Tutorial 6: Uniform Open Channel Flow

Question 1

For the trapezoidal channel shown in Figure 1, calculate

(i) The uniform flow velocity and discharge if $h = 2$m, $S_0 = 1{:}2000$ and $k_s = 0.49$mm.

(ii) The uniform flow depth, h, if $Q = 20\ \mathrm{m}^3/s$, $S_0 = 1{:}2000$ and $k_s = 0.49$mm.

Assume water at 10°C.

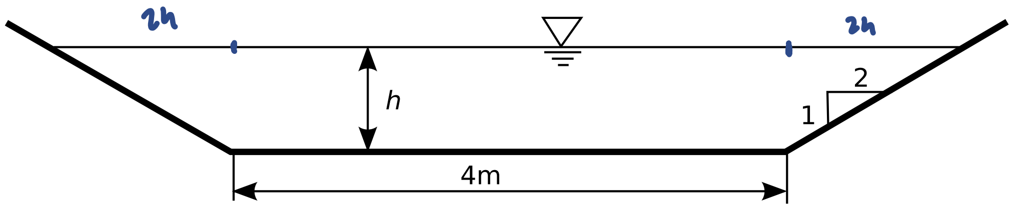

Figure 1: Open channel cross section

Numerical results: (i) $U = 2.02$m/s and $Q = 32.32\mathrm{m}^3$/s. (ii) Find h through iteration (not helpful to state result here).

(i) Using Darcy Weisbach Model:

$$U = \left(\frac{8g}{f}\right)^{1/2} R^{1/2} S^{1/2}, \text{ where } \frac{1}{f^{1/2}} \approx -2\log_{10}\left(\frac{ks/R}{14.84}\right)$$

$$\approx -2\log_{10}\left(\frac{0.49\times10^{-3}/1.2361}{14.84}\right)$$

$$R = \frac{A}{P} = \frac{\frac{1}{2}(4+4+4h)(h)}{4+2\sqrt{4h^2+h^2}} = 1.2361\ \mathrm{m}$$

$$f \approx 0.012$$

$$\therefore U = \left(\frac{8\times9.81}{0.012}\right)^{1/2} \times 1.2361^{1/2} \times \frac{1}{2000}^{1/2} = 2.02\ \mathrm{m/s}$$

doing till this step is enough, but remember that this assumption is based on Re is very big, so we shall check whether it is true or not

$ks/4R = 1\times10^{-4}$

$Re = \frac{UD}{\nu} = 2.65\times10^{6}$, reading from moody diagram ~ slightly increased but is ok.

$$Q = UA = 2.02 \times \frac{1}{2}(4+4+4h)(h) = 32.32\ \mathrm{m}^3/s$$

(ii) remember to find h_n

if rectangular + wide

$$h_n = \left(\frac{fq^2}{8gS_0}\right)^{1/3}$$

(no iteration needed)

if not rectangular & wide.

$$F(h_n) = \frac{Q}{A} - \left(\frac{8g}{f}\right)^{1/2}\left(\frac{A}{P}\right)^{1/2} S^{1/2} = 0$$

(iteration needed)

let $h_{n,0} = 1m$,

$A = \frac{1}{2}(4+4+4(1))(1) = 6$

$P = 4 + 2\sqrt{4(1)^2+1^2} = 8.47$

$R = A/P = 0.708$

$f \approx 0.014$

$V = 1.41$

$F(h_n) = 1.9243$

let $h_{n,1} = 1.5m$,

$A = 10.5 m^2$

$P = 10.7$

$R = A/P = 0.981$

$f \approx 0.013$

$V = 1.72$

$F(h_n) = 0.0184$

let $h_{n,2} = 1.6m$

$A = 11.52$

$P = 11.2$

$R = 1.03$

$f \approx 0.013$

$V = 1.76$

$F(h_n) = -0.027$

$\therefore h_n \approx 1.55 m$

again, check Re whether it is big.

$Re = \frac{VD}{\nu} = 5.5 \times 10^6$ (ok)

Question 2

The cross section of a combined sewer / storm water channel leading to a treatment plant is shown in Figure 2. To avoid overloading the plant during a storm, some of the flow is to spill into an interceptor when the depth y reaches 0.91m. The Manning coefficient is $n = 0.015$ s/m$^{1/3}$ and the bed slope is $S_0 = 0.005$.

(i) Find the discharge Q in the sewer when spilling begins.

(ii) What is the equivalent Darcy-Weisbach friction factor f for this channel?

(iii) What is the equivalent Chezy coefficient C [m$^{1/2}$/s] for this channel?

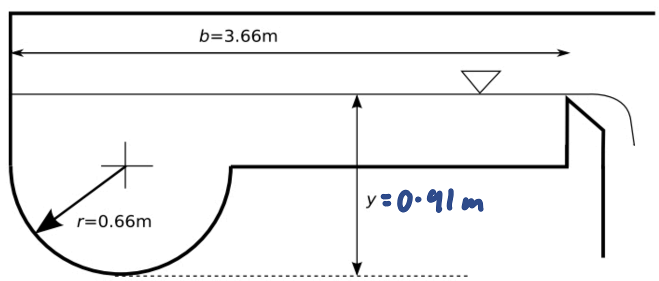

Figure 2: Sewer / storm water system at the point of spilling

Numerical results: (i) $Q = 3.57\text{m}^3/\text{s}$. (ii) $f = 0.026$. (iii) $C = 55.3\text{m}^{1/2}/\text{s}$.

(i) find Q when $h_n = y = 0.91\,m$,

$$U = \frac{1}{n} R^{2/3} S^{1/2} \qquad R = \frac{A}{P} = \frac{0.5\pi r^2 + (y-r)\times b}{\pi r + 2(y-r) + (b-2r)} = \frac{1.5992}{4.9135} = 0.3255$$

$$= \frac{1}{0.015}(0.3255)^{2/3}(0.005)^{1/2}$$

$$= 2.2306\,m/s$$

$$Q = UA = 2.2306 \times 1.5992 = 3.567\,m^3/s$$

(ii) $$\frac{1}{f^{1/2}} \approx -2\log\left(\frac{k_s/R}{14.84}\right)$$

the issue with this is we don't know what k_s is.

but from data sheet we can obtain the following formula:

$$f = \frac{8gn^2}{R^{1/3}} = 0.2567$$

(iii) from data sheets,

$$f = \frac{8g}{C^2} \qquad C = \sqrt{\frac{8g}{f}} = 55.293$$

Question 3

The critical depth for the circular sewer shown in Figure 3 is $h_c = 0.69$m. The sewer diameter is $2r = 1.52$m. Find the discharge for $h = h_c$.

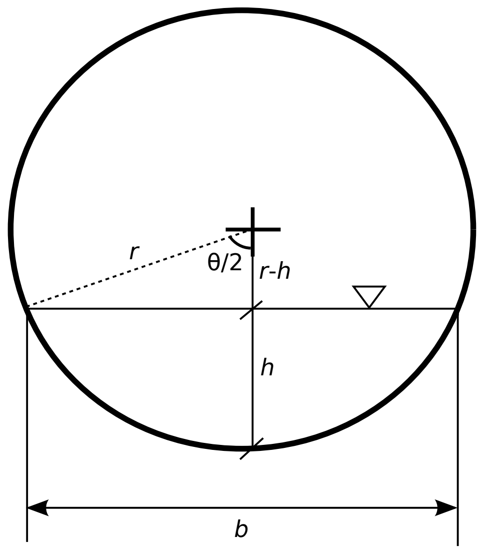

Figure 3: Circular sewer

Numerical results: $Q = 1.82\text{m}^3/\text{s}$.

aim: find Q when h=hc

at first I think I should use:

$U=\left(\frac{8g}{f}\right)^{1/2}R^{1/2}S^{1/2}$ but I realised we are missing too much inputs: f and S

(f requires ks which is not obtainable)

h=hc means critical

at critical depth: $\frac{dE(h)}{dh}=0$ and $Fr=1$

↳ $E(h)=h+\frac{q^2}{2gh^2}$ ↳ $Fr=\frac{U}{\sqrt{gh_m}}$, $Q=UA$

↳ we can definitely get q with this ↳ we can definitely get q with this too.

↳ This one seems easier.

$1.0=\frac{Q/A}{\sqrt{gh}}$ ← only missing A!

this h is hm!

$h_m = A/w$
$= A/b$

$b=\sqrt{r^2-(r-h)^2}\times 2$

$Q=A\sqrt{gh}=1.825$

$A=\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta$
$=0.801$

$\cos\frac{\theta}{2}=\frac{r-h}{r}$
$=\frac{\frac{1.52}{2}-0.69}{\frac{1.52}{2}}$

$\theta=169.43^\circ$