Jonah Tuckman
March 2, 2020

1. Formulate Sudoku as a linear programming problem (you can do either 4x4 or 9x9 Sudoku).

I will first list the constraints of a 4x4 sudoku.

In each row in the range 1 to 4, the possible digit in the box must be independent of all others in the row, meaning that each digit {1 … 4} may occur only once in each row.

In each column in the range 1 to 4, the possible digit in the box must be independent of all others in the column, meaning that each digit {1 … 4} may occur only once in each column.

In each 2x2 square which contains a root box in the corner of the square, each possible digit in the square must be independent of all others in the square, meaning that each digit {1 … 4} may occur only once in each square.

Our function of action would be the need to fill in all blocks while keeping these constraints to be valid.
A valid bit would be sparked based on the summation of the rows, columns and squares. If the summation of a fully filled row, column or square were to be not equal to 10, then there would be an issue as the values are not 4,3,2,1. Thus this valid bit is sparked when this is not equal and the current solution possibility would fail.

2. Consider the 3 × 3 constraint matrix A where the first row is 1, 2, 3, the second row is 4, 5, 6 and the third row 7, 8, 9 (thus it's the numbers 1 through 3^2). Let the vector b equal $(1, 1, 1)^T$ Find all basic feasible solutions to Ax = b with x ≥ 0.

If we are to assume there is a basic feasible solution, then Ax = b with x being a non negative vector and b being equal to a vector with values all equal to 1. We can see that each number in the second row of the matrix is larger then each number in the first row. The same can be said about the third row to the second row. Based on our assumption we want the dot product of the rows to be equal. This is not possible as they are not equal in their values and are consistently unequal in the same fashion. Thus the prior need for the weighted sum to equal 1 is not possible and disproved.

3. Let's revisit the chess problem from class. Consider an n × n chess board. We want to put down n queens and maximize the number of pawns that can be safely placed on the board. Set this up as a linear programming problem.

The constraint that we have for this problem is that we are putting down a fixed number of queens. The problem states that that number is n and thus that is the constraint number of queens placed.
In terms of the attacking pattern of a queen to a pawn, we must think about the safety of a pawn in each situation. When a queen is placed, the pawns are not safe if they are in the diagonal lane of the queen (n row up/down, n row over), they are not safe if they are in the same row (horizontally) as the queen or the same column (vertically) as the queen.

From there we can describe our problem through the achievement of our goal given the understood constraints. We will be maximizing the number of pawns placed, and thus our strategy will be to set up a system that places our n queens on the board and from there checks the number of safe spaces to set down a pawn.

We will have a data structure to hold the board of the current optimal solution and a count variable to hold the highest amount of pawns. When placing different solutions of queens on the board, and calculating the number of safe pawns, we will compare it to our maximum value of pawns found so far and if it exceeds that amount we will change the saved board to be the current and then update the max value.

We will continue this process until we have tested each assortment of queens.

4. Consider the following Linear Programming problem: $x_j \geq 0$.
(Did not show the matrix as could not list it effectively in this format).
Find (or prove one does not exist) an optimal solution.

If each value (x_1, x_2, x_3, x_4, x_5) are all greater than or equal to the zero vector then $Ax = b$ would contain no possible solutions.
We can then manipulate the columns through subtracting and adding them and find that there is an inconsistency that presents us with the understanding that there are no optimal solutions.