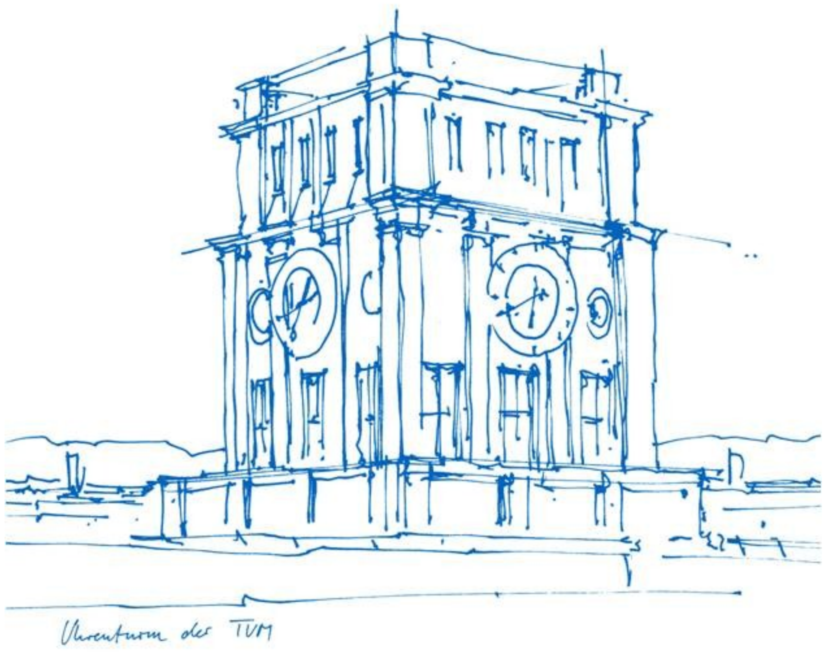
Modeling and Control of Legged Robots

SS 2024

L6: Stability and Balance



Dr.-Ing. J. Rogelio Guadarrama Olvera

Technical University of Munich

School of Computation, Information and Technology

Chair of Cognitive Systems

Munich 11. June 2023

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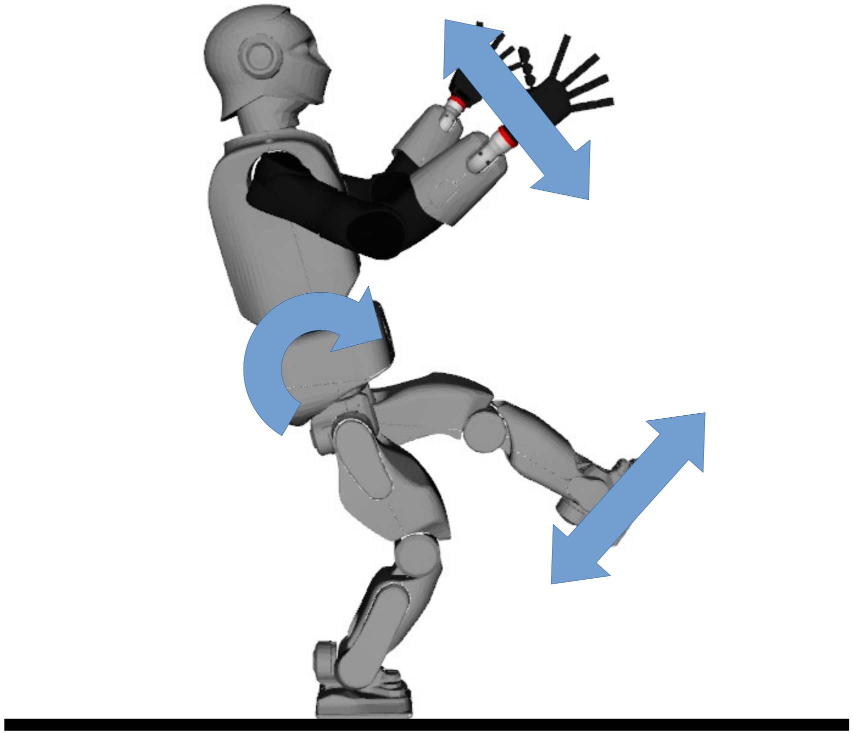

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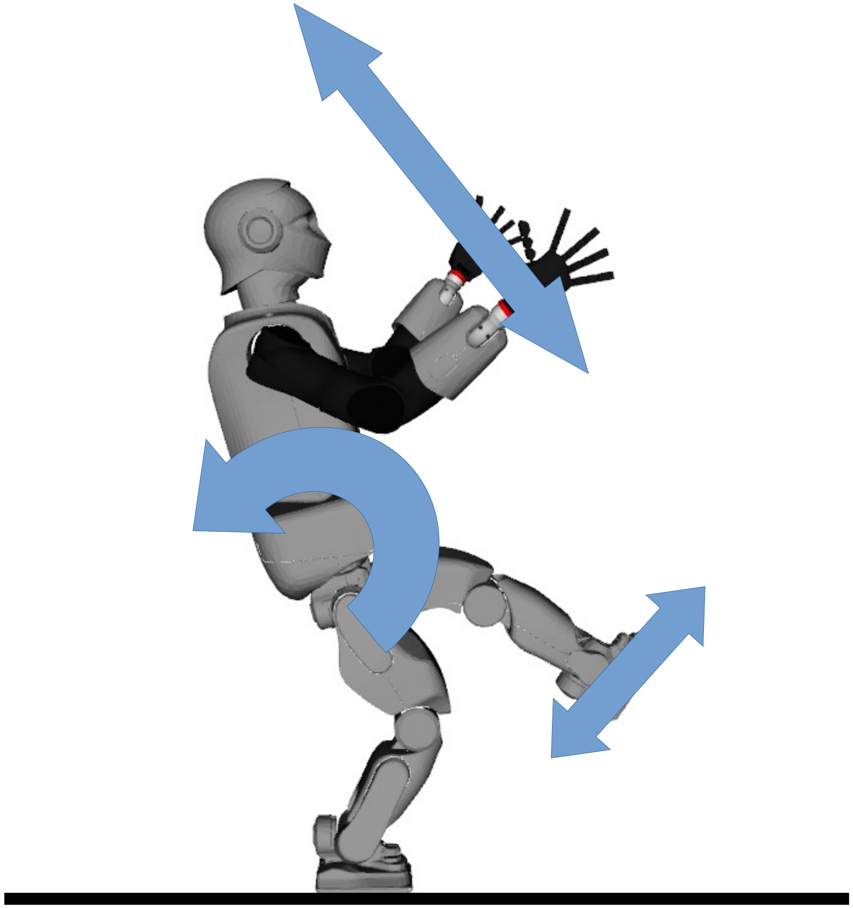

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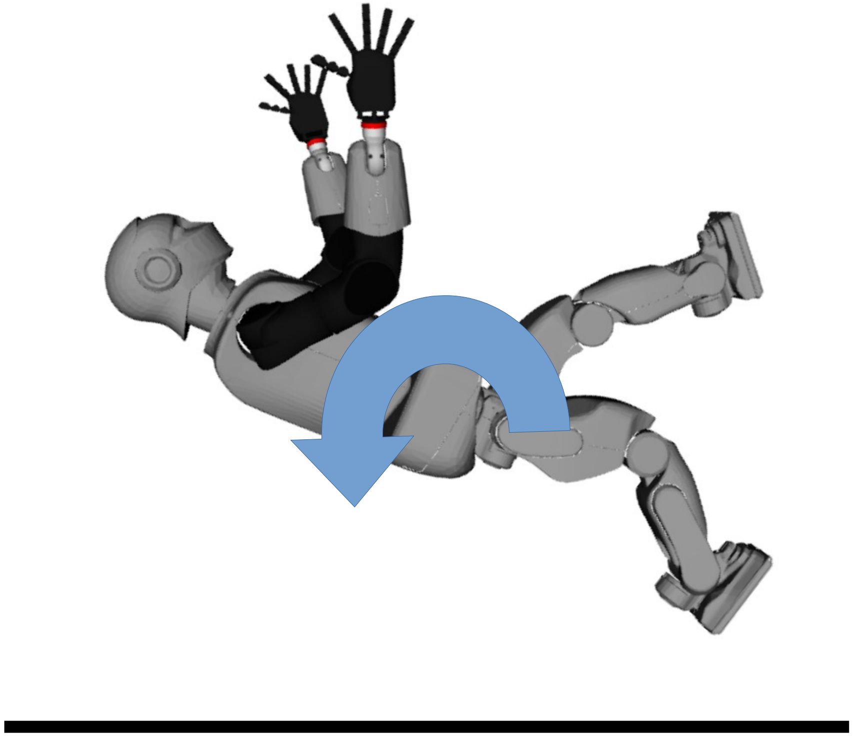

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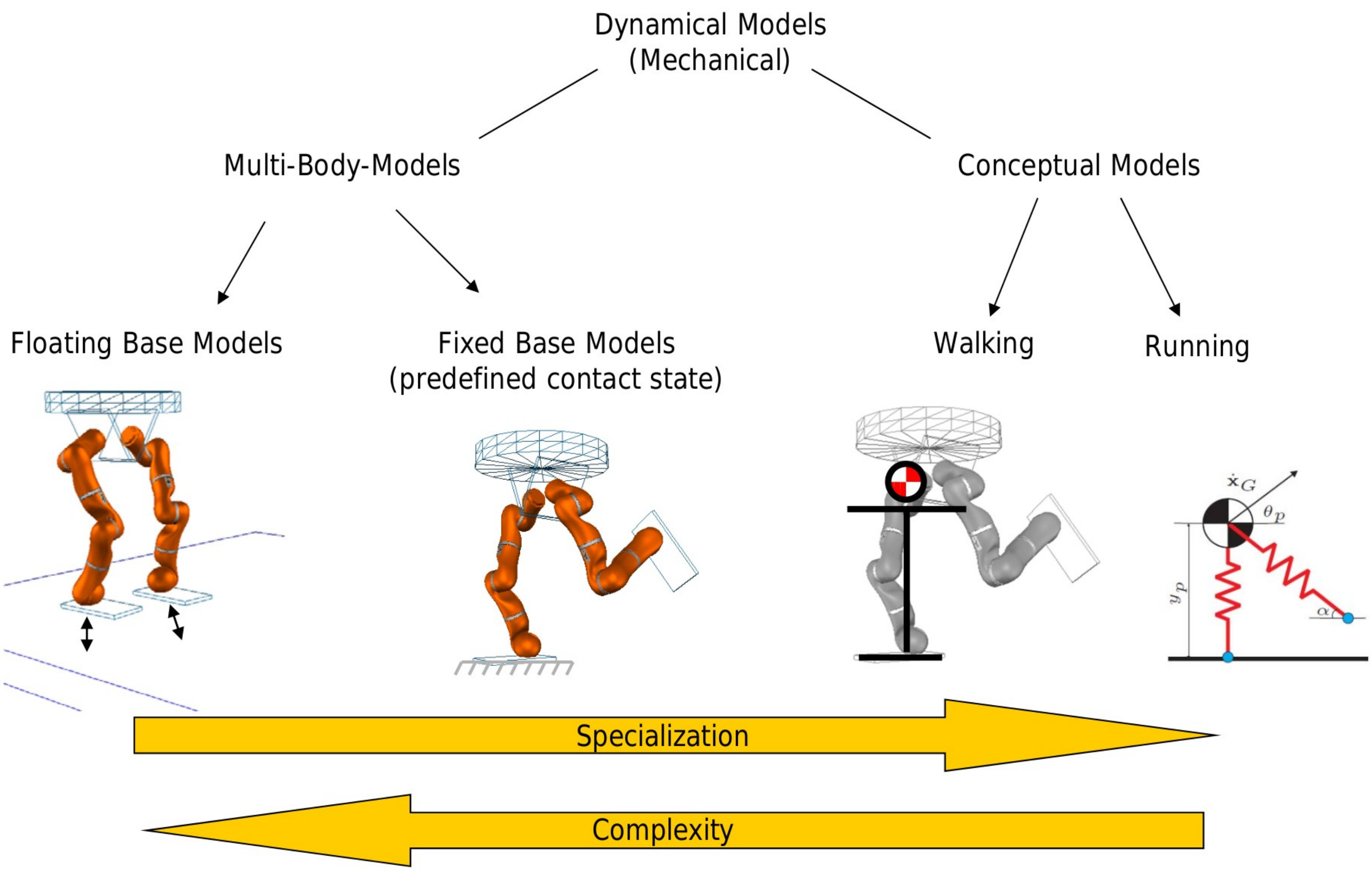

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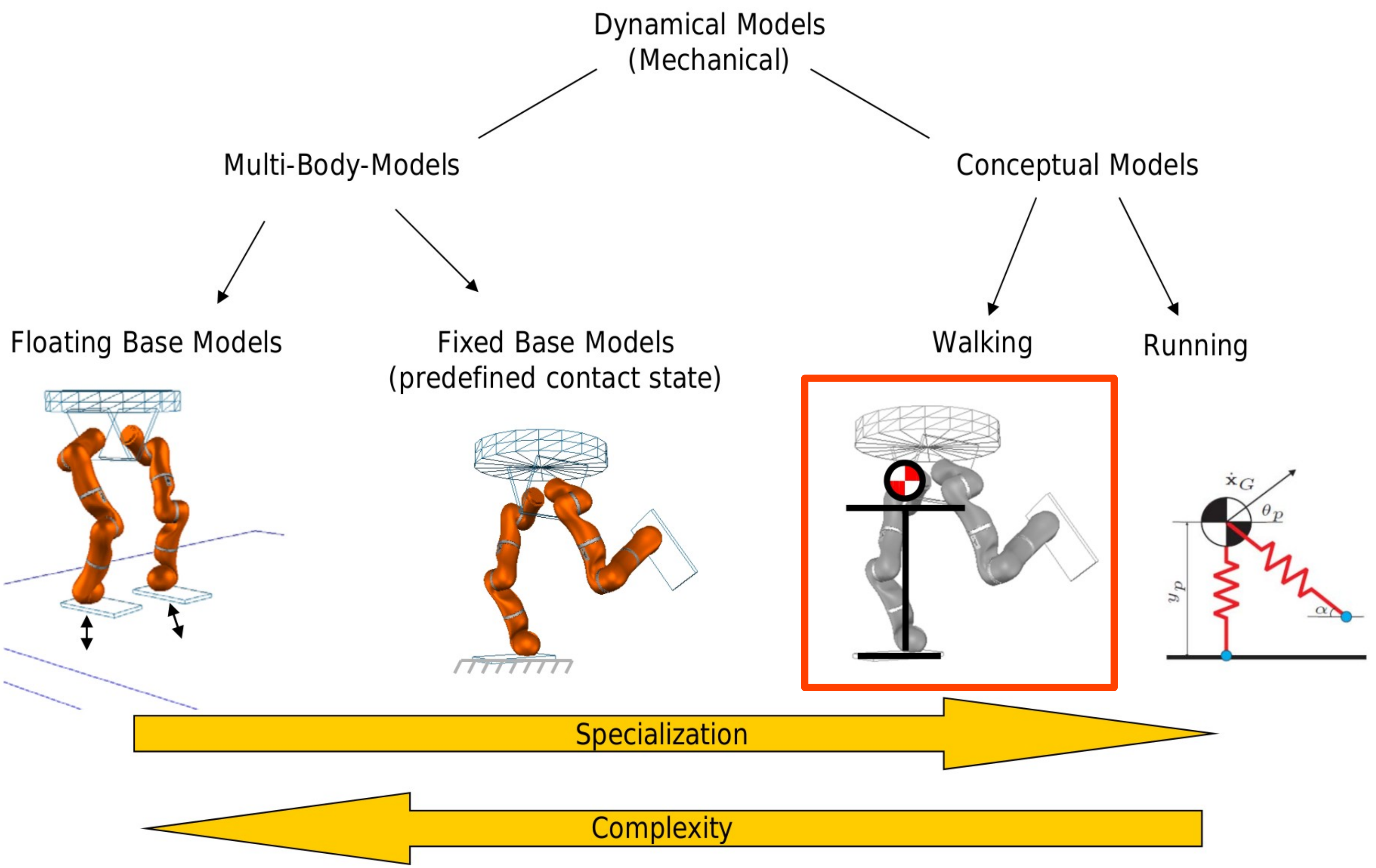

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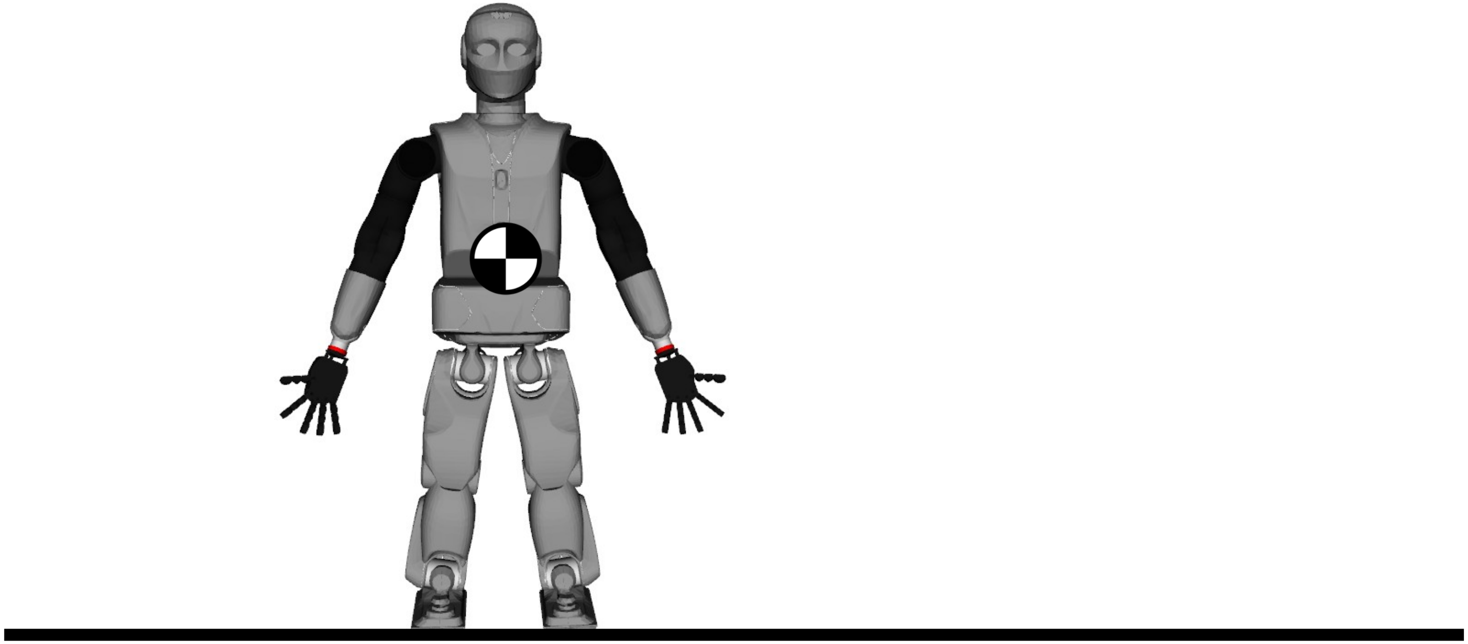

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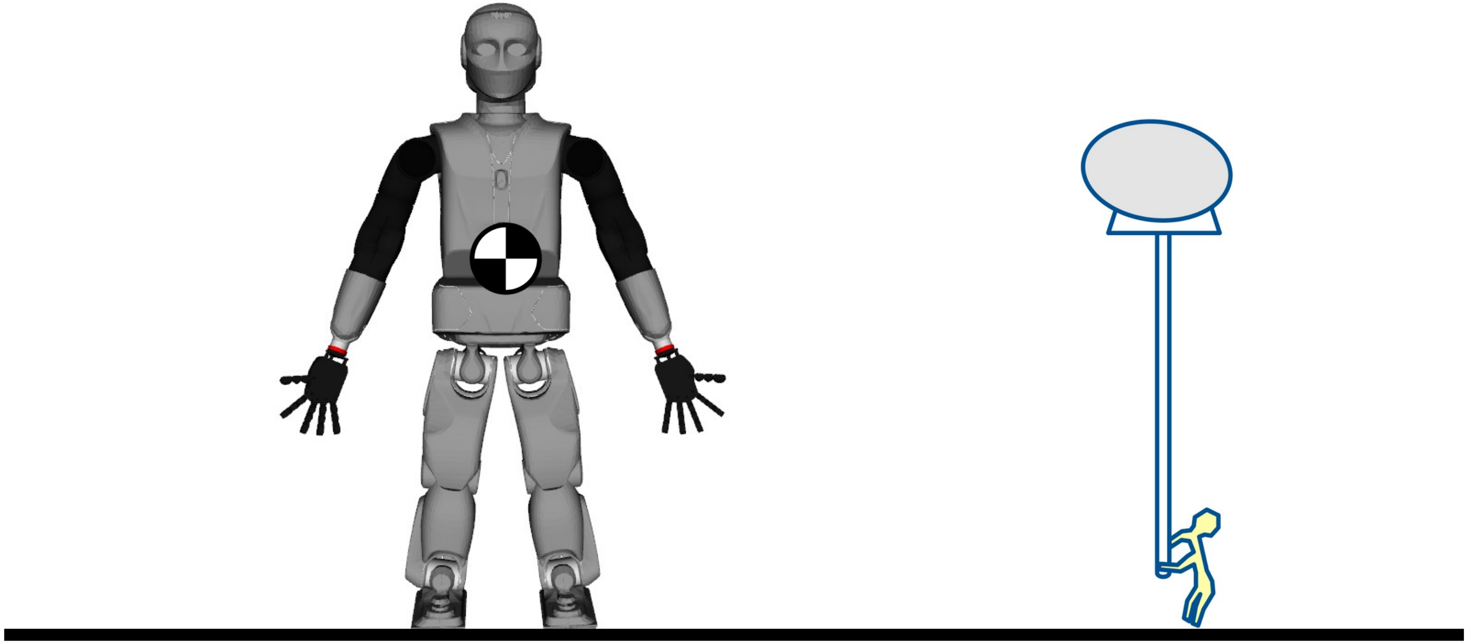

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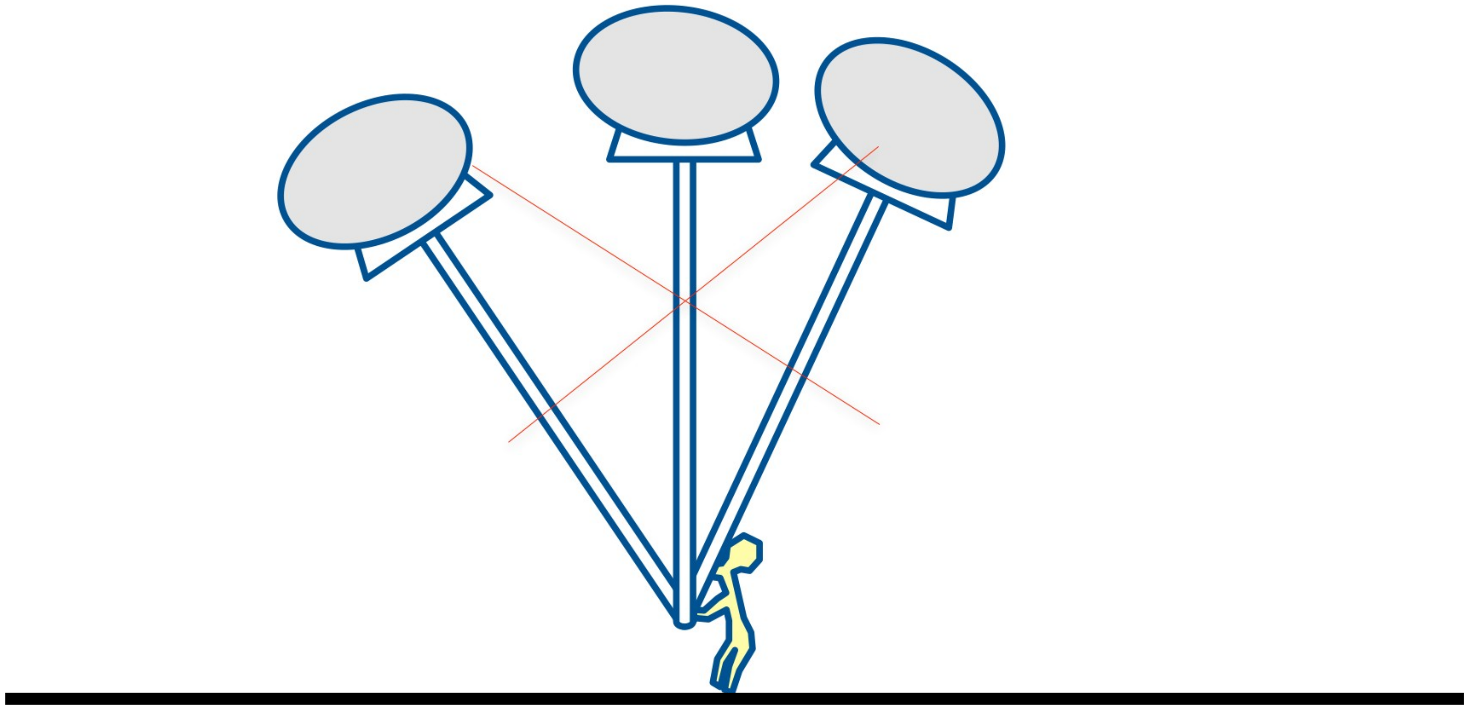

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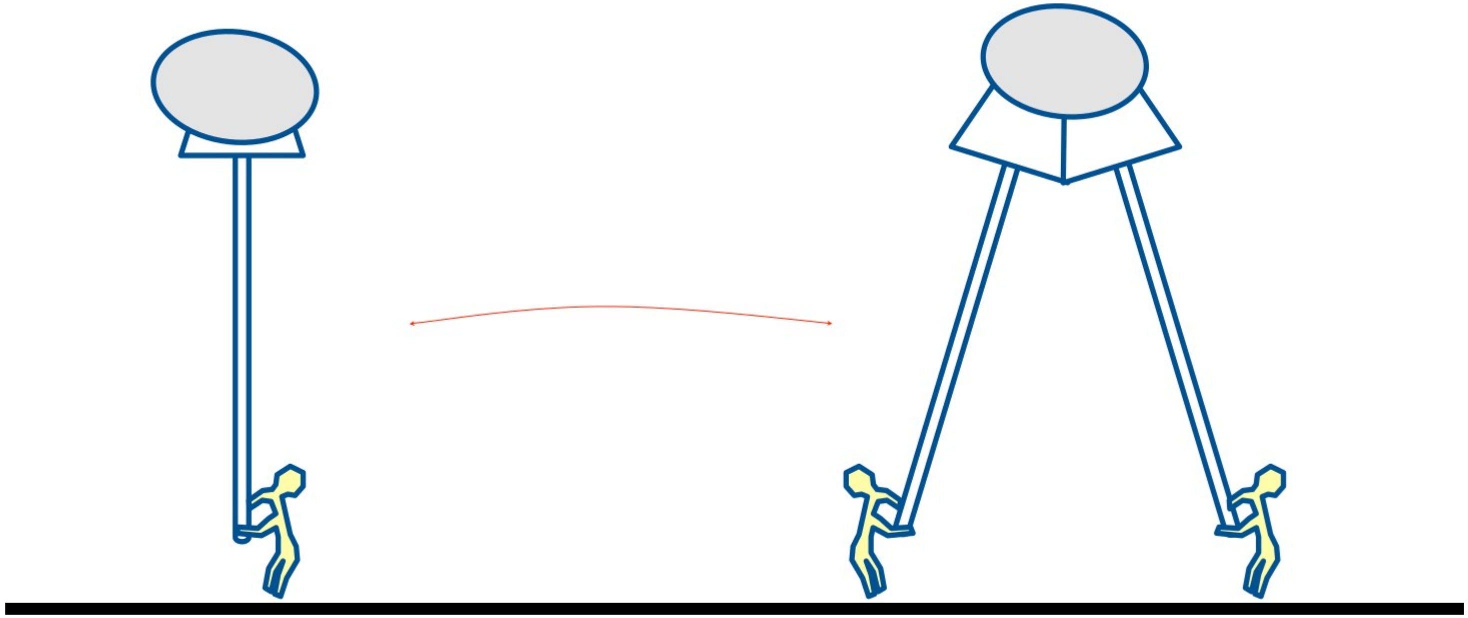
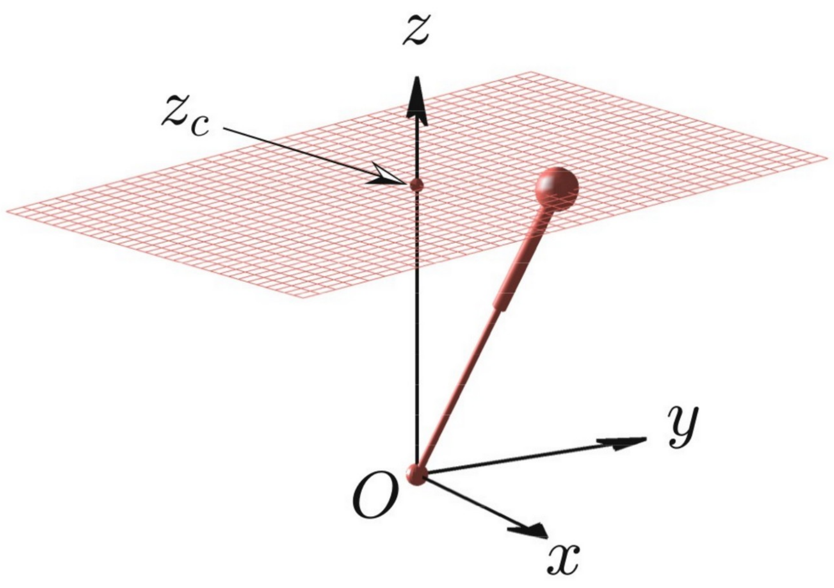


Introduction

Walking as a Linear Inverted Pendulum (LIPM)

- The most basic model for walking.
- Constant hip height.
- Natural frequency depending only on the height.



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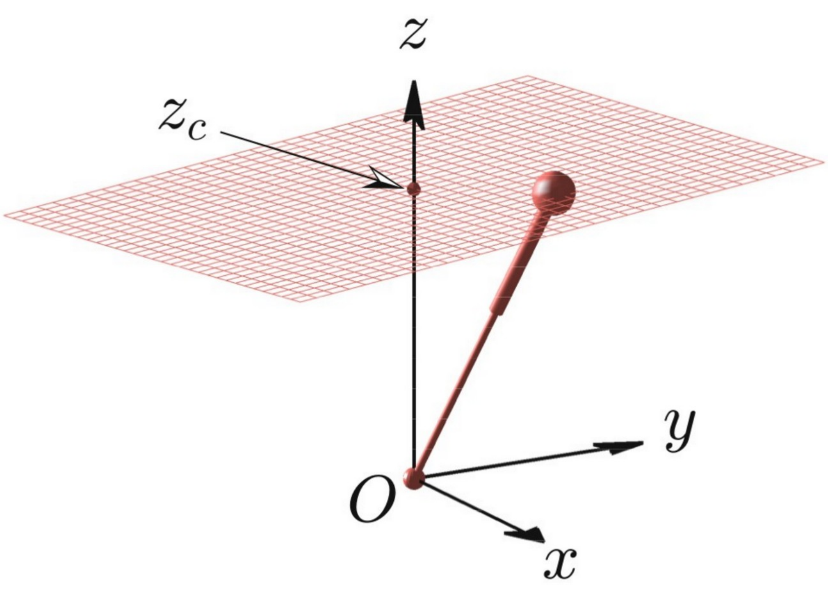
$$\ddot{\theta} = \frac{g}{zc} sin(\theta)$$



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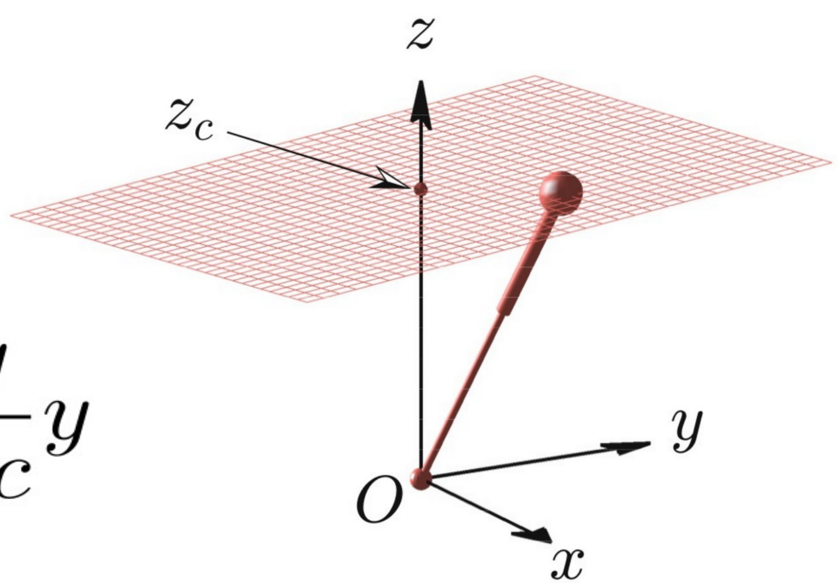
$$\ddot{x} = \frac{g}{zc} x \qquad \ddot{y} = \frac{g}{zc} y$$



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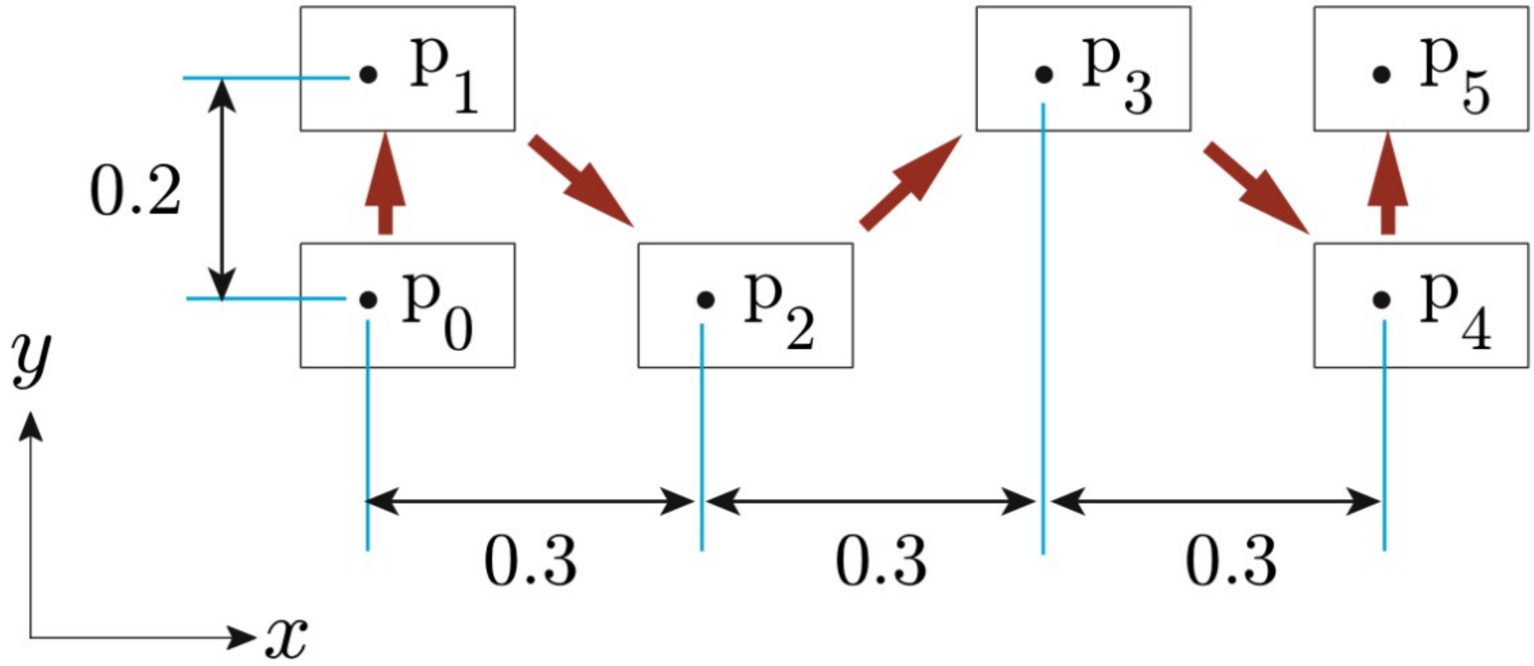

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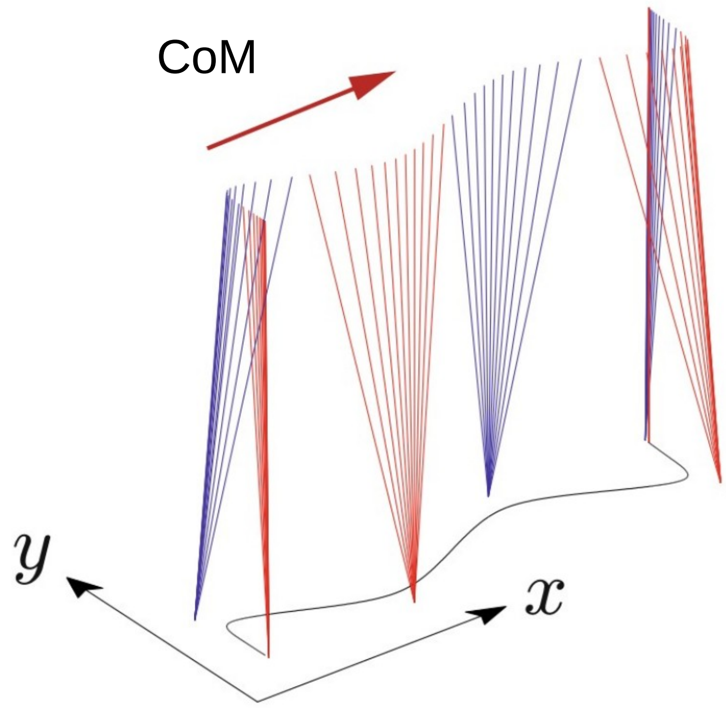

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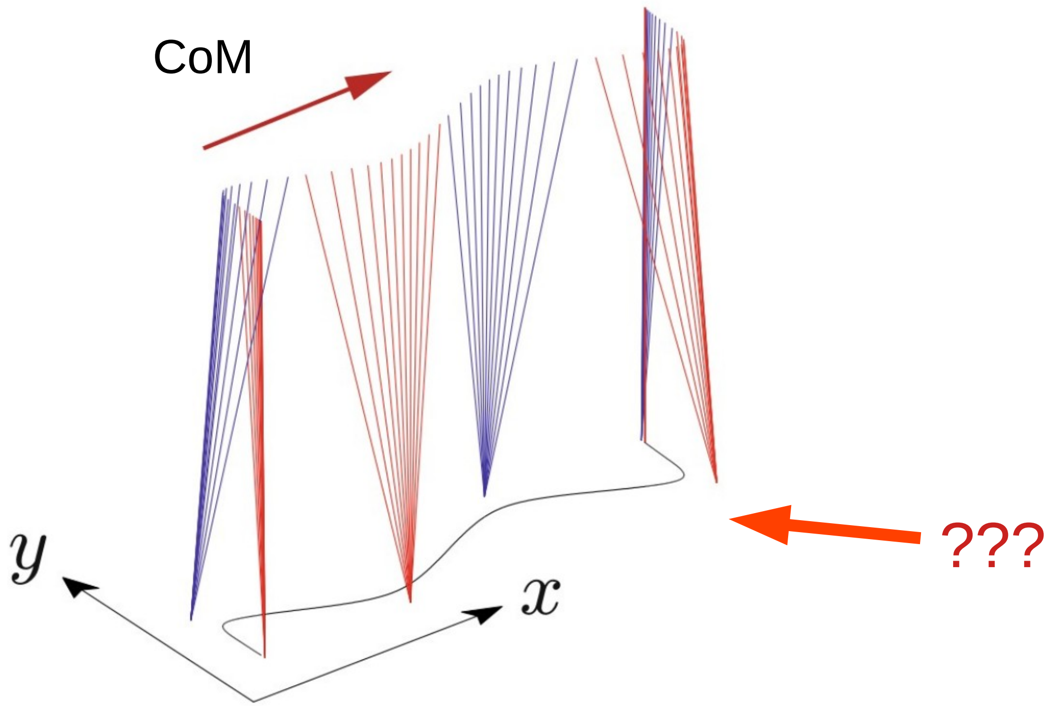

Zero Moment Point (ZMP)

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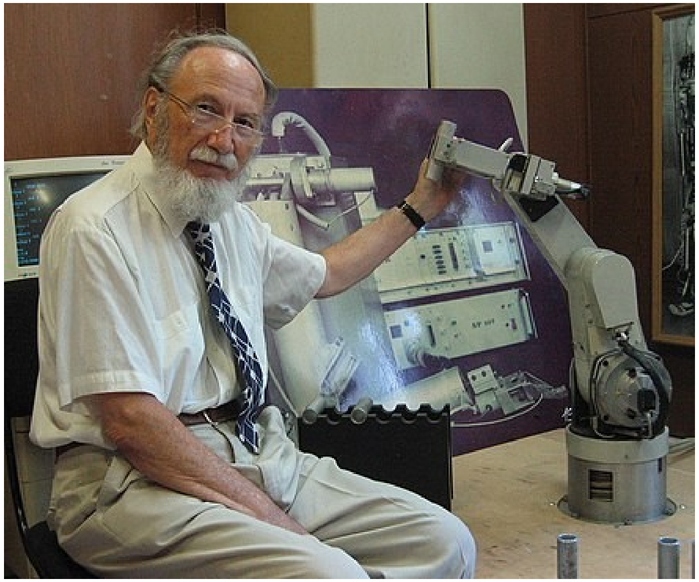

Miomir Vukobratović 1931 - 2012

Vukobratović, Miomir / Stepanenko, J.
On the stability of anthropomorphic systems, 1972

Mathematical biosciences , Vol. 15, No. 1-2
Elsevier p. 1-37

Zero Moment Point (ZMP)

Point where the ground reaction moment is 0.

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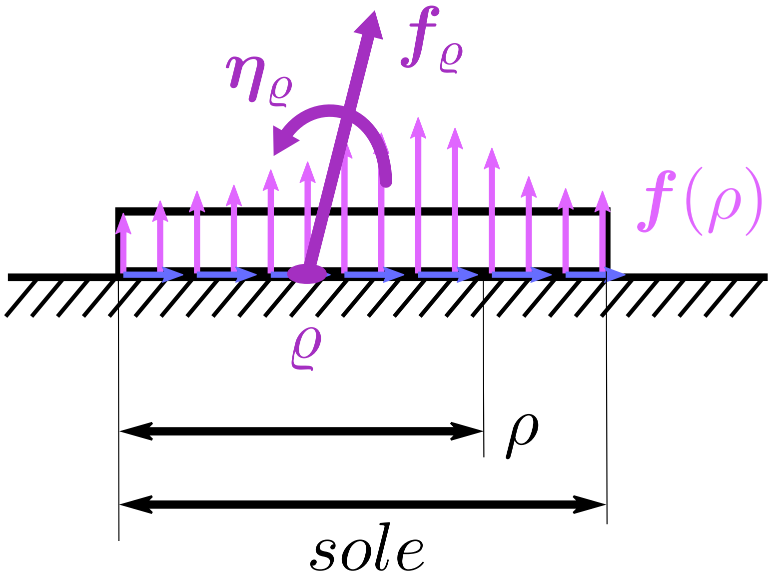

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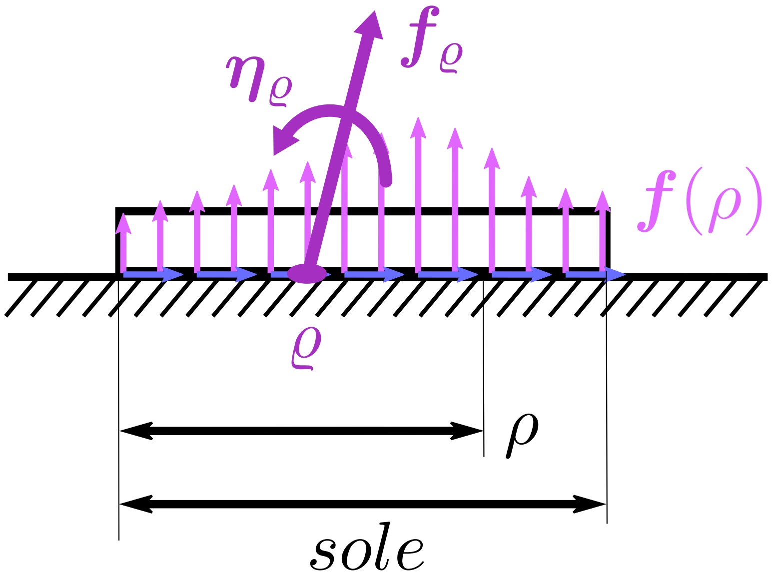

$$\boldsymbol{f}_{\varrho} = \int_{sole} \boldsymbol{f}(\rho)\mathrm{d}\rho$$

$$\boldsymbol{\eta}_{\varrho} = \int_{sole} (\rho - \varrho)\boldsymbol{f}(\rho)\mathrm{d}\rho$$

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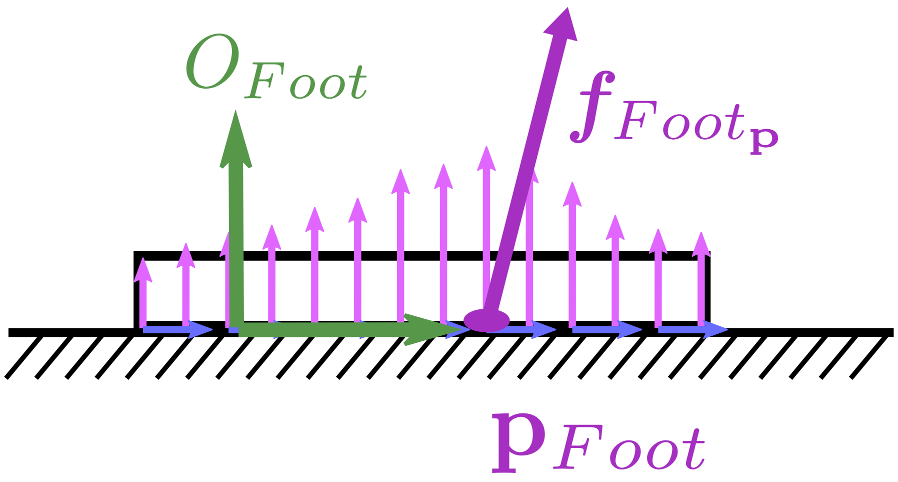

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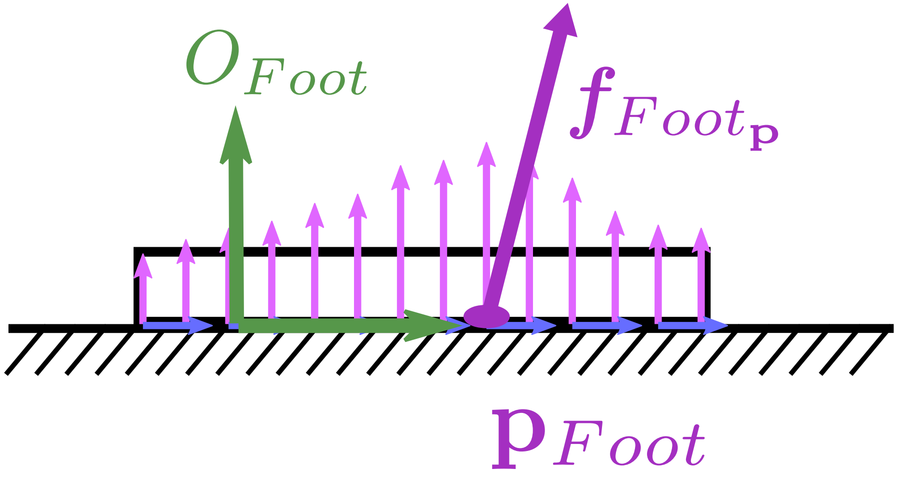

$$\mathrm{p}_{Foot} = \frac{\int_{sole} \rho \boldsymbol{f}(\rho) \mathrm{d}\rho}{\int_{sole} \boldsymbol{f}(\rho) \mathrm{d}\rho}$$

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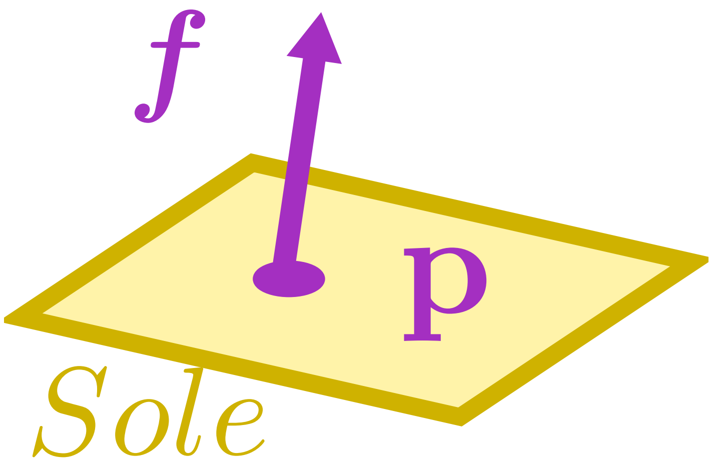

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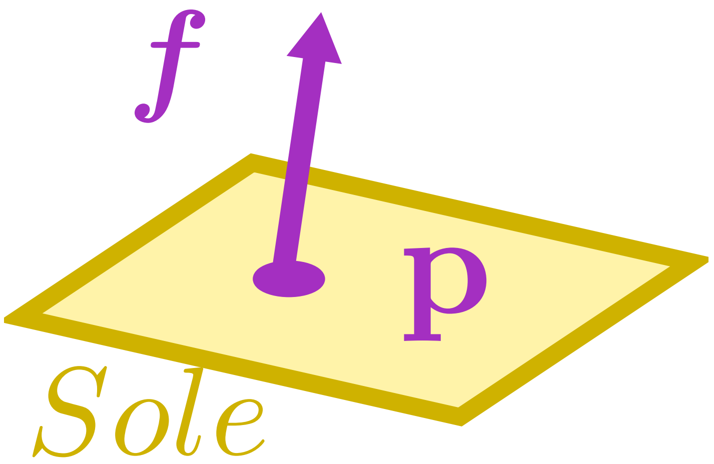

$$Foot_x = \frac{\int_{sole} x\boldsymbol{f}(x,y)\boldsymbol{d}sole}{\int_{sole} \boldsymbol{f}(x,y)\boldsymbol{d}sole}$$

$$Foot_y = \frac{\int_{sole} y\boldsymbol{f}(x,y)\boldsymbol{d}sole}{\int_{sole} \boldsymbol{f}(x,y)\boldsymbol{d}sole}$$

Zero Moment Point (ZMP)

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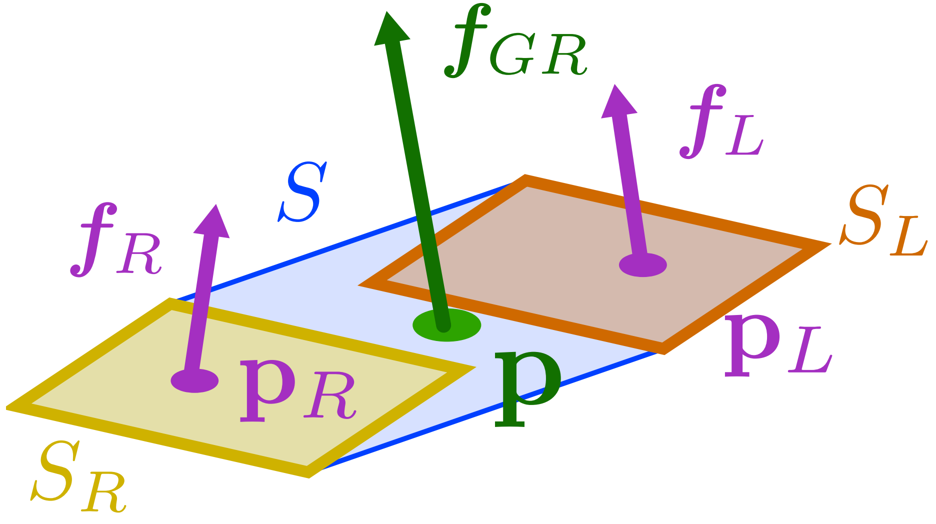

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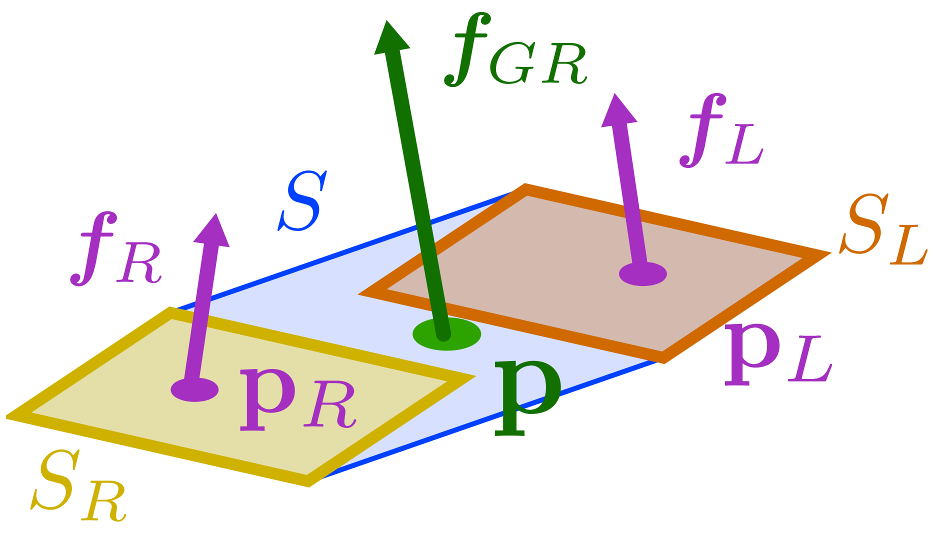
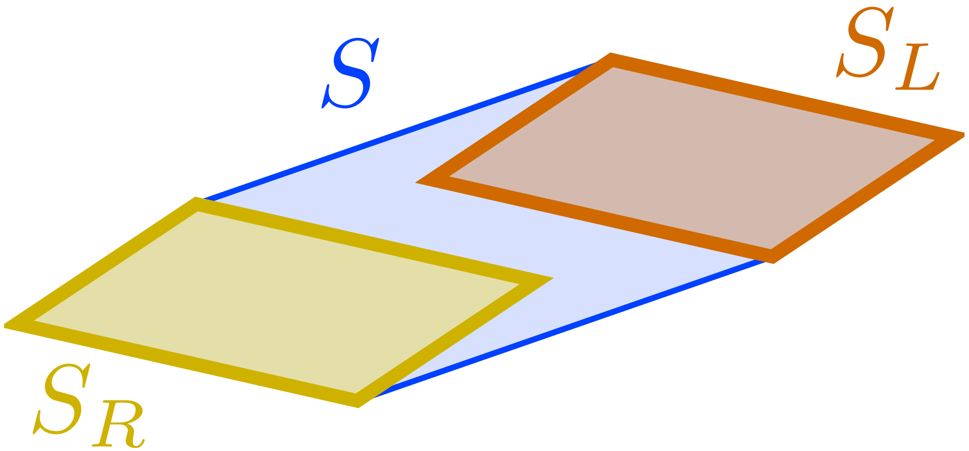


$$\boldsymbol{p}_x = \frac{\boldsymbol{p}_{R_x}\boldsymbol{f}_{R_z} + \boldsymbol{p}_{L_x}\boldsymbol{f}_{L_z}}{\boldsymbol{f}_{R_z} + \boldsymbol{f}_{L_z}}$$

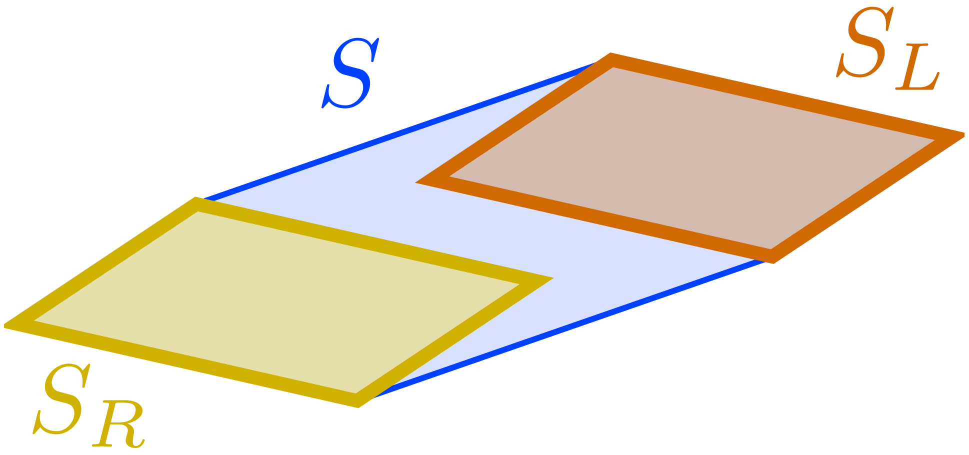
$$\boldsymbol{p}_y = \frac{\boldsymbol{p}_{R_y}\boldsymbol{f}_{R_z} + \boldsymbol{p}_{L_y}\boldsymbol{f}_{L_z}}{\boldsymbol{f}_{R_z} + \boldsymbol{f}_{L_z}}$$

Supporting Polygon (SP)



Supporting Polygon (SP)

The minimum area convex hull that contains all the contact points.



Measurement of ZMP and SP

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The classic method: Ankle FT- sensors

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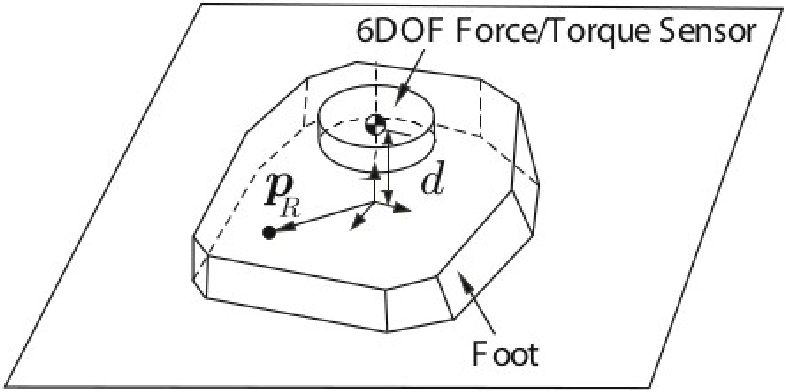

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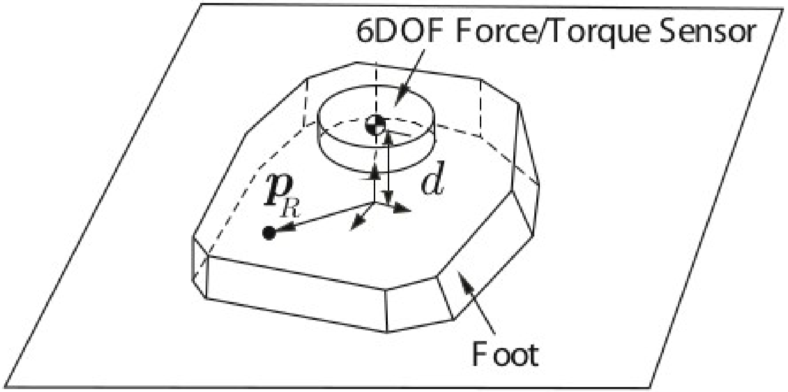

$$\boldsymbol{p}_x = \frac{-\boldsymbol{\tau}_y - \boldsymbol{f}_x d}{\boldsymbol{f}_z}$$

$$\boldsymbol{p}_y = \frac{\boldsymbol{\tau}_x - \boldsymbol{f}_y d}{\boldsymbol{f}_z}$$

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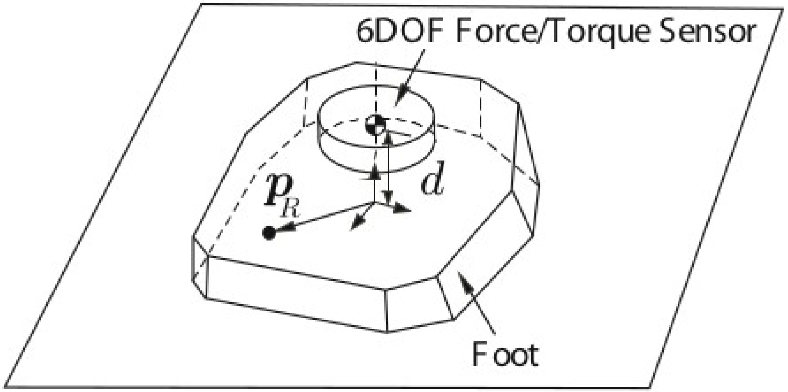

$$\boldsymbol{p}_x = \frac{-\boldsymbol{\tau}_y - \boldsymbol{f}_x d}{\boldsymbol{f}_z}$$

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$$\boldsymbol{p} = \begin{bmatrix} \boldsymbol{p}_x \\ \boldsymbol{p}_y \\ 0 \end{bmatrix}$$

Measurement of ZMP and SP

ZMP depends on the ground reaction forces.

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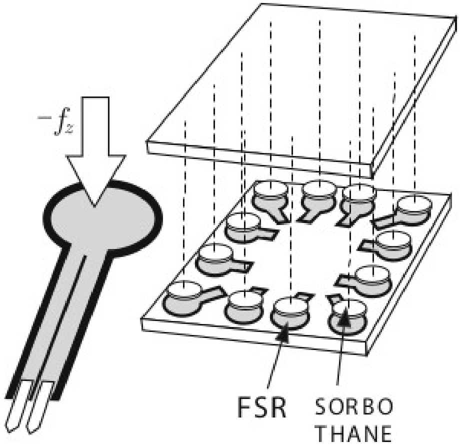
(a) H5

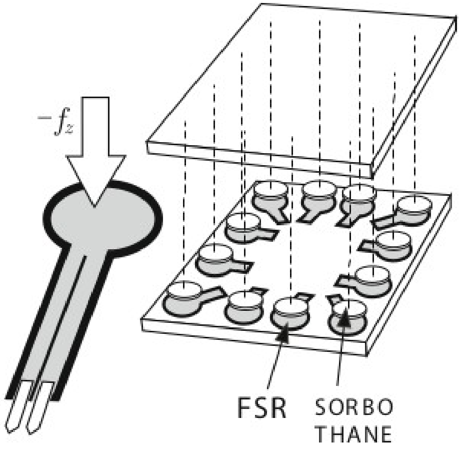


(b) Foot design of H5

Measurement of ZMP and SP

ZMP depends on the ground reaction forces.

(a) H5



(b) Foot design of H5

$$\boldsymbol{p}_x = \frac{\sum_{j=1}^{N} \boldsymbol{p}_{j\,x} \boldsymbol{f}_{j\,z}}{\sum_{j=1}^{N} \boldsymbol{f}_{j\,z}}$$

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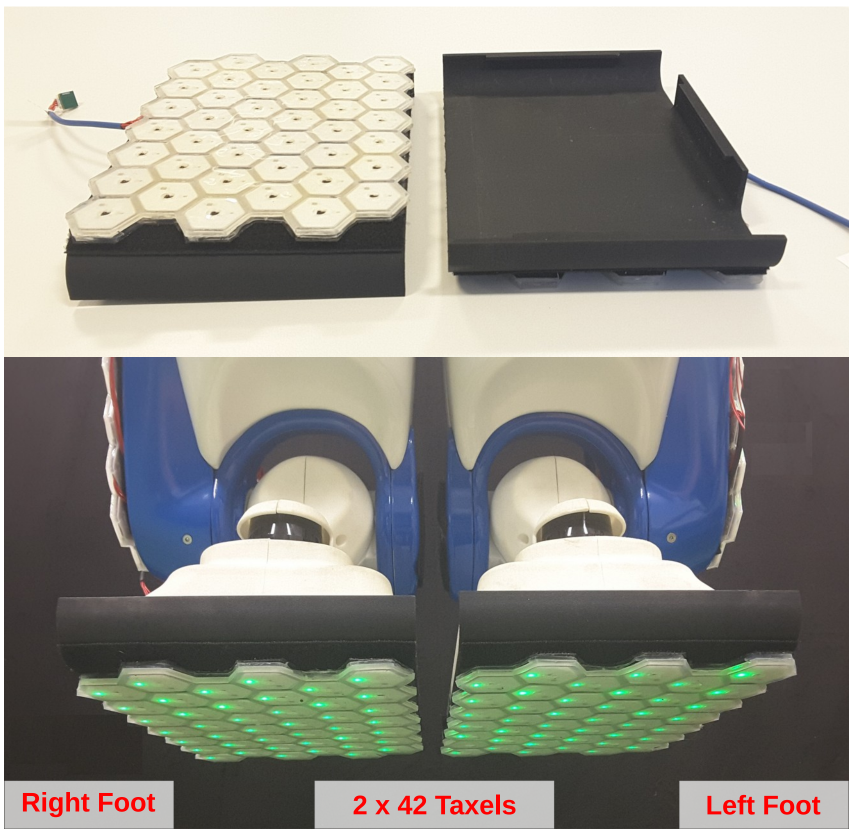
Measurement of ZMP and SP

ZMP depends on the ground reaction forces.



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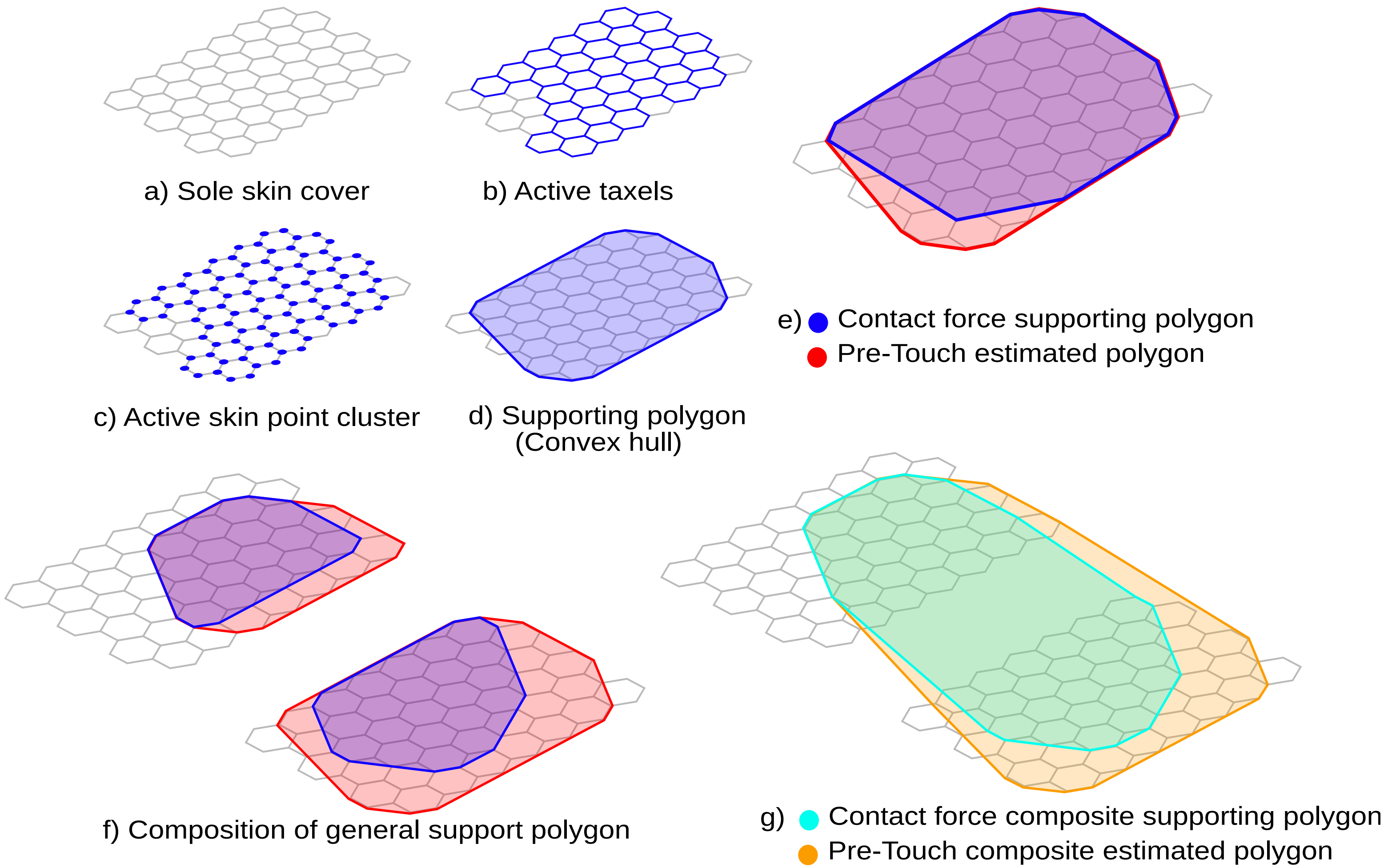

Quiz time !!!

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- **Is the ZMP the projection of the CoM over the ground?**

Quiz time !!!

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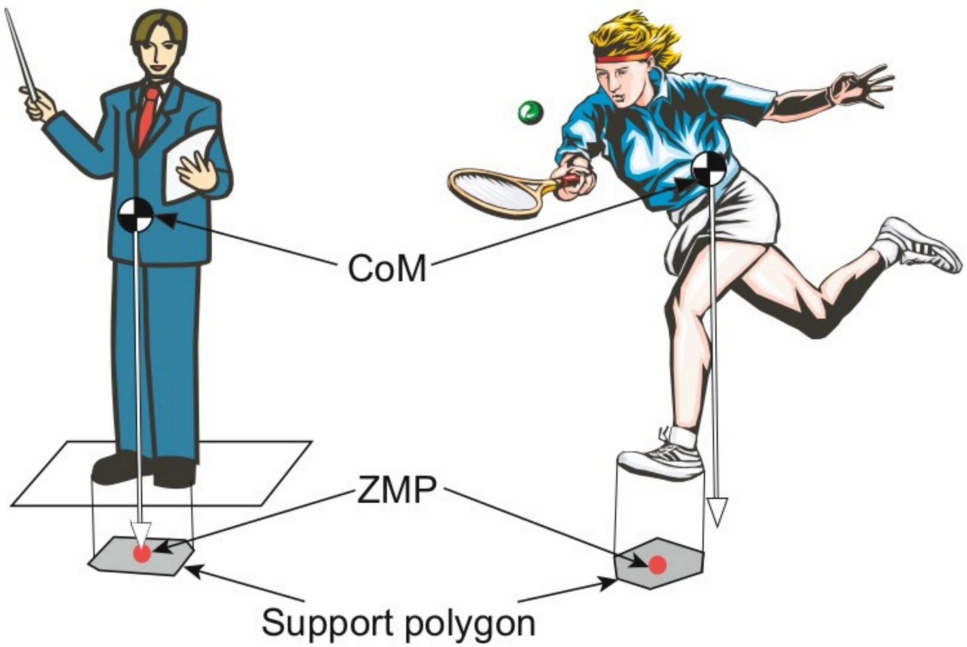
Only in static conditions. When the CoM is moving, the ZMP can be static if no external forces are applied to the upper body. The ZMP is the point where the LIPM model has its fulcrum.



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No, the ZMP only exist inside the supporting region in static contacts (no sliding).

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- **Can the ZMP be exactly at the border of the supporting region?**

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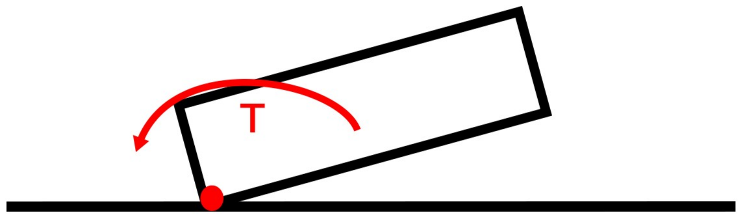
- **Can the ZMP be outside the supporting region?**

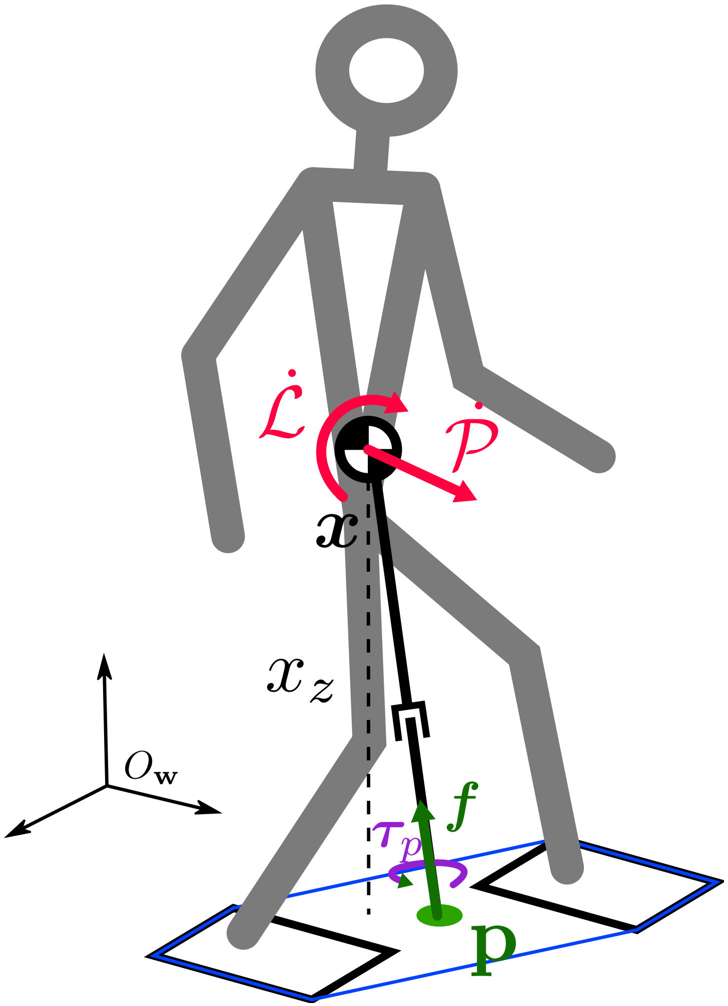
No, the ZMP only exist inside the supporting region in static contacts (no sliding).

- **Can the ZMP be exactly at the border of the supporting region?**

No, if the foot is tilting (border only contact) the moment at the contact point is not zero (because the body is already tilting)



ZMP and Momentum



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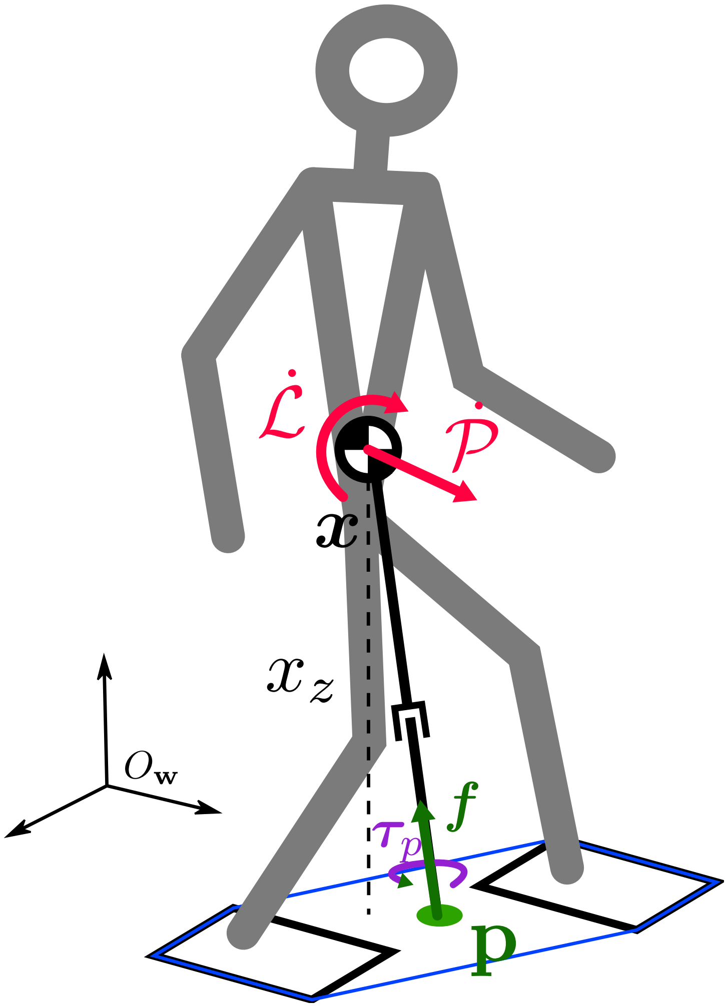
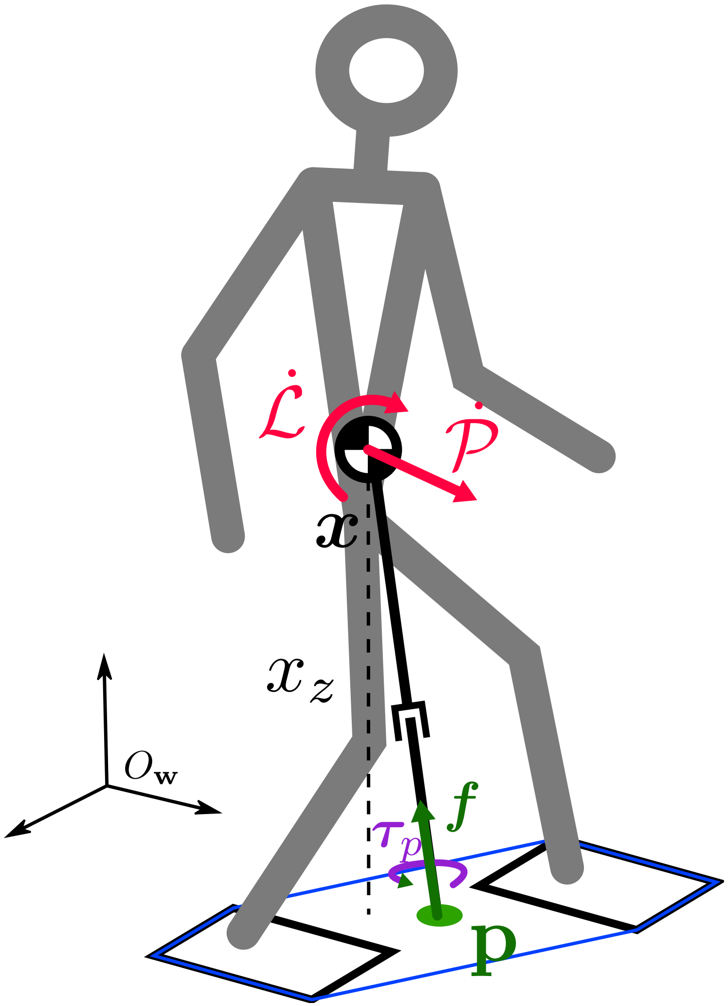


$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}_p$$

ZMP and Momentum



$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}_p$$

Linear $$\dot{\mathcal{P}} = m\boldsymbol{g} + \boldsymbol{f}$$

Angular $$\dot{\mathcal{L}} = \boldsymbol{x} \times m\boldsymbol{g} + \boldsymbol{\tau}$$

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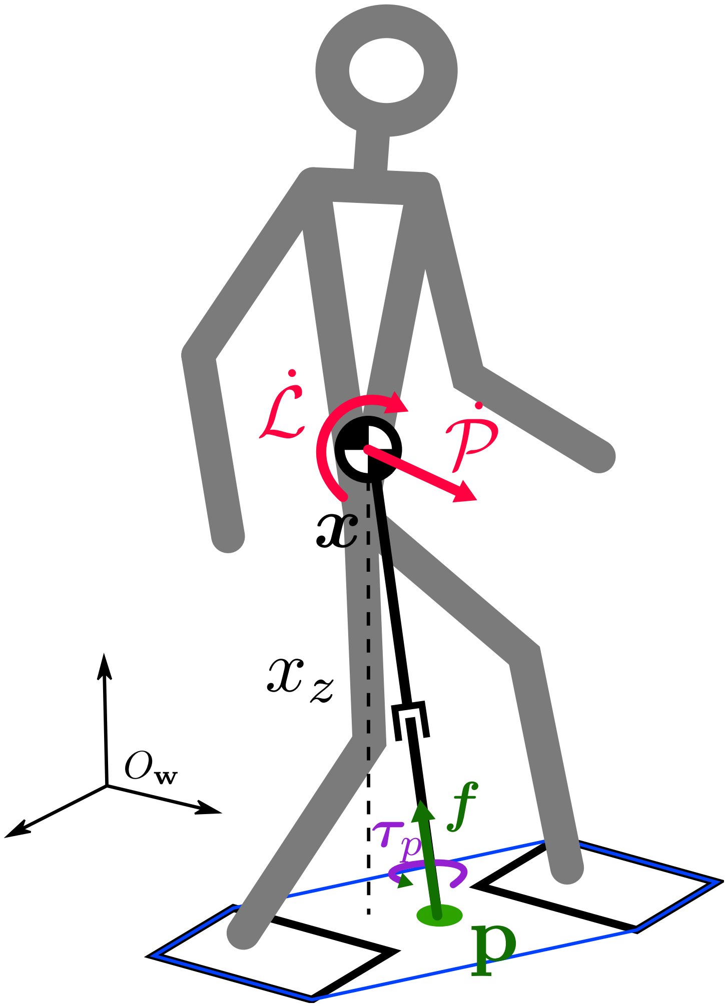
ZMP and Momentum



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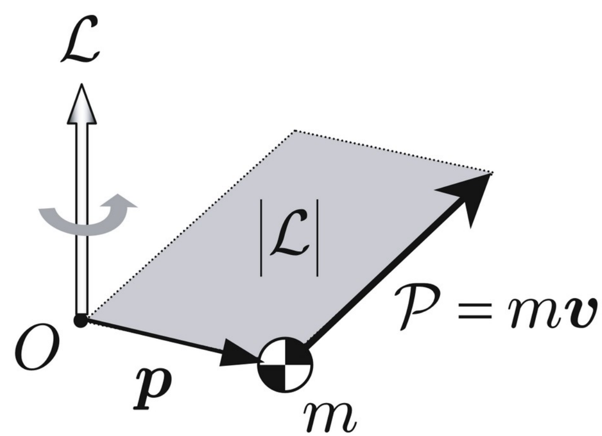

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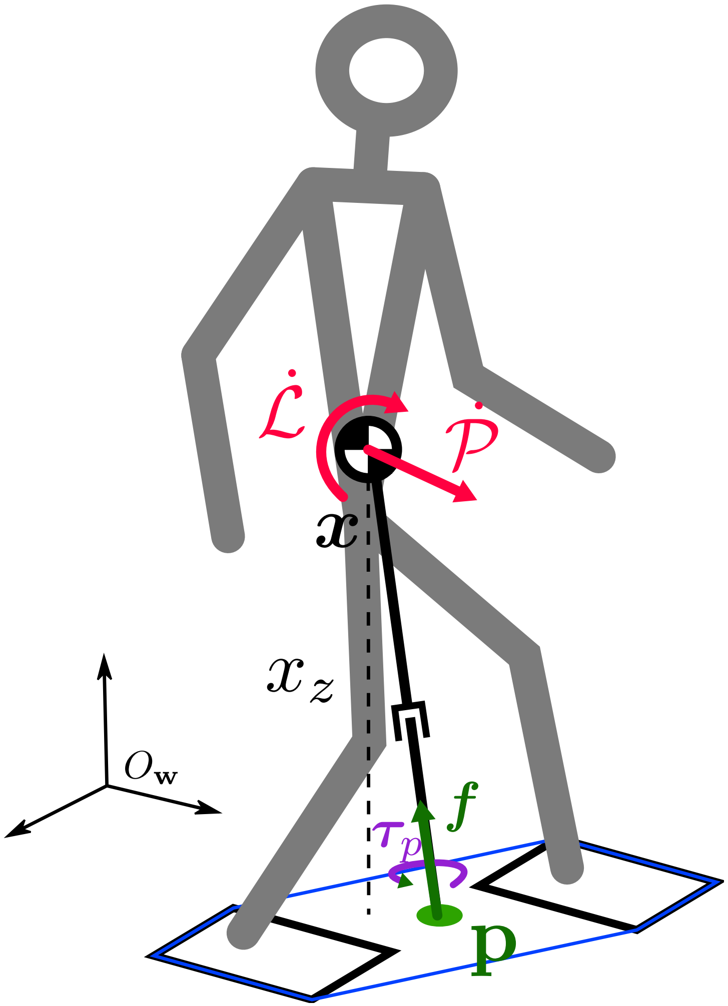


$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}_p$$

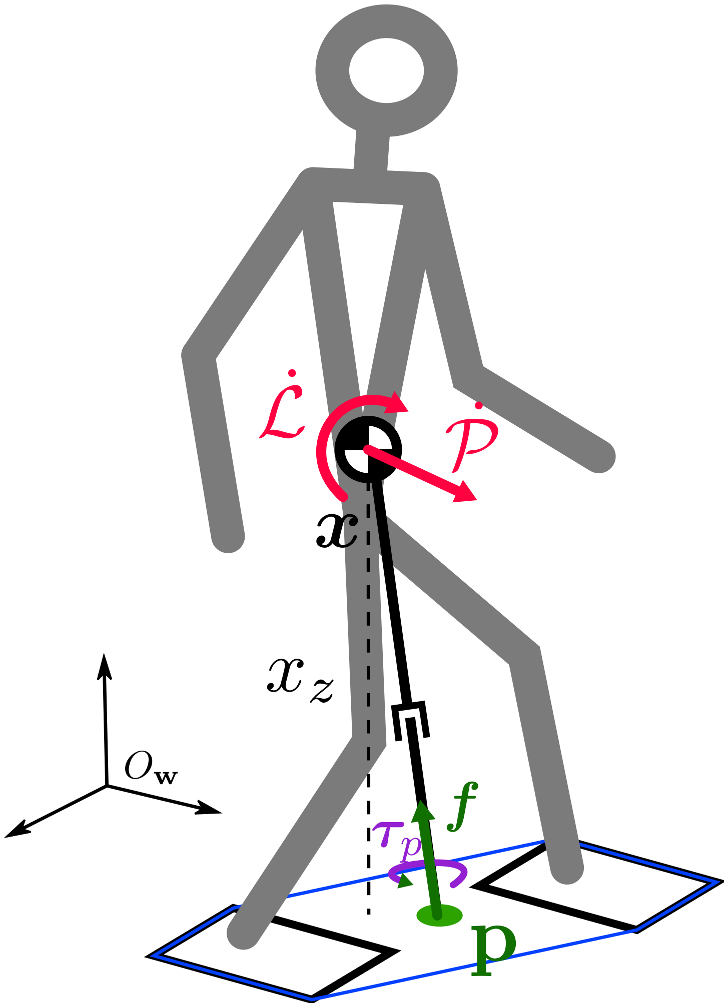
Linear $\dot{\mathcal{P}} = m\boldsymbol{g} + \boldsymbol{f}$

Angular $\dot{\mathcal{L}} = \boldsymbol{x} \times m\boldsymbol{g} + \boldsymbol{\tau}$

$$\boldsymbol{p}_x = \frac{mg\boldsymbol{x}_x + \boldsymbol{p}_z\dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{mg + \dot{\mathcal{P}}_z}$$

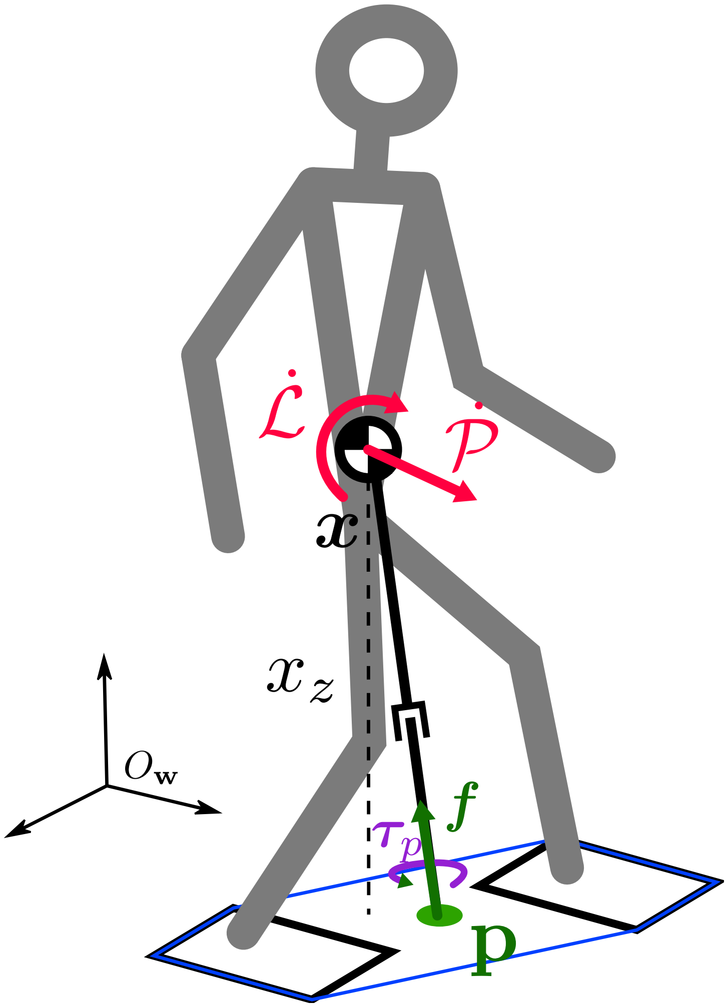
$$\boldsymbol{p}_y = \frac{mg\boldsymbol{x}_y + \boldsymbol{p}_z\dot{\mathcal{P}}_y - \dot{\mathcal{L}}_x}{mg + \dot{\mathcal{P}}_z}$$

ZMP and Momentum



Estimation of ZMP with no force sensors

ZMP and Momentum

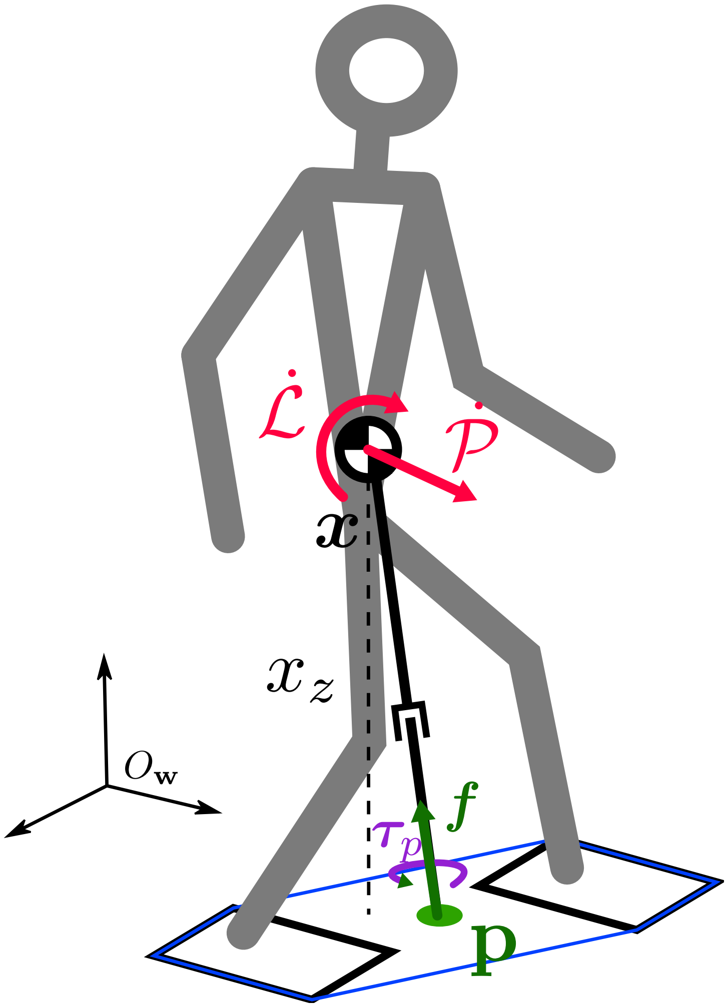


Estimation of ZMP with no force sensors

$$\mathcal{P} = m\dot{\boldsymbol{x}}$$

$$\mathcal{L} = \boldsymbol{x} \times m\dot{\boldsymbol{x}}$$

ZMP and Momentum



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$$\mathcal{P} = m\dot{\boldsymbol{x}}$$

$$\mathcal{L} = \boldsymbol{x} \times m\dot{\boldsymbol{x}}$$

$$\boldsymbol{p}_x = \boldsymbol{x}_x - \frac{(\boldsymbol{x}_z - \boldsymbol{p}_z)\ddot{\boldsymbol{x}}_x}{\ddot{\boldsymbol{x}}_z + g}$$

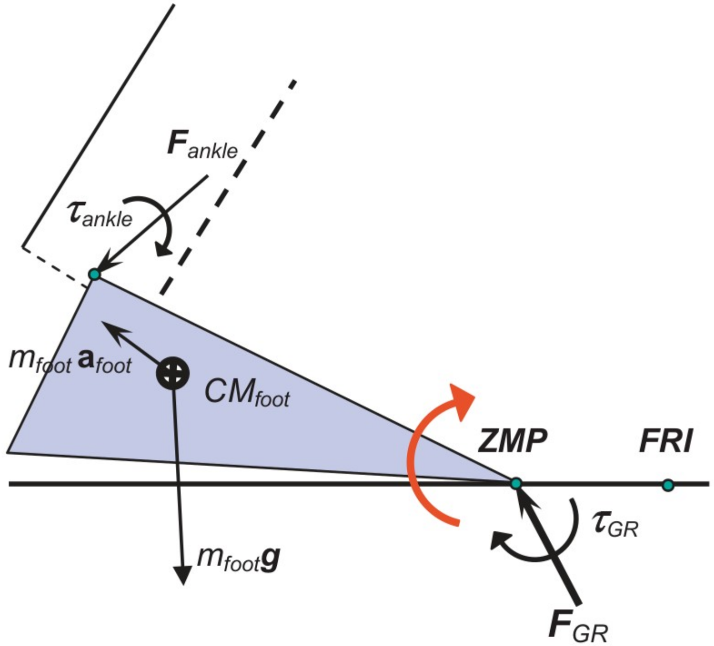
$$\boldsymbol{p}_y = \boldsymbol{x}_y - \frac{(\boldsymbol{x}_z - \boldsymbol{p}_z)\ddot{\boldsymbol{x}}_y}{\ddot{\boldsymbol{x}}_z + g}$$

Foot Rotation Indicator (FRI)

[Goswami 1999]

Foot Rotation Indicator (FRI)

The FRI is the point where the ground reaction force would have to act to keep the foot from accelerating.



[Goswami 1999]

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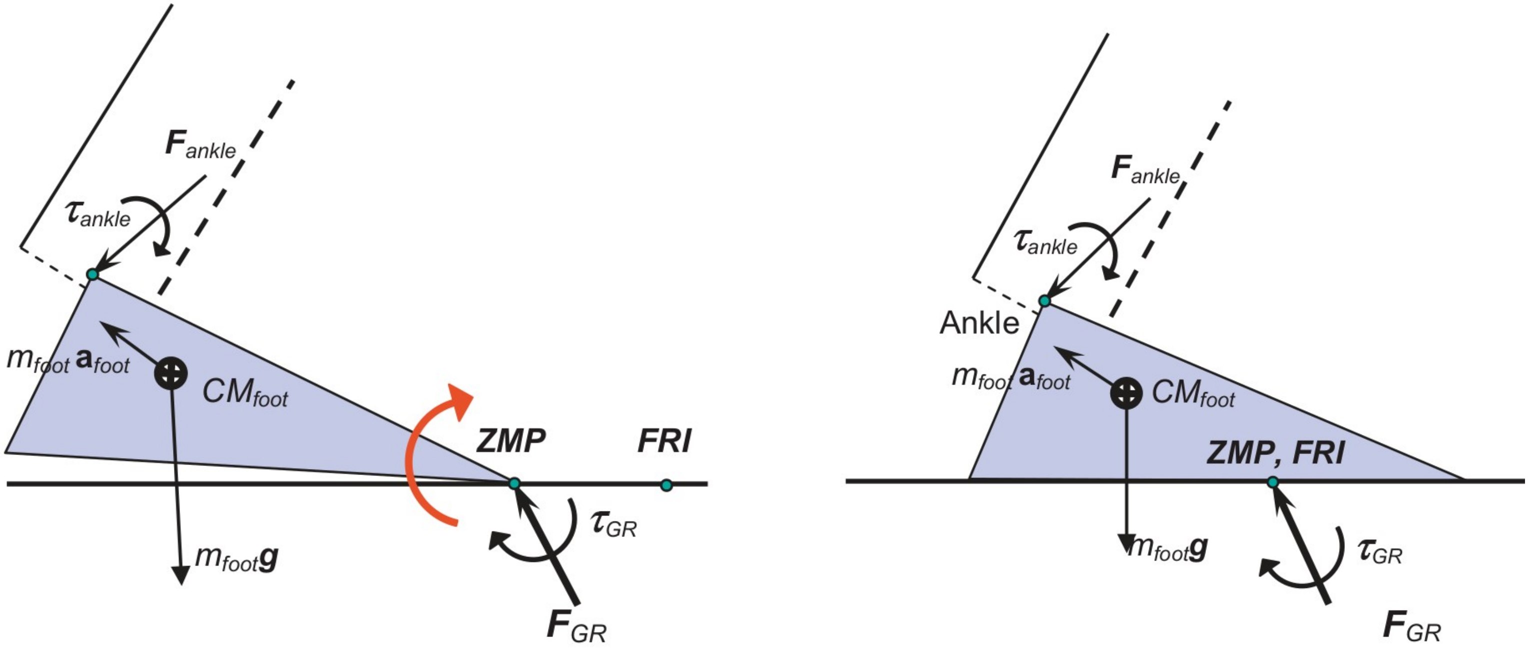
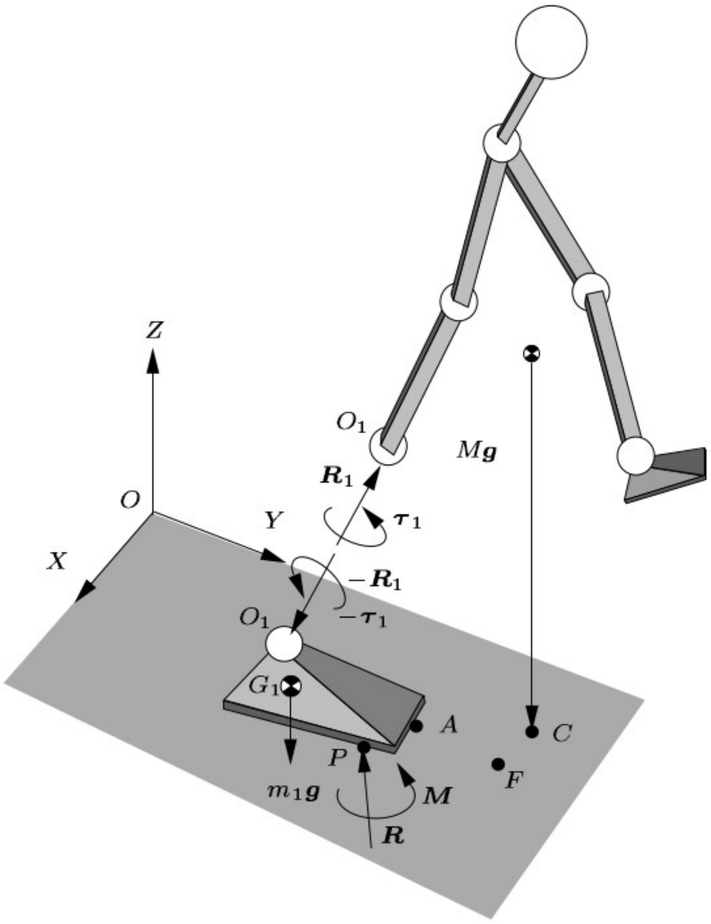


When the foot is stationary, the FRI coincides with the ZMP.

[Goswami 1999]

Foot Rotation Indicator (FRI)



$$OF_x = \frac{m_1 OG_{1y} g + \sum_{i=2}^{n} m_i OG_{iy}(a_{iz} + g)}{m_1 g + \sum_{i=2}^{n} m_i (a_{iz} + g)} - \frac{\sum_{i=2}^{n} m_i OG_{iz} a_{iy} + \sum_{i=2}^{n} \dot{H}_{Gix}}{m_1 g + \sum_{i=2}^{n} m_i (a_{iz} + g)},$$

$$OF_y = \frac{m_1 OG_{1x} g + \sum_{i=2}^{n} m_i OG_{ix}(a_{iz} + g)}{m_1 g + \sum_{i=2}^{n} m_i (a_{iz} + g)} - \frac{\sum_{i=2}^{n} m_i OG_{iz} a_{ix} - \sum_{i=2}^{n} \dot{H}_{Giy}}{m_1 g + \sum_{i=2}^{n} m_i (a_{iz} + g)}.$$

[Goswami 1999, Postural Stability of Biped Robots and the Foot-Rotation Indicator (FRI) Point]

Centroidal Moment Pivot (CMP)

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The point where a line parallel to the ground reaction force, passing through the CoM, intersects with the external contact surface.

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The CMP is the point where the ground reaction force would have to act to keep the horizontal component of the whole-body angular momentum constant.

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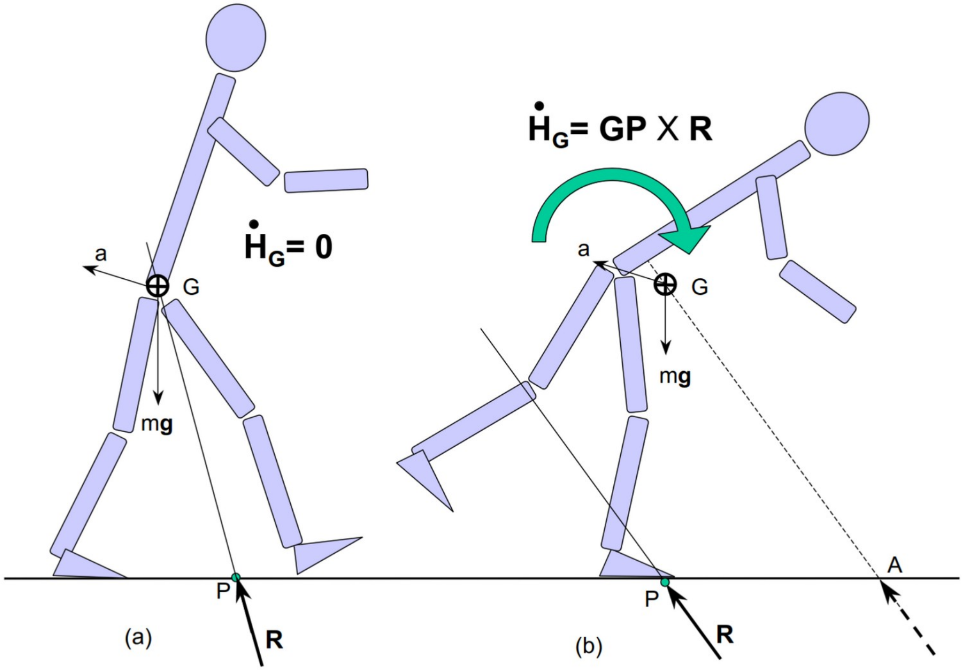
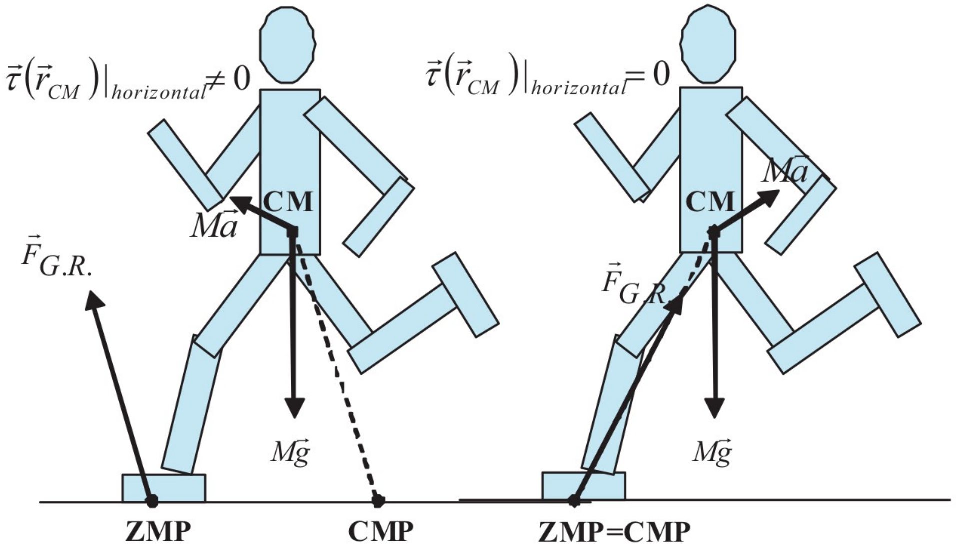


[Goswami & Kallem, 2004, Rate of change of angular momentum and balance maintenance of biped robots]

Centroidal Moment Pivot (CMP)



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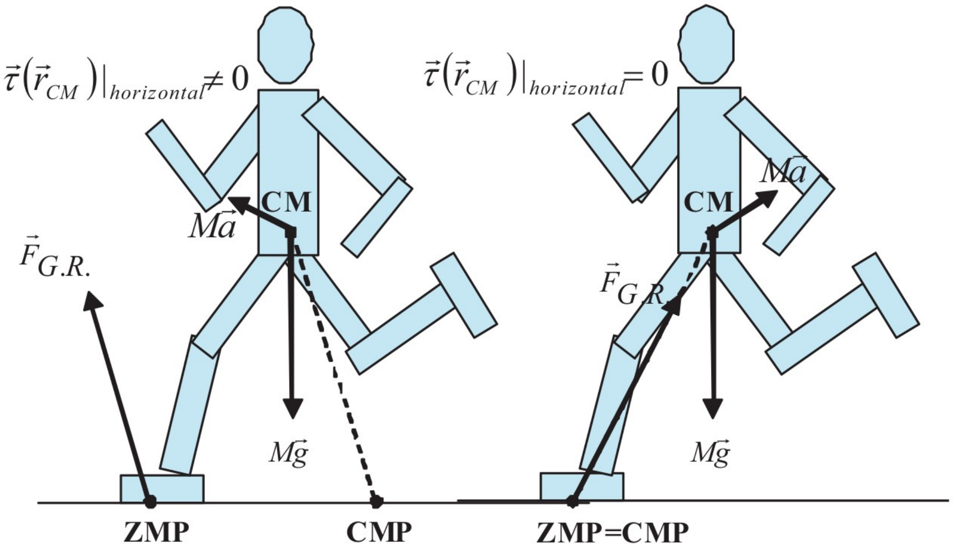
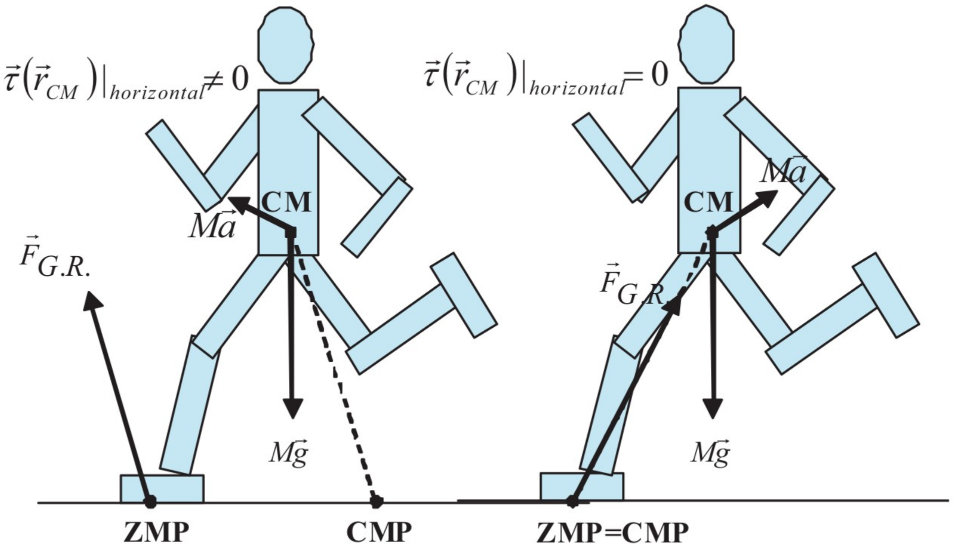


$$(\boldsymbol{r}_{CMP} - \boldsymbol{x}) \times \boldsymbol{f}_{GR} = 0$$

Centroidal Moment Pivot (CMP)

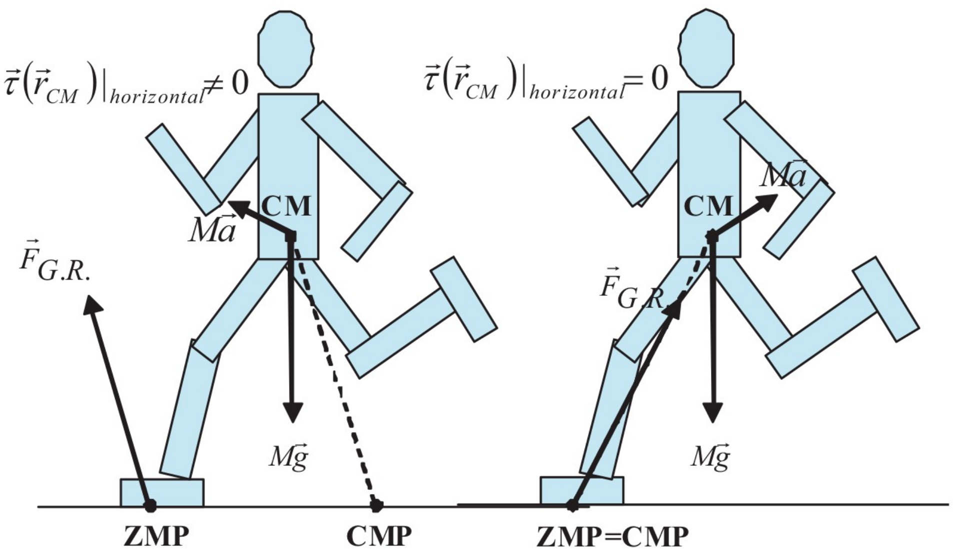


$$(\boldsymbol{r}_{CMP} - \boldsymbol{x}) \times \boldsymbol{f}_{GR} = 0$$

$$\boldsymbol{r}_x = \boldsymbol{x}_x - \frac{\boldsymbol{f}_x}{\boldsymbol{f}_z}\boldsymbol{x}_z$$

$$\boldsymbol{r}_y = \boldsymbol{x}_y - \frac{\boldsymbol{f}_y}{\boldsymbol{f}_z}\boldsymbol{x}_z$$

Centroidal Moment Pivot (CMP)



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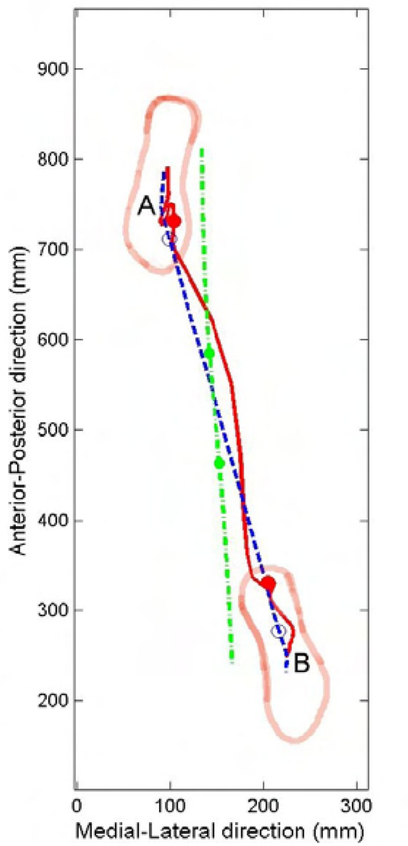
$$\boldsymbol{r}_y = \boldsymbol{x}_y - \frac{\boldsymbol{f}_y}{\boldsymbol{f}_z}\boldsymbol{x}_z$$

From ZMP $$\boldsymbol{r} = \boldsymbol{p} - \frac{\boldsymbol{\tau}}{\boldsymbol{f}_z}$$

Centroidal Moment Pivot (CMP)

ZMP, CMP, and CoM trajectories in Human walking.



CoM $\boldsymbol{x}$

ZMP $\boldsymbol{p}$

CMP $\boldsymbol{r}$

[Popovic & Hofmann, 2004, Angular Momentum Regulation during Human Walking: Biomechanics and Control]

Capture Point (CP)

A Capture Point is a point on the ground where the robot can step to in order to bring itself to a complete stop.

[Pratt et al, 2006]

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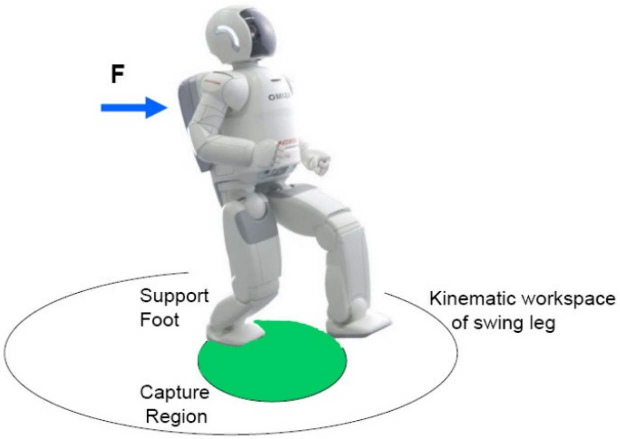

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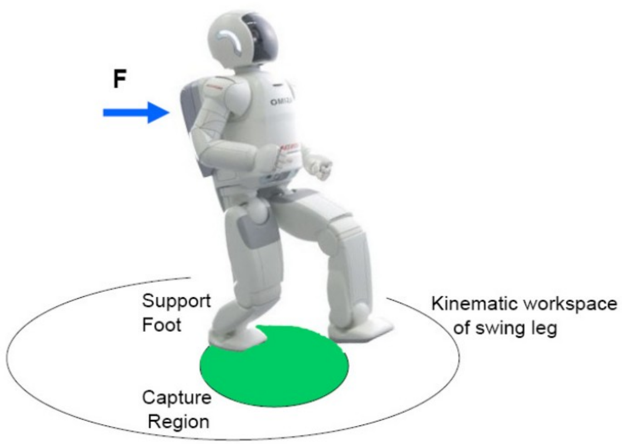

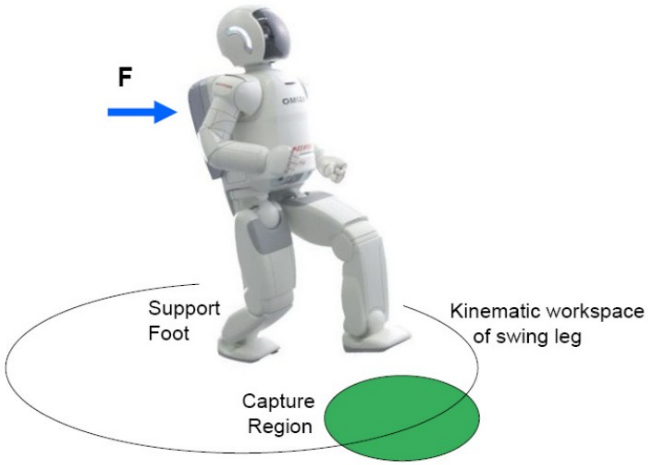

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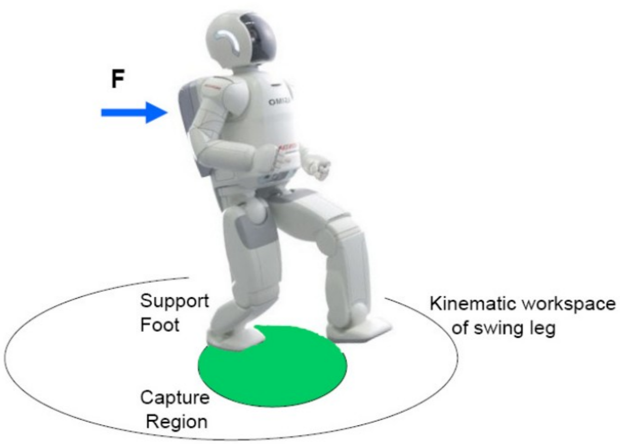

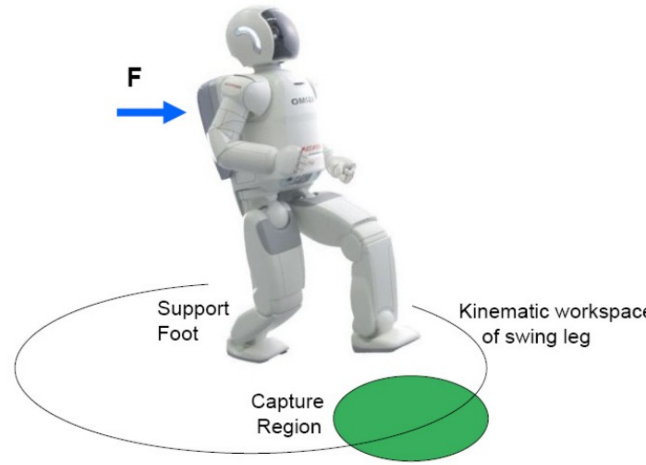

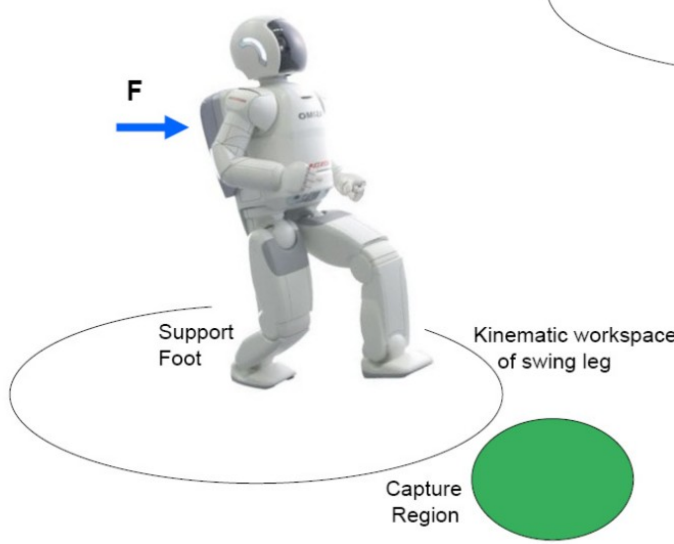

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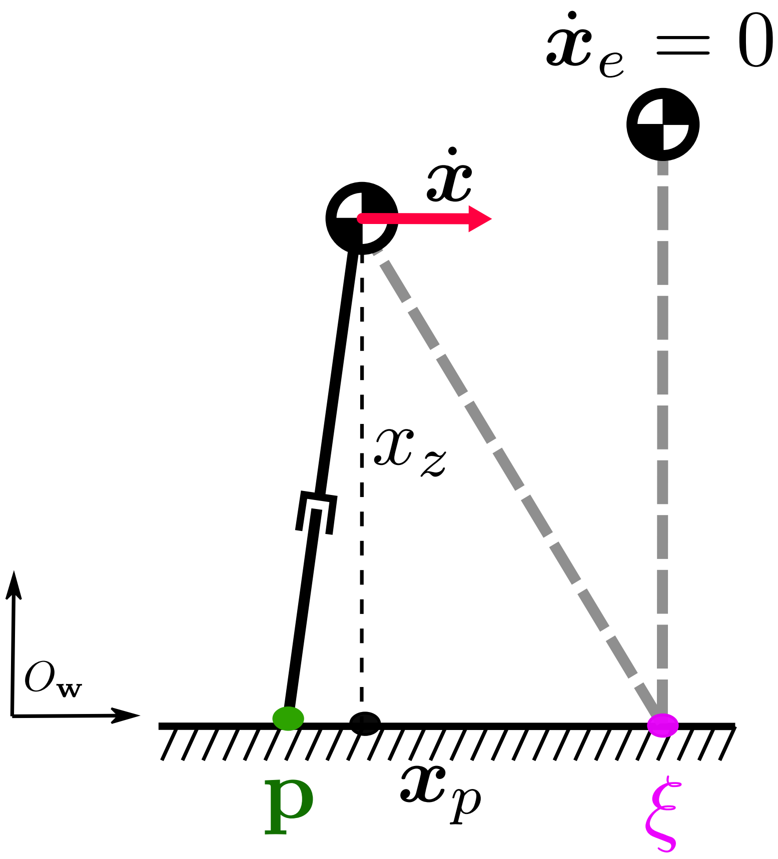

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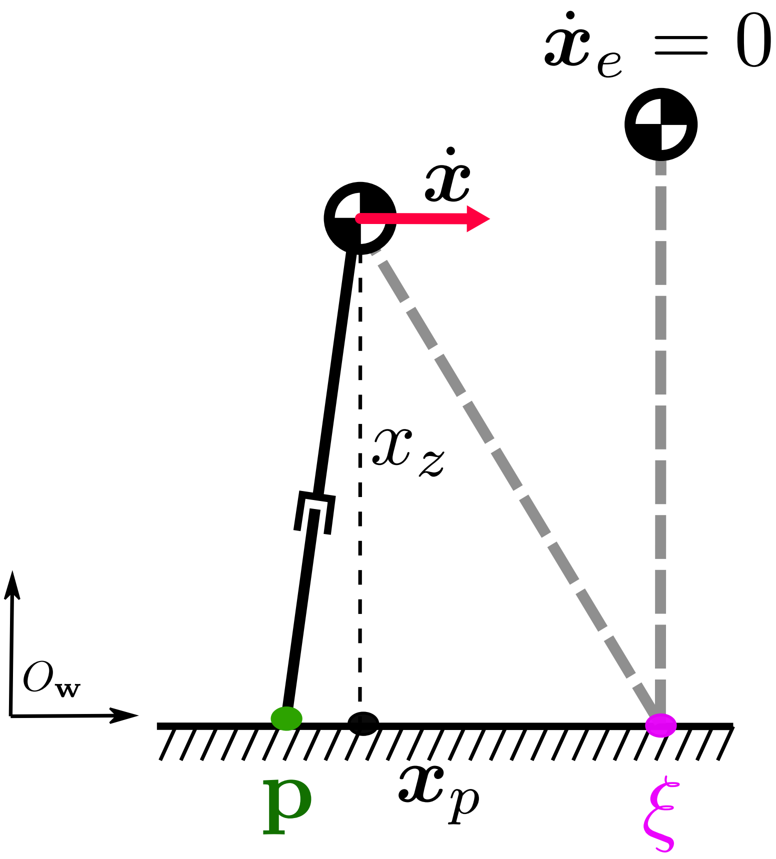

$$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p})$$

$$\boldsymbol{\omega} = \sqrt{\frac{g}{\boldsymbol{x}_z}}$$

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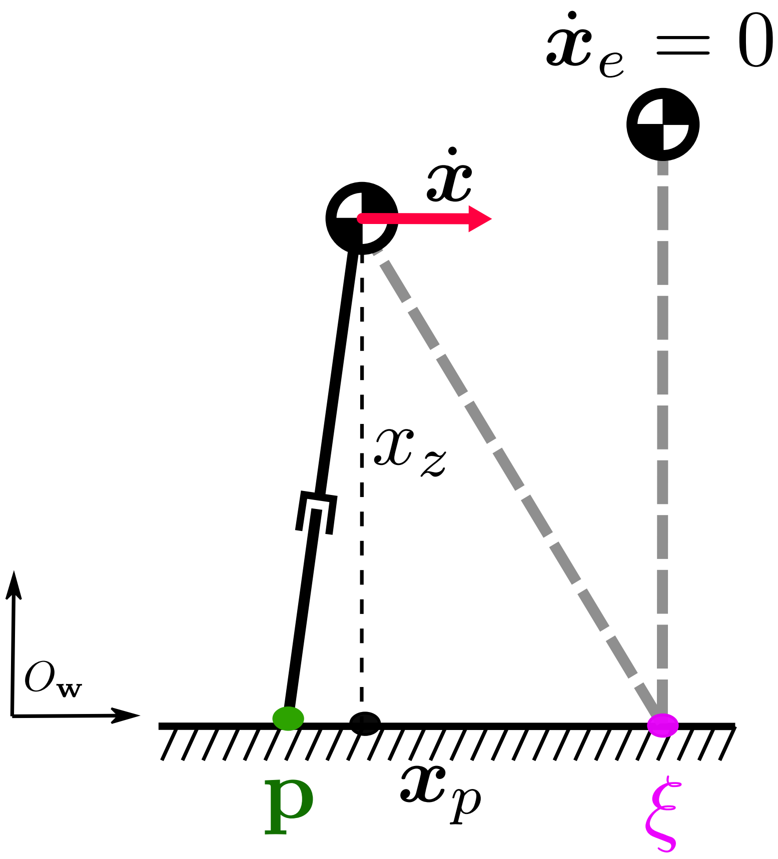

$$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p})$$

$$\boldsymbol{\omega} = \sqrt{\frac{g}{\boldsymbol{x}_z}}$$

$$\boldsymbol{\xi} = \boldsymbol{x}_p + \frac{\dot{\boldsymbol{x}}_p}{\boldsymbol{\omega}}$$

Capture Point (CP)

Assuming $\boldsymbol{p}$ constant: $\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p})$ $\boldsymbol{\omega} = \sqrt{\dfrac{g}{\boldsymbol{x}_z}}$

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$$\boldsymbol{x}_p(t) = \boldsymbol{p} + \frac{e^{\omega t}}{2}\left[\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right] + \frac{e^{-\omega t}}{2}\left[\boldsymbol{x}_p(0) - \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right]$$

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$$t \to \infty,\ \boldsymbol{x}_p \to \boldsymbol{p}$$

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$$t \rightarrow \infty, \; \boldsymbol{x}_p \rightarrow \boldsymbol{p}$$

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$$t \to \infty,\ \boldsymbol{x}_p \to \boldsymbol{p}$$

$$\boldsymbol{x}_p(\infty) = \boldsymbol{p} + \frac{e^{\omega\infty}}{2}\left[\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right]$$

Capture Point (CP)

Assuming $\boldsymbol{p}$ constant: $$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p}) \qquad \boldsymbol{\omega} = \sqrt{\frac{g}{\boldsymbol{x}_z}}$$

$$\boldsymbol{x}_p(t) = \boldsymbol{p} + \frac{e^{\omega t}}{2}\left[\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right] + \cancelto{0}{\frac{e^{-\omega t}}{2}\left[\boldsymbol{x}_p(0) - \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right]}$$

$$t \rightarrow \infty, \;\; \boldsymbol{x}_p \rightarrow \boldsymbol{p}$$

$$\boldsymbol{x}_p(\infty) = \boldsymbol{p} + \frac{e^{\omega\infty}}{2}\left[\underbrace{\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega}}_{\boldsymbol{\xi}} - \boldsymbol{p}\right]$$

Divergent Component of Motion (DCM)

Capture Point (CP)

Assuming $\boldsymbol{p}$ constant: $\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p})$ $\quad \boldsymbol{\omega} = \sqrt{\dfrac{g}{\boldsymbol{x}_z}}$

$$\boldsymbol{x}_p(t) = \boldsymbol{p} + \frac{e^{\omega t}}{2}\left[\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right] + \cancelto{0}{\frac{e^{-\omega t}}{2}\left[\boldsymbol{x}_p(0) - \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right]}$$

$$t \to \infty, \ \boldsymbol{x}_p \to \boldsymbol{p}$$

$$\boldsymbol{x}_p(\infty) = \boldsymbol{p} + \frac{e^{\omega\infty}}{2}\left[\underbrace{\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega}}_{\boldsymbol{\xi}} - \boldsymbol{p}\right]$$

Divergent Component of Motion (DCM)

$$\boldsymbol{x}_p(\infty) = \boldsymbol{p}$$

Only if

$$\boldsymbol{\xi}(\infty) = \boldsymbol{p}$$

Capture Point (CP)

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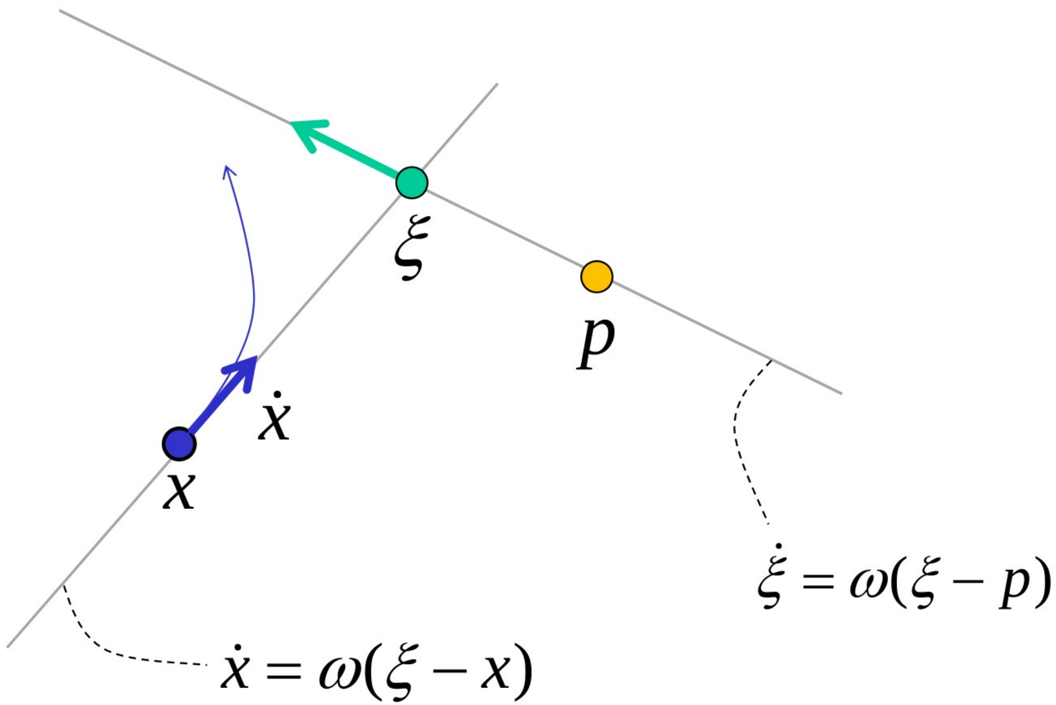
$$\boldsymbol{x}_p(t) = \boldsymbol{p} + \frac{e^{\omega t}}{2}\left[\boldsymbol{x}_p(0) + \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right] + \cancelto{0}{\frac{e^{-\omega t}}{2}\left[\boldsymbol{x}_p(0) - \frac{\dot{\boldsymbol{x}}_p(0)}{\omega} - \boldsymbol{p}\right]}$$

We can thus interpret the capture point as a point where the robot should step (shift its ZMP) in order to come (asymptotically) to a stop.

Capture Point (CP)



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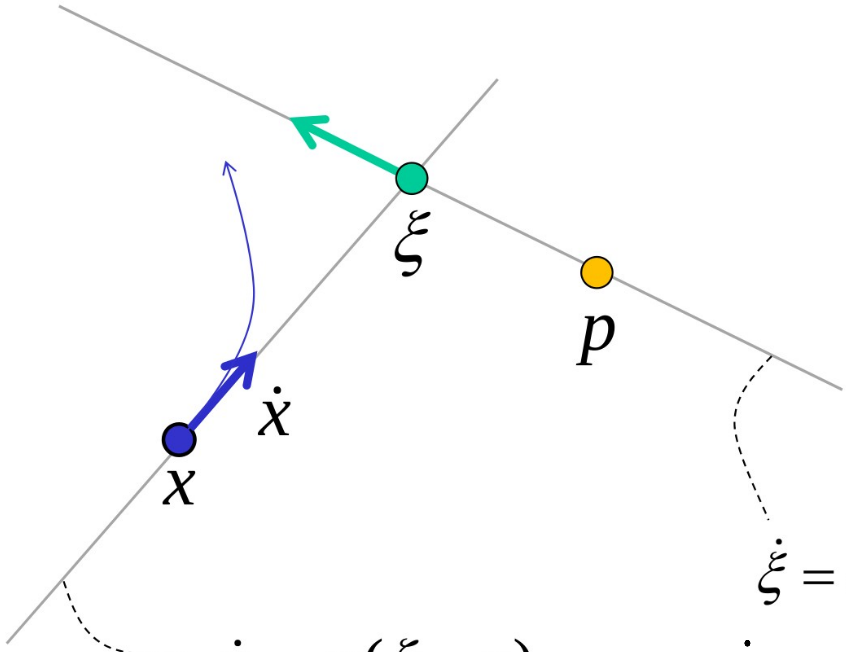
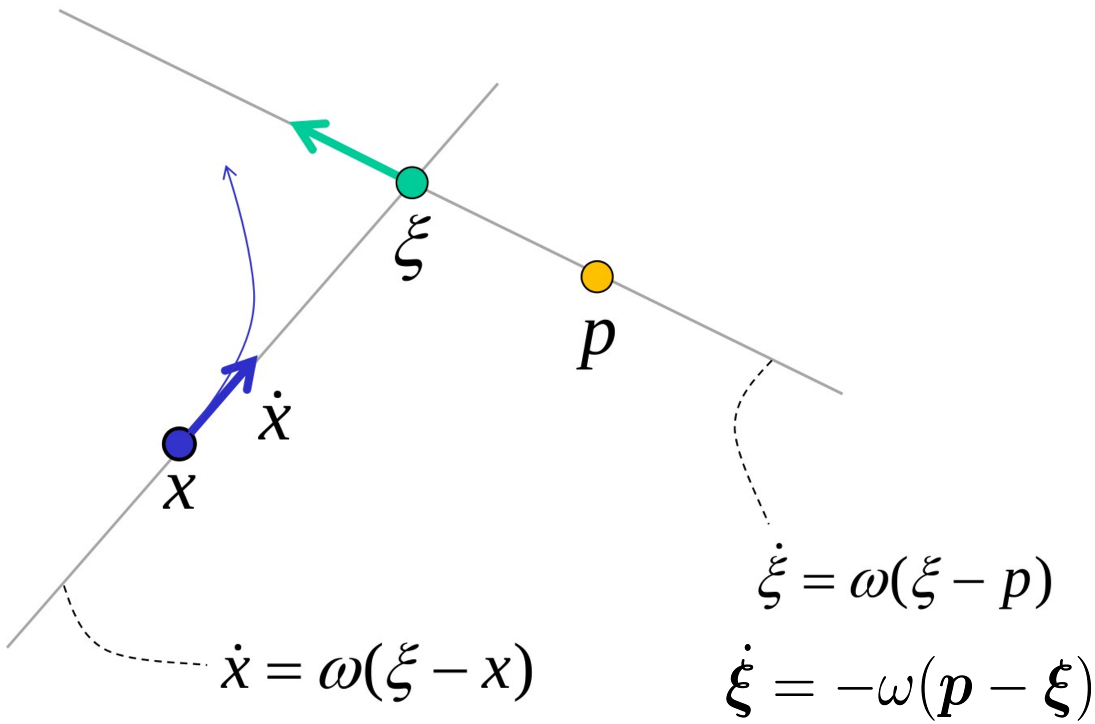


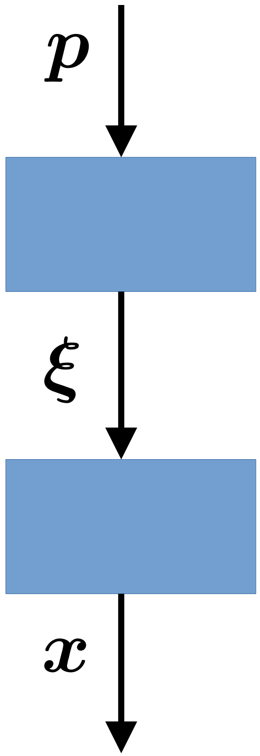
$$\dot{\xi} = \omega(\xi - p)$$

$$\dot{x} = \omega(\xi - x)$$

$$\dot{\boldsymbol{\xi}} = -\omega(\boldsymbol{p} - \boldsymbol{\xi})$$

Capture Point (CP)





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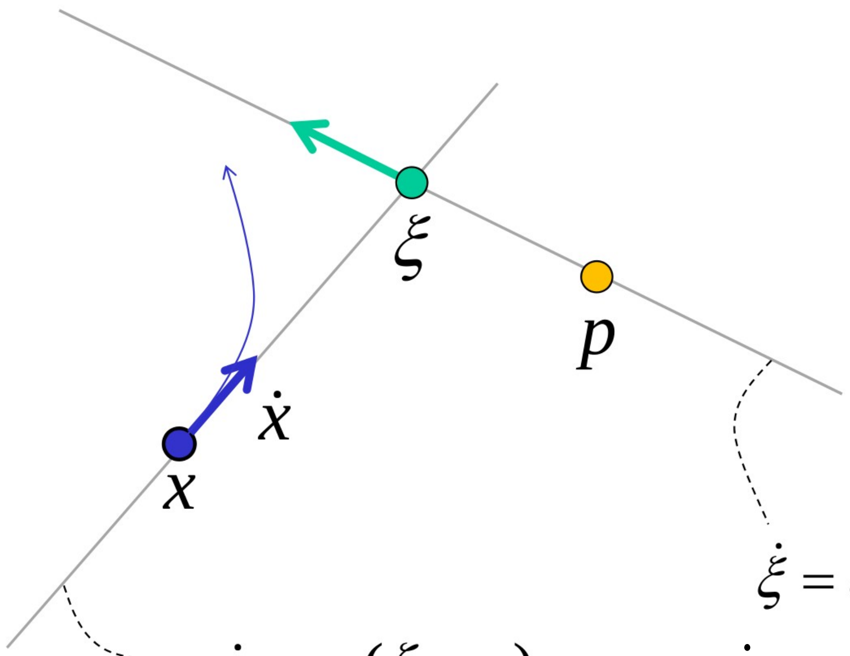
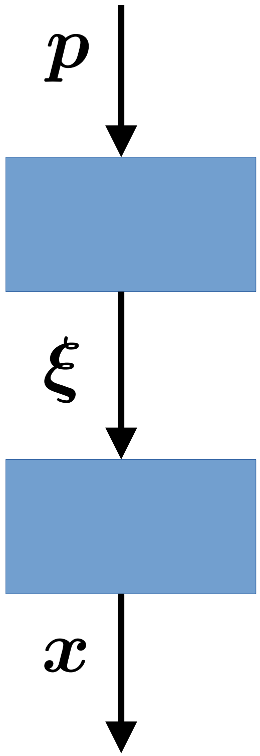


$$\dot{\xi} = \omega(\xi - p)$$

$$\dot{x} = \omega(\xi - x)$$

$$\dot{\boldsymbol{\xi}} = -\omega(\boldsymbol{p} - \boldsymbol{\xi})$$



The capture point diverges away from the ZMP while the center of mass is attracted to the capture point

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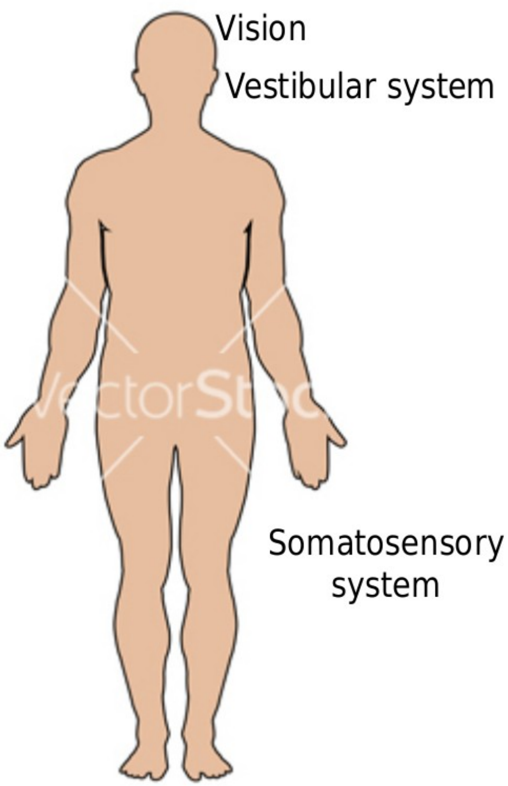
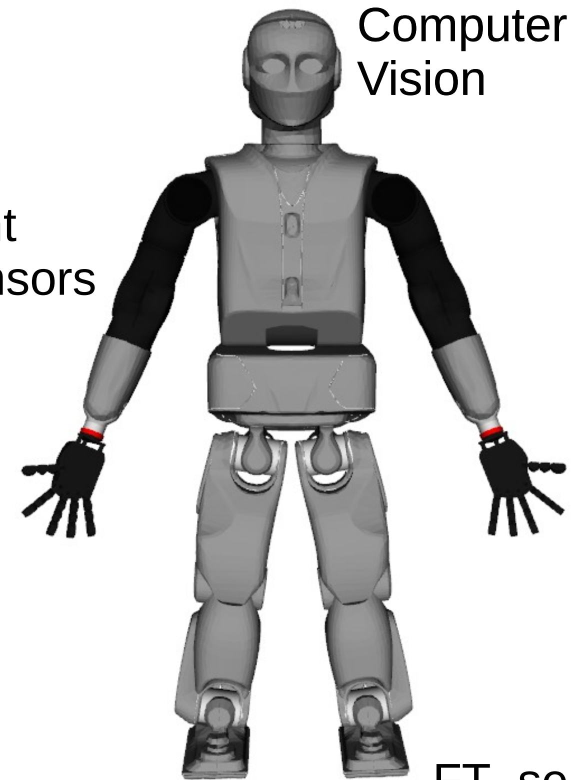
Balance

“Balance” is a generic term describing the ability to control the body posture in order to prevent falling.





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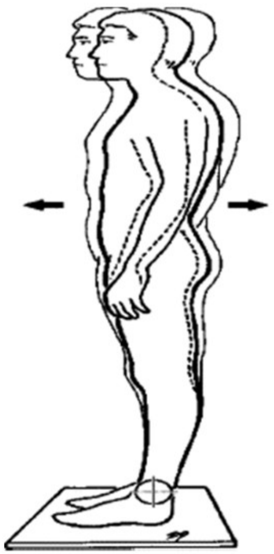
Balance

Balance

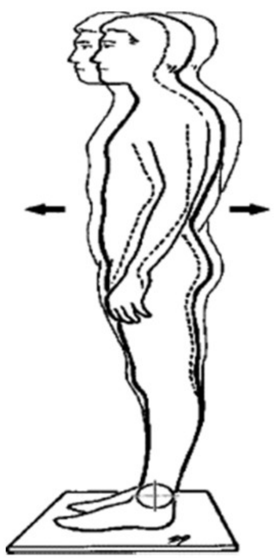
Small
Perturbation



Ankle Strategy

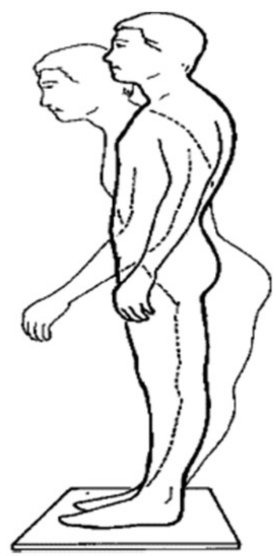
Balance

Small Perturbation



Ankle Strategy

Medium Perturbation



Hip Strategy

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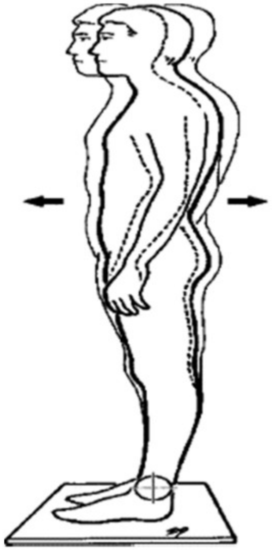
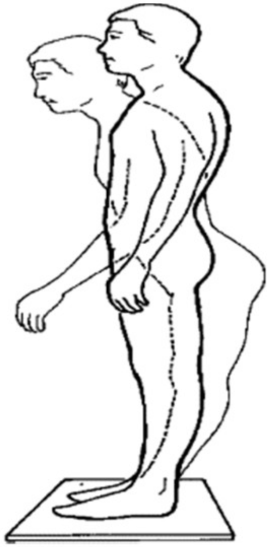
Balance

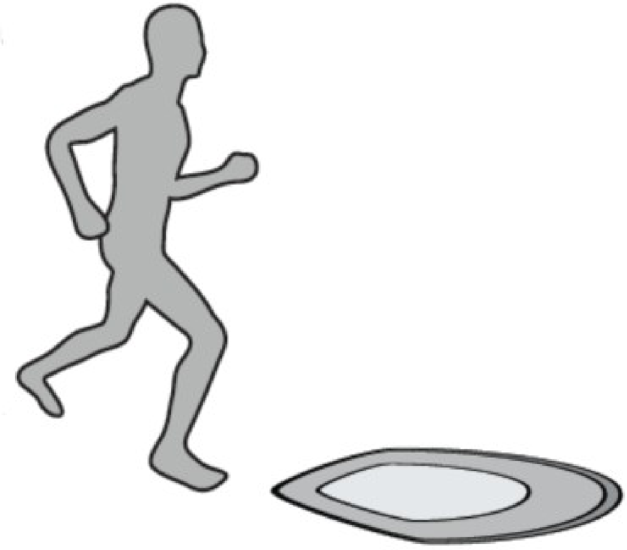
Small Perturbation	Medium Perturbation	Large Perturbation



Ankle Strategy

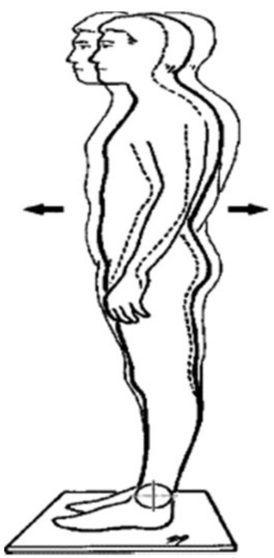
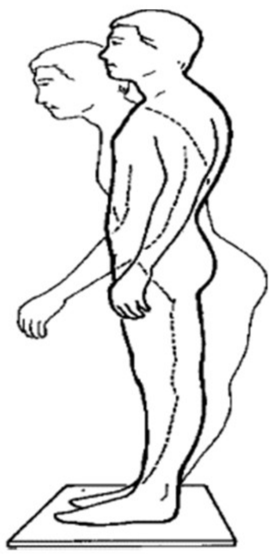
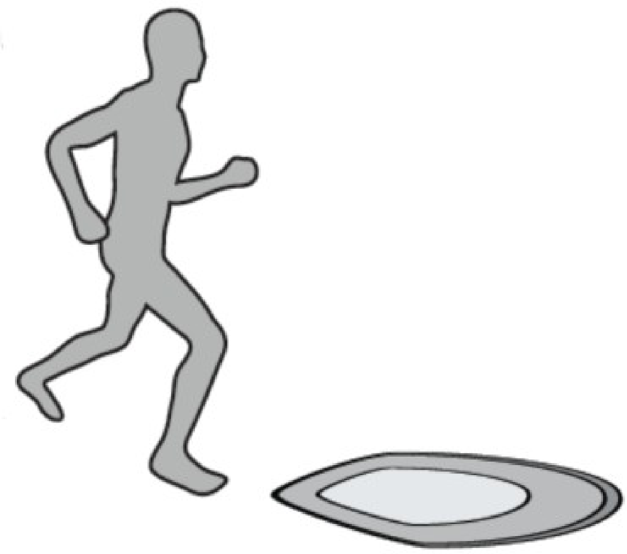


Hip Strategy

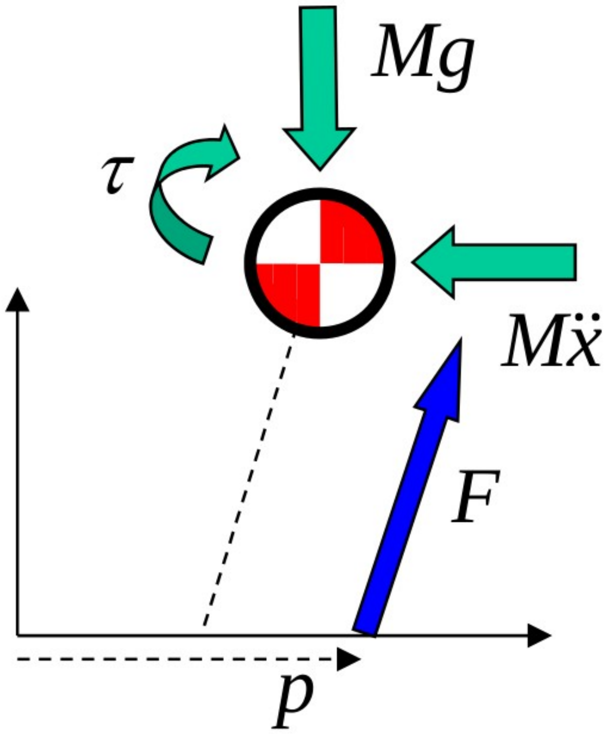


Step Strategy

Balance

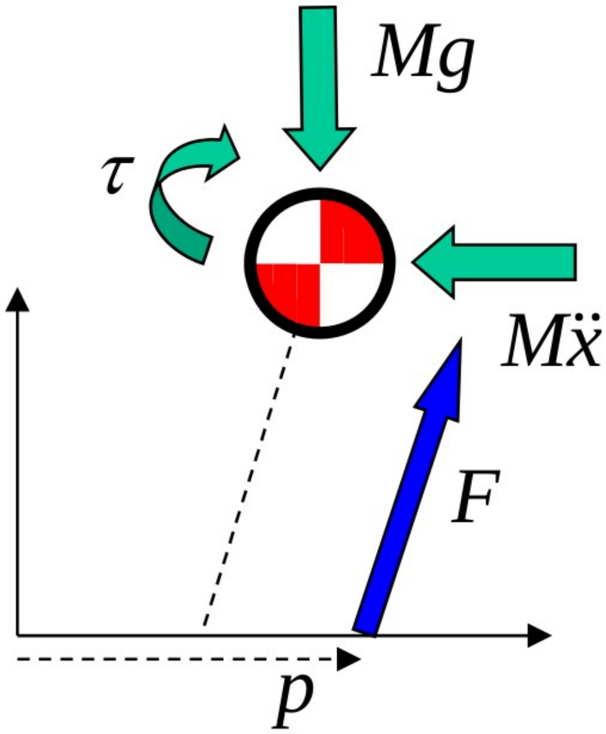
	Small Perturbation	Medium Perturbation	Large Perturbation
			
	Ankle Strategy	Hip Strategy	Step Strategy
Robot Behavior	LIPM ZMP Control	Angular Momentum Control	Capturability Control

Ankle strategy



[Choi et al., 2007]

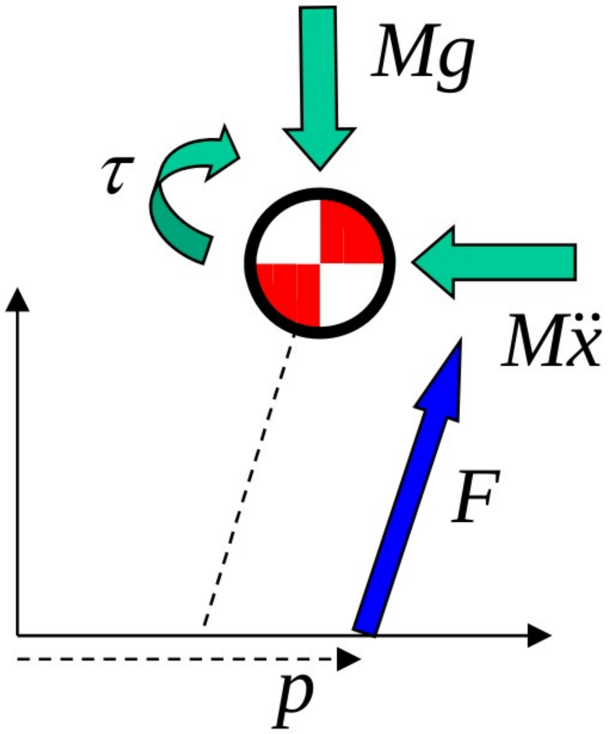
Ankle strategy



$$\ddot{\boldsymbol{x}} = \frac{g}{\boldsymbol{x}_z}(\boldsymbol{x} - \boldsymbol{p}) + \frac{\tau}{M\boldsymbol{x}_z}$$

[Choi et al., 2007]

Ankle strategy

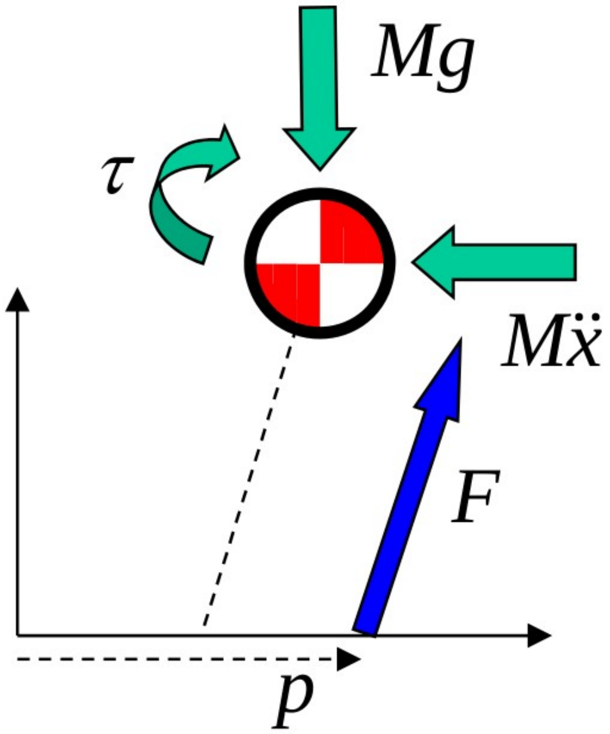


$$\ddot{\boldsymbol{x}} = \frac{g}{\boldsymbol{x}_z}(\boldsymbol{x} - \boldsymbol{p}) + \frac{\tau}{M\boldsymbol{x}_z}$$

$$\tau = M\ddot{\boldsymbol{x}} - g\boldsymbol{x}$$

[Choi et al., 2007]

Ankle strategy



$$\ddot{\boldsymbol{x}} = \frac{g}{\boldsymbol{x}_z}(\boldsymbol{x} - \boldsymbol{p}) + \frac{\tau}{M\boldsymbol{x}_z}$$

$$\tau = M\ddot{\boldsymbol{x}} - g\boldsymbol{x}$$

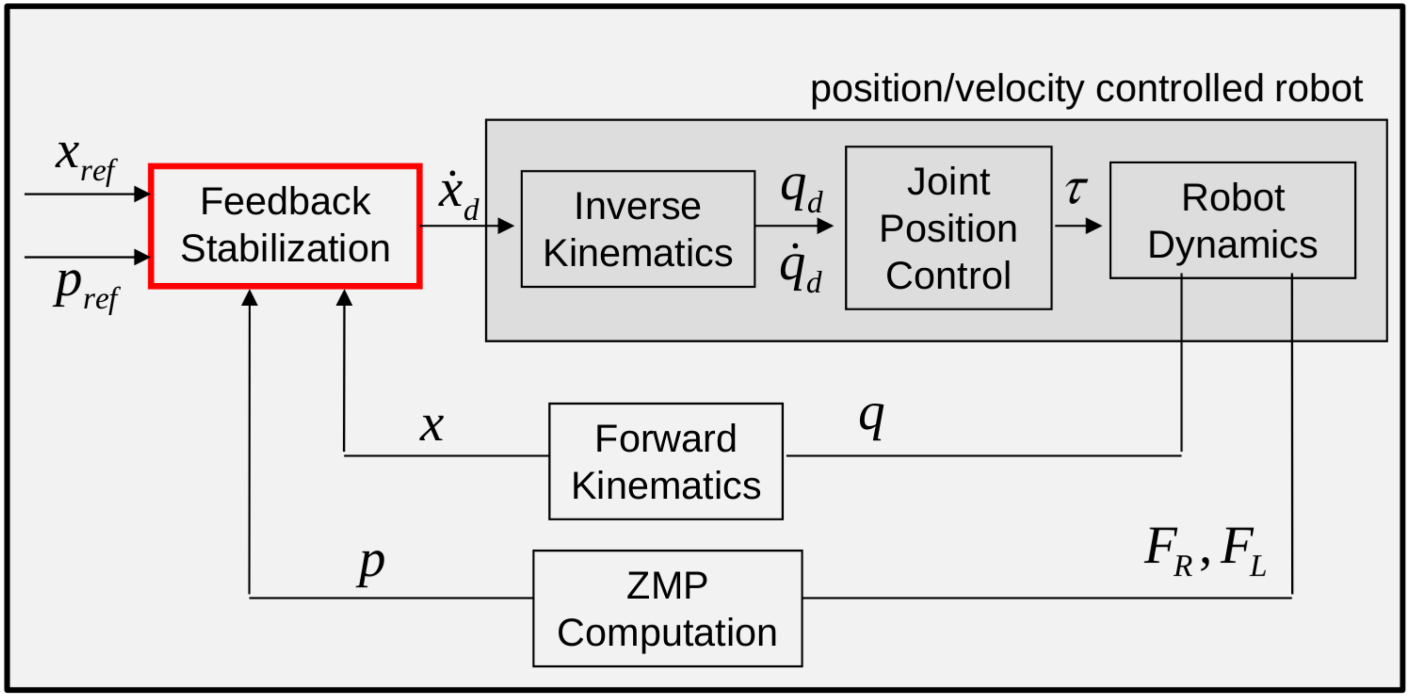
$$\dot{\boldsymbol{x}}_d = \dot{\boldsymbol{x}}_{ref} - \mathbf{K}_x(\boldsymbol{x}_d - \boldsymbol{x}_{ref}) + \mathbf{K}_p(\boldsymbol{p} - \boldsymbol{p}_{ref})$$

[Choi et al., 2007]

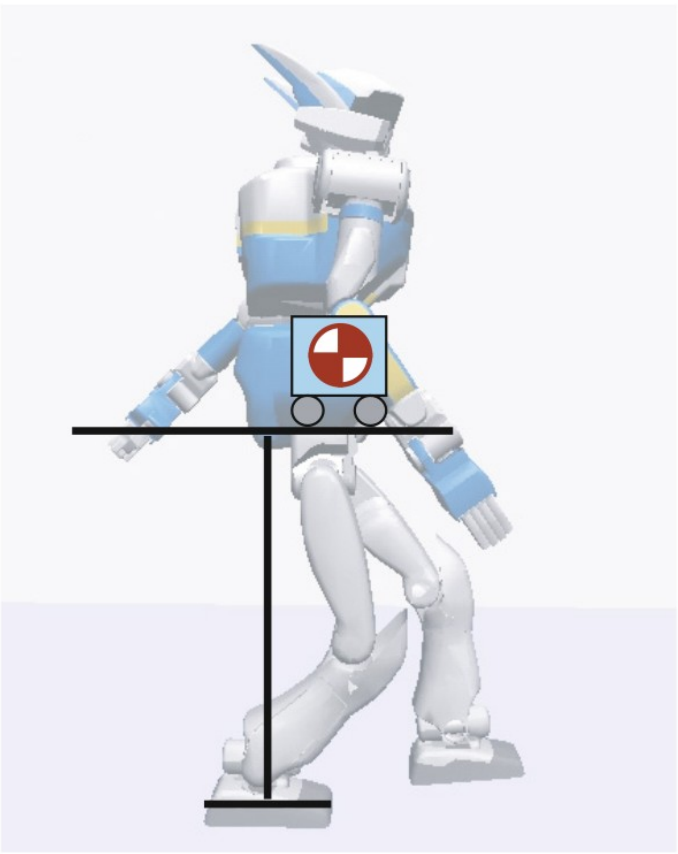
Ankle strategy

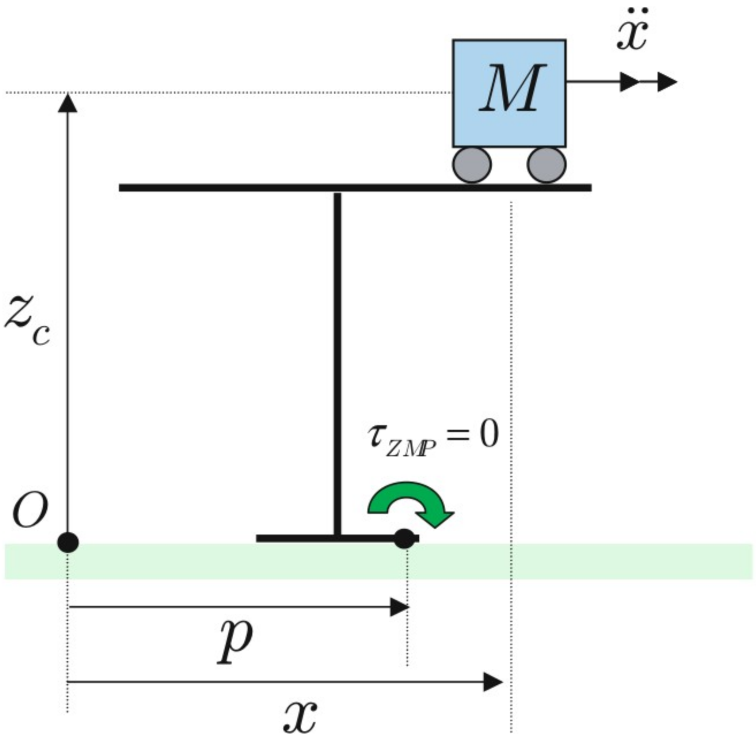
$$\dot{\boldsymbol{x}}_d = \dot{\boldsymbol{x}}_{ref} - \mathbf{K}_x(\boldsymbol{x}_d - \boldsymbol{x}_{ref}) + \mathbf{K}_p(\boldsymbol{p} - \boldsymbol{p}_{ref})$$

Stable if $0 < \mathbf{K}_p < \omega < \mathbf{K}_x$

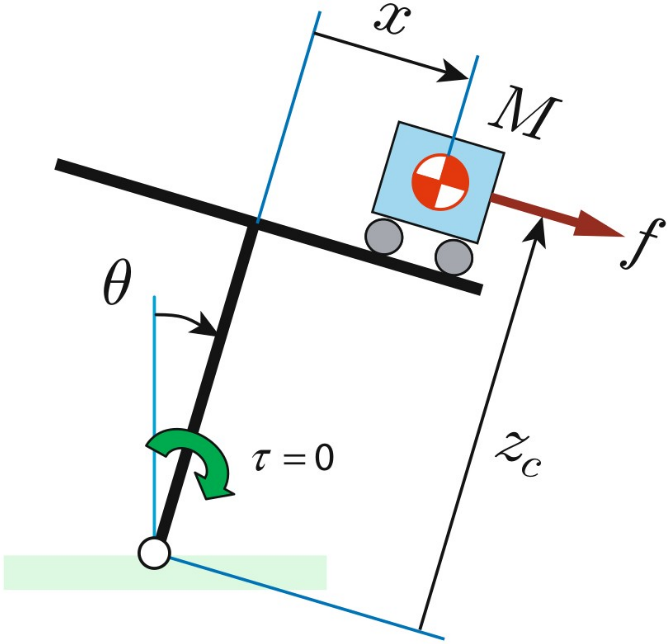


Ankle strategy (Cart table model)

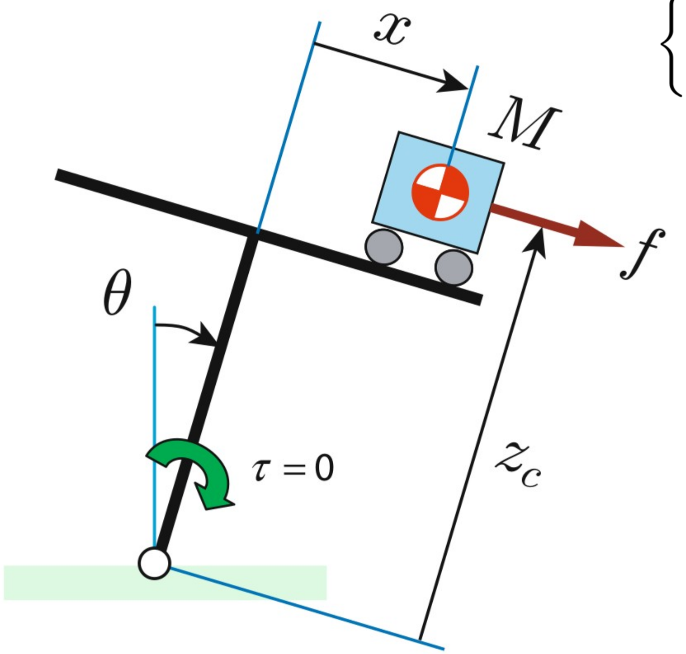




Ankle strategy (Cart table model)

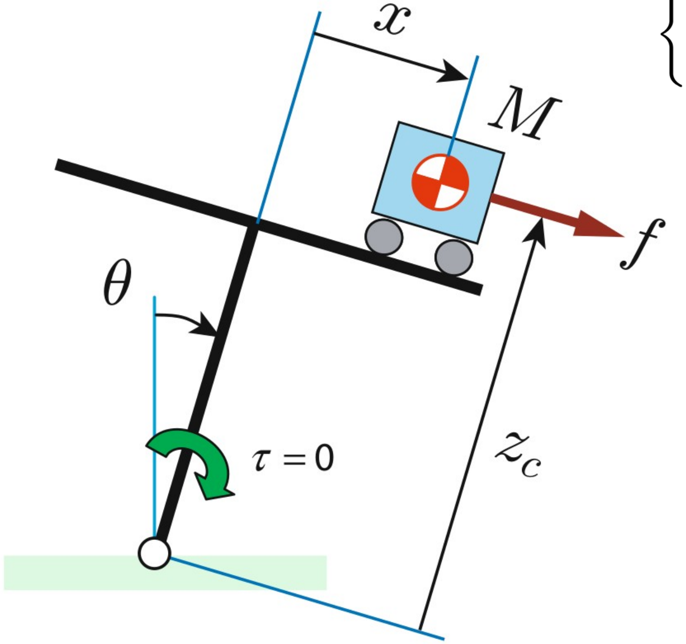


Ankle strategy (Cart table model)



$$\begin{cases} (x^2 + z_c^2)\ddot{\theta} + \ddot{x}z_c - g(z_c \sin\theta + x\cos\theta) + 2x\dot{x}\dot{\theta} = \tau/M \\ \ddot{x} + \ddot{z}_c - \dot{\theta}^2 x + g\sin\theta = f/M \end{cases}$$

Ankle strategy (Cart table model)



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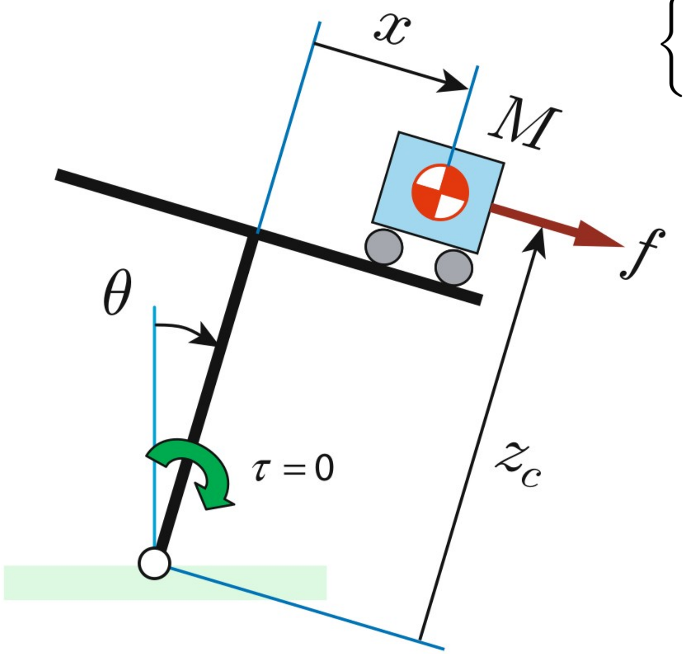
Linearizing around $\theta, \dot{\theta} = 0$

And setting $\tau = 0$

Ankle strategy (Cart table model)



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$$(x^2 + z_c^2)\ddot{\theta} = gx + gz_c\theta - z_c\ddot{x}.$$

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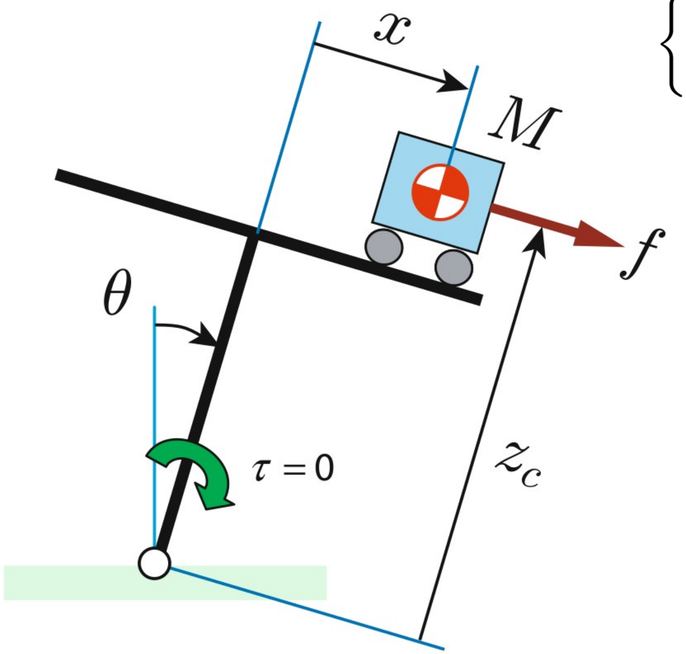

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Then we can define a target CoM

$$\ddot{x}_d = \frac{g}{z_c}(x_d - p_d)$$

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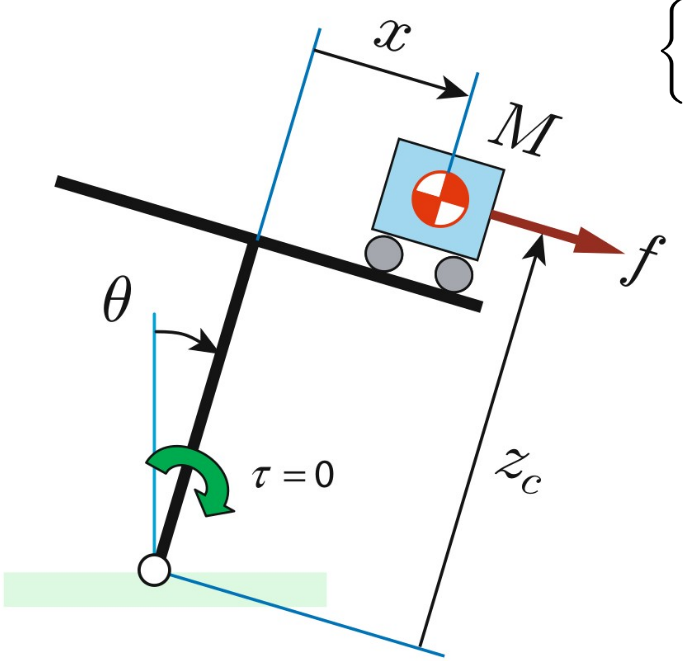
Ankle strategy (Cart table model)



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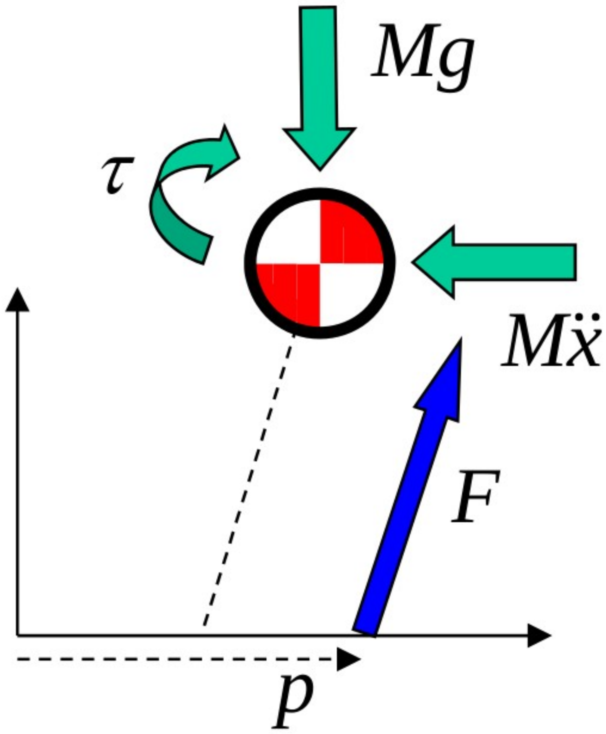
Then we can define a target CoM

$$\ddot{x}_d = \frac{g}{z_c}(x_d - p_d)$$

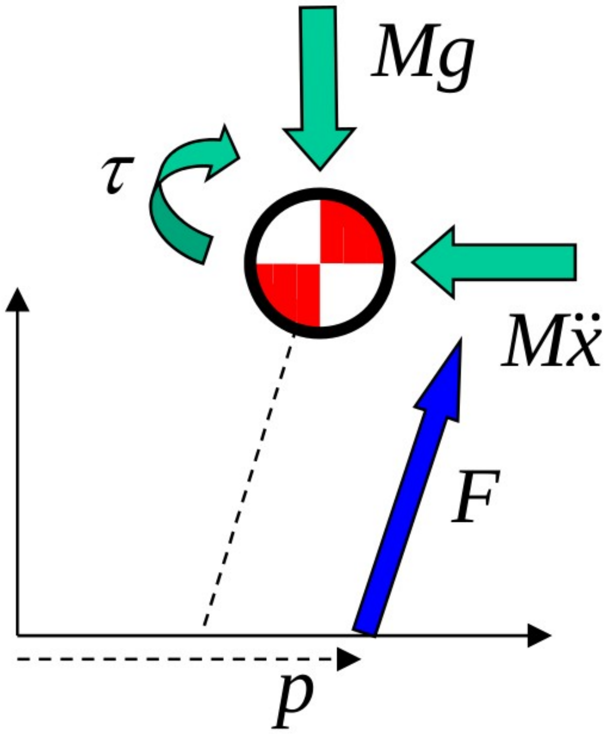
$$\ddot{\theta} = \frac{gz_c}{x_d^2 + z_c^2}\theta + \frac{g}{x_d^2 + z_c^2}\, p_d$$

Hip Strategy



$$\ddot{\boldsymbol{x}} = \frac{g}{\boldsymbol{x}_z}(\boldsymbol{x} - \boldsymbol{p}) + \frac{\tau}{M\boldsymbol{x}_z}$$

Hip Strategy



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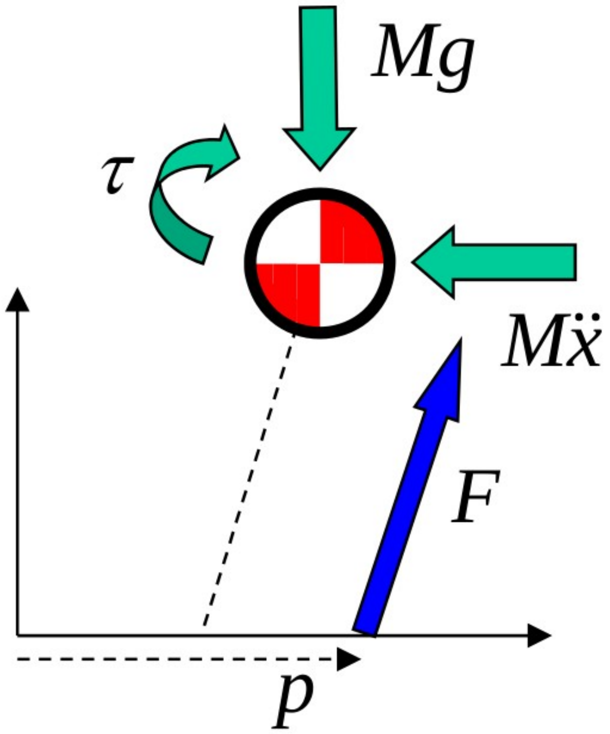
Hip Strategy

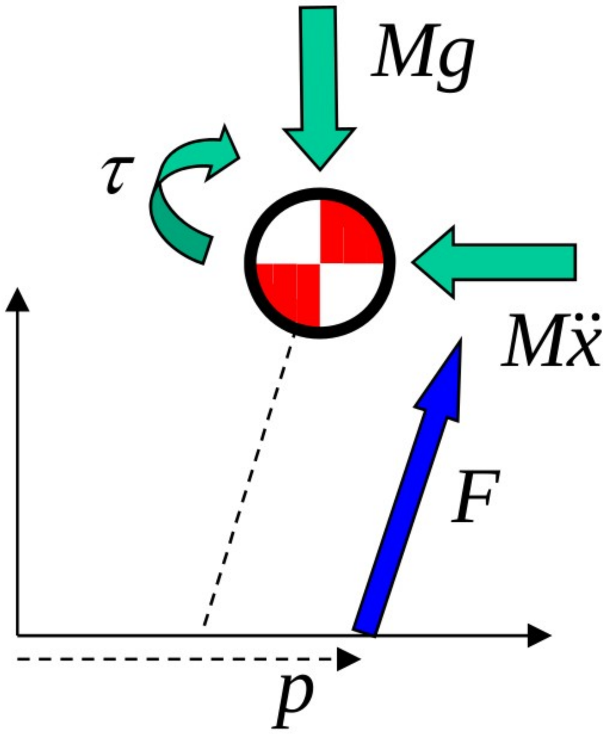


$$\ddot{\boldsymbol{x}} = \frac{g}{\boldsymbol{x}_z}(\boldsymbol{x} - \boldsymbol{p}) + \frac{\tau}{M\boldsymbol{x}_z}$$

$$\mathbf{I}\dot{\omega} = \tau$$

$$\boldsymbol{p} = -\frac{\boldsymbol{x}_z\ddot{\boldsymbol{x}}}{g} + \boldsymbol{x} + \frac{\mathbf{I}\dot{\omega}}{gM}$$

Hip Strategy



$$\ddot{\boldsymbol{x}} = \frac{g}{\boldsymbol{x}_z}(\boldsymbol{x} - \boldsymbol{p}) + \frac{\tau}{M\boldsymbol{x}_z}$$

$$\mathbf{I}\dot{\omega} = \tau$$

$$\boldsymbol{p} = -\frac{\boldsymbol{x}_z\ddot{\boldsymbol{x}}}{g} + \boldsymbol{x} + \frac{\mathbf{I}\dot{\omega}}{gM}$$

$$\boldsymbol{\tau} = \mathbf{K}_\theta(\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \mathbf{K}_\omega\dot{\omega}$$

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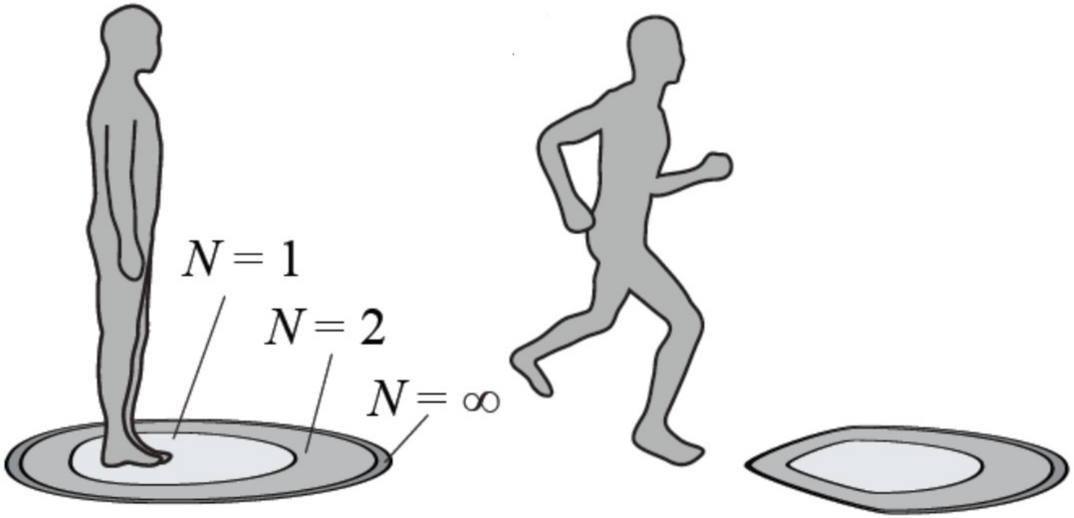
Capturability After N steps

Capturability after N steps

Assumptions:

- Constant step time T
- Maximal step size l_{max}
- Max distance between CP and ZMP at the beginning of swing leg d_N
- Point foot $d_0 = 0$



Capturability After N steps

$$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p}) \qquad \boldsymbol{\xi} = \boldsymbol{x}_p + \frac{\dot{\boldsymbol{x}}_p}{\boldsymbol{\omega}}$$

Capturability After N steps

$$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p}) \qquad \boldsymbol{\xi} = \boldsymbol{x}_p + \frac{\dot{\boldsymbol{x}}_p}{\boldsymbol{\omega}}$$

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \boldsymbol{p})$$

Capturability After N steps

$$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p}) \qquad \boldsymbol{\xi} = \boldsymbol{x}_p + \frac{\dot{\boldsymbol{x}}_p}{\boldsymbol{\omega}}$$

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \boldsymbol{p})$$

Solving for $\boldsymbol{\xi}$

$$\boldsymbol{\xi}(t) - \boldsymbol{p} = (\boldsymbol{\xi}(0) - \boldsymbol{p})\, e^{\omega t}$$

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Capturability After N steps

$$\ddot{\boldsymbol{x}} = \omega^2(\boldsymbol{x}_p - \boldsymbol{p}) \qquad \boldsymbol{\xi} = \boldsymbol{x}_p + \frac{\dot{\boldsymbol{x}}_p}{\boldsymbol{\omega}}$$

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \boldsymbol{p})$$

Solving for $\boldsymbol{\xi}$

$$\boldsymbol{\xi}(t) - \boldsymbol{p} = (\boldsymbol{\xi}(0) - \boldsymbol{p})\, e^{\omega t}$$

For the step time T

$$\boldsymbol{\xi}(T) - \boldsymbol{p} = (\boldsymbol{\xi}(0) - \boldsymbol{p})\, e^{\omega T}$$

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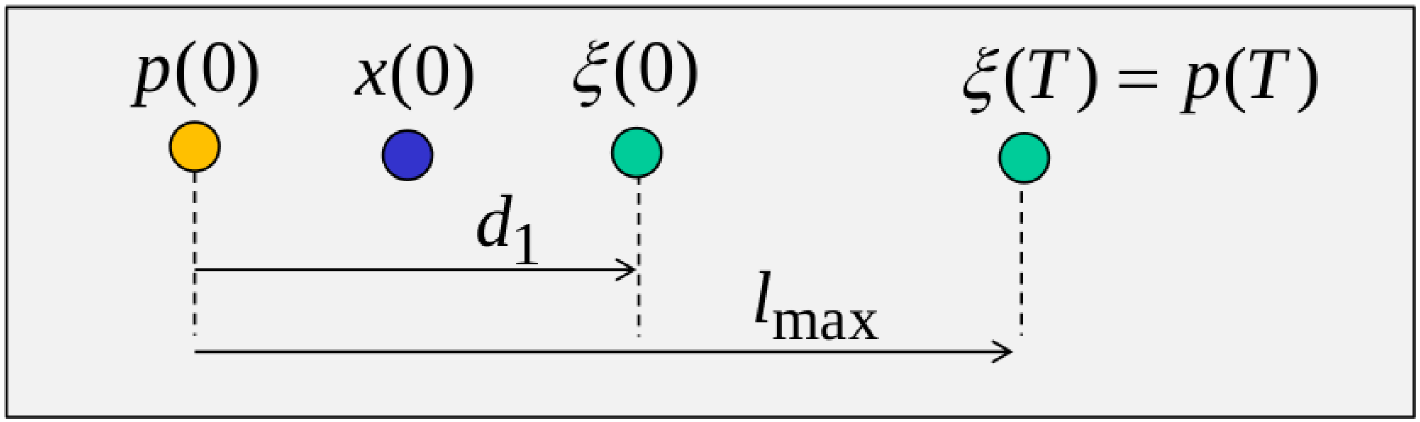
Capturability After N steps

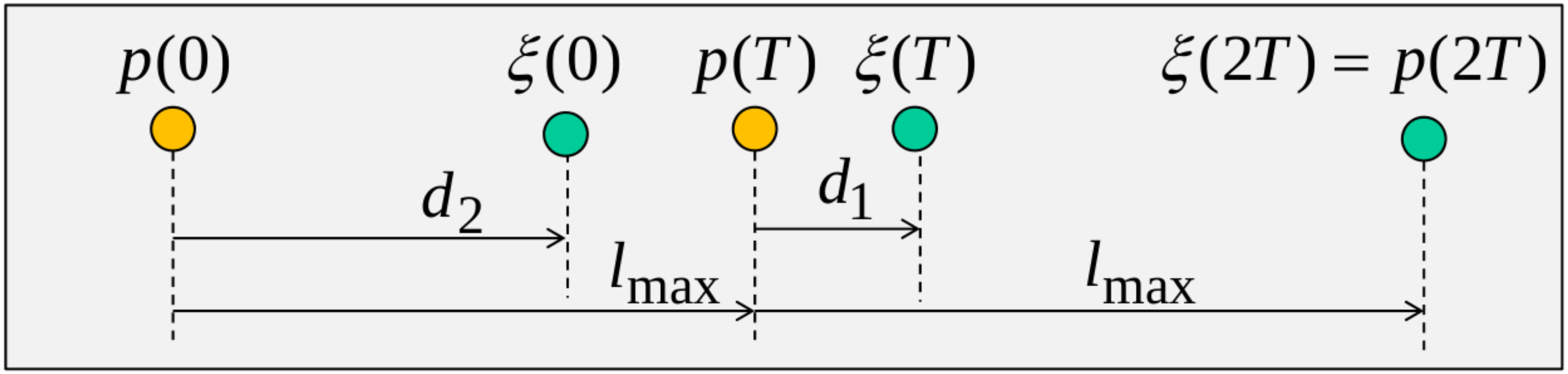
$d_1:$



$$\xi(t) - p = \left(\xi(0) - p\right)e^{\omega t} \quad \Longrightarrow \quad d_1 = l_{\max} e^{-\omega T}$$

Capturability After N steps

$d_2:$



$$\xi(t) - p = (\xi(0) - p)e^{\omega t} \Longrightarrow \xi(T) - p(0) = d_2 e^{\omega T} = l_{\max} + d_1$$

$$\Longrightarrow \boxed{d_2 = (l_{\max} + d_1)e^{-\omega T}}$$

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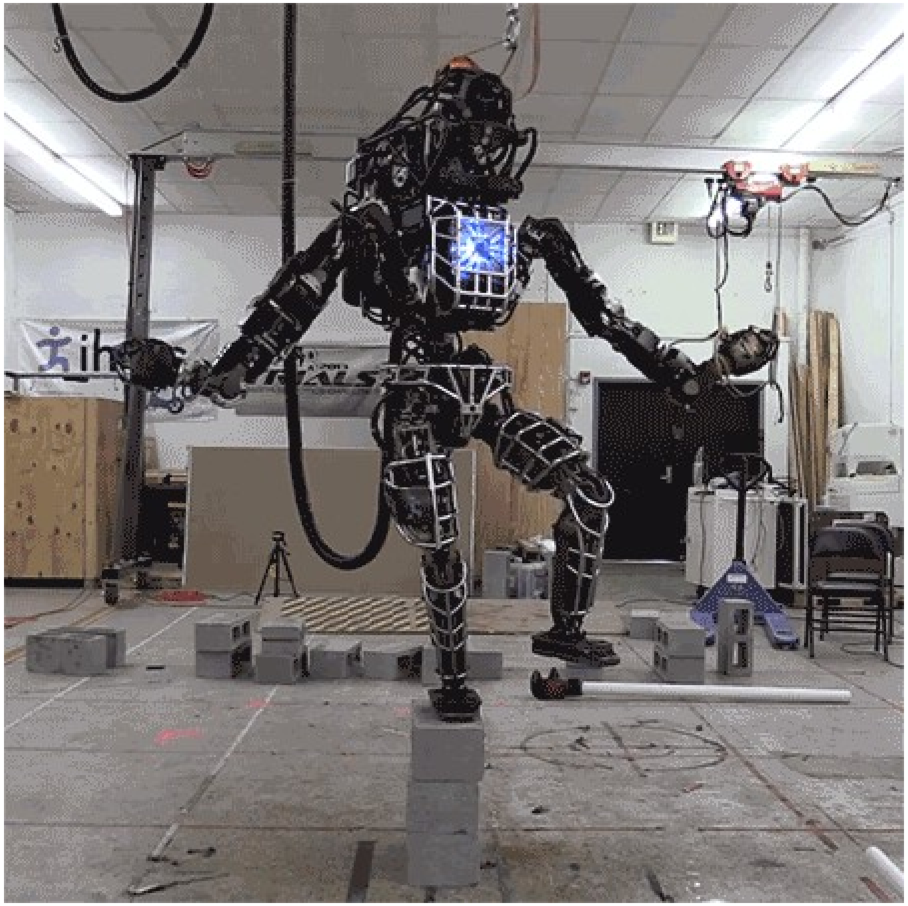
Capturability After N steps

d_n : Recursive computation: $\boxed{d_n = (d_{n-1} + l_{\max})e^{-\omega T}}$ $d_0 = 0$

Questions?



Next session:

Tutorial 6: Stability and Balance