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February 10, 2020

HOMEWORK DUE MONDAY February 10:

#1: Investigate the Euclidean algorithm for various choices of x and y. What values cause it to take a long time? A short time? For problems like this you need to figure out what is the right metric to measure success. For example, if $x < y$ and it takes s steps, a good measure might be $s/\log_2(x)$.

The Euclidean algorithm will take the longest when our various choices for x and y represent large numbers. The shortest time would be adjacent fibonacci numbers.

We know that $x < y$ and we are using $\log_2(x)$ for our scaling.

We can assume that $x < y < 2x$ because at 2x then it will take more steps then the corresponding y which would provide a remainder.

Through our understanding of runtime, we can say that there will be a worse and best case runtime.

This will travel in some range from (y-x, x) for the same reason as stated above.

This ratio of worst runtime is thus $r = x / (y - x)$ which also equals y/x.

This gives us $y^2 - xy = x^2$

$= y^2 - xy - x^2 = 0$

Because $y = rx$, we can manipulate the above statement to be

$x^2(r^2 - 2 - 1) = 0$

We can simplify this to show that r is equal to the golden mean and thus the fibonacci numbers are the answer to our question.

#2: What is the dimension of the Cantor set?

Dimension d can be seen using the formula given of $c = r^d$

The Cantor set is defined as a set of points laying on a single line segment.

When we take a set and we triple it, we are resulting in two copies of the cantor set. Based on this tripling resulting in two copies of the set, we can understand that in the above definition c is 2 when r is 3.

Thus we understand the dimensions to be $\log_3(2)$

#3: Exercise 3.7.38: Find the optimal solution to the diet problem when the cost function is $\text{Cost}(x1, x2) = x1 + x2$.

To find the optimal solution, we are looking to find the minimal solution of x1 + x2 in which x > 0, this is meaning the minimal cost.

I will refer to the vector x, representing how much food we are buying, as *x* and vector b, representing how much nutrition is needed, as *b*. The matrix A will have the notation of being bold, **A**, this will represent the combination of nutrients and foods**.**

We will use the constraint **A***x* >= *b*.

x is consistent of x1 and x2 and *b* is made up of b1 and b2.

Our Matrix A will be equal to: **A** = $[a_{11} \quad a_{12}]$
$[a_{21} \quad a_{22}]$ (2x2 matrix)

Using strategies given in Linear Algebra we can build the linear inequalities:

$$a_{11}x_1 + a_{12}x_2 > b_1$$

$$a_{21}x_1 + a_{22}x_2 > b_2$$

These linear inequalities provide us with a vertex on the x axis, on the y axis and one where the two meet. We can add these three vertices and add their coordinates together to find the lowest cost optimal solution.

#4: Exercise 3.7.39. There are three vertices on the boundary of the polygon (of feasible solutions); we have seen two choices of cost functions that lead to two of the three points being optimal solutions; find a linear cost function which has the third vertex as an optimal solution.

In the text book we see that in figure 5 the cost of food2 is greatly outweighed by that of food1 and they have the same nutrients so the cost function solution is pretty self explanatory as it is optimal to only buy food2 for the full amount of nutrients needed.

In figure 6 things are a bit harder. The cost is now the same. We now must find an equilibrium between the two shown by the intersection of the two vertices which indicates that we are buying a certain portion of each food.

If we would like the third vertex to be the optimal solution, as stated above, we could use a linear cost function that shows food1 costing much less than food2. An example of this is:

$$Cost(x_1, x_2) = x_1 + 100000000x_2$$

#5: Exercise 3.7.40. Generalize the diet problem to the case when there are three or four types of food, and each food contains one of three items a person needs daily to live (for example,

calcium, iron, and protein). The region of feasible solutions will now be a subset of R . Show that an optimal solution is again a point on the boundary.

Need to show the diet problem when we are generalizing based on *n* foods. If we have *n* foods, with *m* types of nutrition, we need to minimize the cost in similar fashions as past problems.

Our solution will be a set of points in the set of real numbers. In this set of real numbers, each point is representative of a combination of one of the n foods which is cost minimized and will satisfy the amount of nutrients, *m,* that one needs. This is based on the similar constraint used above that $\mathbf{A}x >= b$. A is an m * n matrix in this case that shows how much of our necessary *m* nutrients is from each of our *n* foods. *b* is a vector that is made up of how much nutrients is needed and the understanding that *x* is greater than or equal to the zero vector.

#6: the diet problem with two products and two constraints led us to an infinite region, and then searching for the cheapest diet led us to a vertex point. Modify the diet problem by adding additional constraints so that, in general, we have a region of finite volume, and again show that the optimal point is at a vertex. Your constraints should be reasonable, and you should justify their inclusion.

A constraint in terms of cost can be a variety of things, one could be a business provides lunch for the office each day. In this case it is a similar thought as the diet problem where the *m* that we are thinking of is a sufficient amount of food for those in the office and our cost would need to be less than the food budget.

$$Cost(x_1,x_2) < \text{FoodBudget}$$

This is also a finite region as above. Our optimal point is still one of our vertices which are found in our linear cost function. This constraint limits the amount of points in our space as we are now eliminating all that have enough nutrients / adequately fill those eating, yet are limited solely to those points that do no go over the allocated budget. Any point that sufficiently fills everyone and is less than the budget in our space is fine.