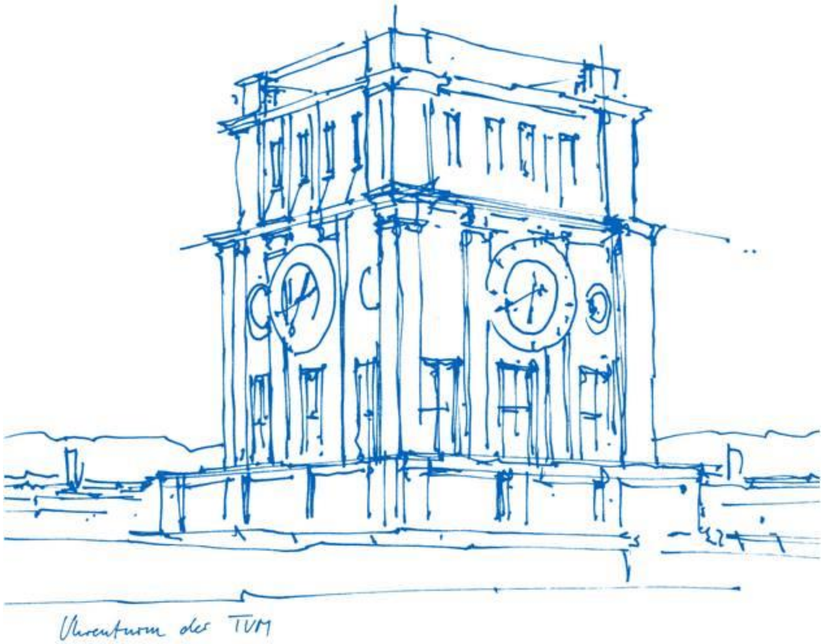
Modeling and Control of Legged Robots
SS 2024

L5: 3D Vision for Robotics



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Munich 04. June 2024

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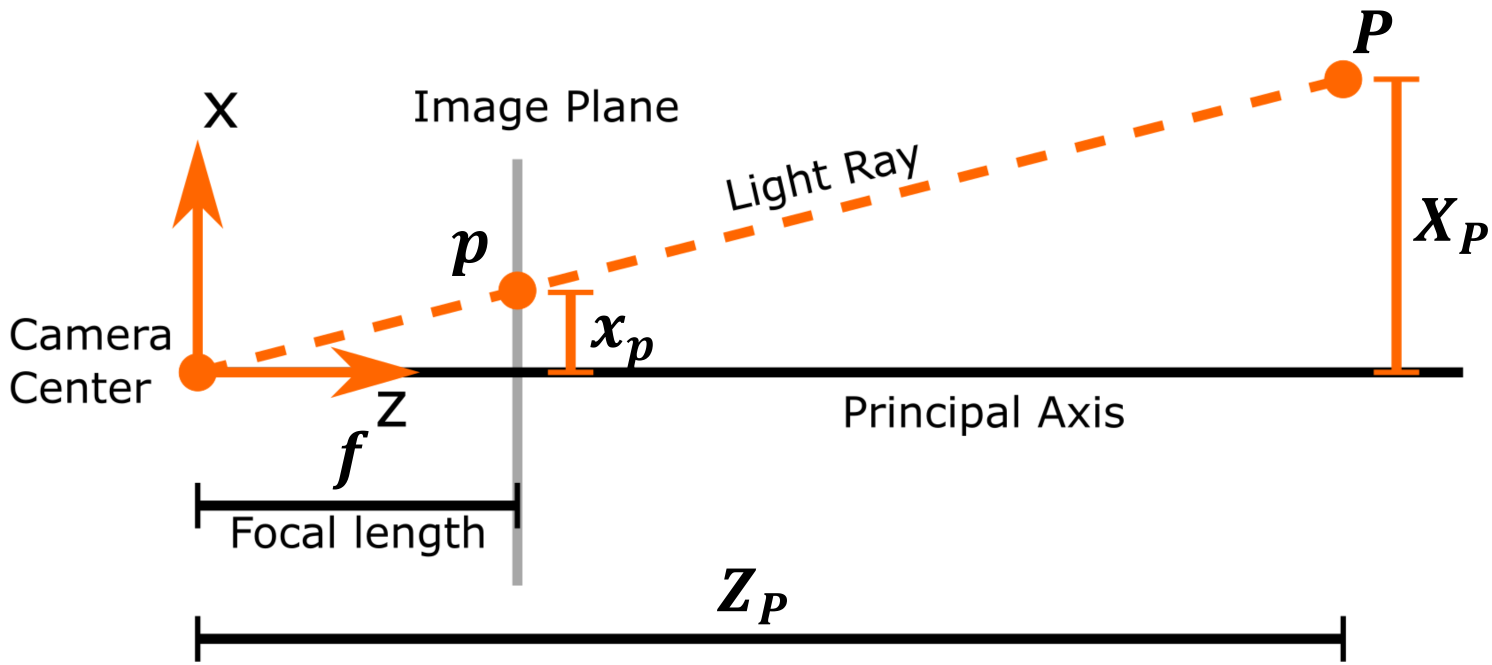
Introduction to Camera Projection



In the camera coordinate (x, y, z) :

$$\frac{x_p}{f} = \frac{X_P}{Z_P} \qquad \frac{y_p}{f} = \frac{Y_P}{Z_P}$$

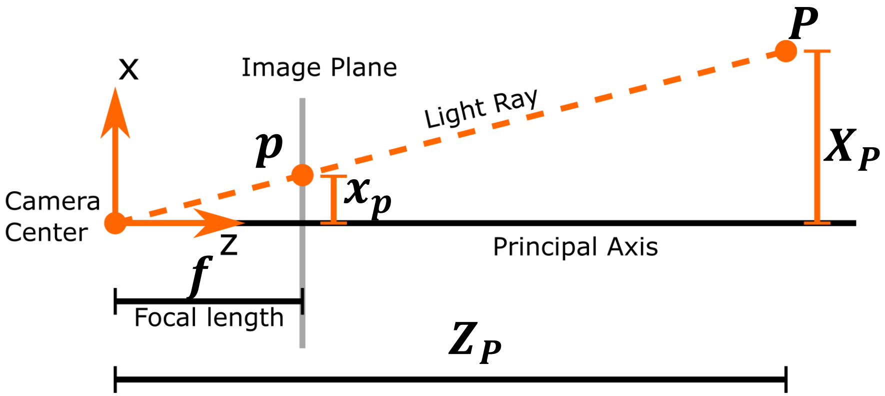
Introduction to Camera Projection

In the camera coordinate (x, y, z) :

$$\frac{x_p}{f} = \frac{X_P}{Z_P} \qquad \frac{y_p}{f} = \frac{Y_P}{Z_P}$$

$$\begin{pmatrix} x_p \\ y_p \end{pmatrix} = \begin{pmatrix} fX_P/Z_P \\ fY_P/Z_P \end{pmatrix}$$

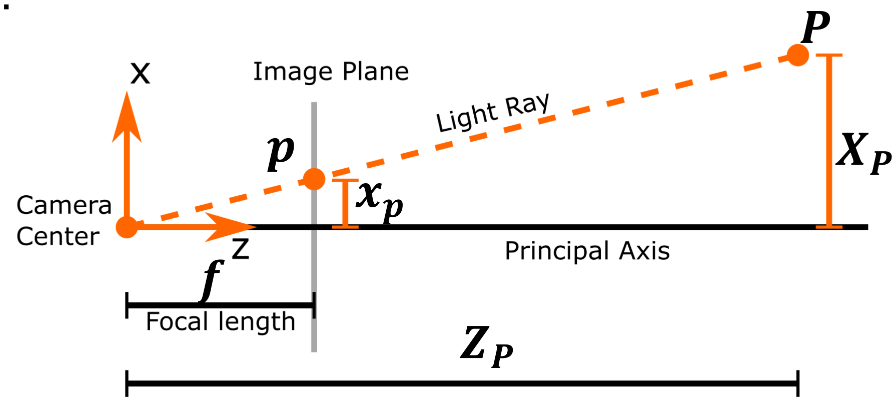
$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_P/Z_P \\ Y_P/Z_P \\ 1 \end{bmatrix}$$



Introduction to Camera Projection

In the camera coordinate (x, y, z) :

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_P/Z_P \\ Y_P/Z_P \\ 1 \end{bmatrix}$$



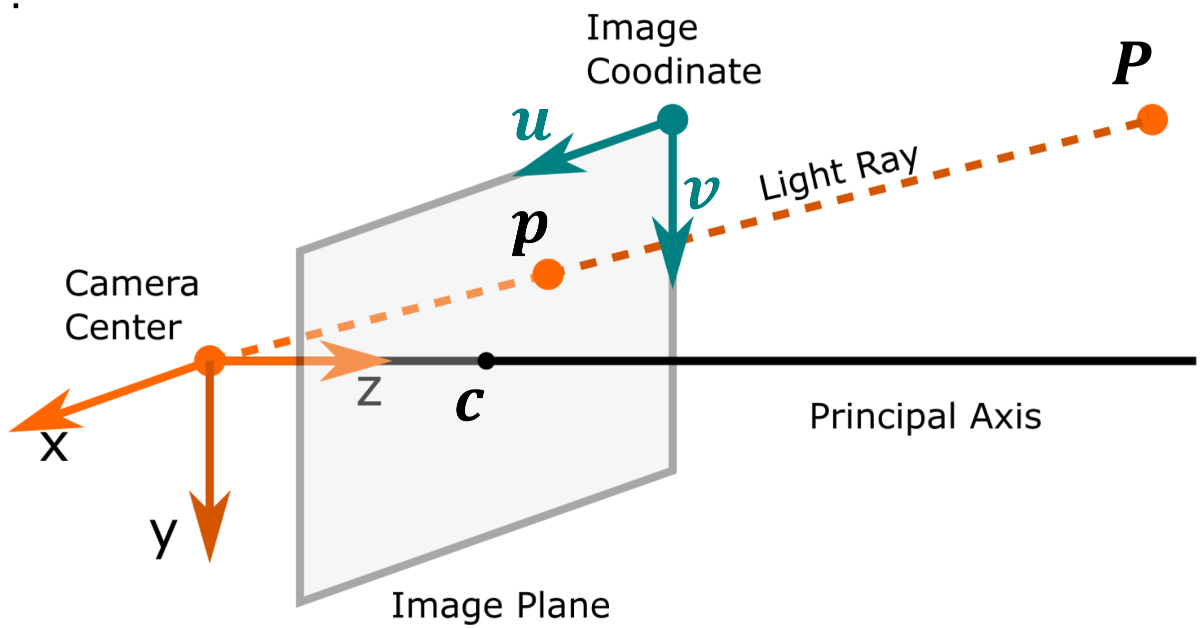
- In perspective projection, 3D points in **camera coordinates** are mapped to the image plane by dividing them by their **Z** component and multiplying with the focal length
- After the projection it is not possible to recover the distance of the 3D point from the image.

Introduction to Camera Projection

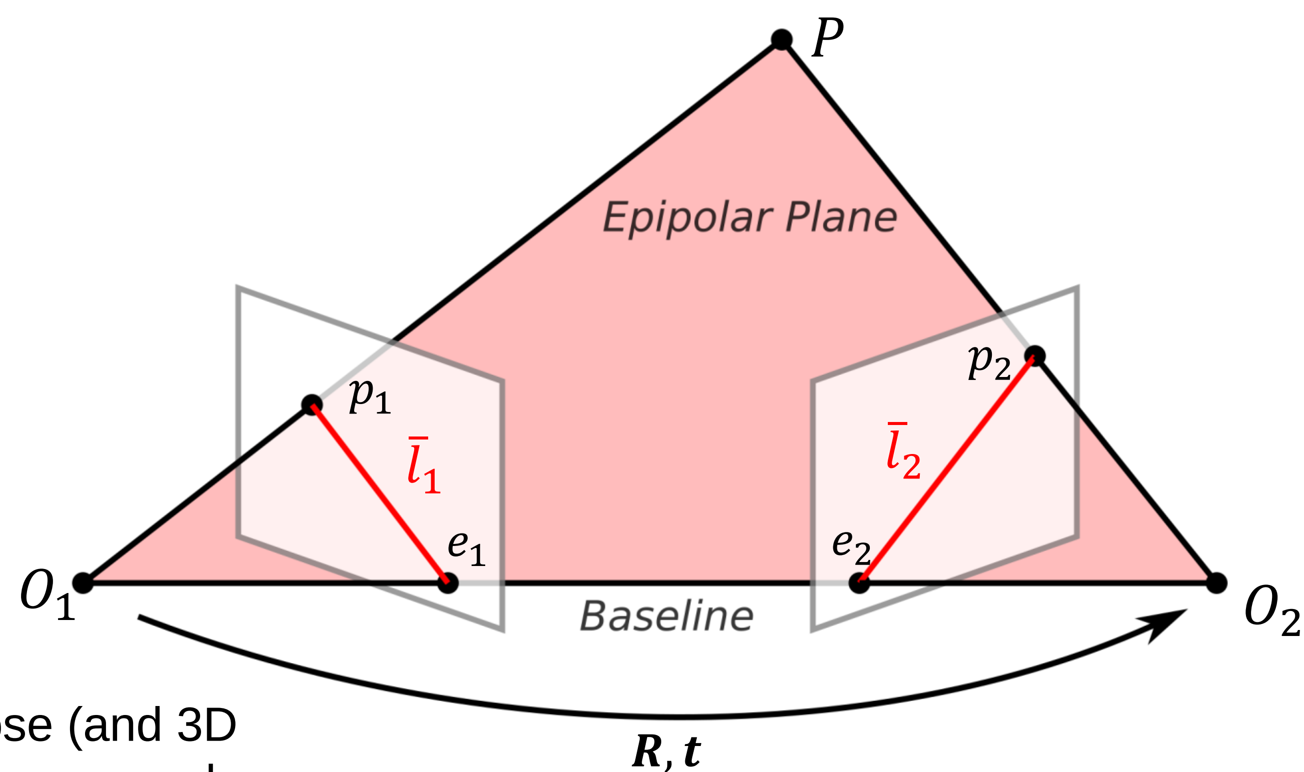
In the image coordinate (u, v) :

$$\begin{pmatrix} u_p \\ v_p \end{pmatrix} = \begin{pmatrix} \frac{fX_P}{Z_P} + c_x \\ \frac{fY_P}{Z_P} + c_y \end{pmatrix}$$

$$\begin{bmatrix} u_p \\ v_p \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_P/Z_P \\ Y_P/Z_P \\ 1 \end{bmatrix}$$



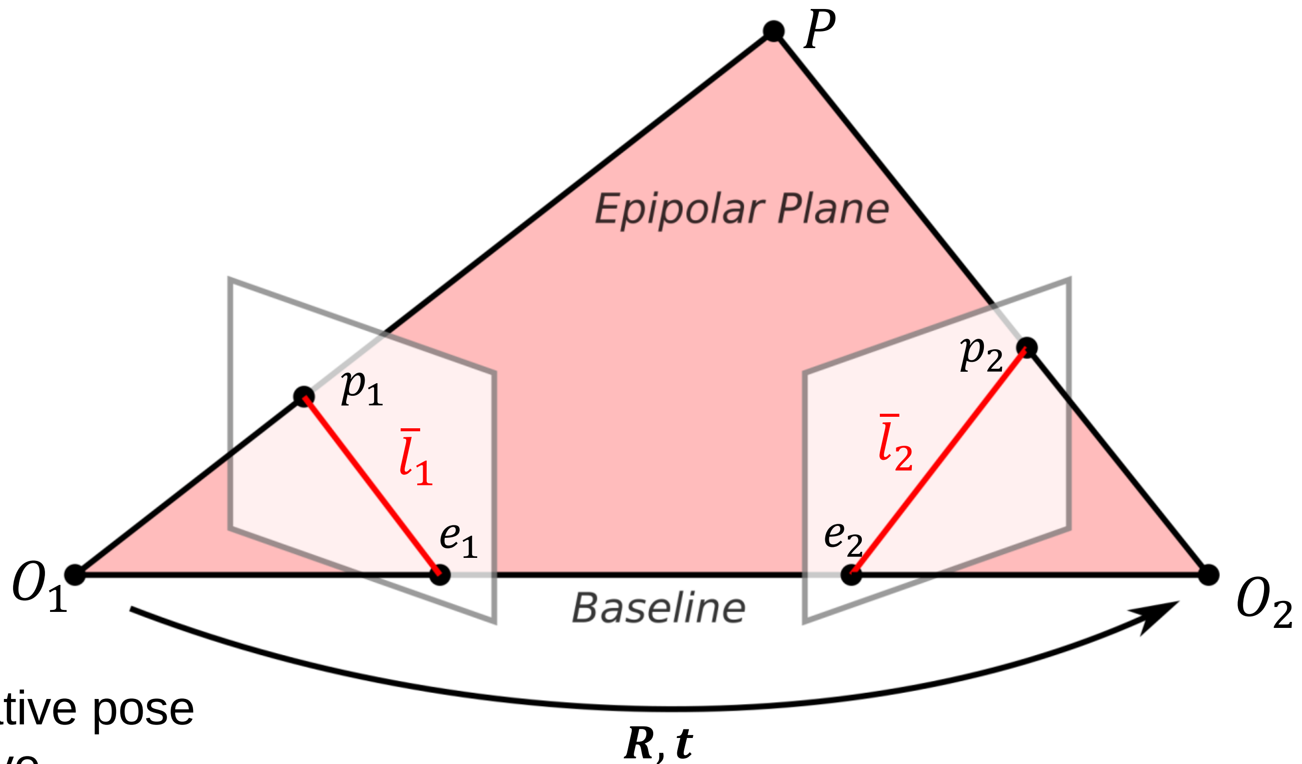
Epipolar Geometry



Recovery of camera pose (and 3D structure) from image correspondences. The required relationships are described by the two-view **epipolar geometry**.

Epipolar lines: $\bar{l}_1, \bar{l}_2$
Epipoles: e_1, e_2

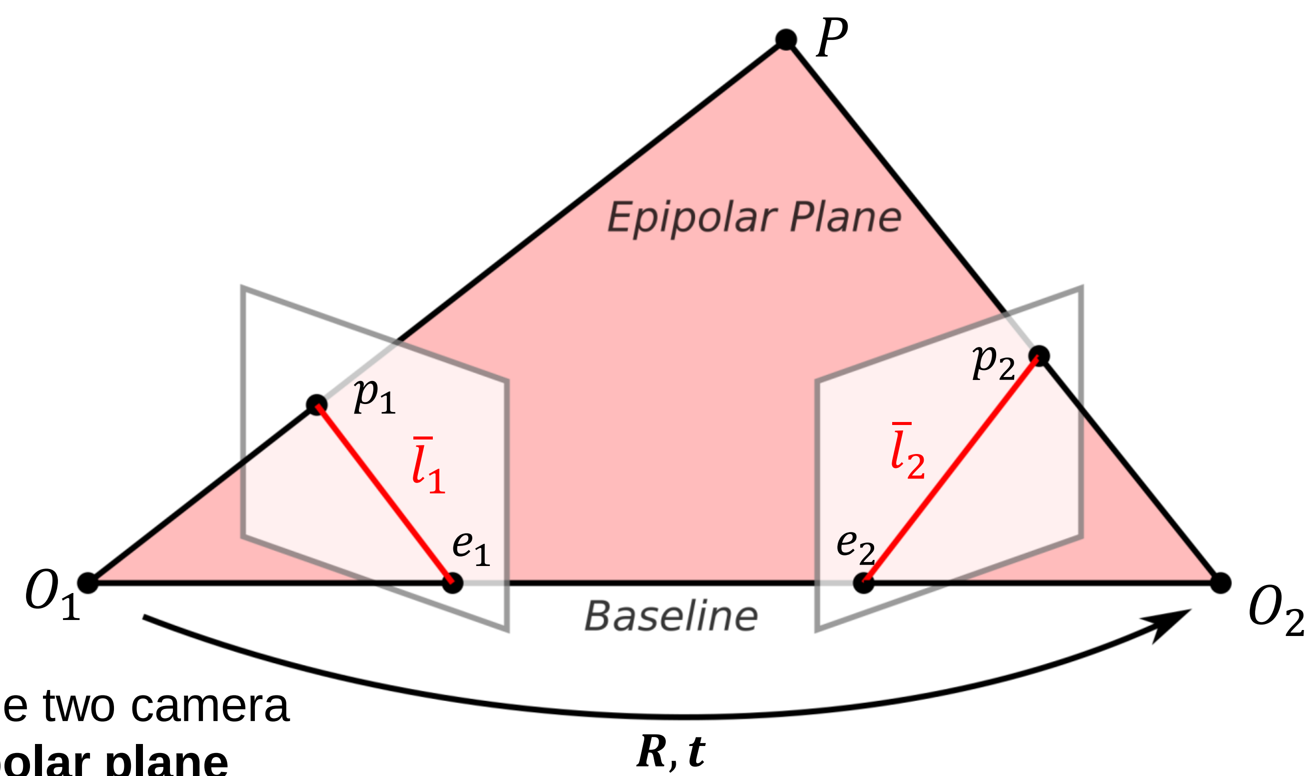
Epipolar Geometry



R and **t** denote the relative pose between two perspective cameras

Epipolar lines: $\bar{l}_1, \bar{l}_2$
Epipoles: e_1, e_2

Epipolar Geometry



The 3D point **P** and the two camera centers span the **epipolar plane**

The correspondence of pixel p_1 in image 2 must lie on the **epipolar line** $\bar{l}_2$ in image 2

Epipolar lines: $\bar{l}_1, \bar{l}_2$
Epipoles: e_1, e_2

Epipolar Geometry

$$\tilde{P}_2 = \boldsymbol{R}\tilde{P}_1 + \boldsymbol{t}$$

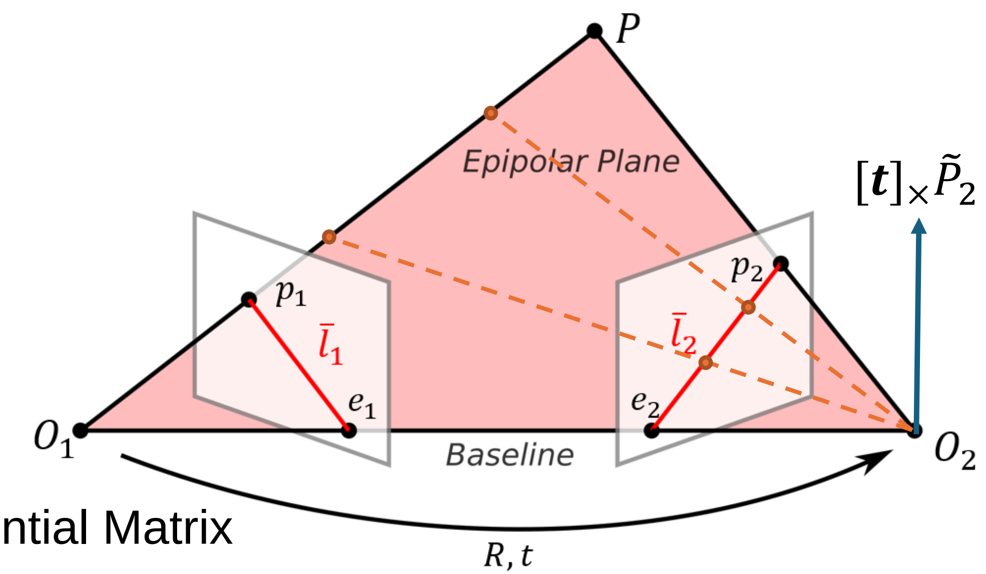
$$[\boldsymbol{t}]_\times \tilde{P}_2 = [\boldsymbol{t}]_\times \boldsymbol{R}\tilde{P}_1$$

$$\tilde{P}_2^T [\boldsymbol{t}]_\times \tilde{P}_2 = \tilde{P}_2^T [\boldsymbol{t}]_\times \boldsymbol{R}\tilde{P}_1 = 0$$

$$\boxed{\boldsymbol{E} = [\boldsymbol{t}]_\times \boldsymbol{R}}$$

Essential Matrix

$$\tilde{P}_2^T \boldsymbol{E}\tilde{P}_1 = 0$$



Epipolar lines: $\bar{l}_1, \bar{l}_2$

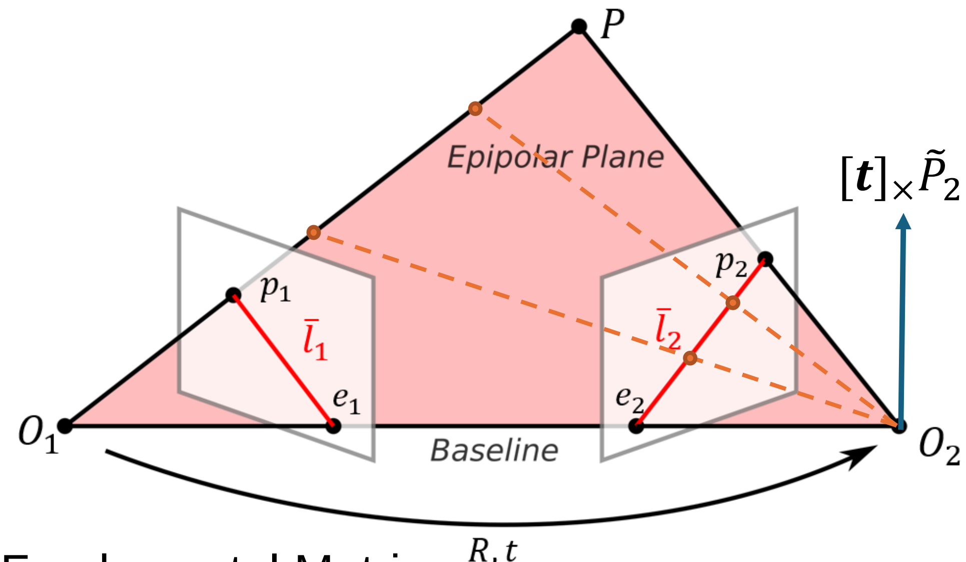
Epipoles: e_1, e_2

Epipolar Geometry

$$p_1 = K\tilde{P}_1,\, p_2 = K\tilde{P}_2$$

$$\tilde{P}_1 = K^{-1}p_1,\, \tilde{P}_2 = K^{-1}p_2$$

$$p_2^T {K^{-1}}^T E K^{-1} p_1$$



$$\boxed{\boldsymbol{F} = {K^{-1}}^T \boldsymbol{E} K^{-1}} \qquad p_2^T \boldsymbol{F} p_1 = 0 \qquad \text{Fundamental Matrix}$$

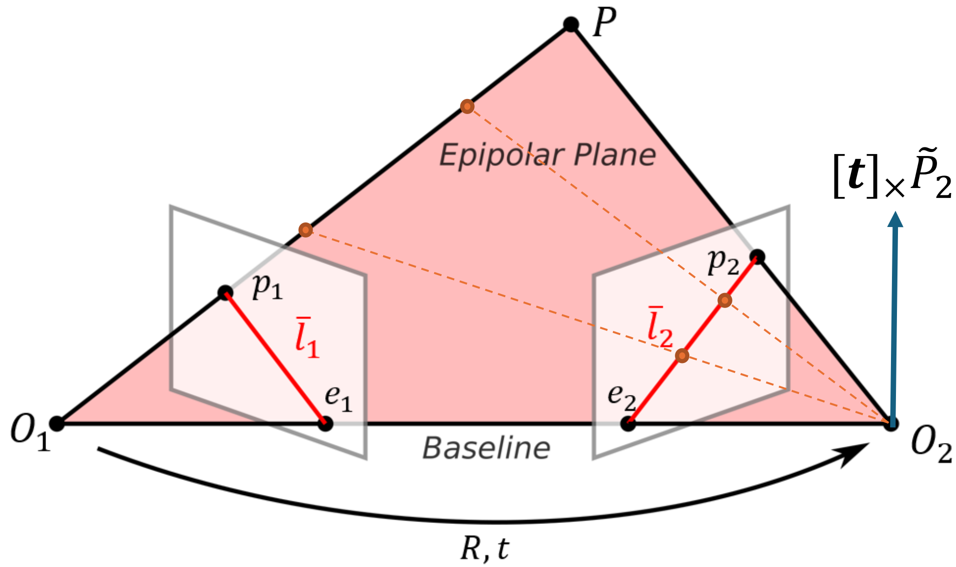
Epipolar lines: $\bar{l}_1, \bar{l}_2$
Epipoles: e_1, e_2

$$\bar{l}_2 = \boldsymbol{F} p_1 \text{ , } \bar{l}_1 = p_2^T \boldsymbol{F} \text{ , } p_2^T \bar{l}_2 = 0 \text{ , } \bar{l}_1 p_1 = 0$$

Estimating the Essential Matrix

$$\tilde{P}_2^T \boldsymbol{E} \tilde{P}_1 = 0$$

$$x_1 x_2 e_{11} + y_1 x_2 e_{12} + x_2 e_{13}$$
$$+ x_1 y_2 e_{21} + y_1 y_2 e_{22} + y_2 e_{23}$$
$$+ x_1 e_{31} + y_1 e_{32} + e_3 = 0$$



As $\boldsymbol{E}$ is homogeneous we use singular value decomposition to constrain the scale.
Note that some terms are products of two image measurements and hence amplify measurement noise asymmetrically. Thus, the normalized 8-point algorithm whitens the observations to have zero-mean and unit variance before the calculation and back-transforms the matrix recovered by SVD accordingly.

Estimating the Translation and Rotation

From $\boldsymbol{E}$, we can recover the direction of the translation vector $\boldsymbol{t}$. We have:

$$\hat{\boldsymbol{t}}^T \boldsymbol{E} = \hat{\boldsymbol{t}}^T [\boldsymbol{t}]_\times \boldsymbol{R} = 0$$

Thus, $\boldsymbol{E}$ is singular and we obtain $\hat{\boldsymbol{t}}$ as the left singular vector associated with singular value 0. In practice the singular value will not be exactly 0 due to measurement noise, and we choose the smallest one. The other two singular values are roughly equal

$$\boldsymbol{E} = \boldsymbol{U\Sigma V}^T,\ \boldsymbol{\Sigma} = diag(1, 1, 0) \text{ to scale}$$

Estimating the Translation and Rotation

Two possible choices of the rotation matrix $\boldsymbol{R}$:

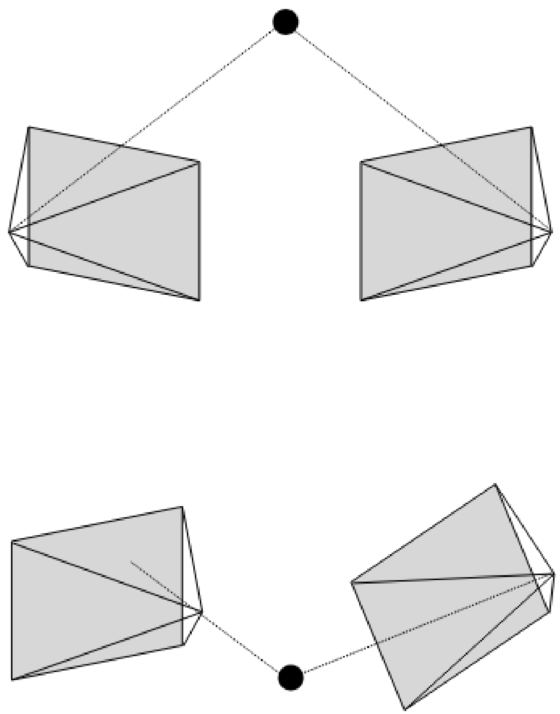
$$\boldsymbol{R_1} = \boldsymbol{UZV}^T \text{ or } \boldsymbol{R_1} = \boldsymbol{UZ^TV}^T, \text{ where } \boldsymbol{Z} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two possible choices of translation direction $\hat{\boldsymbol{t}}$

$$\hat{\boldsymbol{t}} = \boldsymbol{u}_3 \text{ or } -\boldsymbol{u}_3, \text{ where } \boldsymbol{u}_3 \text{ is the last column of } \boldsymbol{U}$$

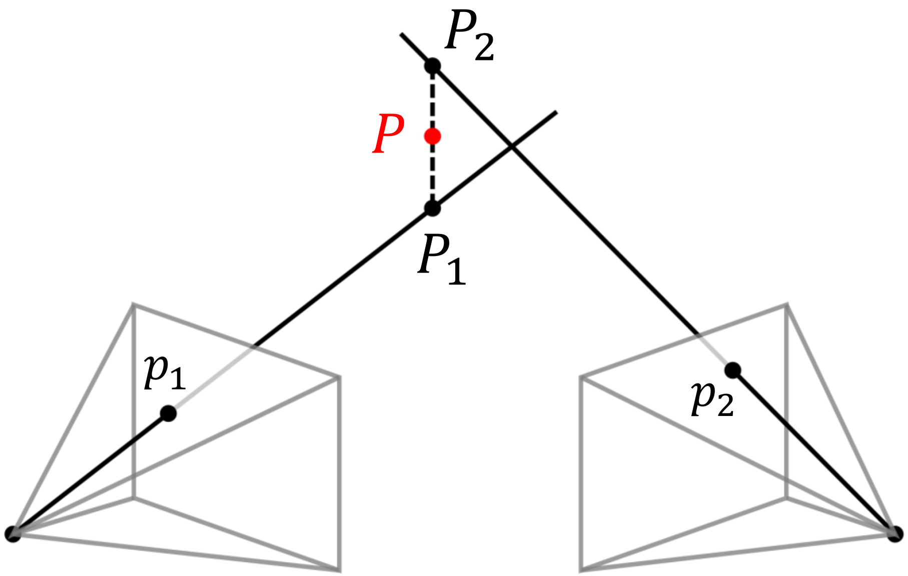
Estimating the Translation and Rotation

Four possible solutions: can be solved by point triangulation



Triangulation

How to recover 3D geometry with known intrinsic and extrinsic?



Given noisy 2D image correspondings p_1 and p_2, the two light rays might not intersect.
We want to recover the 3D point P that is closest to the two rays.

Triangulation

In the projection process $p_i = K\tilde{P}_i$, both sides are homogeneous, they have the same direction but different magnitude.
To account for this, we consider the cross product $p_i \times K\tilde{P}_i = 0$, in a rowwise form:

$$\underbrace{\begin{bmatrix} x_i k_{i3} - k_{i1} \\ y_i k_{i3} - k_{i2} \end{bmatrix}}_{\boldsymbol{A}_i} \tilde{P}_i = 0$$

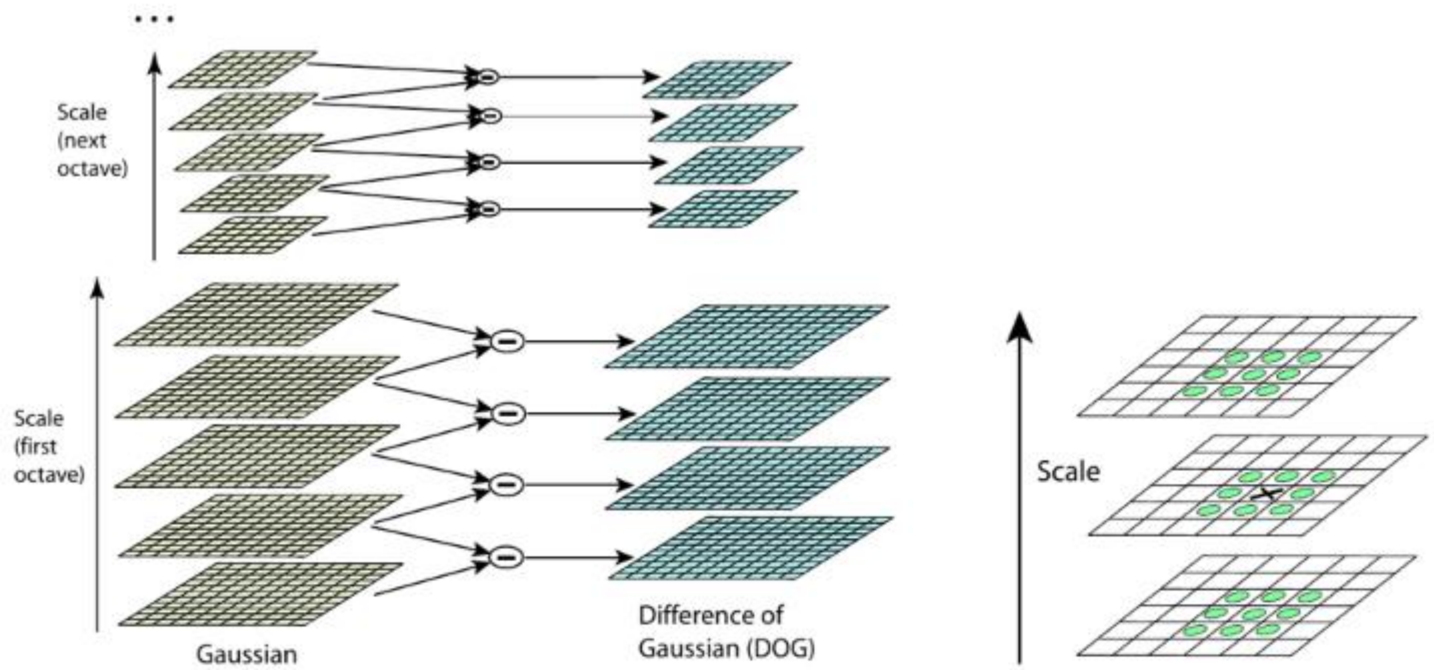
Stacking N ≥ 2 observations of a point, we obtain a linear system $\boldsymbol{A}\tilde{P}_i = 0$.
As $\tilde{P}_i$ is homogeneous this leads to a constrained least squares problem. The solution to this problem is the **right singular vector** corresponding to the smallest singular value of $\boldsymbol{A}$.

Feature Points

Detection and description

- Features should be invariant to perspective effects and illumination
- The same point should have similar vectors independent of pose/viewpoint
- Plain RGB/intensity patches will not have this property, we need something better

Feature Points:
Scale Invariant Feature Transform (SIFT)



- SIFT constructs a scale space by iteratively filtering the image with a Gaussian
- Adjacent scales are subtracted, yielding Difference of Gaussian (DoG) images
- Interest points (=blobs) are detected as extrema in the resulting scale space

Feature Points:
Scale Invariant Feature Transform (SIFT)

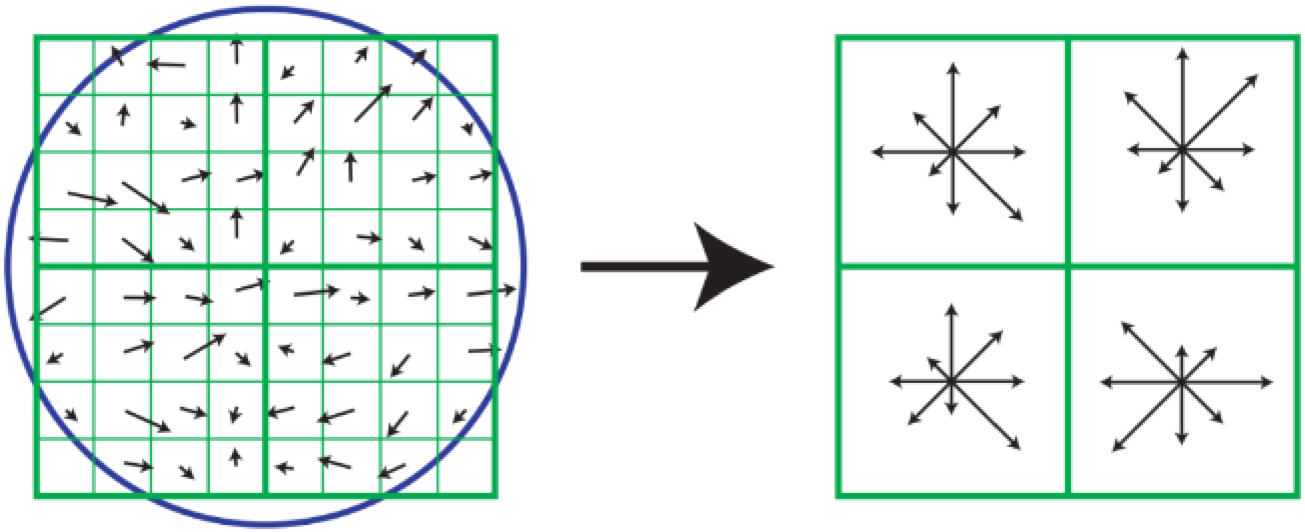
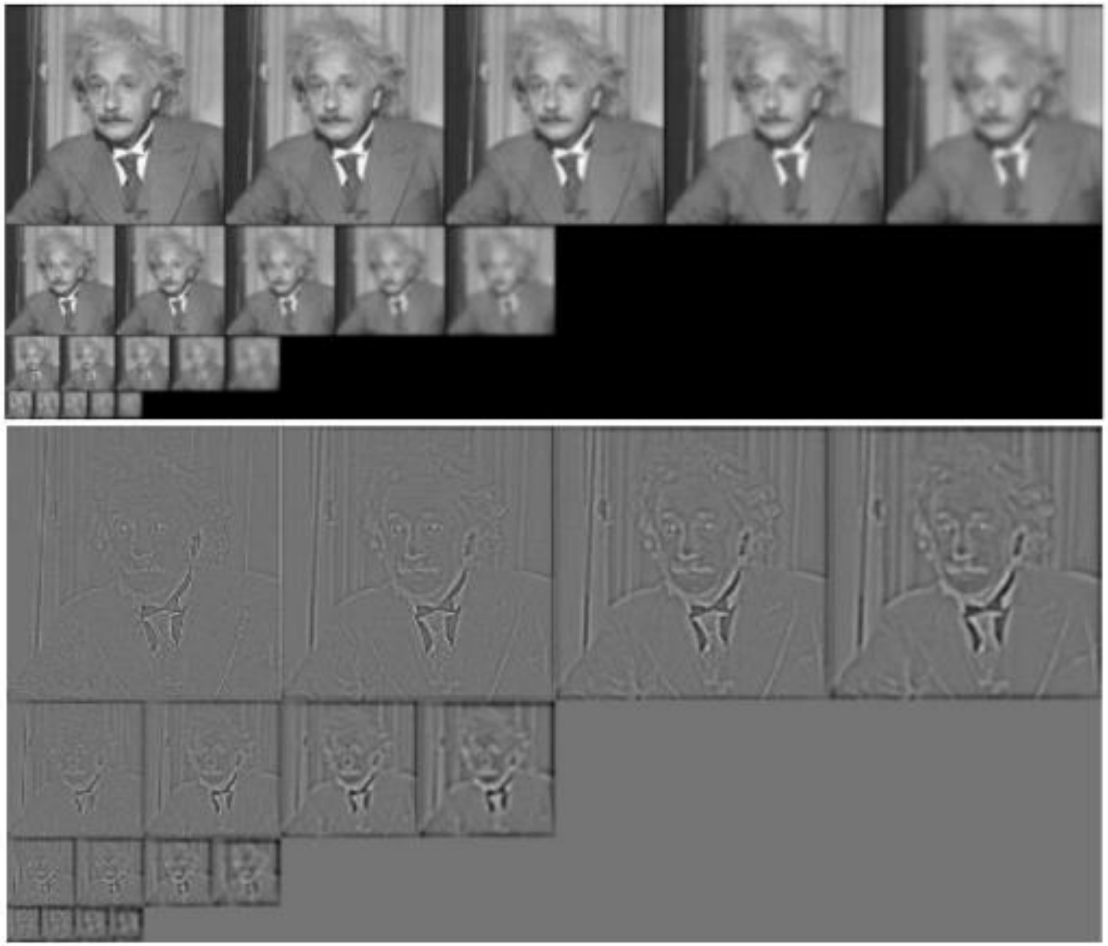


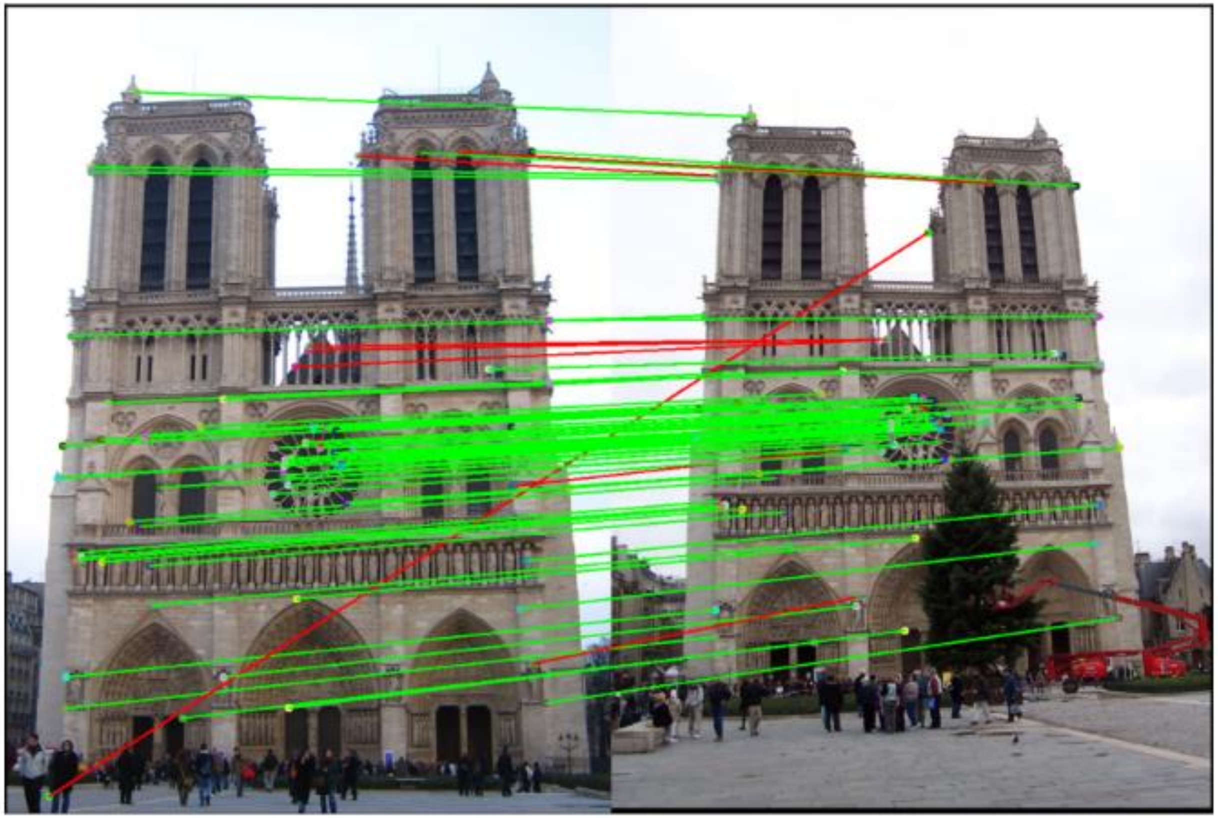
Image gradients Keypoint descriptor

- SIFT rotates the descriptor to align with the dominant gradient orientation
- Gradient histograms are computed for local sub-regions of the descriptor
- All histograms are concatenated and normalized to form a 128D feature vector

Feature Points:
Scale Invariant Feature Transform (SIFT)



Feature Points:
Example



Example of two images with SIFT correspondences (green: correct, red: wrong)

Feature Points: Detection and Description

- Many algorithms for feature detection and description have been developed, e.g., SIFT, SURF, U-SURF, BRISK, ORB, FAST, and recently deep learning based ones
- SIFT was a seminal work due to its invariance and robustness which revolutionized recognition and in particular matching and enabled the development of large-scale SfM techniques which we discuss in this lecture
- Despite >20 years old, SIFT is still used today (e.g., in the SfM pipeline COLMAP)
- Feature correspondences can be retrieved with efficient nearest neighbor search
- Ambiguous matches are typically filtered by computing the ratio of distance from the closest neighbor to the distance of the second closest
- A large ratio (>0.8) indicates that the found match might not be the correct one

Tutorial:
Requirements

Python 3, numpy, opencv >= 3.1x

Contents:

1. Feature points detection: SIFT and OBR
2. Feature matching: Brute Force, KNN.
3. Estimating the essential matrix
4. Decompose the estimated essential matrix: singular values, vectors
5. Estimating translation and rotation matrix
6. Get the angle errors for the translation direction and rotation matrix.