

# Homework 1

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DATE

1.7.6.

$$a \cdot b = \frac{(a+b)^2 - a^2 - b^2}{2}$$

Bab. • → Find Square root. S.

- Guess Pos.  $x_0$

- Improve guess,  $x_1 = \frac{x_0 + s(x_0)}{2}$

- Iterate until convergence

$$x_{n+1} = \frac{x_n + s(x_n)}{2}$$

$$\& x_n \approx x_{n+1}$$

This is less optimal than the Babylonian method. There are more computations needed (6) and the Babylonian requires  $y_0$ .

1.7.7.	.1	.9	1.7	2.5	3.3	4.1	4.8
	.2	1.0	1.8	2.6	3.4	4.2	4.9
	.3	1.1	1.9	2.7	3.5	4.3	5.0
	.4	1.2	2.0	2.8	3.6	4.4	
	.5	1.3	2.1	2.9	3.7	4.5	
	.6	1.4	2.2	3.0	3.8	4.6	
	.7	1.5	2.3	3.1	3.9	4.7	
	.8	1.6	2.4	3.2	4.0		

1.7.7

lookup table is the same as  
the cumulative distribution function  
table.  $Z$  and  $\Phi(\Phi)$  tables.

Not writing it out but have  
it in front of me.

(It is probably the most famous  
lookup table in existence)

1. We know more accurate units (to the hundredth  
than .1 in the  $Z$  table.

Thus one strategy is taking a  
row of values (0.1, 0.2, ..., 0.9), averaging  
them out and using this value for each  
tenth value.

↳ low accuracy at lower values that  
have higher range.

2. Taking average of prior and following  
responses.

↳ similar accuracy restriction as  
above.

1.7.18.  $a_1$  and  $a_2$  pos. #s

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Prove Arithmetic mean  $\geq$  geom. mean.

$$\text{Arith. mean} = \frac{a_1 + a_2}{2}$$

$$\text{geom. mean} = \sqrt{a_1 a_2}$$

to show: arith - geom  $\geq 0$

$$a - g = \frac{a_1 + a_2}{2} - \sqrt{a_1 a_2} = \frac{a_1 + a_2 - 2\sqrt{a_1 a_2}}{2}$$
$$= \frac{(\sqrt{a_1} - \sqrt{a_2})^2}{2}$$

this will always be positive

So arith - geom  $\geq 0$  showing

that arith. mean is always  
greater or equal to geom mean.

When equal?

when  $a_1 = a_2$

arith - geom = 0 thus

equal.

1.7.3u. Yes, we can multiply  
any # of square matrices  
using Strassen's Algorithm.

This is possible through the  
splitting, ie

$$\begin{aligned}ABC &= (AB) \times C \\ABCD &= ((AB) \times C) \times D\end{aligned}$$

1 Brute Force  $n \times n = O(n^3)$

Time

M Brute Force  $= (m-1)n^3 = O((m-1)n^3)$

Non Brute Force  $= O((m-1)n^{\log_2 7})$

much faster

1.07.26. Prove # mult required by  
fast mult to compute  $X^n$  is  
at most  $\lceil \log_2(n) \rceil$

At the lowest level, the  
number of steps needed to multiply  
in fast multiplication is dependent  
on the # of 1's in the binary.

To reach  $X^n$  we continually square  
 $X$  until we approach  $X^n$ .  
To compute this number we need  
 $\log_2(n)$  steps.

Then multiplying these values by  
each other we get  $\lceil \log_2(n) \rceil$   
as we had expected.