Beta function : $B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

Gamma function: $\Gamma(x)=(x-1)!$

Beta distribution $\Rightarrow \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot\theta^{\alpha-1}\cdot(1-\theta)^{\beta-1}=\frac{1}{B(\alpha+\beta)}\cdot\theta^{\alpha-1}\cdot(1-\theta)^{\beta-1}$

$=B(\theta\,|\,\alpha,\beta)$

Binomial distribution $\Rightarrow p(\theta|x)=\frac{N!}{(N-x)!\,x!}\cdot\theta^{x}\cdot(1-\theta)^{N-x}\longrightarrow$ likelihood

*如果 Prior distribution 和 likelihood function 可使 Posterior distribution 和 Prior distribution 有相同形式 那 Prior distribution 和 likelihood function conjugate

$P(x|\theta)\propto$ Prior * likelihood $=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot\theta^{\alpha-1}\cdot(1-\theta)^{\beta-1}\frac{N!}{(N-x)!\,x!}\cdot\theta^{x}\cdot(1-\theta)^{N-x}$

$P(x|\theta)$: Posterior; $\propto$: 正比; $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ → constant; $\frac{N!}{(N-x)!\,x!}$ → constant

$\propto\theta^{\alpha-1}\cdot(1-\theta)^{\beta-1}\cdot\theta^{x}\cdot(1-\theta)^{N-x}\cdot C$ → constant

$\propto\theta^{\alpha-1+x}\cdot(1-\theta)^{\beta-1+N-x}\cdot\frac{\Gamma(\alpha+\beta+N)}{\Gamma(\alpha+x)\Gamma(b+N-x)}$

$=B(\theta\,|\,\alpha+x,\beta-x+N)$