Assignment 1

Linear Regression

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A DSAA5020 Assignment

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Problem 1

Find the closed form solution for linear regression that minimizes the mean square error (MSE)..

Solution. For Linear Regression loss function MSE, we have:

$$
\begin{aligned}
L(w) &= \frac{1}{2}\sum_{i=1}^{n}(w^T x_i - y_i)^2 \\
&= \frac{1}{2}(Xw - y)^T(Xw - y) \\
&= \frac{1}{2}(w^T X^T X w - y^T X w - w^T X^T y + y^T y) \\
&= \frac{1}{2}(w^T X^T X w - 2y^T X w + y^T y)
\end{aligned}
$$

Finding the first-order partial derivative (gradient) of L(w):

$$
\begin{aligned}
\nabla_w L(w) &= \frac{\partial L(w)}{\partial w} \\
&= \frac{\partial}{\partial w}[\frac{1}{2}(w^T X^T X w - 2w^T X^T y + y^T y)] \\
&= X^T X w - X^T y \\
&= X^T(Xw - y)
\end{aligned}
$$

let $\nabla_w L(w) = 0$, we have:

$$
\begin{aligned}
\nabla_w L(w) &= 0 \\
X^T X w - X^T y &= 0
\end{aligned}
$$

Finding the secound-order partial derivative (gradient) of L(w):

$$
\begin{aligned}
H(L(w)) &= \frac{\partial(\nabla X^T(Xw - y))}{\partial w} \\
&= X^T X
\end{aligned}
$$

Because $X^T X \succcurlyeq 0$ so that, when $\nabla_w L(w) = 0$, w has its minimized value:

$$
w^* = (X^T X)^{-1} X^T y
$$