DAT158 – Machine learning and advanced algorithms – Høgskulen på Vestlandet

Compulsory exercise, Algorithms number 2

Due date: November 15th 2023

(At most three students per group. Does **not** count in the final grade)

Problem 1 (Number 5, Problems 2)

Given 7 jobs with corresponding processing times.

Job number	1	2	3	4	5	6	7
Processing time	4	7	2	9	12	5	3

The jobs will be processes on 3 identical machines without interruption. All jobs are available at time 0. (Model on slide 13, Chapter 2)

a) Use the local search algorithm where we have all jobs on machine 1 in the beginning (in the same order as they are given). When you move a job, move it to the machine it can start earliest. I there is a tie, use the machine with the lowest number. What is the result, both time when the last job finish and the actual schedule?
b) Use the Greedy algorithm on the same data. When have all jobs finished?
c) Use the Longest processing time rule. When have all jobs finished?
d) Give a lower bound on the best solution (Give a short explanation)?
e) Is it possible to achieve the answer in d) (Explain)?

Problem 2 (Number 6, Problems 2)

The adjacency matrix of a graph with 6 vertices is given below. The triangle inequality holds for the whole graph.

0	4	5	10	11	7
4	0	9	6	7	9
5	9	0	9	6	2
10	6	9	0	3	9
11	7	6	3	0	6
7	9	2	9	6	0

a) Verify that the triangle inequality holds when going from vertex 1 to vertex 5.

b) Run the nearest addition algorithm on the graph. If there are more than one candidate to add in a step, use the vertex with the smallest number. It is sufficient to have fours nodes in the tour (skipping the last two iteration through the loop).

c) Run the Double tree algorithm on the graph. If there are more than one minimum spanning tree, you should run it on every tree.

d) Run Christofides' algorithm on the graph. If there are more than one minimum spanning tree, you should run it on every tree.

Problem 3 (Number 2, Problem 3)

a) Run the dynamic programming algorithm (page 58 or Chapter 3, slide 6) on the following knapsack instance with B = 10. Show A(i) twice (before and after removing dominating pairs) for each iteration through the loop.

Item	Size	Value
1	3	5
2	2	3
3	2	2
4	4	5
5	3	4
6	1	2

b) What is the maximum number of elements in A(i) expressed in terms of B?

c) Let n be the number of items in the knapsack problem and let v_i be the value of item i. Furthermore let $V = \sum_{i=1}^{n} v_i$. What is the maximum of elements in A(i) expressed in terms of V?

Problem 4 (Number 6, Problem 3)

Consider the problem of minimizing the sum of completion times on a single machine given the data below (start of Chapter 4):

Job number	Release time	Processing time
1	0	6
2	0	5
3	1	3
4	2	1
5	3	4
6	4	2
7	19	2

Use the approximation algorithm based on the Shortest Remaining Processing Time for preemptive scheduling (Chapter 4, slide 7) to find an approximate solution to the nonpreemptive problem.

Problem 5 (Number 3, Problems 4)

a) Implement the randomized algorithm for MAX SAT where we set each x_i to true independently with probability ½ (it is very simple).

b) Assign values to $x_1, \ldots, x_5$ by the derandomized version of the algorithm based on methods of conditional expectations (page 103 – 104) to the following clauses

$$
\begin{aligned}
&C_1 = 3(x_1 \vee \overline{x_2} \vee x_3)\\
&C_2 = 4(x_1 \vee \overline{x_3})\\
&C_3 = 1(\overline{x_1} \vee x_2 \vee \overline{x_5})\\
&C_4 = 3(\overline{x_1} \vee x_3)\\
&C_5 = 2(x_2 \vee \overline{x_4})\\
&C_6 = 7(\overline{x_2} \vee x_5 \vee x_9)\\
&C_7 = 2(x_3 \vee \overline{x_4})\\
&C_8 = 3(\overline{x_4} \vee \overline{x_5} \vee x_{21})\\
&C_9 = 5(x_5 \vee \overline{x_{12}})\\
&\cdots
\end{aligned}
$$

The clauses $C_1, \ldots, C_9$ are the only clauses containing the variables $x_1, \ldots, x_5$.

Problem 6 (Number 4, Problems 4)

We have the following MAX SAT problem

$$C_1 = 4(x_1 \vee \overline{x_3}),\ C_2 = 3(\overline{x_1} \vee x_4), C_3 = 2(\overline{x_2} \vee x_4), C_4 = 1(\overline{x_2} \vee \overline{x_3} \vee x_4)$$

a) Find the ILP (IP) formulation of the problem above.

b) Find the LP formulation of the problem.

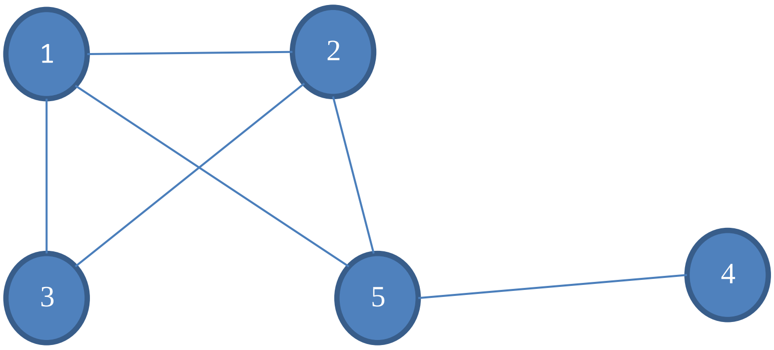
Problem 7

The graph below is 3-colorable. Use the algorithm that colors a 3-colorable graph with $O(\sqrt{n})$ colors, Chapter 6 to color the graph. Every time you have a choice between nodes, choose the lowered number one. Notice, it is important to understand the algorithm, not find the best coloring.

	1	2	3	4	5	6	7	8	9
1			1	1		1		1	
2							1		1
3	1			1		1			
4	1		1						
5						1			1
6	1		1		1			1	
7		1						1	1
8	1					1	1		
9		1			1		1		

Problem 8

A vertex cover of a graph is a subset of the vertices, $C \subseteq V$ such that every edge has at least one endpoint in C, that is $\forall(i.j) \in E$, either $i \in C$ or $j \in C$. We want to find a minimum vertex cover.



a) Formulate the problem above as an integer linear program (ILP).

b) Formulate the LP relaxation of the ILP above.

c) Formulate the Dual version of the LP problem above.

d) We have a large graph with 10.000 vertices. We solve the LP-problem and get the solution 500 (number of vertices in the vertex cover). What can we say about the solution of the original problem in a)?