Title of the document

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Url to GitHub reposotory: https://github.com/Jonaproitz/H21_Project_2_3150

PROBLEM 1.

Given

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x) \tag{1}$$

with the definition $\hat{x} = x/L$, such that

$$\frac{d\hat{x}}{dx} = \frac{1}{L} \implies dx = L d\hat{x}$$

Equation 1 can be written

$$\gamma \frac{d^2 u(\hat{x})}{L^2 d\hat{x}^2} = -F u(\hat{x}) \implies \frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\frac{FL^2}{\gamma} u(\hat{x}) = -\lambda u(\hat{x})$$

with $\lambda = FL^2/\gamma$. ■

PROBLEM 2.

For an arbitrary composite matrix $A = BC$ the transpose of $A^T = (BC)^T = C^T B^T$. Hence for $\vec{w}_i = U\vec{v}_i$

$$\vec{w}_i^T \vec{w}_j = \vec{v}_i^T U^T U \vec{v}_j = \vec{v}_i^T \vec{v}_i = \delta_{i,j}$$

as $U^T U = U^{-1} U = I$

PROBLEM 3.

The calculated values is to be found below. From table 13 to 26 the eigenvalue of the numerical and analytical solution can be found. The expected analytical and the numerical value is are very close to each other.

TABLE I. Eigenvalue number 1.

arma::eig_sym	Analytic	Difference
-74.2949	-74.2949	0

TABLE II. Eigenvector number 1.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	1.11022e-16
-0.417907	-0.417907	1.11022e-16
-0.521121	-0.521121	3.33067e-16
-0.521121	-0.521121	3.33067e-16
-0.417907	-0.417907	3.33067e-16
-0.231921	-0.231921	3.33067e-16

TABLE III. Eigenvalue number 2.

arma::eig_sym	Analytic	Difference
-47.102	-47.102	2.13163e-14

TABLE IV. Eigenvector number 2.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	1.66533e-16
-0.521121	-0.521121	1.11022e-16
-0.231921	-0.231921	8.32667e-17
0.231921	0.231921	1.94289e-16
0.521121	0.521121	2.22045e-16
0.417907	0.417907	2.77556e-16

TABLE V. Eigenvalue number 3.

arma::eig_sym	Analytic	Difference
-7.80705	-7.80705	5.32907e-15

TABLE VI. Eigenvector number 3.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	4.44089e-16
-0.231921	-0.231921	1.66533e-16
0.417907	0.417907	0
0.417907	0.417907	2.77556e-16
-0.231921	-0.231921	6.66134e-16
-0.521121	-0.521121	2.22045e-16

TABLE VII. Eigenvalue number 4.

arma::eig_sym	Analytic	Difference
35.8071	35.8071	2.13163e-14

TABLE VIII. Eigenvector number 4.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	2.22045e-16
0.231921	0.231921	2.77556e-16
0.417907	0.417907	1.66533e-16
-0.417907	-0.417907	1.66533e-16
-0.231921	-0.231921	1.66533e-16
0.521121	0.521121	2.22045e-16

TABLE IX. Eigenvalue number 5.

arma::eig_sym	Analytic	Difference
75.102	75.102	0

TABLE X. Eigenvector number 5.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	1.11022e-16
0.521121	0.521121	6.66134e-16
-0.231921	-0.231921	3.05311e-16
-0.231921	-0.231921	1.38778e-16
0.521121	0.521121	2.22045e-16
-0.417907	-0.417907	3.88578e-16

TABLE XI. Eigenvalue number 6.

arma::eig_sym	Analytic	Difference
102.295	102.295	0

TABLE XII. Eigenvector number 6.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	0
0.417907	0.417907	1.11022e-16
-0.521121	-0.521121	3.33067e-16
0.521121	0.521121	1.11022e-16
-0.417907	-0.417907	3.33067e-16
0.231921	0.231921	1.94289e-16

PROBLEM 4.

The script found at the url at the top of the page returns 0.7. Which is to be expected as the returned value is supposed to be the highest absolute value not on the diagonal of the matrix.

PROBLEM 5.

The calculated values is to be found below. From table 13 to 26 the eigenvalue of the numerical and analytical solution can be found. The expected analytical and the numerical value is are very close to each other. These values are somewhat less accurate than in problem 3.

TABLE XIII. Eigenvalue number 1.

arma::eig_sym	Analytic	Difference
-74.2949	-74.2949	2.84217e-14

TABLE XIV. Eigenvector number 1.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	4.72591e-11
-0.417907	-0.417907	4.07815e-11
-0.521121	-0.521121	2.62265e-11
-0.521121	-0.521121	1.81499e-11
-0.417907	-0.417907	5.89315e-11
-0.231921	-0.231921	3.27045e-11

TABLE XV. Eigenvalue number 2.

arma::eig_sym	Analytic	Difference
-47.102	-47.102	7.10543e-14

TABLE XVI. Eigenvector number 2.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	2.61546e-11
-0.521121	-0.521121	3.27957e-11
-0.231921	-0.231921	5.88911e-11
0.231921	0.231921	4.07412e-11
0.521121	0.521121	4.73492e-11
0.417907	0.417907	1.80767e-11

TABLE XVII. Eigenvalue number 3.

arma::eig_sym	Analytic	Difference
-7.80705	-7.80705	6.21725e-15

TABLE XVIII. Eigenvector number 3.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	9.02611e-14
-0.231921	-0.231921	4.03566e-14
0.417907	0.417907	7.25531e-14
0.417907	0.417907	7.27196e-14
-0.231921	-0.231921	4.06897e-14
-0.521121	-0.521121	9.09273e-14

TABLE XIX. Eigenvalue number 4.

arma::eig_sym	Analytic	Difference
35.8071	35.8071	1.42109e-14

TABLE XX. Eigenvector number 4.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	9.09273e-14
0.231921	0.231921	4.03844e-14
0.417907	0.417907	7.28306e-14
-0.417907	-0.417907	7.27196e-14
-0.231921	-0.231921	4.06897e-14
0.521121	0.521121	9.03722e-14

TABLE XXI. Eigenvalue number 5.

arma::eig_sym	Analytic	Difference
75.102	75.102	4.26326e-14

TABLE XXII. Eigenvector number 5.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	2.63001e-11
0.521121	0.521121	3.26146e-11
-0.231921	-0.231921	5.89711e-11
-0.231921	-0.231921	4.08214e-11
0.521121	0.521121	4.71678e-11
-0.417907	-0.417907	1.82227e-11

TABLE XXIII. Eigenvalue number 6.

arma::eig_sym	Analytic	Difference
102.295	102.295	7.10543e-14

TABLE XXIV. Eigenvector number 6.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	4.72589e-11
0.417907	0.417907	4.07812e-11
-0.521121	-0.521121	2.62262e-11
0.521121	0.521121	1.815e-11
-0.417907	-0.417907	5.89307e-11
0.231921	0.231921	3.27042e-11

PROBLEM 6.

The change of the number of iterations behaves like the quadratic function. This was expected

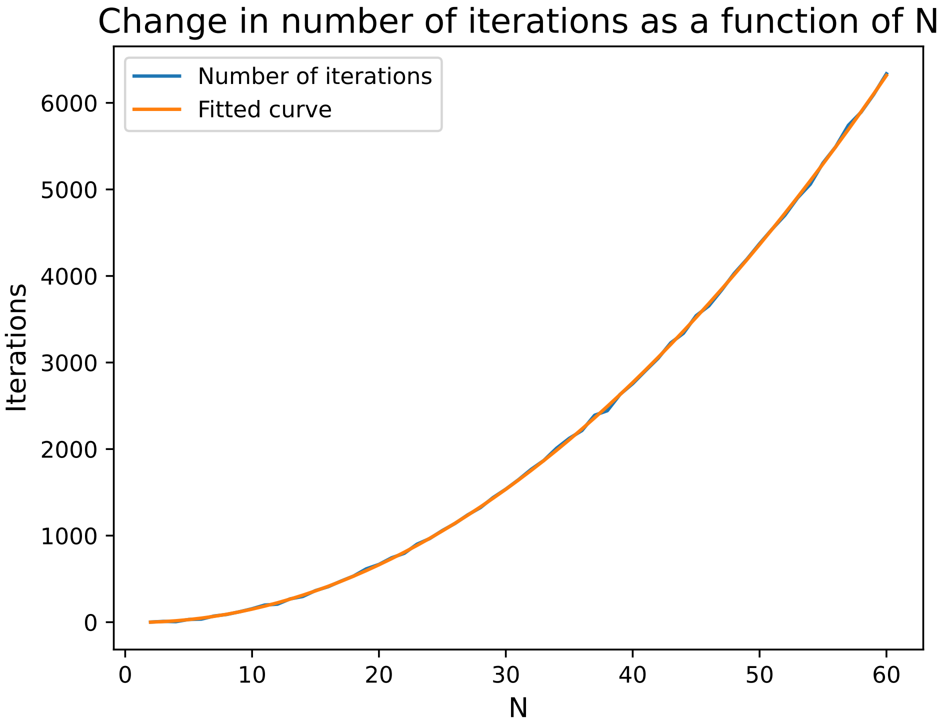

FIG. 1.

B

We expect nothing to change as the algorithm will adjust all values in the matrix. This may be verified by running the algorithm for a dense matrix.

PROBLEM 7.

The Eigenvectors plotted for n = 10 and for n = 100 plotted with the analytical solution. As expected the N = 100 solution is alot smoother and a more accurate representation of the analytical solution.

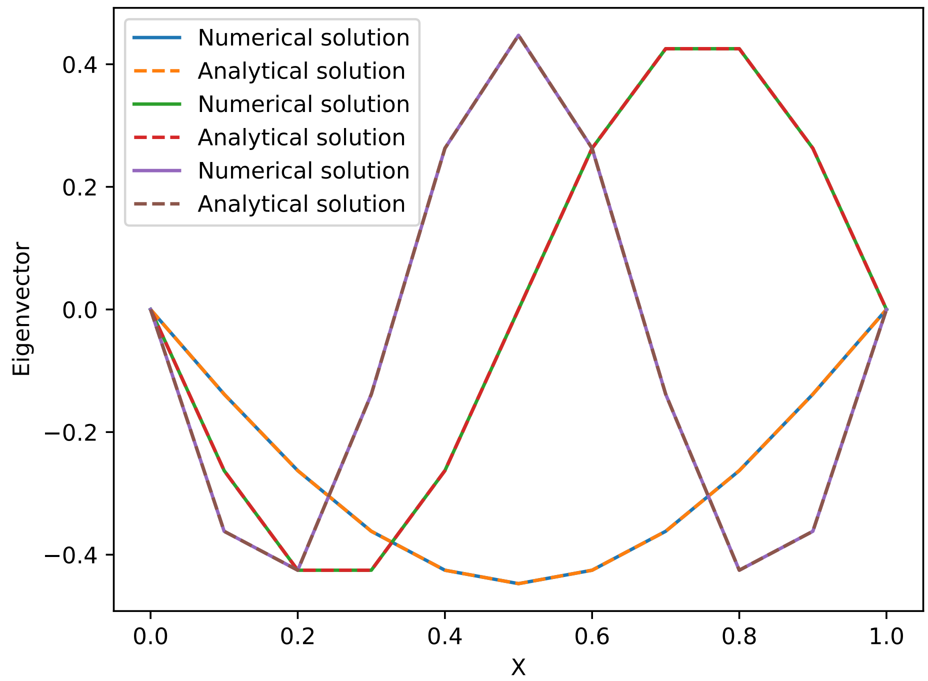

FIG. 2. Plotted for N = 10

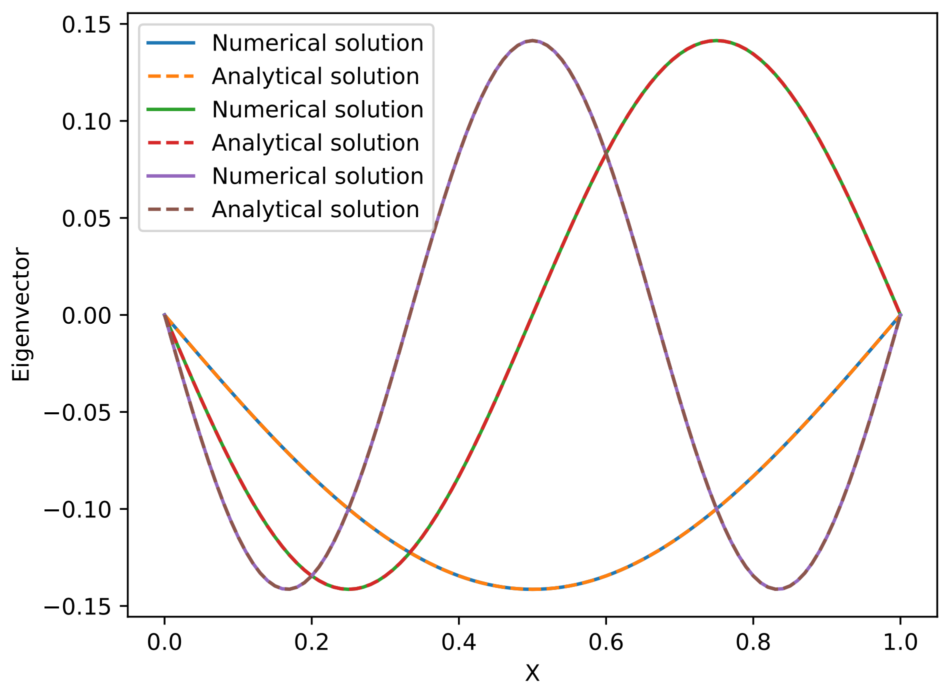

FIG. 3. Plotted for N = 100.