FYS3150 oppgavesett 1

Jon Aleksander Prøitz and Marius Torsvoll

Relevant code can be found at: https://github.com/Jonaproitz/Project_1

PROBLEM 1.

Given

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \tag{1}$$

Inserting $x = 0$

$$u(0) = 1 - (1 - e^{-10}) \cdot 0 - e^{-10 \cdot 0} = 1 - 0 - 1 = 0$$

and $x = 1$

$$u(1) = 1 - (1 - e^{-10}) \cdot 1 - e^{-10 \cdot 1} = 1 - 1 + e^{-10} - e^{-10} = 0$$

furthermore the one-dimensional possion equation can be written

$$-\frac{d^2u}{dx^2} = -\frac{d^2}{dx^2}\left(1 - (1 - e^{-10})x - e^{-10x}\right) = -\frac{d}{dx}\left((1 - e^{-10}) + 10e^{-10x}\right) = 100e^{-10x} = f(x)$$

Hence equation 1 is an exact solution to our problem. ■

PROBLEM 2.

Se githublink

PROBLEM 3.

The one-dimensional poisson equation can be written

$$-\frac{d^2u}{dx^2} = \frac{u(x-h) + 2u(x) - u(x+h)}{h^2} + O(h^2) = f(x)$$

Discretizing x with a given distance h between each distinct value then gives

$$\begin{aligned} x &\to x_0, x_1, x_2, ..., x_m \\ u(x) &\to u_0, u_1, u_2, ..., u_m \\ f(x) &\to f_0, f_1, f_2, ..., f_m \end{aligned}$$

with $u_i = v_i$, such that

$$-\frac{d^2v_i}{dx^2} = -v_{i-1} + 2v_i - v_{i+1} = f_i h^2$$

PROBLEM 4.

The set of equations from problem 3 can be written as

$$\begin{aligned} -v_0 + 2v_1 - v_2 &= h^2 f_1 \\ -v_1 + 2v_2 - v_3 &= h^2 f_2 \\ -v_2 + 2v_3 - v_4 &= h^2 f_3 \\ &\vdots \\ -v_{m-2} + 2v_{m-1} - v_m &= h^2 f_{m-1} \end{aligned}$$

Wich for

$$\begin{aligned} g_1 =& h^2 f_1 + v_0 \\ g_2 =& h^2 f_2 \\ g_3 =& h^2 f_3 \\ &\vdots \\ g_{m-2} =& h^2 f_{m-2} \\ g_{m-1} =& h^2 f_{m-1} + v_m \end{aligned}$$

can be written as the matrix equation

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \dots \\ -1 & 2 & -1 & 0 & 0 & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{m-2} \\ v_{m-1} \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_{m-2} \\ g_{m-1} \end{pmatrix}$$

on the form $A\vec{v} = \vec{g}$.

PROBLEM 5

a

When finding the matrix, A, in problem 4 it is assumed that v_0 and v_m are known. Hence theese values are not calculated and

$$n = m - 2$$

b

ACKNOWLEDGMENTS

Vi ønsker å takke det norske folk for deres støtte og hjelp i dette krevende arbeidet.