interferometry-for-dummies
- Two jupyter notebooks are there, try not to peek at the solutions before you've given it a fair shake
- Make sure you have jupyter notebook, astropy, numpy, matplotlib
- CARTA/DS9 will be useful for inspecting FITS images