Interferometry for dummies

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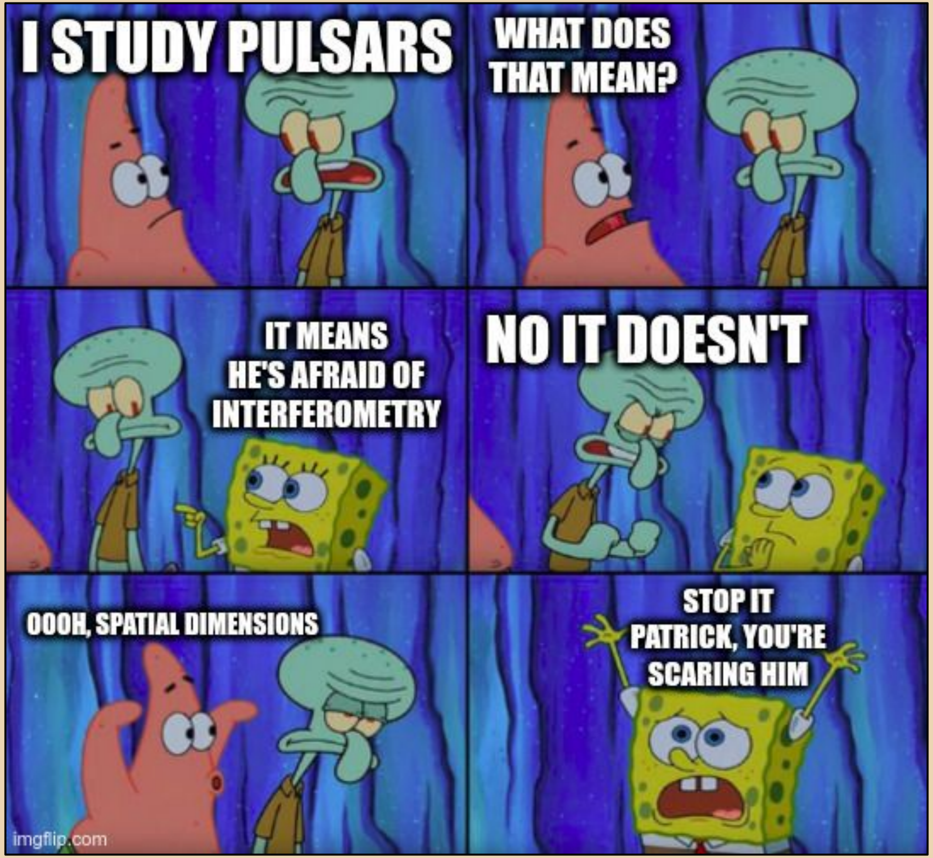
Outline

- Introduction
- Interferometry essentials
- Fourier transforms (without the math)
- Two element interferometer
- N element interferometers & the UV plane
  - Westerbork
  - MeerKAT
- Synthesis imaging
- Basics of CLEAN

Some references

- Essential Radio Astronomy by Condon & Ransom
  - https://www.cv.nrao.edu/~sransom/web/xxx.html
- Synthesis imaging in radio astronomy
  - https://leo.phys.unm.edu/g̃btaylor/astr423/s98book.pdf
- CASA tutorials or documentation
  - https://casaguides.nrao.edu/index.php?title=VLA_CASA_Imaging-CASA6.5.2
  - https://casadocs.readthedocs.io/en/stable/notebooks/synthesis_imaging.html
- VLA workshops
  - 2019: https://science.nrao.edu/science/meetings/2019/vla-data-reduction
  - 2021: https://science.nrao.edu/science/meetings/2021/vla-data-reduction
- Simulating interferometric observations
  - https://github.com/crpurcell/friendlyVRI

Interferometry essentials



Interferometry essentials

Two things required to understand most of radio astronomy

- Some basic Fourier transforms
- The relation between physical scale and scale in the Fourier space

$$\theta = \frac{\lambda}{D}$$

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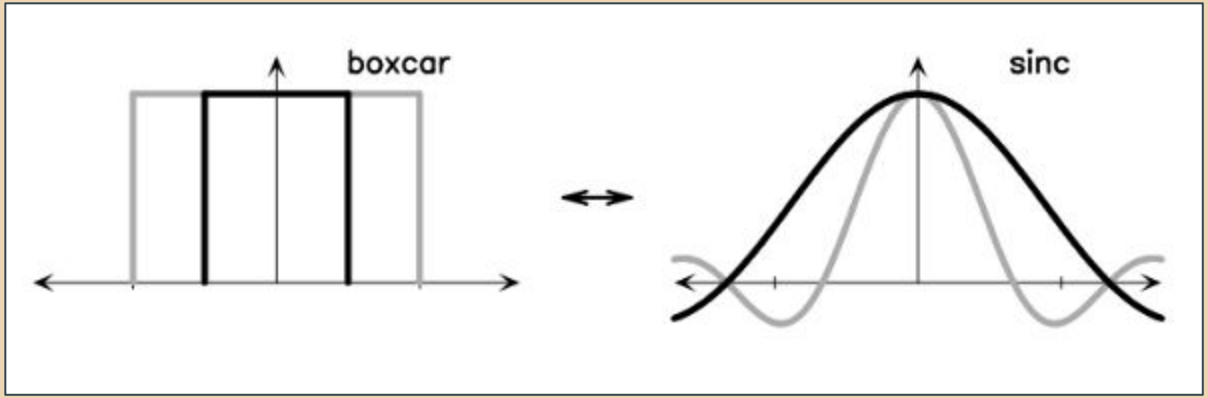
Some added concepts for interferometry

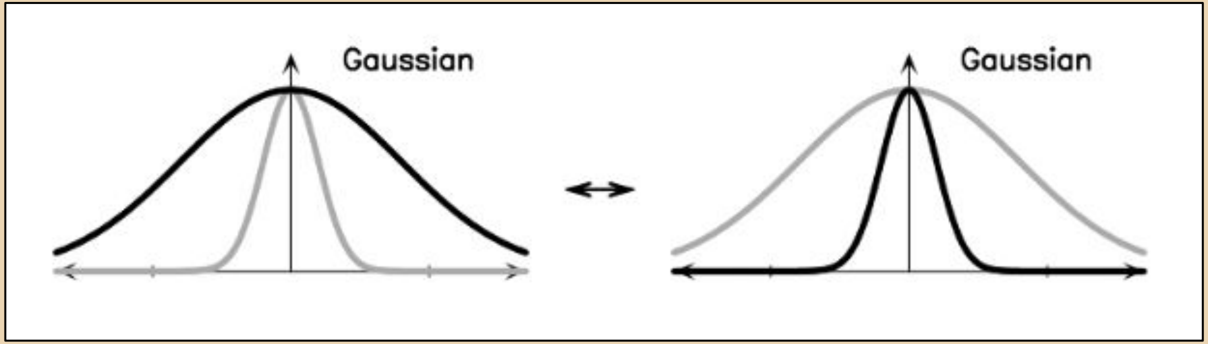
- The UV plane
- Image synthesis
- Deconvolution

Fourier transforms (without the math)

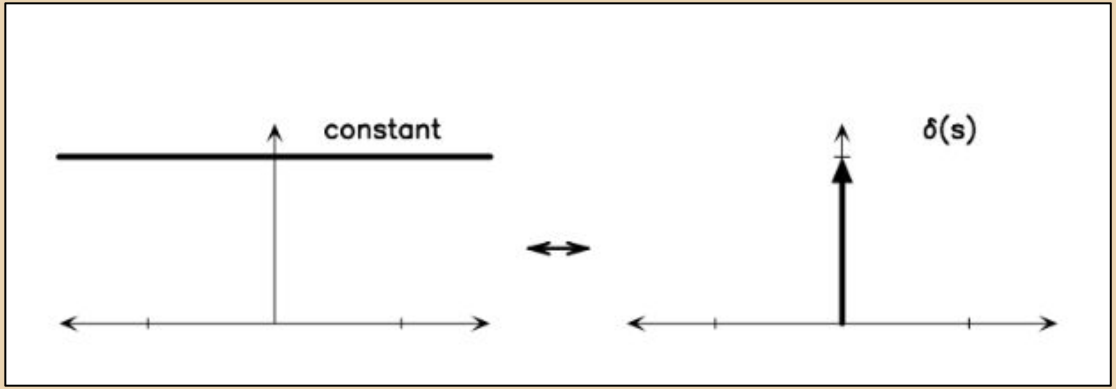
- Basically everything in radio astronomy involves Fourier transforms
    - Antenna responses
    - Interferometric aperture synthesis
    - Source morphologies
- However: most things are (geometrically) simple
- Knowing some Fourier transforms by heart will help you identify things

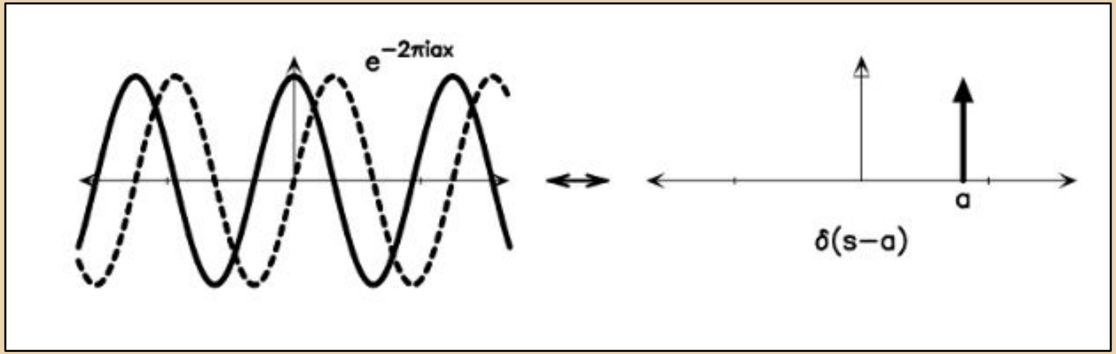
Fourier transforms (without the math)





Fourier transforms (without the math)





Basic Fourier theorems (now a little math)

Addition theorem: Addition in real space is addition in Fourier space

$$f(x) + g(x) \leftrightarrow F(s) + G(s)$$

Similarity theorem: Big in real space is small in Fourier space (and vice versa)

$$f(ax) \leftrightarrow \frac{F(s/a)}{|a|}$$

(very related to $\theta = \frac{\lambda}{D}$)

Convolution theorem: Convolution in real space is multiplication in Fourier space

$$f(x) \otimes g(x) \leftrightarrow F(s) \cdot G(s)$$

Cross-correlation theorem: Cross- correlation in real space is multiplication in Fourier space with one of the signals complex conjugated

$$f(x) \star g(x) \leftrightarrow \bar{F}(s) \cdot G(s)$$

Single dish

- Single completely filled aperture
- Only one resolution defined by size of dish

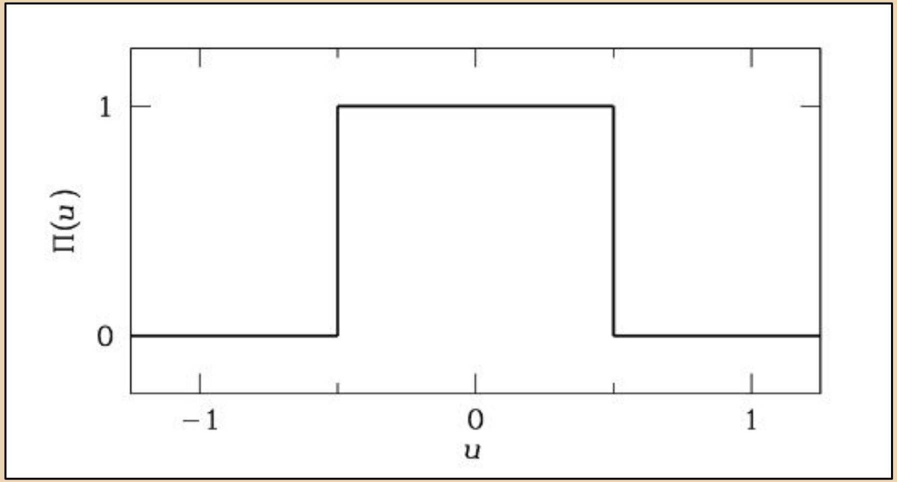
$$\theta = \frac{\lambda}{D}$$

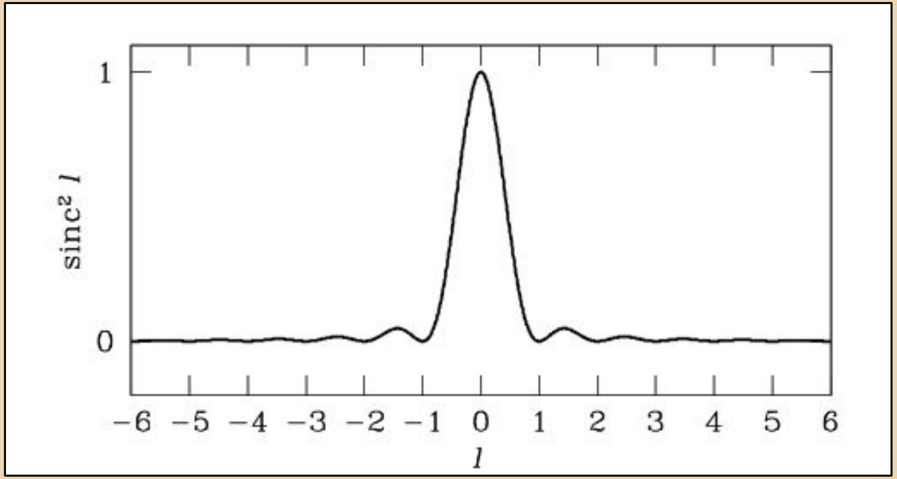
- Works well if no spatial resolution is required
  - Time domain things like pulsars
  - Frequency domain things like emission/absorption lines
- Image can be created by scanning across the sky
  - Limited resolution (even with very big dish)
  - Slow (and slower if your resolution is better)

Credit: Wikipedia

Single dish - Beam

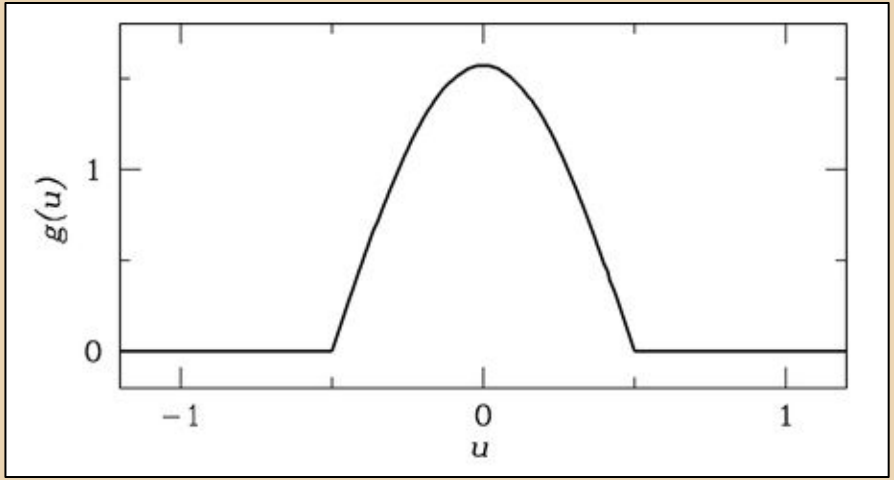
Beam pattern of single dish → square of Fourier transform of illumination pattern

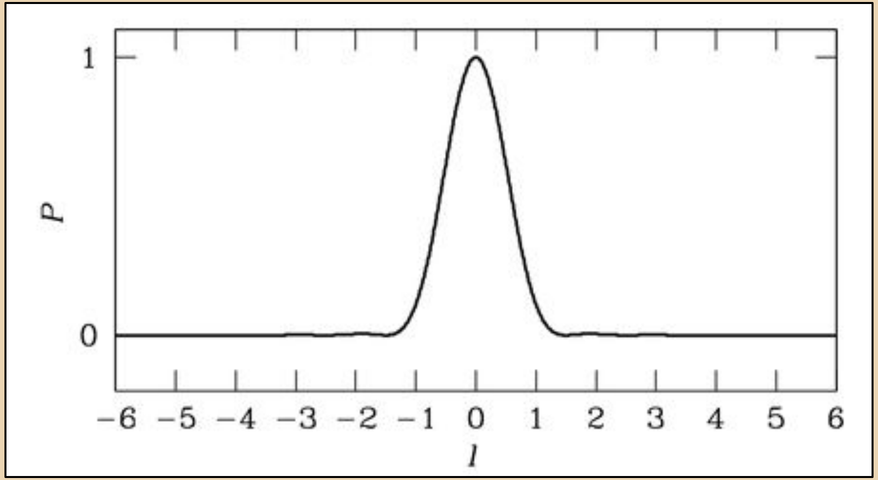




Single dish - Beam

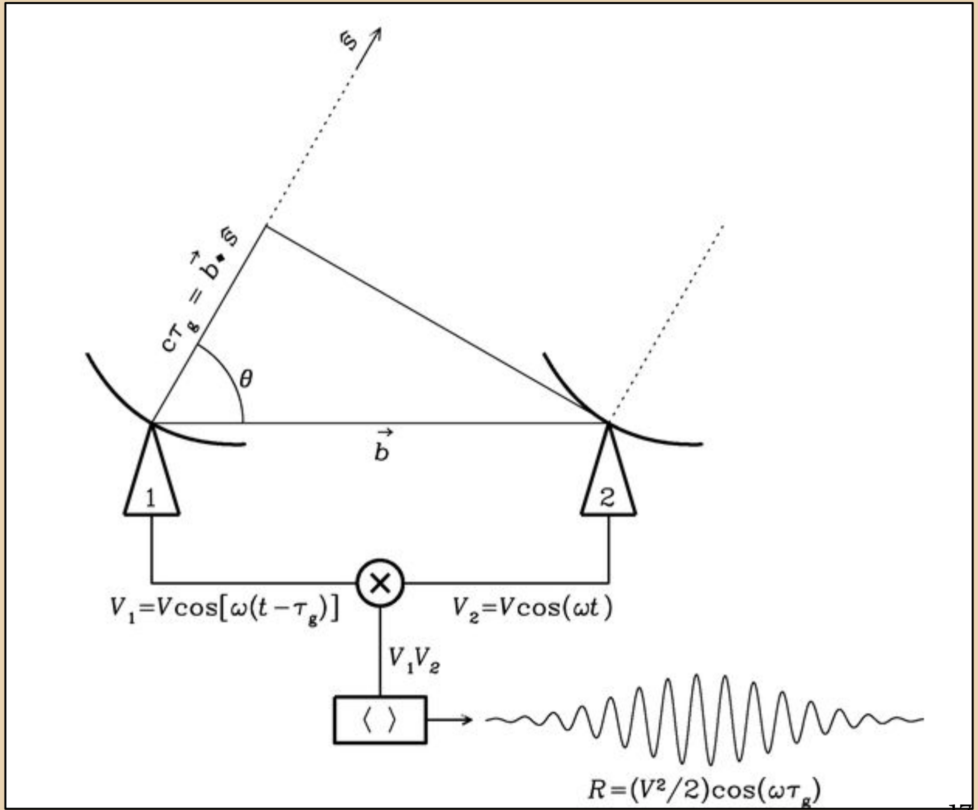
Taper illumination to control sidelobes





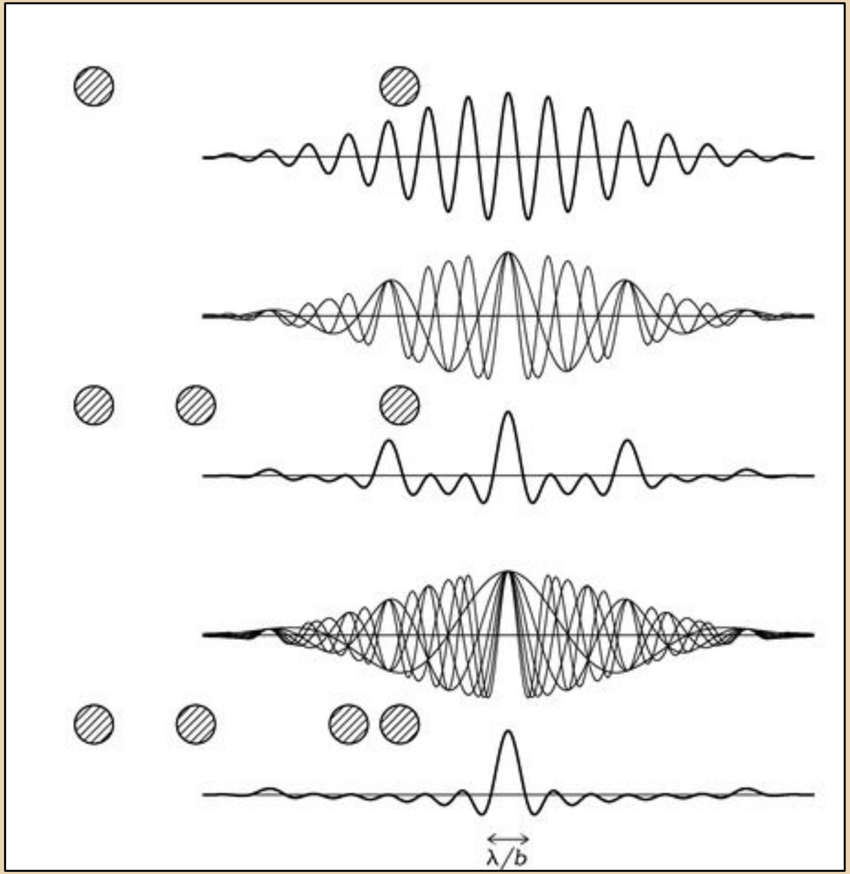
Two element interferometer

- Simplest interferometer
- Cross-correlates the signals coming from the sky
- Correlator multiplies and averages voltages from antennas, which eliminates all signals not common between antennas
- Resulting signal characterised by geometrical delay τ_g, determined by the **baseline** between the dishes, commonly referred to as **fringe**



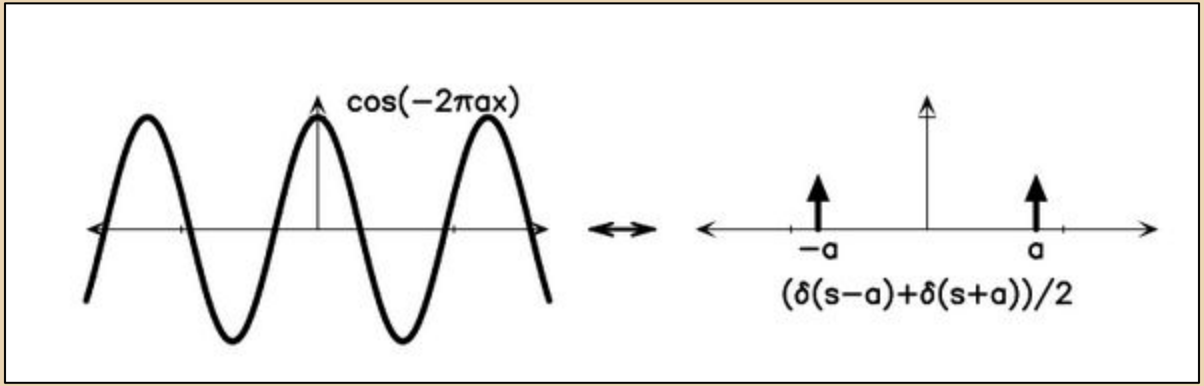
N-element interferometer

- Adding more baselines constrains the spatial origin of the signal
- Basic principles stay the same, an interferometer with **N** elements can be characterised as **N(N-1)/2** two element interferometers
- **Synthesized beam** approaches a Gaussian with size $\theta = \frac{\lambda}{B}$ with **B** the longest baseline



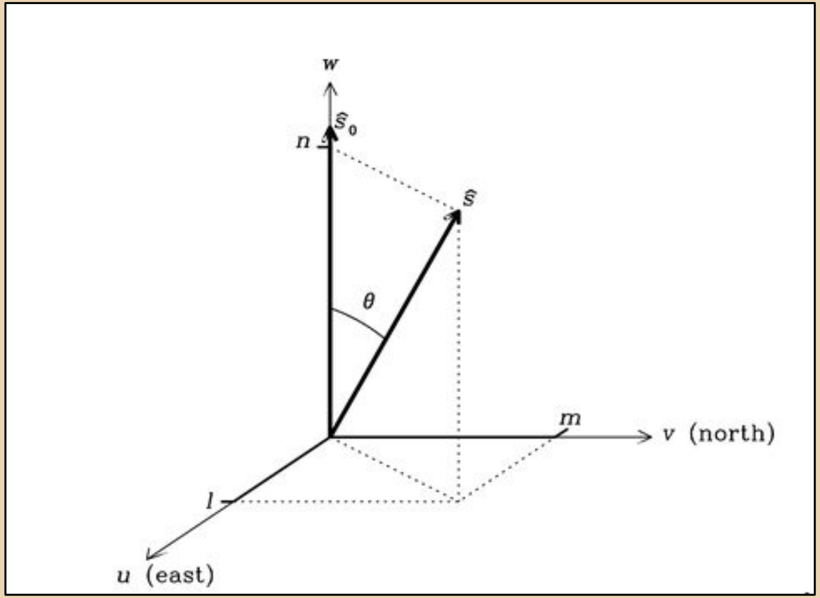
N-element interferometer

- Smallest resolvable scale determined by longest baseline
- Largest resolvable scale determined by shortest baseline
- Thing that we the antennas are pointing at is the **phase center**
- How to figure out the **synthesized beam** pattern? (spoiler: it's Fourier transforms)



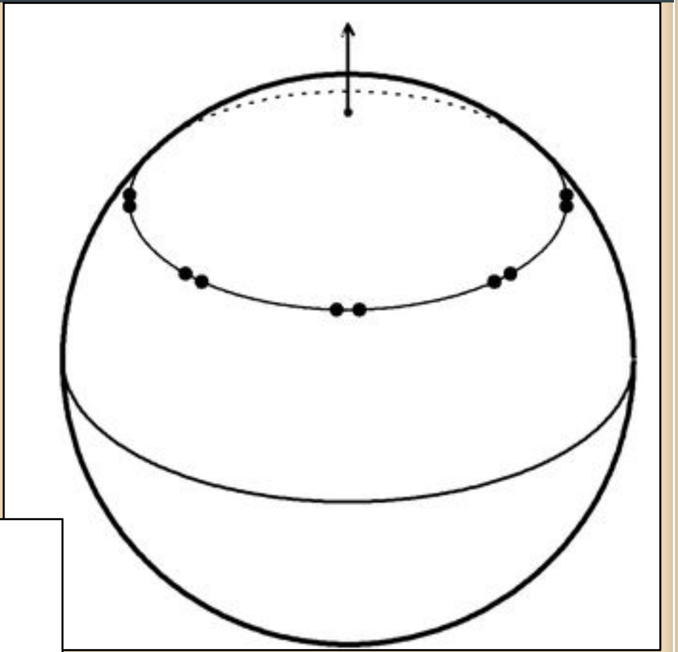
Going 2D: The (u,v) plane

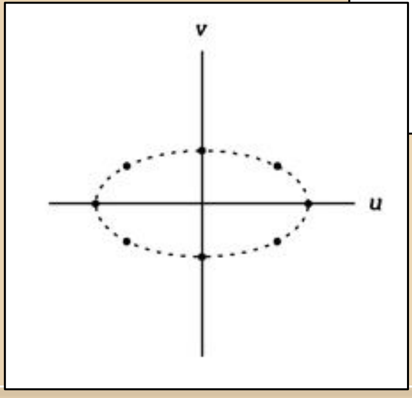
- Represent baselines as points in (u,v) space
- Each baseline adds two points
- Most conveniently described in units of λ
- The synthesized beam of the interferometer is then the Fourier transform of the (u,v) plane
- To match spatial dimensions, the complete coordinate system is **(u,v,w)**, with matching coordinates **(l,m,n)** describing the direction of the source



Rotation aperture synthesis

- How to increase coverage of the (u,v) plane?
- Let the earth do it for you
- As the earth rotates, baselines trace out a path in (u,v) space
- A 12-hour integration will maximally fill the (u,v) plane (or less time depending on array configuration)





Let's keep it 2D: Why we don't like w

- The signal measured at a single point in (u,v,w) space is called a **visibility**

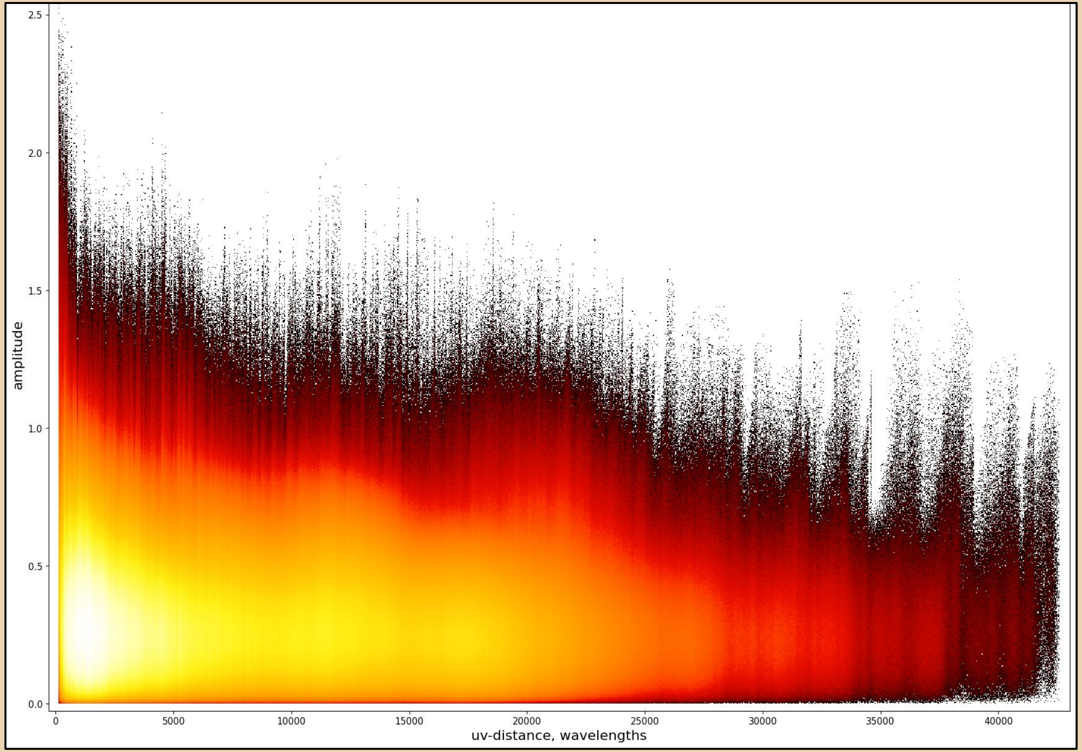
$$\mathcal{V}(u,v,w) = \int\int \frac{I_\nu(l,m)}{1-l^2-m^2}\exp[-2i\pi(ul+vm+wn)]\mathrm{d}l\mathrm{d}m$$

- This is not a 3D Fourier transform, so we don't like it
- To make it a Fourier transform, we have to neglect w, so assume our array is 2D

$$\mathcal{V}(u,v) = \int\int I_\nu(l,m)\exp[-2i\pi(ul+vm)]\mathrm{d}l\mathrm{d}m$$

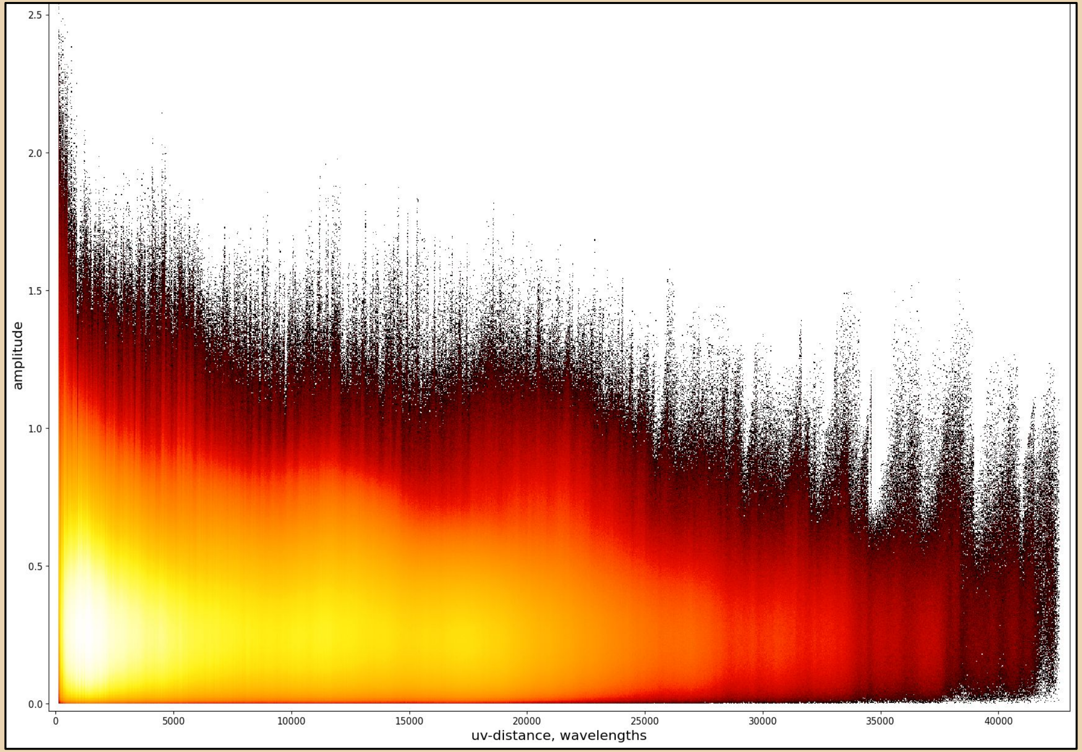
- If we cannot neglect w, we use **w-projection** or **faceting**

Quick test



amplitude
uv-distance, wavelengths
0.0
0.5
1.0
1.5
2.0
2.5
0
5000
10000
15000
20000
25000
30000
35000
40000

Quick test



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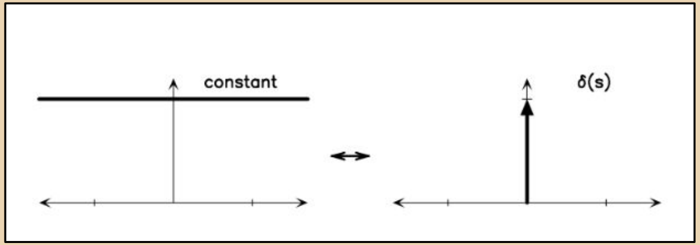
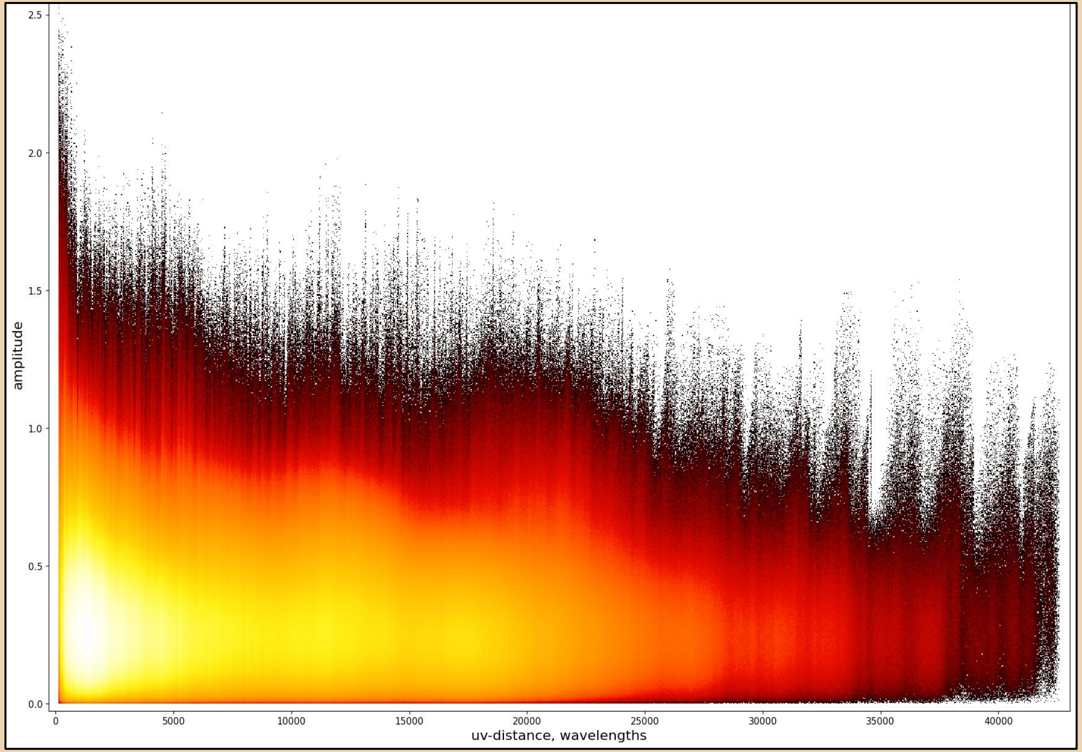


constant
δ(s)

Quick test



amplitude
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0.5
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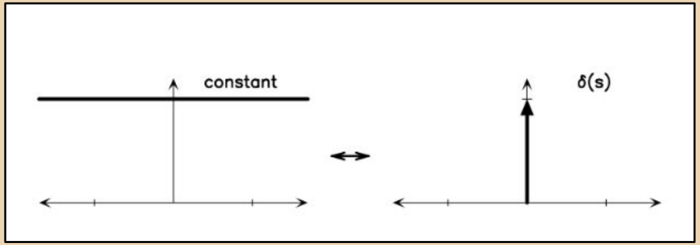

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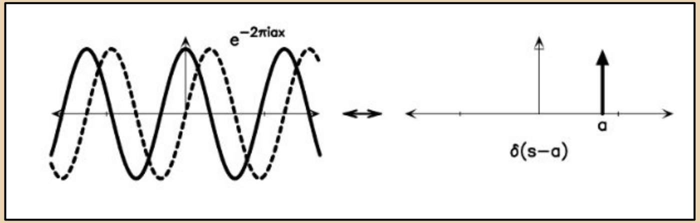
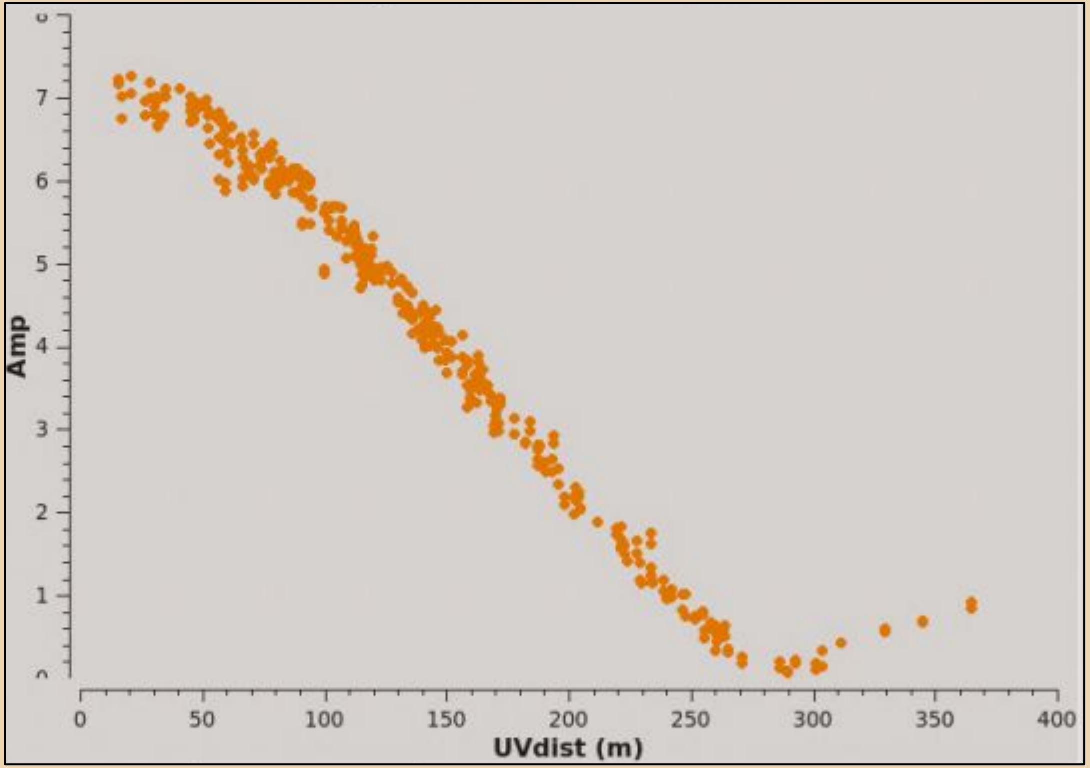
δ(s)

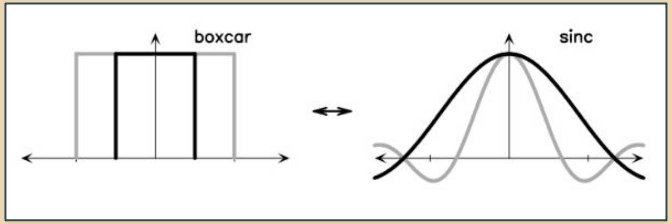


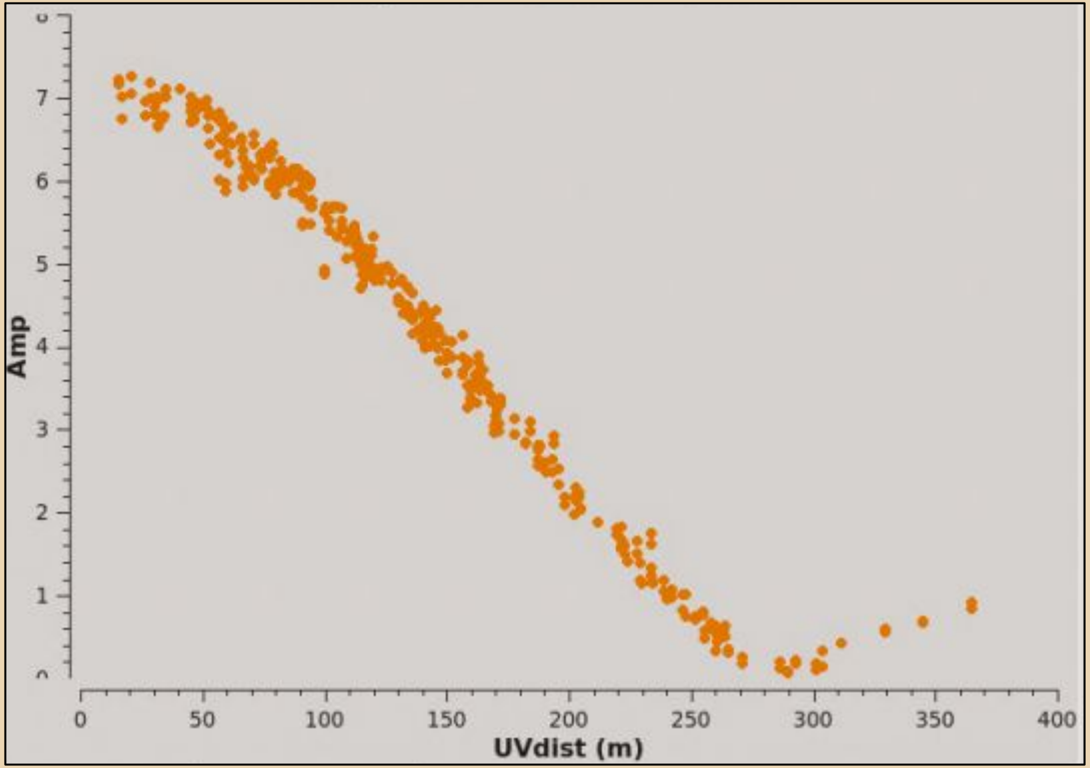
$e^{-2\pi iax}$
a
δ(s−a)

Quick test



Quick test


boxcar
sinc

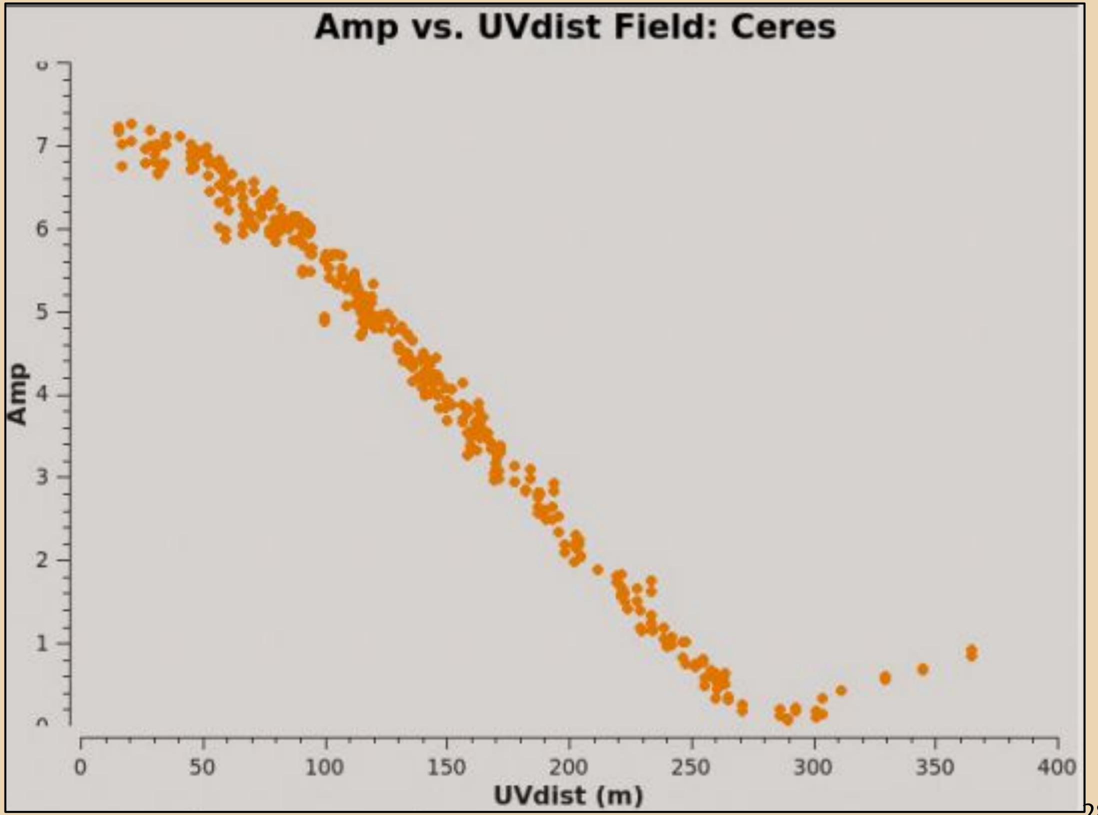

Amp
7
6
5
4
3
2
1
0
50
100
150
200
250
300
350
400
UVdist (m)

Quick test

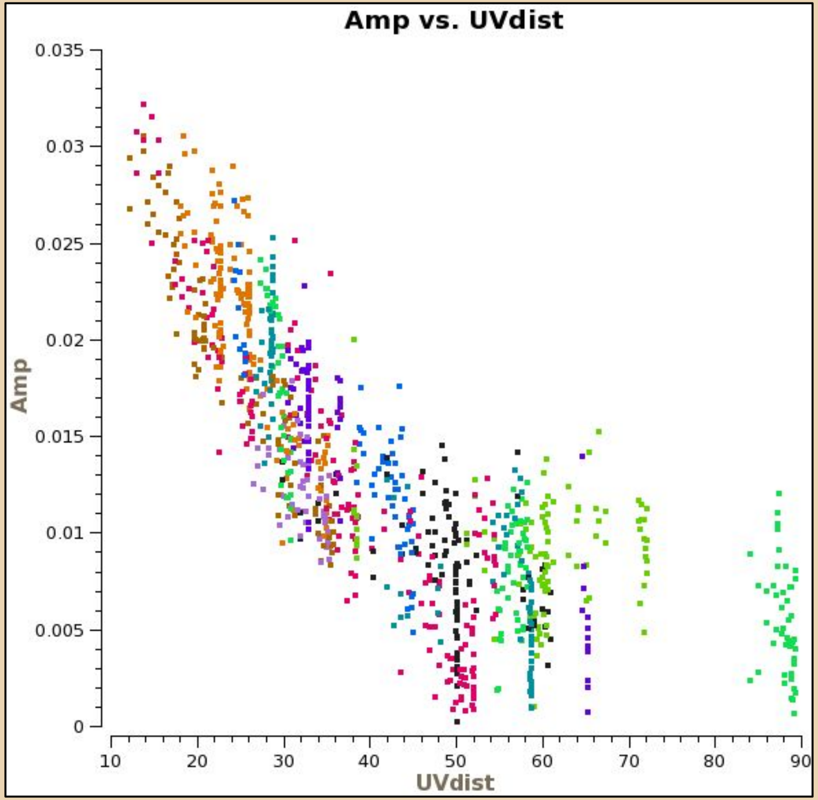
$\nu = 372$ GHz; $\lambda = 800$ μm

First null at: 280 m = 350 kλ

Size of object: **θ = 0.6”**



UV distance plot is your first clue



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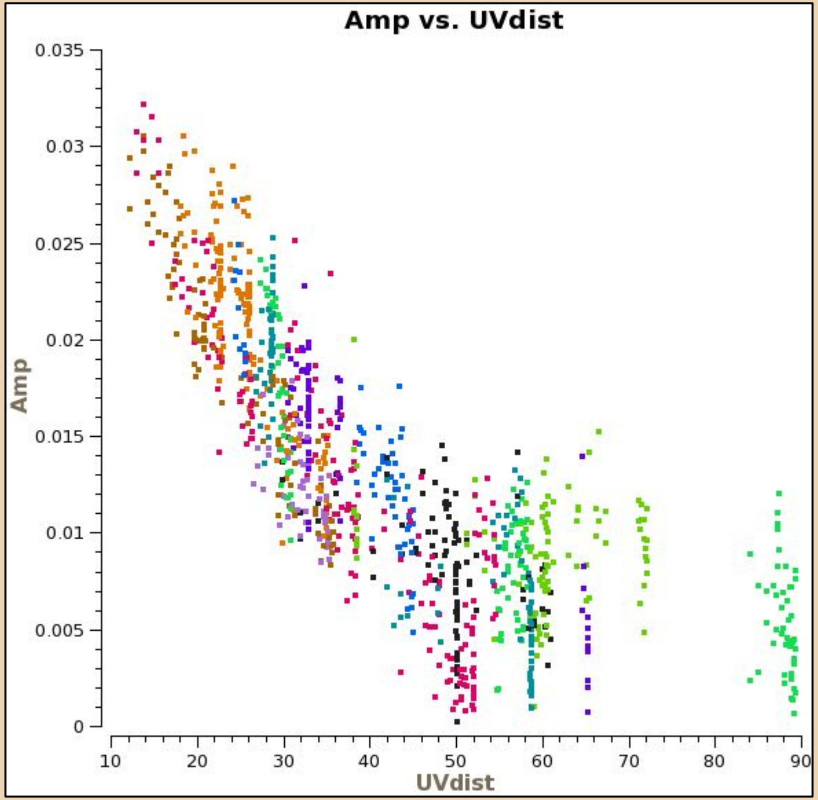

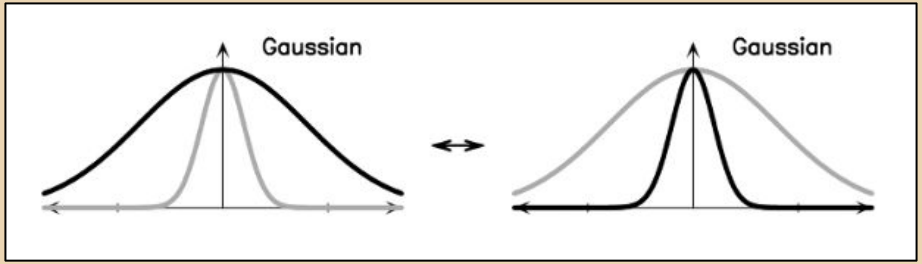

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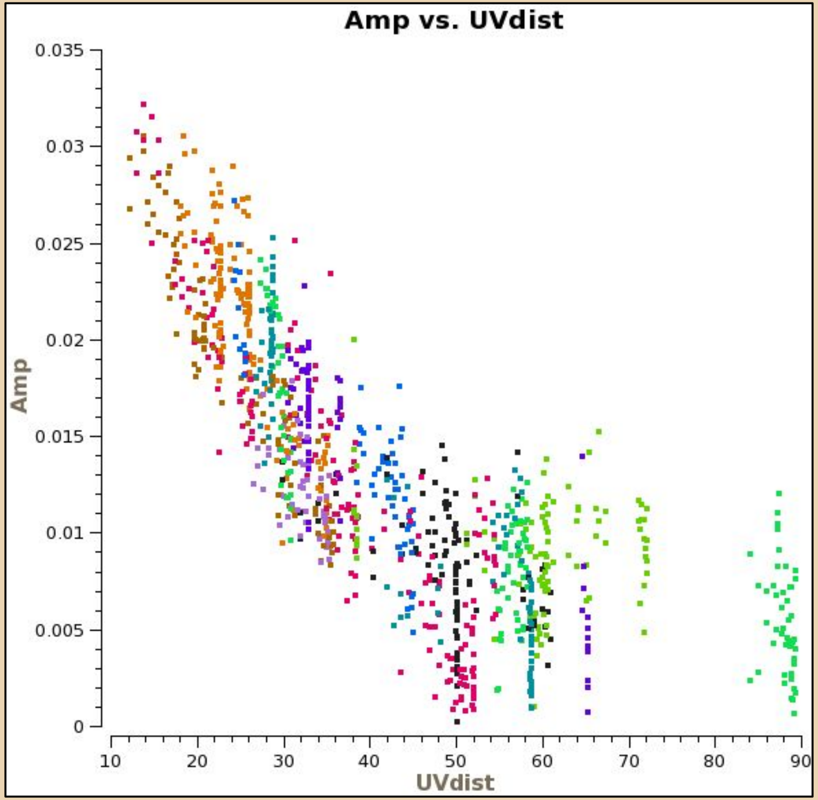
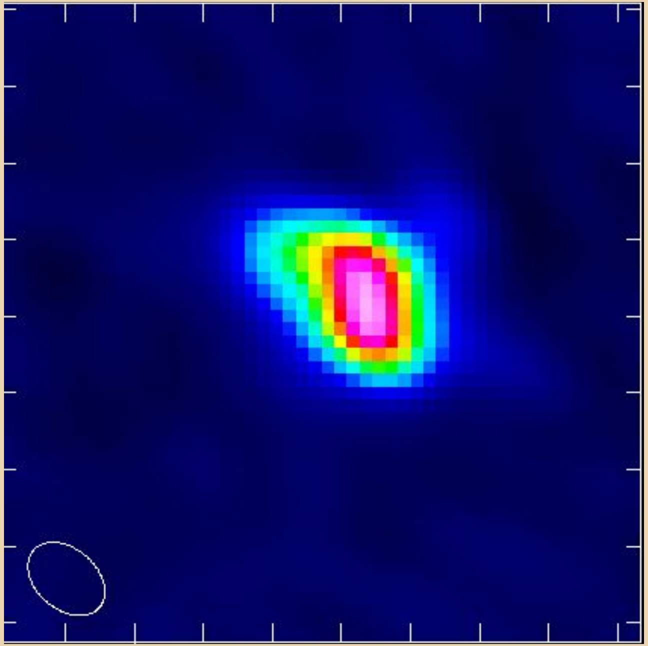
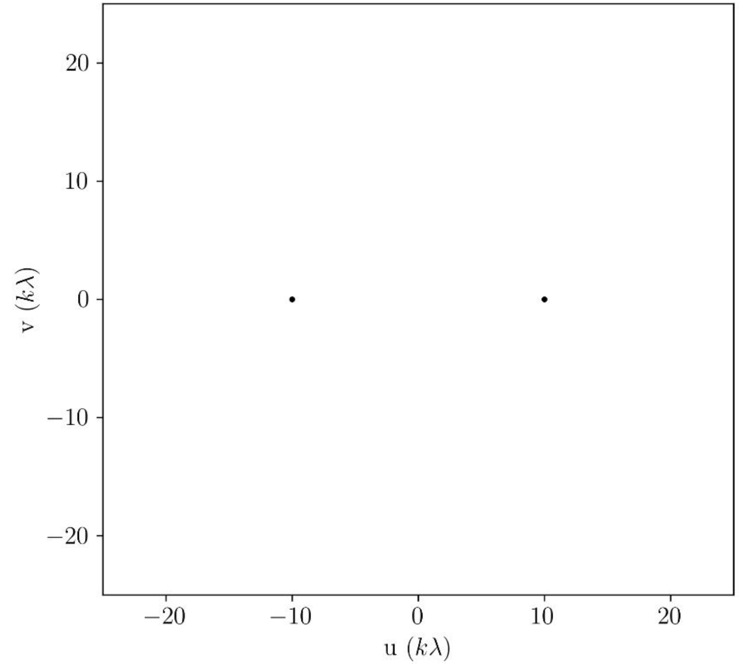
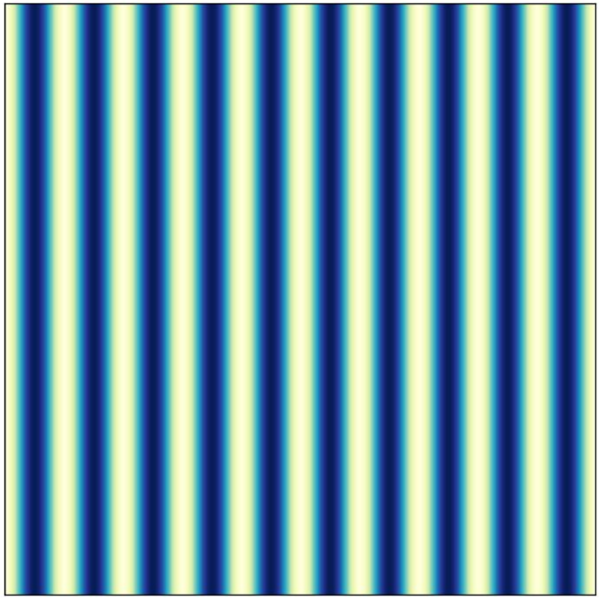


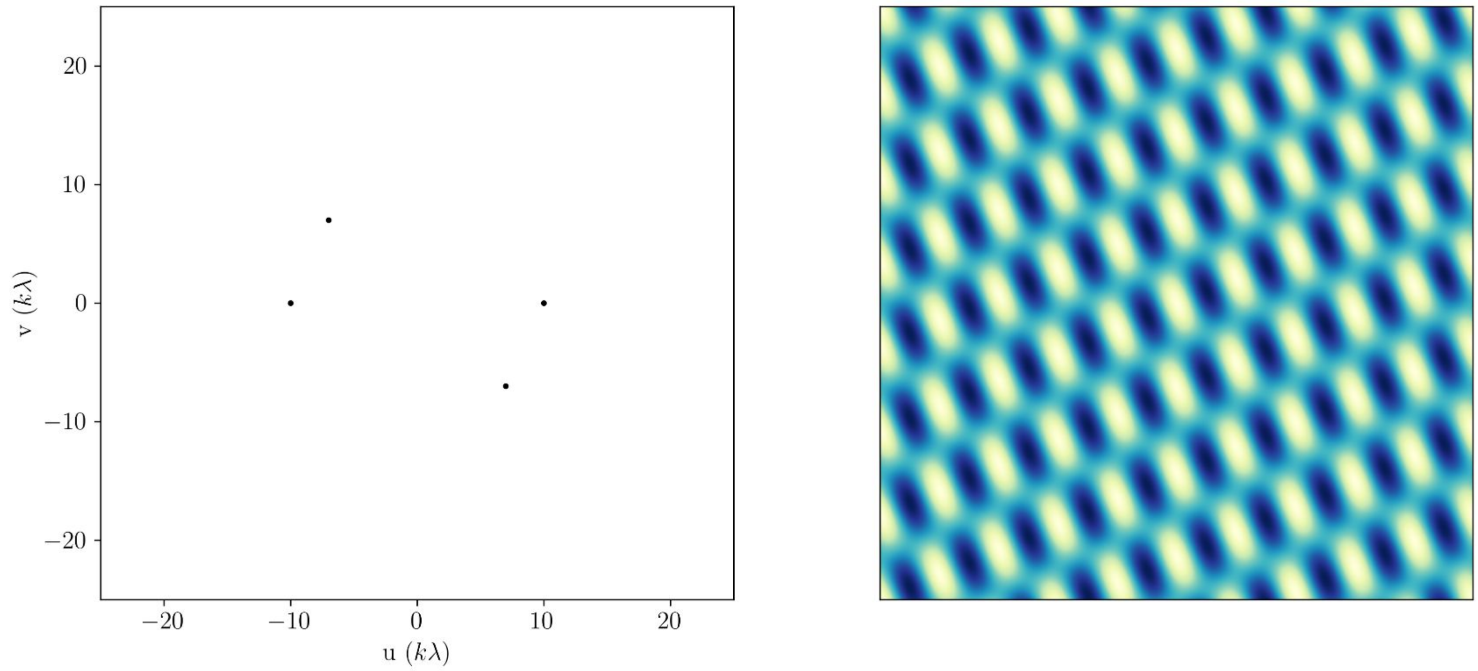


Filling up the UV plane





Filling up the UV plane



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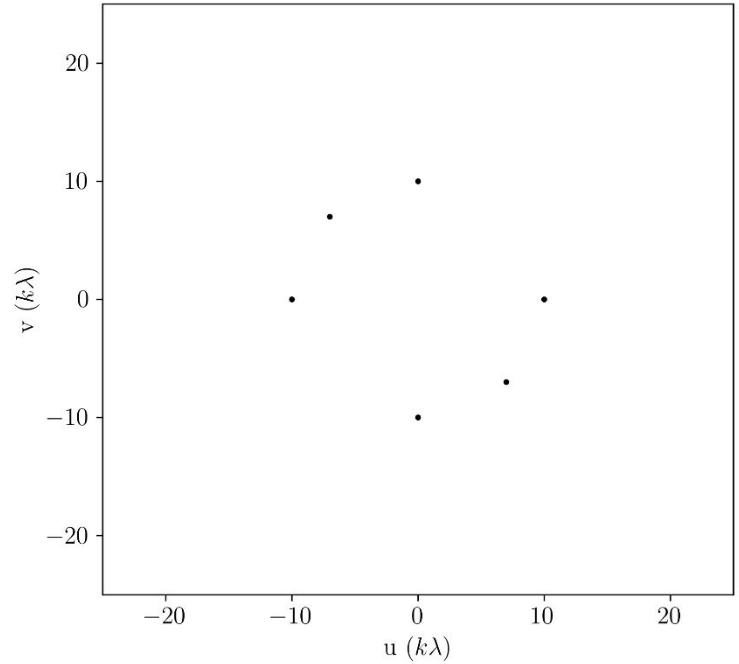

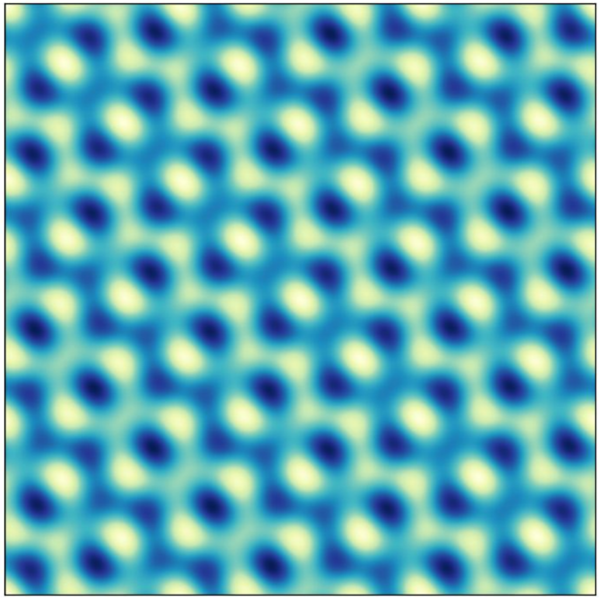

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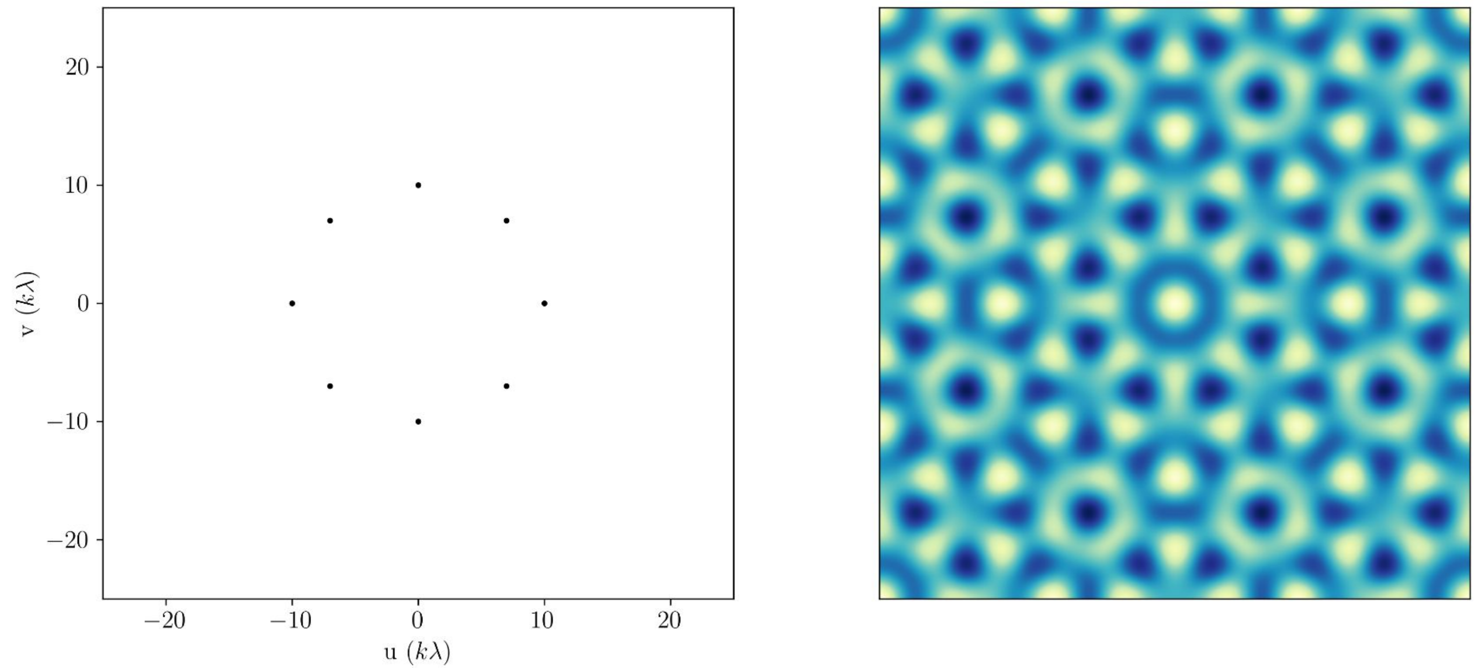

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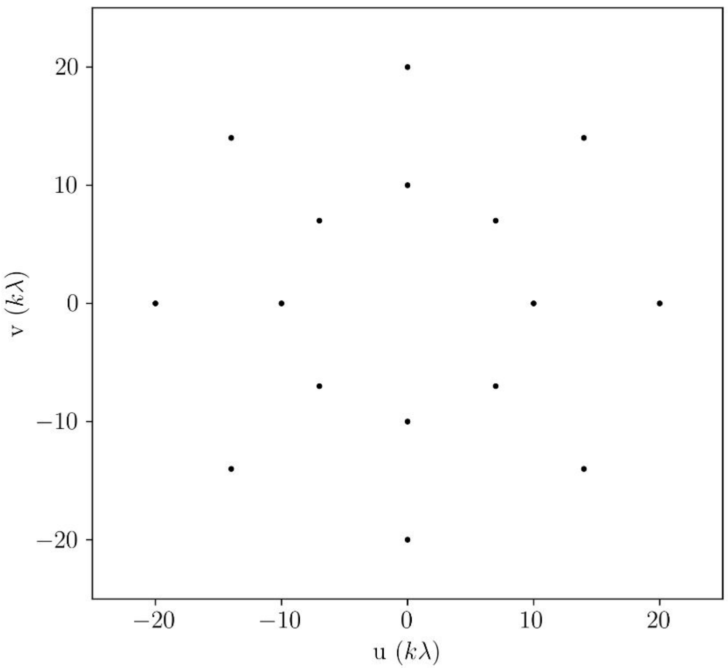

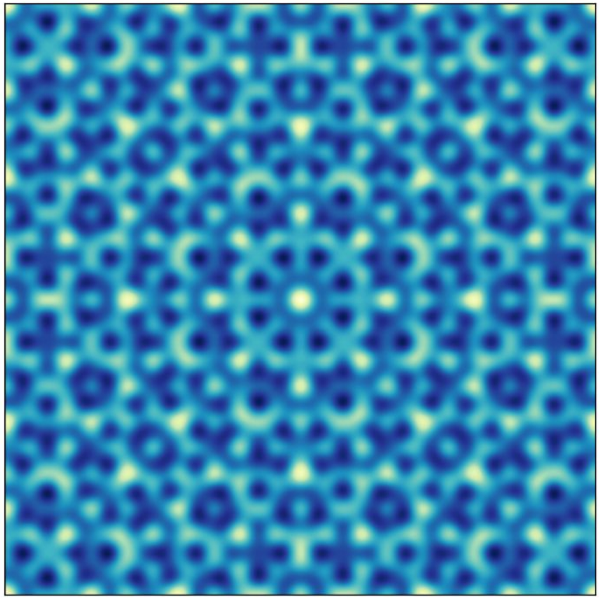

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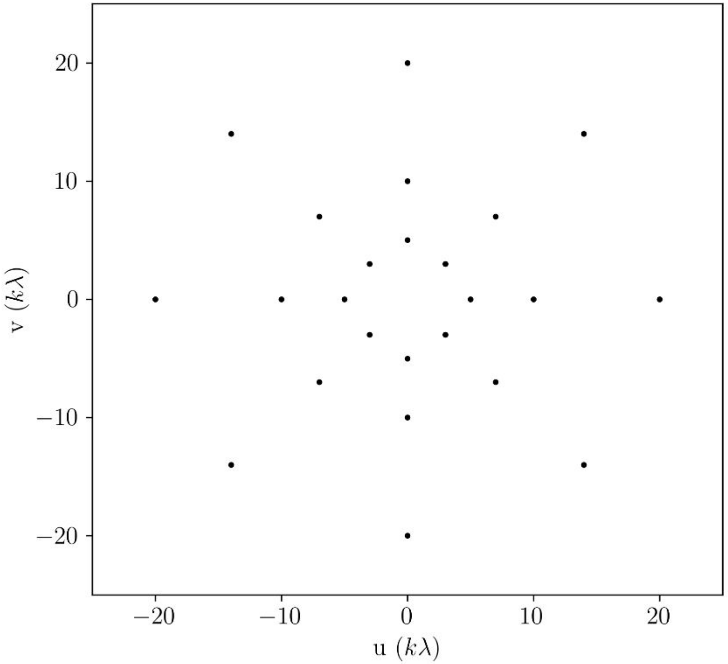
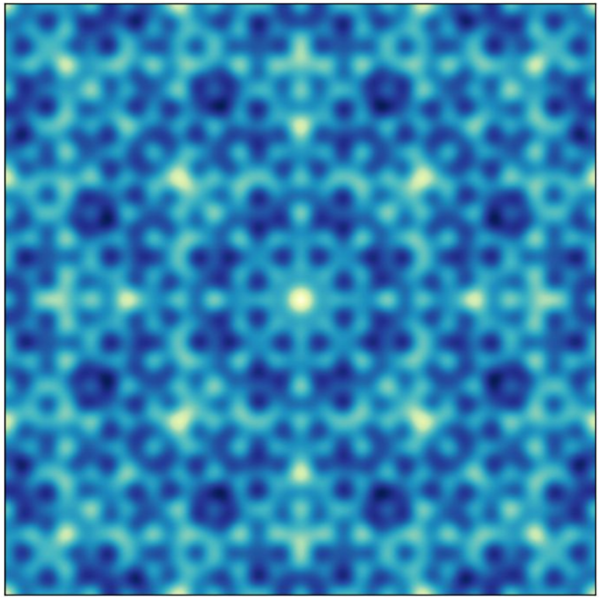
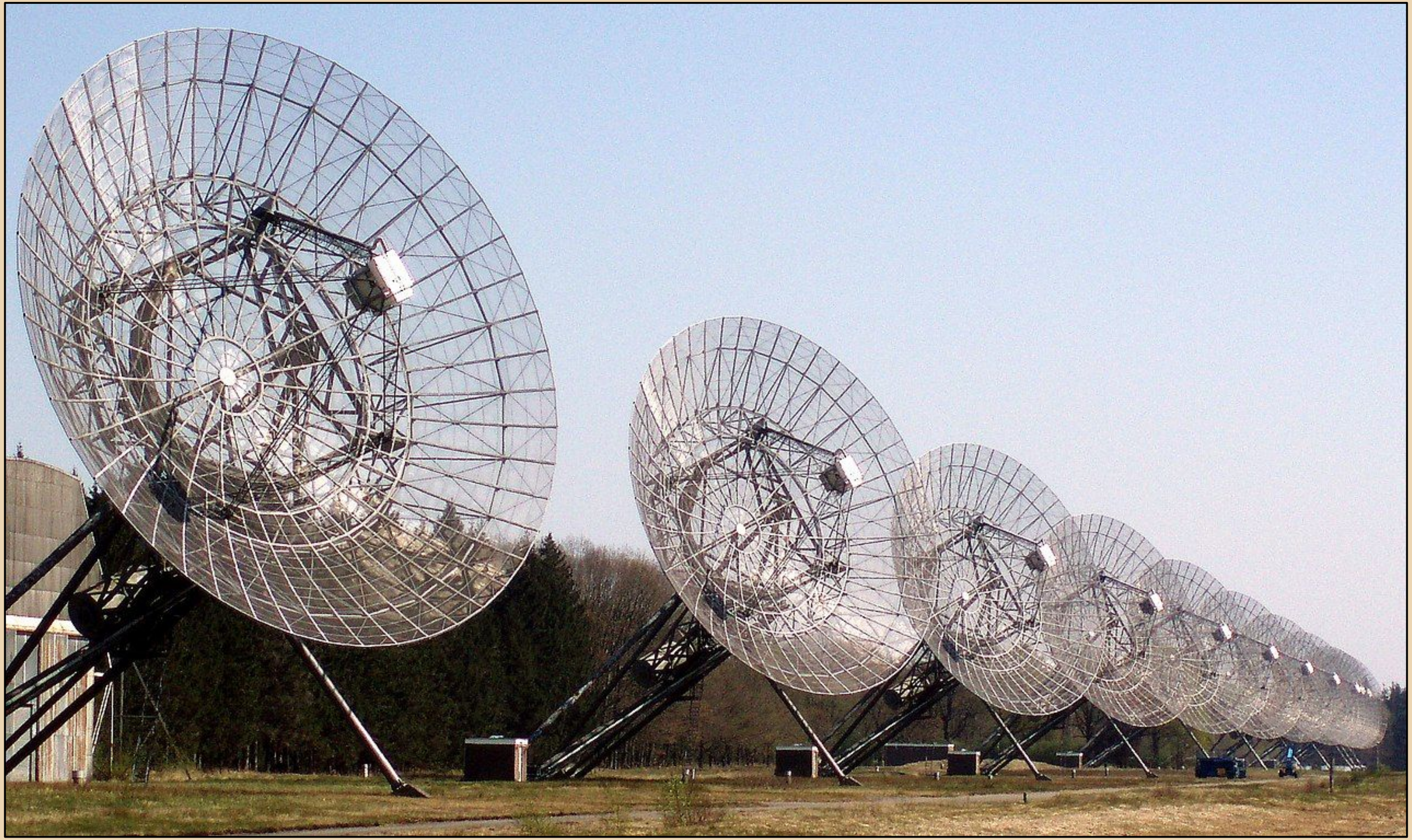




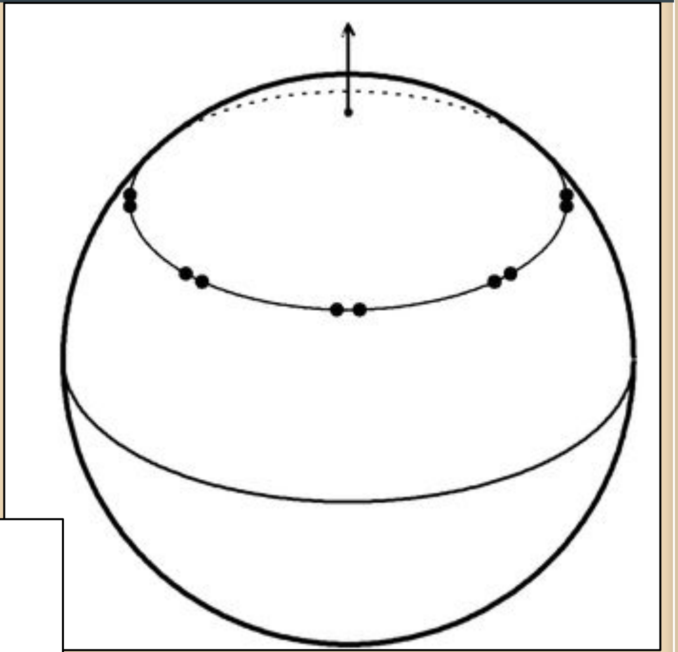
Westerbork

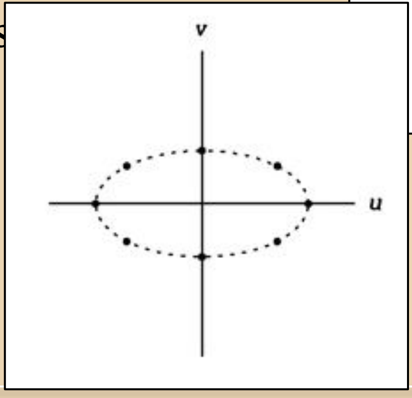
Credit: Wikipedia



Westerbork

- First light in 1970
- Fourteen dishes
  - D = 25 m
- Array is perfectly East-West aligned forming a line
- Antennas spread over 2.8 km
  - Min baseline: B = 144 m
  - Max baseline: B = 2.8 km
- Add earth rotation to create an ellips
- Fourier transform is perfectly 2D

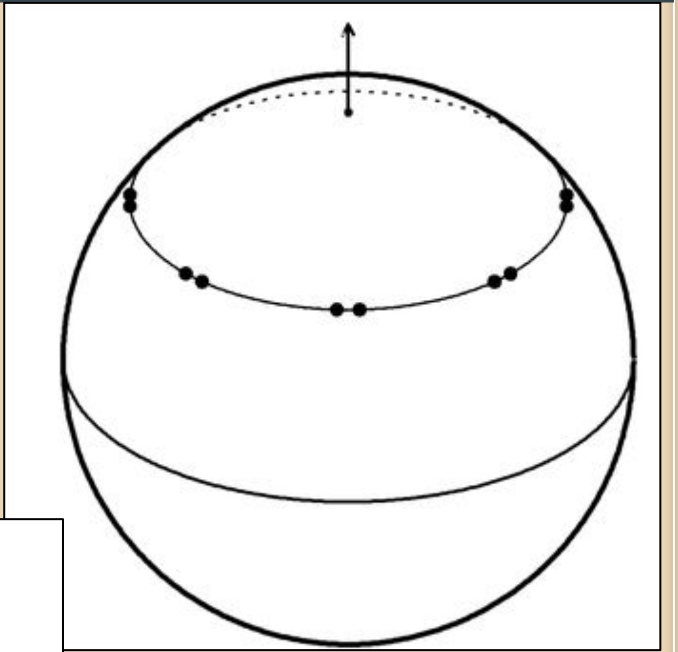


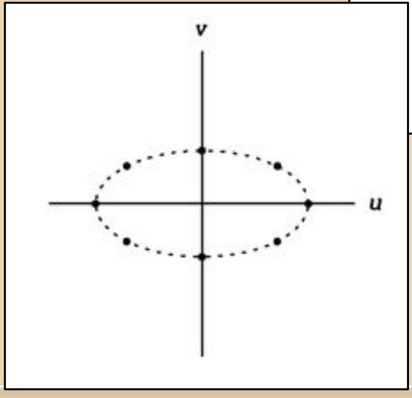


Westerbork

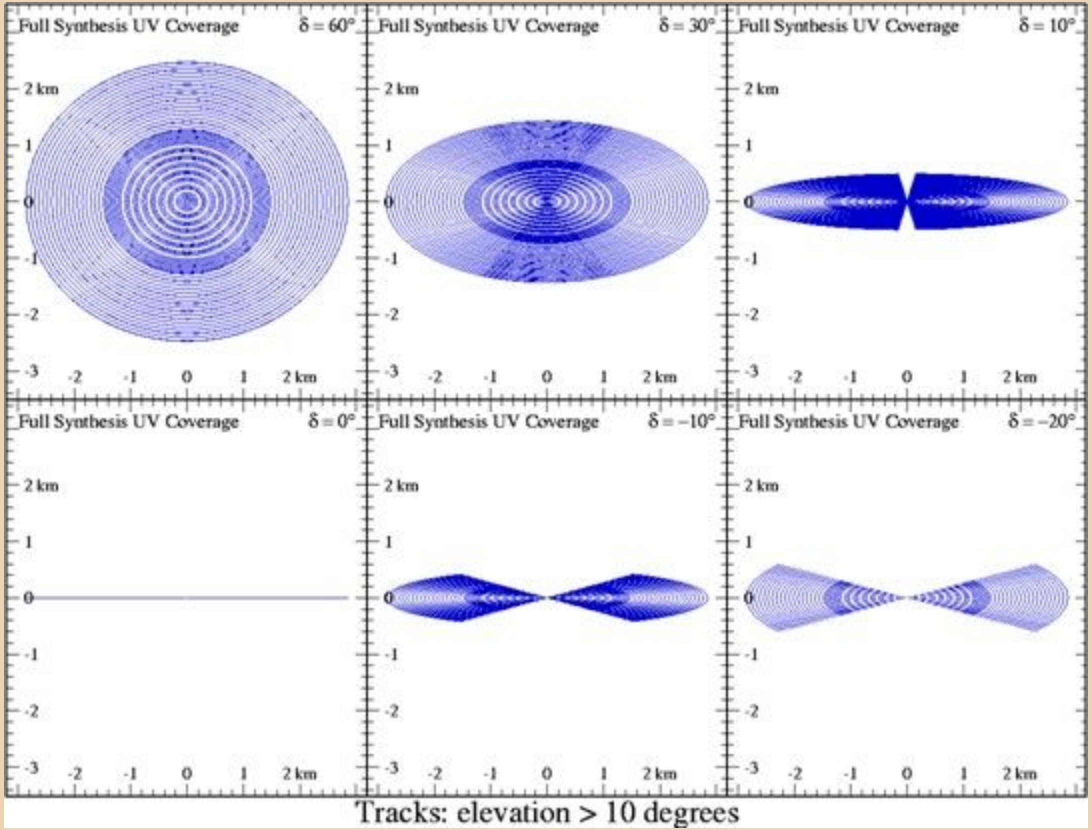
$$\lambda = 21 \text{ cm}$$

- Fourteen dishes
  - D = 25 m; **θ = 0.5°**
- Array is perfectly East-West aligned forming a line
- Antennas spread over 2.8 km
  - Min baseline: B = 144 m; **θ = 5'**
  - Max baseline: B = 2.8 km; **θ = 15"**
- Add earth rotation to create an ellipse in UV-plane
- Fourier transform is perfectly 2D





Westerbork



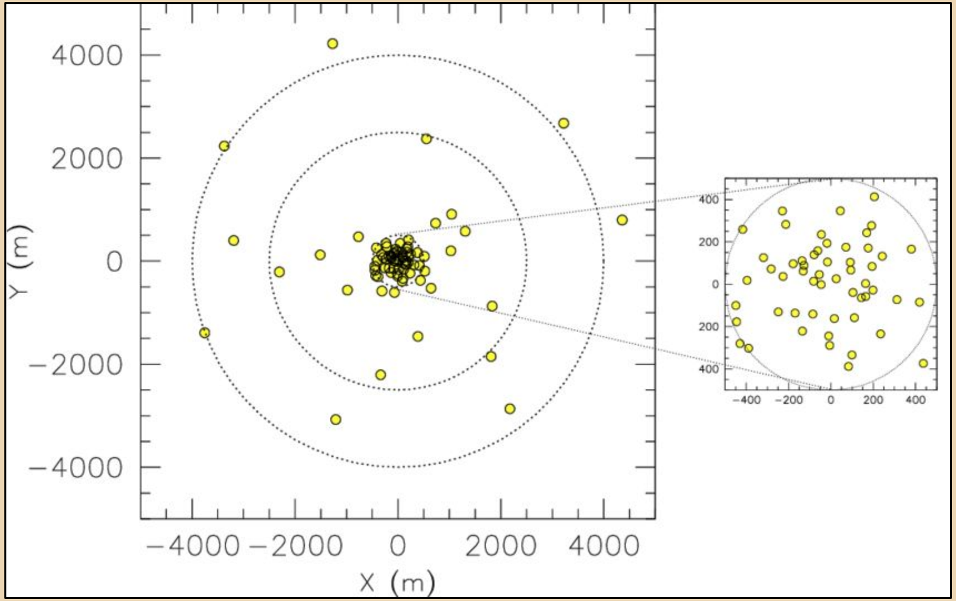
http://www.aoc.nrao.edu/events/synthesis/2006/lectures/TuesdayJune20/Cohen.pdf

MeerKAT

Credit: SARAO

MeerKAT

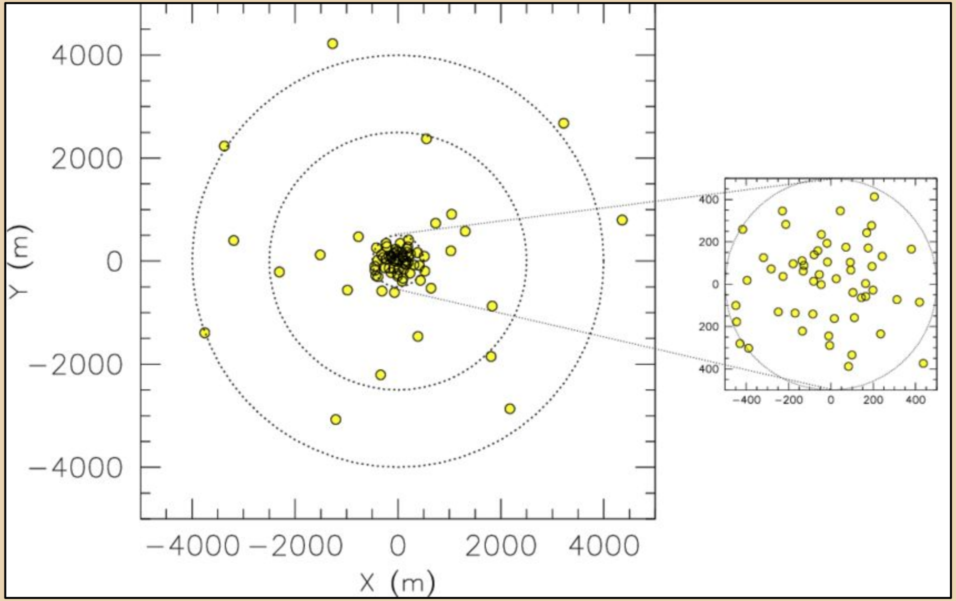
- First light in 2018
- 64 dishes
  - D = 13.5 m
- Many antennas in small core, with longer baselines
  - Min baseline: B = 20 m
  - Max baseline: B = 8 km
- Sensitive to extended emission due to short baselines
- Semi-random spacings between telescopes to get better UV-coverage



MeerKAT

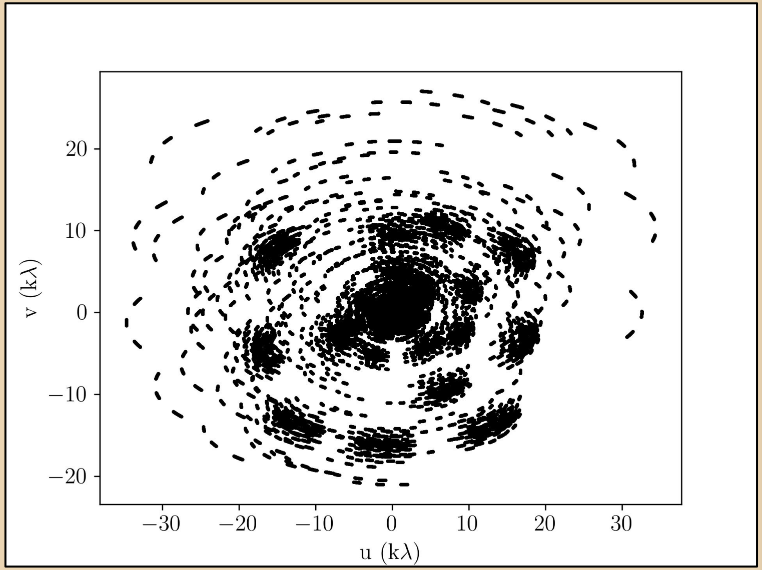
$$\lambda = 21 \text{ cm}$$

- 64 dishes
  - D = 13.5 m; **θ = 0.9°**
- Many antennas in small core, with sparser longer baselines
  - Min baseline: B = 20 m; **θ = 0.6°**
  - Max baseline: B = 8 km; **θ = 5.4”**
- Sensitive to extended emission due to short baselines
- Semi-random spacings between telescopes to get better UV-coverage

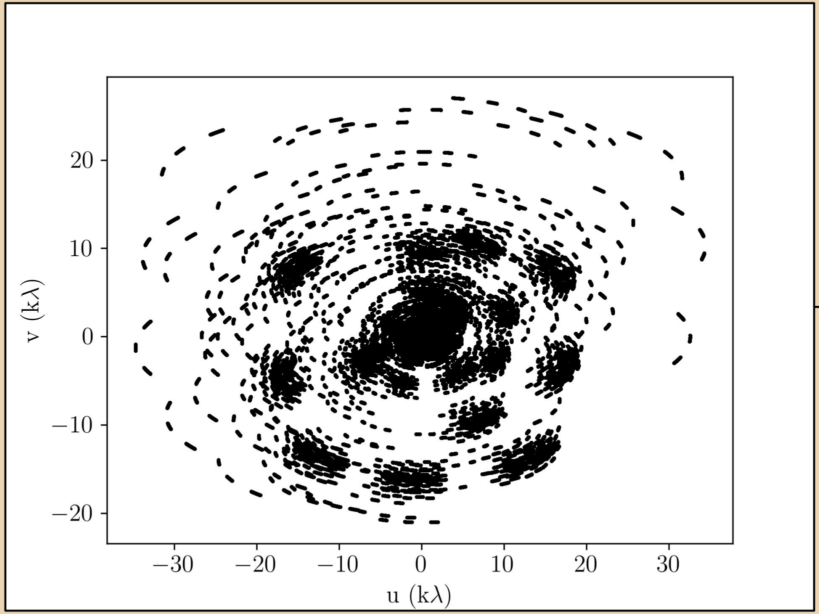


Weighting the UV-coverage

- In order to compute the synthesized beam, UV-coverage must be Fourier transformed, and therefore **gridded**
- Many pixels will contain multiple points, how to combine them together?
- **Weighting**
    - **Natural**: sum all visibilities in cell, emphasizes shorter baselines, worse resolution, better sensitivity
    - **Uniform**: correct cell for number of visibilities, emphasizes longer baselines, better resolution, worse sensitivity
    - **Briggs** : Anywhere between natural and uniform, based on **robust** parameter

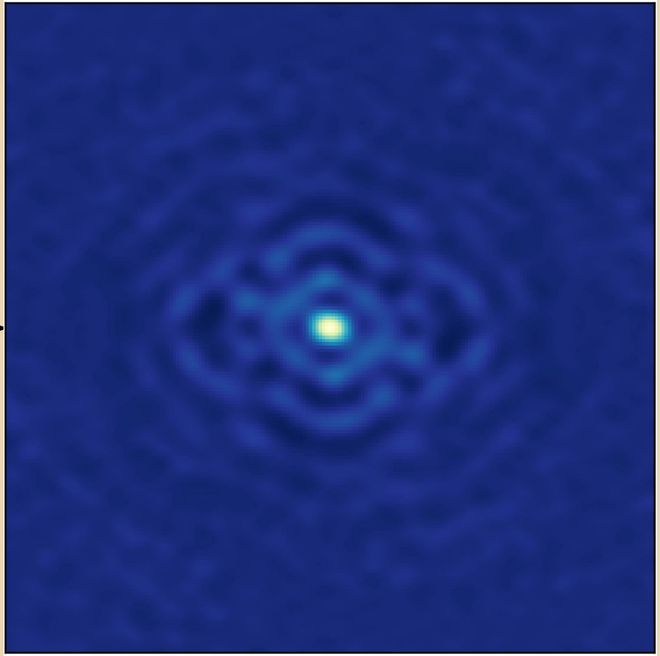


The final image



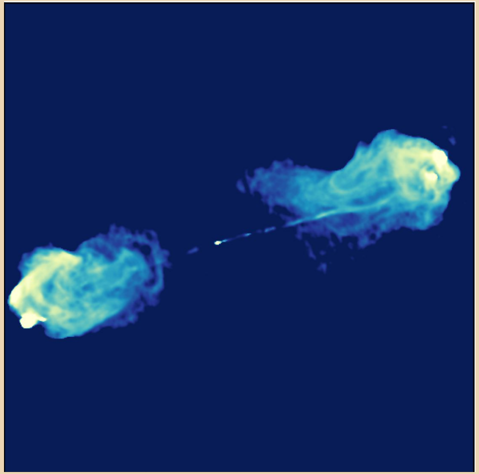
UV coverage

$\mathcal{F}$



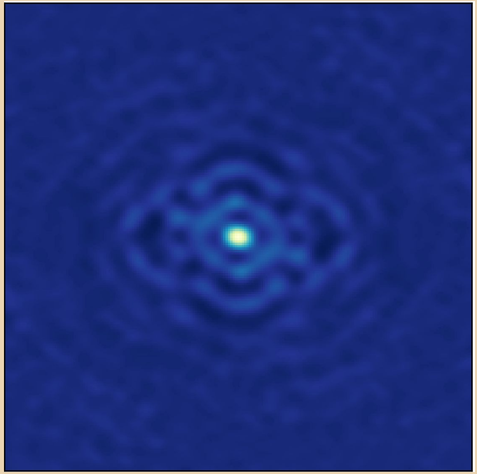
PSF/Synthesized beam/Dirty beam

The final image



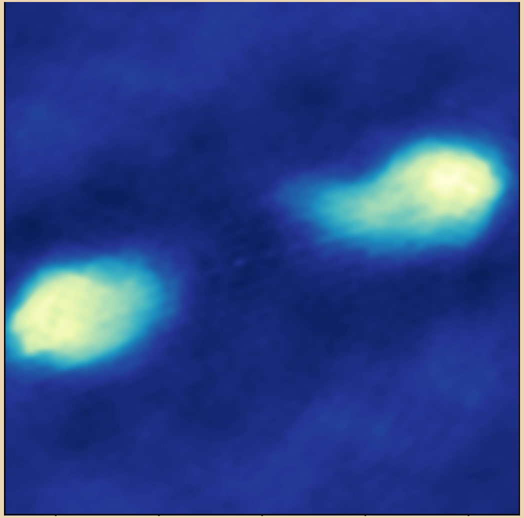
Sky intensity

$\star$



Dirty beam

$=$



Dirty image

Image deconvolution

- Fourier transform the observed visibilities $\mathcal{V}(u, v)$ to get dirty image $I_\nu^D(l, m)$
- We are essentially observing with missing information on various scales, which is represented by the shape of the dirty beam/synthesised beam/PSF
- If we would have no further knowledge of the sources in the sky, this is the best that we can do
- However: things in nature follow certain rules
  - Emission cannot be negative
  - Things should be smoothly varying
  - Many sources are unresolved (=smaller than the beam)
- Thus: we can build a **model** of the sky, with which we can **deconvolve** the image

Högbom CLEAN algorithm

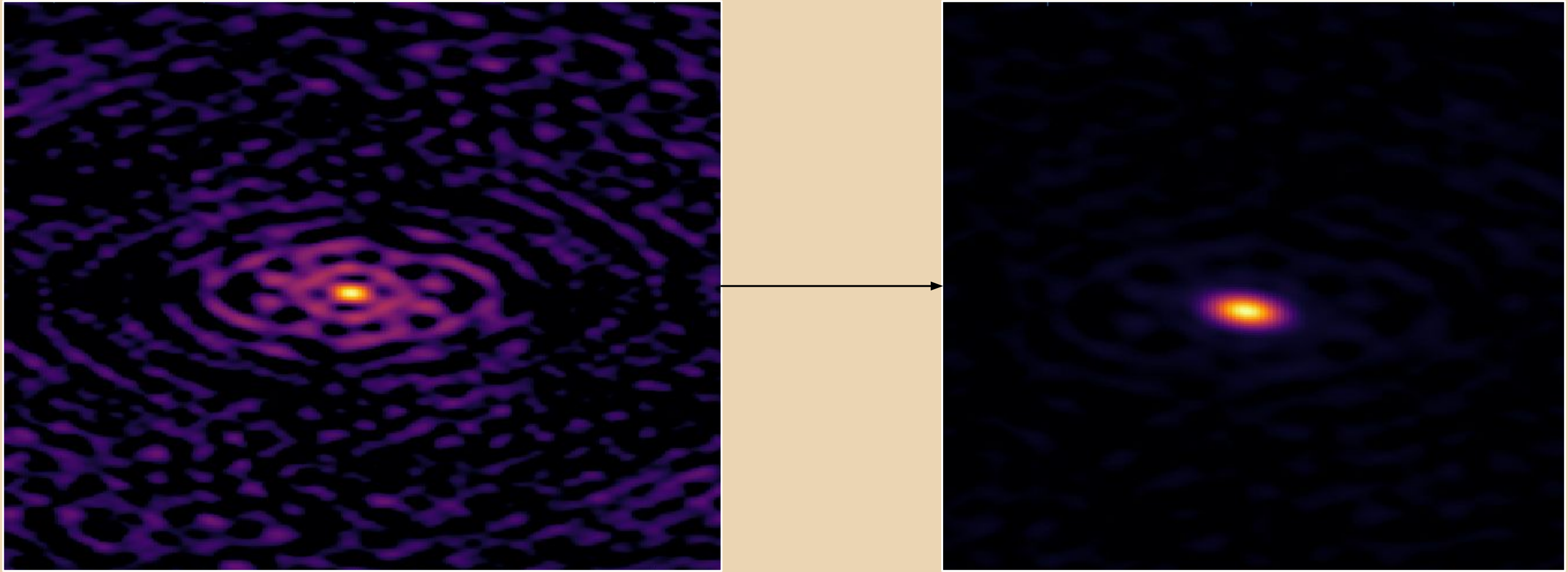
1. Find the brightest pixel in the dirty image
2. Subtract from the dirty image, at the position of the pixel, the dirty beam multiplied by the brightness of the pixel and a gain factor
3. Add the subtracted brightness, at the position of the pixel, to the point-source model image
4. Repeat 1-3 until either:
    a. The highest peak is below a user-specified threshold
    b. A user-specified number of iterations is reached
5. Convolve the point-source model with the CLEAN beam (usually an elliptical Gaussian fitted to the central lobe of the dirty beam) to create CLEAN image
6. Add the CLEAN image and the residual dirty image together

Usual extensions to CLEAN

- **Multi-frequency synthesis**: for larger bandwidth, model emission as a function of frequency by fitting Taylor polynomials
- **Multi-scale:** Model the emission at various scales, not just as a collection of point sources (useful for extended emission)
- **Faceting:** Divide the image into a number of facets with individual phase centers (to minimize w-term)
- **W-projection:** Use a discrete number of w-planes during gridding to account for w-term
- **A-projection:** Correct for variability in time and baselines

CLEAN implementations

- AIPS (old), CASA (standard), WSclean (state of the art)



Do it yourself

- Go to: https://github.com/JonahDW/Interferometry-for-dummies
- Two jupyter notebooks are there, try not to peek at the solutions before you've given it a fair shake