Intrinsic Alignment in Cosmology

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Abstract. A brief literature review of intrinsic alignment (IA) in cosmology.

Contents

Abbreviations Used

EFT: effective field theory

BPs: bias parameters

IA: intrinsic alignment

LIMD: linear in matter density

SC gauge: synchronous-comoving gauge

SPT: standard perturbation theory

Symbols Used

a: scale factor

z, $\bar{z}$, $\tilde{z}$: redshift (background, observed)

η: conformal time

$\mathcal{H}$: conformal Hubble parameter ($\mathcal{H} = aH$)

Ω_m: matter density parameter

$D(z)$: linear growth factor, $\quad f \equiv \mathrm{d}\ln D/\mathrm{d}\ln a$

$\mathbf{x}$, $\mathbf{q}$: Eulerian, Lagrangian position

$\mathbf{v}$, θ: peculiar velocity, velocity divergence ($\theta = \partial_i v^i$)

s^i: Lagrangian displacement

χ, $\tilde{\chi}$: comoving radial distance (true, observed)

$\hat{\mathbf{n}}$: line-of-sight unit vector

P^{ij}: sky projector ($P^{ij} = \delta^{ij} - \hat{n}^i \hat{n}^j$)

$\partial_\parallel$, $\partial_{\perp i}$: derivatives along / transverse to $\hat{\mathbf{n}}$

Φ: Newtonian gravitational potential

δ_m: matter density contrast

δ_g, δ_h: galaxy, halo density contrast

K_{ij}: tidal tensor $K_{ij} \equiv \partial_i \partial_j \nabla^{-2} \delta_m - \frac{1}{3} \delta_{ij} \delta_m$

$\Pi^{[n]}_{ij}$: convective-time-derivative operator hierarchy (EFT basis)

$b_{\mathcal{O}}$: bias / IA coefficients for operator $\mathcal{O}$

R_*: non-locality scale (higher-derivative suppression scale)

Q_{ij}: 3D shape (symmetric, trace-free) tensor of a galaxy

γ_{ij}: projected (spin-2) shear on the sky

γ^I_{ij}: intrinsic shear (IA) contribution

κ: lensing convergence, $\quad \hat{\kappa}$ for observed estimator

Δx^μ: mapping between true and observed source position

δz: perturbation to observed redshift

n_g, $\bar{n}_g$, $\tilde{n}_g$: proper, background, observed galaxy number density

ϵ, ϵ_{ij}: IA stochastic (scalar / tensor) noise fields

$\varepsilon_{\mu\nu\rho\sigma}$: Levi-Civita tensor

$\langle \cdots \rangle$: ensemble average; prime ($'$) means Dirac delta removed

δ_{D}: Dirac delta function

$P^{(m)}_{\ell\ell'}(k)$: helicity power spectra of spin-2 components

1 Introduction and Motivation

TODO

2 Halo Bias and EFT

In this section we introduce the much more established field of galaxy bias and an EFT of galaxy bias. This will directly translate to the EFT used in section ??. We consider a ΛCDM universe with a perturbed FLRW metric unless stated otherwise.

2.1 What is bias

Say we look at a the universe at redshift z. Given a perturbation to the matter density $\delta(\mathbf{x}, z)$ we would like to know the perturbation to the density of some other tracer. In this section we will consider the relative number overdensity of dark matter halos, δ_h, but the results are applicable to any tracer in general, notably to galaxy number density. We then model the relation between these two as

$$\delta_h(\mathbf{x}, z) = \sum_{\mathcal{O}} b_{\mathcal{O}}(z)\mathcal{O}(\delta)(\mathbf{x}, z), \tag{2.1}$$

where we sum over some set of operators $\{\mathcal{O}\}$ that map δ to a new function of position and time and the $b_{\mathcal{O}}$ are the BPs. Crucially, they only depend on z and not on $\mathbf{x}$. 4 questions arise naturally from this model:

1. What is the motivation to use this model? Why is this expected to be accurate up to a (reasonably small) scale cutoff with the right choice of operators and values of BPs?
2. How can we choose a set of operators that is "complete"? I.e. that the model is correct in the sense of the previous question.
3. How do the bias parameters evolves over time?
4. How do we determine bias parameters? Both theoretically and empiricially given a dataset from a real life survey or a simulation.

The rest of this section will be devoted to summarizing the answers to these questions.

2.2 Relating bias parameters from different times

We consider a simple case where a population of galaxies instantly formed at some time τ_*. This is sufficient because the evolution equations we obtain will then work as a "Green's function" for the evolution equations of bias parameters in general. We simply integrate over the time evolved bias parameters weighted by the rate of galaxy formation at the integrand time.

We assume that whatever tracer we consider moves with the fluid. This immediately gives the continuity equation:

$$\frac{D}{D\tau}\delta_g = -\theta(1 + \delta_g), \tag{2.2}$$

where

$$\frac{D}{D\tau} = \frac{\partial}{\partial\tau} + v^i\frac{\partial}{\partial x^i} \tag{2.3}$$

is the convective time derivative, v^i is the peculiar velocity of the cosmic matter fluid, and $\theta = \partial_i v^i$ is the velocity divergence. To evolve δ_g we also need evolution equations for δ and v^i. They are given by the continuity equation (again)

$$\frac{D}{D\tau}\delta = -\theta(1+\delta), \tag{2.4}$$

and the Euler equation,

$$\frac{D}{D\tau}\theta = -\mathcal{H}\theta - (\partial^i v^j)^2 - \frac{3}{2}\Omega_m \mathcal{H}^2 \delta. \tag{2.5}$$

We proceed by dividing equations 2.4 and 2.2 by $1+\delta$ and $1+\delta_g$, respectively to get

$$\frac{1}{1+\delta_g}\frac{D}{D\tau}\delta_g = \frac{1}{1+\delta}\frac{D}{D\tau}\delta. \tag{2.6}$$

This can be solved by switching to Lagrangian coordinates to get

$$\ln(1+\delta_g(\mathbf{x}(\tau),\tau)) = \ln(1+\delta(\mathbf{x}(\tau),\tau)) + \ln(\frac{1+\delta_g(\mathbf{x}(\tau),\tau)}{1+\delta(\mathbf{x}(\tau),\tau)}), \quad \tau > \tau_*, \tag{2.7}$$

which can be written more simply as

$$1+\delta_g|_\tau = \frac{1+\delta|_\tau}{1+\delta|_{\tau_*}}(1+\delta_g|_{\tau_*}). \tag{2.8}$$

This equation can be made usefull by expanding order by order. Up to second order we get

$$1+\delta_g^{(1)}(\mathbf{x},\tau)+\delta_g^{(2)}(\mathbf{x},\tau) = 1+\delta^{(1)}-\delta_*^{(1)}+\delta_{g*}^{(1)}+\delta^{(2)}-\delta(2)_*+\delta_{g*}^{(2)}+[\delta_*^{(1)}]^2-\delta^{(1)}\delta_*^{(1)}+\left[\delta^{(1)}-\delta_*^{(1)}\right]\delta_{g*}^{(1)}. \tag{2.9}$$

Now, the difference between $\mathbf{x}$ and $\mathbf{x}_*$ is first order in perturbations as well. We can Taylor expand around $\mathbf{x}$ and use that the displacement, $\mathbf{x}-\mathbf{x}_*$ evolves according to the growth factor D to get

$$\delta_g^{(1)}(x,\tau) = \left(1+\frac{D_*}{D}\left[b_1^*-1\right]\right)\delta^{(1)}(x,\tau)+\varepsilon^* \tag{2.10}$$

$$\delta_g^{(2)}(x,\tau) = \left\{1+\left[b_1^*-1\right]\left(\frac{D_*}{D}\right)^2\right\}\delta^{(2)}+\left\{\frac{D_*}{D}[b_1^*-1]-\left(\frac{D_*}{D}\right)^2[b_1^*-1]+\tfrac{1}{2}b_2^*\left(\frac{D_*}{D}\right)^2\right\}[\delta^{(1)}]^2 \tag{2.11}$$

$$+\,b_K^{*2}\left(\frac{D_*}{D}\right)^2[K_{ij}^{(1)}]^2+\left(\frac{D_*}{D}-1\right)\frac{D_*}{D}[b_1^*-1]\,s^i_{(1)}\partial_i\delta^{(1)}-\left(\frac{D_*}{D}-1\right)\varepsilon^*\delta^{(1)} \tag{2.12}$$

$$+\left(\frac{D_*}{D}-1\right)s^i_{(1)}\partial_i\varepsilon^*\,. \tag{2.13}$$

Here, ε is a stochastic parameter added in and $s^i = x^i - x^i_*$ is the displacement. One can then extract the bias parameters at current time (Eularian time) by comparing to the bias expansion at Eularian time and matching terms to get

$$b_1^E = 1+\frac{D_*}{D}(b_1^*-1) \tag{2.14}$$

$$b_2^E = b_2^*\left(\frac{D_*}{D}\right)^2+\frac{8}{21}\left(1-\frac{D_*}{D}\right)(b_1^E-1) \tag{2.15}$$

$$b_{K^2}^E = b_{K^2}^*\left(\frac{D_*}{D}\right)^2-\frac{2}{7}\left(1-\frac{D_*}{D}\right)(b_1^E-1) \tag{2.16}$$

Clearly this can be easily generalized to any order.

2.3 EFT of bias parameters

Main ref: [1] We would like to find a "basis" of bias parameters up to a given order in perturbation theory. We will assume that (1) gravitation is described by general relativity, (2) that we can neglect the impact of massive neutrinos and dark energy perturbations, (3) that initial conditions are Gaussian and adiabatic.

We will assume that the only quantities relevant for the formation of galaxies are local on some scale R_* (which is much larger than the size of a galaxy / whatever tracer we are considering). According to the equivalence principle, the only effects that can be measured by a freely falling observer (i.e. forming galaxy) is the second derivatives of the gravitational potential, $\partial_i \partial_j \Phi$. Higher derivatives will be higher order contributions, because each derivative on a scale of R_* contributes a factor of k/R_*, which for large enough scales will be much smaller than one.

Now, the galaxy density, δ_g will in a very general sense depend on the history of $\partial_i \partial_j \Phi$ along the fluid trajectory. In other words:

$$\partial_g(\mathbf{x}, \tau) \supset \int^\tau d\tau' f_{\mathcal{O}}(\tau, \tau') \mathcal{O}(\mathbf{x}_{fl}(\tau'), \tau'), \tag{2.17}$$

where f_O is some kernel that we don't need to bother defining. We can then Taylor expand to get

$$\partial_g(\mathbf{x}, \tau) \supset \left[\int^\tau d\tau' f_{\mathcal{O}}(\tau, \tau')\right] \mathcal{O}(\mathbf{x}_{fl}(\tau), \tau) + \left[\int^\tau d\tau' (\tau' - \tau) f_{\mathcal{O}}(\tau, \tau')\right] \frac{D}{D\tau} \mathcal{O}(\mathbf{x}_{fl}(\tau), \tau) + \cdots, \tag{2.18}$$

where $D/D\tau$ is the convective derivative along the fluid flow. In order to define a basis we thus also have to include time derivatives $D/D\tau$ of the second derivatives of the potential. It now seems like we would need an infinite set of operators to form a basis. This can be avoided by noticing at finite order in perturbation theory, only a finite amount of these operators are linearly independent of one another.

In order to show the above statement, we will first switch to Lagrangian coordinates $\mathbf{q} = \mathbf{x}_{fl}(\tau = 0)$ where the convective time derivative simplifies to $\partial/\partial\tau$. We also assume that the n-th order growth factor is given by the linear growth factor to the n-th power. This is only strictly valid for an EdS (flat matter dominated universe) but is still very accurate for other cosmologies such as Λ CDM. Consider some operator in the basis, $O_L(\mathbf{q}, \tau)$ which has contributions at various orders, $O^i_L(\mathbf{q}, \tau)$, then

$$\left(\frac{D}{D\tau}\right)^n O_L(\mathbf{q}, \tau)|^{(n)} = \left(\frac{\partial}{\partial\tau}\right)^n O_L(\mathbf{q}, \tau)|^{(n)} = \sum_{i=1}^n \left(\frac{d^n}{d\tau^n} D^i(\tau)\right) O^i_L(\mathbf{q}, \tau_0). \tag{2.19}$$

Next, although higher order contributions start being linearly dependent on the lower order contributions after some order, as shown above, it is worth noting that these higher order contributions are in general no longer local in $\partial_i \partial_j \phi$. This is immediately obvious if we notice that the convective time derivative contains a spatial derivative that generates higher order (k/R_*) terms. It turns out that, due to symmetry requirements, we only need to consider $\partial_i \partial_j / \nabla^2$ acting on powers of $\partial_k \partial_l \Phi$ in order to get all the new terms created by the extra spatial derivatives.

Constructing a *local* Eularian basis out of $\partial_i\partial_j\Phi(\mathbf{x},\tau)$ and its convective time derivatives can be done explicitly as first shown in [2]. First define

$$\Pi^{[1]}_{ij}(\mathbf{x},\tau) = \frac{2}{3\Omega_m\mathcal{H}^2}\partial_i\partial_j\Phi(\mathbf{x},\tau) = K_{ij}(\mathbf{x},\tau) + \frac{1}{3}\delta_{ij}\delta(\mathbf{x},\tau) \tag{2.20}$$

and

$$\Pi^{[n]}_{ij} = \frac{1}{(n-1)!}\left[\frac{1}{\mathcal{H}f}\frac{D}{D\tau}\Pi^{[n-1]}_{ij} - (n-1)\Pi^{[n-1]}_{ij}\right], \tag{2.21}$$

as given by ref. [2]. Then take into account that $\mathrm{tr}\Pi^{[n]}$ can be written in terms of lower order operators through the Eularian fluid equations.

To get all terms at order n, we then write products of $\Pi^{[k]}$ where $k < n$ and the total order adds up to n or less and take traces of the different possible combinations of each product.

Thusfar we have only considered an expansion local in the second derivatives of the gravitational potential. Higher order derivatives can and should also be considered. Assume that all relevant nonlocal effects are limited to some scale, the "scale of nonlocality" R_*. Following the reasoning in [3], nonlocal contributions to the tracer number density at $\mathbf{x}$ will come from terms at $\mathbf{x}+\mathbf{y}$, where $y \sim R_*$. Take, for example, $\delta(\mathbf{x}+\mathbf{y})$. Through a Taylor expansion at $\mathbf{x}$ we thus find that this contribution can be written as

$$b_{\delta(\mathbf{x}+\mathbf{y})}\delta(\mathbf{x}+\mathbf{y}) = b_{\delta(\mathbf{x}+\mathbf{y})}(\delta(\mathbf{x}) + \mathbf{y}\cdot\vec{\nabla}\delta(\mathbf{x}) + y^2\nabla^2\delta(\mathbf{x}) + \cdots).$$

Terms with an odd order of derivatives will not contribute due to isotropy. On the other hand, $y^2\nabla^2 \sim R_*^2k^2$ in Fourier space. We thus see that for scales much larger than the non-locality scale (the regime in which we expect biasing to work anyway), higher derivative terms are surpressed through the factor $R_*^{2n}k^{2n} \ll 1$, where n is the order of the derivative. Note that we assume that the BP's themselves are not much larger than 1, and if we find that that is not the case then there's a sign that our perturbative expansion has stopped working for the scales considered. When aiming to find a full basis for our expansion up to some order, we thus need to consider the tracer in question to know up to which order in derivatives and perturbation theory to work.

3 Spin-2 Fields and Their Projections

It is well known how to construct powerspectra from scalar quantities. Constructing them for a spin-2 field requires more care. In particular, we would like to find a way to define powerspectra that is independent of the orientation of the observer.

We model galaxies in 3D space as ellipsoids where we do not care about the overal size. This means they can be described according to the 3×3 symmetric trace-free tensor Q_{ij} given by

$$Q_{ij} = \mathrm{tf}\left(\frac{\int \mathrm{d}^3x\rho(\mathbf{x})x^ix^j}{\int \mathrm{d}^3x\rho(\mathbf{x})}\right), \tag{3.1}$$

where we assume for simplicity that the galaxy's center of mass is at $x^i = 0$ and $\mathrm{tf}(\cdot)$ means taking the trace free part. The trace corresponds to the overal size of the galaxy and is not of interest to us. This corresponds roughly to g_{ij} from [4]. Q_{ij} at some point can transform under SO(3).

Specifically, it transforms under the spin-2 representation, which can immediately be seen because Q_{ij} is a 5 dimensional vector space and there is only one irreducible representation of SO(3) of dimension $2\ell + 1$, with in this case $\ell = 2$. This is the classification theorem. Alternatively, it is clear that under a SO(3) rotation given by R, we get $Q_{ij} \to R_{ia} R_{jb} Q_{ab}$. One can compare this to the spin-2 representation of SO(3) to see that they are the same.

We observe shapes as projected onto the sky. If we take $\mathbf{n}$ as our line of sight with associated projection matrix P^{ij}, then

$$\gamma_{ij} := \mathrm{tf}_P \left(P^{ik} P^{jl} g_{kl}(\mathbf{x}, z) \right) \tag{3.2}$$

$$= \frac{1}{2} \left(P^{ik} P^{jl} + P^{il} P^{jk} - P^{ij} P^{kl} \right) g_{kl}(\mathbf{x}, z) \tag{3.3}$$

$$=: P^{ijkl} g_{kl}(\mathbf{x}, z). \tag{3.4}$$

In the last line we simply define P^{ijkl} as the combination of P^{ij} from the line above. Going from line 1 to line 2 follows by using that in the projected space, the metric is P^{ij} instead of δ^{ij} and the trace of an arbitrary tensor M_{ij} thus becomes $P^{ij} M_{ij}$.

γ_{ij} is again symmetric, trace-free, and transforms like a spin-2 representation, now of SO(2).

3.1 Spherical tensor decomposition

Going back to the non-projected Q_{ij}, it's clear that this tensor is defined at each point in space, $\mathbf{x}$. Now look at the Fourier transform, $Q_{ij}(\mathbf{k})$. For a mode in a given direction, $\hat{\mathbf{k}}$, we can decompose $Q_{ij}(\mathbf{k})$ through the spherical tensors $(\mathrm{Y}_2^{(m)})(\hat{\mathbf{k}})_{ij}$, with $m = -2, -1, 0, 1, 2$ denoting the helicity. We get

$$(\mathrm{Y}_2^{(0)})(\hat{\mathbf{k}})_{ij} = \sqrt{3/2}(\hat{\mathbf{k}}_i \hat{\mathbf{k}}_j - \frac{1}{3}\delta_{ij}) \tag{3.5}$$

$$(\mathrm{Y}_2^{(\pm 1)})(\hat{\mathbf{k}})_{ij} = \sqrt{1/2}(\hat{\mathbf{k}}_i \mathbf{e}_j^{\pm} + \hat{\mathbf{k}}_j \mathbf{e}_i^{\pm}) \tag{3.6}$$

$$(\mathrm{Y}_2^{(\pm 1)})(\hat{\mathbf{k}})_{ij} = \mathbf{e}_i^{\pm} \mathbf{e}_j^{\pm} \tag{3.7}$$

with $e^{\pm} =$ ...and $i, j = 1, 2, 3$. The unit vectors e_1 and e_2 are defined by ... For a rotation around $\hat{\mathbf{k}}$ by an angle ϕ the spherical tensors transform as

$$(\mathrm{Y}_2^{(m)})(\hat{\mathbf{k}})_{ij} \to e^{im\phi} (\mathrm{Y}_2^{(m)})(\hat{\mathbf{k}})_{ij}.$$

These spherical tensors have a number of other usefull properties which we will not get into right now.

3.2 Defining Power Spectra

We can now define the powerspectrum as a $3 \times 3 \times 3 \times 3$ tensor given by

$$\langle Q_{ij}(\mathbf{k}) Q_{ij}(\mathbf{k}') = (2\pi)^3 \delta^D(\mathbf{k} + \mathbf{k}') P_{ijkl}^{QQ}(\mathbf{k}). \tag{3.8}$$

This definition is the most obvious, however because Q_{ij} transforms as a tensor under rotations, it leaves $P^{QQ}_{ijkl}(\mathbf{k})$ dependent on the direction of $\mathbf{k}$. We can instead work with the decomposition of Q_{ij} in the spherical tensors:

$$Q_{ij}(\mathbf{k}) = \frac{1}{3}\delta_{ij}Q_0^0(\mathbf{k}) + \sum_{m=-2}^{2} Q_2^m(\mathbf{k})Y_{ij}^{(m)}(\hat{\mathbf{k}}), \tag{3.9}$$

where we now allow Q_{ij} to also have a trace / scalar component Q_0^0. We can then define the following powerspectra:

$$\langle S_l^{(m)}(\mathbf{k})S_{l'}^{(m')}(\mathbf{k}')\rangle = (2\pi)^3\delta_{mm'}\delta^D(\mathbf{k}+\mathbf{k}')P_{ll'}^{(m)}(k). \tag{3.10}$$

When the powerspectra are defined in this way, they are scalars. There are 7 independent spectra, however we also have by the properties of the spherical tensors that

$$P_{ll'}^{(m)}(k)^* = P_{ll'}^{(-m)}(k). \tag{3.11}$$

If we also enfore invariance under parity transformations, then we find that the powerspectra are real and thus that

$$P_{ll'}^{(m)}(k) = P_{ll'}^{(-m)}(k). \tag{3.12}$$

This reduces the amount of independent spectra to 5.

3.3 Noise power spectra

In accordance with the previous subsection, there are 3 power spectra due to stochastic / noise terms in the EFT of galaxy IA. They come from combinations of scalar / galaxy size noise and spin-2 / symmetric traceless tensor / galaxy shear noise.

$$\langle \epsilon(\mathbf{k})\epsilon(\mathbf{k}')\rangle' = P_\epsilon^s(k), \tag{3.13}$$

$$\langle \epsilon_{ij}(\mathbf{k})\epsilon(\mathbf{k}')\rangle' = (\mathbf{k}_i\mathbf{k}_j - \frac{1}{3}\delta_{ij}k^2)P_\epsilon^{gs}(k), \tag{3.14}$$

$$\langle \epsilon_{ij}(\mathbf{k})\epsilon_{kl}(\mathbf{k}')\rangle' = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jl} - \frac{2}{3}\delta_{ij}\delta_{kl})P_\epsilon^g(k). \tag{3.15}$$

In all three cases $P_\epsilon^{\cdots}(k) = c^{\cdots} + \mathcal{O}(R^2k^2)$. The forms of the spectra are again constructed to satisfy transformation properties, symmetry, and tracelessness. For the scalar-scalar and tensor-tensor spectra the above equations correspond to white noise with some corrections terms that only become significant if the scales we are looking at $(1/k)$ become comparable to the scale of nonlocality of galaxy formation (R). For the tensor-scalar correlation there is no 'white noise' (or term independent of k). To see why, consider what would happen if $P_\epsilon^{gs}(k)$ would have a k^{-2} term.

$$\langle \epsilon_{ij}(\mathbf{k})\epsilon(\mathbf{k}')\rangle' \supset \left(\frac{\mathbf{k}_i\mathbf{k}_j}{k^2} - \frac{1}{3}\delta_{ij}\right) \xrightarrow{\mathcal{F}^{-1}} \propto (\partial_i\partial_j\nabla^{-2} - \frac{1}{3}\delta_{ij})\delta^3(\mathbf{x}) \supset \propto \frac{1}{x^3}. \tag{3.16}$$

This means that we have nonlocality on scales larger than the nonlocality scale of galaxy IA, which shouldn't be possible.

3.4 E and B modes of spin-2 field

A field of spin-2 tensors on the sphere can also be decomposed into E and B modes. E modes correspond to a divergence like field while B modes create a curl like field. Their key property is that, under a parity transformation, B modes change sign while E modes stay the same.

To calculate these modes, first decompose the spin-2 field into spin-2 spherical harmonics. We are now working on the sphere, so the spin-2 tensors, i.e. the shear, is given by a complex number ${}_{\pm 2}X(\hat{\mathbf{n}})$. It transforms under a local rotation by an angle α as

$$ {}_{\pm 2}X(\hat{\mathbf{n}}) \to e^{\mp 2i\psi}\, {}_{\pm 2}X(\hat{\mathbf{n}}). \tag{3.17}$$

The decomposition to spin 2 spherical harmonics is given through the usual inner product. We thus obtain spin $a_{\pm 2,lm}$'s. We define the E and B modes through their spherical harmonic coefficients as

$$E_{lm} = -\frac{1}{2}\left(a_{2,lm} + a_{-2,lm}\right) \tag{3.18}$$

$$B_{lm} = \frac{i}{2}\left(a_{2,lm} - a_{-2,lm}\right). \tag{3.19}$$

4 EFT of Galaxy IA

4.1 Local Deterministic Terms

To get the EFT of galaxy IA, we use $\Pi^{[n]}$ again (equation ...) but instead of making scalar combinations we make symmetric trace free combinations of the form

$$Q_{ij}(\mathbf{x}, \eta) = \sum_{\mathcal{O}} b_{\mathcal{O}}(\eta)\mathcal{O}_{ij}(\mathbf{x}, \eta). \tag{4.1}$$

Up to order 3 we get the following local deterministic contributions [5]:

1st order: $\mathrm{tf}[\Pi^{[1]}]_{ij}$

2nd order: $\mathrm{tf}[\Pi^{[2]}]_{ij}$, $\mathrm{tf}[(\Pi^{[1]})^2]_{ij}$, $\mathrm{tf}[\Pi^{[1]}]_{ij}\mathrm{tr}\Pi^{[1]}$

3rd order: $\mathrm{tf}[\Pi^{[3]}]_{ij}$, $\mathrm{tf}[(\Pi^{[1]}\Pi^{[2]})]_{ij}$, $\mathrm{tf}[\Pi^{[2]}]_{ij}\mathrm{tr}\Pi^{[1]}$, $\mathrm{tf}[(\Pi^{[1]})^3]_{ij}$, $\mathrm{tf}[(\Pi^{[1]})^2]_{ij}\mathrm{tr}\Pi^{[1]}$, $\mathrm{tf}[\Pi^{[1]}]_{ij}(\mathrm{tr}\Pi^{[1]})^2$, $\mathrm{tf}[\Pi^{[1]}]_{ij}\mathrm{tr}((\Pi^{[1]})^2)$

We do not need to include factors of the form $\mathrm{tr}(\Pi^{[n]})$ for $n > 1$. This was first argued and shown explicitly up to $n = 3$ in [2]. In short, we do not need to include this factor because by using the continuity, Euler, and Poisson equations one can write $\mathrm{tr}(\Pi^{[n]})$ as other terms included elsewhere in the bias expansion.

4.2 Non-local deterministic Terms

As explained in subsection ... we should also consider non-local terms, which are surpressed by orders of $(R_* k)^2$ (or $(R_* \nabla)^2$ in real space). At one loop order (when calculating power spectra) it suffices to go up to first order in nonlocality [2], in which case the only new term in our basis is

$$R_*^2 \nabla^2 \mathrm{tf}(\Pi^[1])_{ij}.$$

The ratio between non-local and SPT contributions to a power spectrum can be calculated as ... TODO here: explain this, justify why only one order is needed, give indication to what order of SPT this corresponds

4.3 Stochastic Terms

Due to our lack of knowledge about small scale modes, we get stochastic terms in our basis. In analogy to stochastic contributions to galaxy bias [1], we generalize equation 4.1 to [5]

$$g_{ij}(\mathbf{x}, \tau) = \sum_O \left[b_O^{(g)}(\tau) + \epsilon_O(\mathbf{x}, \tau) \right] O_{ij}(\mathbf{x}, \tau) + \epsilon_{ij}^O(\mathbf{x}, \tau) O(\mathbf{x}, \tau) + \epsilon_{ij}(\mathbf{x}, \tau). \tag{4.2}$$

The ϵ fields are uncorrelated with the O_{ij}, have vanishing expectation value and are completely described by their one-point distribution, i.e. $\langle \epsilon_O(\mathbf{x}) \epsilon_{O'}(\mathbf{x}') \rangle \propto \delta(\mathbf{x} - \mathbf{x}')$. Each ϵ_{ij} respects the same symmetries as g_{ij}. The O's are the scalar terms that one would consider in e.g. a galaxy bias EFT. ϵ_{ij} describes the leading order shape noise (when projected to the sky), see also subsection ... . In general, the stochastic contributions up to third order are given by

1st order: ϵ_{ij},

2nd order: $\epsilon_{ij}^{\delta} \mathrm{tr}\left[\Pi^[1]\right]$, $\epsilon_{\Pi^{[1]}} \mathrm{tf}\left[\pi^{[1]}\right]_{ij}$,

3rd order: $\epsilon_{ij}^{\delta^2} \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)$, $\epsilon_{ij}^{K^2} \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right]$, $\epsilon_{\Pi^{[2]}} \mathrm{tf}\left[\Pi^{[2]}\right]_{ij}$, $\epsilon_{\left[\Pi^{[1]}\right]^2} \mathrm{tf}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}$,
$\epsilon_{\Pi^{[1]}\Pi^{[2]}} \mathrm{tf}\left[\Pi^{[1]}\right]_{ij} \mathrm{tr}\left[\Pi^{[1]}\right]$.

Here K_{ij} is the tidal tensor. It is the traceless part of the hessian of the gravitational potential. In comoving coordinates it is defined as

$$K_{ij}(\mathbf{x}, \tau) := \left(\frac{\partial_i \partial_j}{\nabla^2} - \frac{1}{3} \delta_{ij} \right) \delta(\mathbf{x}, \tau). \tag{4.3}$$

4.4 Selection Effects

Observed number counts and shapes can also depend on the orientation of the galaxy w.r.t. the line of sight. This is, for example, because galaxies stretched along the line of sight appear more luminious and thus are detected more often than other galaxies at similar distances and of similar intrinsic luminosity. Ref. [1] provides the complete enumeration of these terms for galaxy number density bias, while ref. [5] did the same for galaxy shape density bias. The exact contributions fall out of the scope of this review and can be found in [5]. They all consist of contractions of $\hat{\mathbf{n}}^k\hat{\mathbf{n}}^l$ with $\Pi^{[n]}_{kl}$

TODO here still:

1. also mention renormalization of coefficients

4.5 Physical Motivation - Tidal alignment and tidal torquing model

TODO here:

1. Physically motivate linear and quadratic terms by summarizing the argument in [6].

4.6 Connection to E and B modes

The 3D galaxy IA field (so not projected) is invariant under parity transformations up to stochastic contributions. This is because $\Pi^{[n]}$ is invariant under parity transformations. When projecting to the sky, one still obtains B-modes (due to the projection, obviously). To calculate the projected powerspectra, start with the powerspectrum of the 3D galaxy IA field, then project it to the sky using eq. ... . One can show (see section 2.6 of [4]) that

$$P_{\delta E}(k,\mu) = \frac{1}{2}\sqrt{\frac{3}{2}}(1-\mu^2)P^{(0)}_{02}(k), \tag{4.4}$$

$$P_{EE}(k,\mu) = \frac{3}{8}(1-\mu^2)^2P^{(0)}_{22}(k) + \frac{1}{2}\mu^2(1-\mu^2)P^{(1)}_{22}(k) + \frac{1}{8}(1+\mu^2)^2P^{(2)}_{22}(k), \tag{4.5}$$

$$P_{BB}(k,\mu) = \frac{1}{2}(1-\mu^2)P^{(1)}_{22}(k) + \frac{1}{2}\mu^2P^{(2)}_{22}(k). \tag{4.6}$$

Here $\mu = \hat{\mathbf{n}}\cdot\hat{\mathbf{n}}$ is the cosine of the angle between $\hat{\mathbf{n}}$ and $\hat{\mathbf{k}}$ and δ is the usual fractional matter density perturbation.

At leading order in the bias expansion (and ignoring stochastic contributions) the B-mode power spectrum vanishes. This is also the case for the LA and NLA models [4]. We thus require an EFT taken to atleast 2nd order to model B-modes of galaxy IA. As usual, the EB correlator vanishes due to invariance under parity transformations. Parity breaking primordial graviational waves would in general cause a nonzero EB correlator. This would thus serve as a smoking gun for parity violating models of inflation [7].

4.7 Results for Three-Dimensional IA Power Spectra

The helicity power spectra can be seperated as

$$P^{(m)}_{\ell\ell'}(k) = [P^{(m)}_{\ell\ell'}]_{\mathrm{L+H.D.}}(k) + [P^{(m)}_{\ell\ell'}]_{(22)}(k) + [P^{(m)}_{\ell\ell'}]_{(13)+(31)}(k) + [P^{(m)}_{\ell\ell'}]_{\epsilon}(k). \tag{4.7}$$

The first term corresponds to leading order and higher derivative contributions (see subsection ...) and the last term corresponds to the noise spectra (see subsection 3.3). There is no (12) term because this would correspond to contributions like $\langle\delta^3\rangle$ which vanish or, if we do not assume non-gaussianity, are very small.

5 Lensing corrections to observed galaxy density

Based on [3]. Unlensed photon geodesic through the centre ($x^\mu = 0$) given by

$$x^\mu(\chi) = (\eta_0 - \chi, \hat{\mathbf{n}}\chi), \quad \chi \text{ affine parameter.}$$

Therefore, if we observe a (in general) lensed light ray in direction $\hat{n}^i$ corresponding to an object with redshift $\tilde{z}$ corresponding to the comoving radial distance $\tilde{\chi}(\tilde{z})$, then the observed location of the object is given by $\tilde{x}^\mu$ with

$$\tilde{x}^0 = \eta_0 - \tilde{\chi}, \quad \tilde{x}^i = \hat{n}^i\tilde{\chi}. \tag{5.1}$$

Now define the difference between the observed and the actual object location as the true location minus the observed location, i.e.

$$\Delta x^\mu = x^\mu - \tilde{x}^\mu. \tag{5.2}$$

We also define δz through the inferred emission scale factor $\tilde{a} = 1/(1+\tilde{z})$ as

$$\frac{a(x^0)}{\tilde{a}} = 1 + \delta z \implies \bar{z} - \tilde{z} = -(1+\tilde{z})\delta z. \tag{5.3}$$

In other words, δz is the the difference between the observed redshift and the redshift $\bar{z}$ that would be observed in an unperturbed universe.

For the rest of the section, we work in the SC gauge (what is the significance of this? should I say more about it?)

5.1 Observed galaxy number density

The observed number of galaxies in a volume $\tilde{V}$ defined in terms of the observed coordinates is given gauge-invariantly as

$$\begin{aligned} N &= \int_{\tilde{V}} J^\mu d\tilde{V}_\mu && (5.4)\\ &= \int_{\tilde{V}} \sqrt{-g}n_g u^\mu \epsilon_{\mu\alpha\beta\gamma}\frac{\partial x^\alpha}{\partial \tilde{x}^1}\frac{\partial x^\beta}{\partial \tilde{x}^2}\frac{\partial x^\gamma}{\partial \tilde{x}^3}d\tilde{x}^1 d\tilde{x}^2 d\tilde{x}^3 && (5.5)\end{aligned}$$

where $J^\mu = \sqrt{-g}n_g u^\mu$ is the galaxy number 4-current density, $\tilde{V}$ is a hypersurface in 4-space, i.e. a volume in 3-space, and $d\tilde{V}_\mu$ is the normal vector of $\tilde{V}$ with area $\mathrm{d}\tilde{V}$. It is mathematically as

$$\mathrm{d}\tilde{V}_\mu = \underbrace{\epsilon_{\mu 123}dx^1dx^2dx^3}_{\text{in true coordinates}} = \underbrace{\epsilon_{\mu\alpha\beta\gamma}\frac{\partial x^\alpha}{\partial \tilde{x}^1}\frac{\partial x^\beta}{\partial \tilde{x}^2}\frac{\partial x^\gamma}{\partial \tilde{x}^3}d\tilde{x}^1 d\tilde{x}^2 d\tilde{x}^3}_{\text{in observed coordinates}}. \tag{5.6}$$

Finally, n_g is the proper number density of galaxies, i.e. as measured by an observer moving with the galaxies, and u_μ is the 4-velocity of galaxies (we assume zero velocity dispersion) (check this statement).

In the SC gauge, the 4-velocities reduce to $(1/a, 0, 0, 0)$. We thus get

$$N = \int_{\tilde{V}} \sqrt{-g}n_g(x^\alpha)\frac{1}{a(x^0)}\left|\frac{\partial x^i}{\partial \tilde{x}^j}\right| d^3\tilde{\mathbf{x}}, \quad \text{(SC gauge)}. \tag{5.7}$$

One can show that, to first order,

$$\left|\frac{\partial x^i}{\partial \tilde{x}^j}\right| = 1 + \frac{\partial \Delta x^i}{\partial \tilde{x}^i}, \quad \sqrt{-g} = a^4\left(1 + \frac{1}{2}\delta g^\mu_\mu\right). \tag{5.8}$$

Also,

$$a^3(\bar{z})n_g(\mathbf{x}, \bar{z}) = a^3(\bar{z})\bar{n}_g(\bar{z})[1 + \delta_g^{sc}(\mathbf{x}, \bar{z})], \tag{5.9}$$

where we assumed that $\langle \tilde{z} \rangle = \langle \bar{z} \rangle$ and we define δ_g^{sc} as the galaxy number density perturbation in the comoving frame. Additionally, up to first order,

$$a^3(z)n_g(\mathbf{x}, \bar{z}) = a^3(\tilde{z})\bar{n}_g(\tilde{z})[1 + \delta_g^{sc}(\mathbf{x}, \tilde{z})] - \underbrace{(1+\tilde{z})\frac{d(a^3\bar{n}_g)}{dz}\bigg|_{z=\tilde{z}}\delta z}_{:=b_c} \tag{5.10}$$

Now for the important part. The observed galaxy density is defined via the number of galaxies N observed in a volume $\tilde{V}$ as

$$\int_{\tilde{V}} a^3(\tilde{z})\tilde{n}_g(\tilde{\mathbf{x}}, \tilde{z})d^3\tilde{\mathbf{x}} = N. \tag{5.11}$$

Equating the integrand to that of equation 5.7 gives

$$a^3(\tilde{z})\tilde{n}_g(\tilde{\mathbf{x}}, \tilde{z}) = \sqrt{-g}\frac{1}{a(\bar{z})}n_g(\mathbf{x}, \bar{z})\left|\frac{\partial x^i}{\partial \tilde{x}^j}\right| \tag{5.12}$$

using the earlier expansions it can then be shown that we obtain the relation

$$\tilde{\delta}_g = \delta_g^{sc} + b_c\delta z + 2\frac{\Delta x_\parallel}{\tilde{\chi}} + \partial_\parallel \Delta x_\parallel - 2\hat{\kappa}, \quad \hat{\kappa} := -\frac{1}{2}\partial_{\perp i}\Delta x^i_\perp. \tag{5.13}$$

Here "parallel" means parallel to the $\hat{n}$ axis.

Now what is the significance of these corrections? If you take the fourier transform of equation 5.13, you get additional terms that look like the terms obtained from nonzero f_{NL}, i.e.

$$\ldots \tag{5.14}$$

The effect corresponds to a f_{NL} of order 1. These corrections are thus only relevant for extremely large volume surveys, such as Euclid.

6 Calibration and Validation with Simulations

TODO here:

1. Explain validity of EFT of IA based on [4]
2. Take a look at hydrodynamic simulations [8, 9]

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