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Wayne S. Desarbo
Pennsylvania State University

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Restricted principal components analysis for marketing research

Wayne S. DeSarbo
Marketing Department, Smeal College of Business, Pennsylvania State University, University Park, Pennsylvania, USA
Robert E. Hausman
K5 Analytic LLC, Bridgewater, New Jersey, USA, and
Jeffrey M. Kukitz
Marketing Department, Smeal College of Business, Pennsylvania State University, University Park, Pennsylvania, USA

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Abstract
Purpose – Principal components analysis (PCA) is one of the foremost multivariate methods utilized in marketing and business research for data reduction, latent variable modeling, multicollinearity resolution, etc. However, while its optimal properties make PCA solutions unique, interpreting the results of such analyses can be problematic. A plethora of rotation methods are available for such interpretive uses, but there is no theory as to which rotation method should be applied in any given social science problem. In addition, different rotational procedures typically render different interpretive results. The paper aims to introduce restricted PCA (RPCA), which attempts to optimally derive latent components whose coefficients are integer-constrained (e.g.: $\{-1,0,1\}$, $\{0,1\}$, etc.).

Design/methodology/approach – The paper presents two algorithms for deriving efficient solutions for RPCA: an augmented branch and bound algorithm for sequential extraction, and a combinatorial optimization procedure for simultaneous extraction of these constrained components. The paper then contrasts the traditional PCA-derived solution with those obtained from both proposed RPCA procedures with respect to a published data set of psychographic variables collected from potential buyers of the Dodge Viper sports car.

Findings – This constraint results in solutions which are easily interpretable with no need for rotation. In addition, the proposed procedure can enhance data reduction efforts since fewer raw variables define each derived component.

Originality/value – The paper provides two algorithms for estimating RPCA solutions from empirical data.

Keywords Multivariate analysis, Data reduction, Data analysis, Market research methods

Paper type Research paper

Introduction
Virtually every marketing research textbook currently available in the market contains a section on the basis and use of principal components analysis (PCA) in marketing (Malhotra, 1996). The central premise behind traditional PCA is to reduce the dimensionality of a given two-way data set consisting of a large number of interrelated variables all measured on the same set of subjects, while retaining as much as possible of the variation present in the data set. This is attained by transforming to a new set of composite variates called principal components which are orthogonal and typically ordered in terms of the amount of variation explained in all of the original variables.

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The PCA formulation is set up as a constrained optimization problem and reduces to an eigenstructure analysis of the sample covariance or correlation matrix as to be shown later. Priesendorfer and Mobley (1988) and Jolliffe (2002) state that Beltrami (1873) and Jordan (1874) independently derived the singular value decomposition in a form that underlies PCA. However, it is generally accepted that Pearson (1901) and Hotelling (1933) provided the earliest descriptions of the procedure now known as PCA. Jackson (1991) and Jolliffe (2002) discuss the optimal algebraic and geometric properties of PCA.

PCA in marketing research is most often used to explore dependence structures in multivariate data (Belk, 1985; Belk *et al.*, 1984; Childers *et al.*, 1985; Jaccard *et al.*, 1986; Kale, 1986; Newman and Sheth, 1985; Zaichowsky, 1985). For example, a researcher may be interested in exploring the underlying structure of various attributes of designated brands and their importance in impacting overall preference. Alternatively, PCA is often utilized in resolving problems with multicollinearity (Rangaswamy and Krishnamurthi, 1991) where a reduced set of principal components is often employed in a subsequent regression framework to explain some designated dependent variable. As stated by Dillon *et al.* (1989, p. 106):

> . . . the objective is to identify a small number of dimensions that purportedly provide a succinct and meaningful interpretation of the structure underlying the data. Once the salient dimensions are discovered, component scores for individuals are computed to capture major sources of variation in the original data. Subsequently, these scores may be subject to further analysis.

Or, one can utilize PCA as a preliminary confirmatory analysis to test the underlying structure of a given data set (Didow *et al.*, 1985). Hunter and Takane (2002) list a number of different types of more recent applications of PCA in the areas of psychiatric evaluation, family composition preference, psychological adjustment, culture assessment, etc.

As noted by Dillon *et al.* (1989), such PCA scores have been gainfully employed in marketing in a number of different ways. PCA scores have been used by Hauser and Koppelman (1979) and by Holbrook and Huber (1979) in generating perceptual product spaces. Urban and Hauser (1980) employed PCA scores in preference regression and logit analysis for predicting consumer choice. Andreasen and Belk (1980) used component scores as input to multiple regression analysis to understand the factors that best predicted likely future attendance at the performing arts. Johansson *et al.* (1985) utilized PCA scores as input into a simultaneous equation model. Holbrook and Moore (1981) used PCA scores as dependent variables in individual level conjoint analyses in the investigation of verbal vs pictorial presentations. PCA scores have also been employed as input for a cluster analysis by Fuse *et al.* (1984) to develop a typology of individual search strategies among purchasers of new automobiles, and in an ANOVA by Midgley (1983) to examine patterns of interpersonal information seeking for the purchase of a symbolic product. Thus, PCA has been traditionally used as both a primary (as a final analysis) and secondary (as input to other analyses) analysis in marketing.

While traditional PCA has been very useful for a variety of different research endeavors in marketing, a number of issues have been noted in the literature documenting the associated difficulties of implementation and interpretation. While PCA possesses attractive optimal and uniqueness properties, the construction of principal components as linear combinations of all the measured variables means that interpretation is not always easy. One way to aid the interpretation of PCA results is to rotate the components as is done with factor loadings in factor analysis (Acito and Anderson, 1986). Richman (1986, 1987), Jolliffe (1987), Rencher (1995) all provide various types of rotations, both orthogonal and oblique, that are available for use in PCA rotation. They also discuss the associated problems with such rotations in terms of the different criteria they optimize and the fact that different interpretive results are often derived. In addition, other problems have been noted (Jolliffe, 2002). PCA successively maximizes variance accounted for. When rotation is utilized, the total variance within the rotated subspace remains unchanged; it is still the maximum that can be achieved overall, but it is redistributed amongst the rotated components more evenly than before rotation. This indicates, as Jolliffe (2002) notes, that information about the nature of any really dominant components may be lost, unless they somehow satisfy the criterion optimized by the particular rotation procedure utilized. Finally, the choice of the number of principal components to retain has a large effect on the results after rotation. As illustrated in Jolliffe (2002), interpreting the most important dimensions for a data set is clearly difficult if those components appear, disappear, and possibly reappear as one alters the number of principal components to retain.

A number of solutions have been provided to resolve these interpretation and implementation issues with PCA by employing various types of constraints or restrictions. Takane and Shibayama (1991), Takane *et al.* (1995), and Hunter and Takane (2002) propose a constrained principal components approach combining regression and PCA in embedding additional data for subjects and/or stimuli into PCA solutions via linear constraints. Hausman (1982) proposed an integer programming solution for optimally selecting the individual elements or coefficients for each derived principal component as integer values in a restricted set (e.g. $\{-1,0,+1\}$ or $\{+1,0\}$) akin to what DeSarbo *et al.* (1982) proposed for canonical correlation analysis. Successive restricted integer valued principal components are extracted sequentially, each optimizing a variance accounted for measure. Vines (2000) generalized this approach without restricting the integer set solution using a sequence of simplicity preserving transformations. Each transformation chooses a pair of vectors and rotates them orthogonally in such a way that the variance associated with the currently higher variance component of the pair is increased at the expense of the lower variance component. The iterative algorithm stops when no non-trivial simplicity preserving transformation leads to an improvement in variance. Finally, Jolliffe (2002) and Jolliffe *et al.* (2003) proposed a hybrid of variable selection and shrinkage estimators called the simplified component technique, which attempts to shrink some of the PCA coefficients exactly to zero in defining the components.

This manuscript introduces two approaches for simplifying principal components based on restricting the coefficients to integer values akin to Hausman (1982). The augmented branch and bound (B&B) algorithm to be described shortly computationally improves upon the Hausman (1982) proposed integer programming procedure by sequentially deriving optimal restricted components. Unlike in Vines, no *ad hoc* tuning parameter needs be set for estimation. As an alternative, a combinatorial λ-opt optimization procedure is also provided which estimates all k components simultaneously. This has the advantage of extracting a complete set of components simultaneously instead of sequentially where often times later components which explain smaller portions of the variance become sparse with respect to non-zero coefficients.

The next section gives the technical details of the restricted PCA (RPCA) augmented B&B procedure for sequential extraction of restricted components. Followed by a section that provides a technical description of the λ-opt combinatorial optimization algorithm developed for simultaneous extraction of restricted components. The penultimate section describes an application in marketing research in examining the psychographic characteristics of potential consumers interested in buying the Dodge Viper sports car where solutions derived from both RPCA procedures are contrasted with that obtained from traditional PCA and hierarchical cluster analysis. Finally, the last section provides a discussion/summary of the paper, as well as directions for future research.

Restricted principal components analysis – a branch and bound algorithm for sequential extraction

Definitions

As mentioned, PCA is a technique used to reduce the dimensionality (p variables) of the data $x = [x_1, \ldots, x_p]$ while retaining as much information as possible. More specifically, the first principal component is traditionally defined as that linear combination of the random variables, $y_1 = a_1^T x$, that has maximum variance, subject to the standardizing constraint $a_1^T a_1 = 1$. The coefficient vector a_1 can be obtained as the first characteristic vector corresponding to the largest characteristic root of $\sum$ the covariance matrix of x (one can also employ the correlation matrix instead). The variance of $a_1^T x$ is that largest characteristic root.

We prefer an alternative, but equivalent, definition provided by Rao (1964), Okamoto (1969), Hausman (1982), and others. The first principal component is defined as that linear combination $y_1 = a_1^T x$ that maximizes:

$$\phi_1(a_1) = \sum_{i=1}^{p} \sigma_i^2 R^2(y_1; x_i),$$

where $R^2(y_1; x_i)$ is the squared correlation between y_1 and x_i, and σ_i^2 is the variance of x_i. It is not difficult to show that this definition is equivalent to the more traditional one.

$\phi_1(a_1)$ is the variance explained by the first principal component. It is also useful to note that $\phi_1(a_1)$ may be written as the difference between the traces of the original covariance matrix, $\sum$ and the partial covariance matrix of x given y_1 which we denote as $\sum_1$. Thus, the first principal component is found by maximizing:

$$\phi_1(a_1) = \mathrm{tr}\{\Sigma\} - \mathrm{tr}\{\Sigma_1\} \quad = \mathrm{tr}\left\{\Sigma - \left(\Sigma - \frac{\Sigma a_1 a_1^T \Sigma}{a_1^T \Sigma a_1}\right)\right\} \quad = \frac{a_1^T \Sigma^2 a_1}{a_1^T \Sigma a_1}. \qquad (1)$$

After the first component is obtained, the second is defined as the linear combination of the variables that explains the most variation in $\sum_1$. It may be computed as the first characteristic vector of $\sum_1$, or equivalently, the second characteristic vector of $\sum$. Additional components are defined in a similar manner.

The first restricted principal component

In RPCA, the same $\phi_1(a_1)$ is maximized, but with additional constraints on the elements of a_1. Specifically, these elements are required to belong to a small pre-specified integer set, Θ. The objective is to render the components more easily interpreted. Toward that end, we find two sets of particular use: $\{-1,0,1\}$ and $\{0,1\}$. With the first of these, the components become simple sums and differences of the elements of x. With the second, the components are simply sums of subsets of the elements of x. Of course, any other restricted set could be utilized as well, and the proposed procedure is sufficiently flexible to allow any discrete representation.

If the number of variables, p, is small, each RPCA component can be obtained by merely examining all allowable vectors a_1. However, as p increases, the number of possible vectors rapidly becomes too large. In general, there are $|\Theta|^p$ possible vectors. (Although in the case of $\Theta = \{-1,0,1\}$, only $(3^p - 1)/2$ vectors need to be tried since a_1 and $-a_1$ are equivalent, and (0,0,0) is not considered as feasible.) In order to overcome this problem, we propose a modified B&B technique.

Such a discrete optimization problem can be conceptualized via a tree structure. Each node in the tree corresponds to a particular optimization problem. The problem at the root node is the PCA problem with no constraints. At the next level, each node corresponds to the same problem, but with the first element of a_1 constrained to some fixed value. Corresponding to the $|\Theta|$ possible values, we have $|\Theta|$ nodes. At each subsequent level, one more coefficient is constrained, so that at level $p + 1$ all the coefficients are fixed. The value of each node is the maximal value of $\phi_1(a_1)$ obtained for that node's problem. For node i, denote that value as ϕ_{1i}. Our RPCA solution is then identified by the leaf (level $p + 1$) node with the greatest ϕ_{1i}.

If one had to evaluate all the problems in this tree, there would be no advantage to creating the tree. But note that the value at each node is an upper bound on the value of any node below it since as one moves down the tree constraints are only added. This fact allows large portions of the tree to remain unevaluated. For example, suppose in the course of evaluating nodes, one finds a final node A that has a value of, say, 2. And suppose there is another node B somewhere in the tree that has already been evaluated and found to have the value 1.9. Then there is no need to evaluate any descendents of B since none of them can possibly improve upon node A.

This leads to Hausman's (1982) algorithm for finding the optimal final node:

(1) Evaluate the root node.

(2) Among all evaluated nodes having no evaluated children, find the node with the greatest value.

(3) If this node is a leaf node, then it is the solution. Stop.

(4) Otherwise, evaluate the children of that node and go to Step 2.

The remaining issue is how to efficiently evaluate ϕ_{1i} for each node i. Note first that $\phi_1(a_1)$ is invariant under scale transformations of a_1. Thus, rather than constraining the first k elements of a_1 to be specific members of Θ we can instead require that they be proportional to those specific members of Θ. That is, we can simply require that $a_1 = Tv$ for some v where T has the form:

$$T = \begin{bmatrix} t & 0 \\ 0 & I \end{bmatrix}. \quad (2)$$

The k-vector t specifies the constrained values of the first k elements of a_1, I is a $(p-k)\times(p-k)$ identity matrix, and v is a $(p-k+1)$ vector which will be chosen to maximize $\phi_1(Tv)$. Thus, ϕ_{1i}, the value of node i, is the solution to:

$$\underset{v}{\mathrm{Max}}\left\{\phi_{1i} = \frac{v^T T^T \Sigma^2 T v}{v^T T^T \Sigma T v}\right\}. \quad (3)$$

The solution to this problem is the largest characteristic root of $T^T\Sigma^2 T$ with respect to $T^T\Sigma T$, that is, the largest root of the determinantal equation:

$$\left| T^T\Sigma^2 T - \phi_{1i} T^T \Sigma T \right| = 0. \quad (4)$$

Additional RPCA components

The first RPCA component is obtained by executing the algorithm specified above. A second RPCA component may be obtained as in standard PCA, by computing Σ_1, the partial covariance matrix of x given the first RPCA component, $y_1 = a_1^T x$, and then applying the above algorithm to Σ_1. This process may be repeated until p RPCA components have been obtained that account for all the variance in the system.

There are two drawbacks to this approach. First, unlike standard PCA analyses, there is no guarantee that the vectors of loadings defining components will be orthogonal. This may make it considerably difficult for the analyst to interpret the derived components. Second, after several components have been extracted, there are often many potential candidate components that are equivalent in that they all account for roughly the same amount of variance. For these reasons, it is often useful to add a constraint that each RPCA component has a coefficient vector orthogonal to those of the previous RPCA components. While the addition of this constraint can only limit even further the variance explained, in our tests with various datasets we have never found this decrease to surpass 7 percent.

The orthogonality constraint for the $k+1$st-RPCA component can be written as $Aa_{k+1} = 0$, where A is the $k \times p$ matrix whose rows are the first k-RPCA loadings, a_{k+1} is the $k+1$st RPCA component, and 0 is a k-vector of zeros. In the sub-problem for a particular node in the B&B tree, we have $a_{k+1} = Tv$, and so the orthogonality constraint is incorporated into that sub-problem by adding the constraint $ATv = 0$. Thus, the sub-problem is now:

$$\underset{v}{\mathrm{Max}}\left\{\phi_{k+1,i} = \frac{v^T T^T \Sigma_k^2 T v}{v^T T^T \Sigma_k T v}\right\} \quad \text{s.t.} \;\; ATv = 0, \quad (5)$$

where Σ_k is the partial covariance matrix of x given the first k-RPCA components.

Now suppose AT is $k \times m$ and has rank r. If $r = m$, then v must be the null vector and so the value of the node is zero. Otherwise, $r < m$, let $(AT)^*$ be any $m \times (m-r)$ matrix of full column rank $m - r$ such that:

$$(AT)(AT)^{*} = 0. \tag{6}$$

Then:

$$ATv = 0 \tag{7}$$

and:

$$(\exists z) \quad \text{s.t} \quad v = (AT)^{*} z \tag{8}$$

both define $p - r$ dimensional linear subspaces for v. Furthermore, since:

$$v = (AT)^{*} z \tag{9}$$

implies:

$$ATv = AT(AT)^{*} z = 0, \tag{10}$$

they must both define the same subspace.

Thus, the sub-problem with the orthogonality constraint can be rewritten as:

$$\operatorname*{Max}_{z} \left\{ \phi_{k+1,i} = \frac{v^{T} T^{T} \Sigma_{k}^{2} T v}{v^{T} T^{T} \Sigma_{k} T v} \right\} \quad \text{s.t } v = (AT)^{*} z \tag{11}$$

or equivalently:

$$\operatorname*{Max}_{z} \left\{ \phi_{k+1,i} = \frac{z^{T} (AT)^{*} T^{T} \Sigma_{k}^{2} T (AT)^{*} z}{z^{T} (AT)^{*} T^{T} \Sigma_{k} T (AT)^{*} z} \right\}. \tag{12}$$

The maximal value of $\phi_{k+1,i}$ (the value of node i) is then the largest root of the determinantal equation:

$$\left| (AT)^{*} T^{T} \Sigma^{2} T (AT)^{*} - \phi_{k+1,i} (AT)^{*} T^{T} \Sigma T (AT)^{*} \right| = 0. \tag{13}$$

Efficiency issues

The B&B algorithm described above adapted from Hausman (1982) works fine for small to medium-sized problems, but the tree can grow far too large for efficient solution when performing RPCA with more variables. For these situations, we have devised two modifications that help to obtain solutions in a reasonable period of time. Both techniques work to keep the tree relatively thin so that we reach the leaf nodes more quickly. These techniques can be used individually or in tandem.

Adding depth bias. In the B&B tree, the value of each node is an upper bound on the values of all nodes below it. At each step, we select the leaf node in the currently evaluated tree having the greatest value. If that node is a leaf node in the complete tree (that is, all coefficients have fixed values in Θ), then we have found our solution. If not, we expand the tree by creating and evaluating the immediate children of that node.

The problem that can arise is that if there are, as is usually the case, many final solutions that are similar in terms of their variance explained, then the evaluated tree can be very bushy with a large number of nodes at each level examined before proceeding to the next level. In order to minimize this behavior, we propose adding a slight bias toward looking at nodes further down the tree rather than widening the tree at a higher level.

In practice, we add a small amount $n\alpha$ to the value of each node, where n is the number of levels that node is removed from the root, and α is typically on the order of 0.001. We have found that while this can lead to non-optimal solutions, the variation explained by those solutions is still well within 1 percent of the variation explained by the optimal RPCA.

Randomizing B&B ordering. Another issue that can cause the tree to be bushy is the ordering of the variables. The algorithm as described above splits first on the first variable, then on the second, and so on. With a good ordering, the tree may expand almost exclusively downward, perhaps solving a 50 variable problem by evaluating well under 1,000 nodes. With a poor ordering for the same problem, the algorithm may evaluate several million nodes without arriving at a final solution. At first, we experimented with various heuristics for identifying a good ordering, and then solved the problem using that ordering. Sometimes the number of evaluated nodes was greatly decreased, but in other cases the opposite was true. Since, there were often several orders of magnitude difference in the resources required depending upon the ordering, we decided to take a different approach. Instead of deciding on a particular ordering up front, we randomly order the variables and then try to solve for the RPC in a reasonable time ("reasonable" is defined by a user-specified maximum number of node evaluations). If the final solution is not found, the variable ordering is re-randomized and we try once more for the solution. This continues until either the solution is found or a pre-specified number of randomizations have been attempted.

Restricted principal components analysis – a combinatorial optimization approach for simultaneous extraction

An alternative to complete enumeration or B&B for solving the RPCA problem is a modification of the Lin and Kernighan (1973) combinatorial optimization algorithm presented initially to solve the traveling salesman problem, here modified and tailored to the RPCA problem. The algorithm attempts to generate a number of local optima by examining changes in a total of m elements in the $p \times k$ matrix ($\underline{a}$) at a time. The steps of the modified algorithm adapted for use in RPCA are as follows:

(1) Set $K = 0$; select m from $\{1, 2, \ldots, kp\}$; Set maximum number of iterations (MAXIT).

(2) Generate random feasible $\underline{a}$ vectors.

(3) Set $K = K + 1$.

(4) Evaluate the objective function (z = the total variance-accounted-for values computed across all k components) and let $z^* = z$.

(5) Generate a random map, i.e. a random permutation of the first p positive integers. This map indicates (randomly) the order in which elements will be changed m at a time.

(6) Try to improve – one attempts to evaluate changes in $\underline{a}$ m at a time according to the random map until either one improves ($z > z^*$) or one has evaluated all possible m changes according to the map without improvement. If there is

improvement, then set $z^* = z$, store the $\underline{a}$ solution that resulted in that z, and go to step 5. If no improvement results ($z \leq z^*$), store $\underline{a}$ and z-value. If $K < \mathrm{MAXIT}$, go to step 2, otherwise output results and stop.

The algorithm is much quicker to run than a complete enumeration routine or the previously described modified B&B-RPCA procedure. Lin and Kernighan (1973) estimate computer costs for the algorithm as a function of the number of parameters and describe some of its interesting mathematical properties for the traveling salesman problem. While the procedure does not guarantee a globally optimum result, setting MAXIT $\geq$ 1,000 in our problem has always resulted in obtaining the globally optimum solution at least once for $m = 1$ or 2. Obviously, the larger one makes m and MAXIT, the higher the probability of finding the globally optimal result, but at a much higher computer cost.

Another desirable feature of the algorithm is that it finds MAXIT local optimum solutions. This gives the analyst the opportunity of examining a number of "good" solutions enabling him to perhaps make his own trade-off between a z-value and say coefficients set equal to zero. Or, perhaps one solution may be much more interpretable than the others, given the nature of the application at hand. At any rate, examining a number of "good" solutions is one of the distinct advantages of this method.

For the extraction of more than one factor, one can embed the orthogonality constraints in the algorithm as well to avoid repetitive factor extraction. These (as well as any other constraints the user wishes to enforce for a particular application) are screened in step (ii) of this λ-opt algorithm to ensure satisfaction of all constraints prior to the computation of the objective function. In some RPCA applications such as the one presented below, the efficiency of the combinatorial search can be markedly improved given the characteristics of the desired constraints. For example, in applications where one is interested in restricting a_{ij} ε {0.1} with orthogonality constraints, the resulting search can be vectorized by item or variable in $\underline{\mathbf{I}}_p$, the p × p identity matrix, since orthogonality here would require only one non-zero element per component per item/variable. Thus, instead of searching over the $p \times k$ cells of $\underline{a}$, the search reduces to just p options for each item/variable row of $\underline{a}$.

Application: the Dodge Viper

Background[1]

In the late 1980s, the Chrysler Corporation found itself in a dire situation. Its best-selling product, the Dodge Ram (truck), had a meager 3.8 percent market share. In an effort to rebound from this sagging market share, and facing stiff competition from both domestic and foreign competitors, the company decided that it needed to make a radical statement to the marketplace. That statement came in the form of the Dodge Viper: a high-performance, expensive sports car. The Viper would give Chrysler a product in its portfolio that had the ability to compete with vehicles like Chevrolet's Corvette, Porsche's 911 Carrera, and Ford's Mustang Cobra.

Originally unveiled as a concept car in early 1989, Dodge's mission in developing the Viper was to create "the ultimate American sports car." In order to achieve this goal, the Viper would need to be created by a team that was focused solely on performance. With this mission in mind, Team Viper set out to build a lightweight, rear-wheel-drive sports car chassis around the high-performance ten-cylinder engine of the Dodge Ram.

Three years after the original concept model was shown to the public, in early 1992, the first Viper RT-10 production cars were rolled off the assembly line. These RT-10s were met with much anticipation from car enthusiasts, and the vehicles did not fail to impress. In 1993, Team Viper would make history by pushing the RT-10 from 0 to 100 and back to zero in less than 15 seconds. Chrysler's limited production of these vehicles ensured that the Viper would appeal to a very specific segment of sports car enthusiasts.

In 1996, Dodge released its second generation Viper: the GTS Coupe. The 1996 calendar year saw the production of 1,700 Viper GTSs. Although over 90 percent of the GTS was new, it looked very similar to the RT-10, maintaining several of the original's outrageous design features. However, the weight of the GTS had been reduced dramatically through alterations made to several parts of the car, including the suspension, the engine, the cooling system, and the frame. The GTS Coupe was intended to appeal to a broader market segment, including professionals with an interest in performance cars, as well as pure performance enthusiasts.

The year 2003 saw the release of the third generation of the Viper: the SRT-10. Still following its original mission to be "the ultimate American sports car," the Viper SRT-10 is equipped with a 505 cubic inch, 8.3 liter all-aluminum big block engine, delivering 500 horsepower and 525 ft-lbs of torque. These attributes made the SRT-10 substantially more powerful than competitors such as the Chevrolet Corvette, Ford Mustang Cobra, and Porsche 911 Carerra. Further, evolution of the Viper continues to date.

The study

As stated in 1989, Dodge produced a concept version of this automobile which was featured in a number of auto shows throughout the USA. During this period, this manufacturer wanted an idea on what type of market exists for such a car priced to be around $60,000. Initially, they hypothesized that the car would be attractive to a Yuppie crowd – highly educated, affluent baby boomers who tended to prefer imported vehicles. Malhotra (1996) reports of a psychographic study conducted for the Dodge Viper at auto shows amongst targeted consumers. After a series of in-depth personal interviews with consumers, a list of some 30 psychographic questions was constructed to measure important constructs such as patriotism, styling, prestige, personality, etc. These appear in Table I and are measured on nine-point Likert scales where (1 – definitely disagree, ..., 9 – definitely agree). In addition, an intention/likelihood to buy question was asked in an effort to understand how these psychological constructs impacted buying intentions. Knowledge of these key constructs could drive advertising decisions and the positioning of the automobile. Targeted respondents were interviewed at such auto shows and some 400 completed surveys were collected.

Traditional principal components analysis results

Table II presents the results of a traditional PCA on these data (correlations amongst the 30 psychographic items listed in Table I). One elbow appears at $k = 3$ components, and another at $k = 9$ components. Using the "eigenvalue greater than one" rule,

1. I am in very good physical condition	(v2)
2. When I must choose between the two, I usually dress for fashion, not comfort	(v3)
3. I have more stylish clothes than most of my friends	(v4)
4. I want to look a little different from others	(v5)
5. Life is too short not to take some gambles	(v6)
6. I am not concerned about the ozone layer	(v7)
7. I think the government is doing too much to control pollution	(v8)
8. Basically, society today is fine	(v9)
9. I do not have time to volunteer for charities	(v10)
10. Our family is not too heavily in debt today	(v11)
11. I like to pay cash for everything I buy	(v12)
12. I pretty much spend for today and let tomorrow bring what it will	(v13)
13. I use credit cards because I can pay the bill off slowly	(v14)
14. I seldom use coupons when I shop	(v15)
15. Interest rates are low enough to allow me to buy what I want	(v16)
16. I have more self-confidence than most of my friends	(v17)
17. I like to be considered a leader	(v18)
18. Others often ask me to help them out of a jam	(v19)
19. Children are the most important things in a marriage	(v20)
20. I would rather spend a quiet evening at home than go out to a party	(v21)
21. American-made cars could not compare with foreign-made cars	(v22)
22. The government should restrict imports of products from Japan	(v23)
23. Americans should always try to buy American products	(v24)
24. I would like to take a trip around the world	(v25)
25. I wish I could leave my present life and do something entirely different	(v26)
26. I am usually among the first to try new products	(v27)
27. I like to work hard and play hard	(v28)
28. Skeptical predictions are usually wrong	(v29)
29. I can do anything I set my mind to	(v30)
30. Five years from now, my income will be a lot higher than it is now	(v31)

Source: Case and data taken from Malhotra (1996, pp. 762-4)

Table I. Psychographic items

Malhotra (1996) extracts some nine principal components that account for 78.53 percent of the total variance in this data.

Tables III and IV present the unrotated and varimax rotated PCA loadings for the $k = 9$ PCA solution. Malhotra (1996) labels these varimax rotated factors as shown in Table V.

As mentioned earlier, the questionnaire also includes an intention to buy measure as the marketing group wanted to understand how to properly position the Viper. Table VI presents the corresponding regression analysis on these PCA scores for the nine derived components. Table VI indicates that the total regression equation is significant with an $R^2 = 0.522$. As seen in Table VI, only the first and eighth components (financial concerns and traditionalism) are not significant. Overall, this model suggests that the major psychographic constructs explaining likelihood of purchase are adventurous, style, and optimism. As Malhotra (1996) concludes, marketing communications should be designed to appeal to these three major traits.

Note, as mentioned before in the introduction section, one of the difficulties associated with traditional PCA analysis is the fact that while the original extracted

Table II.
PCA eigenvalue variance decomposition

	Initial eigenvalues			Extraction sums of squared loadings			Rotation sums of squared loadings		
Component	Total	Percentage of variance	Cumulative percentage	Total	Percentage of variance	Cumulative percentage	Total	Percentage of variance	Cumulative percentage
1	4.746	15.818	15.818	4.746	15.818	15.818	4.723	15.743	15.743
2	4.011	13.370	29.188	4.011	13.370	29.188	2.813	9.376	25.119
3	2.809	9.362	38.550	2.809	9.362	38.550	2.812	9.374	34.493
4	2.685	8.949	47.499	2.685	8.949	47.499	2.734	9.115	43.608
5	2.393	7.976	55.475	2.393	7.976	55.475	2.728	9.095	52.703
6	2.293	7.643	63.118	2.293	7.643	63.118	2.560	8.534	61.237
7	1.984	6.613	69.731	1.984	6.613	69.731	2.488	8.294	69.531
8	1.614	5.379	75.110	1.614	5.379	75.110	1.638	5.459	74.990
9	1.026	3.421	78.531	1.026	3.421	78.531	1.062	3.540	78.531
10	0.931	3.104	81.635						
11	0.777	2.591	84.226						
12	0.755	2.516	86.742						
13	0.706	2.354	89.095						
14	0.524	1.746	90.841						
15	0.392	1.308	92.149						
16	0.360	1.201	93.350						
17	0.332	1.106	94.456						
18	0.268	0.895	95.351						
19	0.245	0.818	96.169						
20	0.205	0.685	96.854						
21	0.186	0.620	97.474						
22	0.163	0.544	98.018						
23	0.139	0.462	98.480						
24	0.093	0.310	98.791						
25	0.080	0.267	99.058						
26	0.075	0.252	99.309						
27	0.064	0.215	99.524						
28	0.054	0.181	99.705						
29	0.051	0.170	99.875						
30	0.038	0.125	100.000						

	Component								
	1	2	3	4	5	6	7	8	9
V2	0.047	0.475	− 0.190	0.482	0.268	0.505	0.006	− 0.070	0.048
V3	0.072	0.472	− 0.180	0.477	0.265	0.514	− 0.021	− 0.062	0.034
V4	0.097	0.444	− 0.211	0.473	0.266	0.384	− 0.031	− 0.007	0.053
V5	0.048	0.363	− 0.181	0.438	0.231	0.201	− 0.011	− 0.056	− 0.019
V6	0.044	0.476	0.035	− 0.003	− 0.184	0.065	0.144	− 0.001	− 0.286
V7	0.046	0.306	0.611	0.263	− 0.229	− 0.041	− 0.016	0.053	− 0.424
V8	0.084	0.277	0.677	0.264	− 0.290	− 0.047	− 0.057	0.028	− 0.319
V9	0.084	0.211	0.747	0.252	− 0.242	0.021	− 0.164	0.017	0.357
V10	0.076	0.210	0.735	0.264	− 0.252	0.030	− 0.183	0.011	0.346
V11	0.889	− 0.074	0.010	− 0.053	0.026	− 0.002	0.057	− 0.042	− 0.048
V12	0.896	− 0.097	− 0.007	− 0.041	0.016	− 0.003	0.019	− 0.030	− 0.019
V13	0.929	− 0.085	0.003	− 0.048	0.042	− 0.025	0.038	− 0.054	− 0.023
V14	0.934	− 0.058	0.013	− 0.030	0.039	− 0.009	0.029	− 0.024	− 0.013
V15	0.870	− 0.055	− 0.021	− 0.010	0.031	− 0.021	− 0.048	− 0.002	− 0.013
V16	0.761	− 0.034	− 0.048	− 0.036	0.025	− 0.006	− 0.040	0.061	0.046
V17	− 0.013	0.340	0.298	− 0.129	0.626	− 0.329	0.324	0.000	0.032
V18	− 0.041	0.347	0.299	− 0.118	0.650	− 0.408	0.270	0.003	0.020
V19	− 0.043	0.266	0.286	− 0.050	0.658	− 0.340	0.260	0.052	0.045
V20	0.047	− 0.034	0.002	− 0.009	0.046	0.164	0.070	0.882	− 0.028
V21	0.046	− 0.046	− 0.014	− 0.033	0.010	0.171	0.106	0.874	0.058
V22	− 0.041	0.424	0.164	− 0.708	0.060	0.397	− 0.224	− 0.040	0.005
V23	0.009	0.456	0.140	− 0.704	0.090	0.380	− 0.224	− 0.036	− 0.008
V24	0.043	0.415	0.121	− 0.672	0.073	0.374	− 0.241	− 0.033	0.017
V25	− 0.001	0.539	− 0.122	− 0.123	− 0.403	0.022	0.600	− 0.036	− 0.065
V26	0.037	0.528	− 0.181	− 0.127	− 0.412	0.011	0.602	− 0.046	− 0.059
V27	0.085	0.382	− 0.189	− 0.095	− 0.386	− 0.071	0.465	0.011	0.236
V28	0.087	0.181	− 0.041	− 0.032	− 0.216	− 0.110	0.121	− 0.023	0.575
V29	0.062	0.633	− 0.316	0.057	− 0.122	− 0.475	− 0.417	0.094	− 0.032
V30	0.072	0.614	− 0.341	0.035	− 0.127	− 0.502	− 0.399	0.105	− 0.007
V31	0.052	0.568	− 0.327	0.026	− 0.057	− 0.437	− 0.426	0.122	0.010

Table III. PCA unrotated components

components are unique due to their sequential optimality property, interpretation is typically enhanced by employing various rotational methods such as varimax (above), quartimax, equamax, oblimin, etc. Such rotations spread the cumulative variance accounted for among the derived rotated factors so that they are no longer sequentially optimal. In addition, different rotations can result in different interpretations of the derived factors. As an illustration of this, Table VII depicts the derived rotated component loadings after equamax rotation in SPSS. While many of the earlier components are in fact quite similar to those from the varimax solution presented earlier in Table IV, some of the later components differ substantially with respect to interpretation. (This same comparison was also performed with factor analysis and oblimin rotation where again different results were obtained.) Without substantial theory, it is difficult often times to justify one particular solution over another. This problem also creates difficulties with respect to the particular marketing strategy to employ, as the subsequent regression analysis will differ with what is shown is Table VI.

Table IV.
PCA varimax rotated components

	Component 1	2	3	4	5	6	7	8	9
V2	−0.013	0.907	0.022	0.054	0.005	0.072	0.004	0.001	0.006
V3	0.012	0.905	0.035	0.069	0.013	0.053	−0.009	0.008	−0.009
V4	0.040	0.826	0.007	−0.002	0.094	0.036	0.027	0.043	0.015
V5	0.005	0.648	0.006	−0.093	0.129	0.036	0.072	−0.034	−0.045
V6	0.005	0.162	0.196	0.156	0.164	0.445	0.019	−0.003	−0.236
V7	0.000	0.029	0.764	−0.027	0.051	0.165	0.073	0.011	−0.392
V8	0.034	−0.011	0.837	−0.022	0.039	0.131	0.032	−0.016	−0.284
V9	0.020	0.030	0.859	0.050	−0.041	−0.087	0.034	−0.004	0.367
V10	0.011	0.041	0.858	0.048	−0.032	−0.095	0.011	−0.011	0.356
V11	0.896	−0.005	0.013	0.001	−0.038	0.037	0.004	−0.012	−0.021
V12	0.902	−0.007	0.006	−0.010	−0.021	0.002	−0.028	−0.002	0.007
V13	0.937	−0.008	0.005	−0.011	−0.021	0.011	0.012	−0.027	0.003
V14	0.937	0.019	0.029	−0.004	−0.010	0.013	0.010	0.004	0.013
V15	0.871	0.027	0.020	−0.007	0.052	−0.036	−0.028	0.015	0.009
V16	0.758	0.025	−0.010	0.017	0.061	−0.023	−0.028	0.079	0.065
V17	0.001	0.037	0.035	0.080	0.011	0.063	0.903	−0.016	−0.001
V18	−0.025	0.011	0.042	0.056	0.082	0.016	0.935	−0.032	−0.015
V19	−0.028	0.054	0.032	0.006	0.020	−0.045	0.877	0.028	−0.001
V20	0.026	0.016	0.003	0.006	−0.015	−0.015	0.001	0.901	−0.056
V21	0.025	−0.004	−0.018	0.009	−0.040	0.018	−0.017	0.900	0.035
V22	−0.048	−0.014	0.027	0.955	0.020	0.074	0.042	0.003	−0.003
V23	0.001	0.011	0.010	0.955	0.054	0.082	0.070	0.006	−0.016
V24	0.035	0.010	0.004	0.915	0.055	0.056	0.034	0.008	0.009
V25	−0.044	0.061	0.033	0.079	0.043	0.912	0.031	−0.005	0.038
V26	−0.004	0.061	−0.015	0.063	0.064	0.923	0.008	−0.015	0.048
V27	0.045	−0.002	−0.036	−0.010	0.101	0.708	−0.020	0.030	0.328
V28	0.053	−0.023	0.045	−0.006	0.105	0.221	−0.005	−0.021	0.618
V29	0.005	0.101	0.036	0.041	0.950	0.117	0.034	−0.037	0.020
V30	0.018	0.070	0.002	0.028	0.955	0.126	0.038	−0.028	0.047
V31	0.003	0.098	−0.021	0.062	0.896	0.047	0.046	0.000	0.048

Table V.

Component	Label
1	Financial concerns
2	Style consciousness
3	Societal concern
4	Patriotism
5	Optimism
6	Adventurous
7	Opinion leadership
8	Traditionalism
9	Intensity

Restricted principal components analysis results

We conducted RPCA analysis for $k = 9$ factors using both the sequential modified B&B and the λ-opt combinatorial optimization *simultaneous* procedures for restricted sets {0,1} and {−1,0,1}. Based upon the simple structure displayed

Table VI.
Regression model for PCA scores

Model summary

Model	*R*	R^2	*Adjusted* R^2	*Std error of the estimate*
1	0.723(a)	0.522	0.511	1.808

ANOVA

Model		*Sum of squares*	*df*	*Mean square*	*F*	*Sig.*
1	Regression	1,395.267	9	155.030	47.401	0.000(a)
	Residual	1,275.531	390	3.271		
	Total	2,670.797	399			

Coefficients

		Unstandardized coefficients		*Standardized coefficients*		
Model		*B*	*Std error*	β	*t*	*Sig.*
1	(Constant)	5.023	0.090		55.544	0.000
	PCA1	0.111	0.091	0.043	1.225	0.221
	PCA2	0.774	0.091	0.299	8.550	0.000
	PCA3	0.385	0.091	0.149	4.249	0.000
	PCA4	0.388	0.091	0.150	4.282	0.000
	PCA5	0.762	0.091	0.294	8.413	0.000
	PCA6	1.371	0.091	0.530	15.143	0.000
	PCA7	0.201	0.091	0.078	2.225	0.027
	PCA8	0.096	0.091	0.037	1.063	0.288
	PCA9	−0.278	0.091	−0.108	−3.073	0.002

earlier in Tables III and IV with basically positive loadings and the subsequent interpretation, we report the results of the two procedures for the {0,1} restricted set with the orthogonality constraints. Note, for this particular combination of user specified options (binary $\underline{a}$ and orthogonality), items are only allowed to load up on one factor. Table VIII presents the RPCA results for the modified B&B procedure which extracts the restricted components sequentially. As mentioned earlier, and seen here in this application, one of the potential problems associated with this sequential extraction is that it tends to result in sparse components (those defined on few items) in the later stages of extraction. As shown in Table VIII here, the first two factors tend to consume most of the items with non-zero loadings. Derived RPCA factors 3 and 4 are only defined in terms of two items each, and the remaining five factors (*P*5-*P*9) only each possess one non-zero loading. In fact, no other extracted RPCA factors are possible here given the orthogonality constraint. The total variance accounted for by these nine RPCA components is 71.9 percent, which is approximately 6.6 percentage points lower than PCA. As seen, the sequential procedure tends to load up optimal components early on in the extraction process that account for more of the variation.

Let us now examine and contrast the λ-opt simultaneous procedure for similar binary $\underline{a}$ and orthogonality constraints. Table IX contrasts the comparative fits by component for the traditional varimax rotated PCA solution and the combinatorial λ-opt RPCA simultaneous solution both run in $k = 9$ components. As seen, one loses only 0.014 in explained variance between the two solutions, which improves upon the B&B extracted RPCA solution. Thus, the constraints result in a trivial loss of fit. Table IX also presents the λ-opt derived RPCA nine component solution. As seen, the interpretation is much cleaner and crisper than the sequential RPCA and PCA

	Component								
	1	2	3	4	5	6	7	8	9
V2	−0.016	0.906	0.015	0.056	0.069	0.008	0.025	0.025	0.002
V3	0.009	0.904	0.022	0.070	0.047	−0.005	0.028	0.042	0.010
V4	0.036	0.825	0.103	−0.001	0.036	0.031	0.019	0.006	0.044
V5	0.004	0.647	0.134	−0.091	0.025	0.074	−0.025	0.053	−0.033
V6	0.004	0.159	0.156	0.165	0.380	0.018	−0.041	0.383	−0.004
V7	−0.003	0.021	0.030	−0.024	0.061	0.071	0.324	0.812	0.010
V8	0.030	−0.020	0.022	−0.022	0.046	0.030	0.452	0.770	−0.017
V9	0.013	0.019	−0.031	0.034	−0.042	0.037	0.903	0.255	−0.005
V10	0.004	0.030	−0.022	0.033	−0.052	0.014	0.897	0.262	−0.012
v11	0.896	−0.003	−0.035	0.001	0.035	−0.002	0.001	0.029	−0.002
v12	0.902	−0.005	−0.018	−0.011	0.006	−0.033	0.016	−0.001	0.008
v13	0.937	−0.005	−0.017	−0.012	0.014	0.006	0.013	0.003	−0.016
v14	0.936	0.022	−0.006	−0.006	0.017	0.004	0.037	0.011	0.015
v15	0.870	0.028	0.056	−0.009	−0.032	−0.033	0.031	0.002	0.025
v16	0.757	0.026	0.067	0.014	−0.009	−0.032	0.041	−0.057	0.087
v17	0.007	0.033	0.011	0.080	0.058	0.903	0.018	0.038	−0.018
v18	−0.020	0.006	0.081	0.056	0.009	0.935	0.016	0.047	−0.033
v19	−0.023	0.049	0.020	0.005	−0.046	0.877	0.025	0.019	0.027
V20	0.016	0.015	−0.014	0.006	−0.024	0.002	−0.029	0.040	0.901
v21	0.014	−0.004	−0.035	0.007	0.027	−0.016	0.009	−0.037	0.900
v22	−0.047	−0.016	0.019	0.956	0.060	0.043	0.029	0.022	0.004
v23	0.002	0.009	0.054	0.956	0.066	0.071	0.008	0.024	0.007
v24	0.036	0.008	0.056	0.915	0.046	0.034	0.021	−0.003	0.009
v25	−0.046	0.061	0.048	0.091	0.899	0.032	−0.040	0.148	−0.008
v26	−0.006	0.061	0.069	0.076	0.913	0.009	−0.072	0.113	−0.017
v27	0.042	−0.002	0.118	−0.005	0.758	−0.019	0.106	−0.146	0.027
v28	0.049	−0.026	0.132	−0.015	0.334	−0.001	0.400	−0.394	−0.024
v29	0.002	0.090	0.951	0.043	0.105	0.034	0.010	0.061	−0.041
V30	0.015	0.060	0.957	0.030	0.121	0.039	0.000	0.022	−0.031
v31	0.000	0.088	0.899	0.063	0.045	0.047	−0.007	−0.010	−0.003

Table VII.
PCA equamax rotated components

solutions as each factor is only influenced by a small number of psychographic items. The interpretation of these nine derived factors is also very similar in meaning to those derived from traditional PCA after varimax rotation (as such, we will keep these same interpretive labels as a convenience).

Table X presents the corresponding regression results when regressing these nine simultaneous RPCA scores on the intention to buy dependent variable. In Table X, we see a significant regression equation with $R^2 = 0.485$, a drop of only 0.037 from that of Table VI. Table X presents a slightly different result where there seems more clarity in terms of the impact of these derived factors or likelihood of buying a Viper. Now, restricted components No. $P1$, No. $P7$, and No. $P8$ are non-significant; No. $P3$ and No. $P4$ are weakly significant; and No. $P6$, No. $P2$, No. $P9$ and No. $P5$ are strongly significant. Thus, advertising emphasis should be placed on adventurousness, style, optimism, and intensity in the promotion of the Viper. Note, one of the clear benefits derived here from such a constrained analysis is that the computation of the RPCA components and focus can be reduced to 12 of the 30 raw psychographic items involved in the

Table VIII. RPCA sequential extracted components

Component	Roots 1	2	3	4	5	6	7	8	9
Root	4.67279	3.73445	2.56192	2.49698	2.04624	1.98669	1.72238	1.34985	0.999614
Percentage of variance	15.6	12.4	8.5	8.3	6.8	6.6	5.7	4.5	3.3
Cumulative percentage	15.6	28.0	36.6	44.9	51.7	58.3	64.1	68.6	71.9
Nodes	2,085	370,639	40,740	10,852	20,026	11,991	10,756	5,416	7,428
	P1	*P2*	*P3*	*P4*	*P5*	*P6*	*P7*	*P8*	*P9*
V2	1	0	0	0	0	0	0	0	0
V3	0	1	0	0	0	0	0	0	0
V4	0	1	0	0	0	0	0	0	0
V5	0	1	0	0	0	0	0	0	0
V6	0	1	0	0	0	0	0	0	0
V7	0	1	0	0	0	0	0	0	0
V8	0	0	1	0	0	0	0	0	0
V9	0	0	1	0	0	0	0	0	0
V10	0	1	0	0	0	0	0	0	0
V11	1	0	0	0	0	0	0	0	0
V12	1	0	0	0	0	0	0	0	0
V13	1	0	0	0	0	0	0	0	0
V14	1	0	0	0	0	0	0	0	0
V15	1	0	0	0	0	0	0	0	0
V16	1	0	0	0	0	0	0	0	0
V17	0	1	0	0	0	0	0	0	0
V18	0	0	0	0	1	0	0	0	0
V19	0	1	0	0	0	0	0	0	0
V20	0	0	0	0	0	0	0	1	0
V21	0	1	0	0	0	0	0	0	0
V22	0	0	0	1	0	0	0	0	0
V23	0	1	0	0	0	0	0	0	0
V24	0	0	0	1	0	0	0	0	0
V25	0	1	0	0	0	0	0	0	0
V26	0	0	0	0	0	0	1	0	0
V27	0	1	0	0	0	0	0	0	0
V28	0	0	0	0	0	0	0	0	1
V29	0	1	0	0	0	0	0	0	0
V30	0	0	0	0	0	1	0	0	0
V31	0	1	0	0	0	0	0	0	0

Table IX.
Simultaneous RPCA model fit comparison

	Component: 1	2	3	4	5	6	7	8	9	Total
PCA-varimax	0.157	0.094	0.093	0.091	0.091	0.085	0.083	0.055	0.035	0.785
RPCA	0.156	0.093	0.092	0.090	0.090	0.083	0.078	0.054	0.036	0.771
	*P*4	*P*1	*P*7	*P*5	*P*8	*P*3	*P*6	*P*2	*P*9	
V2	0	0	0	0	1	0	0	0	0	
V3	0	0	0	0	1	0	0	0	0	
V4	0	0	0	0	1	0	0	0	0	
V5	0	0	0	0	1	0	0	0	0	
V6	0	0	0	0	0	0	0	0	1	
V7	0	0	0	0	0	0	1	0	0	
V8	0	0	0	0	0	0	1	0	0	
V9	0	0	0	0	0	0	1	0	0	
V10	0	0	0	0	0	0	1	0	0	
V11	0	0	0	0	0	0	0	1	0	
V12	0	0	0	0	0	0	0	1	0	
V13	0	0	0	0	0	0	0	1	0	
V14	0	0	0	0	0	0	0	1	0	
V15	0	0	0	0	0	0	0	1	0	
V16	0	0	0	0	0	0	0	1	0	
V17	0	0	0	0	0	1	0	0	0	
V18	0	0	0	0	0	1	0	0	0	
V19	0	0	0	0	0	1	0	0	0	
V20	0	0	0	1	0	0	0	0	0	
V21	0	0	0	1	0	0	0	0	0	
V22	0	1	0	0	0	0	0	0	0	
V23	0	1	0	0	0	0	0	0	0	
V24	0	1	0	0	0	0	0	0	0	
V25	0	0	1	0	0	0	0	0	0	
V26	0	0	1	0	0	0	0	0	0	
V27	0	0	1	0	0	0	0	0	0	
V28	0	0	0	0	0	0	0	0	1	
V29	1	0	0	0	0	0	0	0	0	
V30	1	0	0	0	0	0	0	0	0	
V31	1	0	0	0	0	0	0	0	0	

computation of the four dominant restricted factors, as opposed to the need for all 30 items in the computation of any reduced set of traditional PCA components.

Comparison with hierarchical clustering results

As a final comparison to these RPCA solutions given the binary nature of the RPCA optimization problem in this application, we performed a hierarchical cluster analysis on the input variables using the nearest neighbor procedure in SPSS. If we select a nine cluster solution, the resulting dendrogram will allocate the 30 input variables into the resulting nine clusters to mimic the binary, nine component RPCA solution. Note, here too as in the case of PCA and choice of rotations, one obtains different results depending upon which clustering method is utilized in this hierarchical analysis, and there is no a priori theory to guide us as to which one to apply for this application. Figure 1 shows the resulting dendrogram obtained in this analysis. The nine derived clusters are depicted in Table XI. While we see some similarities with our RPCA

Model summary							
Model		R	R^2	Adjusted R^2	Std error of the estimate		
1		0.696(a)	0.485	0.473	1.87796		
ANOVA							
Model		Sum of squares	Df	Mean square	F	Sig.	
1	Regression	1,295.376	9	143.931	40.811	0.000(a)	
	Residual	1,375.421	390	3.527			
	Total	2,670.797	399				
Coefficients							
		Unstandardized coefficients		Standardized coefficients			
Model		B	Std error	β	t	Sig.	
1	(Constant)	−7.336	0.860		−8.533	0.000	
	$P4$	0.048	0.021	0.083	2.252	0.025	
	$P1$	0.008	0.011	0.027	0.729	0.466	
	$P7$	0.035	0.023	0.056	1.513	0.131	
	$P5$	0.121	0.022	0.206	5.409	0.000	
	$P8$	0.046	0.034	0.049	1.333	0.183	
	$P3$	0.046	0.019	0.091	2.461	0.014	
	$P6$	0.228	0.026	0.345	8.794	0.000	
	$P2$	0.121	0.019	0.231	6.216	0.000	
	$P9$	0.262	0.044	0.240	6.016	0.000	

Table X. Restricted PCA regression model

solution, two singleton clusters result involving clusters 2 (v6) and 9 (v28). Thus, one cannot reproduce the optimal RPCA solutions via traditional methods.

Discussion

We have provided two algorithms for estimating RPCA solutions from empirical data. A discussion of the technical details of each have been given, as well as an application in marketing regarding the psychographic profiles of buying intenders for the Dodge Viper sports car. As shown, very little goodness of fit is sacrificed in enhancing the interpretability of the derived solutions as contrasted against traditional PCA. The major benefits with such restricted analysis involves ease of interpretation of the derived latent components as well as being able to focus on fewer raw variable items in defining psychological constructs in subsequent analyses (as illustrated in the Viper application, where only 12 of the 30 raw psychographic items collected in the survey were really necessary in the final regression analysis). Contrast this to the traditional PCA analyses where the results vary as to the type of rotation utilized and where all 30 items are required for the calculation of PCA scores and final regression analysis.

Additional research is required in a number of related areas. One, there is no statistical theory developed for the number of RPCA components to be selected. In the application provided, Malhotra (1996) utilized a scree plot heuristic for traditional PCA to determine the number of components which we utilized for RPCA. Two, comparative Monte Carlo simulations need to be conducted on a number of fronts here. For example, it would be valuable to understand the circumstances under which such restricted analyses more seriously detract from variance accounted for as compared to traditional PCA fits. Also, simulations with synthetic data whose structure is known a

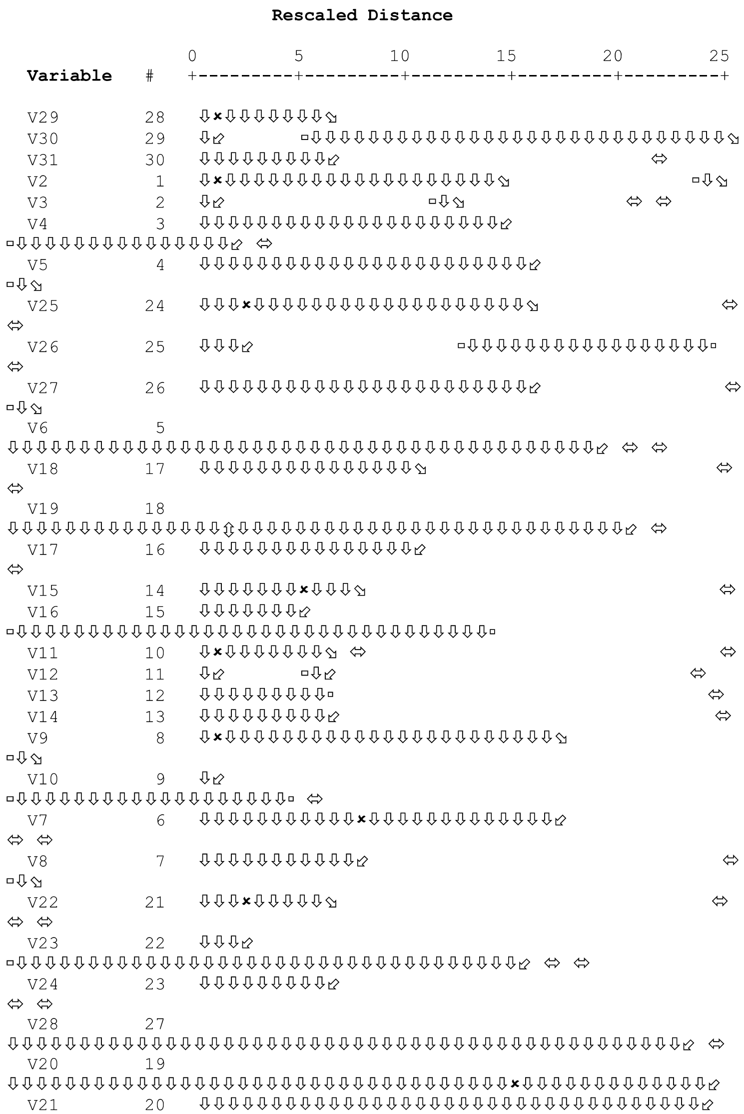

Figure 1.
Nearest neighbor hierarchical clustering results

priori is needed to fully describe the performance of RPCA in recovering known structures, as well as to contrast the performance between the two algorithms provided in the manuscript. That is, when is sequential fitting more appropriate than simultaneous fitting? Intuitively, the answer to this question would appear to be related to the number of raw items or variables included in the analysis. Three additional applications with real data need to be undertaken to fully examine the usefulness of the proposed methods. Four, attempts to embed RPCA in current statistical software packages (e.g. SAS, SPSS, etc.) should be undertaken to increase its

Item	Cluster membership Cluster
V2	1
V3	1
V4	1
V5	1
V6	2
V7	3
V8	3
V9	3
V10	3
V11	4
V12	4
V13	4
V14	4
V15	4
V16	4
V17	5
V18	5
V19	5
V20	6
V21	6
V22	7
V23	7
V24	7
V25	8
V26	8
V27	8
V28	9
V29	1
V30	1
V31	1

Table XI. Hierarchical clustering nine cluster solution

dissemination across both academic and practitioner spheres as specialized software is now required to perform such analyses. Finally, generalization of such restricted analyses to other multivariate procedures (e.g. discriminant analysis and multidimensional scaling) would prove to also be a fruitful area to explore.

Note

1. The reported facts relating to the Dodge Viper's background were acquired via the five web sites denoted in the references Chrysler's Viper Strikes (2004), Dodge Viper (2004), Dodge Viper official web site (2004), "Dodge Viper SRT-10 gives new meaning to extreme performance" (2004) and The complete Dodge Viper informational and multimedia site (2004).

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About the authors

Wayne S. DeSarbo is the Mary Jean and Frank P. Smeal Distinguished Professor of marketing at the Smeal College of Business at the Pennsylvania State University at University Park, Pennsylvania. He obtained his PhD in marketing and statistics from the University of Pennsylvania, and completed post doctorate work in operations research and econometrics there. He has published over 130 articles in such journals as the *Journal of Marketing Research*, *Psychometrika*, *Journal of Consumer Research*, *Journal of Mathematical Psychology*, *Marketing Science*, *Journal of Classification*, *Journal of Marketing, Management Science*, and *Decision Sciences*. His methodological interests lie in multidimensional scaling, classification, and multivariate statistics, especially as they pertain to substantive marketing problems in positioning, market structure, consumer choice, market segmentation, and competitive strategy. Wayne S. DeSarbo is the corresponding author and can be contacted at: wsd6@psu.edu

Robert E. Hausman is a President of the K5 Consulting Firm. He earned his PhD in statistics from the Wharton School of the University of Pennsylvania. His research interests involve the area of Multivariate Statistics.

Jeffrey M. Kukitz is a Senior Research Associate at Pennsylvania State University. He has an MBA degree from the Smeal College of Business at Penn State and more than seven years of experience in marketing research and consulting. For the past six years, he has served in various capacities in research and instruction with the Pennsylvania State University and as a Consultant with Analytika Marketing Sciences Inc.