Chair for Cognitive Systems
Department of Electrical and Computer Engineering
Technische Universität München

Modeling and Control of Legged Robots
T~~6~~: Trajectory Generation

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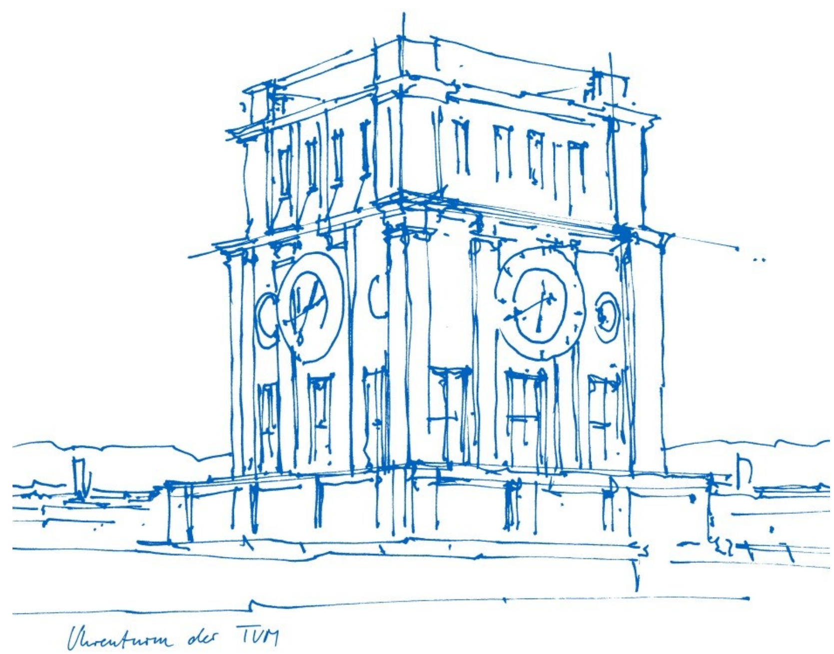
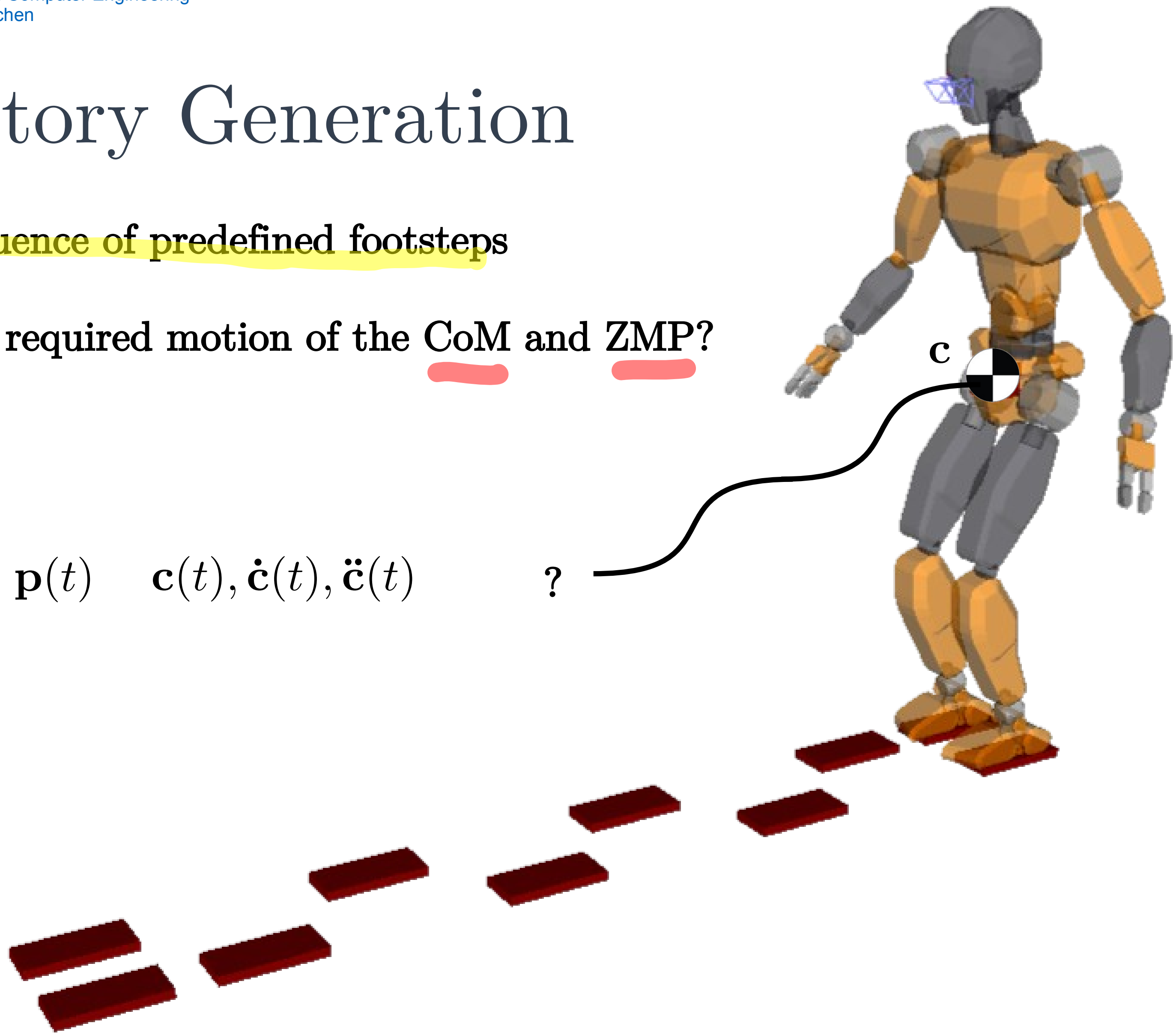


Trajectory Generation

Given a sequence of predefined footsteps

What is the required motion of the CoM and ZMP?

$\mathbf{p}(t) \quad \mathbf{c}(t), \dot{\mathbf{c}}(t), \ddot{\mathbf{c}}(t)$



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Trajectory Generation

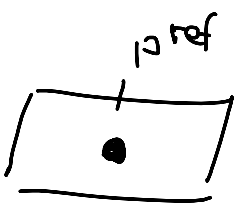
Given a sequence of predefined footsteps

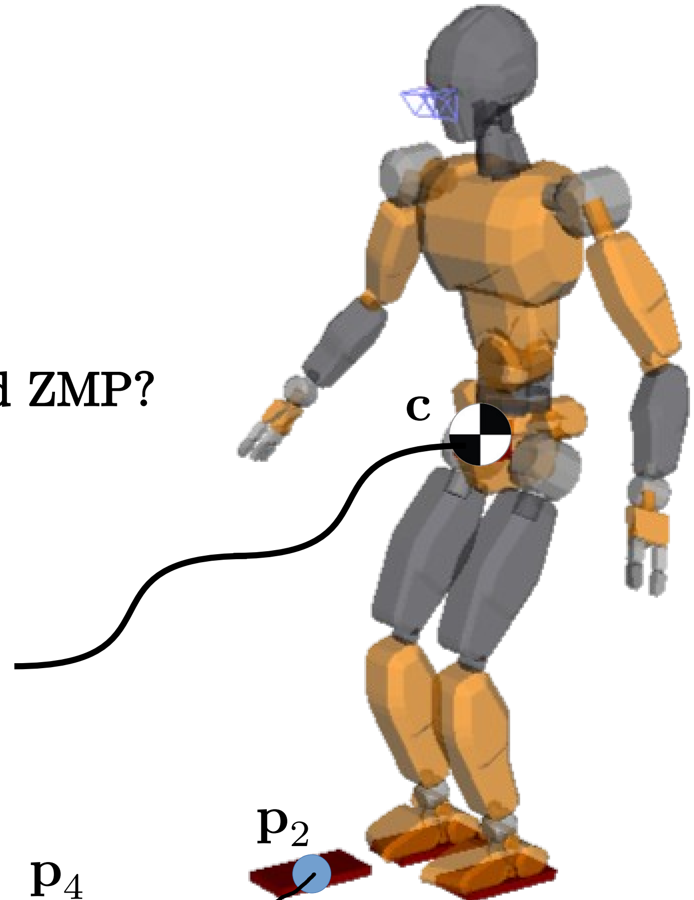
What is the required motion of the CoM and ZMP?

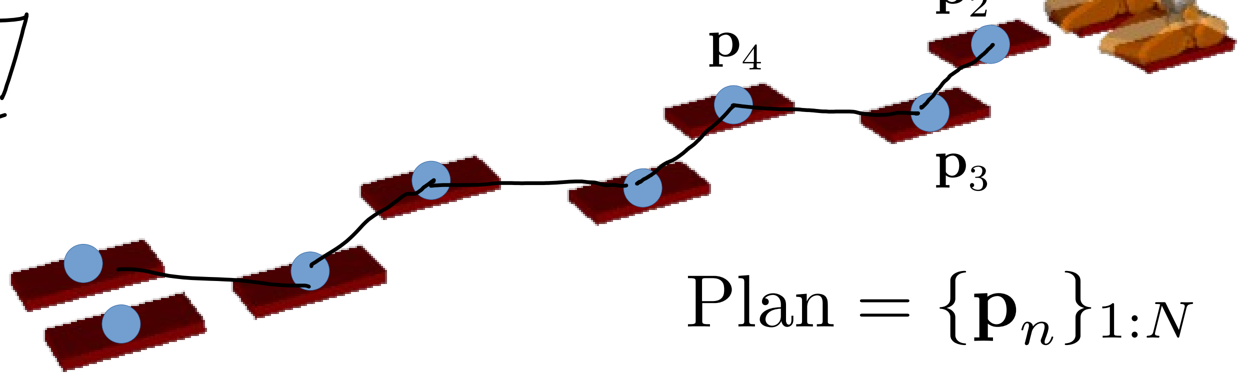
Remember:

ZMP needs to stay within Support polygon

→ ZMP reference are the footstep positions







$$\text{Plan} = \{\mathbf{p}_n\}_{1:N}$$

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Trajectory Generation

Can be formulated as Model Predictive Control (MPC)

ZMP Equation (sec. order eq.)

$$\ddot{c}_{xy} = \frac{g}{h}\left(c_{xy} - p_{xy}\right)$$

Dyn.

CoM pos

ZMP pos

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$$\ddot{c}_{xy} = \frac{g}{h}\left(c_{xy} - p_{xy}\right)$$

ΔT step size

Can be analytically integrated in time to obtain:

pos

vel

$$\underbrace{\begin{bmatrix} c_{xy}^{+} \\ \dot{c}_{xy}^{+} \end{bmatrix}}_{\text{com next step}} = \begin{bmatrix} \cosh(\omega\Delta T) & \omega^{-1}\sinh(\omega\Delta T) \\ \omega\sinh(\omega\Delta T) & \cosh(\omega\Delta T) \end{bmatrix} \underbrace{\begin{bmatrix} c_{xy}^{-} \\ \dot{c}_{xy}^{-} \end{bmatrix}}_{\text{com prev. step.}} + \begin{bmatrix} 1 - \cosh(\omega\Delta T) \\ -\omega\sinh(\omega\Delta T) \end{bmatrix} p_{xy}$$

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Which gives the update: (with discretization step ΔT)

$$\mathbf{x}^{+} = \mathbf{A}\mathbf{x}^{-} + \mathbf{B}u$$

– input is zmp

com state

constan

Matrix

Trajectory Generation

Can be formulated as Model Predictive Control (MPC)

ZMP Equation (sec. order eq.)

$$\ddot{c}_{xy} = \frac{g}{h}\left(c_{xy} - p_{xy}\right)$$

Can be analytically integrated in time to obtain:

$$\begin{bmatrix} c_{xy}^{+} \\ \dot{c}_{xy}^{+} \end{bmatrix} = \begin{bmatrix} \cosh\left(\omega \Delta T\right) & \omega^{-1}\sinh\left(\omega \Delta T\right) \\ \omega\sinh\left(\omega \Delta T\right) & \cosh\left(\omega \Delta T\right) \end{bmatrix} \begin{bmatrix} c_{xy}^{-} \\ \dot{c}_{xy}^{-} \end{bmatrix} + \begin{bmatrix} 1 - \cosh\left(\omega \Delta T\right) \\ -\omega\sinh\left(\omega \Delta T\right) \end{bmatrix} p_{xy}$$

Which gives the update: (with discretization step ΔT)

$$\mathbf{x}^{+} = \mathbf{A}\mathbf{x}^{-} + \mathbf{B}u$$

Control input is ZMP

State CoM

Trajectory Generation

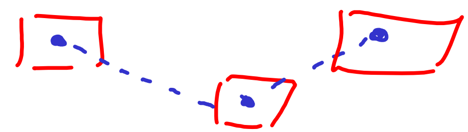
Can be formulated as Model Predictive Control (MPC)

Reference Trajectory

ZMP Positions: $\{\mathbf{p}_t\}_{1:T}$ are the footsteps centers (remain in Support Polygon)

CoM Positions: $\{\mathbf{c}_t\}_{1:T}$ (if known)

CoM Velocity: $\{\dot{\mathbf{c}}_t\}_{1:T}$ (if known)



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Trajectory Generation

Can be formulated as Model Predictive Control (MPC)

Horizon length

Problem:

$$\min_{\mathbf{c}[],\dot{\mathbf{c}}[],\mathbf{p}[]} \sum_{k}^{K} \underbrace{\frac{\beta}{2}\left\|c_k - c_k^{ref}\right\|^2}_{\text{com pos}} + \underbrace{\frac{\gamma}{2}\left\|\dot{c}_k - \dot{c}_k^{ref}\right\|^2}_{\text{com vel}} + \underbrace{\frac{\alpha}{2}\left\|u_k - u_k^{ref}\right\|^2}_{\text{zmp pos}}$$

subject to

α, β, γ are user defined gains

Trajectory Generation

Can be formulated as Model Predictive Control (MPC)

Problem:

$$\min_{\mathbf{c}[],\dot{\mathbf{c}}[],\mathbf{p}[]} \sum_k \frac{\beta}{2} \left\| c_k - c_k^{ref} \right\|^2 + \frac{\gamma}{2} \left\| \dot{c}_k - \dot{c}_k^{ref} \right\|^2 + \frac{\alpha}{2} \left\| u_k - u_k^{ref} \right\|^2$$

subject to

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$ Equality: Respect the discretized Dynamics

α, β, γ are user defined gains

Trajectory Generation

Can be formulated as Model Predictive Control (MPC)

Problem:

$$\min_{\mathbf{c}[],\dot{\mathbf{c}}[],\mathbf{p}[]} \sum_k \frac{\beta}{2} \left\| c_k - c_k^{ref} \right\|^2 + \frac{\gamma}{2} \left\| \dot{c}_k - \dot{c}_k^{ref} \right\|^2 + \frac{\alpha}{2} \left\| u_k - u_k^{ref} \right\|^2$$

subject to

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

Equality: Respect the discretized Dynamics

$$\mathbf{x}_0 = \mathbf{x}_{\text{init}}$$
$$\mathbf{x}_N = \mathbf{x}_{\text{terminal}}$$

Initial and terminal conditions

α, β, γ are user defined gains

Trajectory Generation

Can be formulated as Model Predictive Control (MPC)

Problem:

$$\min_{\mathbf{c}[],\dot{\mathbf{c}}[],\mathbf{p}[]} \sum_k \frac{\beta}{2} \left\| c_k - c_k^{ref} \right\|^2 + \frac{\gamma}{2} \left\| \dot{c}_k - \dot{c}_k^{ref} \right\|^2 + \frac{\alpha}{2} \left\| u_k - u_k^{ref} \right\|^2$$

subject to

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

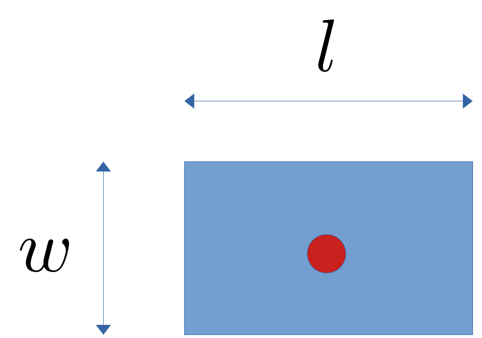
$$\mathbf{x}_0 = \mathbf{x}_{\text{init}}$$

$$\mathbf{x}_N = \mathbf{x}_{\text{terminal}}$$

known

$$\underbrace{p_k - \frac{s}{2}}_{\text{known}} < u_k < p_k + \frac{s}{2}$$

α, β, γ are user defined gains



$$s = \{w, l\}$$

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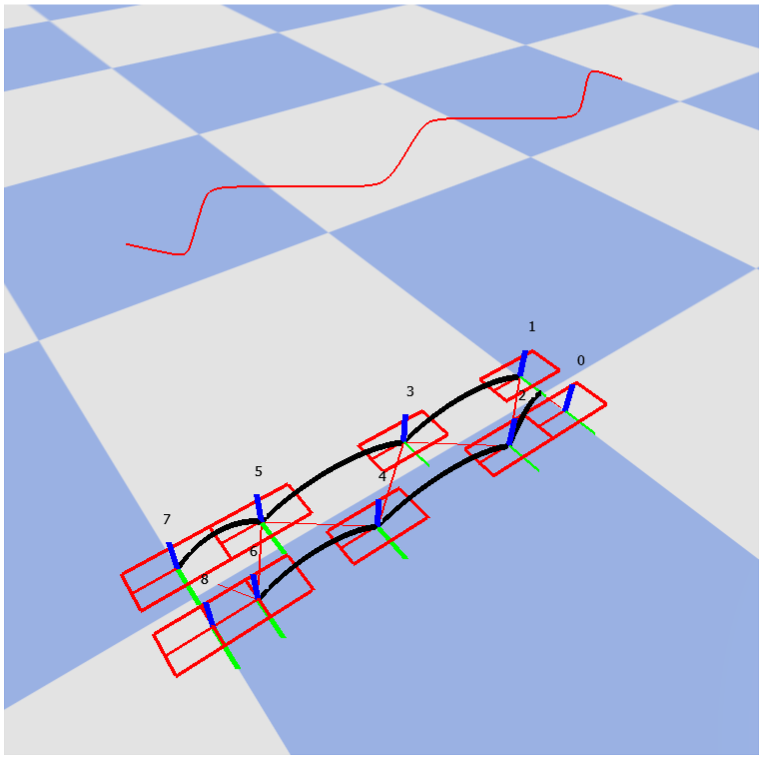

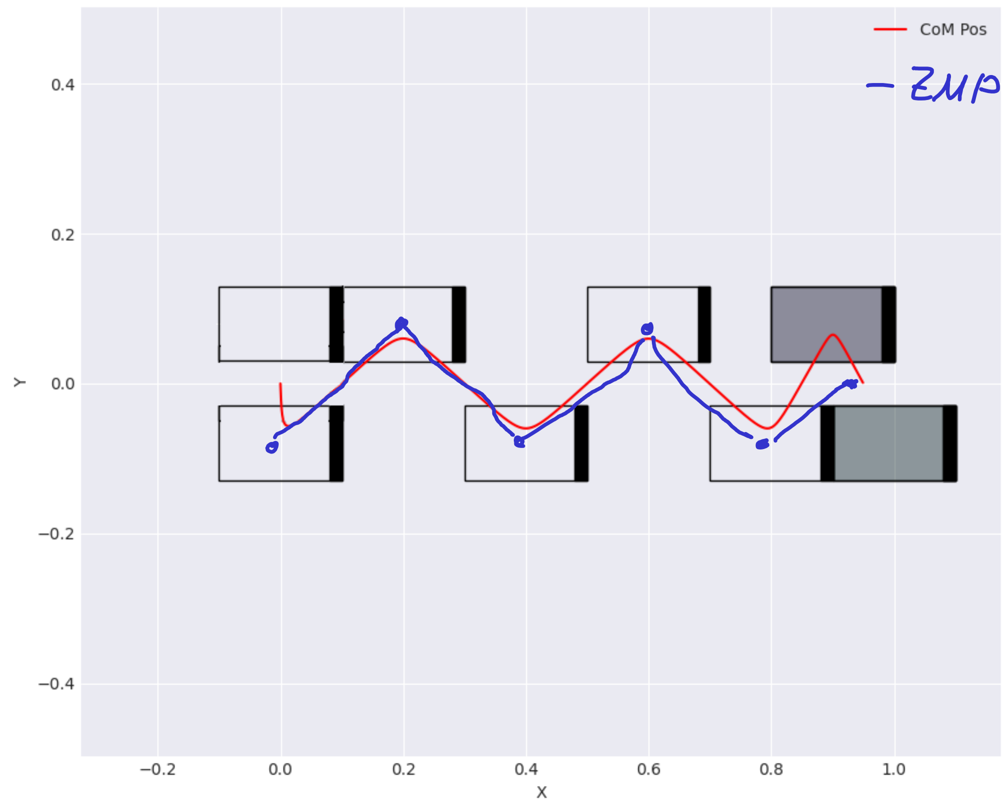

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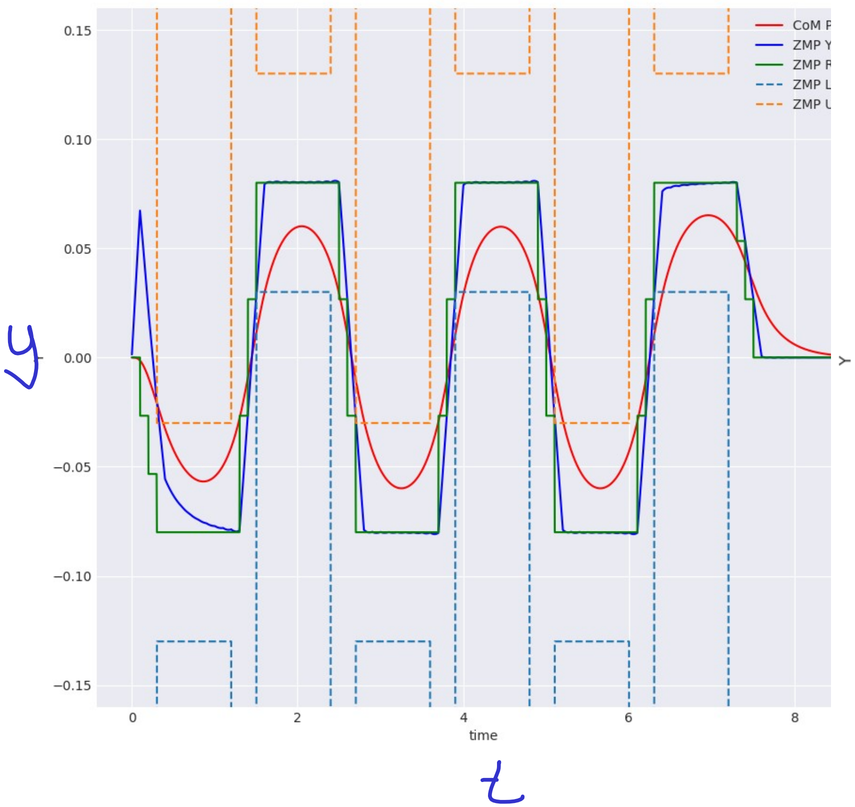

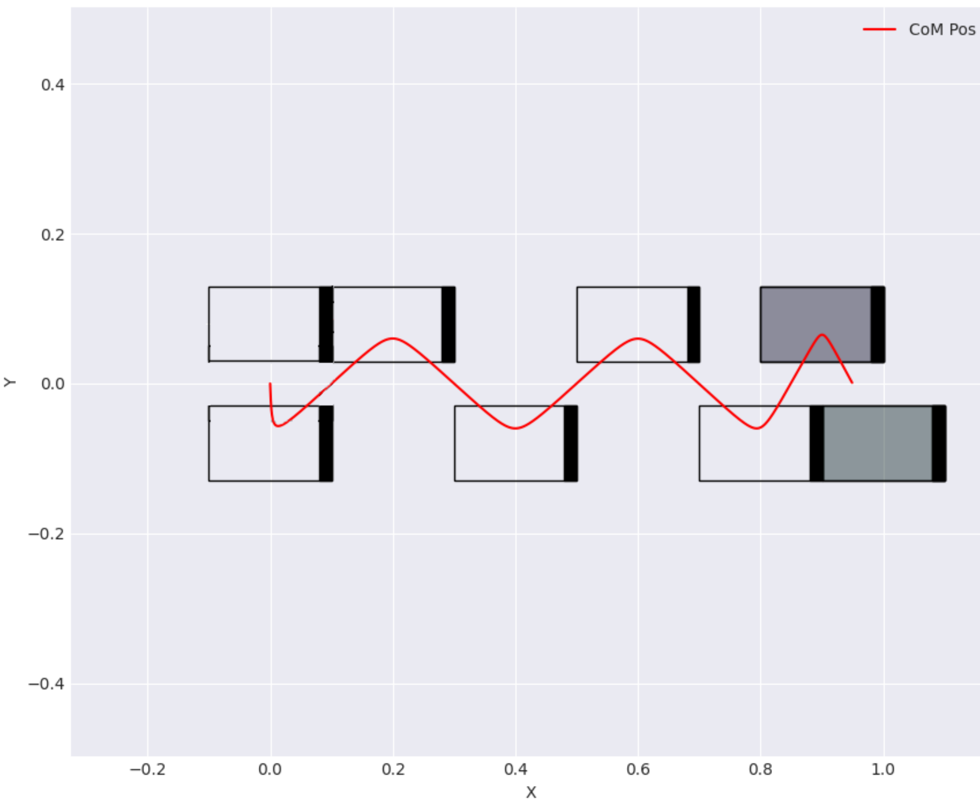

Continious Sys:

$$\dot{x} = f(x, u)$$

Solver: Discritized

$$x_{k+1} = x_k + \delta t \, \dot{x}_k$$

$$x_{k+1} = x_k + \delta t \, f(x_k, u_k)$$

Equality constrain
to inforce dynamics