Jonah's science combiner scribble

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1 AC Fringe Tracking

A fringe intensity on a detector as a function of delay x can be modelled as:

$$i(x,\kappa) = F_0[1 + |\gamma|\cos(\kappa x - \Phi)] \tag{1}$$

where F_0 is the total flux from the object, $|\gamma| = V$ is the complex coherence modulus/visibility, κ is the angular wavenumber ($\kappa = 2\pi/\lambda$) and Φ is the phase delay from the astrophysical source corrupted by the atmosphere ($\Phi = \phi_{obj} + \phi_{atm}$).

If observing in polychromatic light, with a rectangular bandpass with centre $\bar{\kappa}$ and width $\Delta\lambda$, we have:

$$i(x,\bar{\kappa}) = F_0\left[1 + V\,\mathrm{sinc}\left(\frac{\pi x}{\Delta\lambda}\right)\cos(\bar{\kappa}x - \Phi)\right]$$

We can also add a bit of dispersion into the system, by making one of the inputs slightly longer than the other. Lets assume that we extend one length by Δ_{glass}. Then, to make sure the coherence packet has a phase of zero at λ_0 (central wavelength channel?), we can extend the vacuum length by:

$$\Delta_{vac} = n_{group}(\lambda_0)\Delta_{glass}$$

The group index is given by:

$$n_{group}(\lambda) = n(\lambda) - \lambda\frac{\mathrm{d}n(\lambda)}{\mathrm{d}\lambda}$$

where $n(\lambda)$ is given by the Sellmeier equation:

$$n(\lambda)^2 = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}$$

Could simplify this maths here... may do later

This extra dispersion will then induce a wavelength based phase change. For a subchannel with central angular wavenumber $\bar{\kappa}_j$ and width $\Delta\kappa_j$ given by:

$$\begin{aligned}
\phi_{disp}(\bar{\kappa}_j) &= OPD \cdot \bar{\kappa}_j \\
&= \left(n\left(\frac{2\pi}{\bar{\kappa}_j}\right)\Delta_{glass} - \Delta_{vac}\right)\bar{\kappa}_j \\
&= \left(n\left(\frac{2\pi}{\bar{\kappa}_j}\right)\Delta_{glass} - n_{group}\left(\frac{2\pi}{\kappa_0}\right)\Delta_{glass}\right)\bar{\kappa}_j \\
&= \left(n\left(\frac{2\pi}{\bar{\kappa}_j}\right) - n_{group}\left(\frac{2\pi}{\kappa_0}\right)\right)\Delta_{glass}\bar{\kappa}_j
\end{aligned}$$

The intensity can then be written as:

$$i(x,\bar{\kappa}_j) = F_0\left[1 + V\,\mathrm{sinc}\left(\frac{\pi x}{\Delta\lambda_j}\right)\cos(\bar{\kappa}_j x - \phi_{disp}(\bar{\kappa}_j) - \Phi)\right]$$

An AC combiner measures two quantities per channel (with centre $\bar{\kappa}_i$ and width $\Delta\lambda_i$), which has the phase averaged over the n subchannels:

$$A_i(x, \bar{\kappa}_i) = F_0 \left[1 + V \operatorname{sinc}\left(\frac{\pi x}{\Delta\lambda_i}\right) \cos\left(\frac{1}{n}\sum_j (\bar{\kappa}_j x - \phi_{disp}(\bar{\kappa}_j)) - \Phi\right)\right]$$
$$C_i(x, \bar{\kappa}_i) = F_0 \left[1 + V \operatorname{sinc}\left(\frac{\pi x}{\Delta\lambda_i}\right) \cos\left(\frac{1}{n}\sum_j (\bar{\kappa}_j x - \phi_{disp}(\bar{\kappa}_j)) - \Phi + \pi\right)\right]$$

Together, these quantities can be used to obtain the real component of the complex coherence (hereby called R_γ):

$$R_{\gamma,i} = \frac{A_i - C_i}{A_i + C_i} = V \operatorname{sinc}\left(\frac{\pi x}{\Delta\lambda_i}\right) \cos\left(\frac{1}{n}\sum_j (\bar{\kappa}_j x - \phi_{disp}(\bar{\kappa}_j)) - \Phi\right)$$

The goal of the fringe tracker is to take a measurement of R_γ and return the value of x without knowing Φ or V. Then, the delay line can be adjusted so as to reduce the value of x to zero in order to maximise visibility. The question is how to do this...

Possible solution: Would attempting to minimise the value of

$$\chi^2 = \sum_i \frac{\operatorname{sinc}\left(\frac{\pi x}{\Delta\lambda_i}\right) \cos\left(\frac{1}{n}\sum_j (\bar{\kappa}_j x - \phi_{disp}(\bar{\kappa}_j))\right) - R_{\gamma,i}}{\sigma_i^2}$$

over x, where σ is the gaussian error in R_γ, work?

Simulation note: Can roughly generate noise from the signal to noise ratio by random sampling a gaussian with mean of the correct signal, and a standard deviation equal to $\sigma \approx i(x, \bar{\kappa})/SNR$

2 Tricoupler Length

2.1 2-Coupler

We use a 2-coupler to identify the coupling coefficients before looking at the tricoupler.

The coupling equation is given by:

$$\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}z} = i\mathbf{A}\mathbf{b}$$

where $\mathbf{A}$ is:

$$\mathbf{A} = \begin{bmatrix} \beta_0 + C_{11} & C_{12} \\ C_{21} & \beta_0 + C_{22} \end{bmatrix}$$

and $\beta_0 = \kappa n_0$ is the propagation constant for both fibers.

Making the substitution $c_{ij} = \frac{C_{ij}}{\beta_0}$ then we have:

$$\mathbf{A} = \beta_0 \mathbf{A}' = \beta_0 \begin{bmatrix} 1 + c_{11} & c_{12} \\ c_{21} & 1 + c_{22} \end{bmatrix}$$

We can ignore the diagonal c_{11} and c_{22} terms due to them being much smaller than 1. Then, setting $\delta = c_{12} = c_{21}$ due to symmetry and we have:

$$\mathbf{A} = \beta_0 \mathbf{A}' = \beta_0 \begin{bmatrix} 1 & \delta \\ \delta & 1 \end{bmatrix}$$

We then assume a solution:

$$\mathbf{b}_j = \mathbf{v}_j e^{i\beta_j z}$$

Which produces an eigenvector equation:

$$\mathbf{A}'\mathbf{b}_j = \frac{\beta_j}{\beta_0}\mathbf{b}_j$$

The eigenvalues (effective propagation constants) for the 2-coupler are:

$$\beta_j = \beta_0\{1+\delta, 1-\delta\}$$

The difference in the effective propagation constants is also given by:

$$\begin{aligned}
\Delta\beta_{\text{eff}} &= \kappa\Delta n_{\text{eff}} \\
&= \beta_0(1+\delta-(1-\delta)) \\
&= \kappa n_0 2\delta \\
\delta &= \frac{\Delta n_{\text{eff}}}{2n_0}
\end{aligned}$$

Hence we can calculate the coupling coefficient simply by measuring the difference in effective indices of the two different modes. This can be done e.g using Rsoft

2.2 Tri-coupler

We now turn to the tri-coupler, which has a coupling matrix (calculated similarly) of

$$\mathbf{A} = \beta_0\mathbf{A}' = \beta_0 \begin{bmatrix} 1 & \delta & \delta \\ \delta & 1 & \delta \\ \delta & \delta & 1 \end{bmatrix}$$

The eigenvalues of this system are:

$$\beta_j = \beta_0\{1-\delta, 1+2\delta, 1-\delta\} = \kappa\{n_0 - \frac{\Delta n_{\text{eff}}}{2}, n_0 + \Delta n_{\text{eff}}, n_0 - \frac{\Delta n_{\text{eff}}}{2}\}$$

with eigenvectors:

$$v_j = \{\frac{1}{\sqrt{2}}[-1,1,0], \frac{1}{\sqrt{3}}[1,1,1], \frac{1}{\sqrt{6}}[-1,-1,2]\}$$

The solution is hence:

$$\mathbf{b}(z) = \sum_{j=1}^{3} a_j v_j e^{i\beta_j z}$$

Now, we can identify the length of the coupler, z_{len}, such that if light is injected into one fiber, the output light is distributed evenly between all three. That is $|\mathbf{b}(z_{\text{len}})| = \frac{1}{\sqrt{3}}[1,1,1]$

The input $\mathbf{b}$ vector is:

$$\mathbf{b}(0) = a_1v_1 + a_2v_2 + a_3v_3 = [1,0,0]$$

Solving this through row reduction gives coefficients:

$$a_j = \{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{6}}\}$$

Using Eagle2000 glass (with a refractive index of 1.507), and requiring a 4.5 μm diameter with V number of 2.2 at 600 nm, we find that the cladding index must be approximately 1.504. The average index is hence 1.5055.

With $n_0 = 1.5055$, $\kappa = \frac{2\pi}{0.6\mu\text{m}}$ and $\Delta n_{\text{eff}} = 1.50551 - 1.50494 = 5.7\times10^{-4}$, we obtain $z_{\text{len}} = 351.5\mu$m.

We can also plot what happens if we have an input of $\mathbf{b}(0) = \frac{1}{\sqrt{2}}[1,0,e^{i\phi}]$ and then see how $|\mathbf{b}(z_{\text{len}})|$ changes as ϕ is varied:

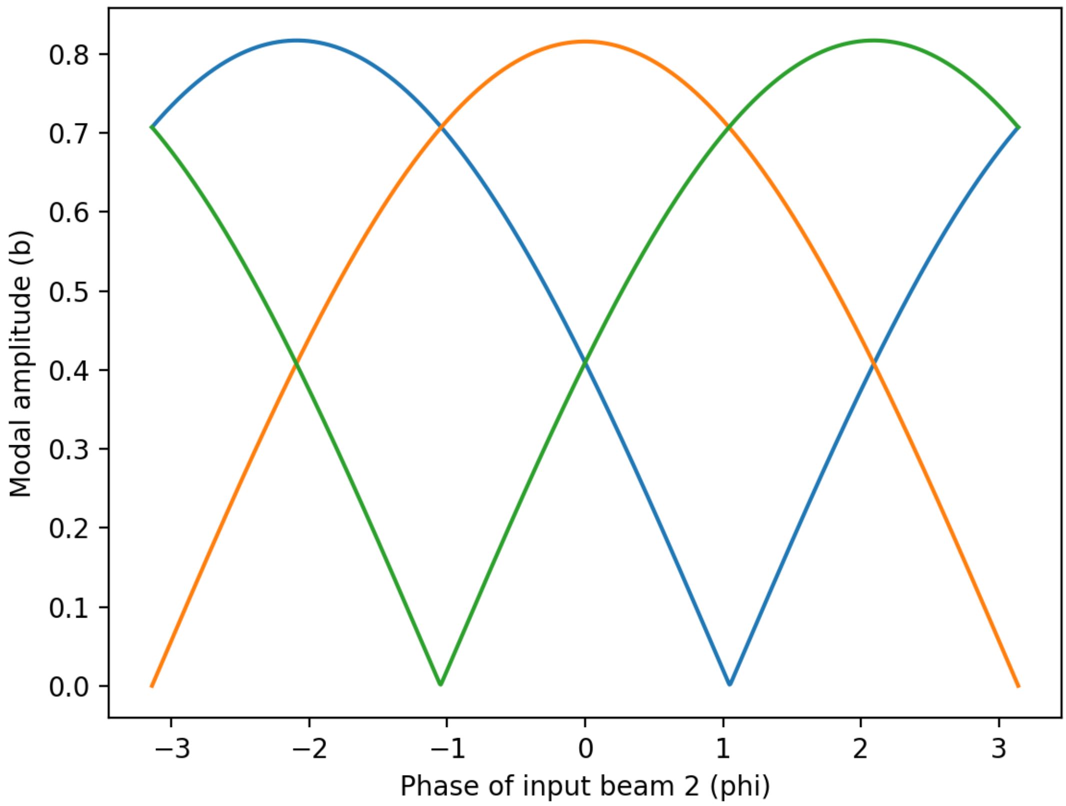

Figure 1: Caption

3 Tricoupler Fringe Tracking

Let's recall that the polychromatic intensity of a fringe pattern is given by

$$i(x, \bar{\kappa}) = F_0[1 + V\,\mathrm{sinc}(\frac{\Delta\kappa x}{2})\cos(\bar{\kappa}x - \Phi)]$$

Let's assume that the tricoupler introduces a $\pm 2\pi/3$ phase shift for two of the outputs:

$$\begin{aligned}
A(x, \bar{\kappa}_i) &= F_0[1 + V\,\mathrm{sinc}(\frac{\Delta\kappa_i x}{2})\cos(\bar{\kappa}_i x - \Phi)] \\
B(x, \bar{\kappa}_i) &= F_0[1 + V\,\mathrm{sinc}(\frac{\Delta\kappa_i x}{2})\cos\left(\bar{\kappa}_i x - \Phi + \frac{2\pi}{3}\right)] \\
C(x, \bar{\kappa}_i) &= F_0[1 + V\,\mathrm{sinc}(\frac{\Delta\kappa_i x}{2})\cos\left(\bar{\kappa}_i x - \Phi + \frac{4\pi}{3}\right)]
\end{aligned}$$

We can then derive the full complex coherence from this:

$$\begin{aligned}
\gamma &= \frac{3A + \sqrt{3}i(C - B)}{A + B + C} - 1 \\
&= V\,\mathrm{sinc}(\frac{\Delta\kappa_i x}{2})e^{i(\bar{\kappa}_i x - \Phi)}
\end{aligned}$$