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Given an even integer $n > 0$, what is the sum of the even integers greater than zero and less than or equal to n? The sum is $0.25(n^2 + 2n)$. Prove this inductively. Think about what the inductive step needs to be.

Here we are trying to prove that $2 + 4 + \ldots + n = 0.25(n^2 + 2n)$. We will do so by using a proof by induction.

Our base case is 2, since 2 is the smallest even number greater than 0. Putting 2 into $0.25(n^2 + 2n)$ we get 2, which is correct.

Since the base case holds, we now want to show that it also holds for for some case $k + 2$ (We are proving it for $k + 2$ instead of $k + 1$ because $k + 1$ would be odd)

We need to show that $2 + 4 + \ldots k + (k + 2) = 0.25((k^2 + 2k) + 2(k + 2))$.

On the lefthand side we can substitute $2 + 4 + \ldots + k$ for $0.25(k^2 + 2k)$ by the inductive hypothesis. This gives us $0.25(k^2 + 2k) + (k + 2)$, which can be rewritten as $0.25(k^2 + 6k + 8)$.

The righthand side, $0.25((k^2 + 2k) + 2(k + 2))$, can be rewritten as $0.25((k^2 + 6k + 8)$.

Since the lefthand and righthand sides are equal, the theorem is true.

(2) Points and Line Segments Suppose I draw n points in a vertical column and n points in a second vertical column. I then draw a line segment between each point in the first column and each point in the second column. How many line segments will there be in total, as a function of n? Prove this inductively.

1 row = 1 line

$1^2 + 1 - 1 = 1$w

$1 + (2(1) + 2) = 5$

2 rows = 5 lines

$2^2 + 2 - 1 = 5$

$5 + (2(2) + 2) = 11$

c

3 rows = 11 lines

$3^3 + 3 - 1 = 11$

Each point in the first column connects to another point in the same row: n

Each point connects to all other points in the other column: n^2

Remove the overcounted connection: -1

Using the above information, I propose that the function L(n), which gives the total number of lines, exists such that $L(n)= n^2 + n - 1$.

For the base case of 1, we get $n^2 + n - 1 = 1^2 + 1 - 1 = 1$, which is true.

Now we need to prove it holds for some cases k + 1.

We can write $L(k + 1) = (k^2 + k - 1) + (k + 1)$. We can see that adding 1 new row adds 1 line across, lines for the left column to the points on the opposite side and lines for the right column to the points of the opposite side. We can rewrite L(k + 1) as L(k) + 2k + 2. Using the inductive hypothesis we can rewrite it again as $(k^2 + k - 1) + (k + 1)$.

Since we can see that the lefthand and righthand sides are still equivalent, the theorem holds.