Economics Letters 118 (2013) 265–268

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A Markov-switching model with component structure for US GNP

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ARTICLE INFO

Article history:
Received 4 October 2012
Received in revised form
25 October 2012
Accepted 29 October 2012
Available online 5 November 2012

Keywords:
Business cycles
Great moderation
Markov-switching models
US GNP

ABSTRACT

The two-regime Markov-switching model that James Hamilton estimated for US real GNP up to 1984 does not survive extension of the data set. To allow for the 'Great Moderation' we require a mean and variance regime that evolve separately. The Markov-switching component model is proposed as a way to avoid estimating a fragile four-regime model. The resulting model captures business cycles and structural change in the variance well.

1. Introduction

Hamilton (1989) introduced the Markov-Switching model for time-series data, which became a popular approach for time series that are subject to regimes, such as business cycles. He illustrated the model with an application to US real GNP, showing how well it captures the NBER classification.

Unfortunately, extending the data beyond 1984 causes a breakdown of the Hamilton model. The reason is that by about 1985 we enter a period of relatively stable GNP growth, now often called the 'Great Moderation'. Regime-switching models lost some of their appeal during that period. Of course, the financial crisis put an abrupt halt to this in 2007.

There are two questions of interest. First, can we extend the Markov-switching model beyond 1984? The second is whether the current economic crisis is happening during a period of low volatility, or has the Great Moderation ended with the financial crisis?

To allow for fairly frequent recessions, as well as long periods of stable variance, requires a four regime Markov-switching model: recessions and expansions before as well as during the Great Moderation. However, such a model has four means, four variances and 12 free transition probabilities to estimate, which may be too much flexibility. Instead, we propose a model that has two mean and two variance regimes that can switch as separate components. This model corresponds better to the stylized facts of post-war economic history.

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doi:10.1016/j.econlet.2012.10.035

2. The Markov-switching model

In the Markov-switching model, the unobserved random variable S_t follows a Markov chain, defined by transition probabilities between the S regimes:

$$p_{i|j} = \mathsf{P}\left[S_{t+1} = i | S_t = j\right], \quad i, j = 0, \ldots, S-1.$$

So the probability of moving from state j in one period to state i in the next only depends on the previous state.

A simple example of a Markov-switching autoregression with a single lag is:

$$y_t - \mu(S_t) = \rho\left[y_{t-1} - \mu(S_{t-1})\right] + \epsilon_t, \quad \epsilon_t \sim \mathsf{IIN}[0, \sigma^2].$$

More generally, we write for p lags and S regimes: MS(S)-AR(p). A related model is the Markov-switching dynamic regression (MS-DR):

$$y_t = \nu(S_t) + \alpha y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathsf{IIN}[0, \sigma^2].$$

These are not identical in the Markov-switching setting, because the intercept in the MS-AR(1) model depends on the last two regimes. The variance can also be made regime dependent: $\sigma^2(S_t)$.

The likelihood of the Markov-switching model can be evaluated efficiently using the filtering procedure of Hamilton (1990), while a smoothing algorithm was derived by Kim (1994); see Hamilton (1994, Chapter 22) and Krolzig (1997, Chapter 5) for more details. This enables numerical maximization of the log-likelihood as a function of the parameters in the mean and variance, as well as the transition probabilities $p_{i|j}$. The probabilities are subject to the constraint that they lie between 0 and 1 and sum to unity: $\sum_{i=0}^{S-1} p_{i|j} = 1$. Let $t = 1, \ldots, T$ denote the estimation sample,

$\mathbf{Y}_t^1 = (y_t, \dots, y_t)'$, and θ the vector of parameters. The resulting filtered regime probabilities $\mathsf{P}(S_t|\mathbf{Y}_t^1, \widehat{\theta})$ and smoothed regime probabilities $\mathsf{P}(S_t|\mathbf{Y}_T^1, \widehat{\theta})$ are time varying.

The MS-AR model can be written as a standard Markov-switching model by allowing for all possible intercepts, resulting in N^{p+1} states, again, see Hamilton (1994, Chapter 22). The four regime equivalent of the Hamilton model of US GNP, i.e. the MS(4)-AR(4), has 1024 states!

3. A Markov-switching mean–variance component model

We introduce a separate mean regime S_t^m and variance regime S_t^v. These evolve independently of each other, so have their own transition matrices $\mathbf{P}_m = p_{i|j}^m$ and $\mathbf{P}_v = p_{i|j}^v$ respectively.

For example, a component model with two regimes for both the mean and the variance:

	$S_t^m = 0$	$S_t^m = 1$		$S_t^v = 0$	$S_t^v = 1$
$S_{t+1}^m = 0$:	$p_{0\|0}^m$	$p_{0\|1}^m$	$S_{t+1}^v = 0$	$p_{0\|0}^v$	$p_{0\|1}^v$
$S_{t+1}^m = 1$:	$p_{1\|0}^m$	$p_{1\|1}^m$	$S_{t+1}^v = 1$	$p_{1\|0}^v$	$p_{1\|1}^v$

corresponds to a restricted four regime model:

	$S_t^v = 0$		$S_t^v = 1$	
	$S_t^m = 0$	$S_t^m = 1$	$S_t^m = 0$	$S_t^m = 1$
	$S_t = 0$	$S_t = 1$	$S_t = 2$	$S_t = 3$
$S_{t+1} = 0$ $S_{t+1} = 1$	$p_{0\|0}^v \mathbf{P}^m$		$p_{0\|1}^v \mathbf{P}^m$	
$S_{t+1} = 2$ $S_{t+1} = 3$	$p_{1\|0}^v \mathbf{P}^m$		$p_{1\|1}^v \mathbf{P}^m$	

Instead of four means and variances there are only two of each. Moreover, the transition matrix is restricted to:

$$\mathbf{P} = \mathbf{P}^v \otimes \mathbf{P}^m, \tag{1}$$

which has only four free probabilities, rather than 12 for the unrestricted four-regime model. These models are denoted by MSComp(S^m, S^v) for S^m mean regimes and S^v variance regimes, which need not be the same. The MSComp model can be combined with the AR or DR specification for the dynamics.

Estimation is straightforward when a numerical maximization routine is used, because the model fits in the standard Markov-switching framework with only minor adjustments. We adopt a non-linear programming approach that is based on the feasible SQP algorithm of Lawrence and Tits (2001). The inequality constraints on the transition probabilities are expressed as part of the maximization problem. The mean and variance constraints imposed by the component model are simply substituted out, while the appropriate constraints on $\mathbf{P}^v$ and $\mathbf{P}^m$ automatically entail that $\mathbf{P}$ satisfies the necessary constraints.[1]

4. MS(2)-AR(4) model of US GNP

The data is quarterly US real Gross National Product in billions of chained 2005 dollars available for the period 1947(1)–2012(2). This is seasonally adjusted and available from FRED as series GNPC96.[2]

Table 1
Estimates of MS(2)-AR(4) model using new and old GNP data. Standard errors in parentheses.

	Old data		New data		New data	
	1952(2)–1984(4)		1952(2)–1984(4)		1952(2)–2012(2)	
ϕ_1	0.01	(0.12)	0.10	(0.14)	0.38	(0.08)
ϕ_2	−0.06	(0.14)	−0.05	(0.13)	0.25	(0.08)
ϕ_3	−0.25	(0.11)	−0.15	(0.11)	−0.20	(0.08)
ϕ_4	−0.21	(0.11)	−0.17	(0.11)	−0.09	(0.07)
$\mu(0)$	1.16	(0.07)	1.30	(0.12)	0.86	(0.08)
$\mu(1)$	−0.36	(0.26)	−0.18	(0.26)	−1.30	(0.29)
σ	0.76		0.86		0.73	
$p_{0\|0}$	0.90		0.89		0.96	
$p_{1\|1}$	0.75		0.75		0.28	
Log-likelihood	−181.26		−191.07		−300.60	

First we try to replicate the Hamilton (1989) MS(2)-AR(4) model for the period 1952(2)–1984(4). The dependent variable is

$$y_t = 100\Delta \log(\mathrm{GNP}_t).$$

Because the data has been rebased and revised, we cannot expect to get exactly the same results as Hamilton (1989, Table 1)[3] who uses a much older generation of the data.

Table 1 compares the estimates of the MS(2)-AR(4) model[4]

$$y_t - \mu(S_t) = \phi_1 [y_{t-1} - \mu(S_{t-1})] + \dots + \phi_4 [y_{t-4} - \mu(S_{t-4})] + \epsilon_t, \quad \epsilon_t \sim \mathsf{IIN}[0, \sigma^2],$$

with $S_t \in \{0, 1\}$, using the original data and the most recent data. For the identical sample period, the value of $\widehat{\sigma}$ has risen, reflecting a poorer fit. The autoregressive coefficients were jointly significant on a Wald test at 3% on the old data, but this has changed to 11% for the new data. The means have changed somewhat, but the transition probabilities are essentially the same.

Estimating this model on the full data is unsatisfactory. The smoothed residuals are non-normal and heteroscedastic. Only ten observations are classified into the low-mean regime, which is now very low. The probability of staying there is also very low at 0.28.

5. Markov-switching component model of US GNP

We compare two models. The first is the four-regime MS(4)-DR(2) with switching variance and autoregressive parameters:

$$y_t = \rho_1(S_t) y_{t-1} + \rho_2(S_t) y_{t-2} + \nu(S_t) + \epsilon_t, \quad \epsilon_t \sim \mathsf{IIN}[0, \sigma^2(S_t)].$$

The second model is the Markov-switching component model, with 3 regimes for the mean and two for the variance, MSComp(3, 2), and two regime-dependent lags of the dependent variable:

$$y_t = \rho_1(S_{m,t}) y_{t-1} + \rho_2(S_{m,t}) y_{t-2} + \nu(S_{m,t}) + \epsilon_t, \quad \epsilon_t \sim \mathsf{IIN}[0, \sigma^2(S_{v,t})],$$

with $S_{m,t} \in \{0, 1, 2\}$, $S_{v,t} \in \{0, 1\}$, and the Kronecker structure (1) for the transition matrix.

1 Unconstrained maximization after parameter transformations is also feasible. The EM algorithm, however, does not provide the flexibility to easily add the required constraints that are implied by the component model.

2 Federal Reserve Economic Data, supplied by the St. Louis Fed.

3 Also see Hamilton (1994, Table 22.1, p. 698), which reports the correct mean in the expansion regime. These results are replicated when using the original data set, provided the ergodic probabilities are used to initialize the filter, as was done for all results in Table 1. In the remainder we always use uniform probabilities for this, noting that the impact of this choice is very small.

4 All results are obtained using Ox 7, see Doornik (2012), and implemented in OxMetrics, Doornik and Hendry (2012).

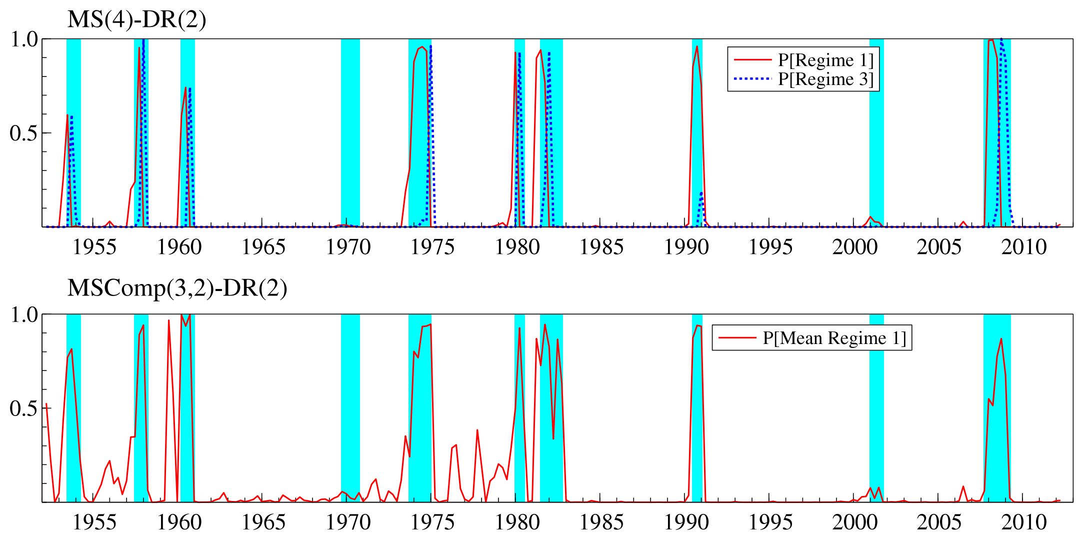

Fig. 1. Top: Estimated smoothed probabilities to be in one of the two low-mean regimes in the MS(4)-DR(2) model. Bottom: Probabilities to be in the low-mean regime for the component model. Both with shading of recessions according to the NBER.

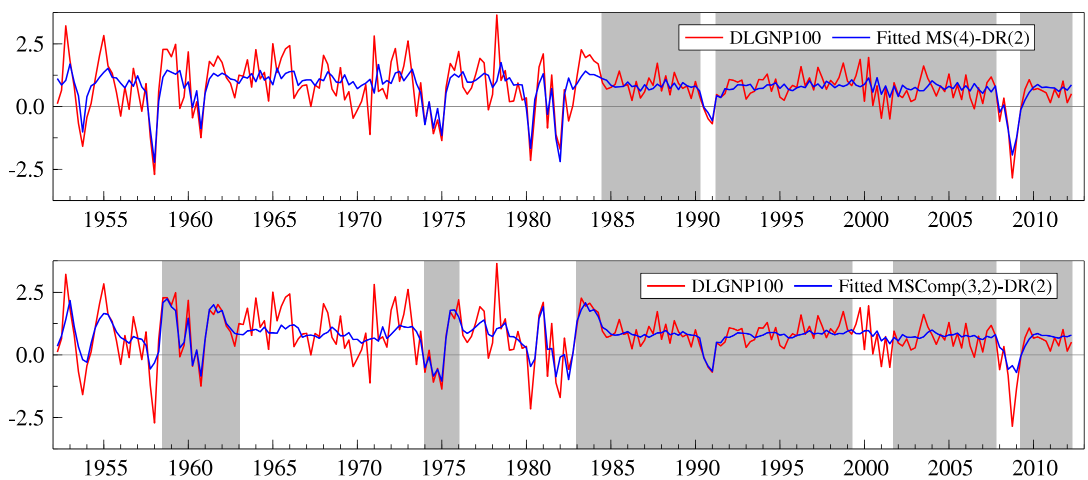

Fig. 2. Top: Actual and fitted values for the MS(4)-DR(2) model, together with shading of regime 0 (low variance and expansion). Bottom: Actual and fitted values for the MSComp(3, 2)-DR(2) model, together with shading of the low-variance regime.

Table 2
Estimates of MS(4)-DR(2) and MSComp(3, 2)-DR(2) models with regime-dependent autoregressive parameters and variances. Estimation over 1952(2)–2012(2); standard errors in parentheses.

	MS(4)-DR(2)		MSComp(3, 2)-DR(2)	
$\rho_1(0)$	−0.03	(0.09)	0.07	(0.08)
$\rho_1(1)$	−0.34	(0.16)	0.05	(0.12)
$\rho_1(2)$	0.26	(0.09)	0.47	(0.10)
$\rho_1(3)$	0.03	(0.22)	–	
$\rho_2(0)$	0.30	(0.07)	0.20	(0.07)
$\rho_2(1)$	0.44	(0.13)	0.39	(0.11)
$\rho_2(2)$	−0.05	(0.09)	−0.19	(0.09)
$\rho_2(3)$	−0.49	(0.25)	–	
$\nu(0)$	0.58	(0.09)	0.59	(0.08)
$\nu(1)$	−0.49	(0.17)	−0.67	(0.14)
$\nu(2)$	0.88	(0.14)	1.28	(0.15)
$\nu(3)$	−1.75	(0.34)	–	
$\sigma(0)$	0.42		0.36	
$\sigma(1)$	0.41		0.95	
$\sigma(2)$	0.91		–	
$\sigma(3)$	0.54		–	
Log-likelihood	−268.26		−275.40	
No. parameters	23		18	

Table 2 reports the estimated coefficients for both models. The parameter count excludes transition probabilities that were estimated at zero (5 in the first case and 1 in the second).

The interpretation of the MS(4)-DR(2) model is somewhat difficult with so many regimes: there are two recession regimes (numbered 1 and 3 here) and one high-variance regime. The top panel in Fig. 1 plots the probabilities of being in the low-mean regimes. There is correspondence to the NBER classification of recessions, which is indicated with the shaded areas.

The last two columns of Table 2 show the estimated coefficients of the model with a mean–variance component structure, MSComp(3, 2)-DR(2). This has fewer coefficients, but a fairly similar log-likelihood. Note that the model is nested in MS(6)-DR(2), but not in MS(4)-DR(2). Now the mean regimes are independent of the variance regimes, and fewer are needed.

The MSComp(3, 2)-DR(2) component model is easier to interpret. There is a low variance and a high variance regime. In addition, there is a recessionary mean and a normal mean. The higher growth regime of the component model always occurs after recessions prior to 1984—it has not occurred since.

The bottom panel of Fig. 1 shows the smoothed probabilities to be in the recession regime for the component model, together with the NBER classification. There is a close correspondence, but again, the recessions of 1970 and 2001 are not found in the current GNP data. The bottom panel of Fig. 2 shows the actual and fitted values, together with shading that indicates the low variance regime. This suggests that the Great Moderation is still ongoing, but that there were two interruptions since 1982, the second of which is also a clear recession. It is difficult to distinguish these events, but, from a statistical perspective, it may also be harder to get out of recession when volatility is low.

6. Conclusion

We introduced the two-component version of the Markov-switching model, allowing the mean and the variance to evolve independently. This is a restricted version of a much larger model, avoiding the problems that beset Markov-switching models with many regimes. The component structure also matches the stylized facts of the post-war macro-economy, namely intermittent recessions and a structural change in the variance, dated at the second quarter of 1982 by our estimates.

The component model provides a satisfactory description of quarterly US real GNP, confirming some of the Hamilton (1989) results for the extended sample. Recession detection was not as pronounced as we hoped, but a multivariate version of the Markov-switching component model may improve this. Surprisingly, the estimates suggest that we are still in a low-variance regime that was interrupted twice, a higher variance in the quarterly period 1999(3)–2001(3), and higher variance with recession in 2008(1)–2009(1).

Markov-switching models with three or more regimes often have too much flexibility. Some regimes may pick out just a few observations, and, relatedly, the likelihood will have several local modes. Imposing structure on the matrix of transition probabilities, as done in the Markov-switching component model, alleviates this. The Markov-switching multifractal volatility model of Calvet and Fisher (2004) imposes structure in a different way. That model can also be combined with a Markov switching model for the mean through the proposed component structure.

Acknowledgments

This research was supported in part by grants from the Open Society Foundations and the Oxford Martin School. The author thanks David Hendry for providing helpful comments.

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