

FYS3150 oppgavesett 1

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Relevant code can be found at: https://github.com/Jonaproitz/Project_1

PROBLEM 1.

Given

$$u(x) = 1 - (1 - e^{-10})x - e^{-10x} \quad (1)$$

Inserting $x = 0$

$$u(0) = 1 - (1 - e^{-10}) \cdot 0 - e^{-10 \cdot 0} = 1 - 0 - 1 = 0$$

and $x = 1$

$$u(1) = 1 - (1 - e^{-10}) \cdot 1 - e^{-10 \cdot 1} = 1 - 1 + e^{-10} - e^{-10} = 0$$

furthermore the one-dimensional poisson equation can be written

$$-\frac{d^2 u}{dx^2} = -\frac{d^2}{dx^2} (1 - (1 - e^{-10})x - e^{-10x}) = -\frac{d}{dx} ((1 - e^{-10}) + 10e^{-10x}) = 100e^{-10x} = f(x)$$

Hence equation 1 is an exact solution to our problem. ■

PROBLEM 2.

Se githublink

PROBLEM 3.

The one-dimensional poisson equation can be written

$$-\frac{d^2 u}{dx^2} = \frac{u(x-h) + 2u(x) - u(x+h)}{h^2} + O(h^2) = f(x)$$

Discretizing x with m values and a given distance h between each distinct value then gives

$$\begin{aligned} x &\rightarrow x_0, x_1, x_2, \dots, x_{m-1} \\ u(x) &\rightarrow u_0, u_1, u_2, \dots, u_{m-1} \\ f(x) &\rightarrow f_0, f_1, f_2, \dots, f_{m-1} \end{aligned}$$

with $u_i = v_i$, such that

$$-\frac{d^2 v_i}{dx^2} = -v_{i-1} + 2v_i - v_{i+1} = f_i h^2$$

PROBLEM 4.

The set of equations from problem 3 can be written as

$$\begin{aligned} -v_0 + 2v_1 - v_2 &= h^2 f_1 \\ -v_1 + 2v_2 - v_3 &= h^2 f_2 \\ -v_2 + 2v_3 - v_4 &= h^2 f_3 \\ &\vdots \\ -v_{m-3} + 2v_{m-2} - v_{m-1} &= h^2 f_{m-2} \end{aligned}$$

Wich for

$$\begin{aligned}
 g_1 &= h^2 f_1 + v_0 \\
 g_2 &= h^2 f_2 \\
 g_3 &= h^2 f_3 \\
 &\vdots \\
 g_{m-3} &= h^2 f_{m-3} \\
 g_{m-1} &= h^2 f_{m-2} + v_{m-1}
 \end{aligned}$$

can be written as the matrix equation

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & \dots \\ -1 & 2 & -1 & 0 & 0 & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{m-3} \\ v_{m-2} \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_{m-3} \\ g_{m-2} \end{pmatrix}$$

on the form $A\vec{v} = \vec{g}$.

PROBLEM 5

a

When finding the matrix, A , in problem 4 it is assumed that v_0 and v_{m-1} are known. Hence these values are not calculated and

$$n = m - 2$$

b

Since $m = n + 2$ and \vec{v}^* is of length m the vector thus has to be given by the following:

$$\vec{v}^* = [v_0, \vec{V}, v_m] \quad (2)$$

This is a result of task 4 as the first and last elements of the equation set is known.

PROBLEM 6

a

A general tridiagonal matrix is given by the following: syntax:

$$\begin{array}{cccccccc|c}
 (R_1) & b_1 & c_1 & 0 & 0 & 0 & \dots & & g_1 \\
 (R_2) & a_2 & b_2 & c_2 & 0 & 0 & \dots & & g_2 \\
 (R_3) & 0 & a_3 & b_3 & c_3 & 0 & \dots & & g_3 \\
 & & & & & & & \ddots & \vdots \\
 (R_n) & 0 & 0 & 0 & 0 & a_n & b_n & & g_n
 \end{array} \quad (3)$$

Forward substitution gives us the following relation:

$$R_2 \rightarrow R_2 - \frac{a_2}{b_1} R_1 = 0 \quad b_2 - \frac{a_2}{b_1} c_1 \quad 0 \quad 0$$

Continuing this substitution with the variables gives us the following results for the variables:

$$\begin{aligned}\tilde{b}_1 &= b_1 \quad \text{and} \quad \tilde{b}_i = b_i - \frac{a_i}{\tilde{b}_{i-1}} c_{i-1}, & \text{For } i = 2, 3, 4, \dots, n \\ \tilde{g}_1 &= g_1 \quad \text{and} \quad \tilde{g}_i = g_i - \frac{a_i}{\tilde{g}_{i-1}} \tilde{g}_{i-1}, & \text{For } i = 2, 3, 4, \dots, n\end{aligned}$$

This then gives us the matrix:

$$\begin{array}{cccccc|c} \tilde{b} & c_1 & 0 & 0 & 0 & \dots & \tilde{g}_1 \\ 0 & \tilde{b}_2 & c_2 & 0 & 0 & \dots & \tilde{g}_2 \\ 0 & 0 & \tilde{b}_3 & c_3 & 0 & \dots & \tilde{g}_3 \\ & & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & & \tilde{b}_n & \tilde{g}_n \end{array}$$

We then see that:

$$R_n = \frac{R_n}{\tilde{b}_n} = 0 \quad 0$$

and that:

$$\frac{\tilde{g}_n}{\tilde{b}_n} = v_n$$

Bacwards substitution then gives us the following:

$$v_i = \frac{\tilde{g}_i - c_i v_{i+1}}{\tilde{b}_i}, \text{ For } i = n-1, n-2, n-3, \dots, 2, 1$$

b

This algorithm is thus:

$$\begin{aligned}\tilde{b}_1 &= b_1 \\ \tilde{b}_i &= b_i - \frac{a_i}{\tilde{b}_{i-1}} c_{i-1}, \text{ For } i = 2, 3, 4 \dots n \\ \tilde{g}_1 &= g_1 \tilde{g}_i = g_i - \frac{a_i}{\tilde{b}_{i-1}} \tilde{g}_{i-1}, \text{ For } i = 2, 3, 4 \dots n \\ v_n &= \frac{\tilde{g}_n}{\tilde{b}_n} \\ v_i &= \frac{\tilde{g}_i - c_i v_{i+1}}{\tilde{b}_i}, \text{ For } i = n-1, n-2, n-3, \dots, 1\end{aligned}$$

The total amount of FLOPs will then be described as the sum of FLOPs per n. Thus the FLOPs will then be described as:

$$8(n-1)(+1)$$

TASK 7

See the GitHub link

TASK 8

See the GitHub link

TASK 9

a

By making the special algorithm we then get the following results.

$$\begin{aligned} \tilde{b}_1 &= 2 \\ \tilde{b}_i &= 2 - \frac{1}{\tilde{b}_{i-1}} \\ \tilde{g}_1 &= g_1 \\ \tilde{g}_i &= g_i + \frac{1}{\tilde{b}_{i-1}} \tilde{g}_{i-1} \\ v_n &= \frac{\tilde{g}_n}{\tilde{b}_n} \\ v_i &= \frac{\tilde{g}_i + v_{i+1}}{\tilde{b}_i} \end{aligned}$$

b

The number of flops for this special algorithm will then be described as:

$$6(n-1)(+1)$$

c

See the [GitHub link](#)

TASK 10

See the [GitHub link](#)

TASK 11

ACKNOWLEDGMENTS

Vi ønsker å takke det norske folk for deres støtte og hjelp i dette krevende arbeidet.