BPRD – Programs as Data

Lecture 3: Parsing (2/3)

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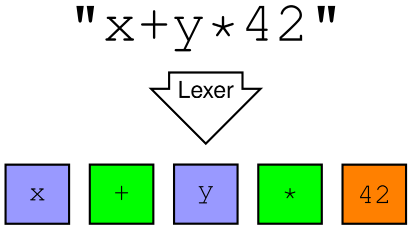
These slides are based on original slides by Niels Hallenberg, Peter Sestoft, David Raymond Christensen and Ahmad Salim Al-Sibahi. Thanks!!!

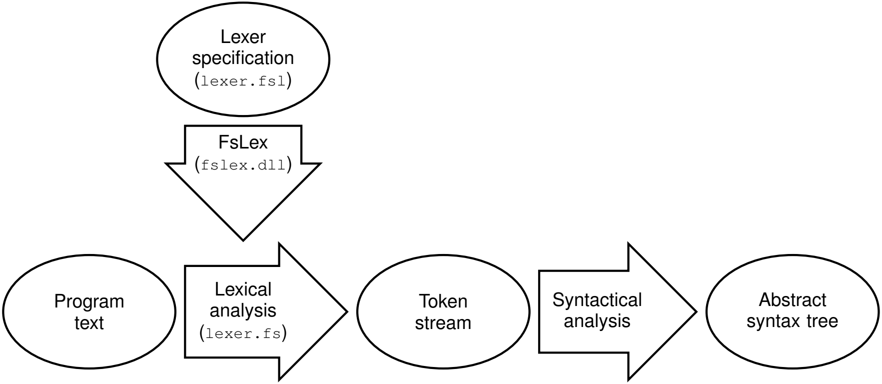
Previous lecture: Lexical analysis

Lexical analysis (Lexer) converts LexBuffer to a token stream



Previous lecture: Lexer generator

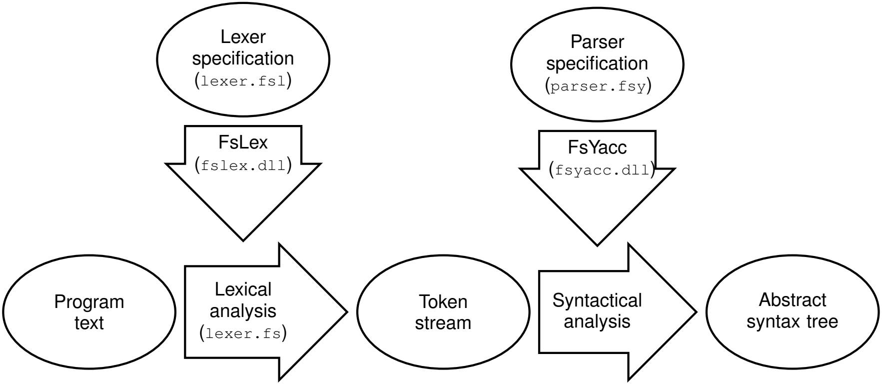
‘ A lexer generator takes as input a lexer specification and outputs a lexer.



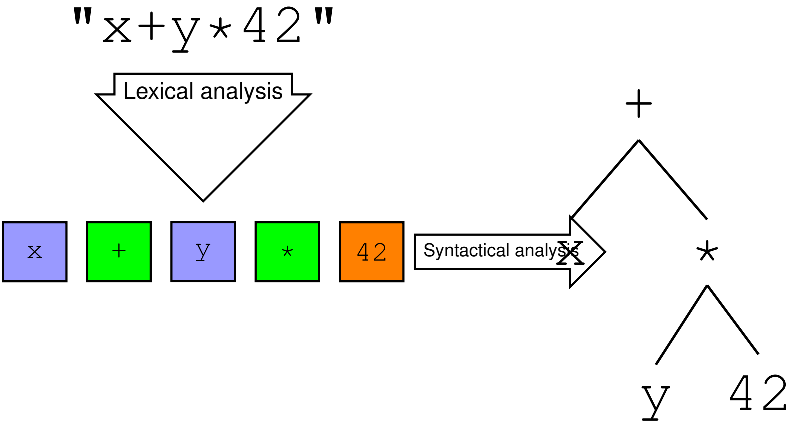
Previous lecture: Regular expression $\rightarrow$ NFA $\rightarrow$ DFA

The lexer (usually) relies on an DFA, which is transformed from the regular expressions. This transformation makes sense thanks to Kleene's Theorem.

Today's lecture



Demo



Anecdote: How I used parsing techniques in my work

First, I used parsing techniques to derive security holes of a programs into logic constraints.

```
void Prog(double x) {
  if (x < 1){
    x = x + 1;
    assert(x < 2);
}}
```

The security hole is transformed into $x < 1 \wedge x + 1 \geq 2$.

Then I solved these constraints via MCMC (Markov chain Monte Carlo), an approach in applied mathematics.

Finally, I found that "x = 0.999 999 999 999 999 9" opened that door.
Demo.

Reference; "Effective floating-point analysis via weak-distance minimization". In ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI), 2019.

`http://zhoulaifu.com/wp-content/papercite-data/pdf/wd.pdf`

Intended learning outcomes for today

1. You know what context-free grammar is and why it matters
   - Derivations
   - Leftmost and Rightmost Derivations
   - Ambiguity
2. You get an intuition of concepts in context-free grammar presented above, and in particular, how to generate a language with in practice
3. You know how to Parse with a grammar
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A grammar in Programming Language Theory describes the syntax of a language, namely, how to form sentences from a language's words.

Which grammar have we already used?

- Grammar in Danish, English: Subject - Verb - Object
- Regular expression: $[1-9][0-9]*$ for positive integers
- Context-free grammar

A grammar in Programming Language Theory describes the syntax of a language, namely, how to form sentences from a language's words.

Which grammar have we already used?

- Grammar in Danish, English: Subject - Verb - Object
- Regular expression: $[1-9][0-9]*$ for positive integers
- Context-free grammar

Context-Free Grammars

Definition (Context-free grammar)

A **context-free grammar**, abbreviated CFG, is a grammar whose rules have the following properties:

- The left-hand side is a single variable.
- The right-hand side is any string of variables and terminals.
- That is, every rule is of the form $A \to w$, where $w \in (V \cup \Sigma)^*$.

Example

Example (Context-free grammar)

Let

- $V = \{E\}$
- $S = E$
- $\Sigma = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, +, *, (,)\}$
- R consists of the rules

$$E \rightarrow E + E \mid E * E \mid (E) \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$$

Examples

Example (Context-free grammars)

Find CFGs for the following languages.

- $\{\mathbf{a}^n\mathbf{b}^n \mid n \geq 0\}$
- $\{\mathbf{a}^n\mathbf{b}^m \mid n \geq m \geq 0\}$
- $\{\mathbf{a}^n\mathbf{b}^m \mid m \geq n \geq 0\}$

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Derivations

Definition (Yields)

A string u **yields** a string v if we can apply a grammar rule to u and get v.

- We write $u \Rightarrow v$.

Derivations

Definition (Derives)

A string u **derives** a string v if there is a sequence

$$u_1, u_2, \ldots, u_k,$$

with $k \geq 1$, where $u = u_1$, $v = u_k$, and

$$u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k.$$

The sequence is called a **derivation**.

- We write $u \stackrel{*}{\Rightarrow} v$.

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Leftmost and Rightmost Derivations

Definition (Leftmost derivation)

A **leftmost derivation** of a string is a derivation in which, at each step, the leftmost variable is replaced with a string.

Definition (Rightmost derivation)

A **rightmost derivation** of a string is a derivation in which, at each step, the rightmost variable is replaced with a string.

Example

Example (Leftmost and rightmost derivations)

Using the grammar

$$S \rightarrow SS \mid \mathbf{a}S\mathbf{b} \mid \mathbf{b}S\mathbf{a} \mid \varepsilon,$$

find leftmost and rightmost derivations of **abab**.

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Ambiguity

- Some grammars provide more than one way to derive a string.
- For example, **abab** can be derived in two different ways using the grammar rules

$$S \rightarrow SS \mid \mathbf{a}S\mathbf{b} \mid \mathbf{b}S\mathbf{a} \mid \varepsilon.$$

Ambiguity

Definition (Ambiguous grammar)

A grammar is **ambiguous** if its language contains a string that has more than one leftmost derivation under that grammar.

Definition (Inherently ambiguous language)

A language is **inherently ambiguous** if every grammar for that language is ambiguous.

Ambiguity

- Consider again the grammar rules

$$S \to SS \mid \mathbf{a}S\mathbf{b} \mid \mathbf{b}S\mathbf{a} \mid \varepsilon.$$

- Suppose that, for every string, we need to associate each **a** in the string with a unique **b** in the string.
- Does the grammar show us how to do this?

Example

Example (Ambiguous grammar)

- Consider the grammar

$$E \to E + E \mid E * E \mid (E) \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}.$$

- Derive the string $\mathbf{a} + \mathbf{b} * \mathbf{c}$ in two different ways.
- Is this grammar ambiguous?

Example

Example (Unambiguous grammar)

The same language can be derived unambiguously from the following grammar.

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$$

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Context-free grammars

See PLC page 45.

```
Main ::= Expr EOF                                (rule A)
Expr ::= NAME                                    (rule B)
       | CSTINT                                  (rule C)
       | - CSTINT                                (rule D)
       | ( Expr )                                (rule E)
       | let NAME = Expr in Expr end             (rule F)
       | Expr *  Expr                            (rule G)
       | Expr + Expr                             (rule H)
       | Expr - Expr                             (rule I)
```

- Nonterminal symbols
- Terminal symbols (from lexer)
- Grammar rules, or Productions (called A–I)
- Start symbol (the nonterminal Main)

Context-free grammars

See PLC page 45.

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       | Expr + Expr                             (rule H)
       | Expr - Expr                             (rule I)
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- Nonterminal symbols
- Terminal symbols (from lexer)
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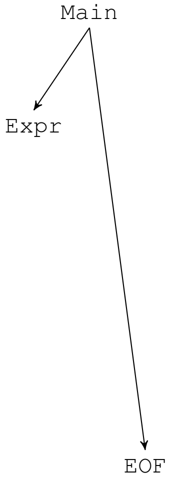
- **Start symbol (the nonterminal Main)**

Derivation: grammar as string generator

	Main
A	Expr EOF
H	Expr + Expr EOF
B	x + Expr EOF
G	x + Expr * Expr EOF
C	x + 52 * EXPR EOF
B	x + 52 * wk EOF

Main

Derivation: grammar as string generator



	Main
A	**Expr EOF**
H	Expr + Expr EOF
B	x + Expr EOF
G	x + Expr * Expr EOF
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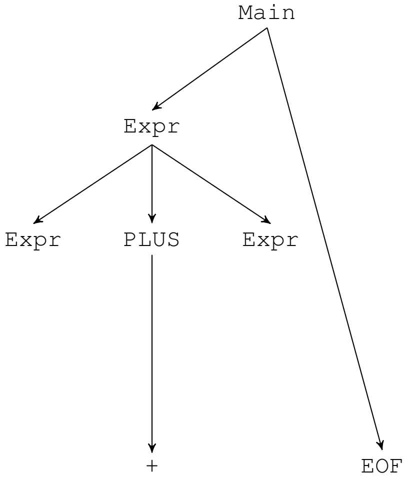

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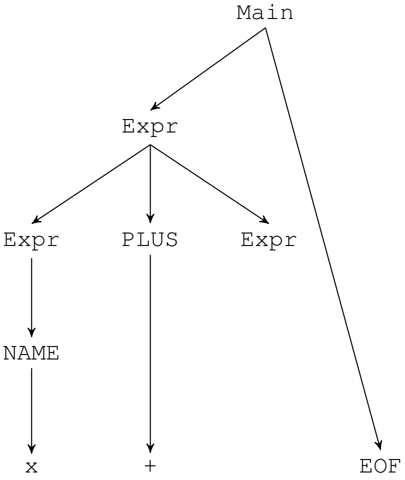
Derivation: grammar as string generator



	Main
A	Expr EOF
H	**Expr** + Expr EOF
B	**x** + Expr EOF
G	x + Expr * Expr EOF
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Derivation: grammar as string generator

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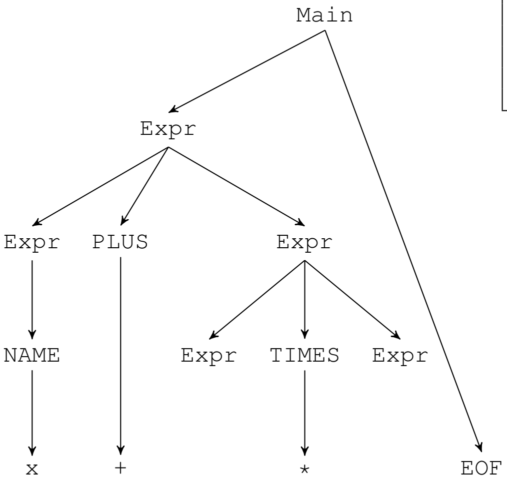

Derivation: grammar as string generator

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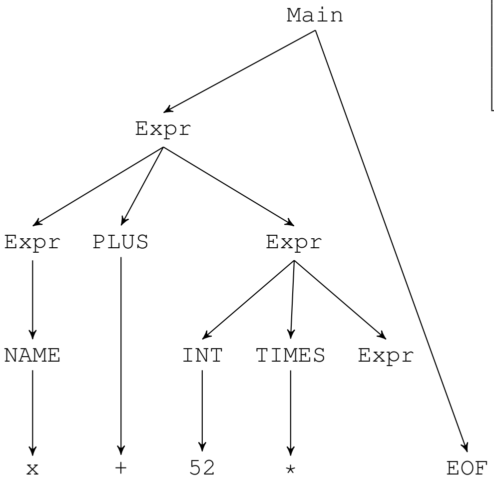

Derivation: grammar as string generator

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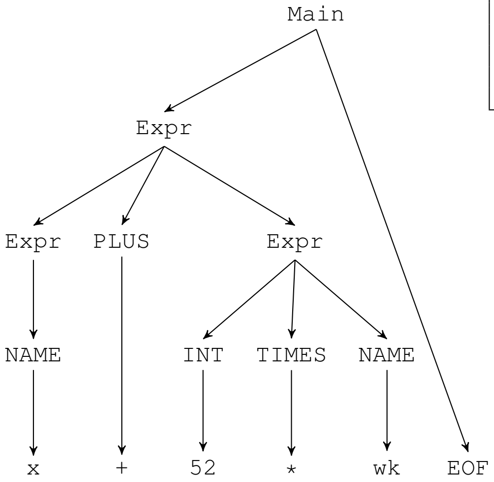

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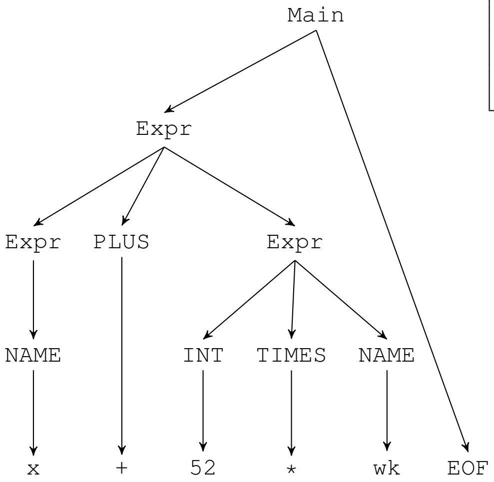

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Parsing is inverse derivation

Parsing

Given a grammar and a string - Determine whether the string can be derived - and if possible, then reconstruct the derivation steps

There are many systematic ways to do this:

- Hand-written top-down parsers (1970)
- Generated bottom-up parsers (1974)
  - Write parser specification
  - Use tool to generate parser

Grammar ambiguity

A grammar is *ambiguous* if there exists a string with more than one derivation tree.

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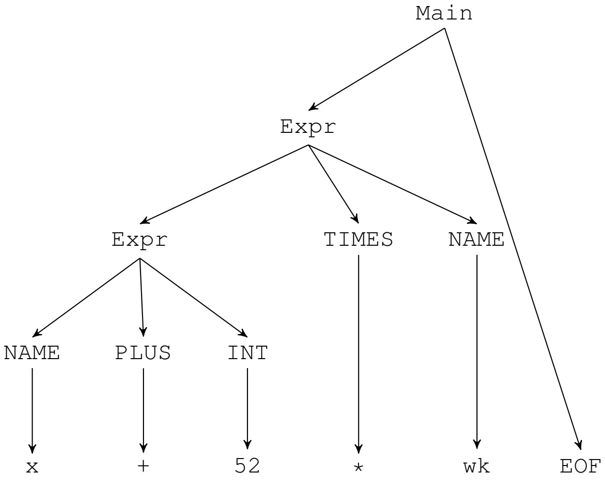

Leftmost and rightmost derivations

Leftmost derivation

Always expand the leftmost nonterminal.
See first example.

Rightmost derivation

Always expand the rightmost nonterminal.
See second example.

Associativity

How to read $x \diamond y \diamond z$?

$\diamond$ is left-associative

$\diamond$ is right-associative

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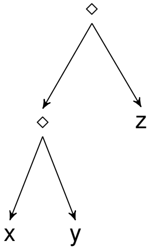
$\diamond$ is left-associative



$\diamond$ is right-associative

Associativity

How to read $x \diamond y \diamond z$?

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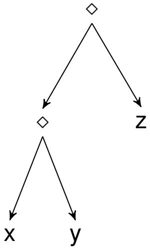
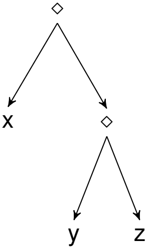


$\diamond$ is right-associative



Precedence

How to read $x \diamond y \bullet z$?

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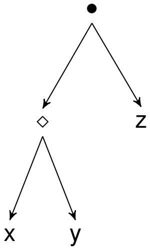

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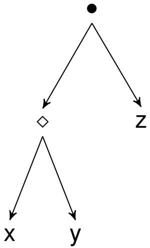

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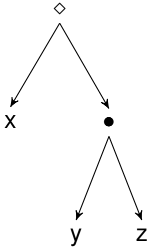

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`ExprPar` as running example

FsLexYacc:
`http://fsprojects.github.io/FsLexYacc/index.html`
Example:
`https://www.itu.dk/people/sestoft/plc/expr.zip`

File `Absyn.fs` defines the type representing the abstract syntax tree that the parser builds.

```
module Absyn

type expr =
| CstI of int
| Var of string
| Let of string * expr * expr
| Prim of string * expr * expr
```

Lexer specifications: ExprLex.fsl

```
rule Token = parse
| [' ' '\t' '\n' '\r']  { Token lexbuf }
| ['0'-'9']+            { CSTINT (...) }
| ['a'-'z''A'-'Z']['a'-'z''A'-'Z''0'-'9']*
{ keyword (...) }
| '+'                   { PLUS  }
| '-'                   { MINUS }
| '*'                   { TIMES }
| '('                   { LPAR  }
| ')'                   { RPAR  }
| eof                   { EOF   }
| _                     { lexerError lexbuf "Bad char" }
```

Lexer specifications: ExprLex.fsl

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Regular Expressions

Lexer specifications: ExprLex.fsl

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{ keyword (...) }
| '+'                    { PLUS  }
| '-'                    { MINUS }
| '*'                    { TIMES }
| '('                    { LPAR  }
| ')'                    { RPAR  }
| eof                    { EOF   }
| _                      { lexerError lexbuf "Bad char" }
```

F# to construct token

Tokens, associativity and precedence in fsyacc (`ExprPar.fsy`)

```
%token <int> CSTINT
%token <string> NAME
%token PLUS MINUS TIMES EQ
%token END IN LET
%token LPAR RPAR
%token EOF

%left MINUS PLUS   /* lowest precedence  */
%left TIMES        /* highest precedence */
```

Tokens, associativity and precedence in fsyacc (`ExprPar.fsy`)

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```

Token specifications - is expanded to a datatype

Tokens, associativity and precedence in fsyacc (`ExprPar.fsy`)

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Tokens carrying data

Tokens, associativity and precedence in fsyacc (`ExprPar.fsy`)

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```

Precedence: `%left`, `%right`, and `%nonassoc` allowed

Tokens, associativity and precedence in fsyacc (`ExprPar.fsy`)

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```

Ordering of groups defines precedence levels

Parser specification (`ExprPar.fsy`)

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                         { $1                 } A
Expr:
    NAME                             { Var $1             } B
  | CSTINT                           { CstI $1            } C
  | MINUS CSTINT                     { CstI (- $2)        } D
  | LPAR Expr RPAR                   { $2                 } E
  | LET NAME EQ Expr IN Expr END     { Let($2, $4, $6)    } F
  | Expr TIMES Expr                  { Prim("*", $1, $3)  } G
  | Expr PLUS  Expr                  { Prim("+", $1, $3)  } H
  | Expr MINUS Expr                  { Prim("-", $1, $3)  } I
```

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  | MINUS CSTINT                      { CstI (- $2)        } D
  | LPAR Expr RPAR                    { $2                 } E
  | LET NAME EQ Expr IN Expr END      { Let($2, $4, $6)    } F
  | Expr TIMES Expr                   { Prim("*", $1, $3) } G
  | Expr PLUS  Expr                   { Prim("+", $1, $3) } H
  | Expr MINUS Expr                   { Prim("-", $1, $3) } I
```

Non-terminals

Parser specification (`ExprPar.fsy`)

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%type <Absyn.expr> Main
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Main:
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Expr:
    NAME                             { Var $1             } B
  | CSTINT                           { CstI $1            } C
  | MINUS CSTINT                     { CstI (- $2)        } D
  | LPAR Expr RPAR                   { $2                 } E
  | LET NAME EQ Expr IN Expr END     { Let($2, $4, $6)    } F
  | Expr TIMES Expr                  { Prim("*", $1, $3)  } G
  | Expr PLUS  Expr                  { Prim("+", $1, $3)  } H
  | Expr MINUS Expr                  { Prim("-", $1, $3)  } I
```

Start symbol

Parser specification (`ExprPar.fsy`)

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                          { $1                  } A
Expr:
    NAME                              { Var $1              } B
  | CSTINT                            { CstI $1             } C
  | MINUS CSTINT                      { CstI (- $2)         } D
  | LPAR Expr RPAR                    { $2                  } E
  | LET NAME EQ Expr IN Expr END      { Let($2, $4, $6)     } F
  | Expr TIMES Expr                   { Prim("*", $1, $3)   } G
  | Expr PLUS  Expr                   { Prim("+", $1, $3)   } H
  | Expr MINUS Expr                   { Prim("-", $1, $3)   } I
```

Semantic actions

Parser specification (`ExprPar.fsy`)

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                           { $1                 } A
Expr:
    NAME                               { Var $1             } B
  | CSTINT                             { CstI $1            } C
  | MINUS CSTINT                       { CstI (- $2)        } D
  | LPAR Expr RPAR                     { $2                 } E
  | LET NAME EQ Expr IN Expr END       { Let($2, $4, $6)    } F
  | Expr TIMES Expr                    { Prim("*", $1, $3)  } G
  | Expr PLUS  Expr                    { Prim("+", $1, $3)  } H
  | Expr MINUS Expr                    { Prim("-", $1, $3)  } I
```

Arguments count from left

Parser specification (`ExprPar.fsy`)

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                         { $1                } A
Expr:
    NAME                             { Var $1            } B
  | CSTINT                           { CstI $1           } C
  | MINUS CSTINT                     { CstI (- $2)       } D
  | LPAR Expr RPAR                   { $2                } E
  | LET NAME EQ Expr IN Expr END     { Let($2, $4, $6)   } F
  | Expr TIMES Expr                  { Prim("*", $1, $3) } G
  | Expr PLUS  Expr                  { Prim("+", $1, $3) } H
  | Expr MINUS Expr                  { Prim("-", $1, $3) } I
```

Arguments count from left

Parser specification (`ExprPar.fsy`)

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                           { $1                   } A
Expr:
    NAME                               { Var $1               } B
  | CSTINT                             { CstI $1              } C
  | MINUS CSTINT                       { CstI (- $2)          } D
  | LPAR Expr RPAR                     { $2                   } E
  | LET NAME EQ Expr IN Expr END       { Let($2, $4, $6)      } F
  | Expr TIMES Expr                    { Prim("*", $1, $3)    } G
  | Expr PLUS  Expr                    { Prim("+", $1, $3)    } H
  | Expr MINUS Expr                    { Prim("-", $1, $3)    } I
```

Arguments count from left

Parser specification (`ExprPar.fsy`)

```
%start Main
%type <Absyn.expr> Main
%%
Main:
    Expr EOF                         { $1                } A
Expr:
    NAME                             { Var $1            } B
  | CSTINT                           { CstI $1           } C
  | MINUS CSTINT                     { CstI (- $2)       } D
  | LPAR Expr RPAR                   { $2                } E
  | LET NAME EQ Expr IN Expr END     { Let($2, $4, $6)   } F
  | Expr TIMES Expr                  { Prim("*", $1, $3) } G
  | Expr PLUS  Expr                  { Prim("+", $1, $3) } H
  | Expr MINUS Expr                  { Prim("-", $1, $3) } I
```

Type annotation - type of value returned by your semantic actions for the `Main` non-terminal. That is the values you have written to the right.

Putting together lexer and parser (`Parse.fs`)

From string to lexbuffer to tokens to abstract syntax tree.
From file `Expr/Parse.fs`:

```
let fromString (str : string) : expr =
    let lexbuf = Lexing.LexBuffer<char>.FromString(str)
    try
      ExprPar.Main ExprLex.Token lexbuf
    with
      | exn -> failwith "Lexing or parsing error ... "
```

Putting together lexer and parser (`Parse.fs`)

From string to lexbuffer to tokens to abstract syntax tree.
From file `Expr/Parse.fs`:

```
let fromString (str : string) : expr =
    let lexbuf = Lexing.LexBuffer<char>.FromString(str)
    try
      ExprPar.Main ExprLex.Token lexbuf
    with
      | exn -> failwith "Lexing or parsing error ... "
```

Entry point in parser

Putting together lexer and parser (`Parse.fs`)

From string to lexbuffer to tokens to abstract syntax tree.
From file `Expr/Parse.fs`:

```
let fromString (str : string) : expr =
    let lexbuf = Lexing.LexBuffer<char>.FromString(str)
    try
      ExprPar.Main ExprLex.Token lexbuf
    with
      | exn -> failwith "Lexing or parsing error ... "
```

Entry point in lexer

Invoking fslex and fsyacc

- Build the lexer and parser vs files `ExprLex.fs` and `ExprPar.fs`
- Compile as modules together with `Absyn.fs` and `Parse.fs`:

```
$ dotnet fsyacc.dll --module ExprPar ExprPar.fsy
$ dotnet fslex.dll --unicode ExprLex.fsl
$ fsharpi -r FsLexYacc.Runtime.dll Absyn.fs
             Expr.fs ExprPar.fs ExprLex.fs Parse.fs
```

- All DLL files above refer to their actual paths.
- Open the Parse module and experiment:

```
open Parse;;
fromString "x + 52 * wk";;
```

Conclusions

1. You know what context-free grammar is and why it matters
   - Derivations
   - Leftmost and Rightmost Derivations
   - Ambiguity
2. You get an intuition of concepts in context-free grammar presented above, and in particular, how to generate a language with in practice
3. You know how to Parse with a grammar
4. You know how to use FsYacc and FsLex to generate a parser