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Question 1:

To prove that the change of base formula between exponents is equal, we will need to prove that the exponents themselves are equal and thus the change between them would no affect the results.

Based on our understanding of logarithmic bases, we know that the solution of something that is base two with x would similarly equal x with that base two raised to the solution (more simply $\log_2 x = y \Rightarrow y = 2^x$).

With this understanding of a logarithm we can look into each of our three logarithms and set them equal to arbitrary values in order to make this shift.

1. $x = b^{v1}$

2. $x = c^{v2}$

3. $b = c^{v3}$

With this understanding, we can reorder our equation based on like terms and using equation 2 and 3 see that $c^{v2} = c^{v1v3}$. With this understanding we can conclude that this could only be true if these arbitrary chosen values were equivalent and thus indicating that the exponents themselves are equivalent.

Question 2: 25 points:

The definition of a basic feasible solution is that there is linear independence between the columns of the vector and the x vector itself.

Our vector has three columns meaning that there are three possibilities for linear independence to be checked (col 1 and 2, col 2 and 3, col 1 and 3).

When checking the row pairings described above we can see that in col 1 and 2 there is no linear independence as the combination of rows would present us with no further solution, thus this does not provide us with a basic solution.

When checking column 1 and 3, we can see that there is indeed a solution. When adding a negative row 1 to row 2 we are presented with

$$\begin{pmatrix} 1 & 0 \end{pmatrix} = (0)$$

$$\begin{pmatrix} 0 & -1 \end{pmatrix} = (2)$$

This provides us with the basic solution of (1, 0, 2).

When checking column 2 and 3 we are able to find a solution by multiplying row 1 by -1. This presents us very simply with a basic solution of (0, 1, 1).

This there are two basic feasible solutions which are described above.

Question 3: 25 points:

When looking at the proof of the simplex method, we assumed that vector b is not a linear combination of less than m columns of the m x n constraint matrix A. Within this assumption we also have specifications for the dimensions of m and n. m must be smaller than n. Also the rows of A are all linearly independent.

Looking at the first assumption primarily, if our vector b were to have less than m positive components then it is in fact a contradiction of the understanding that b is a linear combination of no fewer than m columns of A. This contradiction would prove to break the proof thus these assumptions must stay true.

Question 4: 50 points:

- (a) No all games are not winnable. We could start of with four frogs on each corner of the board meaning that there are no frogs to jump over thus no where for them to go.
- (b) We know that each move must eliminate a frog (you cannot jump without jumping over one, no jumping over empty lilies) thus the amount of moves to win will always be N - 1. There are N frogs and there must be one left when the game is won thus each move will eliminate another frog until the game has ended through one remaining.
- (c) We have N frogs and a constraint of the game needing to be concluded in N-1 moves otherwise there has been an error. If we have gotten to a point where move count = N and there is more than 1 frog left, we have failed.
- We also have a constraint that only one frog can jump at a time (each turn is slotted to hold the jumping of solely one frog).
- The constraint of the technical jumping is that a frog can only make a jump from one lily to another if there is a frog between his departure lily and his empty landing lily. The empty landing lily must firstly be empty, and secondly be no more or less than two spaces from the departure lily. The lily between the departure lily and the landing lily must be occupied by a separate frog (non-empty).

- We must update the value of the jumped over lily as one does. If we are looking at it in a binary sense of occupied or not occupied then we will need to change the value of the previously occupied lily from 1 (occupied) to 0 (free).
- We also have L lilies on the board to be jumped onto by frogs.
- In this problem we will track peg location of the frogs through an understanding of the vertical and horizontal tracking patterns of this 2d array. Our frog will move along this 2d array and be tracked as his location changes, incrementing the turn variable and changing his current location in this array.
- Our objective function is that we want to minimize the amount of moves taken to have only one frog remaining, while ensuring that all of the above constraints and rulings are followed.
- (d) This game would be more challenging if the frogs were broken up into squadrons similar to chess and a frog could not move backwards from its sides directional movement. Similarly to chess all moves would have to be forward moving (other than a selective queen frog on each side who is a separate color and has moving ability). This would mean that the frogs no longer had the freedom of 360 degree movement but rather moves had to be understood by the placement they provide.

Question 5: 25 points:

This is a true statement. We can modify our algorithm to accept binary values (0,1) and be manipulated through a summation of values ranging from 0 to the logarithm base 2 of our constraints of $(317!)^{(2019!)}$. When we take this summation we are binding it to our necessary conditions and associating it with our binary values, this allows us to have a binary integer programming problem which is the goal of the exercise and would be scalable with binary programming practices.

Question 6: 25 points:

- (a) Yes, $x = \theta_1 x_1 + \ldots \theta_n x_n$ is a feasible solution. It is feasible because A is linearly independent this we can find vector x given above to equal vector b in $Ax = b$.
- (b) No this is not true because there are multiple solutions to similar vectors which violates the policy stated.

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Question 7: 25 points:

Yes, this is something that is possible considering that the original values in the problem are integers rather than binary values. We can modify our algorithm to accept binary values (0,1) and be manipulated through a summation of values ranging from 0 to the logarithm base 2 of our constraints of $(317!)^{(2019!)}$. When we take this summation we are binding it to our necessary conditions and associating it with our binary values. We can use the values of our binary conversation to be implicit in the summation of our values. Thus using the binary values as an indication of boolean (true or false) responses to conditions we can associate our restraints with binary values.