Title of the document

Your name(s) here
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PROBLEM 1.

Given

$$\gamma \frac{d^2 u(x)}{dx^2} = -F u(x) \tag{1}$$

with the definition $\hat{x} = x/L$, such that

$$\frac{d\hat{x}}{dx} = \frac{1}{L} \implies dx = L d\hat{x}$$

Equation 1 can be written

$$\gamma \frac{d^2 u(\hat{x})}{L^2 d\hat{x}^2} = -F u(\hat{x}) \implies \frac{d^2 u(\hat{x})}{d\hat{x}^2} = -\frac{F L^2}{\gamma} u(\hat{x}) = -\lambda u(\hat{x})$$

with $\lambda = F L^2 / \gamma$. ∎

PROBLEM 2.

For an arbitrary composite matrix $A = BC$ the transpose of $A^T = (BC)^T = C^T B^T$. Hence for $\vec{w}_i = U \vec{v}_i$

$$\vec{w}_i^T \vec{w}_j = \vec{v}_i^T U^T U \vec{v}_j = \vec{v}_i^T \vec{v}_i = \delta_{i,j}$$

as $U^T U = U^{-1} U = I$

PROBLEM 3.

TABLE I. Eigenvalue number 1.		
arma::sym	Analytic	Difference
-74.2949	-74.2949	0

TABLE II. Eigenvector number 1.		
arma::sym	Analytic	Difference
-0.231921	-0.231921	1.11022e-16
-0.417907	-0.417907	1.11022e-16
-0.521121	-0.521121	3.33067e-16
-0.521121	-0.521121	3.33067e-16
-0.417907	-0.417907	3.33067e-16
-0.231921	-0.231921	3.33067e-16

TABLE III. Eigenvalue number 2.		
arma::sym	Analytic	Difference
-47.102	-47.102	2.13163e-14

TABLE IV. Eigenvector number 2.		
arma::sym	Analytic	Difference
-0.417907	-0.417907	1.66533e-16
-0.521121	-0.521121	1.11022e-16
-0.231921	-0.231921	8.32667e-17
0.231921	0.231921	1.94289e-16
0.521121	0.521121	2.22045e-16
0.417907	0.417907	2.77556e-16

TABLE V. Eigenvalue number 2.

arma::sym	Analytic	Difference
-7.80705	-7.80705	5.32907e-15

TABLE VI. Eigenvector number 3.

arma::sym	Analytic	Difference
-0.521121	-0.521121	4.44089e-16
-0.231921	-0.231921	1.66533e-16
0.417907	0.417907	0
0.417907	0.417907	2.77556e-16
-0.231921	-0.231921	6.66134e-16
-0.521121	-0.521121	2.22045e-16

TABLE VII. Eigenvalue number 4.

arma::sym	Analytic	Difference
35.8071	35.8071	2.13163e-14

TABLE VIII. Eigenvector number 4.

arma::sym	Analytic	Difference
-0.521121	-0.521121	2.22045e-16
0.231921	0.231921	2.77556e-16
0.417907	0.417907	1.66533e-16
-0.417907	-0.417907	1.66533e-16
-0.231921	-0.231921	1.66533e-16
0.521121	0.521121	2.22045e-16

TABLE IX. Eigenvalue number 5.

arma::sym	Analytic	Difference
75.102	75.102	0

TABLE X. Eigenvector number 5.

arma::sym	Analytic	Difference
-0.417907	-0.417907	1.11022e-16
0.521121	0.521121	6.66134e-16
-0.231921	-0.231921	3.05311e-16
-0.231921	-0.231921	1.38778e-16
0.521121	0.521121	2.22045e-16
-0.417907	-0.417907	3.88578e-16

TABLE XI. Eigenvalue number 6.

arma::sym	Analytic	Difference
102.295	102.295	0

TABLE XII. Eigenvector number 6.

arma::sym	Analytic	Difference
-0.231921	-0.231921	0
0.417907	0.417907	1.11022e-16
-0.521121	-0.521121	3.33067e-16
0.521121	0.521121	1.11022e-16
-0.417907	-0.417907	3.33067e-16
0.231921	0.231921	1.94289e-16