Business Cycle and Structural Change: Evidence for Brazil

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August 4, 2025

Abstract

This paper models the Brazilian business cycle using GDP data spanning 44 years. We employ a fusion of two models: the Markov-Switching Mean–Variance Component Model, as introduced by Doornik (2013), and a Markov-Switching model that incorporates a specific transition probability matrix to account for structural permanent breaks in the data-generating process while maintaining accuracy in identifying recessions, similar to Kim and Nelson (1999). To the best of our knowledge, this is the first application of this model to Brazilian GDP. Our analysis agnostically identifies a structural break in the second quarter of 1999, coinciding with the shift from a fixed exchange rate regime to an inflation-targeting framework. Recessions before this break were more frequent and severe but less persistent. Episodes of higher volatility regimes are associated with extreme events such as the COVID-19 pandemic, the subprime crisis, and the Collor Plan. Our model demonstrates a good fit to the data, showing no signs of misspecification and successfully reproducing the CODACE business cycle classification.

JEL codes: E32, C22, C52

Keywords: Markov switching, business cycle

1 Introduction

The economic cycle and long-term growth are two important research areas in macroeconomics. A crucial step in understanding the sources driving these phenomena is to date episodes of rapid growth compared to slow growth or crises. In this context, the Markov-Switching model introduced by Hamilton (1989) became a popular approach for dating business cycles. Using data from 1952 to 1984, Hamilton demonstrated that his model effectively captured the NBER cycle classification. However, as noted by McConnell and Perez-Quiros (2000) and Doornik (2013), Hamilton's findings did not hold when the data set was extended to recent period. This was largely because, around the middle of the 80s, the United States economy started to show a stable GNP growth, often referred to as the "Great Moderation," which led to a breakdown of the Hamilton (1989) model. Doornik (2013) proposed a model with two independent mean and variance regimes that can switch independently named Markov-Switching Mean–Variance Component Model. He showed that his model better corresponded to the stylized facts of post-war USA economic history. Particularly a regime change in the variance can be observed in the data. Kim and Nelson (1999) proposed a model with absorbing states to model structural change in Hamilton (1989) specification.

Hamilton's Markov-Switching model has been applied to various data sets and countries. Chauvet (2002) was among the first to use the Hamilton model to analyze Brazilian business cycles. She modelled Brazilian GDP from the second quarter of 1980 to the first quarter of 2000. But after that Brazilian economy underwent significant changes during the 1990s due to a successful macroeconomic stabilization started in 1993 and implemented throughout the period 1994 and 1995 based on an almost fixed exchange rate regime. By the beginning to 1999 Brazilian Central Bank had to let the Brazilian currency fluctuate. Brazilian government adopted a major change in economic policy by following an inflation target, dirty float exchange rate regime and fiscal consolidation. These structural shifts may have affected the results of her model in a manner like what Doornik (2013) observed for the United States data. This discussion sets the stage for our applied paper.

The main goal of this paper is to model Brazilian business cycle data by merging the frameworks suggested by Doornik (2013) and Kim and Nelson (1999). We model Brazilian quarterly GDP from 1980 to 2024 using the Markov-Switching Mean–Variance Component Model with absorbing states. We introduce an important modification to the mean transition probabilities matrix that allows the model to accommodate potential structural breaks in the data while effectively dating recession episodes. To the best of our knowledge, this is the first application of this setup to Brazilian GDP data.

Our findings suggest that our specification model provides a reasonable fit for the classification of the Brazilian business cycle and offers a more nuanced characterization of the cycle. Instead of merely classifying recessions, the model allows us to label periods as high and low-volatility recessions and between those occurring before and after the structural break. As such, this model offers substantial flexibility in addressing the breaks in Brazil's macroeconomic policy history and data. We agnostically identify a structural break in the second quarter of 1999, coinciding with the abandonment of the fixed exchange rate and the implementation of the inflation-targeting regime. Furthermore, our analysis reveals that recessions occurring before this break were more frequent, intense and less persistent.

The structure of this paper is as follows. In the next section, we present our econometric approach, beginning with Hamilton (1989) Markov-Switching model, followed by the Markov-Switching Mean–Variance Component Model introduced by Doornik (2013), and then discuss the restriction we impose on the transition probabilities matrix similar to Kim and Nelson (1999). We then present and interpret the results of our empirical analysis, comparing them with stylised facts. Finally, in the concluding section, we summarise the insights gained from our study of the Brazilian business cycle.

2 Econometric Approach

We model Brazilian GDP using Markov-switching models. Two variants of this approach are considered: the Markov-Switching Dynamic Regression model (MS-DR) and the Markov-Switching ARIMA model (MS-ARIMA). Both models are discussed below; we choose the latter due to its greater generality.

2.1 The Markov-Switching Dynamic Regression Model

In the Markov-switching model, the unobserved random variable S_t follows a Markov chain, defined by transition probabilities between the S regimes:

$$p_{i|j} = P[S_{t+1} = i | S_t = j], \quad i, j = 0, \ldots, S-1$$

Therefore, the probability of moving from state j in one period to state i in the next period depends only on the previous state.

$$y_t - \mu(S_t) = \rho[y_{t-1} - \mu(S_t)] + \epsilon_t, \quad \epsilon_t \sim IIN[0, \sigma^2]$$

More generally, we write for p lags and S regimes: MS(S)-DR-AR(p). $IIND$ stands for independent and identically Normal distribution. A related model is the Markov-switching dynamic regression (MS-DR):

2.2 The Markov-Switching ARIMA Model

A simple example of a Markov-Switching autoregression with a single lag is:

$$y_t - \mu(S_t) = \rho[y_{t-1} - \mu(S_{t-1})] + \epsilon_t, \quad \epsilon_t \sim IIN[0, \sigma^2]$$

More generally, we write for p lags and S regimes: MS(S)-AR(p). $IIND$ stands for independent and identically Normal distribution. A related model is the Markov-switching dynamic regression (MS-DR):

This specification is the general case and we opt to use that in this paper. The variance can also be regime-dependent: $\sigma^2(S_t)$.

The likelihood of the Markov-switching model can be evaluated efficiently using the filtering procedure of Hamilton (1990). Kim (1994) derived a smoothing algorithm. This enables numerical maximization of the log-likelihood as a function of the parameters in the mean and variance, as well as the transition probabilities $p_{i|j}$. The probabilities are subject to the constraint that they lie between 0 and 1 and sum to unity. Let $t = 1, \ldots, T$ denote the estimation sample, $\mathbf{Y}_t^1 = (y_t, \ldots, y_t)'$, and θ the vector of parameters. The resulting filtered regime probabilities $P(S_t|\mathbf{Y}_t^1, \hat{\theta})$ are time varying.

2.3 The Markov-Switching Mean–Variance Component Model

Doornik (2013) introduces a separate mean regime S_t^m and variance regime S_t^v that evolve independently of each other, so have their transition matrices $\mathbf{P}_m = p_{i|j}^m$ and $\mathbf{P}_v = p_{i|j}^v$ respectively. For example, a component model with two regimes for both the mean and the variance:

	$S_t^m = 0$	$S_t^m = 1$		$S_t^v = 0$	$S_t^v = 1$
$S_{t+1}^m = 0$	$p_{0\|0}^m$	$p_{0\|1}^m$	$S_{t+1}^v = 0$	$p_{0\|0}^v$	$p_{0\|1}^v$
$S_{t+1}^m = 1$	$p_{1\|0}^m$	$p_{1\|1}^m$	$S_{t+1}^v = 1$	$p_{1\|0}^v$	$p_{1\|1}^v$

corresponds to a restricted four-regime model:

	$S_t^v = 0$		$S_t^v = 1$	
	$S_t^m = 0$	$S_t^m = 1$	$S_t^m = 0$	$S_t^m = 1$
	$S_t = 0$	$S_t = 1$	$S_t = 2$	$S_t = 3$
$S_{t+1} = 0$	$p_{0\|0}^v \mathbf{P}^m$		$p_{0\|1}^v \mathbf{P}^m$	
$S_{t+1} = 1$				
$S_{t+1} = 2$	$p_{1\|0}^v \mathbf{P}^m$		$p_{1\|1}^v \mathbf{P}^m$	
$S_{t+1} = 3$				

Instead of four means and variances, there are only two of each. Moreover, the transition matrix is restricted to:

$$\mathbf{P} = \mathbf{P}^v \otimes \mathbf{P}^m \tag{1}$$

which has only four free probabilities, rather than 12 for the unrestricted four-regime model. The term $\otimes$ denotes Kronecker product. Doornik (2013) represents these models by MSComp(S^m,S^v) for S^m mean regimes and S^v variance regimes, which don't need to be the same. The MSComp model can be combined with the AR, MA or DR specifications for the dynamics.

Estimation is straightforward when a numerical maximization routine is used because the model fits in the standard Markov switching framework with only minor adjustments. The inequality constraints on the transition probabilities are expressed as part of the maximization problem. The mean and variance constraints imposed by the component model are simply substituted out, while the appropriate constraints on $\mathbf{P^v}$ and $\mathbf{P^m}$ automatically entail that $\mathbf{P}$ satisfies the necessary constraints.

2.4 The Structural Break Restriction

The novelty of this paper is the configuration of structural change given by to (2) together with (1). We use Markov switching mean variance framework can be leveraged to model endogenously determined structural breaks by modifying the $\mathbf{P^m}$ matrix to incorporate permanent structural change. For instance, a permanent regime shift can be represented using a two-state Markov chain, where state 2 acts as an new structure. In this setup, once the process transitions to state 2, it remains there indefinitely. This is achieved by rewriting the $\mathbf{P^m}$ matrix in a triangular form, ensuring that the transition probabilities reflect the permanence of state 2. In the case of higher-order Markov chains, the concept can be extended using block diagonal matrices, where one block represents an absorbing block, and once the process enters this block, it remains within the regimes defined by that block indefinitely.

$$\mathbf{P^m} = \begin{pmatrix} p_{0|0} & p_{0|1} & 0 & 0 \\ p_{1|0} & p_{1|1} & 0 & 0 \\ 0 & p_{2|1} & p_{2|2} & p_{2|3} \\ 0 & 0 & p_{3|2} & p_{3|3} \end{pmatrix} \tag{2}$$

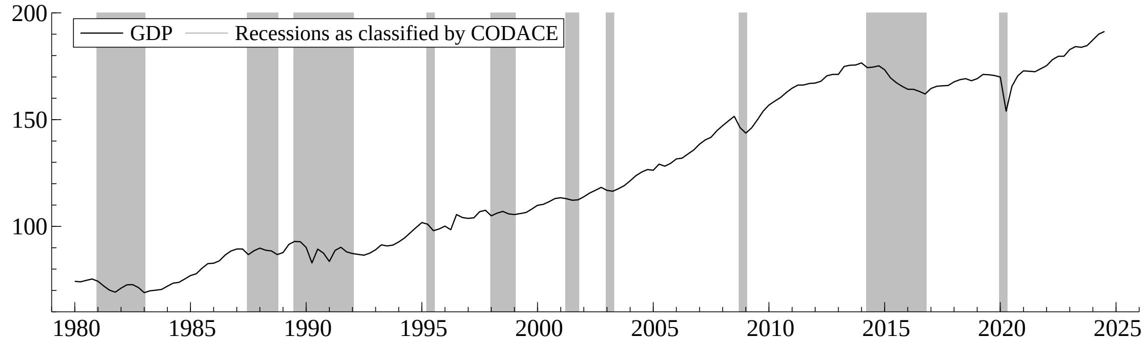
As described in Hamilton (1994), this kind of approach is particularly useful when shifts in economic, financial, or policy regimes are expected to persist over time, or when it is suspected to have a permanent structural change but is uncertain about its exact timing. Incorporating absorbing states provides a more accurate representation of long-term structural changes, such as those resulting from major policy reforms or significant market disruptions. Moreover, this framework enhances the ability to capture and forecast the effects of such regime shifts, leading to more reliable predictions and deeper insights into the system's dynamics under varying conditions. Kim and Nelson (1999) adopts a similar approach but uses Bayesian techniques to estimate the model.

3 Database description

Our empirical analysis uses quarterly data from 1980:1 to 2024:3 (179 observations), collected from IBGE [1]. It is important to note that data from 1980 to 1995 are taken from IBGE (1996), a publication in IBGE's Web Library. From 1996 to 2024, the data come from the Quarterly National Accounts database, which starts in 1996. We constructed the series by chaining the two sources. The data were then seasonally adjusted using the X12-ARIMA method. Figure 1 shows the seasonally adjusted GDP, with shaded areas indicating recession periods as classified by CODACE (2023).

[1] The acronym IBGE stands for the Brazilian Institute of Geography and Statistics.

Figure 1: GDP - Index 1995=100 - Seasonally Adjusted



We acknowledge that some studies use longer quarterly GDP time series, such as Bonelli and Rodrigues (2010) and Pereira and Vieira (2013). However, these series are typically constructed using statistical filtering techniques applied to annual data. The official quarterly GDP series from IBGE only began in 1980. So we restrict our analysis to this period.

4 Results and Discussion

One important question is the choice of the number of regimes. A formal test would be desirable, but this is quite challenging because, under the null, there are unidentified parameters. Hansen (1992) and Garcia (1998) address this problem to develop a likelihood ratio test. Psaradakis and Spagnolo (2003) and Cavicchioli (2014) analyze the use of information criteria to select the number of regimes and the lag order of the model. Since no definitive criterion for regime selection has emerged, we opt to work with four regimes, drawing on prior knowledge of the Brazilian economy and the results of specification tests on the estimated models, as will be demonstrated throughout the paper.

Our benchmark model is an MSComp(4,2)-ARMA(2,2) consisting of four mean regimes and two variance regimes. The regimes in mean and variance switch independently. We allow the AR and MA parameters to vary across these regimes, providing greater flexibility in capturing the system's dynamics. The model is specified as:

$$y_t = \mu(S_t^m) + \phi_1(S_t^m)(y_{t-1} - \mu(S_{t-1}^m)) + \phi_2(S_{t-2}^m)(y_{t-2} - \mu(S_{t-2}^m)) + \epsilon_t + \theta_1(S_{t-1}^m)\epsilon_{t-1} + \theta_2(S_{t-2}^m)\epsilon_{t-2} \quad (3)$$

where $\epsilon_t \sim IIND[0, \sigma^2(S_t^v)]$, $S_t^m \in \{0, 1, 2, 3\}$ and $S_t^v \in \{0, 1\}$ and with the Kronecker structure for the transition matrix. The dependent variable is: $y_t = 100ln(GDP_t/GDP_{t-1})$.

Additionally, we incorporate the structural break hypothesis by specifying the mean transition probability matrix $\mathbf{P^m}$ given by (2).

This results in an almost block-diagonal matrix, where the "second block" is an "absorbing state." Once the process enters this block, it remains there indefinitely. Within each regime, we expect one high mean and one low mean. This setup allows the model to accommodate potential structural breaks in the data and still identify effective recession episodes. Specifically, we anticipate two recession regimes: one in each block, corresponding to periods before and after the structural break.

Our approach is motivated by two key factors. First, as discussed in the data section, our sample was constructed using chained data from two sources. So, the structural break restriction can potentially address significant methodological changes. Second, we believe the Brazilian economy has undergone substantial transformations over the past four decades, particularly due to four major events: re-democratization, the adoption of the Real Plan and inflation stabilization, the adoption of inflation targeting, and the end of the fixed exchange rate regime.

The first column of the following table presents the results from the estimation of this model. For brevity, we refer to the MSComp(4,2)-ARMA(2,2) model as MSC(4,2)-ARMA(2,2) in the table. Several important insights can be drawn from these results. First, the estimated coefficients for the constant $\mu(s^m)$ suggest two expansion regimes (0 and 2) and two recession regimes (1 and 3), where regimes 0 and 1 occur before the structural break, and regimes 2 and 3 occur after. By computing the unconditional mean of the regimes, we find the following quarterly mean GDP growth rates: 0.65% for regime 0, -0.25% for regime 1, 1.54% for regime 2, and -0.17% for regime 3. Notably, the recession regimes appear quite similar in terms of mean GDP contraction, with the recessions after the break being slightly less intense. However, the conditional probabilities and MA coefficients indicate that recessions, before the break occurred, are more frequent but less persistent.

In contrast, the expansion periods are markedly different in terms of unconditional means, with the expansions after the break being nearly twice as intense as those before. This difference is driven by more favorable growth dynamics in regime 2, as suggested by the positive AR coefficients. Moreover, the conditional probabilities suggest that expansion periods are more persistent after the structural break.

Figure 2 displays the smoothed probabilities for the mean regimes. The structural break, or transition from Regime 1 to Regime 2, occurred in the second quarter of 1999. This timing has a sound economic interpretation, which we will explore in the next subsection. Figure 2 clearly shows distinct patterns for the recessions, with contractions before the break being less persistent but more frequent.

Figure 3 shows the smoothed probabilities for the variance regimes, where regime 0 corresponds to low variance and regime 1 corresponds to high variance. As seen in the table, the latter has a variance approximately seven times larger. Therefore, periods in regime 1 are characterized by high volatility in terms of GDP growth. From the transition probabilities, we observe that these periods are relatively rare and transient, mainly occurring in the early 1980s and 1990s. We found only two periods of sustained high variance in GDP growth.

Table 1 also presents the results of two alternative models. The second column displays the outcomes for a MSC(4,2)-ARMA(2,2) model with an unrestricted $\mathbf{P^m}$ matrix. This setup involves estimating more parameters and does not provide significant economic insights due to the lack of structural break restrictions. As a result, the four regimes lack a clear economic interpretation. For these reasons, we opted to show the model presented in the first column.

The third column reports the results of a more parsimonious specification MSC(2,2)-ARMA(2,2) model. While this model is simpler than our benchmark, residual diagnostics were unsatisfactory, as evidenced by the failure of the Portmanteau test. Furthermore, specification tests indicated that higher-order AR and MA models would be more appropriate, which we interpreted as a signal that more than two mean regimes were necessary to capture properly the underlying dynamics.

Figure 2: Smooth probabilities of the mean regimes

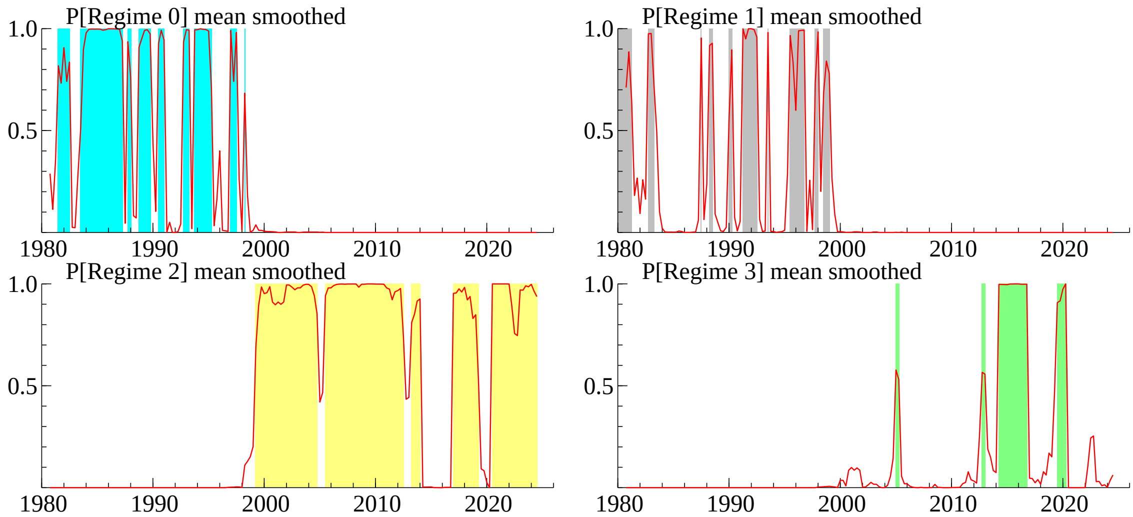


Figure 3: Smooth probabilities of the variance regimes

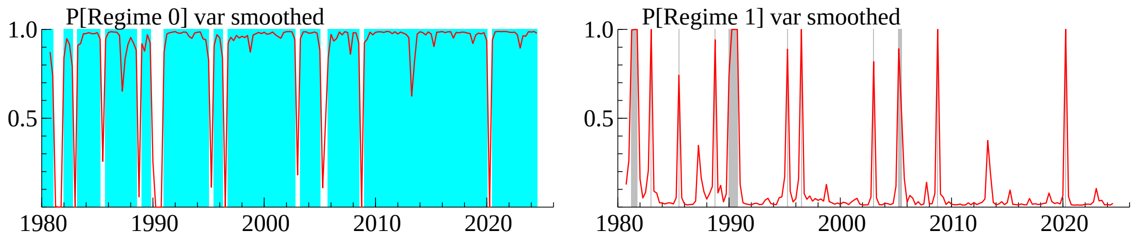


Figure 4: GDP Growth and fitted values

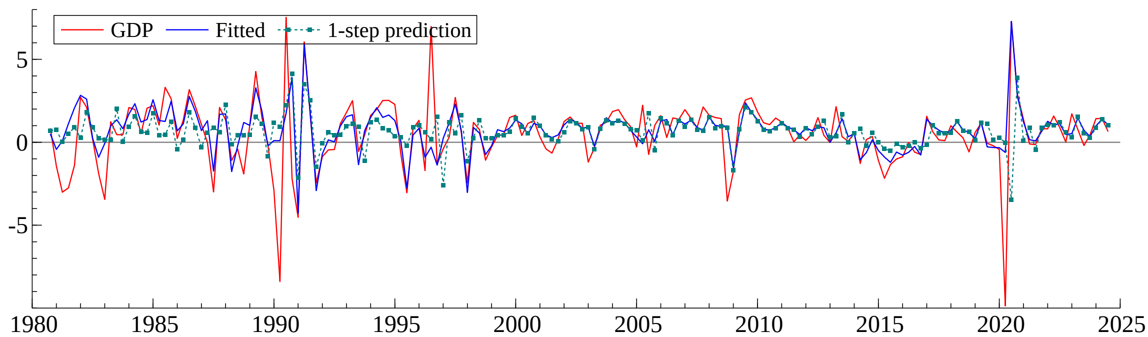


Table 1: Models estimates

	(1)-Structural Change		(2) - 4 regimes		(3) - Two Regimes	
	MSC(4,2)-ARMA(2,2)		MSC(4,2)-ARMA(2,2)		MSC(2,2)-ARMA(2,2)	
	Coef.	S.d	Coef.	S.d	Coef.	S.d
$\phi_1(0)$	-0.13	(0.14)	0.85	(0.13)	0.57	(0.12)
$\phi_1(1)$	-0.22	(0.07)	0.04	(0.17)	-0.38	(0.12)
$\phi_1(2)$	0.71	(0.17)	0.21	(0.10)		
$\phi_1(3)$	-0.44	(0.20)	0.99	(0.24)		
$\phi_2(0)$	-0.70	(0.08)	-0.24	(0.11)	-0.47	(0.10)
$\phi_2(1)$	-0.17	(0.09)	-0.21	(0.09)	-0.20	(0.08)
$\phi_2(2)$	-0.42	(0.10)	-0.36	(0.07)		
$\phi_2(3)$	-0.04	(0.13)	-0.10	(0.12)		
$\theta_1(0)$	0.30	(0.10)	-0.06	(0.11)	-0.21	(0.08)
$\theta_1(1)$	-0.01	(0.15)	0.49	(0.11)	0.79	(0.10)
$\theta_1(2)$	-0.19	(0.24)	-0.96	(0.11)		
$\theta_1(3)$	0.96	(0.08)	0.21	(0.16)		
$\theta_2(0)$	0.11	(0.08)	-0.16	(0.11)	0.14	(0.07)
$\theta_2(1)$	-0.33	(0.05)	-0.10	(0.10)	0.17	(0.09)
$\theta_2(2)$	0.07	(0.09)	0.22	(0.09)		
$\theta_2(3)$	0.23	(0.17)	0.16	(0.19)		
$\mu(0)$	1.20	(0.13)	1.03	(0.18)	1.18	(0.17)
$\mu(1)$	-0.35	(0.20)	-0.47	(0.14)	-0.17	(0.19)
$\mu(2)$	1.09	(0.27)	3.08	(0.29)		
$\mu(3)$	-0.26	(0.15)	-4.47	(0.35)		
$\sigma(0)$	0.60	(0.07)	0.52	(0.04)	0.85	(0.10)
$\sigma(1)$	4.23	(0.81)	1.72	(0.25)	4.20	(1.26)
$p_{m,0\|0}$	0.77	(0.07)	0.92	(0.03)	0.88	(0.05)
$p_{m,2\|0}$			0.04	(0.03)		
$p_{m,0\|1}$	0.34	(0.11)	0.18	(0.07)		
$p_{m,0\|2}$			0.07	(0.05)		
$p_{m,1\|2}$			0.08	(0.05)		
$p_{m,2\|2}$	0.95	(0.04)	0.83	(0.07)		
$p_{m,2\|3}$	0.19	(0.11)	0.45	(0.22)		
$p_{m,3\|3}$			0.17	(0.16)		
$p_{m,1\|1}$	0.63	(0.12)			0.78	(0.08)
$p_{v,0\|0}$	0.91	(0.03)	0.97	(0.02)	0.94	(0.04)
$p_{v,1\|1}$	0.37	(0.14)	0.92	(0.05)	0.46	(0.21)
Restricted $\mathbf{P^m}$	Yes		No		No	
Log-Likelihood	-293.64		-279.32		-311.11	
AIC	3.67		3.54		3.72	
SC	4.19		4.11		4.01	
Normality test	✓		✓		✓	
ARCH 1-1 test	✓		✓		✓	
Portmanteau	✓		✓		*	

Residual diagnostics: **rejects the null with 1%; *rejects the null with 5% and ✓does not reject with 5%. To make it shorter in the table, we call the MSComp(2,2)-ARMA(2,2) model as MSC(2,2)-ARMA(2,2).

4.1 Regime Classification and Characterization

Table 2 presents a detailed classification of the recession periods. It is divided into two parts: the first covers recessions before the structural break whereas the second addresses recessions after the structural break. We also attempt to link the classified recessions to known economic or political events that either marked these periods or may have contributed to the GDP contractions. Our view aligns with well-known events, such as the stabilization plans and the crisis triggered by the inconsistent macroeconomic policy during the Dilma Rousseff administration.[2].

There are some key points to note here. First, we could not establish a clear link to two of the recessions in the Table, which might be considered false positives given that the estimated smoothed probabilities are slightly above 50%. Second, the mean classification did not identify the 2008 financial crisis. It only appeared in the high-variance classification, as shown in Table 3. This suggests that the model views the 2008 event, from Brazil's perspective, as a period of turbulence rather than a recession. Indeed, the quarters before and after the crisis were marked by solid economic growth in Brazil, making this classification reasonable.

Third, our classification of the Covid recession begins in the third quarter of 2019, even though it is commonly understood that the crisis only began in the first quarter of 2020, with the peak occurring in the second quarter of 2020. However, this classification is not incorrect. The abrupt GDP contraction caused by the COVID-19 pandemic made traditional seasonal adjustments less effective. As a result, the GDP decline, after seasonal adjustment, began in the third quarter of 2019, which is why the model identifies this as the onset of the Covid crisis. It is also possible that the Brazilian economy was already experiencing a slowdown just before COVID, and the model may be capturing this trend.

Fourth, by cross-referencing Table 2 with Table 3, we can identify periods of recession and high volatility. These occurred in the early 1980s due to external crises and in the early 1990s during the stabilization plans.

Finally, as mentioned in the previous subsection, the estimation suggests that a structural break occurred in the second quarter of 1999. This is precisely the year when Brazil adopted the inflation-targeting regime and abandoned the fixed exchange rate, solidifying what has been known in Brazil as the "macroeconomic tripod" that is primary surplus, inflation targeting, and a floating exchange rate. The model indicates that this structural break marks the beginning of a sort of "Brazilian Great Moderation" in Brazil, which was only interrupted by the 2014 crisis. Indeed, recessions became much less frequent after the break. Another way to interpret this is by observing that high-variance periods were rare after the structural break.

[2] Officials nominated this policy as "New Economic Matrix" (NEM).

Table 2: Regime Classification - Low Mean/Recession Periods

Window	N. of Quarters.	Aver. Prob.	Event Description
Before Structural Change			
1980(4) - 1981(2)	3	0.747	Figueredo's External Adjustment Plan
1982(4) - 1983(2)	3	0.894	Latin America Debt Crisis
1987(3) - 1987(3)	1	0.954	Stabilization Plans (Cruzado/Bresser/Verão)
1988(2) - 1988(3)	2	0.923	Stabilization Plans (Cruzado/Bresser/Verão)
1990(1) - 1990(2)	2	0.723	Stabilization Plans (Collor I)
1991(2) - 1992(3)	6	0.984	Stabilization Plans (Collor II)
1993(3) - 1993(3)	1	0.981	Real stabilization Plan was elaborated and announced
1995(3) - 1996(4)	6	0.896	Tequila Crisis and Tight Monetary Policy
1997(4) - 1998(1)	2	0.859	Asian and Russian Crisis
1998(3) - 1999(1)	3	0.768	External Debt Crisis/FX Anchor End
Total: 29 quarters (16.48%) with average duration of 2.90 quarters.			
After Structural Change			
2005(1) - 2005(2)	2	0.554	Corruption Scandal
2012(4) - 2013(1)	2	0.562	Protests against Federal Government in major cities.
2014(2) - 2016(4)	11	0.998	"New Economic Matrix" Recession
2019(3) - 2020(2)	4	0.950	Covid
Total: 19 quarters (10.80%) with average duration of 4.75 quarters.			

The first part of the table refers to recession periods classified before the structural break, while the second part of the table refers to recession periods classfied after the structural break..

Table 3: Regime Classification - High Variance/Volatility Periods

Window	N. of Quarters.	Aver. Prob.	Event Description
1981(2) - 1981(4)	3	0.998	Figueredo government
1983(1) - 1983(1)	1	1.000	Soverign Debt Crisis
1985(3) - 1985(3)	1	0.742	No clear reason
1988(4) - 1988(4)	1	0.942	Stabilization Plans
1990(1) - 1990(4)	4	0.936	Collor Plan
1995(2) - 1995(2)	1	0.888	Real Plan
1996(3) - 1996(3)	1	1.000	Tequila Crisis and Tight Monetary Policy
2003(1) - 2003(1)	1	0.818	Lula's First Term Election
2005(2) - 2005(3)	2	0.712	Corruption Scandal
2008(4) - 2008(4)	1	1.000	Great Financial Crisis
2020(2) - 2020(2)	1	1.000	COVID-19 crisis
Total: 17 quarters (9.66%) with average duration of 1.55 quarters.			

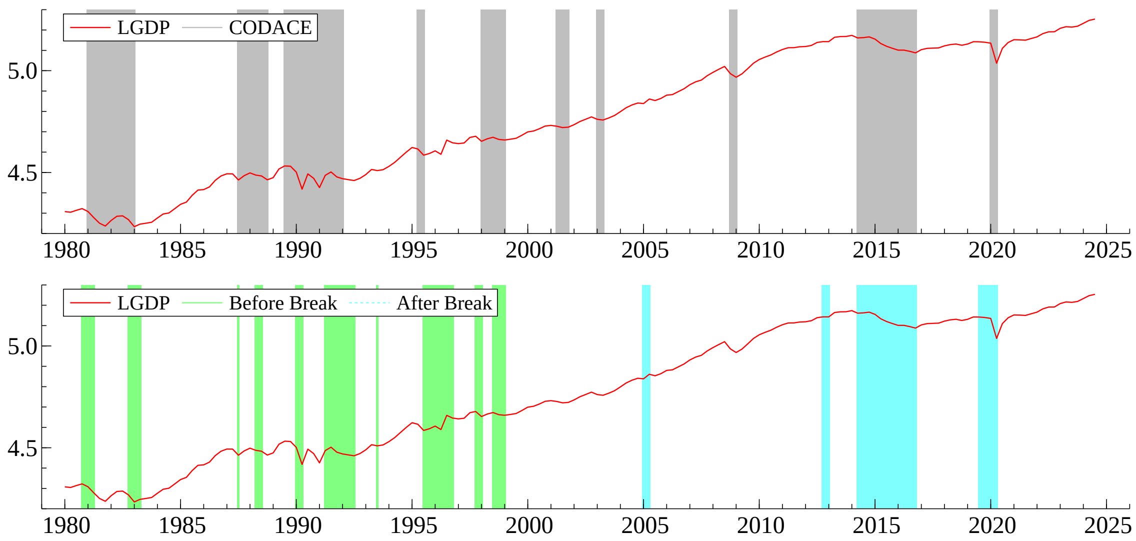
4.2 Comparing with Stylized Facts

This subsection compares our model's classification with CODACE's classification[3]. Figure 4 presents both classifications alongside the logarithm if GDP. The first point to note is that the two classifications are reasonably matched. The Pearson correlation is approximately 0.5, and the methods align in terms of regime classification in 78% of the quarters.

[3]COCADE is the Brazilian business cycle dating committee similar to NBER

As previously mentioned, the 2008 financial crisis stands out as a clear divergence. CODACE classifies it as a recession, while our model categorizes it as a turbulent period between phases of solid growth. Additionally, there are two other points of divergence: the recessions of 2001 and 2003, which are identified by CODACE but not by our model. The 2003 recession is classified by our model as a high-variance event. On the other hand, our model identifies two recessions that CODACE does not: 2005 and 2012. The former is likely related to a unveil corruption scandal that hit the Brazilian Executive and Legislative Federal authorities [4], while the latter might be a false positive. As highlighted before these points they are maybe false positives.

Figure 5: Comparision of Recession Classifications



4.3 Limitations and possible extensions

Threshold autoregressive models, as those pioneered and developed by Tong and Lim (1980), can serve as an alternative to these approaches. Additionally, other methodologies, such as those proposed by Bai and Perron (1998) and Doornik and Hendry (2015), can be employed to test for structural changes.

Further research could include formal tests to determine the optimal number of regimes and assess the presence of absorbing states. Another valuable extension would be applying the model to datasets from other countries.

5 Conclusion

We applied the Markov-Switching Mean–Variance Component Model from Doornik (2013) to Brazilian quarterly GDP data from 1980 to 2024 to study business cycles and to date recessions. In addition, we introduced a significant modification to the transition probabilities matrix, enabling the model to better account for potential structural breaks in the data while still effectively identifying recession episodes. To the best of our knowledge, this is the first application of this modified approach to Brazilian GDP.

Our analysis identified a structural break in the second quarter of 1999, coinciding with the adoption of the inflation-targeting regime and the end of the fixed exchange rate regime. We observed that recessions

[4]The scandal was named "Mensalão" by the press because some authorities and congressman received bribes every month.

before this break were not only more intense and frequent, but also less persistent. Furthermore, we found only two periods of sustained high variance in GDP growth. The independent switching of variance regimes, a key feature of the Doornik (2013) model, appears to have functioned more as a tool for managing outliers than for dating high-variance periods in this application.

Overall, we believe this setup offers a reasonable fit to the stylized facts of the Brazilian macroeconomy over the past four decades, aligning well with the CODACE classification. We also see the potential for improvement through a multivariate version of the model, though the availability of consistent macroeconomic data over such a long period could be challenging for Brazil or to look at higher frequency indicators.

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