Modeling and Control of Legged Robots
SS 2023

L7: Walking

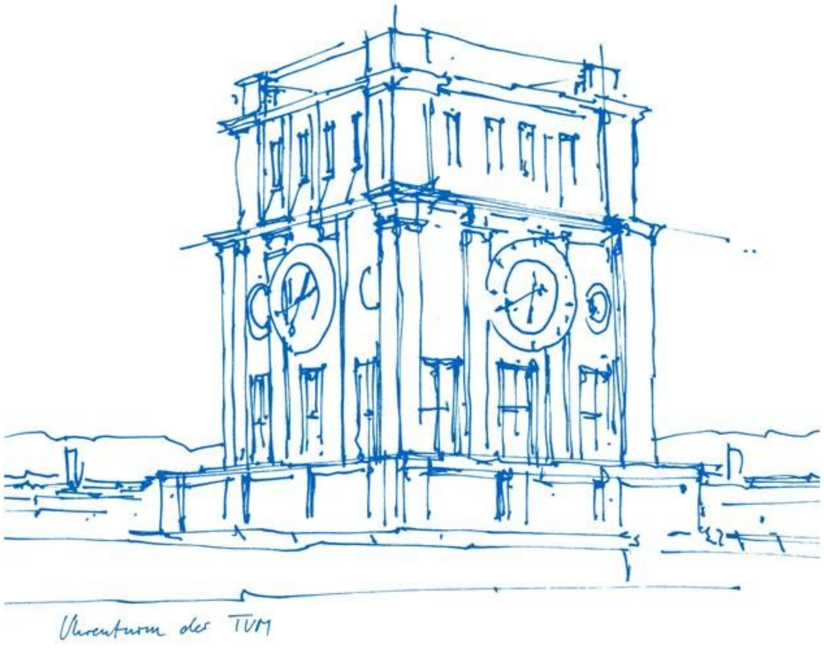
Dr.-Ing. J. Rogelio Guadarrama Olvera

Technical University of Munich

School of Computation, Information and Technology

Chair of Cognitive Systems

Munich 18. June 2024



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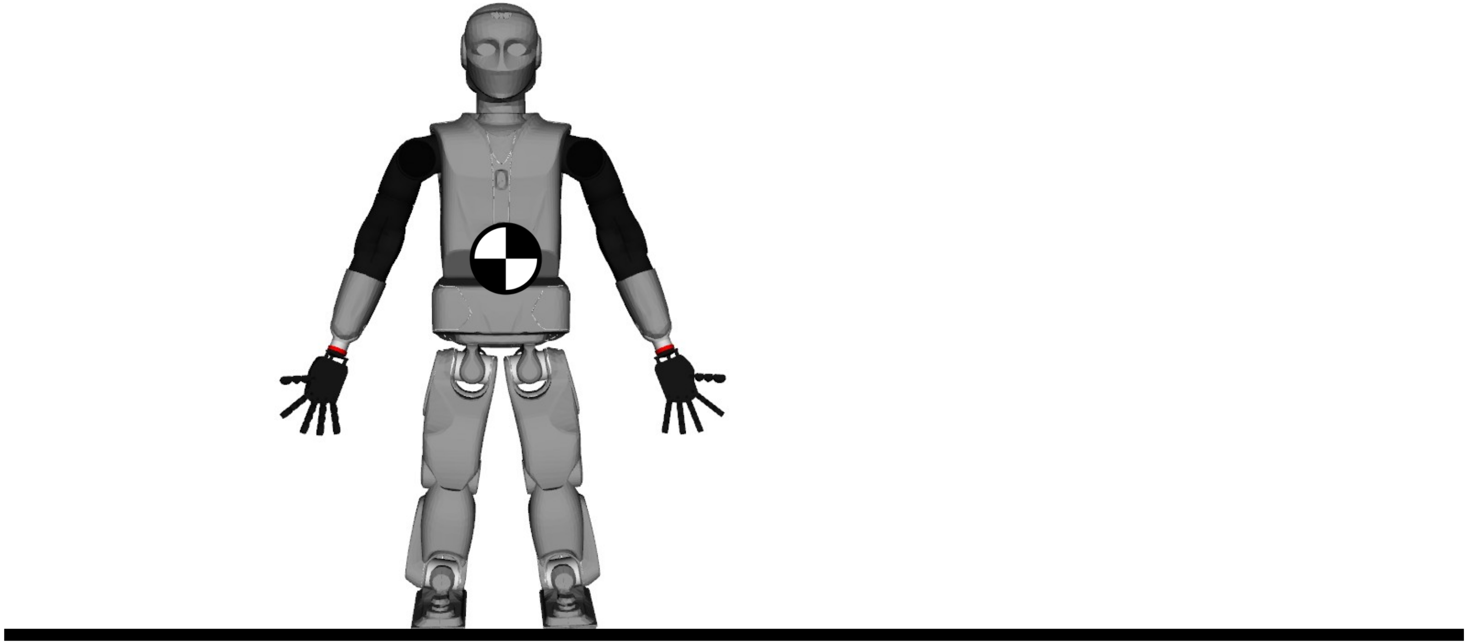
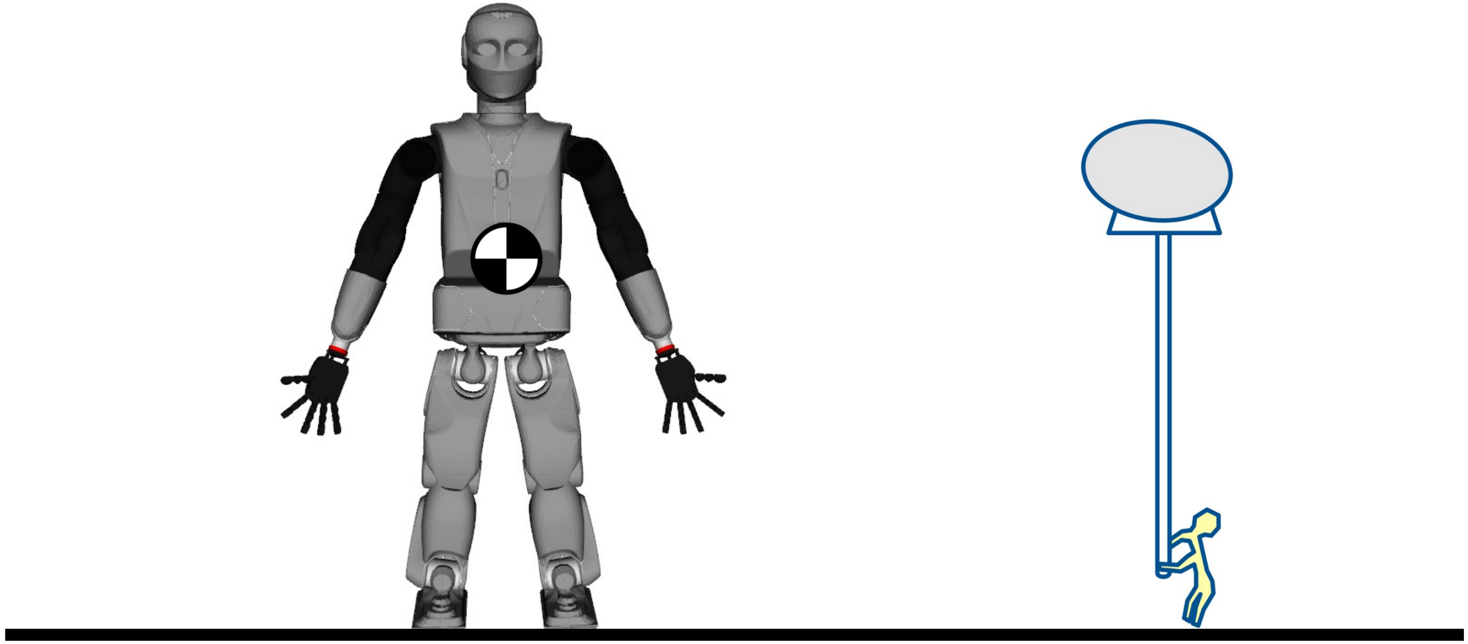
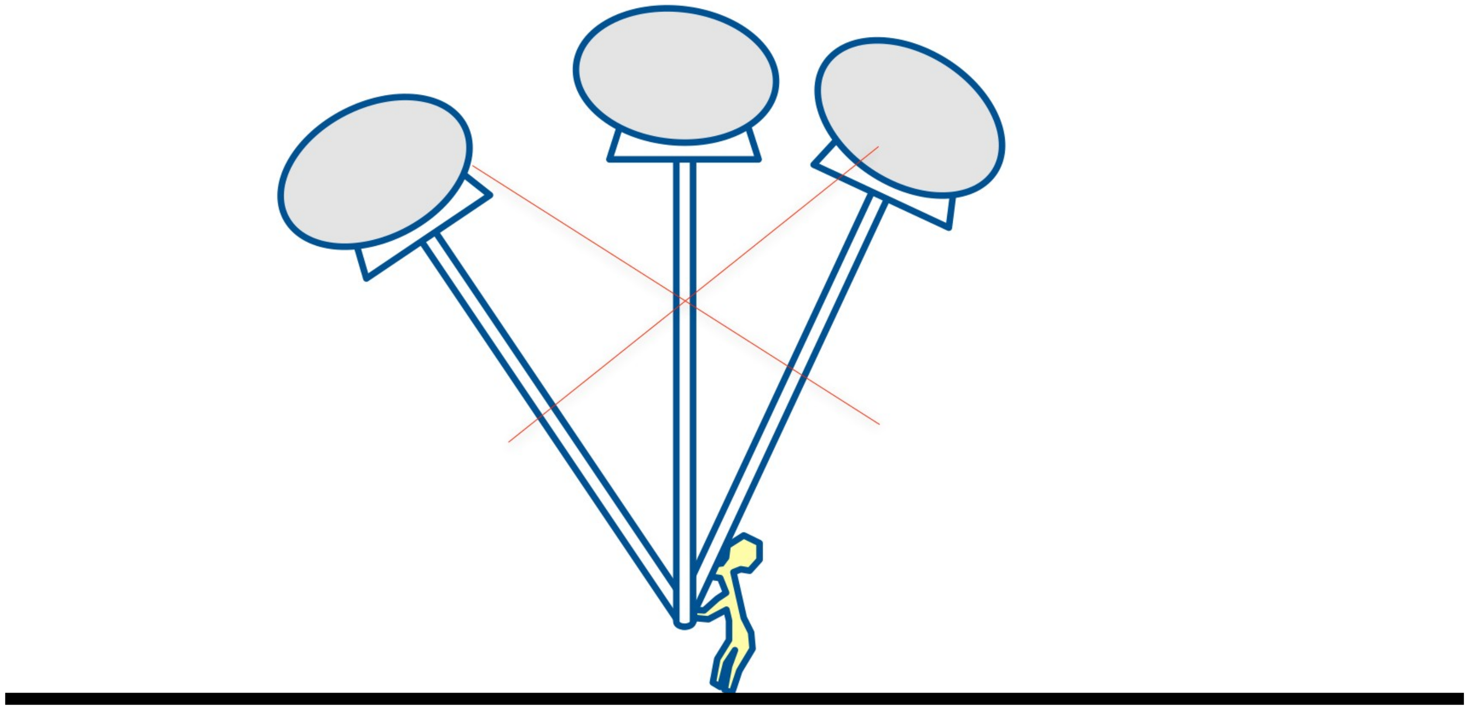


Introduction



Introduction



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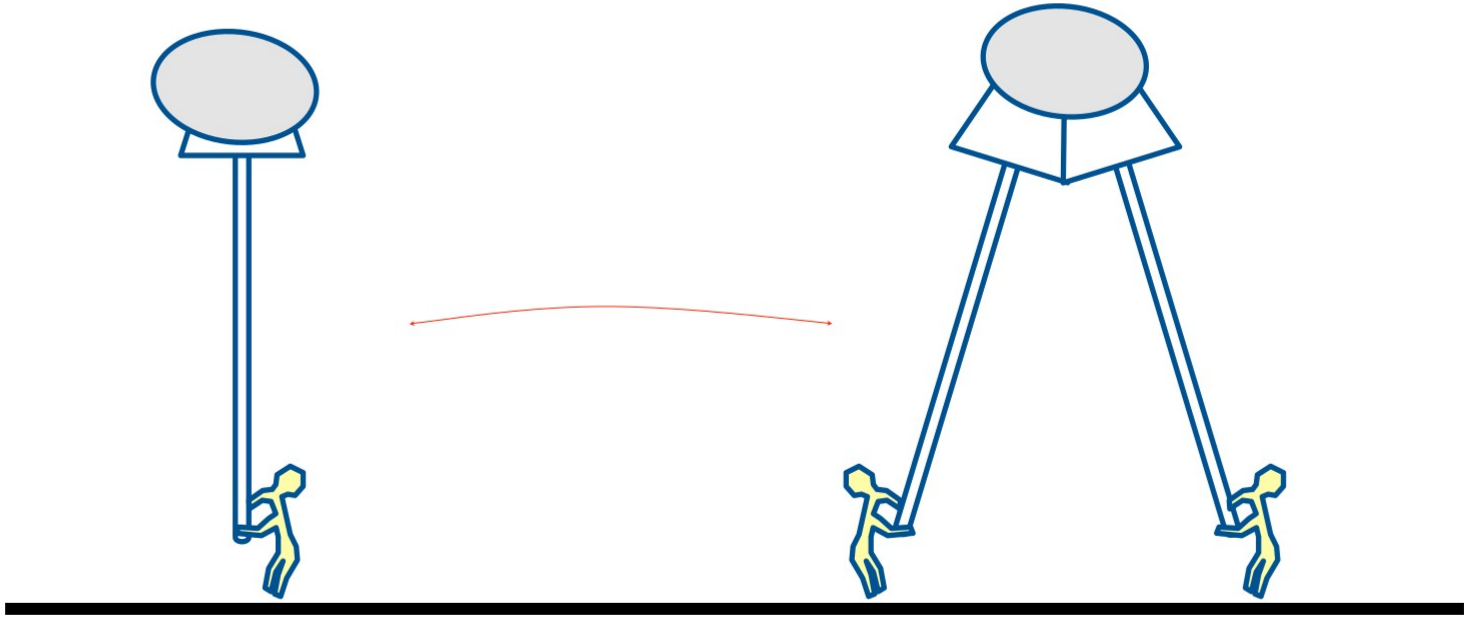
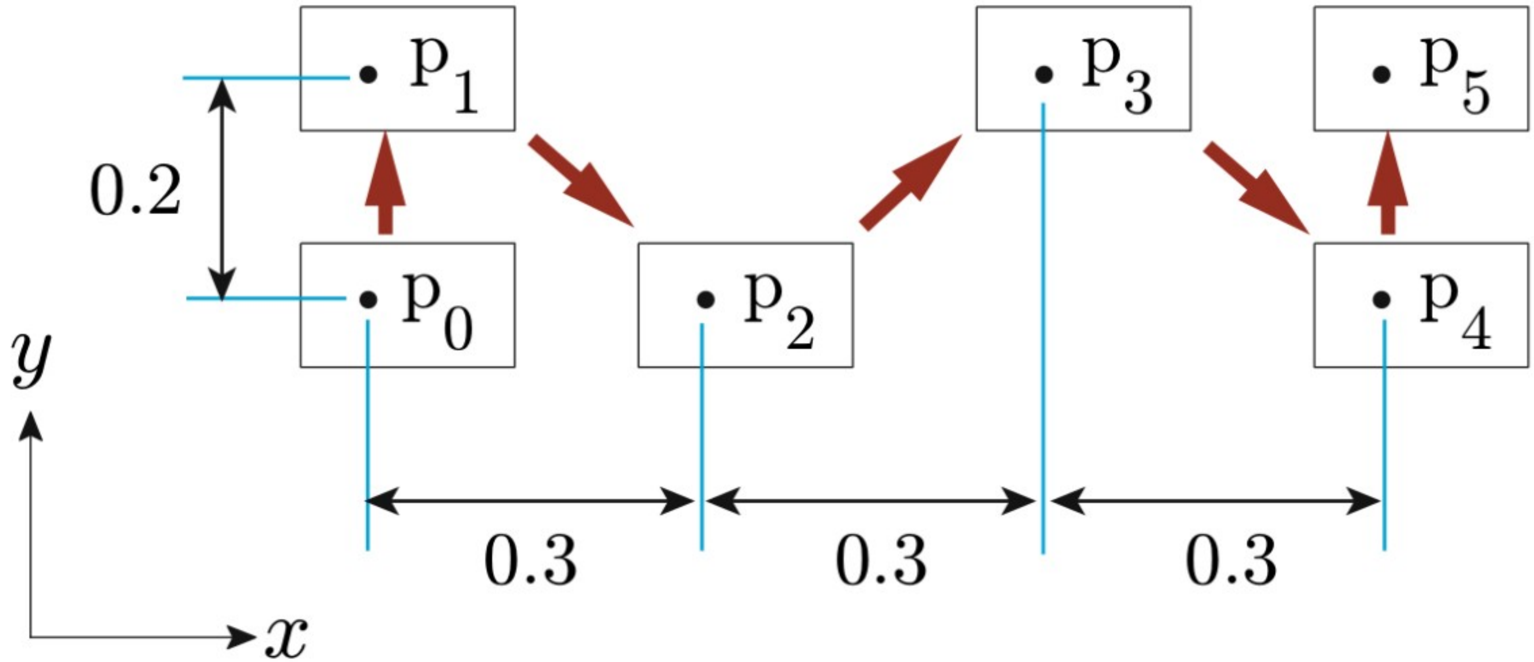


Introduction

Walking as a Linear Inverted Pendulum (LIPM)



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Introduction

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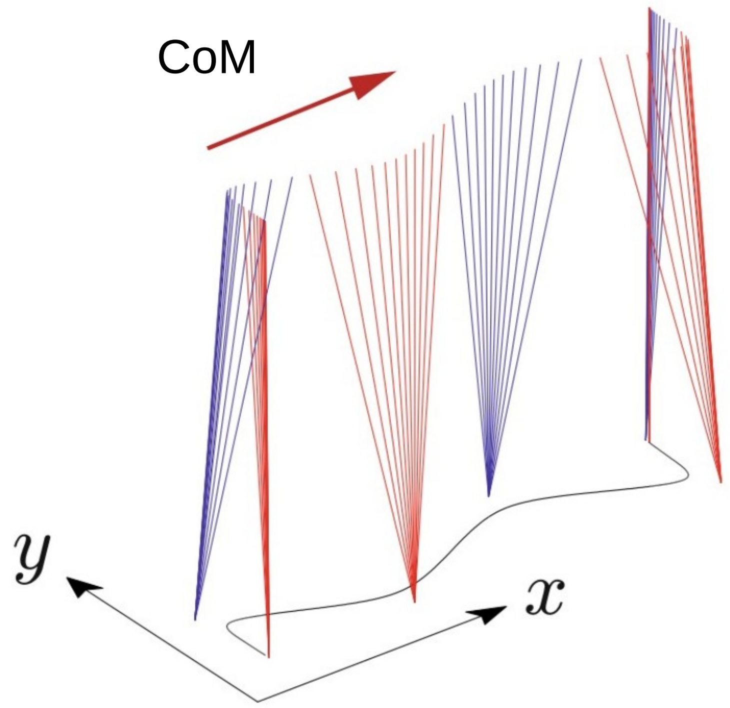

Walking

Oxford Advanced Learner’s Dictionary, Oxford University Press:
Move along at a moderate pace by lifting up and putting down each foot in turn, so that one foot is on the ground while the other is being lifted

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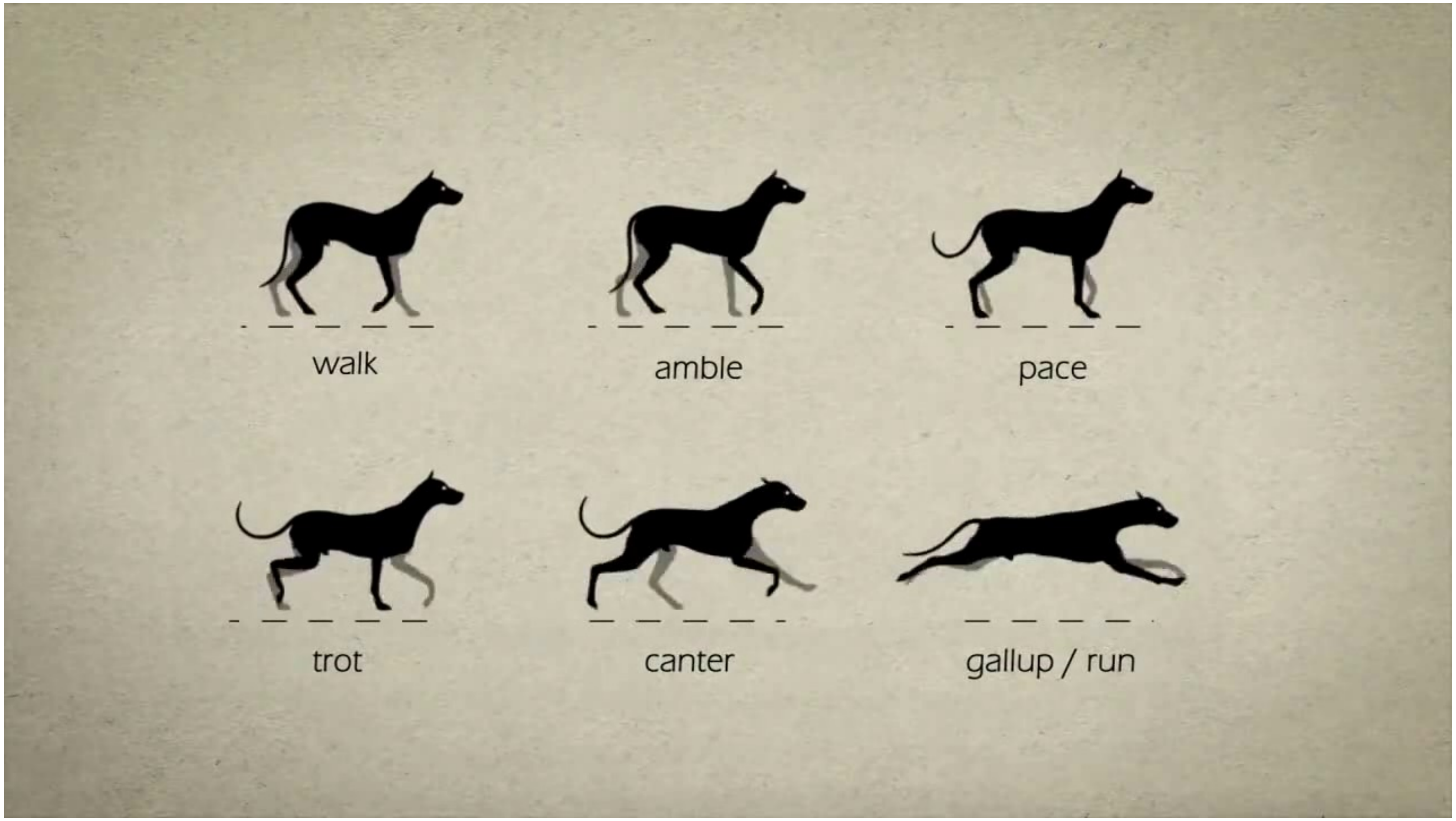
Walking

Running

Walking

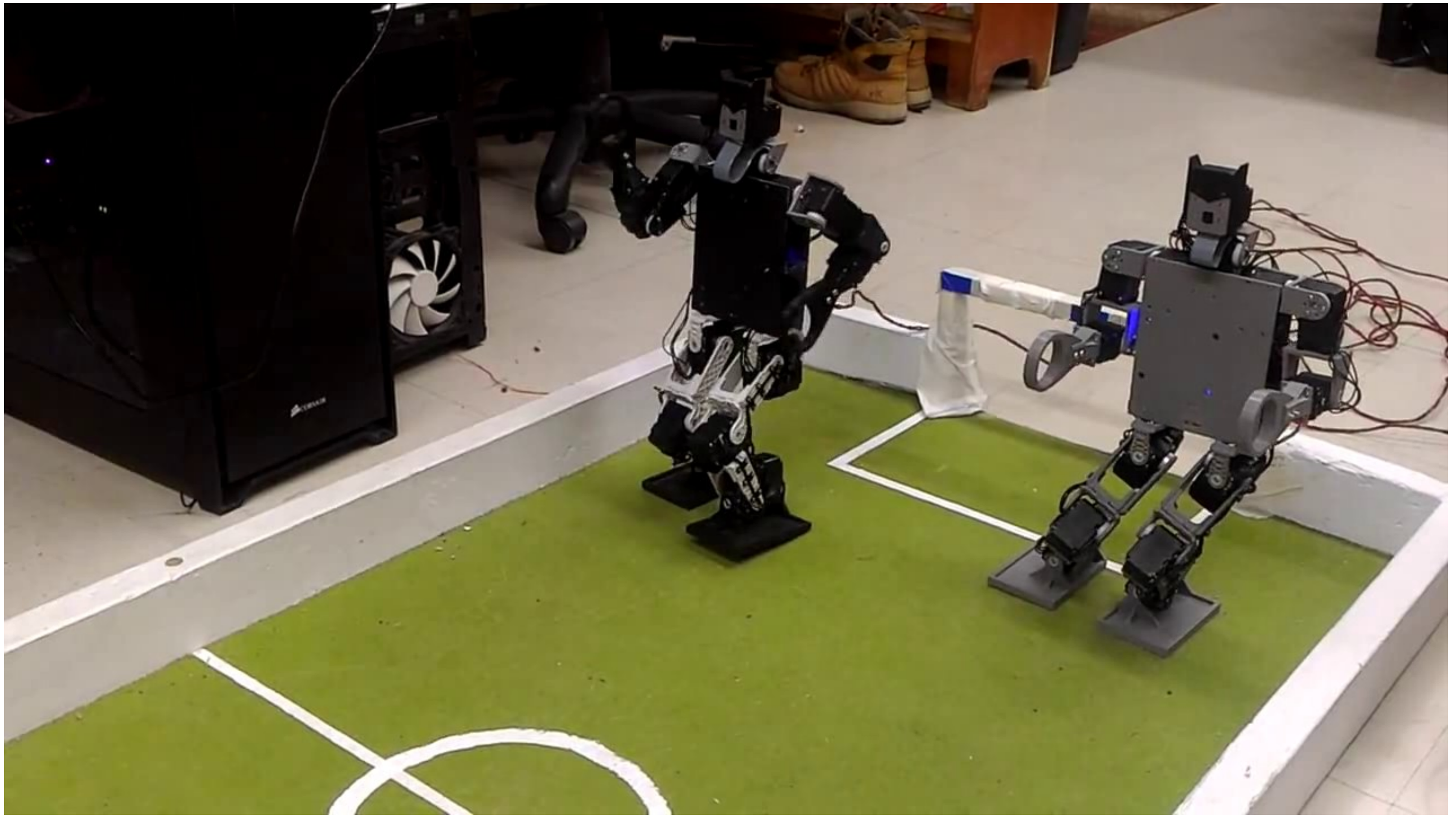
Legged locomotion can take different modalities. For example, Quadrupeds can:



Jamie Capsolas, Animal Gaits for Animators
https://www.youtube.com/watch?v=AZGNjKolAiQ

Walking

For bipeds



Dynamic walking

Static Walking

Walking

For bipeds

Dynamic Walking

- The projection of the CoM on the ground can leave the supporting polygon.
- Posture is dynamically stable.
- Will fall if you freeze the motors.
- High energy efficiency

Static Walking

- The projection of the CoM on the ground never leaves the supporting polygon.
- Posture is always stable.
- Will not fall if you freeze the motors.
- Not efficient.

Walking

For bipeds

- The projection of the CoM on the ground can leave the supporting polygon.
- Posture is dynamically stable.
- Will fall if you freeze the motors.
- High energy efficiency

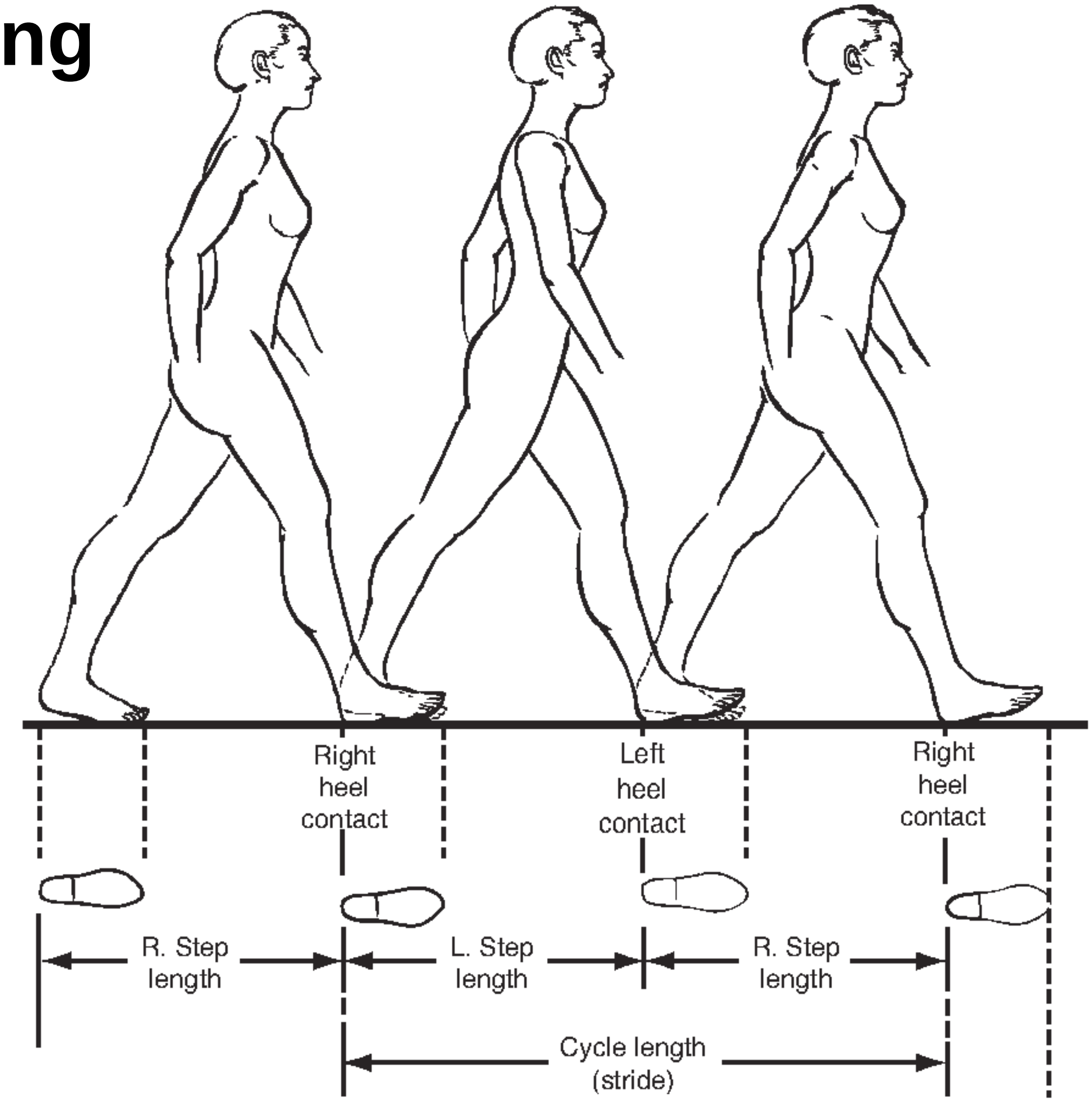
Dynamic Walking

- The projection of the CoM on the ground never leaves the supporting polygon.
- Posture is always stable.
- Will not fall if you freeze the motors.
- Not efficient.

Static Walking

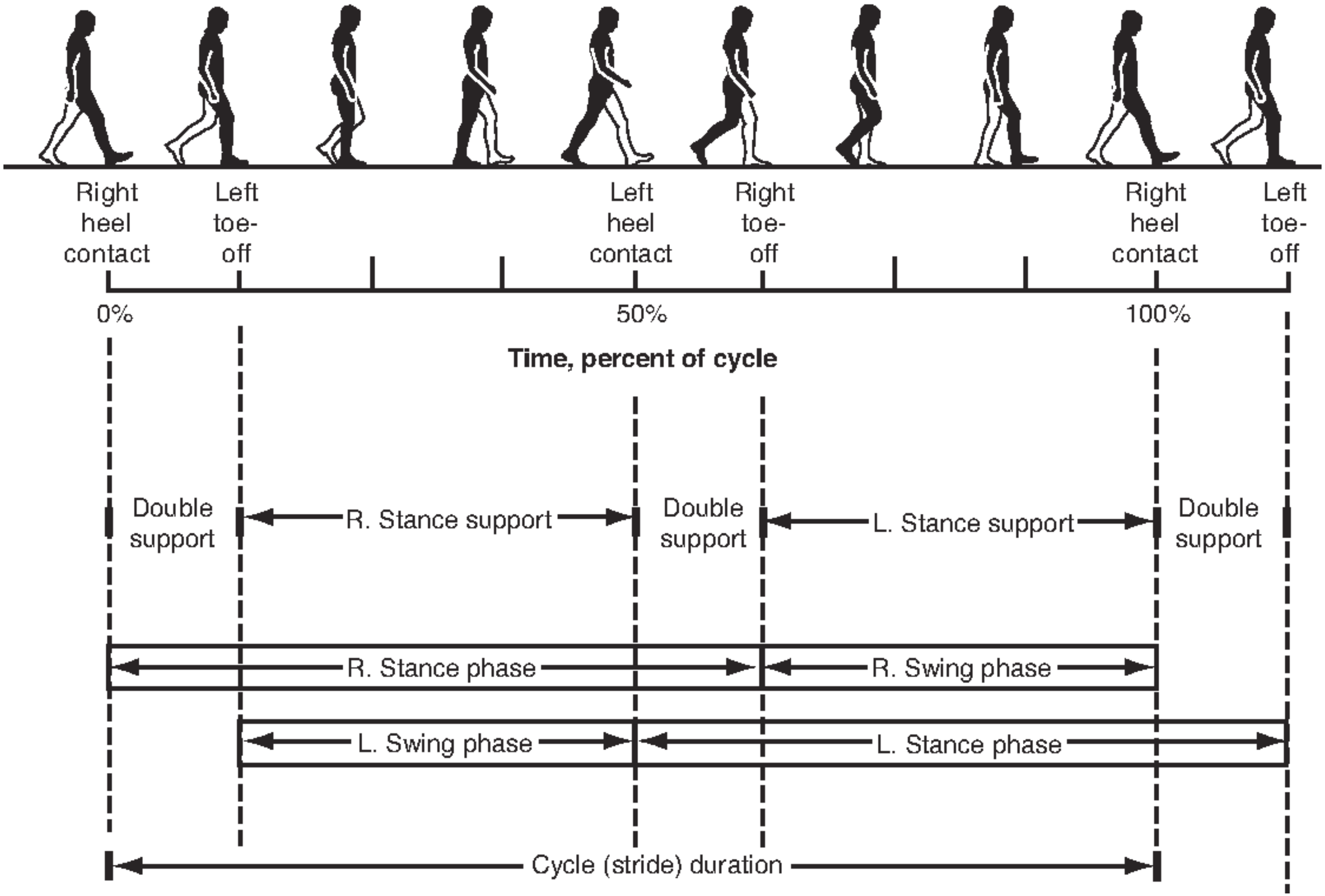
Human Walking Cycle

Space domain

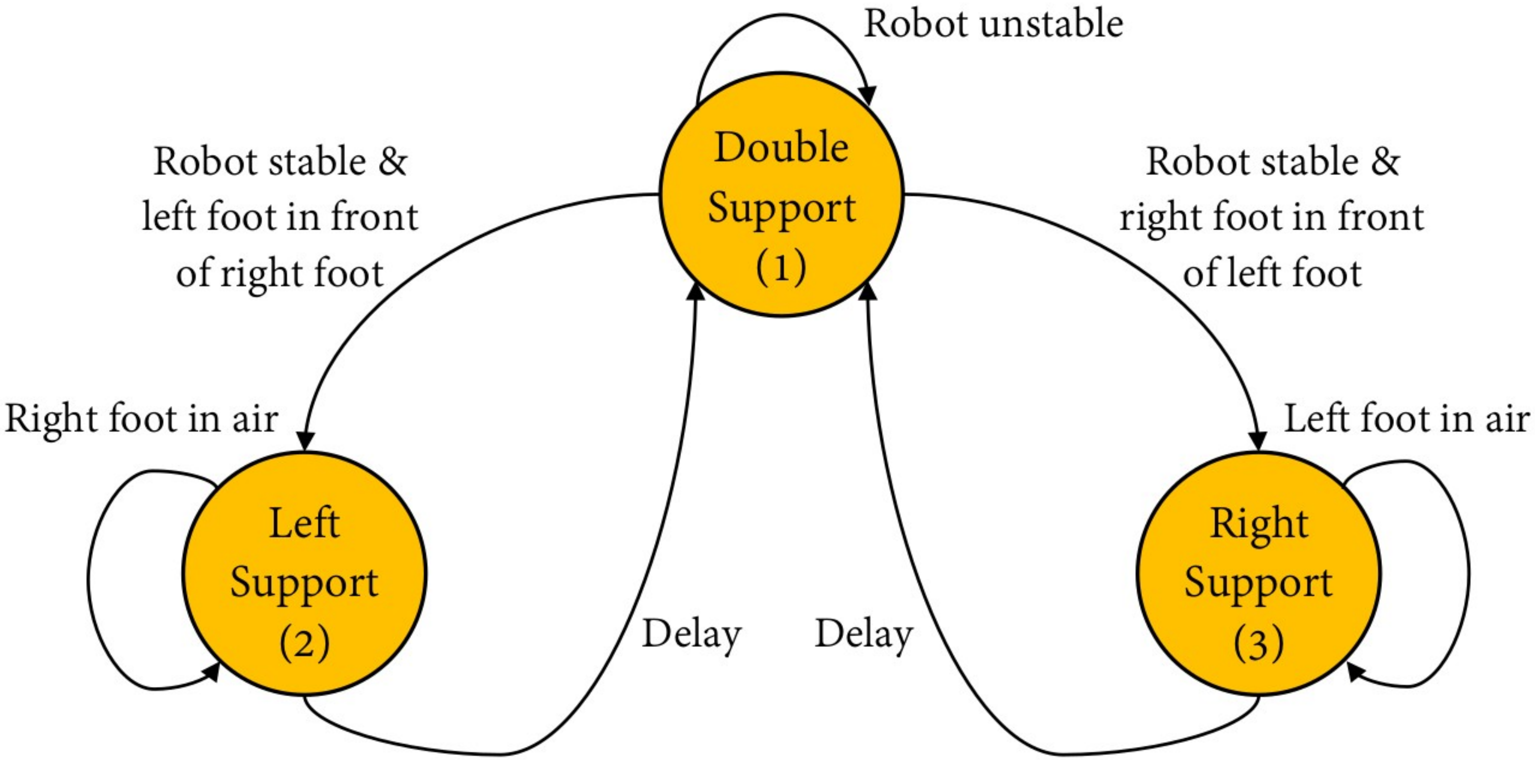


Human Walking Cycle

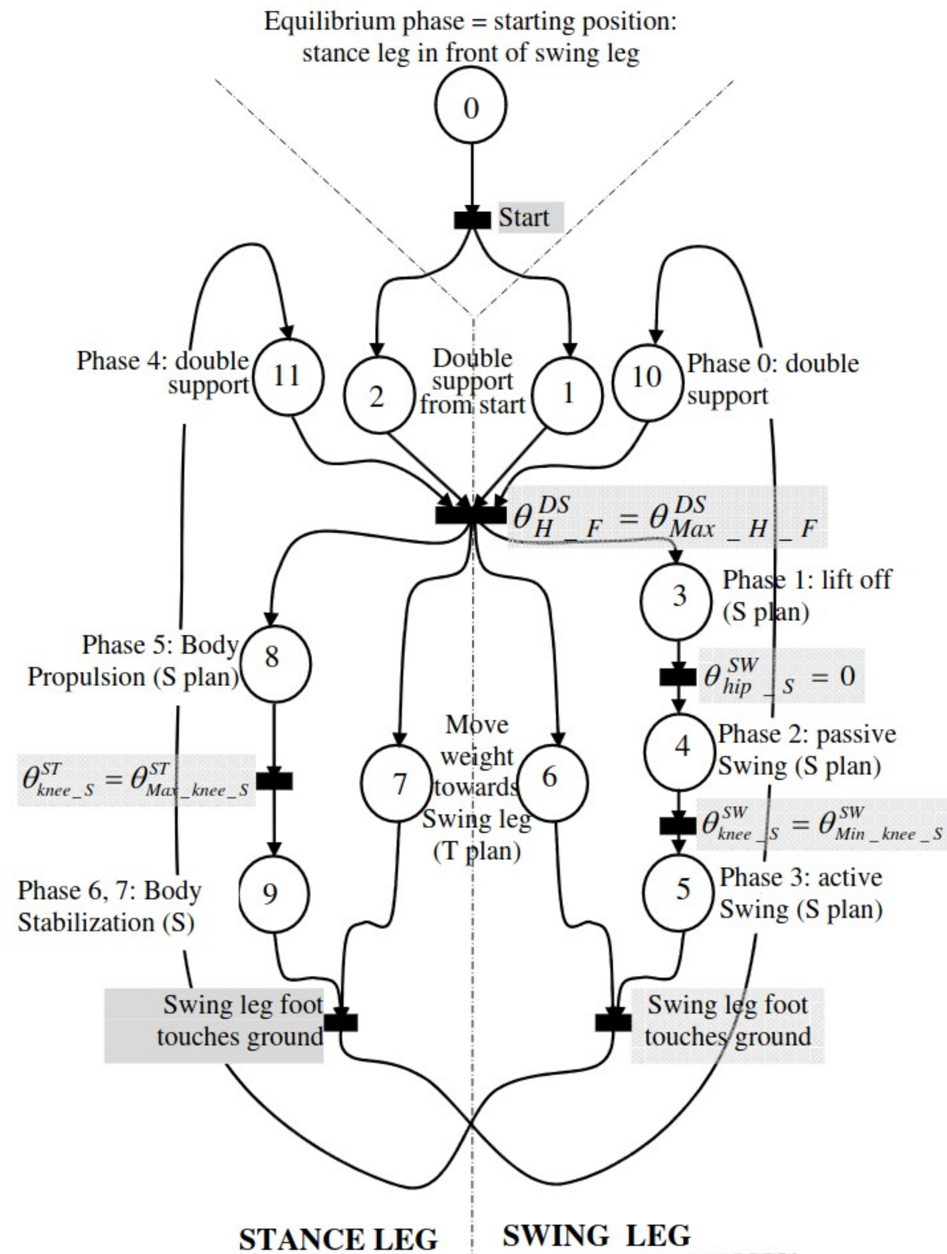
Time domain



Walking Cycle

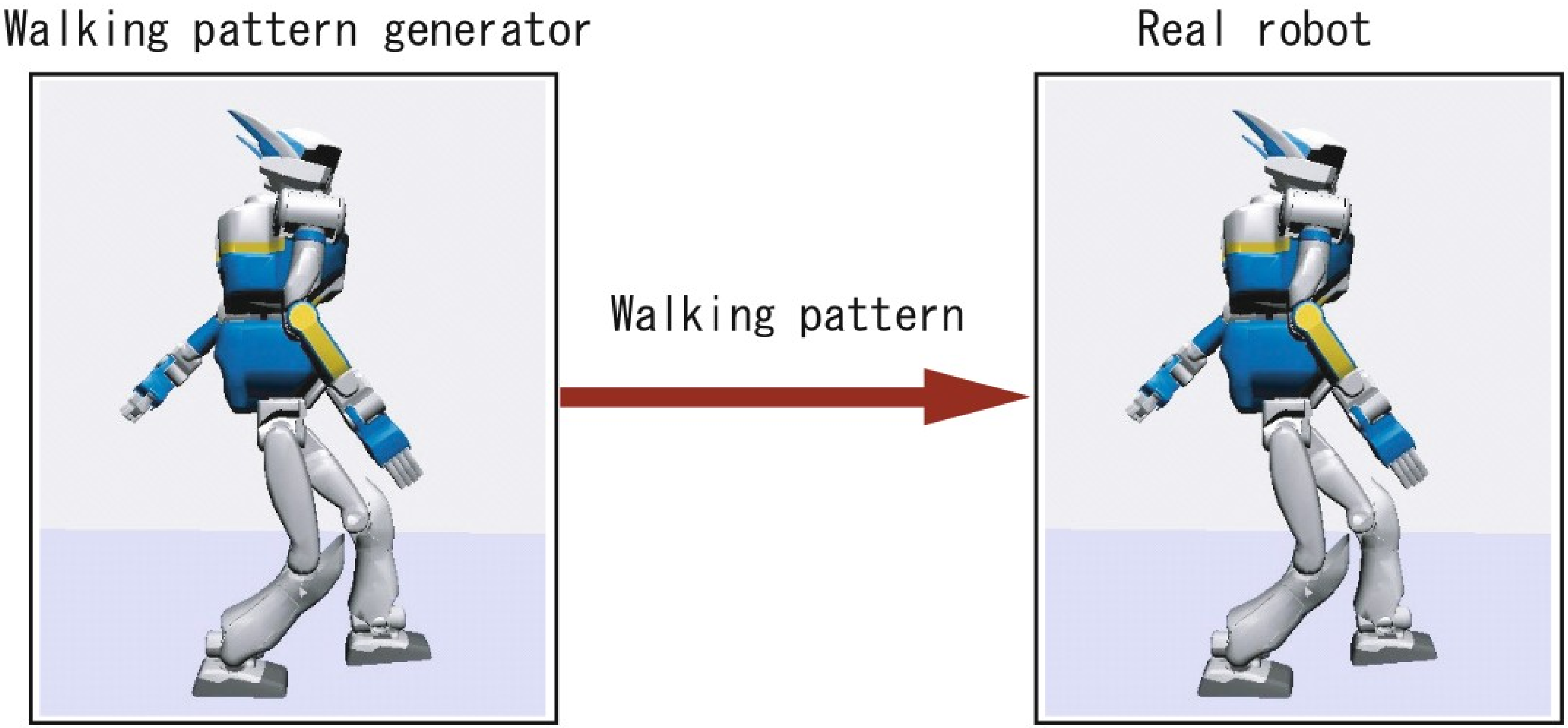


Walking Cycle



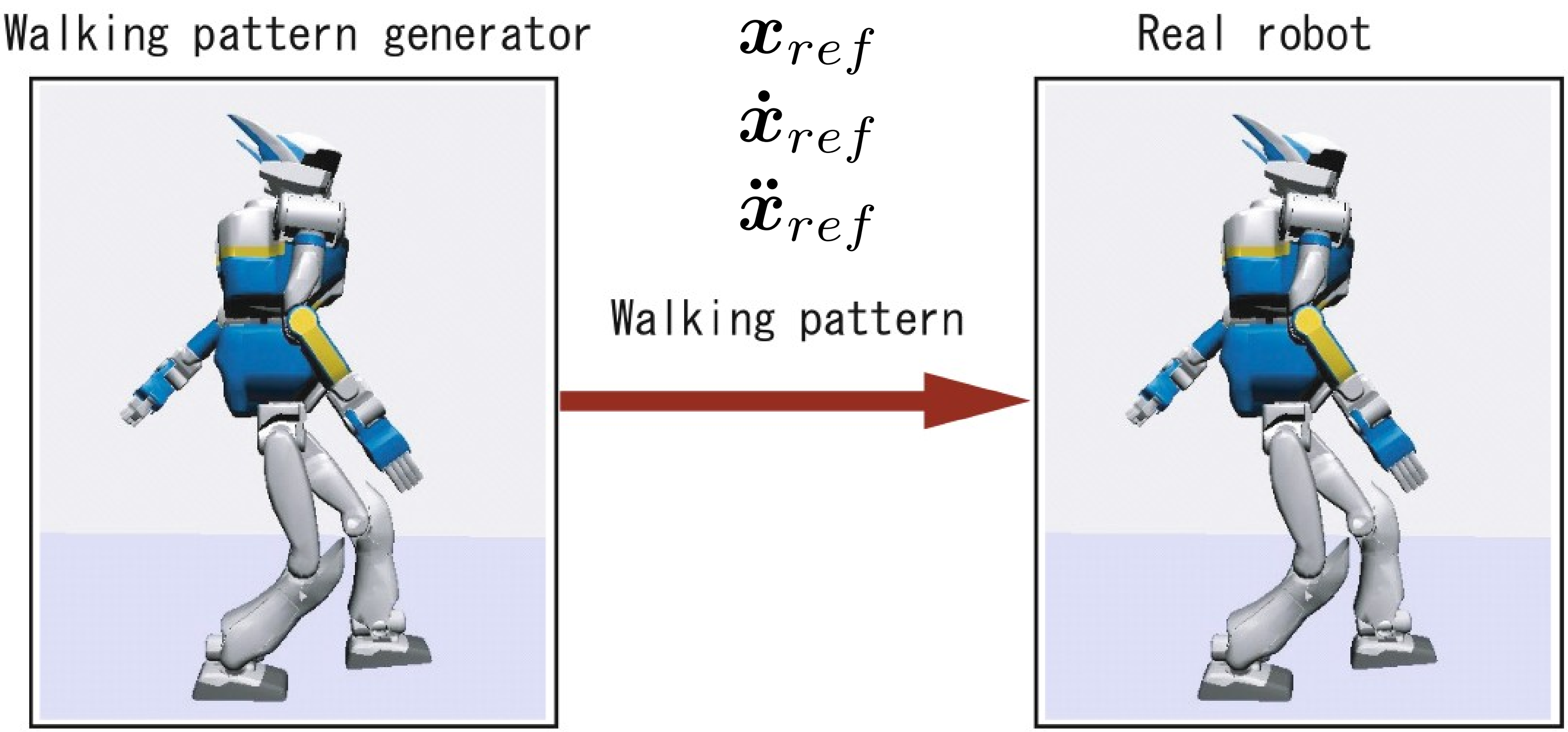
[Serhan et al., 2008]

Walking Motion Generation

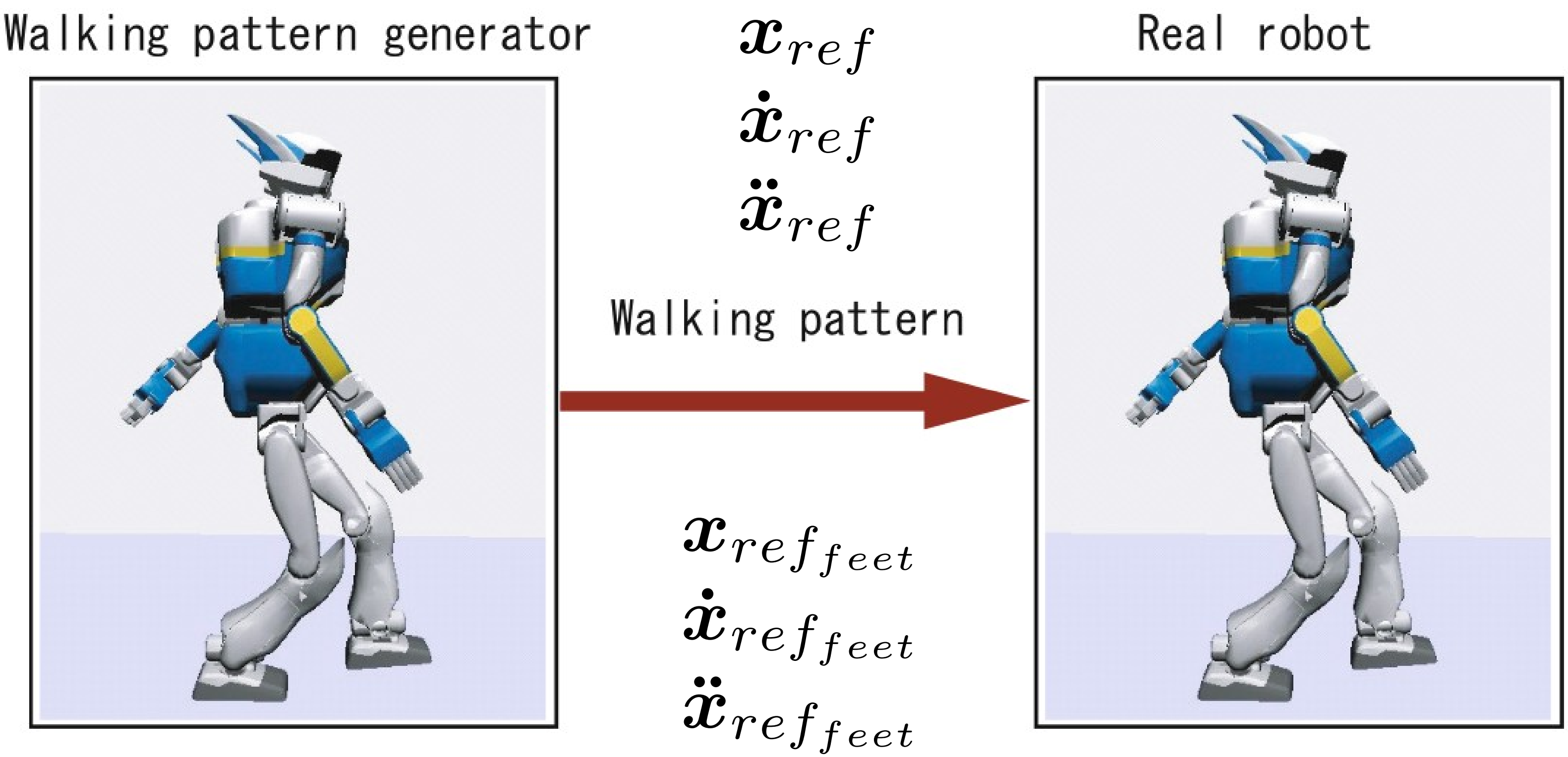


[Kajita et al., *Introduction to humanoid robotics*, Springer 2014]

Walking Motion Generation



Walking Motion Generation



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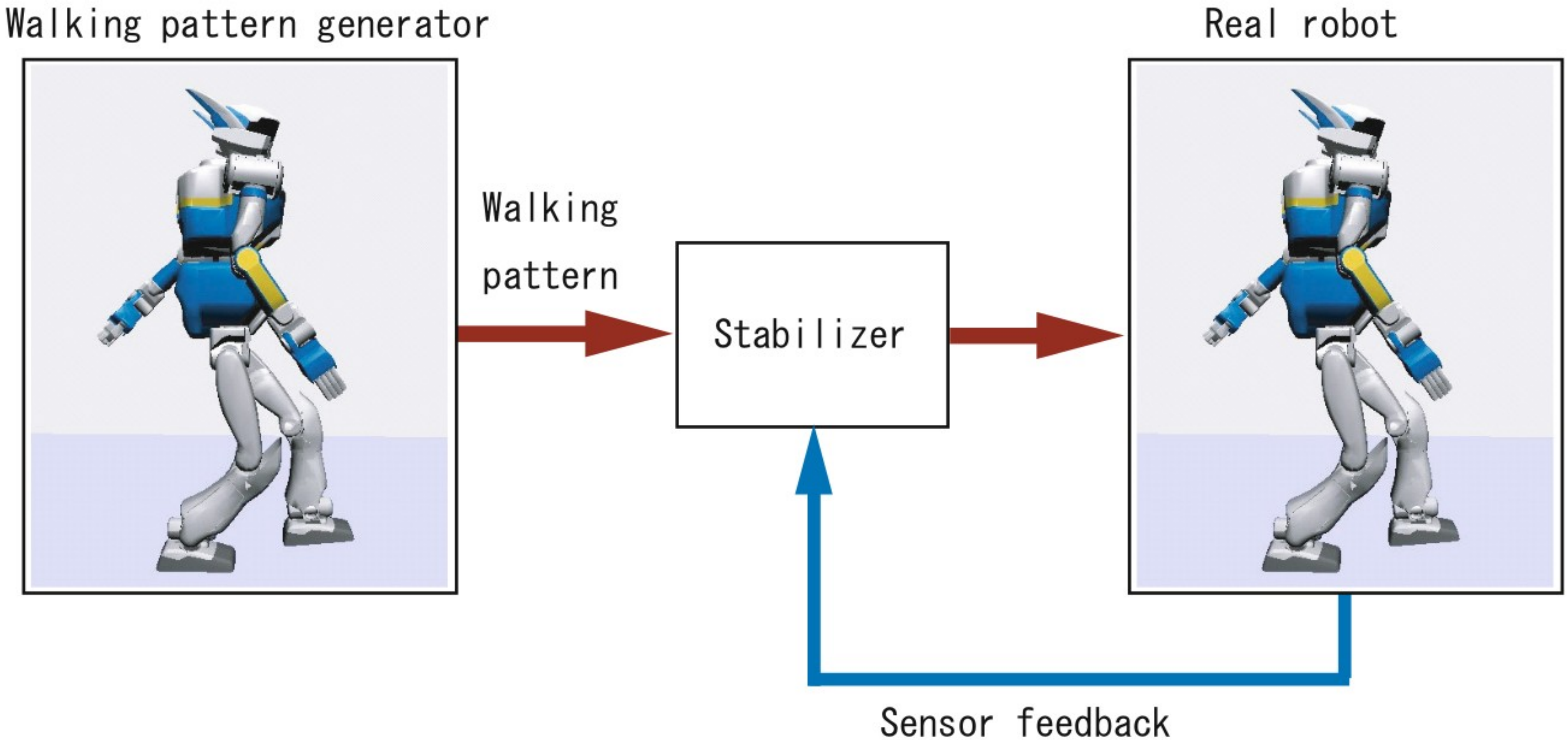
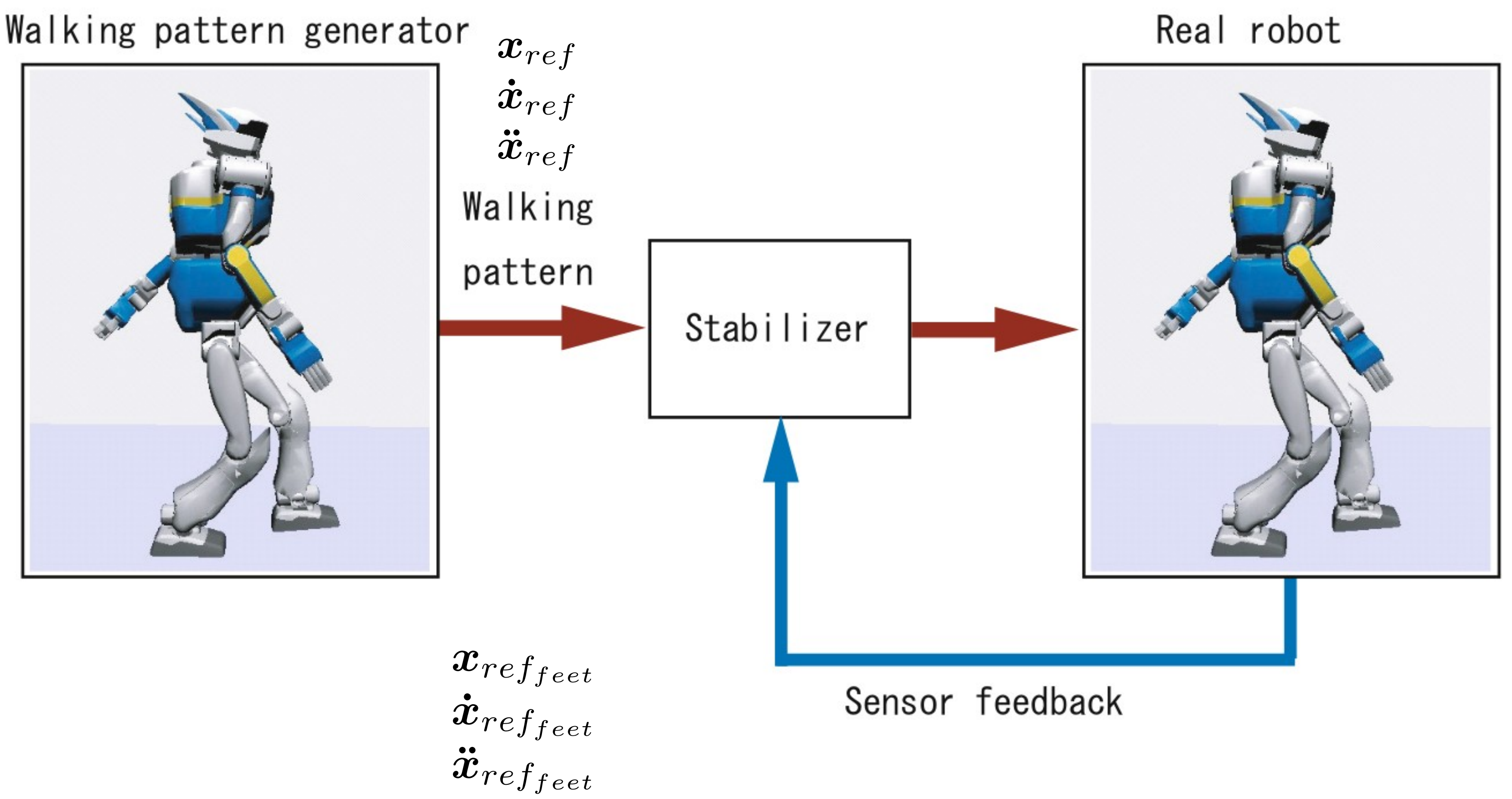
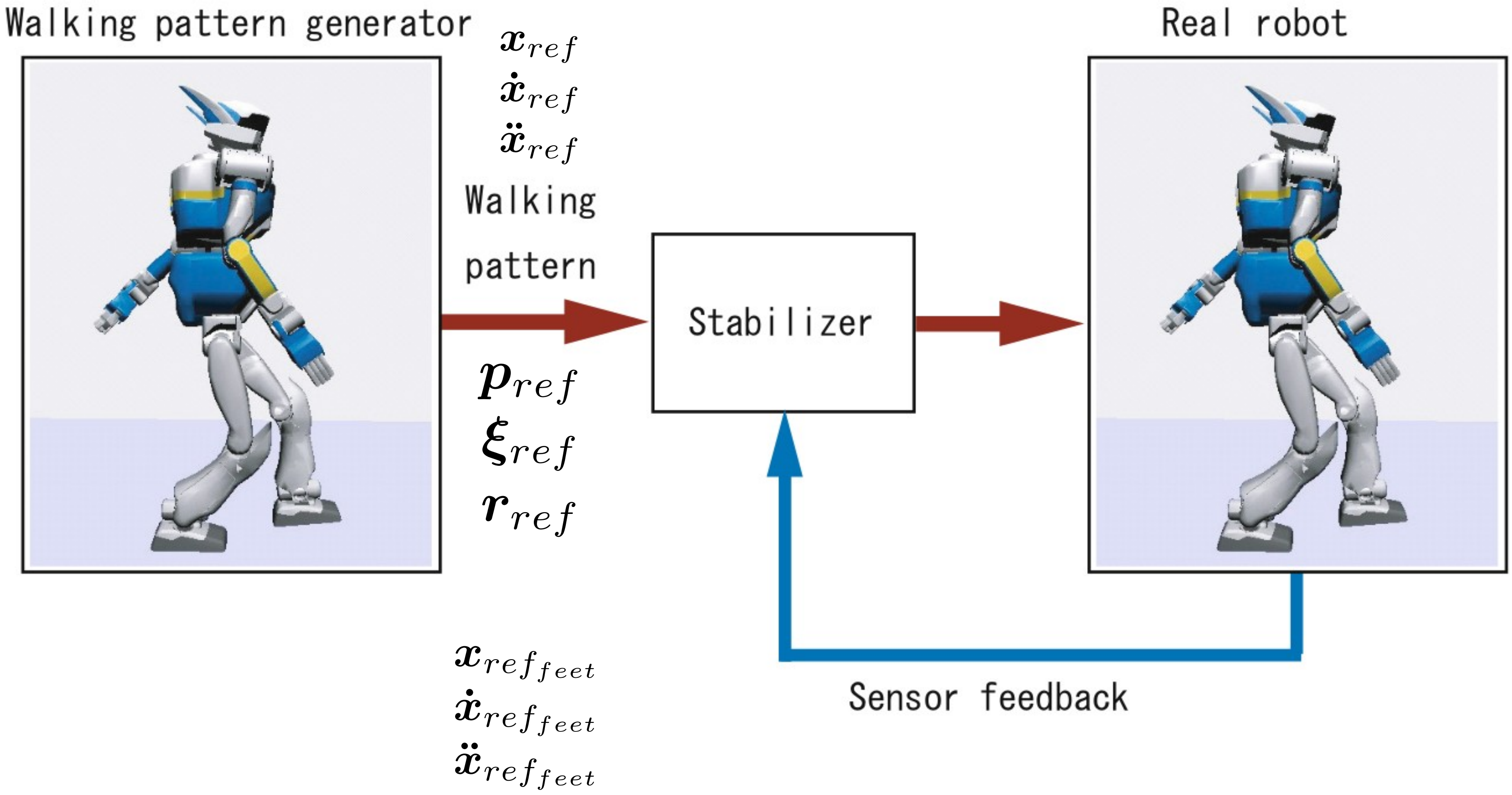


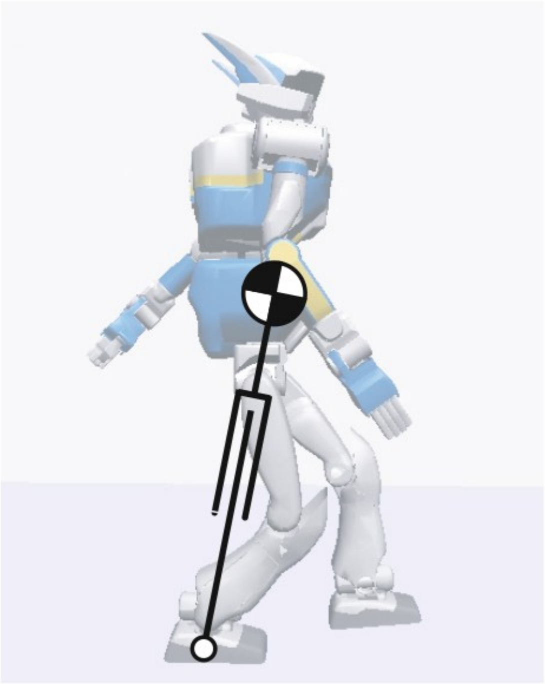
Walking Motion Generation

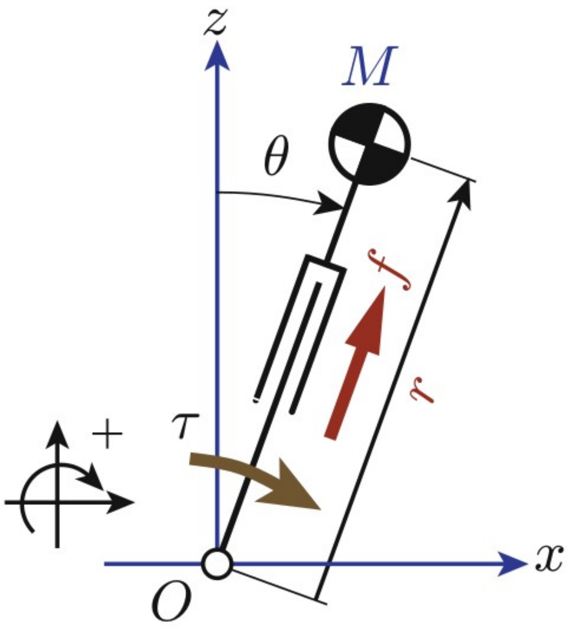


Walking Motion Generation



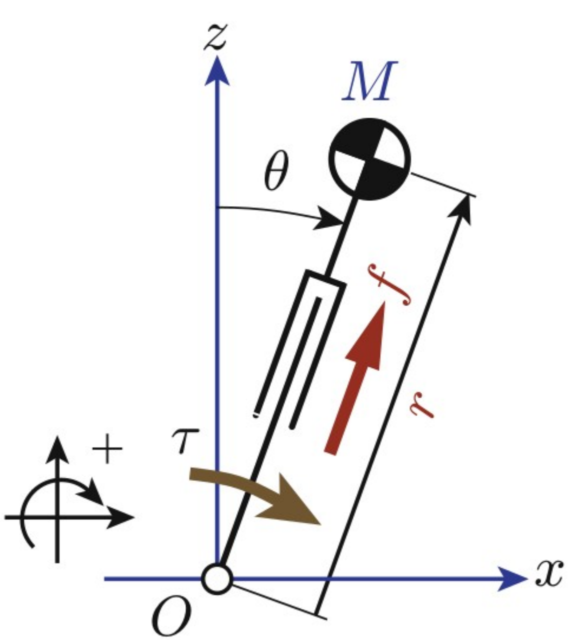
Linear Inverted Pendulum Model

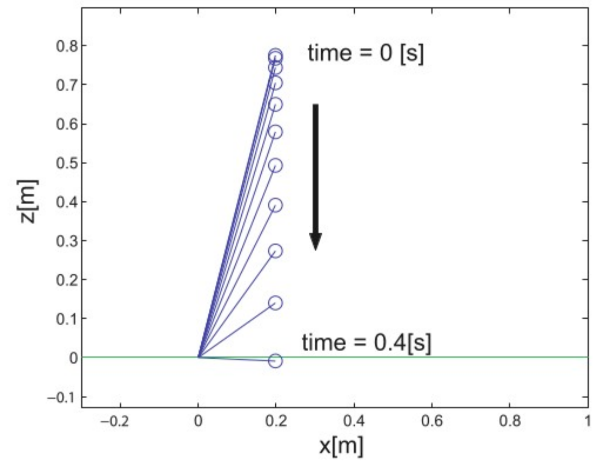




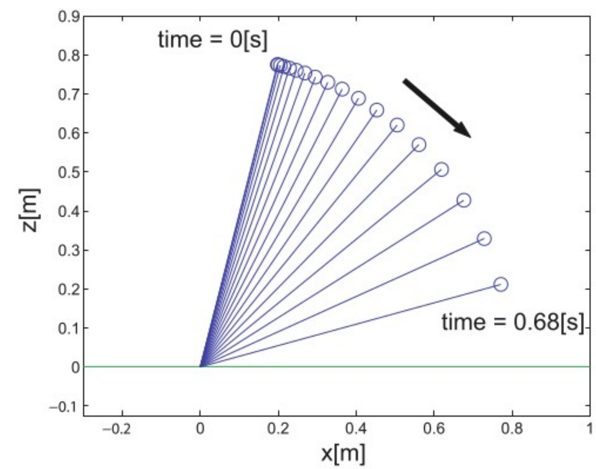
$$\ddot{\boldsymbol{\theta}} = \frac{g}{r} sin(\boldsymbol{\theta})$$

Linear Inverted Pendulum Model

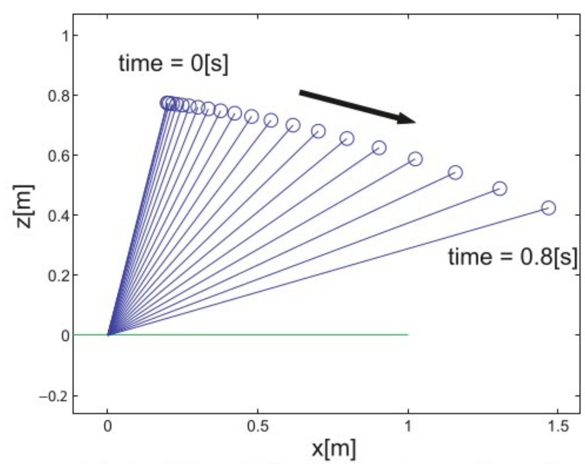




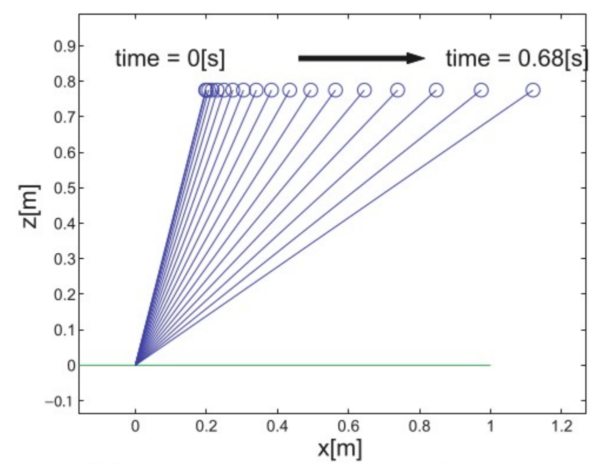
(a) $f = 0$: Free fall of CoM



(b) $f = Mg\cos\theta - Mr\dot{\theta}^2$: Fall down with constant leg length

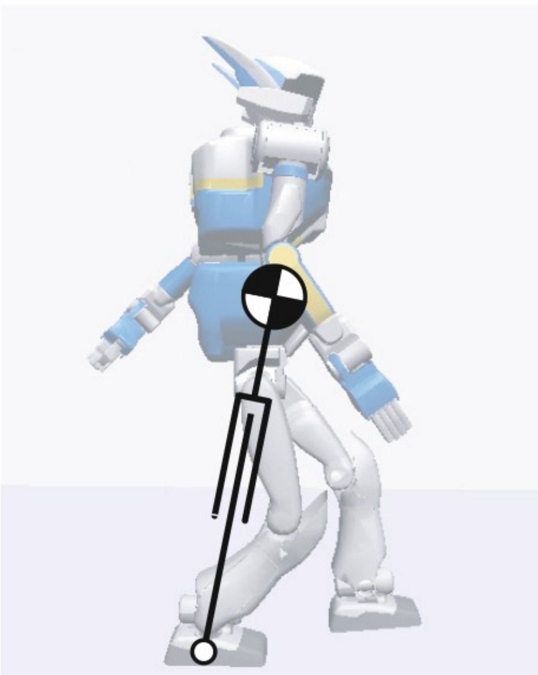


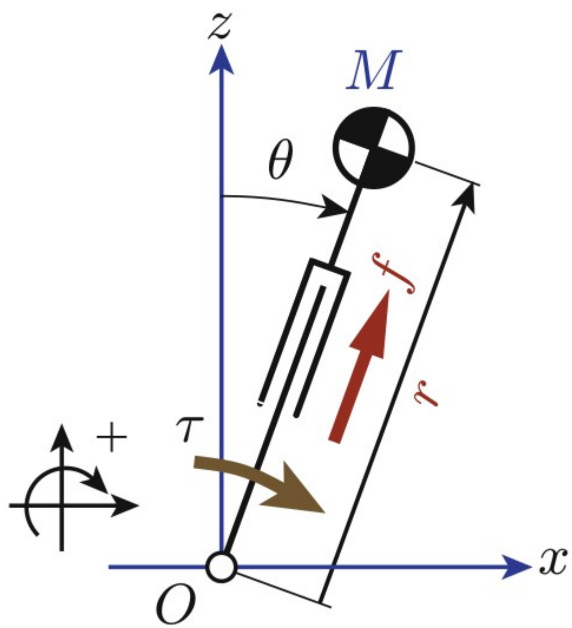
(c) $f = Mg$: Fall down and acceleration



(d) $f = Mg/\cos\theta$: CoM accelerates while keeping the initial height

Linear Inverted Pendulum Model

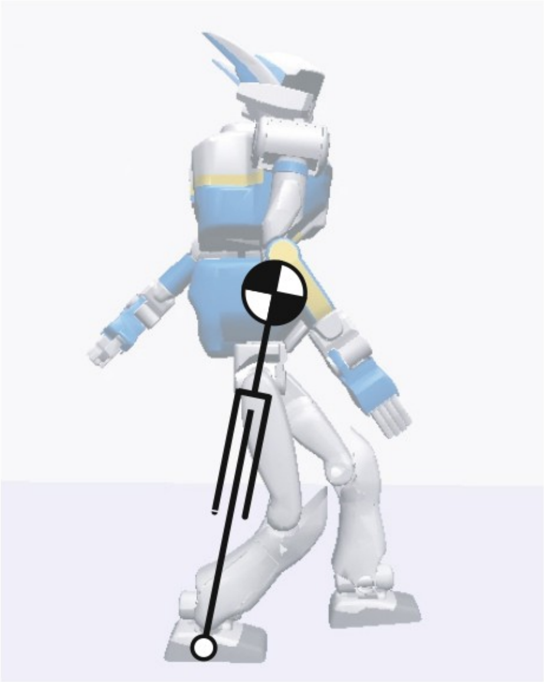


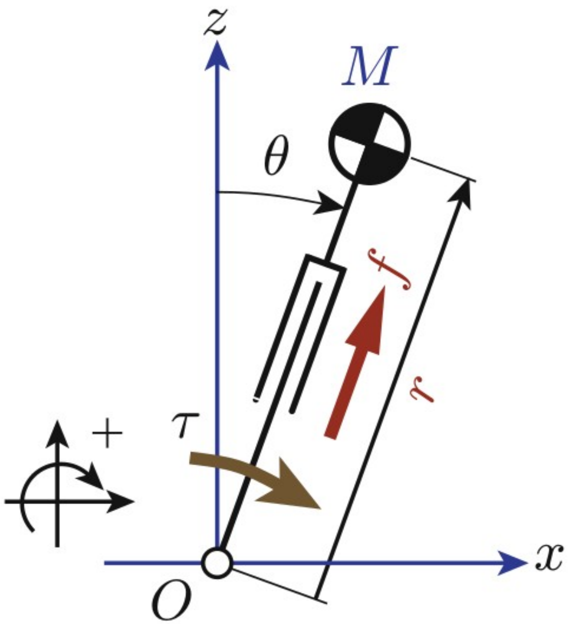


$$\ddot{\boldsymbol{\theta}} = \frac{g}{r} sin(\boldsymbol{\theta})$$

$$\boldsymbol{f} = \frac{Mg}{cos(\boldsymbol{\theta})}$$

Linear Inverted Pendulum Model



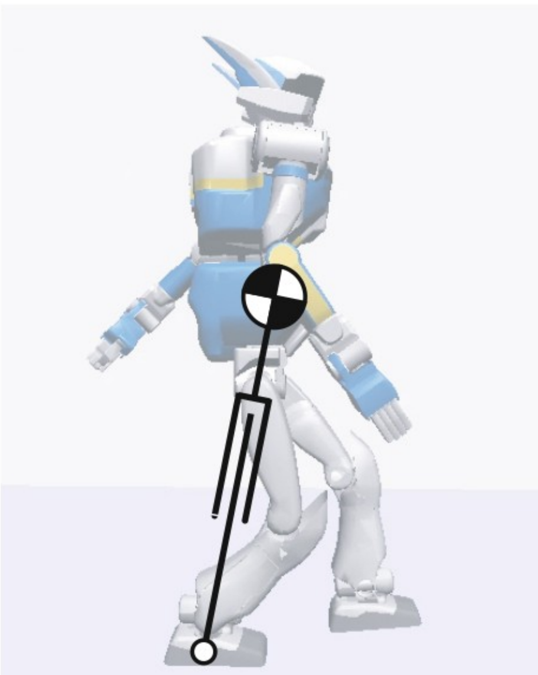


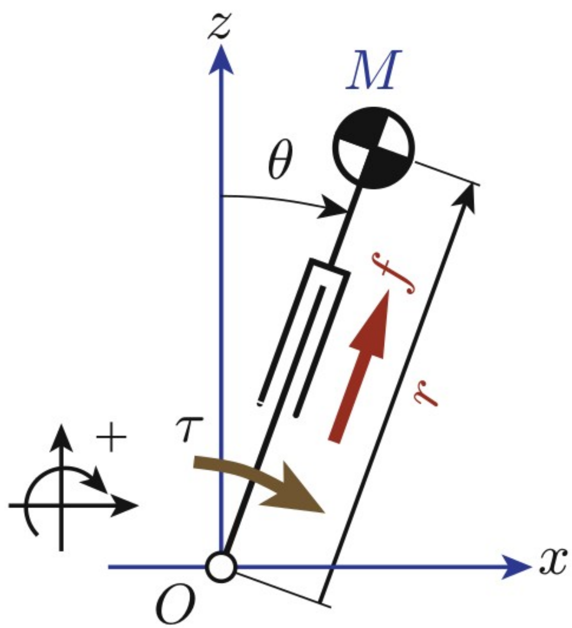
$$\ddot{\boldsymbol{\theta}} = \frac{g}{r} sin(\boldsymbol{\theta})$$

$$\boldsymbol{f} = \frac{Mg}{cos(\boldsymbol{\theta})}$$

$$\ddot{\boldsymbol{x}} = \frac{g}{r} \boldsymbol{x}$$

Linear Inverted Pendulum Model





$$\ddot{\boldsymbol{\theta}} = \frac{g}{r} sin(\boldsymbol{\theta})$$

$$\boldsymbol{f} = \frac{Mg}{cos(\boldsymbol{\theta})}$$

$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x}$$

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Linear Inverted Pendulum Model

$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x}$$

Linear Inverted Pendulum Model

$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x}$$

$$\boldsymbol{x}(t) = \boldsymbol{x}(0)\cosh\left(\frac{t}{T_c}\right) + T_c\dot{\boldsymbol{x}}(0)\sinh\left(\frac{t}{T_c}\right)$$

$$\dot{\boldsymbol{x}}(t) = \frac{\boldsymbol{x}(0)}{T_c}\sinh\left(\frac{t}{T_c}\right) + \dot{\boldsymbol{x}}(0)\cosh\left(\frac{t}{T_c}\right)$$

Linear Inverted Pendulum Model

$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x}$$

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$$T_c = \sqrt{\frac{z}{g}}$$

Linear Inverted Pendulum Model

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$$\dot{\boldsymbol{x}}(t) = \frac{\boldsymbol{x}(0)}{T_c}\sinh\left(\frac{t}{T_c}\right) + \dot{\boldsymbol{x}}(0)\cosh\left(\frac{t}{T_c}\right)$$

$$T_c = \sqrt{\frac{z}{g}} \qquad \omega_c = \sqrt{\frac{g}{z}}$$

Pendulum Natural Frequency

Linear Inverted Pendulum Model

$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x}$$

$$\boldsymbol{x}(t) = \boldsymbol{x}(0)\cosh\left(\frac{t}{T_c}\right) + T_c\dot{\boldsymbol{x}}(0)\sinh\left(\frac{t}{T_c}\right)$$

$$\dot{\boldsymbol{x}}(t) = \frac{\boldsymbol{x}(0)}{T_c}\sinh\left(\frac{t}{T_c}\right) + \dot{\boldsymbol{x}}(0)\cosh\left(\frac{t}{T_c}\right)$$

$$T_c = \sqrt{\frac{z}{g}} \qquad \omega_c = \sqrt{\frac{g}{z}}$$

Pendulum Natural Frequency

$$f_c = \frac{1}{\pi}\sqrt{\frac{g}{z}}$$

Linear Inverted Pendulum Model

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 \cosh\left(\frac{\Delta t}{T_c}\right) + T_c \dot{\boldsymbol{x}}_0 \sinh\left(\frac{\Delta t}{T_c}\right)$$

$$\dot{\boldsymbol{x}}_1 = \frac{\boldsymbol{x}_0}{T_c} \sinh\left(\frac{\Delta t}{T_c}\right) + \dot{\boldsymbol{x}}_0 \cosh\left(\frac{\Delta t}{T_c}\right)$$

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$$\cosh \boldsymbol{x} = \frac{e^{\boldsymbol{x}} + e^{-\boldsymbol{x}}}{2} \qquad \sinh \boldsymbol{x} = \frac{e^{\boldsymbol{x}} - e^{-\boldsymbol{x}}}{2}$$

Linear Inverted Pendulum Model

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$$\boldsymbol{x}_1 = \frac{\boldsymbol{x}_0 + T_c \dot{\boldsymbol{x}}_0}{2} e^{\frac{\Delta t}{T_c}} + \frac{\boldsymbol{x}_0 - T_c \dot{\boldsymbol{x}}_0}{2} e^{-\frac{\Delta t}{T_c}}$$

$$\dot{\boldsymbol{x}}_1 = \frac{\boldsymbol{x}_0 + T_c \dot{\boldsymbol{x}}_0}{2T_c} e^{\frac{\Delta t}{T_c}} - \frac{\boldsymbol{x}_0 - T_c \dot{\boldsymbol{x}}_0}{2T_c} e^{-\frac{\Delta t}{T_c}}$$

Linear Inverted Pendulum Model

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 \cosh\left(\frac{\Delta t}{T_c}\right) + T_c \dot{\boldsymbol{x}}_0 \sinh\left(\frac{\Delta t}{T_c}\right)$$

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$$\boldsymbol{x}_1 + T_c \dot{\boldsymbol{x}}_1 = \left(\boldsymbol{x}_0 + T_c \dot{\boldsymbol{x}}_0\right) e^{\frac{\Delta t}{T_c}}$$

Linear Inverted Pendulum Model

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 \cosh\left(\frac{\Delta t}{T_c}\right) + T_c \dot{\boldsymbol{x}}_0 \sinh\left(\frac{\Delta t}{T_c}\right)$$

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$$\dot{\boldsymbol{x}}_1 = \frac{\boldsymbol{x}_0 + T_c \dot{\boldsymbol{x}}_0}{2T_c} e^{\frac{\Delta t}{T_c}} - \frac{\boldsymbol{x}_0 - T_c \dot{\boldsymbol{x}}_0}{2T_c} e^{-\frac{\Delta t}{T_c}}$$

$$\boldsymbol{x}_1 + T_c \dot{\boldsymbol{x}}_1 = \left(\boldsymbol{x}_0 + T_c \dot{\boldsymbol{x}}_0\right) e^{\frac{\Delta t}{T_c}}$$

$$\Delta t = T_c \ln \frac{\boldsymbol{x}_0 - T_c \dot{\boldsymbol{x}}_0}{\boldsymbol{x}_1 - T_c \dot{\boldsymbol{x}}_1}$$

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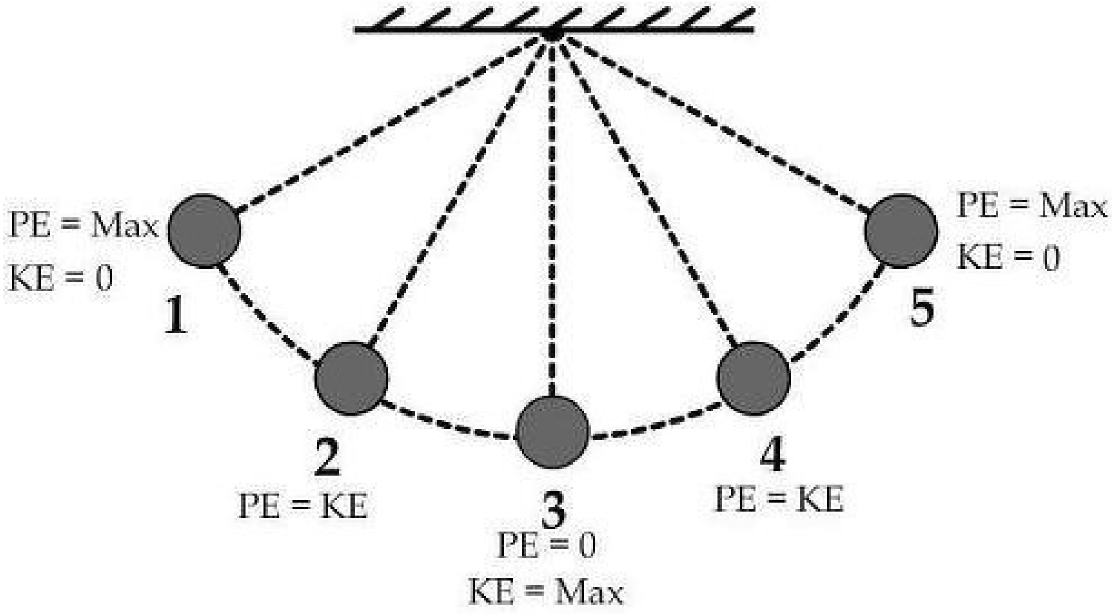
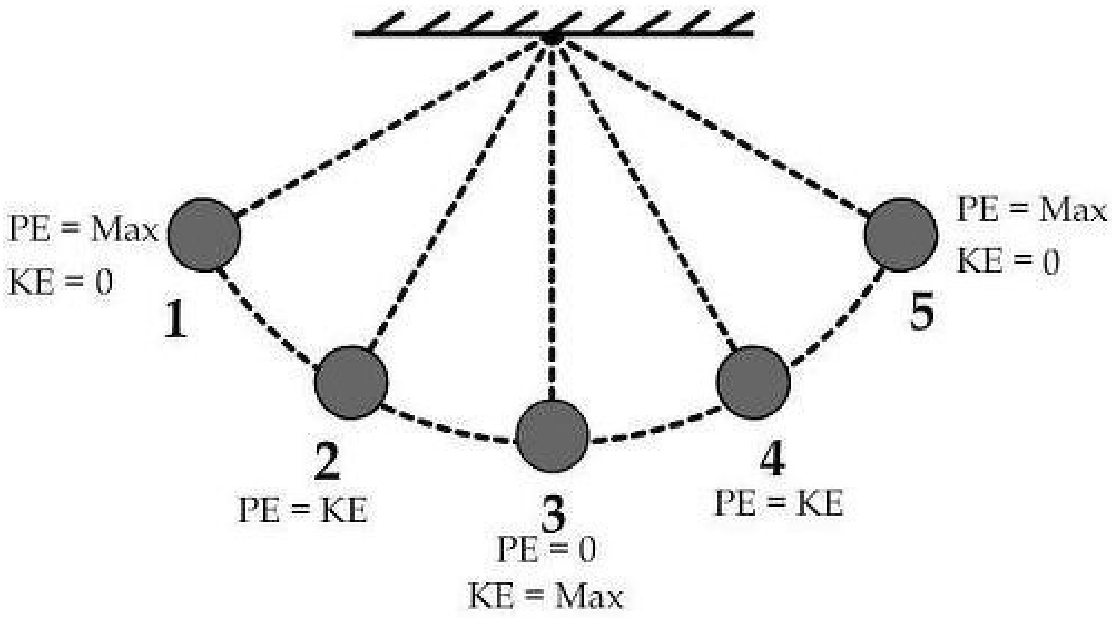
Linear Inverted Pendulum Model



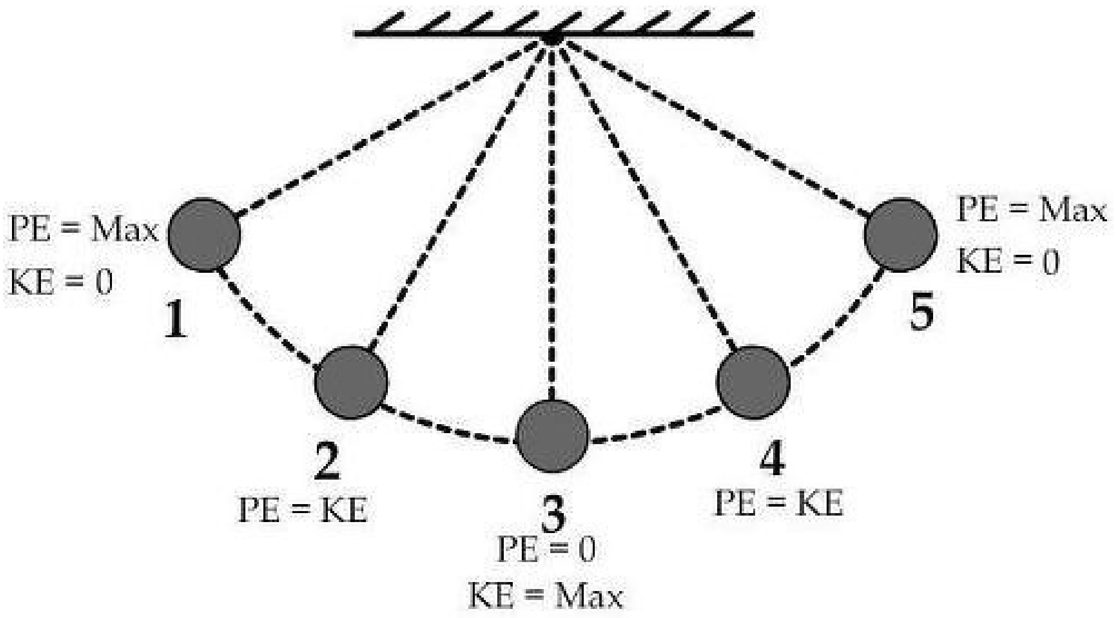
Linear Inverted Pendulum Model



$$KE_{max} = \frac{1}{2} M \dot{\boldsymbol{x}}_{max}$$

$$PE_{max} = M g z_{max}$$

Linear Inverted Pendulum Model



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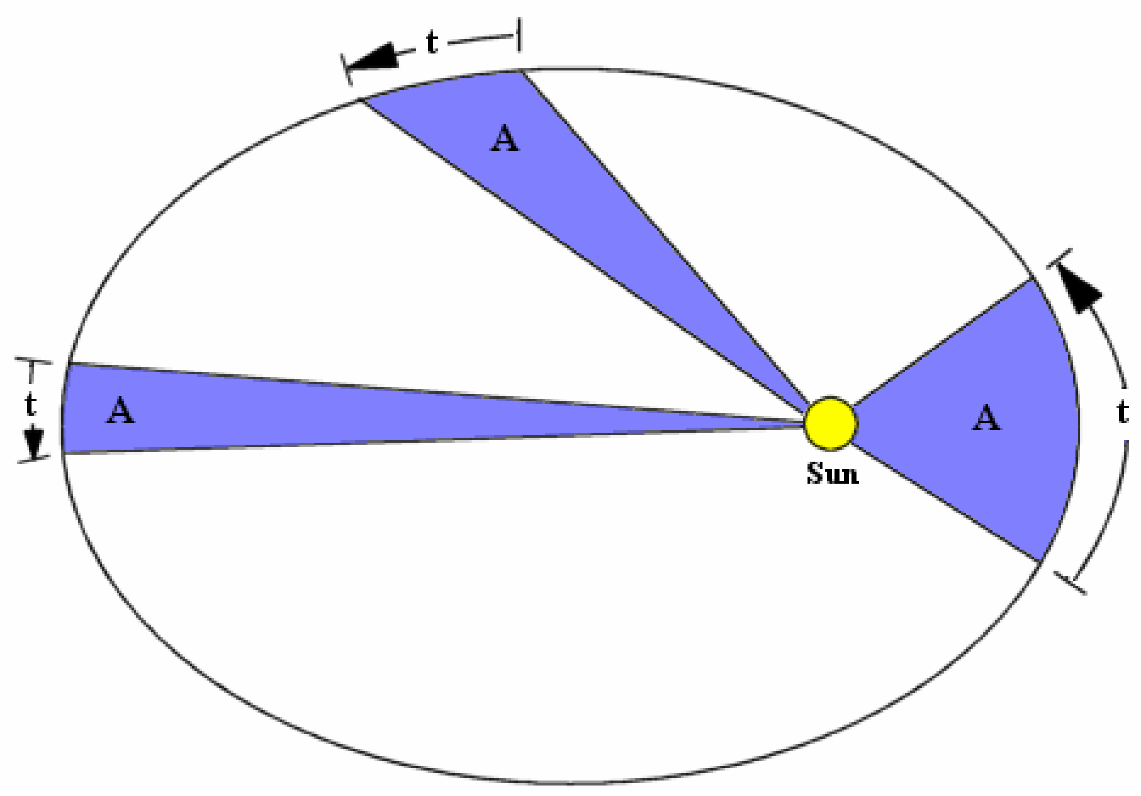
Linear Inverted Pendulum Model



$$KE_{max} = \frac{1}{2} M \dot{\boldsymbol{x}}_{max}$$

$$PE_{max} = M g z_{max}$$

$$KE_{max} = PE_{max}$$

Linear Inverted Pendulum Model

$$\dot{\boldsymbol{x}}\left(\ddot{\boldsymbol{x}} - \frac{g}{z}\boldsymbol{x}\right) = 0$$

$$\int \{\ddot{x}\dot{x} - \frac{g}{z}x\dot{x}\}dt = constant$$

$$\frac{1}{2}\dot{\boldsymbol{x}}^2 - \frac{g}{2z}\boldsymbol{x}^2 = E$$

KE PE

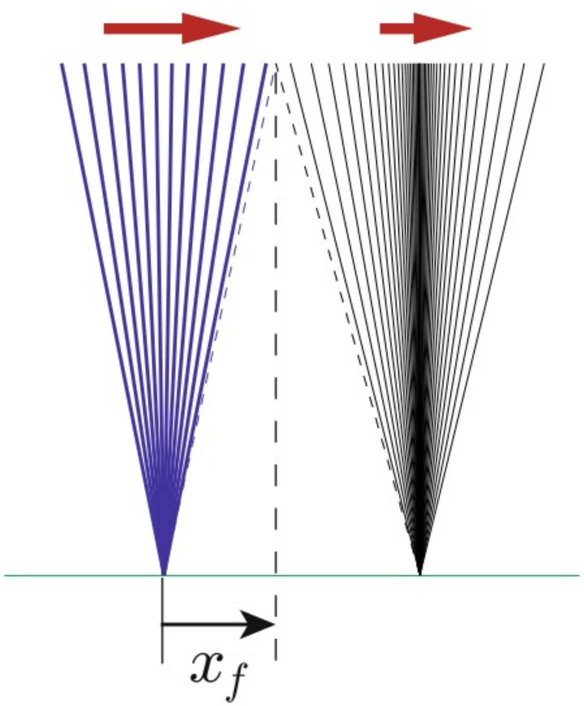
Linear Inverted Pendulum Model

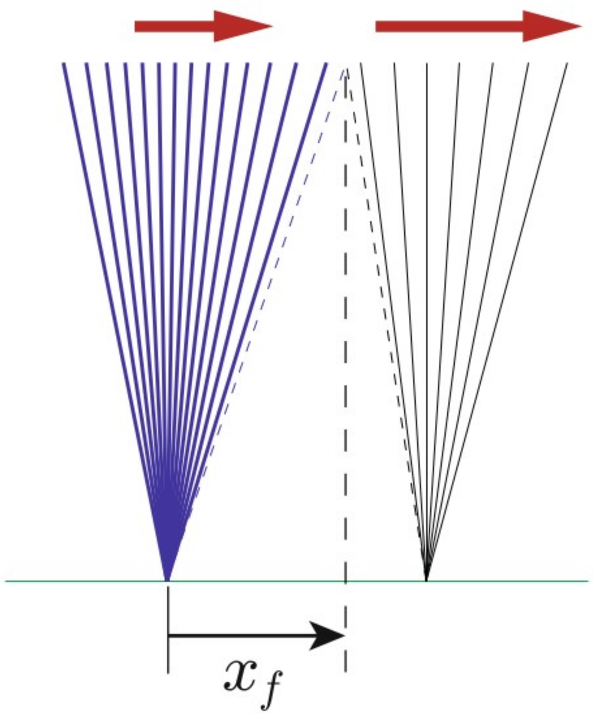
$$\dot{\boldsymbol{x}}\left(\ddot{\boldsymbol{x}}-\frac{g}{z}\boldsymbol{x}\right)=0$$

$$\int\{\ddot{x}\dot{x}-\frac{g}{z}x\dot{x}\}dt=constant$$

$$\frac{1}{2}\dot{\boldsymbol{x}}^2-\frac{g}{2z}\boldsymbol{x}^2=E$$

KE PE





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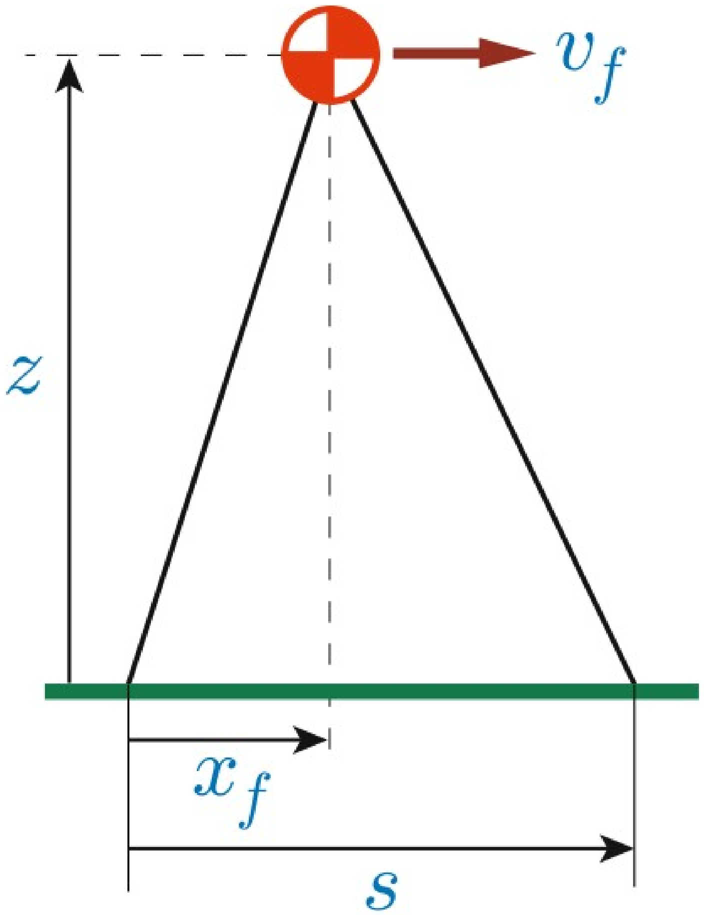
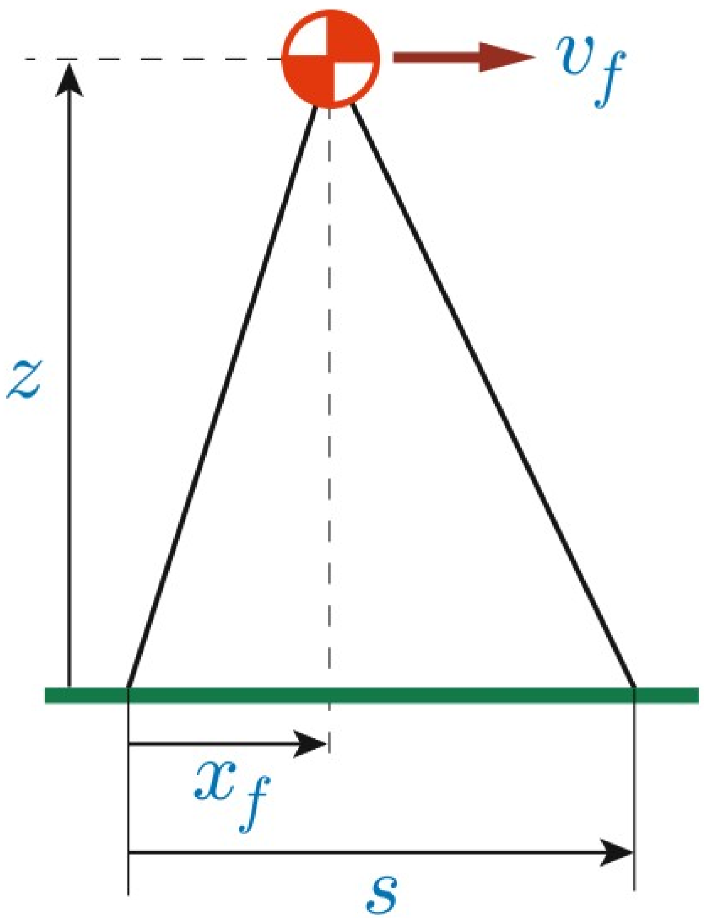
Linear Inverted Pendulum Model



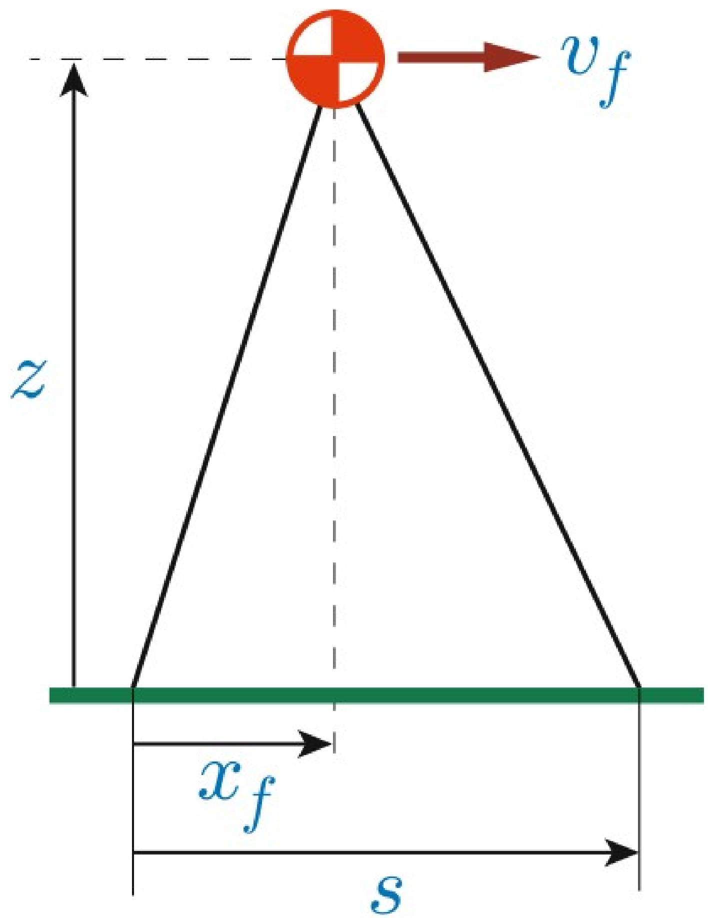
Linear Inverted Pendulum Model



$$E_1 = \frac{1}{2}\boldsymbol{v}_f^2 - \frac{g}{2z}\boldsymbol{x}_f^2$$

$$E_2 = \frac{1}{2}\boldsymbol{v}_f^2 - \frac{g}{2z}\left(\boldsymbol{x}_f - s\right)^2$$

Linear Inverted Pendulum Model



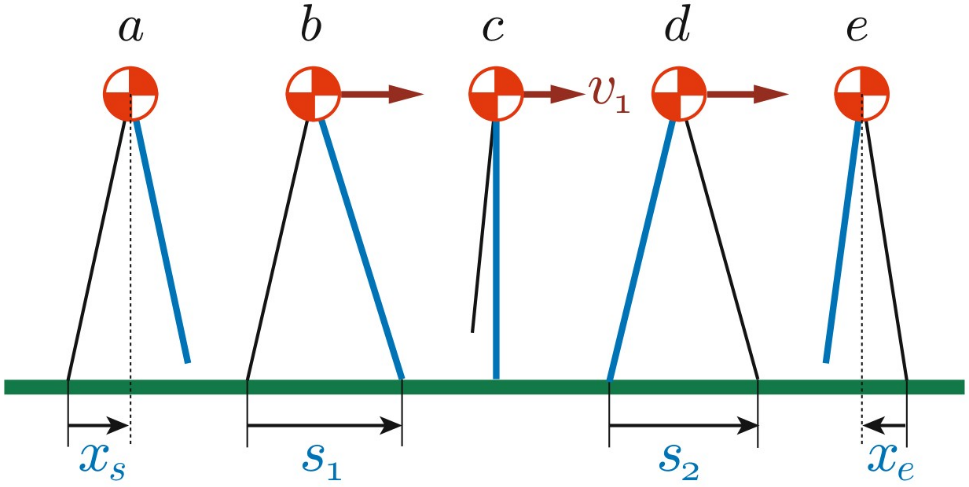
$$E_1 = \frac{1}{2}\boldsymbol{v}_f^2 - \frac{g}{2z}\boldsymbol{x}_f^2$$

$$E_2 = \frac{1}{2}\boldsymbol{v}_f^2 - \frac{g}{2z}\left(\boldsymbol{x}_f - s\right)^2$$

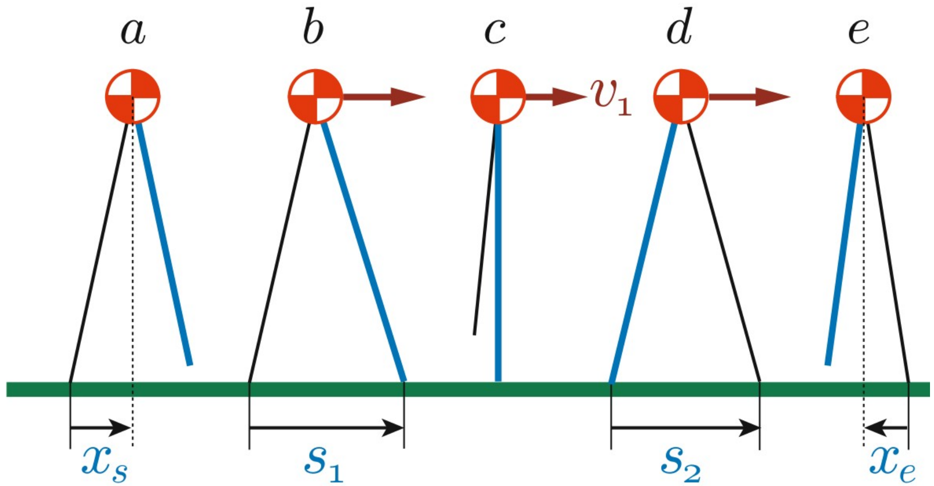
$$\boldsymbol{x}_f = \frac{z}{gs}\left(E_2 - E_1\right) + \frac{s}{2}$$

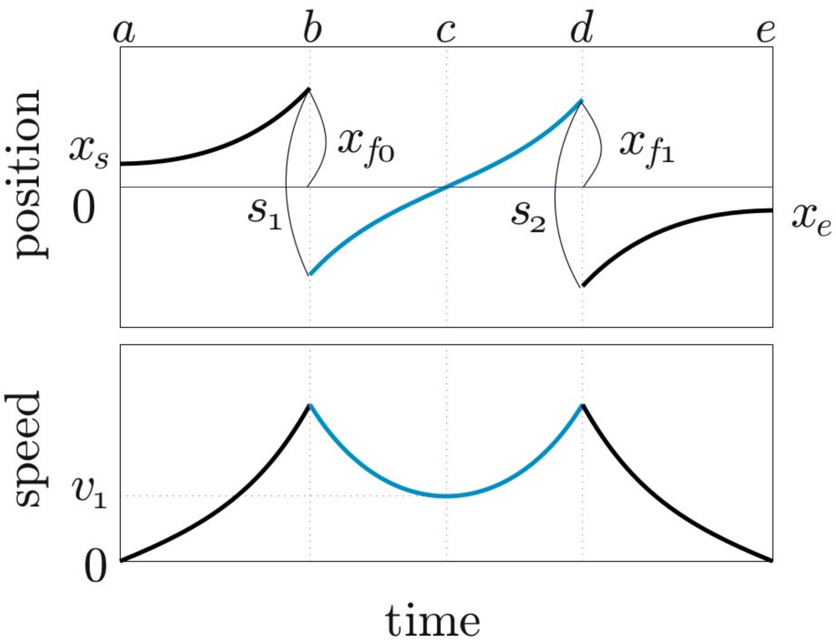
$$\boldsymbol{v}_f = \sqrt{2E_1 + \frac{g}{z}\boldsymbol{x}_f^2}$$

Linear Inverted Pendulum Model



Linear Inverted Pendulum Model





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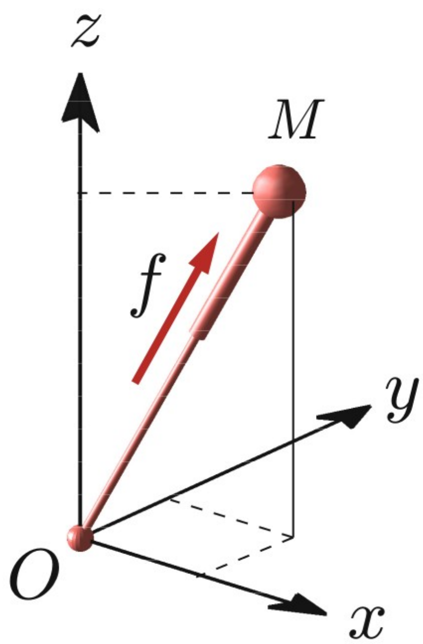
Linear Inverted Pendulum Model 3D



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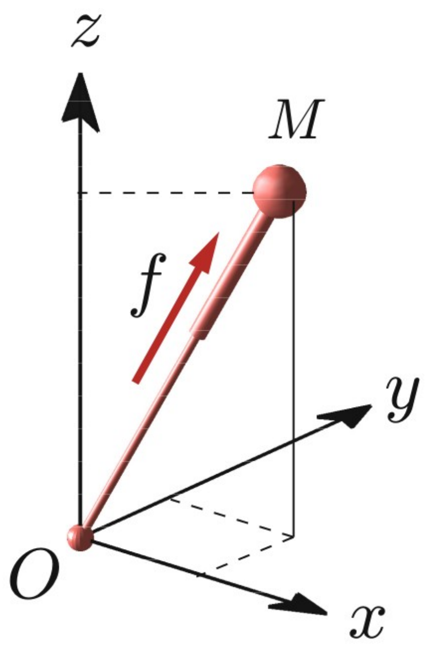
Linear Inverted Pendulum Model 3D



$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x} \quad \ddot{\boldsymbol{y}} = \frac{g}{z}\boldsymbol{y}$$

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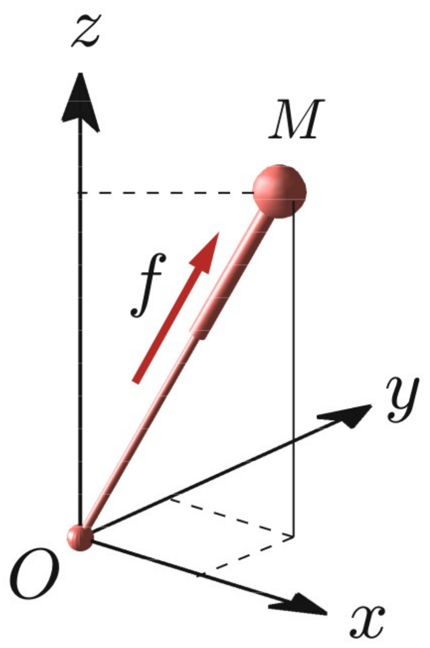
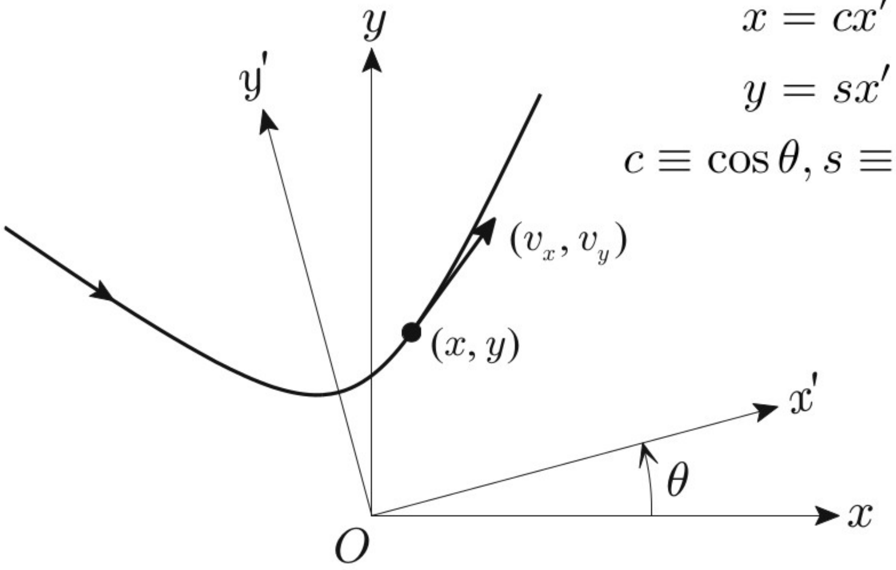
Linear Inverted Pendulum Model 3D





$$x = cx' - sy'$$
$$y = sx' + cy'$$
$$c \equiv \cos\theta, s \equiv \sin\theta.$$

$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x} \quad \ddot{\boldsymbol{y}} = \frac{g}{z}\boldsymbol{y}$$

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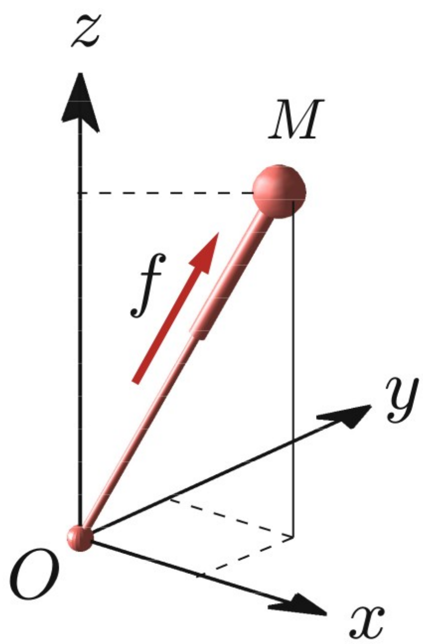
Linear Inverted Pendulum Model 3D



$$\ddot{\boldsymbol{x}} = \frac{g}{z}\boldsymbol{x} \qquad \ddot{\boldsymbol{y}} = \frac{g}{z}\boldsymbol{y}$$

$$x = cx' - sy'$$
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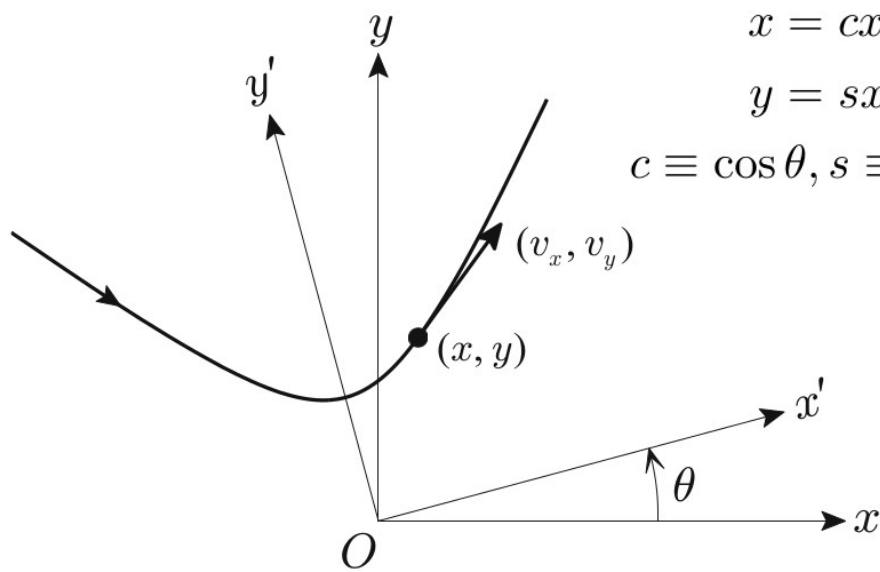

$$\ddot{\boldsymbol{x}}' = \frac{g}{z}\boldsymbol{x}' \qquad \ddot{\boldsymbol{y}}' = \frac{g}{z}\boldsymbol{y}'$$

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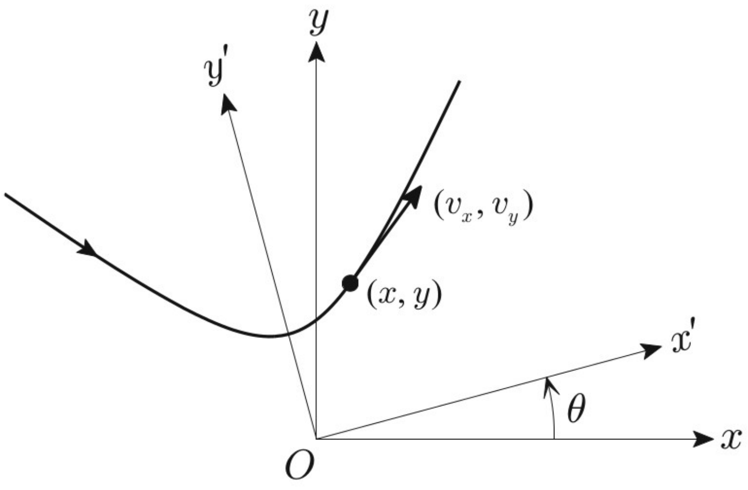
Linear Inverted Pendulum Model 3D

$$E_x' = -\frac{g}{2z}\left(c\boldsymbol{x} + s\boldsymbol{y}\right)^2 + \frac{1}{2}\left(c\dot{\boldsymbol{x}} + s\dot{\boldsymbol{y}}\right)$$



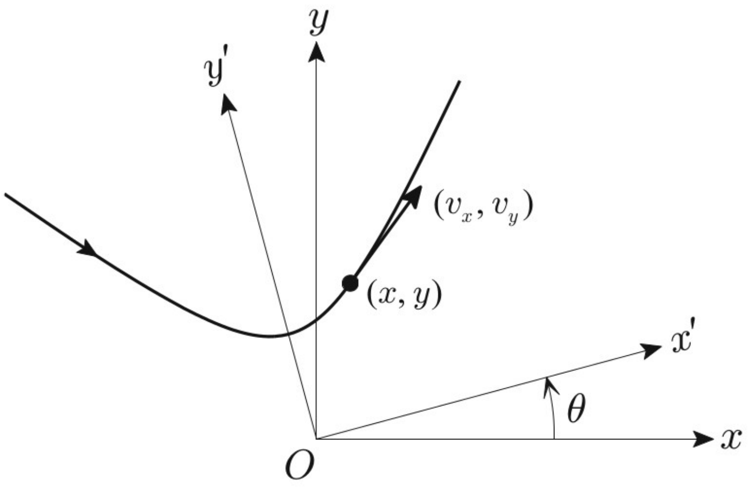
Linear Inverted Pendulum Model 3D

$$E'_x = -\frac{g}{2z}\left(c\boldsymbol{x} + s\boldsymbol{y}\right)^2 + \frac{1}{2}\left(c\dot{\boldsymbol{x}} + s\dot{\boldsymbol{y}}\right)$$

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta$$

$$A = \frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}}$$

$$B = \frac{g}{z}\left(\boldsymbol{x}^2 - \boldsymbol{y}^2\right) - \left(\dot{\boldsymbol{x}}^2 - \dot{\boldsymbol{y}}^2\right)$$



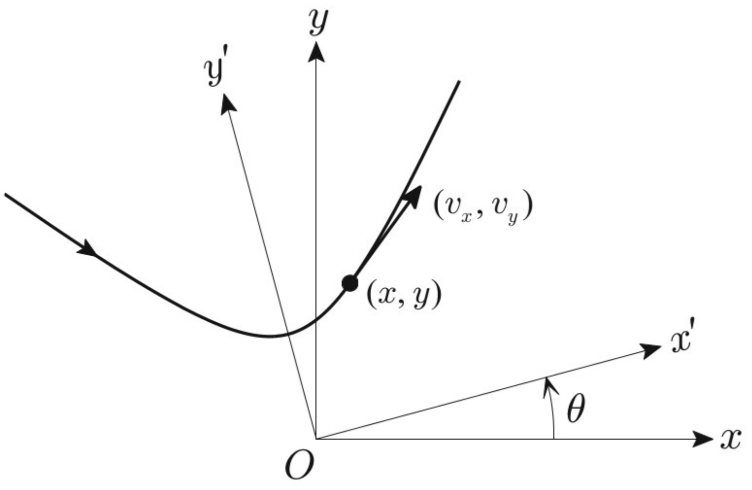
Linear Inverted Pendulum Model 3D

$$E'_x = -\frac{g}{2z}\left(c\boldsymbol{x} + s\boldsymbol{y}\right)^2 + \frac{1}{2}\left(c\dot{\boldsymbol{x}} + s\dot{\boldsymbol{y}}\right)$$

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta$$

$$A = \frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}}$$

$$B = \frac{g}{z}\left(\boldsymbol{x}^2 - \boldsymbol{y}^2\right) - \left(\dot{\boldsymbol{x}}^2 - \dot{\boldsymbol{y}}^2\right)$$



$$\theta = \begin{cases} (1/2)\tan^{-1}(2A/B) & (\text{if } B \neq 0) \\ \pi/4 & (\text{if } A \neq 0, B = 0) \end{cases}$$

Linear Inverted Pendulum Model 3D

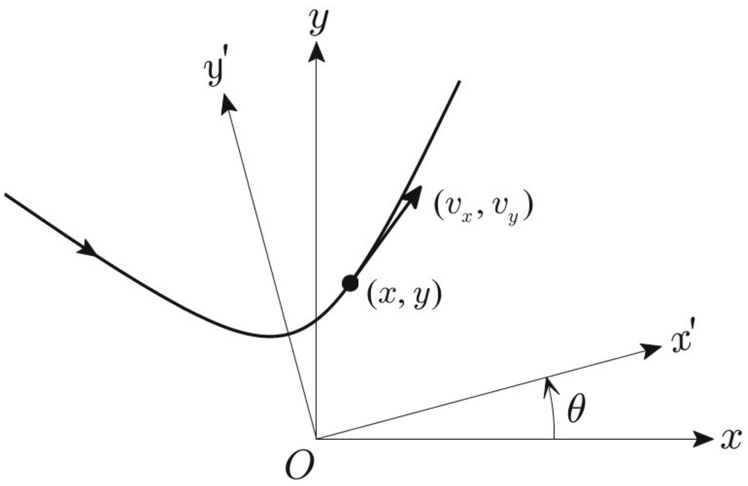
$$E'_x = -\frac{g}{2z}(c\boldsymbol{x} + s\boldsymbol{y})^2 + \frac{1}{2}(c\dot{\boldsymbol{x}} + s\dot{\boldsymbol{y}})$$

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta$$

$$A = \frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}}$$

$$B = \frac{g}{z}\left(\boldsymbol{x}^2 - \boldsymbol{y}^2\right) - \left(\dot{\boldsymbol{x}}^2 - \dot{\boldsymbol{y}}^2\right)$$

$$\theta = \begin{cases} (1/2)\tan^{-1}(2A/B) & (\text{if } B \neq 0) \\ \pi/4 & (\text{if } A \neq 0, B = 0) \end{cases}$$



$$E'_{x_{max}}$$

when

$$\theta = 0$$

Linear Inverted Pendulum Model 3D

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta \qquad\qquad \theta = 0$$

Linear Inverted Pendulum Model 3D

$$\frac{\partial E_x'}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta \qquad\qquad \theta = 0$$

$$\frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}} = 0$$

Linear Inverted Pendulum Model 3D

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta \qquad \theta = 0$$

$$\frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}} = 0$$

$$\dot{\boldsymbol{x}}^2 = 2E_x + \frac{g}{z}\boldsymbol{x}^2 \qquad \dot{\boldsymbol{y}}^2 = 2E_y + \frac{g}{z}\boldsymbol{y}^2$$

Linear Inverted Pendulum Model 3D

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta \qquad \theta = 0$$

$$\frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}} = 0$$

$$\dot{\boldsymbol{x}}^2 = 2E_x + \frac{g}{z}\boldsymbol{x}^2 \qquad \dot{\boldsymbol{y}}^2 = 2E_y + \frac{g}{z}\boldsymbol{y}^2$$

$$\frac{g}{2zE_x}\boldsymbol{x}^2 + \frac{g}{2zE_y}\boldsymbol{y}^2 + 1 = 0$$

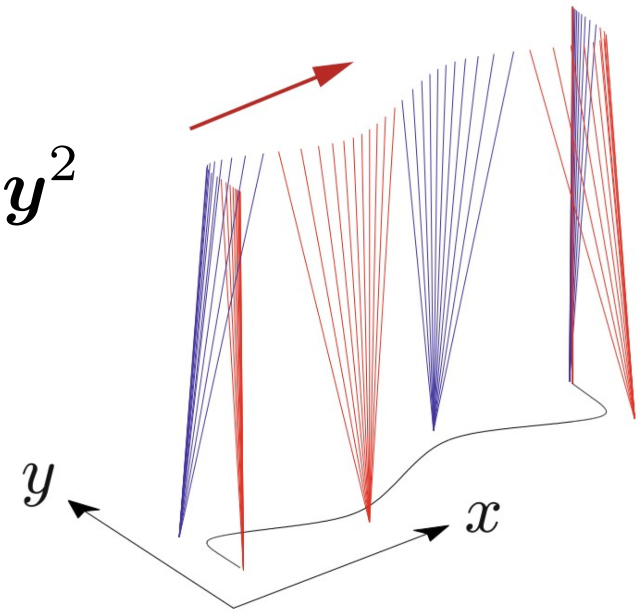
Linear Inverted Pendulum Model 3D

$$\frac{\partial E'_x}{\partial \theta} = -A\cos 2\theta + \frac{B}{2}\sin 2\theta \qquad \theta = 0$$

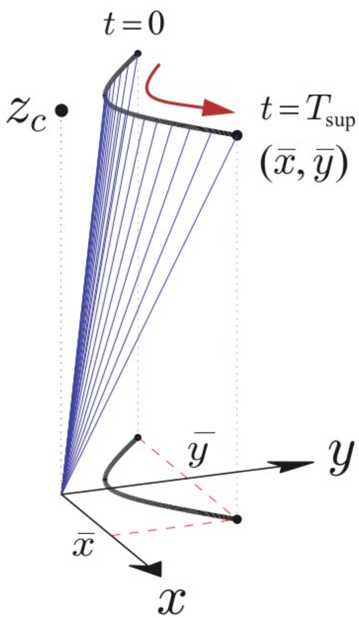
$$\frac{g}{z}\boldsymbol{x}\boldsymbol{y} - \dot{\boldsymbol{x}}\dot{\boldsymbol{y}} = 0$$

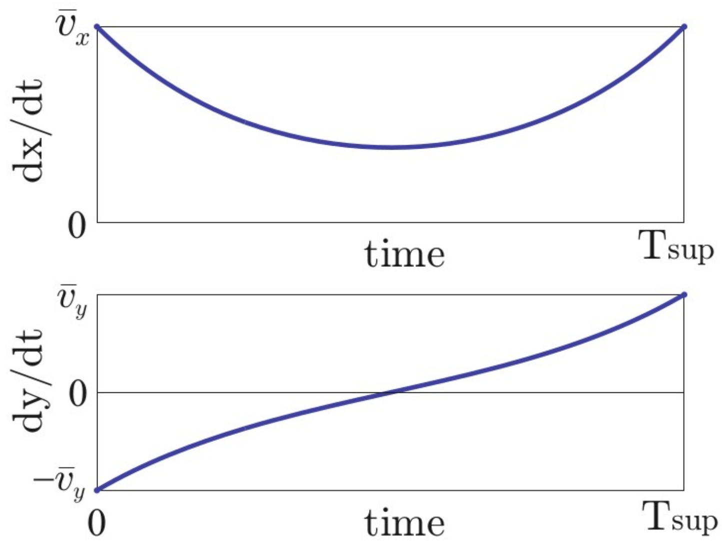
$$\dot{\boldsymbol{x}}^2 = 2E_x + \frac{g}{z}\boldsymbol{x}^2 \qquad \dot{\boldsymbol{y}}^2 = 2E_y + \frac{g}{z}\boldsymbol{y}^2$$

$$\frac{g}{2zE_x}\boldsymbol{x}^2 + \frac{g}{2zE_y}\boldsymbol{y}^2 + 1 = 0$$



Linear Inverted Pendulum Model 3D





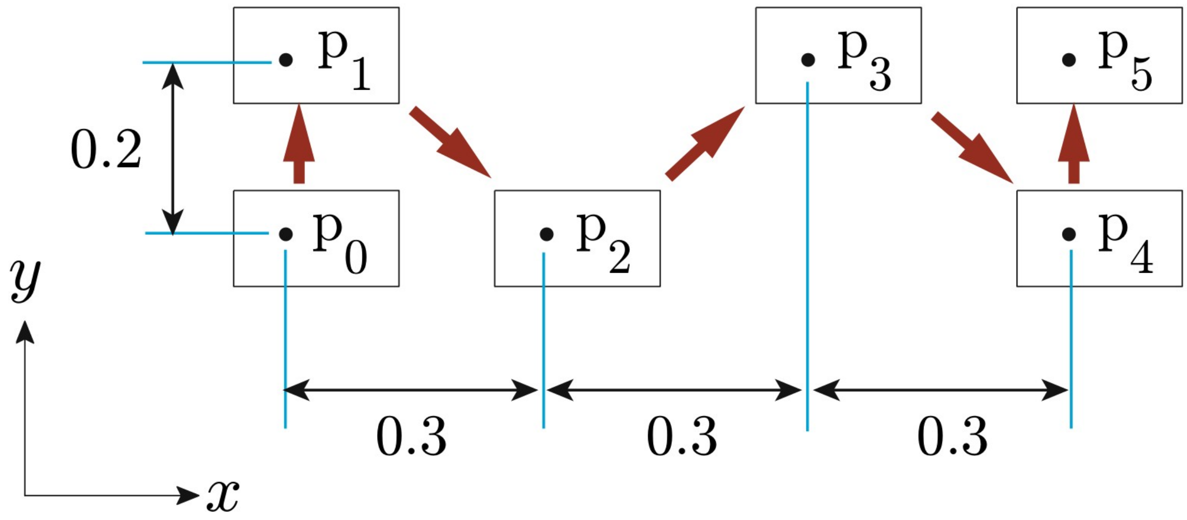
$$\overline{\boldsymbol{x}} = -\overline{\boldsymbol{x}} \cosh\left(\frac{T_{sup}}{T_c}\right) + T_c \overline{\boldsymbol{v}}_x \sinh\left(\frac{T_{sup}}{T_c}\right)$$

$$\overline{\boldsymbol{v}}_x = \frac{\overline{\boldsymbol{x}}\left(\cosh\frac{T_{sup}}{T_c} + 1\right)}{T_c \sinh\frac{T_{sup}}{T_c}}$$

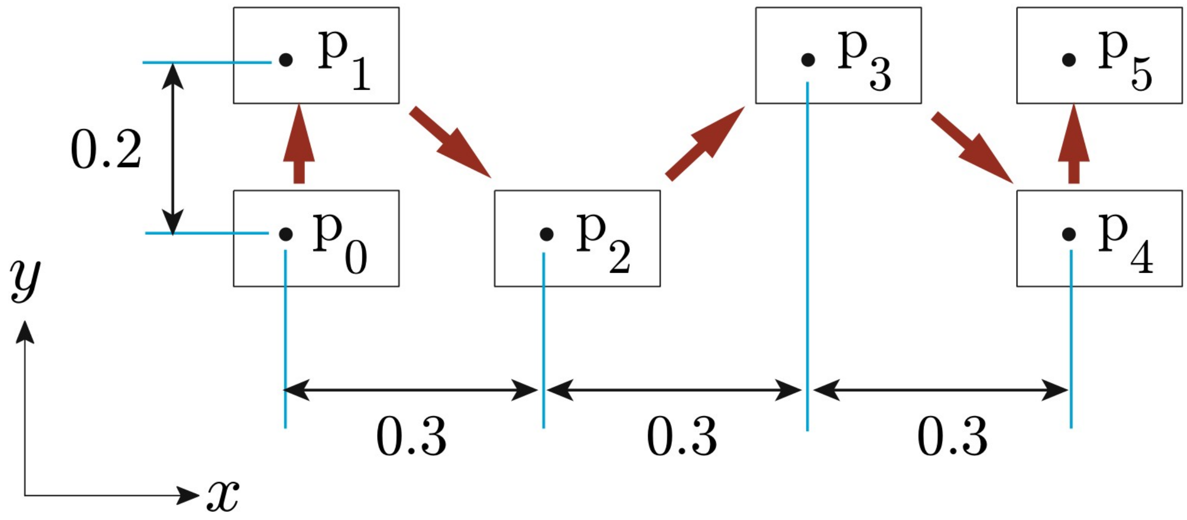
$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{y}} \cosh\left(\frac{T_{sup}}{T_c}\right) - T_c \overline{\boldsymbol{v}}_y \sinh\left(\frac{T_{sup}}{T_c}\right)$$

$$\overline{\boldsymbol{v}}_y = \frac{\overline{\boldsymbol{y}}\left(\cosh\frac{T_{sup}}{T_c} - 1\right)}{T_c \sinh\frac{T_{sup}}{T_c}}$$

Walking Primitives Generation

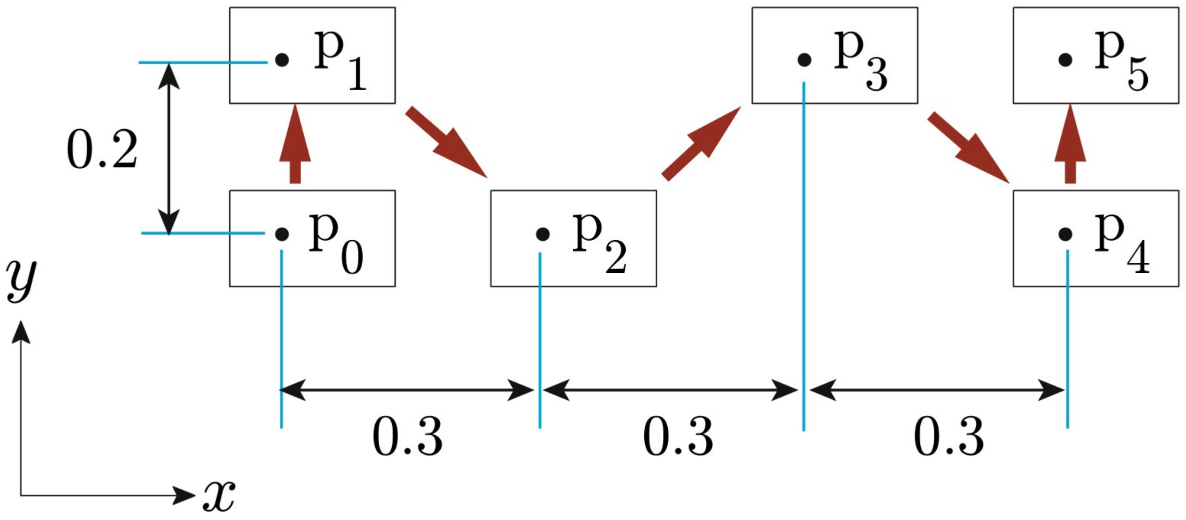


Walking Primitives Generation



$$\begin{bmatrix} p_x^{(n)} \\ p_y^{(n)} \end{bmatrix} = \begin{bmatrix} p_x^{(n-1)} + s_x^{(n)} \\ p_y^{(n-1)} - (-1)^n s_y^{(n)} \end{bmatrix}$$

Walking Primitives Generation



$$\begin{bmatrix} p_x^{(n)} \\ p_y^{(n)} \end{bmatrix} = \begin{bmatrix} p_x^{(n-1)} + s_x^{(n)} \\ p_y^{(n-1)} - (-1)^n s_y^{(n)} \end{bmatrix}$$

n	1	2	3	4	5
$s_x^{(n)}$	0.0	0.3	0.3	0.3	0
$s_y^{(n)}$	0.2	0.2	0.2	0.2	0.2

Walking Primitives Generation

$$\begin{bmatrix} \bar{x}^{(n)} \\ \bar{y}^{(n)} \end{bmatrix} = \begin{bmatrix} s_x^{(n+1)}/2 \\ (-1)^n s_y^{(n+1)}/2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{v}_x^{(n)} \\ \bar{v}_y^{(n)} \end{bmatrix} = \begin{bmatrix} (C+1)/(T_c S)\bar{x}^{(n)} \\ (C-1)/(T_c S)\bar{y}^{(n)} \end{bmatrix} \qquad C \equiv \cosh\frac{T_{sup}}{T_c} \qquad S \equiv \sinh\frac{T_{sup}}{T_c}$$

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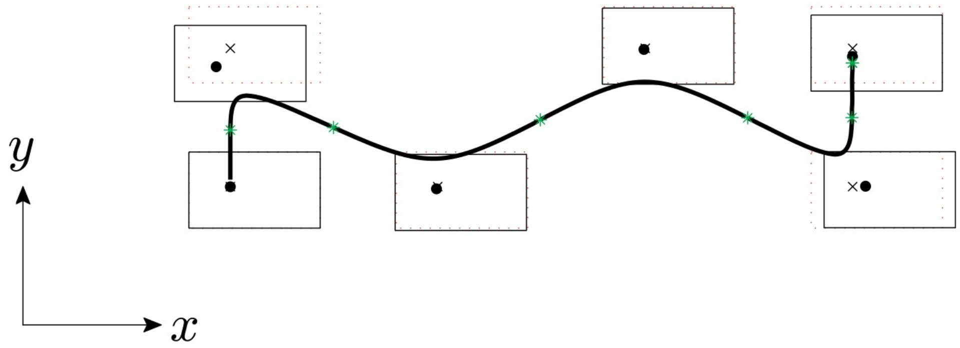
Walking Primitives Generation

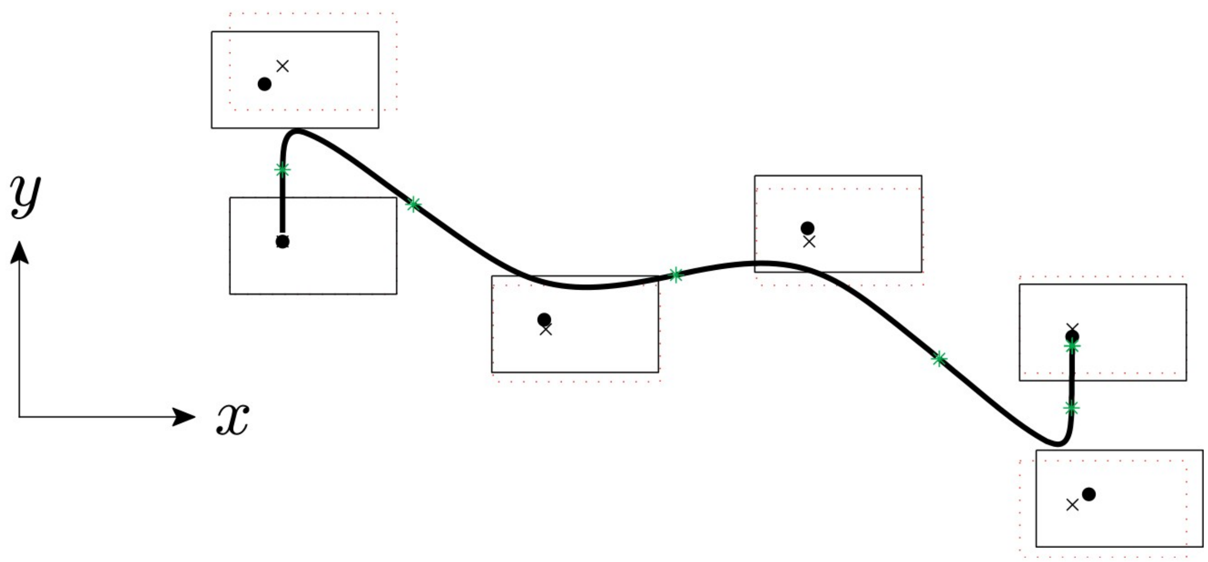
$$\begin{bmatrix} \bar{x}^{(n)} \\ \bar{y}^{(n)} \end{bmatrix} = \begin{bmatrix} s_x^{(n+1)}/2 \\ (-1)^n s_y^{(n+1)}/2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{v}_x^{(n)} \\ \bar{v}_y^{(n)} \end{bmatrix} = \begin{bmatrix} (C+1)/(T_c S)\bar{x}^{(n)} \\ (C-1)/(T_c S)\bar{y}^{(n)} \end{bmatrix} \qquad C \equiv \cosh \frac{T_{sup}}{T_c} \qquad S \equiv \sinh \frac{T_{sup}}{T_c}$$

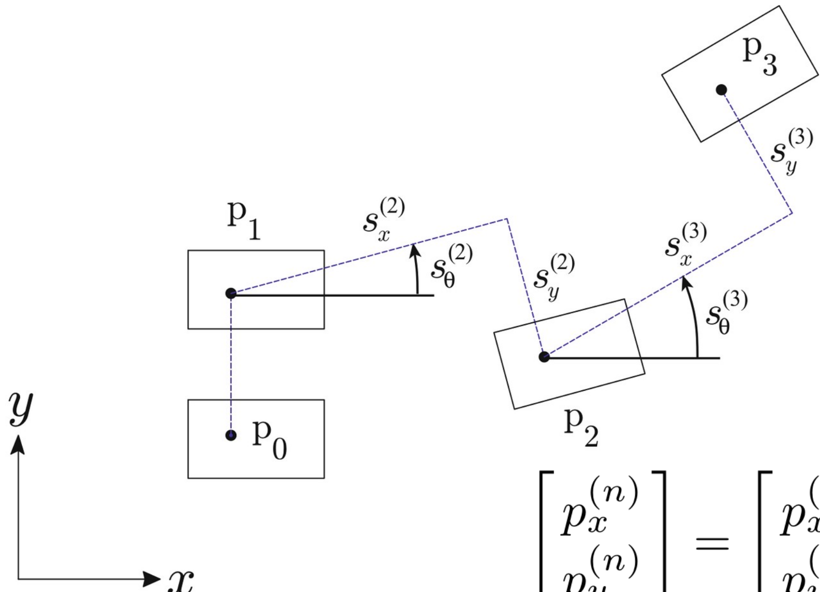


Walking Primitives Generation

n	1	2	3	4	5
$s_x^{(n)}$	0.0	0.2	0.2	0.2	0
$s_y^{(n)}$	0.2	0.3	0.1	0.3	0.2



Walking Primitives Generation



n	1	2	3	4	5
s_x	0.0	0.25	0.25	0.25	0
s_y	0.2	0.2	0.2	0.2	0.2
s_θ	0	20	40	60	60

$$\begin{bmatrix} p_x^{(n)} \\ p_y^{(n)} \end{bmatrix} = \begin{bmatrix} p_x^{(n-1)} \\ p_y^{(n-1)} \end{bmatrix} + \begin{bmatrix} \cos s_\theta^{(n)} & -\sin s_\theta^{(n)} \\ \sin s_\theta^{(n)} & \cos s_\theta^{(n)} \end{bmatrix} \begin{bmatrix} s_x^{(n)} \\ -(-1)^n s_y^{(n)} \end{bmatrix}$$

$$\begin{bmatrix} \bar{x}^{(n)} \\ \bar{y}^{(n)} \end{bmatrix} = \begin{bmatrix} \cos s_\theta^{(n+1)} & -\sin s_\theta^{(n+1)} \\ \sin s_\theta^{(n+1)} & \cos s_\theta^{(n+1)} \end{bmatrix} \begin{bmatrix} s_x^{(n+1)}/2 \\ (-1)^n s_y^{(n+1)}/2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{v}_x^{(n)} \\ \bar{v}_y^{(n)} \end{bmatrix} = \begin{bmatrix} \cos s_\theta^{(n+1)} & -\sin s_\theta^{(n+1)} \\ \sin s_\theta^{(n+1)} & \cos s_\theta^{(n+1)} \end{bmatrix} \begin{bmatrix} (1+C)/(T_c S)\bar{x}^{(n)} \\ (C-1)/(T_c S)\bar{y}^{(n)} \end{bmatrix}$$

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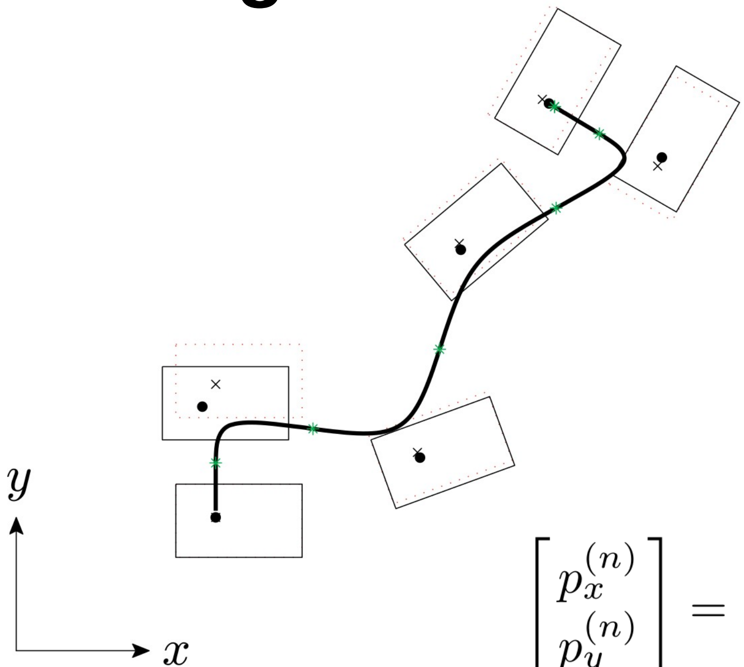
Walking Primitives Generation



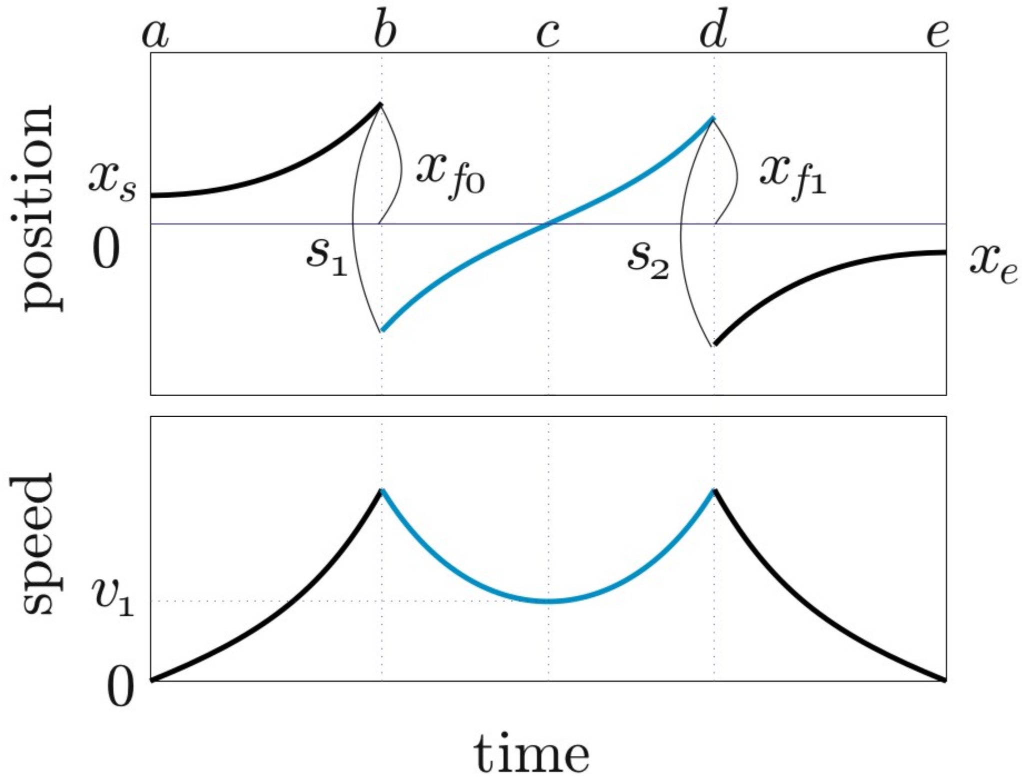
n	1	2	3	4	5
s_x	0.0	0.25	0.25	0.25	0
s_y	0.2	0.2	0.2	0.2	0.2
s_θ	0	20	40	60	60

$$\begin{bmatrix} p_x^{(n)} \\ p_y^{(n)} \end{bmatrix} = \begin{bmatrix} p_x^{(n-1)} \\ p_y^{(n-1)} \end{bmatrix} + \begin{bmatrix} \cos s_\theta^{(n)} & -\sin s_\theta^{(n)} \\ \sin s_\theta^{(n)} & \cos s_\theta^{(n)} \end{bmatrix} \begin{bmatrix} s_x^{(n)} \\ -(-1)^n s_y^{(n)} \end{bmatrix}$$

$$\begin{bmatrix} \bar{x}^{(n)} \\ \bar{y}^{(n)} \end{bmatrix} = \begin{bmatrix} \cos s_\theta^{(n+1)} & -\sin s_\theta^{(n+1)} \\ \sin s_\theta^{(n+1)} & \cos s_\theta^{(n+1)} \end{bmatrix} \begin{bmatrix} s_x^{(n+1)}/2 \\ (-1)^n s_y^{(n+1)}/2 \end{bmatrix}$$

$$\begin{bmatrix} \bar{v}_x^{(n)} \\ \bar{v}_y^{(n)} \end{bmatrix} = \begin{bmatrix} \cos s_\theta^{(n+1)} & -\sin s_\theta^{(n+1)} \\ \sin s_\theta^{(n+1)} & \cos s_\theta^{(n+1)} \end{bmatrix} \begin{bmatrix} (1+C)/(T_c S)\bar{x}^{(n)} \\ (C-1)/(T_c S)\bar{y}^{(n)} \end{bmatrix}$$

Walking Primitives Generation



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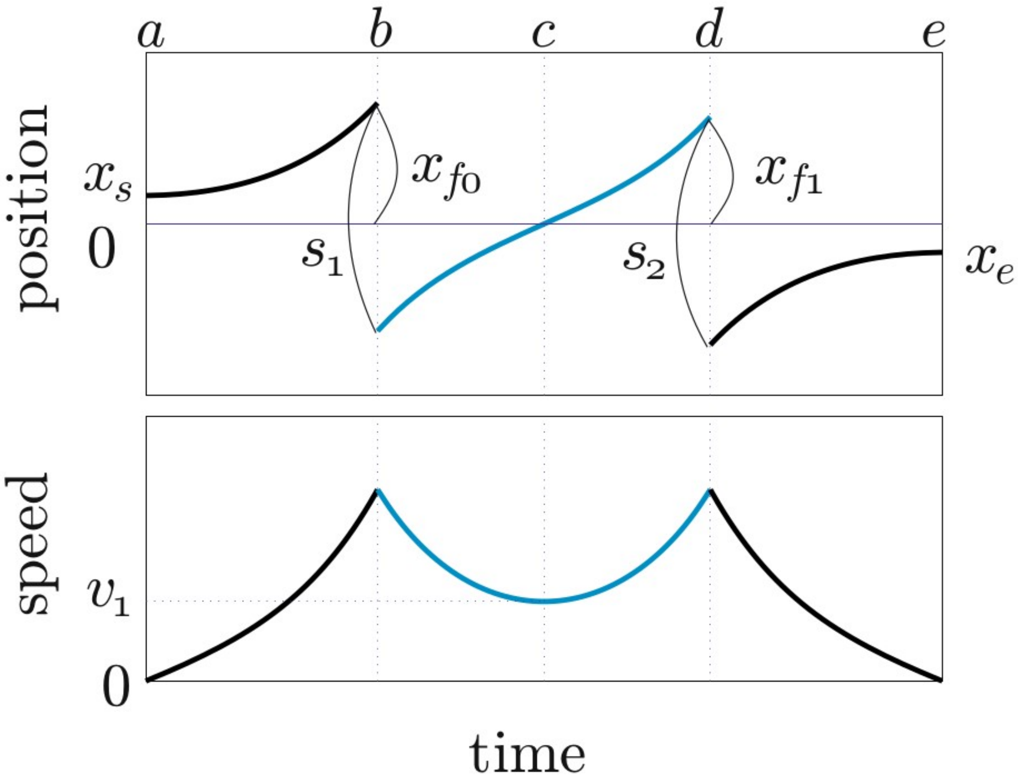
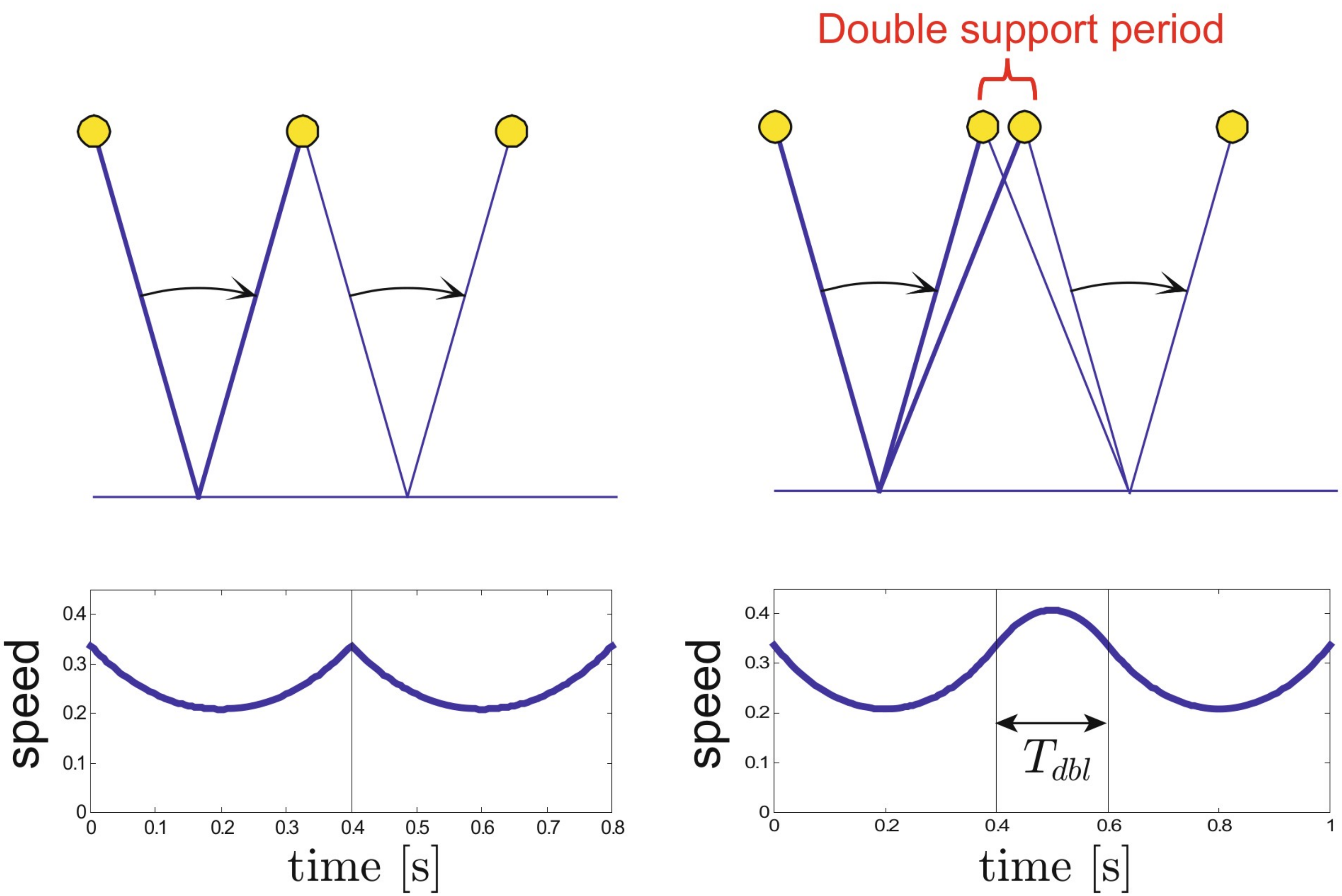
Walking Primitives Generation



This is not continuous !!!

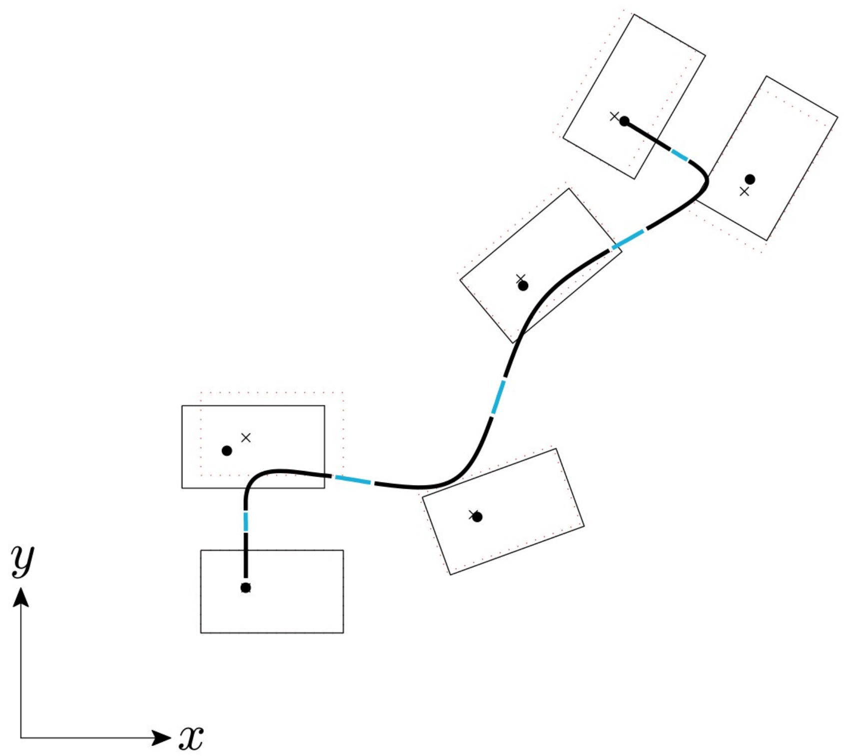
Walking Primitives Generation



Walking Primitives Generation

4th order polynomial (spline) to set initial and final position and velocity.

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$



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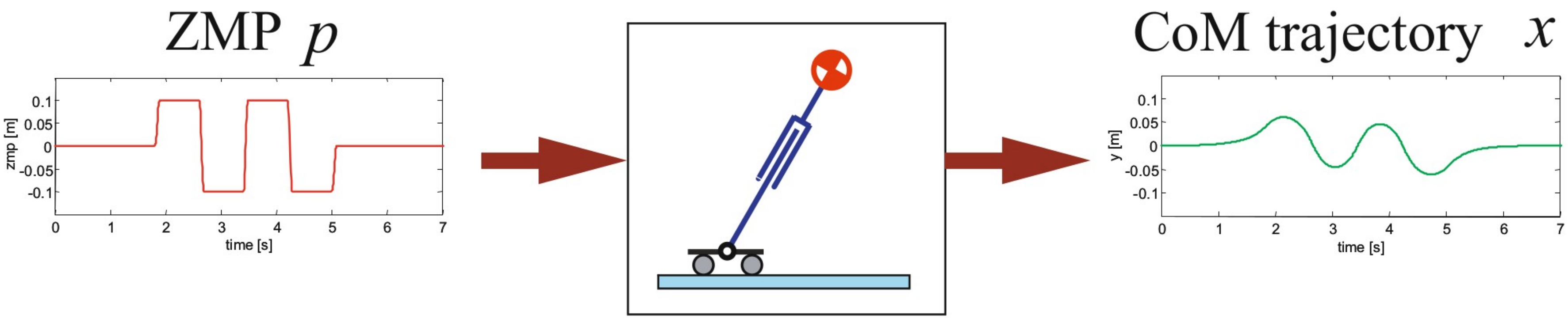
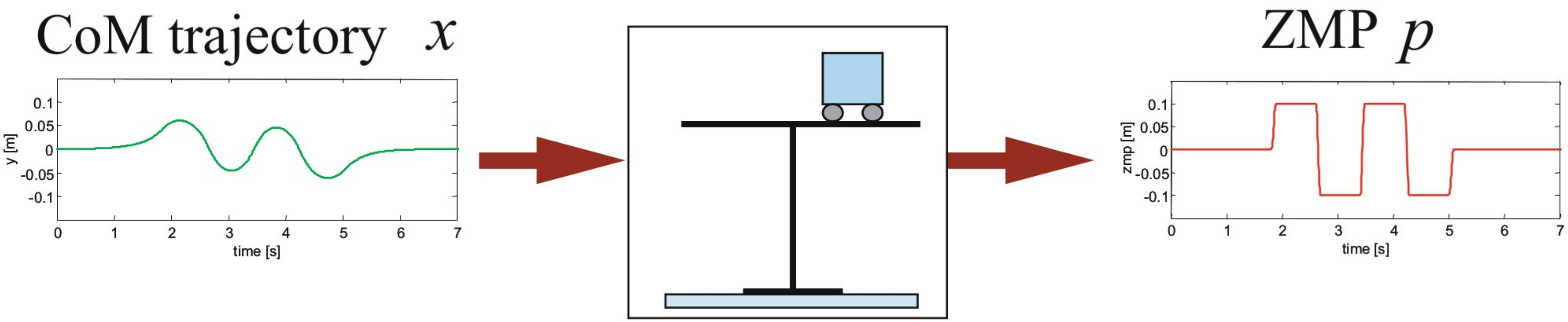
Walking Primitives Generation

- Footsteps used to generate primitives.

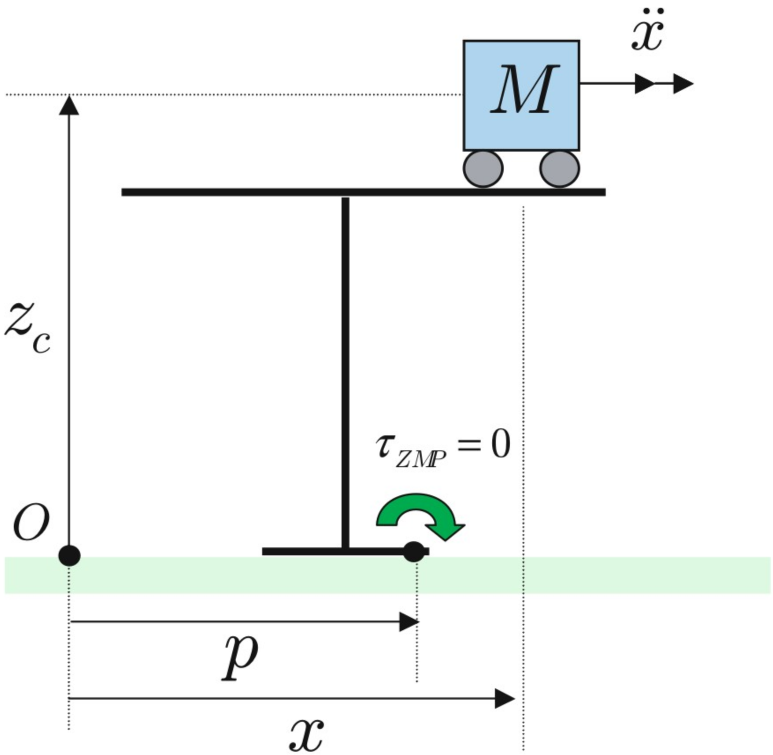


Walking Primitives Generation

- CoM motions to generate ZMP trajectory.

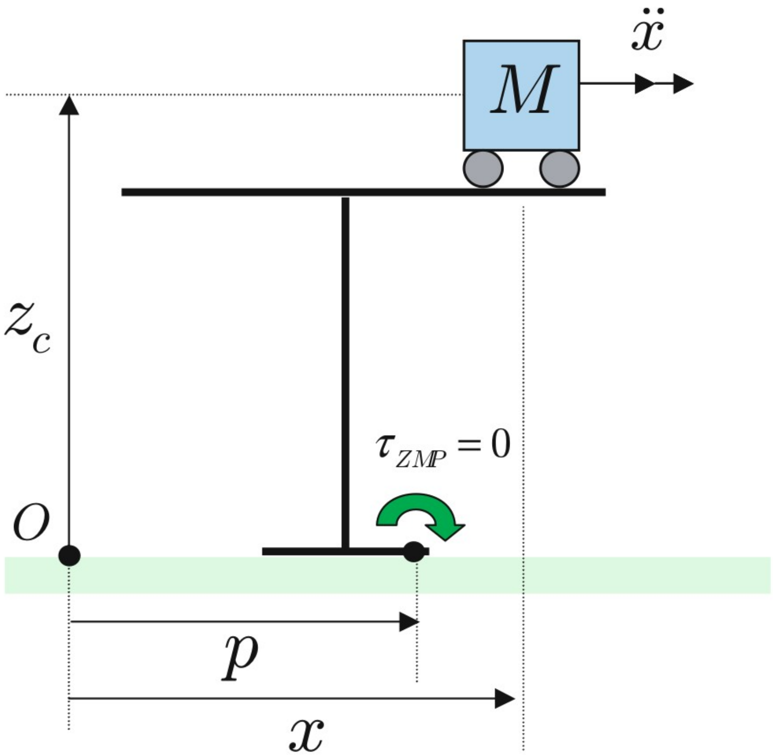


ZMP Based Walking Motion



$$\boldsymbol{p} = \boldsymbol{x} - \frac{z}{g}\ddot{\boldsymbol{x}} \qquad \ddot{\boldsymbol{x}} = \frac{g}{z}(\boldsymbol{x} - \boldsymbol{p})$$

ZMP Based Walking Motion



$$\boldsymbol{p} = \boldsymbol{x} - \frac{z}{g}\ddot{\boldsymbol{x}} \qquad \ddot{\boldsymbol{x}} = \frac{g}{z}(\boldsymbol{x} - \boldsymbol{p})$$

$$\ddot{x}_i = \frac{x_{i-1} - 2x_i + x_{i+1}}{\Delta t^2}$$

$$x_i \equiv x(i\Delta t)$$

$$p_i = ax_{i-1} + bx_i + cx_{i+1}$$

$$a_i \equiv -z_c/(g\Delta t^2)$$

$$b_i \equiv 2z_c/(g\Delta t^2) + 1$$

$$c_i \equiv -z_c/(g\Delta t^2)$$

ZMP Based Walking Motion

$$
\begin{bmatrix} p_1' \\ p_2 \\ \vdots \\ p_{N-1} \\ p_N' \end{bmatrix} = \begin{bmatrix} a_1+b_1 & c_1 & 0 & & & \\ a_2 & b_2 & c_2 & \ddots & & \\ & & & \ddots & & \\ & & & \ddots & a_{N-1} & b_{N-1} & c_{N-1} \\ & & & & 0 & a_N & b_N+c_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix}
$$

$$
p_1' = p_1 + a_1 v_1 \Delta t
$$

$$
p_N' = p_N - c_N v_N \Delta t
$$

ZMP Based Walking Motion

$$\begin{bmatrix} p'_1 \\ p_2 \\ \vdots \\ p_{N-1} \\ p'_N \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & c_1 & 0 & & & \\ a_2 & b_2 & c_2 & \ddots & & \\ & & & \ddots & & \\ & & & \ddots & a_{N-1} & b_{N-1} & c_{N-1} \\ & & & & 0 & a_N & b_N + c_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix}$$

$$p'_1 = p_1 + a_1 v_1 \Delta t$$

$$p'_N = p_N - c_N v_N \Delta t$$

$$\boldsymbol{p} = \boldsymbol{A}\boldsymbol{x}$$

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ZMP Based Walking Motion

$$
\begin{bmatrix} p'_1 \\ p_2 \\ \vdots \\ p_{N-1} \\ p'_N \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & c_1 & 0 & & & \\ a_2 & b_2 & c_2 & \ddots & & \\ & & & \ddots & & \\ & & & \ddots & a_{N-1} & b_{N-1} & c_{N-1} \\ & & & & 0 & a_N & b_N + c_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix}
$$

$$p'_1 = p_1 + a_1 v_1 \Delta t$$

$$p'_N = p_N - c_N v_N \Delta t$$

$$\boldsymbol{p} = \boldsymbol{A}\boldsymbol{x}$$

$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{p}$$

ZMP Based Walking Motion

$$
\begin{bmatrix} p'_1 \\ p_2 \\ \vdots \\ p_{N-1} \\ p'_N \end{bmatrix} = \begin{bmatrix} a_1 + b_1 & c_1 & 0 & & & \\ a_2 & b_2 & c_2 & \ddots & & \\ & & & \ddots & & \\ & & & \ddots & a_{N-1} & b_{N-1} & c_{N-1} \\ & & & & 0 & a_N & b_N + c_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix}
$$

$$p'_1 = p_1 + a_1 v_1 \Delta t$$

$$p'_N = p_N - c_N v_N \Delta t$$

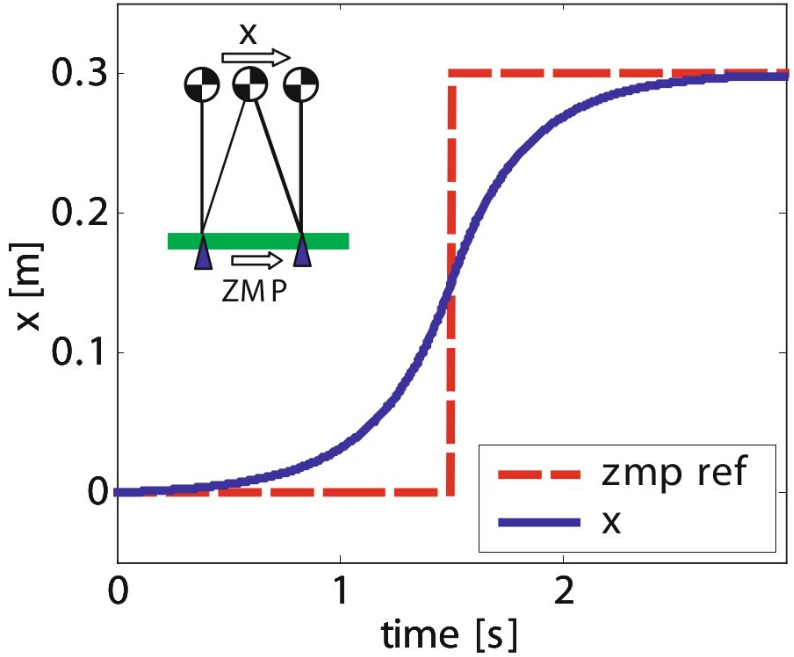
$$\boldsymbol{p} = \boldsymbol{A}\boldsymbol{x}$$

$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{p}$$

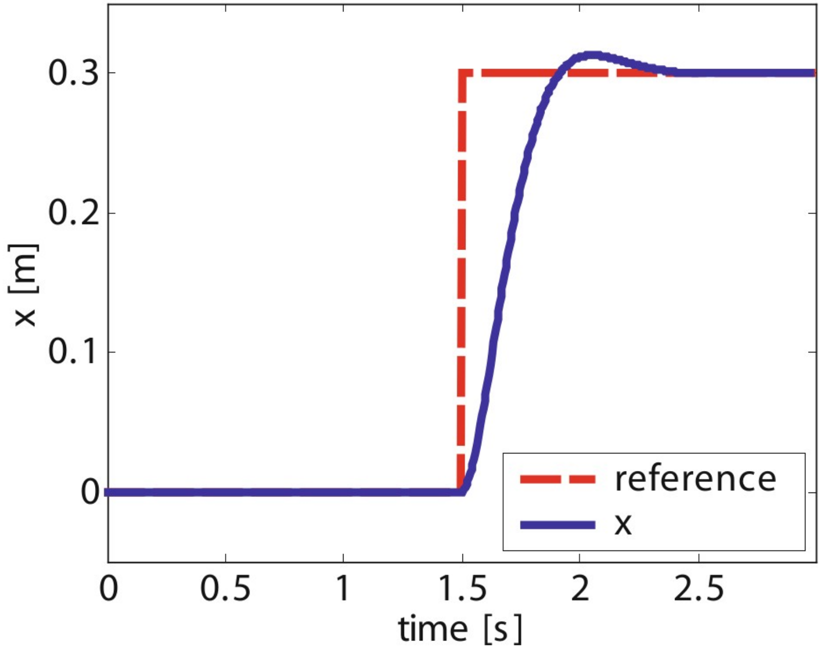
$$\Delta\boldsymbol{x} = \boldsymbol{A}^{-1}(\boldsymbol{p}^* - \boldsymbol{p}^d)$$

$$\boldsymbol{x} := \boldsymbol{x} - \Delta\boldsymbol{x}$$

Model Predictive Control



Walking motion



Classic Servoing

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Model Predictive Control

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{b}u_k \\ p_k = \boldsymbol{c}\boldsymbol{x}_k \end{cases}$$

$$\boldsymbol{x}_k \equiv [\, x(k\Delta t) \; \dot{x}(k\Delta t) \; \ddot{x}(k\Delta t) \,]^T$$

$$u_k \equiv u(k\Delta t),$$

$$p_k \equiv p(k\Delta t),$$

$$\boldsymbol{A} \equiv \begin{bmatrix} 1 & \Delta t & \Delta t^2/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{b} \equiv \begin{bmatrix} \Delta t^3/6 \\ \Delta t^2/2 \\ \Delta t \end{bmatrix}$$

$$\boldsymbol{c} \equiv [\, 1 \; 0 \; -z_c/g \,]$$

Minimize the performance index

$$J = \sum_{j=1}^{\infty} \{ Q(p_j^{ref} - p_j)^2 + R u_j^2 \}$$

$$Q, \; R \in \mathbb{R}^+$$ Weights

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Model Predictive Control

Minimize the tracking control problem

$$J = \sum_{j=1}^{\infty} \{Q(p_j^{ref} - p_j)^2 + Ru_j^2\}$$

$$Q,\ R \in \mathbb{R}^+$$ Weights

$$u_k = -\boldsymbol{K}x_k + [f_1, f_2, \cdots f_N] \begin{bmatrix} p_{k+1}^{ref} \\ \vdots \\ p_{k+N}^{ref} \end{bmatrix}$$

$$\boldsymbol{K} \equiv (R + \boldsymbol{b}^T \boldsymbol{P} \boldsymbol{b})^{-1} \boldsymbol{b}^T \boldsymbol{P} \boldsymbol{A}$$
$$f_i \equiv (R + \boldsymbol{b}^T \boldsymbol{P} \boldsymbol{b})^{-1} \boldsymbol{b}^T (\boldsymbol{A} - \boldsymbol{b}\boldsymbol{K})^{T*(i-1)} \boldsymbol{c}^T Q$$

$$\boldsymbol{P} = \boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A} + \boldsymbol{c}^T Q \boldsymbol{c} - \boldsymbol{A}^T \boldsymbol{P} \boldsymbol{b} (R + \boldsymbol{b}^T \boldsymbol{P} \boldsymbol{b})^{-1} \boldsymbol{b}^T \boldsymbol{P} \boldsymbol{A}$$ → Riccati Equation

Model Predictive Control

Minimize the tracking control problem

$$J = \sum_{j=1}^{\infty} \{Q(p_j^{ref} - p_j)^2 + Ru_j^2\}$$

$$Q,\ R \in \mathbb{R}^+$$ Weights

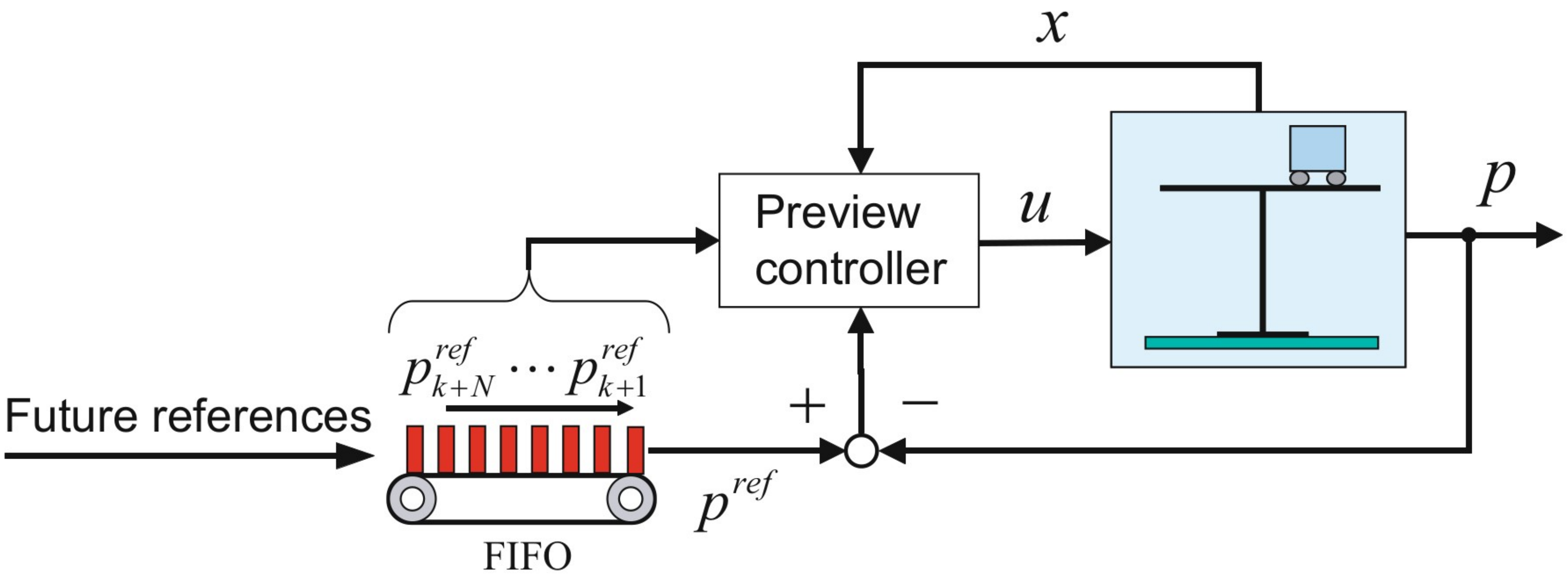
$$u_k = -\boldsymbol{K}x_k + [f_1, f_2, \cdots f_N] \begin{bmatrix} p_{k+1}^{ref} \\ \vdots \\ p_{k+N}^{ref} \end{bmatrix}$$

LQR Design Problem Solution will produce **P** and **K** e. g. dlqr in Matlab

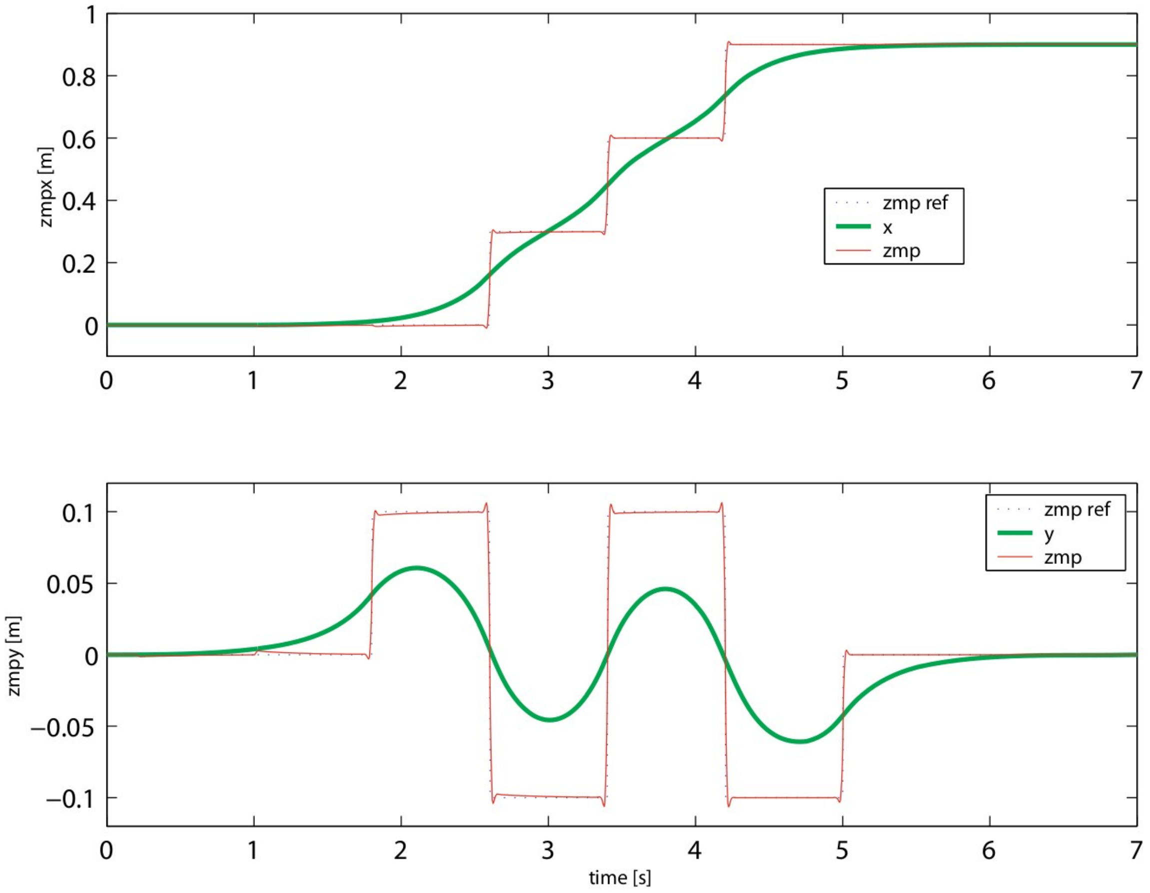
$$\boldsymbol{K} \equiv (R + \boldsymbol{b}^T\boldsymbol{P}\boldsymbol{b})^{-1}\boldsymbol{b}^T\boldsymbol{P}\boldsymbol{A}$$
$$f_i \equiv (R + \boldsymbol{b}^T\boldsymbol{P}\boldsymbol{b})^{-1}\boldsymbol{b}^T(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{K})^{T*(i-1)}\boldsymbol{c}^T Q$$

$$\boldsymbol{P} = \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} + \boldsymbol{c}^T Q\boldsymbol{c} - \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{b}(R + \boldsymbol{b}^T\boldsymbol{P}\boldsymbol{b})^{-1}\boldsymbol{b}^T\boldsymbol{P}\boldsymbol{A}$$ → Riccati Equation

Model Predictive Control

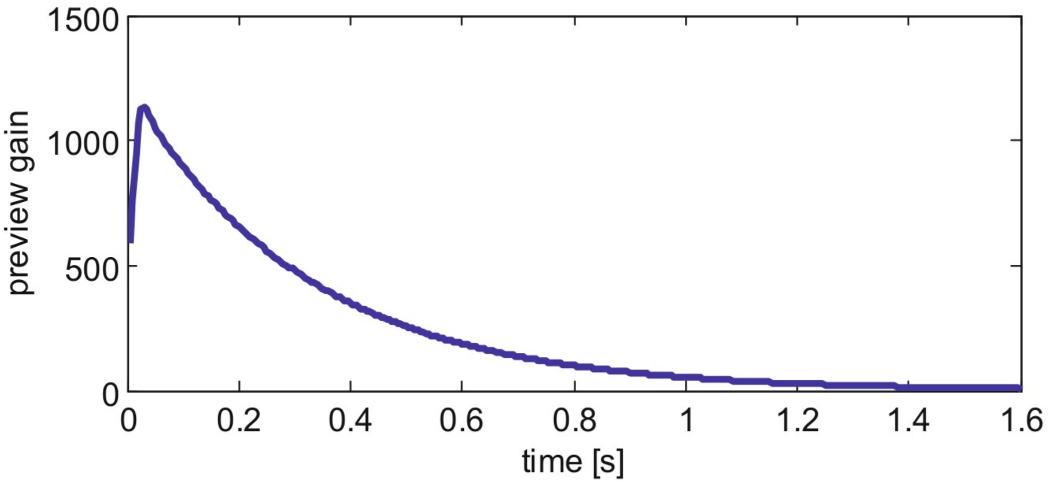


Model Predictive Control



Model Predictive Control

Preview gain



$$\sum_{i=j}^{N+j} \tilde{f}_i$$

DCM Based Walking Motions Generation

$$\boldsymbol{\xi} = \boldsymbol{x} + \frac{\dot{\boldsymbol{x}}}{\omega}$$

$$\dot{\boldsymbol{\xi}} = \dot{\boldsymbol{x}} + \frac{\ddot{\boldsymbol{x}}}{\omega}$$

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \mathbf{p})$$

$$\dot{\boldsymbol{x}} = -\omega(\boldsymbol{x} - \boldsymbol{\xi})$$

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DCM Based Walking Motions Generation

$$\boldsymbol{\xi} = \boldsymbol{x} + \frac{\dot{\boldsymbol{x}}}{\omega}$$

$$\dot{\boldsymbol{\xi}} = \dot{\boldsymbol{x}} + \frac{\ddot{\boldsymbol{x}}}{\omega}$$

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \mathbf{p})$$

$$\dot{\boldsymbol{x}} = -\omega(\boldsymbol{x} - \boldsymbol{\xi})$$

1) Plan footsteps

2) Define Virtual Repellent Points (VRP)

3) Define waypoints for the DCM

4) Interpolate CoM trajectory

DCM Based Walking Motions Generation

$$\boldsymbol{\xi} = \boldsymbol{x} + \frac{\dot{\boldsymbol{x}}}{\omega}$$

$$\dot{\boldsymbol{\xi}} = \dot{\boldsymbol{x}} + \frac{\ddot{\boldsymbol{x}}}{\omega}$$

$$\dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \mathbf{p})$$

$$\dot{\boldsymbol{x}} = -\omega(\boldsymbol{x} - \boldsymbol{\xi})$$

1) Plan footsteps

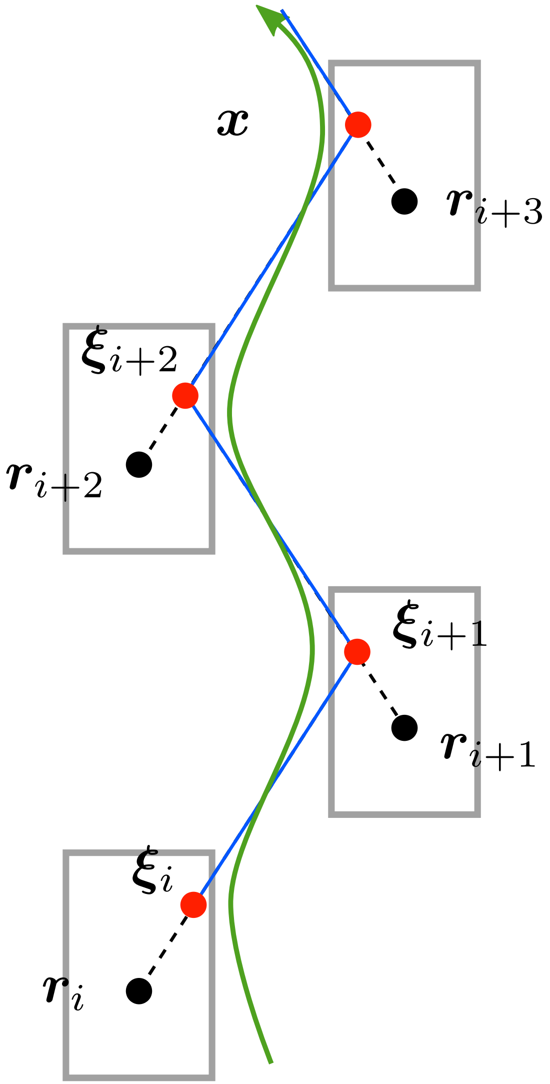
2) Define Virtual Repellent Points (VRP)

3) Define waypoints for the DCM

4) Interpolate CoM trajectory

[Englsberger et al., 2011]

DCM Based Walking Motions Generation



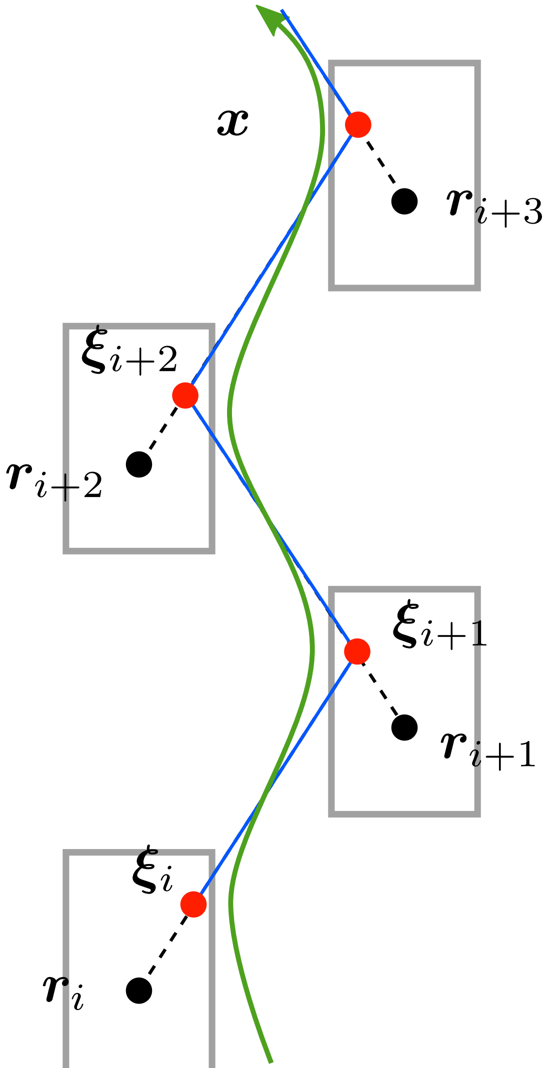
1) Define Footsteps

Recommended at least a horizon of 3 steps.

$$\boldsymbol{r}_1,\ \boldsymbol{r}_2,\ \cdots\ \boldsymbol{r}_N,$$

Including side and angular steps

DCM Based Walking Motions Generation



2) Define Virtual Repellent Points (VRP)

$$\boldsymbol{r}_i = {}^{\mathbf{w}}\,\mathbf{r}_i + \begin{bmatrix} 0 \\ 0 \\ z_x \end{bmatrix}$$

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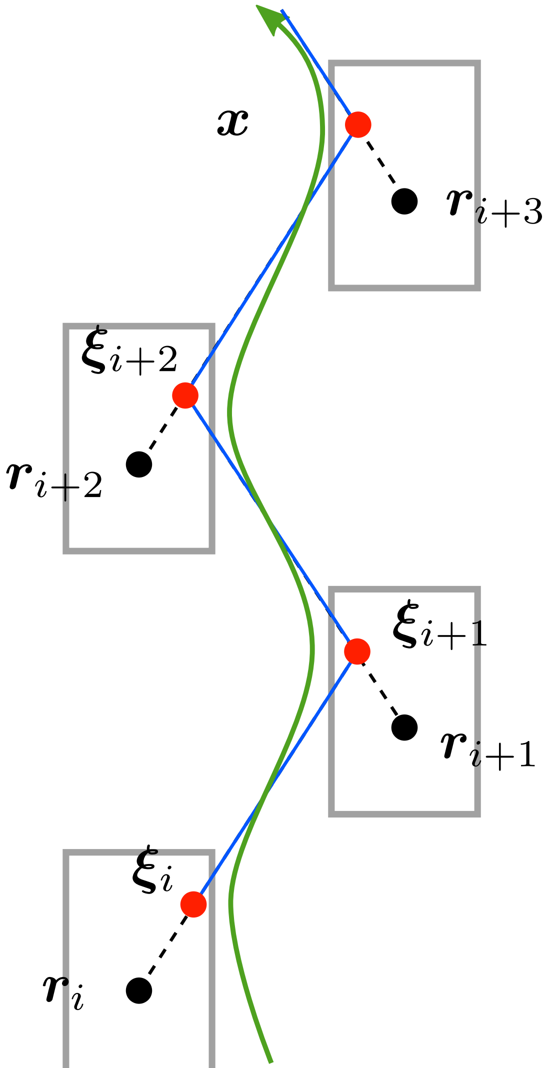
DCM Based Walking Motions Generation



3) Define way-points for the DCM

$$\boldsymbol{\xi}_i = \boldsymbol{r}_{i+1} + e^{\omega t_{step}} \left(\boldsymbol{\xi}_{i+1} + \boldsymbol{r}_{i+1} \right)$$

$$t_{step} = \frac{\omega}{\pi} \qquad \boldsymbol{\omega} = \sqrt{\frac{g}{z}}$$

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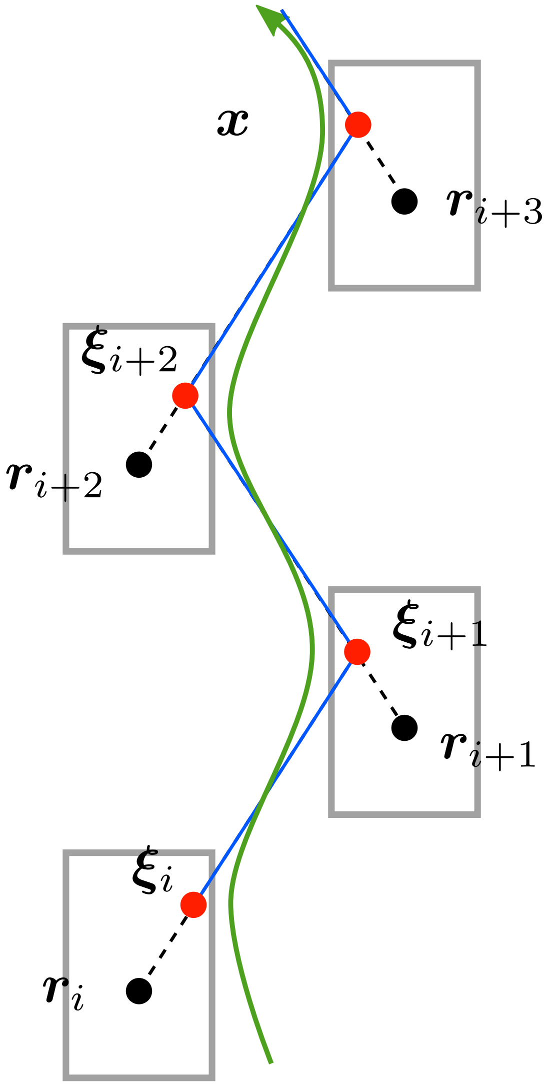
DCM Based Walking Motions Generation

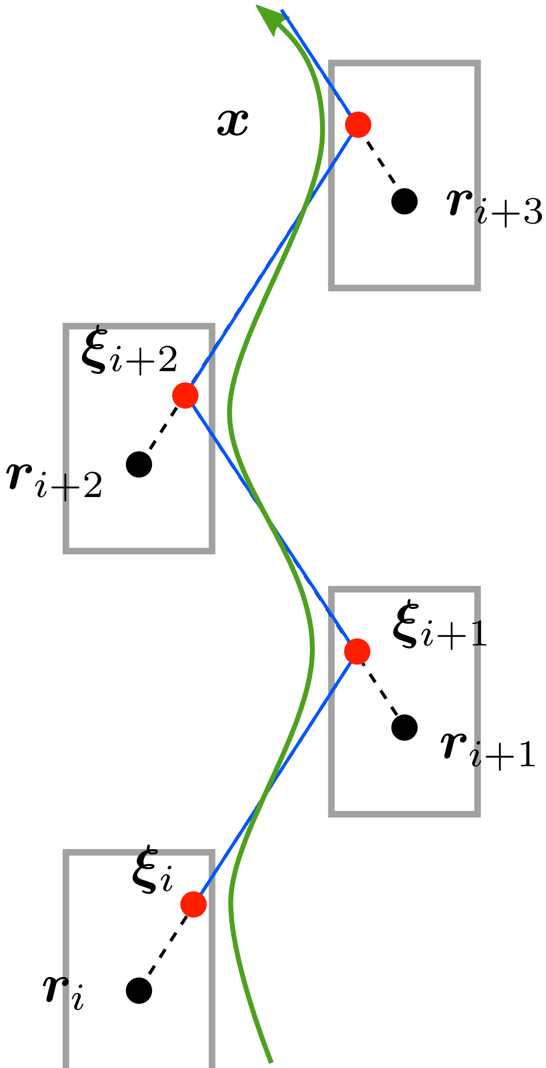


4) Generate CoM Trajectory

$$\boldsymbol{\xi}_r = \boldsymbol{r}_i + e^{\omega(t - t_{step})}\,(\boldsymbol{\xi}_i + \boldsymbol{r}_i)$$

$$\dot{\boldsymbol{x}} = -\omega(\boldsymbol{x} - \boldsymbol{\xi})$$

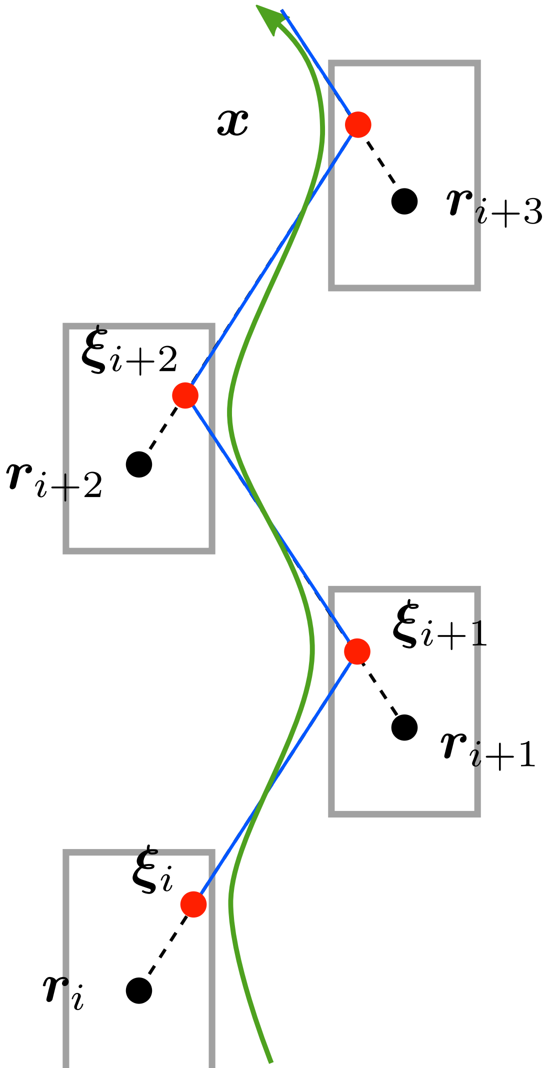
DCM Based Walking Motions Generation



DCM/ZMP balance control

$$\dot{\boldsymbol{\xi}} - \dot{\boldsymbol{\xi}}_r = -k_\xi \left(\boldsymbol{\xi} - \boldsymbol{\xi}_r \right)$$

DCM Based Walking Motions Generation

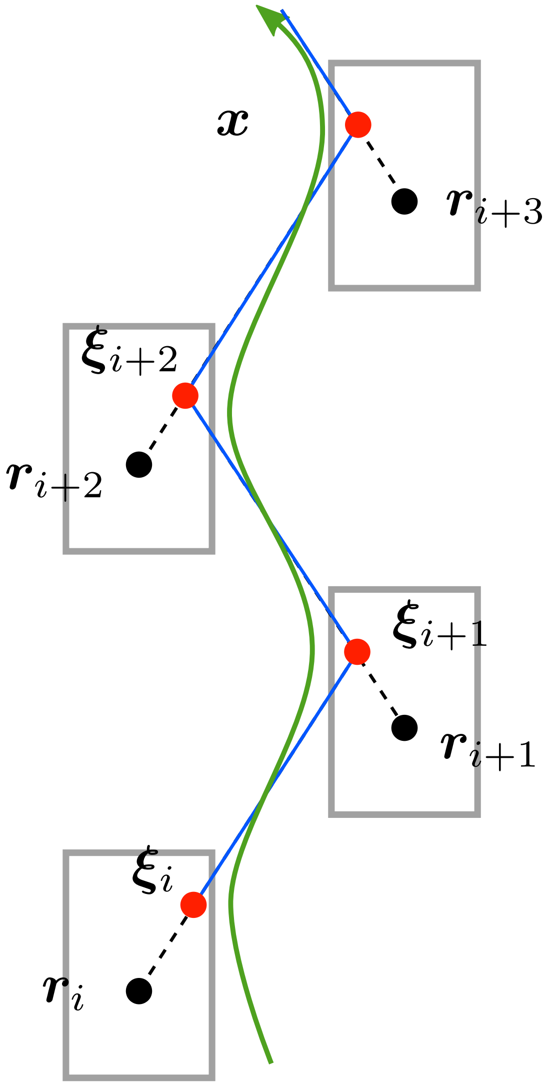


DCM/ZMP balance control

$$\dot{\boldsymbol{\xi}} - \dot{\boldsymbol{\xi}}_r = -k_\xi \left(\boldsymbol{\xi} - \boldsymbol{\xi}_r\right)$$

$$\mathbf{p}_d = \mathbf{p}_r - \left(1 + \frac{k_\xi}{\omega}\right)\left(\boldsymbol{\xi}_r - \boldsymbol{\xi}\right)$$

DCM Based Walking Motions Generation



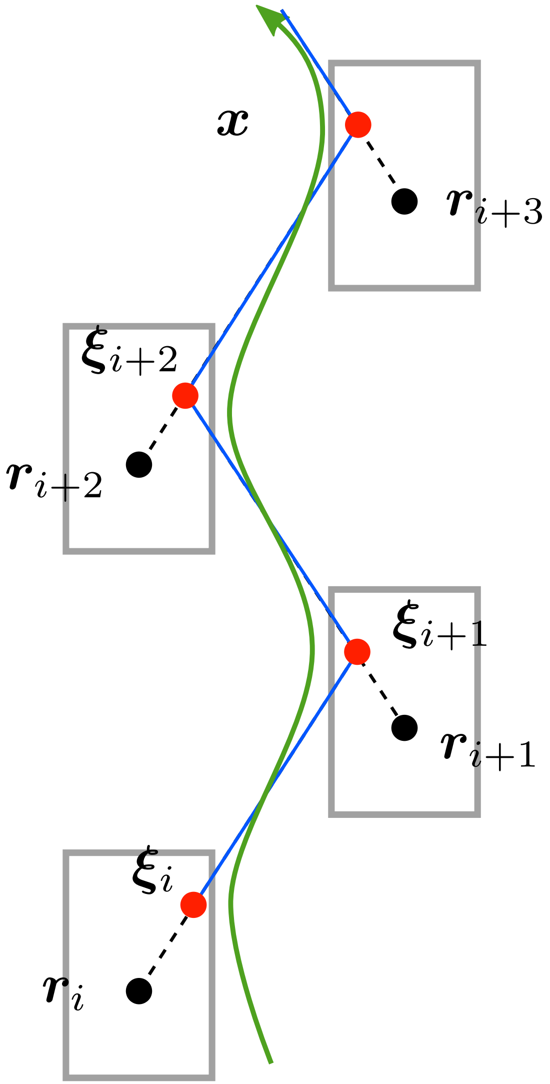
DCM/ZMP balance control

$$\dot{\boldsymbol{\xi}} - \dot{\boldsymbol{\xi}}_r = -k_\xi \left(\boldsymbol{\xi} - \boldsymbol{\xi}_r\right)$$

$$\mathbf{p}_d = \mathbf{p}_r - \left(1 + \frac{k_\xi}{\omega}\right)\left(\boldsymbol{\xi}_r - \boldsymbol{\xi}\right)$$

$$\dot{\mathbf{p}} - \dot{\mathbf{p}}_d = -k_p \left(\mathbf{p} - \mathbf{p}_d\right)$$

DCM Based Walking Motions Generation



DCM/ZMP balance control

$$\dot{\boldsymbol{\xi}} - \dot{\boldsymbol{\xi}}_r = -k_\xi \left(\boldsymbol{\xi} - \boldsymbol{\xi}_r\right)$$

$$\mathbf{p}_d = \mathbf{p}_r - \left(1 + \frac{k_\xi}{\omega}\right)\left(\boldsymbol{\xi}_r - \boldsymbol{\xi}\right)$$

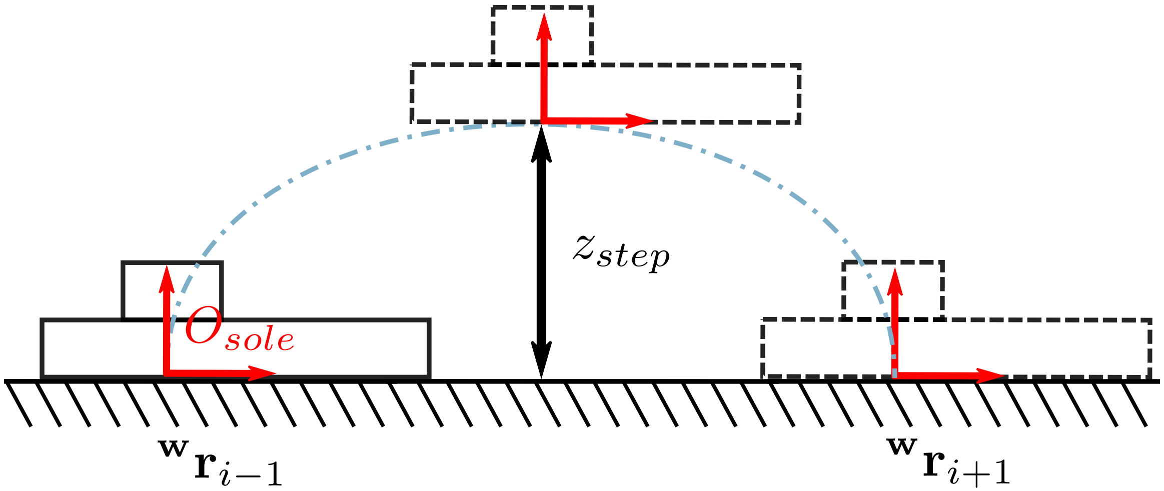
$$\dot{\mathbf{p}} - \dot{\mathbf{p}}_d = -k_p \left(\mathbf{p} - \mathbf{p}_d\right)$$

$$\dot{\boldsymbol{x}}_d = \dot{\mathbf{p}} + k_p(\mathbf{p} - \mathbf{p}_d)$$

Foot Trajectory Generation

$$t_{step} = t_{ss} + t_{ds}$$

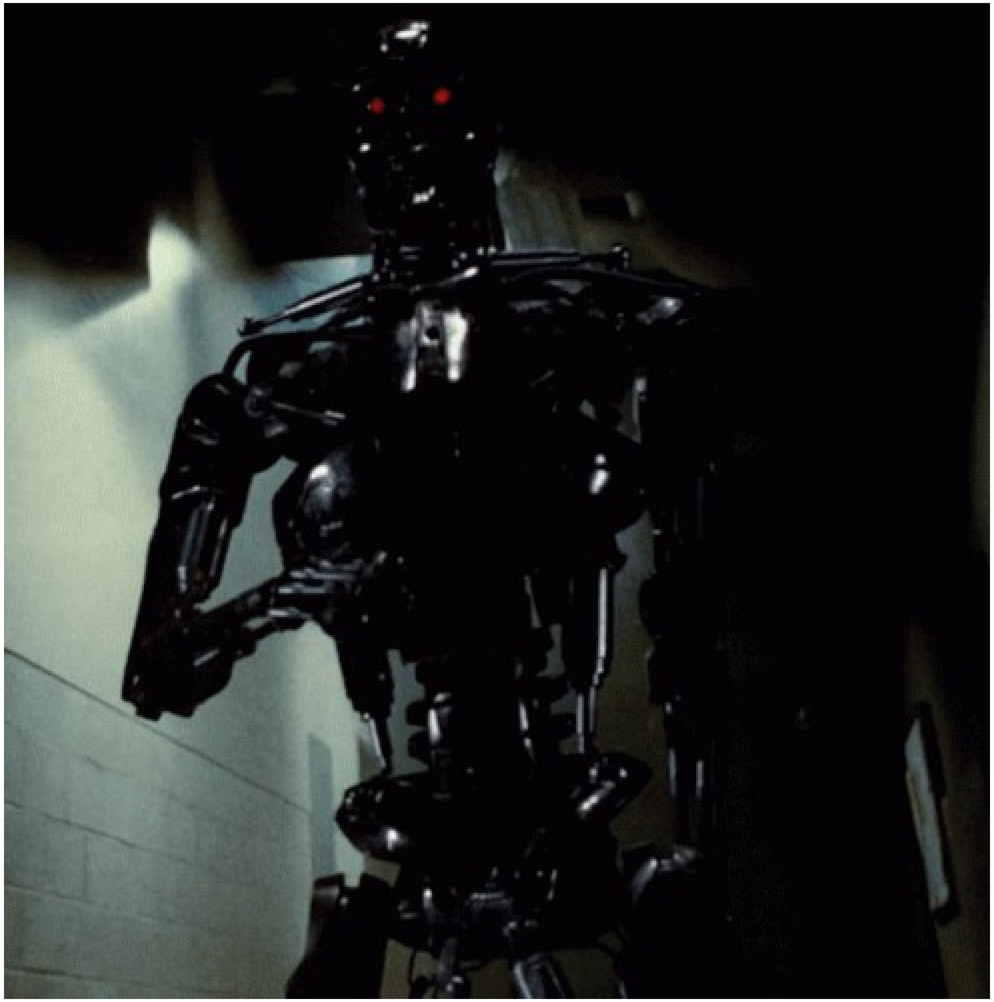
$$\boldsymbol{x}_{foot}(0) = \boldsymbol{r}_{i-1}$$
$$\dot{\boldsymbol{x}}_{foot}(0) = 0$$
$$\ddot{\boldsymbol{x}}_{foot}(0) = 0$$



$$\boldsymbol{x}_{foot}(t_{ss}) = \boldsymbol{r}_{i+1}$$
$$\dot{\boldsymbol{x}}_{foot}(t_{ss}) = 0$$
$$\ddot{\boldsymbol{x}}_{foot}(t_{ss}) = 0$$

$$\boldsymbol{x}\left(\frac{t_{ss}}{2}\right) = \boldsymbol{r_{i-1}} + \frac{1}{2}\left(\boldsymbol{r_{i-1}} + \boldsymbol{r_{i+1}}\right) + \begin{bmatrix} 0 \\ 0 \\ z_{step} \end{bmatrix}$$

Questions?



Next session:

Tutorial 7: Foot Step Planning