Title of the document

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Url to GitHub repository: https://github.com/Jonaproitz/H21_Project_2_3150

PROBLEM 1.

Given

$$\gamma \frac{d^2u(x)}{dx^2} = -Fu(x) \tag{1}$$

with the definition $\hat{x} = x/L$, such that

$$\frac{d\hat{x}}{dx} = \frac{1}{L} \implies dx = Ld\hat{x}$$

Equation 1 can be written

$$\gamma \frac{d^2u(\hat{x})}{L^2 d\hat{x}^2} = -Fu(\hat{x}) \implies \frac{d^2u(\hat{x})}{d\hat{x}^2} = -\frac{FL^2}{\gamma}u(\hat{x}) = -\lambda u(\hat{x})$$

with $\lambda = FL^2/\gamma$. ■

PROBLEM 2.

For an arbitrary composite matrix $A = BC$ the transpose of $A^T = (BC)^T = C^T B^T$. Hence for $\vec{w}_i = U\vec{v}_i$

$$\vec{w}_i^T \vec{w}_j = \vec{v}_i^T U^T U \vec{v}_j = \vec{v}_i^T \vec{v}_i = \delta_{i,j}$$

as $U^T U = U^{-1} U = I$

PROBLEM 3.

TABLE I. Eigenvalue number 1.

arma::eig_sym	Analytic	Difference
-74.2949	-74.2949	0

TABLE II. Eigenvector number 1.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	1.11022e-16
-0.417907	-0.417907	1.11022e-16
-0.521121	-0.521121	3.33067e-16
-0.521121	-0.521121	3.33067e-16
-0.417907	-0.417907	3.33067e-16
-0.231921	-0.231921	3.33067e-16

TABLE III. Eigenvalue number 2.

arma::eig_sym	Analytic	Difference
-47.102	-47.102	2.13163e-14

TABLE IV. Eigenvector number 2.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	1.66533e-16
-0.521121	-0.521121	1.11022e-16
-0.231921	-0.231921	8.32667e-17
0.231921	0.231921	1.94289e-16
0.521121	0.521121	2.22045e-16
0.417907	0.417907	2.77556e-16

TABLE V. Eigenvalue number 3.

arma::eig_sym	Analytic	Difference
-7.80705	-7.80705	5.32907e-15

TABLE VI. Eigenvector number 3.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	4.44089e-16
-0.231921	-0.231921	1.66533e-16
0.417907	0.417907	0
0.417907	0.417907	2.77556e-16
-0.231921	-0.231921	6.66134e-16
-0.521121	-0.521121	2.22045e-16

TABLE VII. Eigenvalue number 4.

arma::eig_sym	Analytic	Difference
35.8071	35.8071	2.13163e-14

TABLE VIII. Eigenvector number 4.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	2.22045e-16
0.231921	0.231921	2.77556e-16
0.417907	0.417907	1.66533e-16
-0.417907	-0.417907	1.66533e-16
-0.231921	-0.231921	1.66533e-16
0.521121	0.521121	2.22045e-16

TABLE IX. Eigenvalue number 5.

arma::eig_sym	Analytic	Difference
75.102	75.102	0

TABLE X. Eigenvector number 5.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	1.11022e-16
0.521121	0.521121	6.66134e-16
-0.231921	-0.231921	3.05311e-16
-0.231921	-0.231921	1.38778e-16
0.521121	0.521121	2.22045e-16
-0.417907	-0.417907	3.88578e-16

TABLE XI. Eigenvalue number 6.

arma::eig_sym	Analytic	Difference
102.295	102.295	0

TABLE XII. Eigenvector number 6.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	0
0.417907	0.417907	1.11022e-16
-0.521121	-0.521121	3.33067e-16
0.521121	0.521121	1.11022e-16
-0.417907	-0.417907	3.33067e-16
0.231921	0.231921	1.94289e-16

PROBLEM 4.

The script found at the url at the top of the page returns 0.7.

PROBLEM 5.

TABLE XIII. Eigenvalue number 1.

arma::eig_sym	Analytic	Difference
-74.2949	-74.2949	2.84217e-14

TABLE XIV. Eigenvector number 1.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	4.72591e-11
-0.417907	-0.417907	4.07815e-11
-0.521121	-0.521121	2.62265e-11
-0.521121	-0.521121	1.81499e-11
-0.417907	-0.417907	5.89315e-11
-0.231921	-0.231921	3.27045e-11

TABLE XV. Eigenvalue number 2.

arma::eig_sym	Analytic	Difference
-47.102	-47.102	7.10543e-14

TABLE XVI. Eigenvector number 2.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	2.61546e-11
-0.521121	-0.521121	3.27957e-11
-0.231921	-0.231921	5.88911e-11
0.231921	0.231921	4.07412e-11
0.521121	0.521121	4.73492e-11
0.417907	0.417907	1.80767e-11

TABLE XVII. Eigenvalue number 3.

arma::eig_sym	Analytic	Difference
-7.80705	-7.80705	6.21725e-15

TABLE XIX. Eigenvalue number 4.

arma::eig_sym	Analytic	Difference
35.8071	35.8071	1.42109e-14

TABLE XVIII. Eigenvector number 3.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	9.02611e-14
-0.231921	-0.231921	4.03566e-14
0.417907	0.417907	7.25531e-14
0.417907	0.417907	7.27196e-14
-0.231921	-0.231921	4.06897e-14
-0.521121	-0.521121	9.09273e-14

TABLE XX. Eigenvector number 4.

arma::eig_sym	Analytic	Difference
-0.521121	-0.521121	9.09273e-14
0.231921	0.231921	4.03844e-14
0.417907	0.417907	7.28306e-14
-0.417907	-0.417907	7.27196e-14
-0.231921	-0.231921	4.06897e-14
0.521121	0.521121	9.03722e-14

TABLE XXI. Eigenvalue number 5.

arma::eig_sym	Analytic	Difference
75.102	75.102	4.26326e-14

TABLE XXIII. Eigenvalue number 6.

arma::eig_sym	Analytic	Difference
102.295	102.295	7.10543e-14

TABLE XXII. Eigenvector number 5.

arma::eig_sym	Analytic	Difference
-0.417907	-0.417907	2.63001e-11
0.521121	0.521121	3.26146e-11
-0.231921	-0.231921	5.89711e-11
-0.231921	-0.231921	4.08214e-11
0.521121	0.521121	4.71678e-11
-0.417907	-0.417907	1.82227e-11

TABLE XXIV. Eigenvector number 6.

arma::eig_sym	Analytic	Difference
-0.231921	-0.231921	4.72589e-11
0.417907	0.417907	4.07812e-11
-0.521121	-0.521121	2.62262e-11
0.521121	0.521121	1.815e-11
-0.417907	-0.417907	5.89307e-11
0.231921	0.231921	3.27042e-11

PROBLEM 6.

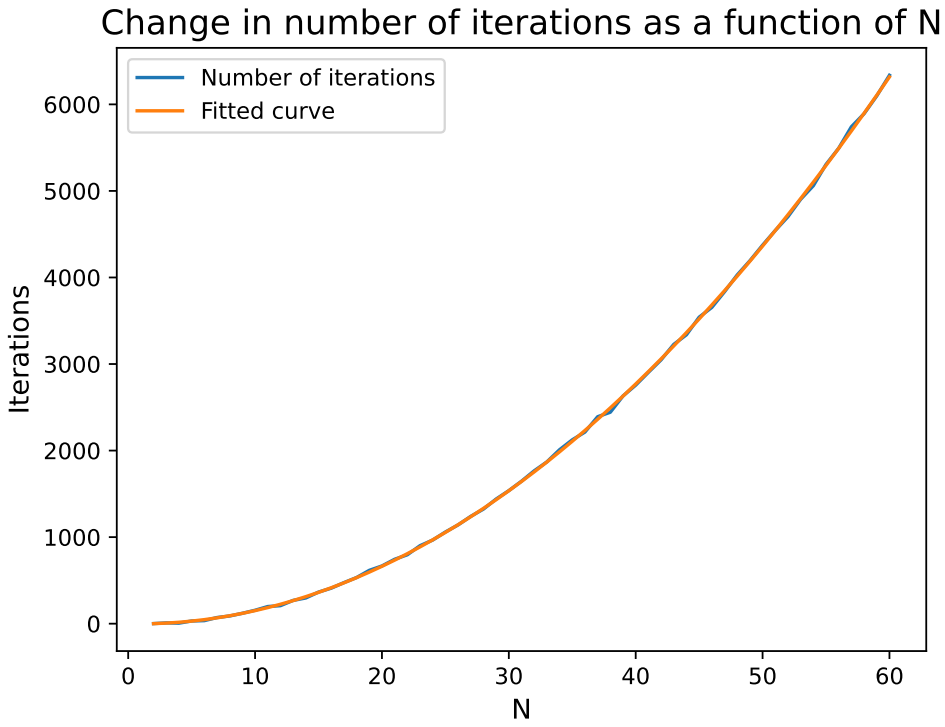
Change in number of iterations as a function of N
Number of iterations
Fitted curve
Iterations
N
0
1000
2000
3000
4000
5000
6000
0
10
20
30
40
50
60

FIG. 1.