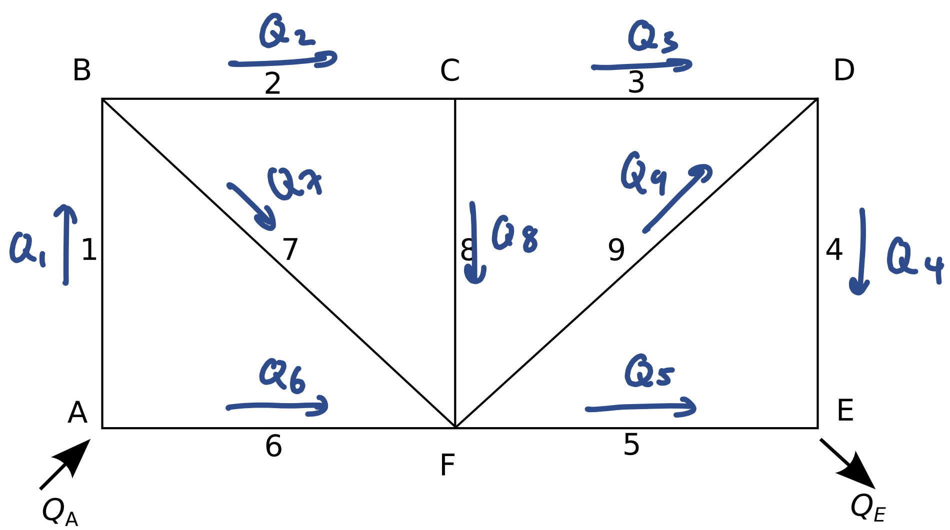
Tutorial 5: Pipe Systems

Question 1

Steady flow of water is established in the pipe network illustrated in Figure 1. The friction resistance coefficients, K_1 through K_9 [s^2/m^5] for pipe segments marked 1 through 9 respectively, are known. Junction inflow Q_A [m^3/s] and junction outflow, Q_E [m^3/s], are also known. You are to formulate (but not solve) the problem to estimate the flow in all pipe segments of the network.

(i) List and clearly identify the unknowns.

(ii) Establish sufficient independent equations to solve for these unknowns.



↻ as positive for each loop

Figure 1: Pipe network with pipes 1-9

mass conservation at each junction:

$Q_A = Q_1 + Q_6 \quad Q_1 = Q_2 + Q_7 \quad Q_6 + Q_7 + Q_8 = Q_9 + Q_5 \quad Q_2 = Q_3 + Q_8$

$Q_5 + Q_4 = Q_E \quad Q_3 + Q_9 = Q_4$

6 equations

overall conservation ($Q_{in} = Q_{out}$)

but -1 (5) independent

$Q_A = Q_E$

Bernoulli around closed loop:

$K_1|Q_1|Q_1 + K_7|Q_7|Q_7 - K_6|Q_6|Q_6 = 0$
$K_2|Q_2|Q_2 + K_8|Q_8|Q_8 - K_7|Q_7|Q_7 = 0$
$K_3|Q_3|Q_3 - K_9|Q_9|Q_9 - K_8|Q_8|Q_8 = 0$
$K_9|Q_9|Q_9 + K_4|Q_4|Q_4 - K_5|Q_5|Q_5 = 0$

4 more independent eqn.

total 9 independent eqn.
($Q_1 - Q_9 \leftarrow$ 9 unknowns)
mathematically closed

Question 2

Water at 10°C flows from the large reservoir A to the large reservoir B via the pipe system shown in Figure 2. What must be the water surface elevation in reservoir A to drive a steady flow of $Q = 0.03\text{m}^3/\text{s}$ in the pipe? Carefully sketch the Hydraulic Grade Line and the Energy Line, marking elevations and changes of grade.

[Ans: i) 12.4 m]

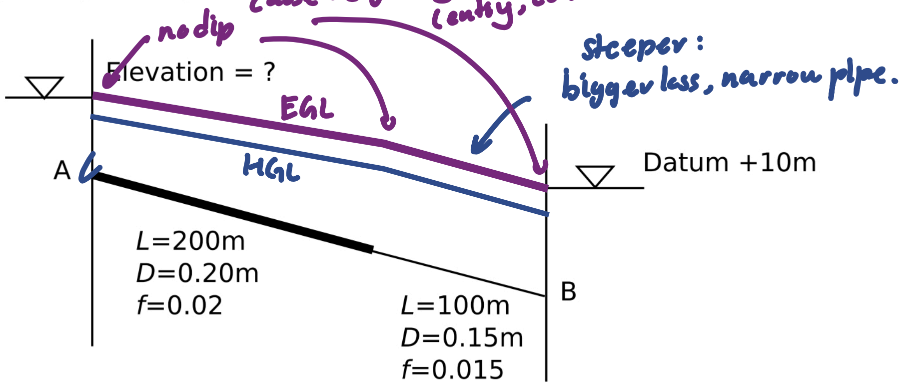


Figure 2: Reservoirs A and B connected by two pipes in series

I would usually do bernoulli, in term of U:

$E_A = E_B + E_{lost}$ where $K_A = \frac{f_A L_A}{D_A}$

$z_A + \frac{p_A}{\rho g} + \frac{U_A^2}{2g} = z_B + \frac{p_B}{\rho g} + \frac{U_B^2}{2g} + K_A U_A^2 + K_B U_B^2$ $K_B = \frac{f_B L_B}{D_B}$

(ignoring minor loss)

but in this subtopic, let us try in term of Q.

$z_A = z_B + K_A Q_A^2 + K_B Q_B^2$, since $Q_A = Q_B$ (series)

$z_A = 10 + (K_A + K_B) Q^2$

to find K_A and K_B:

$K = \frac{8fL}{g\pi^2 D^5}$ (from formula sheets)

$\therefore K_A = 1032.8$, $K_B = 1632.1$

$z_A = 10 + (1032.8 + 1632.1) \times 0.03^2 = 12.40$ m

Question 3

A steady water flow is established between the large reservoirs 1, 2, and 3 as shown in Figure 3. The properties of the reservoir/pipe system are given in Table 1

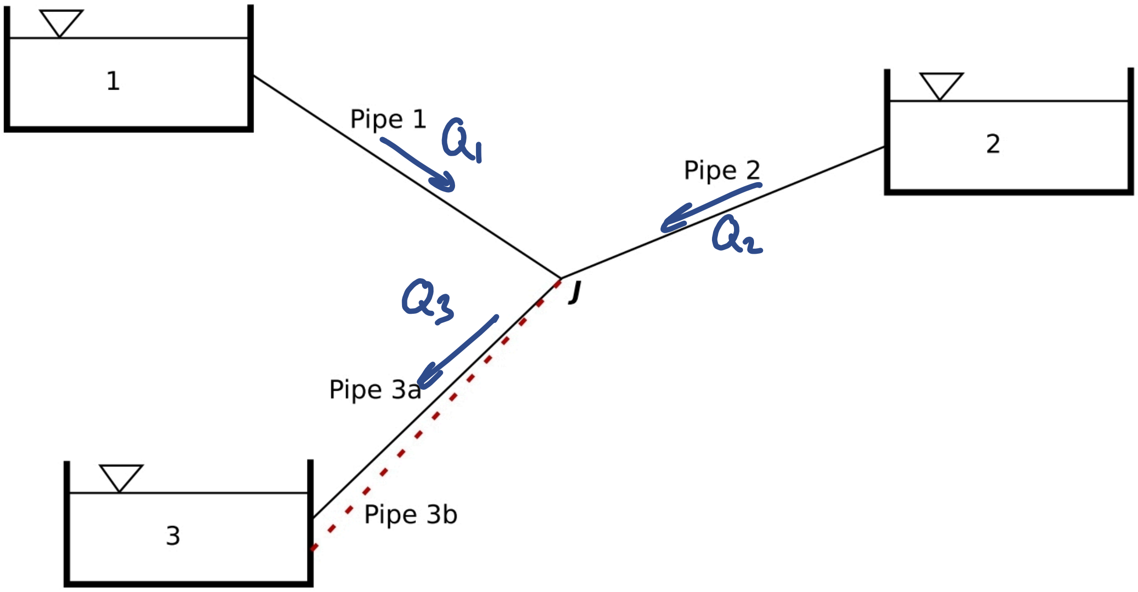


Figure 3: Three reservoirs connected by three/four pipes

i. The water flows through pipes 1, 2 and 3a which have a common junction at J. Determine the steady flow Q from the Junction to reservoir 3. You may neglect minor losses.

ii. In an effort to increase the flow to reservoir 3, a second pipe, 3b, is installed from the junction J to reservoir 3. The pipe diameter and the Darcy-Weisbach friction factor for pipe 3b are $D = 500$mm and $f = 0.018$ respectively. The lengths of pipes 3a and 3b are identical. Determine the revised steady flow Q from the junction J to reservoir 3. You may neglect minor losses.

[Ans: i) 0.0873 m^3/s, ii) 0.261 m^3/s]

Reservoir/Pipe	1	2	3a
Reservoir water surface elevation [m]	+25	+15	-20
Pipe length [m]	5000	7000	6000
Pipe diameter [mm]	300	400	300
Pipe Darcy-Weisbach friction factor [-]	0.020	0.015	0.022

Table 1: Pipe system properties

(i) only z ; $z, \frac{p}{\rho g}, \frac{v^2}{2g}$.

$E_1 = E_J + E_{lost,1}$ $\quad z_1 = H_J + K_1|Q_1|Q_1$ $\quad H_J = z_1 - K_1|Q_1|Q_1$

$E_2 = E_J + E_{lost,2}$ $\rightarrow$ $z_2 = H_J + k_2|Q_2|Q_2$ $\quad H_J = z_2 - k_2|Q_2|Q_2$

$E_J = E_3 + E_{lost,3}$ $\quad H_J = z_3 + k_3|Q_3|Q_3$ $\quad H_J = z_3 + k_3|Q_3|Q_3$

3 eqn. but only two independent as: $Q_1 + Q_2 = Q_3$

energy before = energy after + energy lost.

the only common variable between the 3 eqn. hence iterate by guessing H.

$K = \frac{8fL}{g\pi^2 D^5}$, $\therefore K_1 = 3400.3$, $K_2 = 847.2$, $K_3 = 4488.4$

- Iterative algorithm
  i. Guess H_J
  ii. Calculate Q_1, Q_2 and Q_3
  iii. Check the mass balance
  iv. Adjust H_J

H_J[m]	Q_1 [m^3/s]	Q_2 [m^3/s]	Q_{3a} [m^3/s]	$Q_1+Q_2-Q_{3a}$ [m^3/s]
10	0.0664	0.0768	0.0818	0.0615
15	0.0542	0	0.0883	-0.0341
14	0.0569	0.0344	0.0870	0.0042
14.2	0.0564	0.0307	0.0873	-2.0$\cdot 10^{-4}$
14.19	0.0564	0.0309	0.0873	**2.6$\cdot$10^{-5}**

answer

(ii)

$$E_1 = E_J + E_{lost,1} \qquad z_1 = H_J + K_1|Q_1|Q_1 \qquad H_J = z_1 - K_1|Q_1|Q_1$$
$$E_2 = E_J + E_{lost,2} \rightarrow z_2 = H_J + k_2|Q_2|Q_2 \rightarrow H_J = z_2 - k_2|Q_2|Q_2$$
$$E_J = E_{3a} + E_{lost,3a} \qquad H_J = z_{3a} + k_{3a}|Q_{3a}|Q_{3a} \qquad H_J = z_{3a} + k_{3a}|Q_{3a}|Q_{3a}$$
$$E_J = E_{3b} + E_{lost,3b} \qquad H_J = z_{3b} + k_{3b}|Q_{3b}|Q_{3b} \qquad H_J = z_{3b} + K_{3b}|Q_{3b}|Q_{3b}$$

h_J[m]	Q_1 [m^3/s]	Q_2 [m^3/s]	Q_{3a} [m^3/s]	Q_{3b} [m^3/s]	$Q_1+Q_2-Q_{3a}-Q_{3b}$ [m^3/s]
10	0.0664	0.0768	0.0818	0.3241	-0.2626
-10	0.1015	0.1718	0.0472	0.1871	0.0389
-7.5	0.0978	0.1630	0.0528	0.2092	-0.0013
-7.55	0.0978	0.1631	0.0527	0.2088	-4.87$\cdot$10^{-4}
-7.58	0.0979	0.1633	0.0526	0.2086	**-1.8$\cdot$10^{-5}**

parallel series

$$Q = Q_{3a} + Q_{3b} = 0.261.$$

Question 4

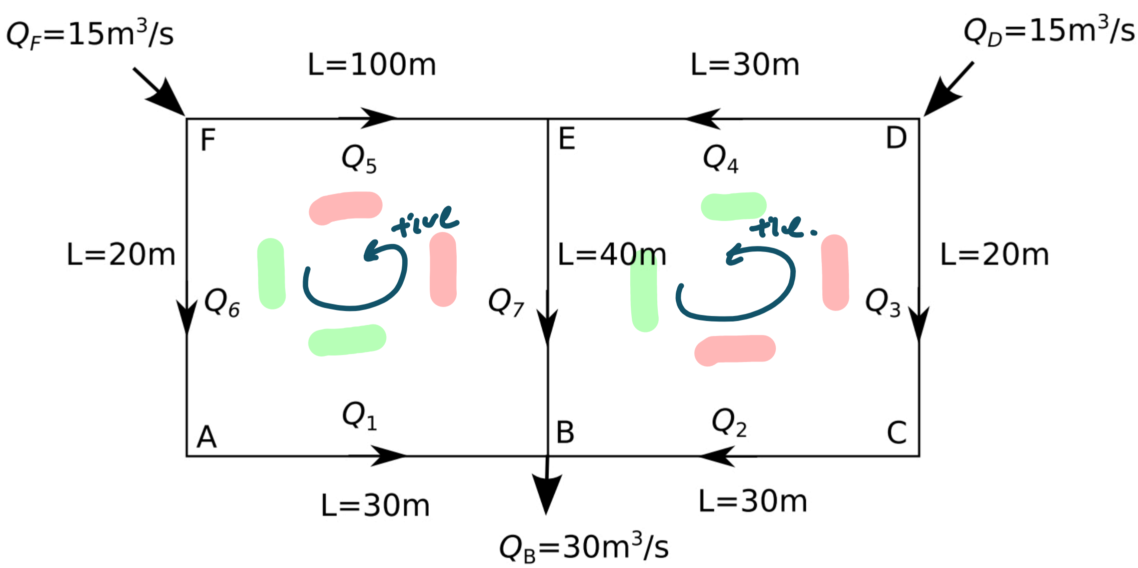
Carefully consider the pipe system in Figure 4.

1. Write down a system of simultaneous equations based on mass conservation, and momentum conservation around each loop. Show that these can be reduced to two simultaneous quadratic equations in two unknowns. Have a think about why you might want to go about tackling the problem by a different route (see the remaining parts of the question for a sensible route to solution).

   The friction factor and diameter for each of the pipes are identical. You may assume $f = 0.01$ and $D = 1$m.

2. Establish the relation between the flows in each of the pipes. In so doing, clearly indicate which flows are equal and which flows are larger than others. (e.g. $Q_x = Q_y$, $Q_x > Q_z$).

3. **Estimate** the flow in all pipes and discuss your estimate. Observe mass continuity!



(i) Choose two variables to express all others in terms of, I chose Q_1 and Q_2. By inspecting the figure:

$Q_1 = Q_6$ $\quad$ $Q_3 = Q_2$ $\quad$ $Q_5 = Q_F - Q_1$ $\quad$ $Q_4 = Q_D - Q_2$

$Q_7 = Q_4 + Q_5 = Q_D - Q_2 + Q_F - Q_1$

$Q_7 = Q_B - Q_1 - Q_2$ but $Q_B = Q_D + Q_F$ so this equation is not independent

Conserving the momentum around the left-hand loop:

$$K_6Q_6^2 + K_1Q_1^2 - K_7Q_7^2 - K_5Q_5^2 = 0$$

Using our results from the conservation of mass to write this in terms of Q_1 and Q_2, gives:

$(K_6 + K_1)Q_1^2 - K_7(Q_D - Q_2 + Q_F - Q_1)^2 - K_5(Q_F - Q_1)^2 = 0$, choosing Q_1 as our variable, gives:

$$(K_6 + K_1 - K_7 - K_5)Q_1^2 + [2K_7(Q_D - Q_2 + Q_F) + 2K_5Q_F]Q_1 + [-K_7(Q_D - Q_2 + Q_F)^2 - K_5Q_F^2]$$

Conserving the momentum around the right-hand loop:

$$K_7Q_7^2 - K_2Q_2^2 - K_3Q_3^2 + K_4Q_4^2 = 0$$

Using our results from the conservation of mass to write this in terms of Q_1 and Q_2; and choosing Q_2 as our variable, gives:

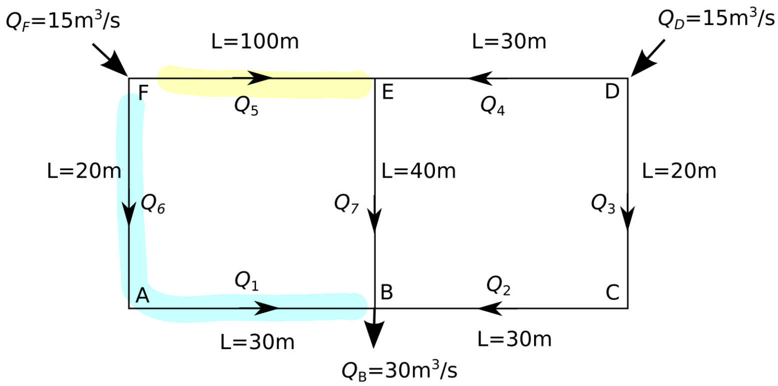
$$(K_7 - K_2 - K_3 + K_4)Q_2^2 + [2K_7(Q_D + Q_F - Q_1) - 2K_4Q_D]Q_2 + [K_7(Q_D + Q_F - Q_1)^2 + K_4Q_D^2]$$

We could then use the quadratic formula to write Q_1 in terms of Q_2 (or vice versa) and then look to sub this expression into our remaining quadratic equation. However, in either case the solution to the quadratic formula contains a term $\sqrt{b^2 - 4ac}$, which can be written (for example choosing to solve our quadratic for Q_2) in the form:

$$Q_2 = (C_1 + C_2 Q_1) \pm [(C_3 + C_4 Q_1)^2 - C_5]^{1/2}$$

where all C_i's are known constants. This can then be substituted back into the quadratic equation for Q_1. Hence we a have a single equation for Q_1 in terms of known constants but solving this requires an iterative scheme and thus it is not energy well spent. Much better to set up and iterative scheme in the first place and allow a computer to do the hard work!

(ii)



$f = 0.01$, $D = 1$
(constant for all pipes)

$Q_1 = Q_6$; $Q_2 = Q_3$; $Q_4 > Q_5$ (cause $L_4 < L_5$); Q_7

\> because $(30+40) > (30+20)$

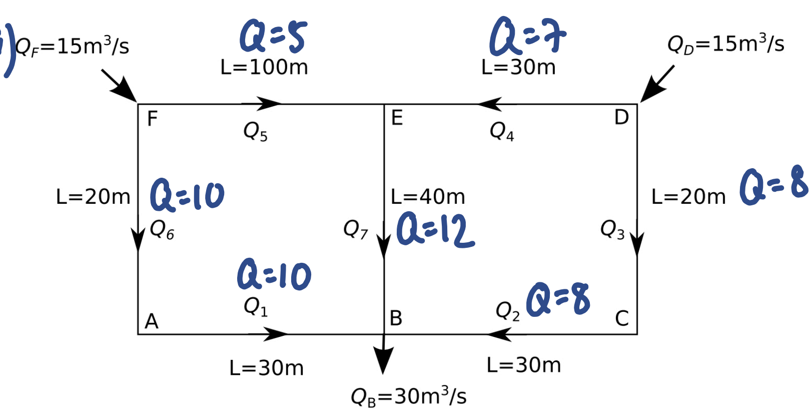
\>

because the other path $Q_5 < Q_4$

although look like an unneccesary step, but it is crucial to get next step easier to do!

do it even the question doesn't tell you to!

(iii)



$$K = \frac{8fL}{g\pi^2 D^5} = \frac{8 \times 0.01}{9.81\pi^2 1^5} L = 8.26 \times 10^{-4} L$$

let $\lambda = 8.26 \times 10^{-4} \rightarrow K = \lambda L$

$$K_6 Q_6^2 + K_1 Q_1^2 - K_7 Q_7^2 - K_5 Q_5^2 = 0$$

$\lambda(20)(10)^2 + \lambda(30)(10)^2 - \lambda(40)(12)^2 - \lambda(100)(5)^2$

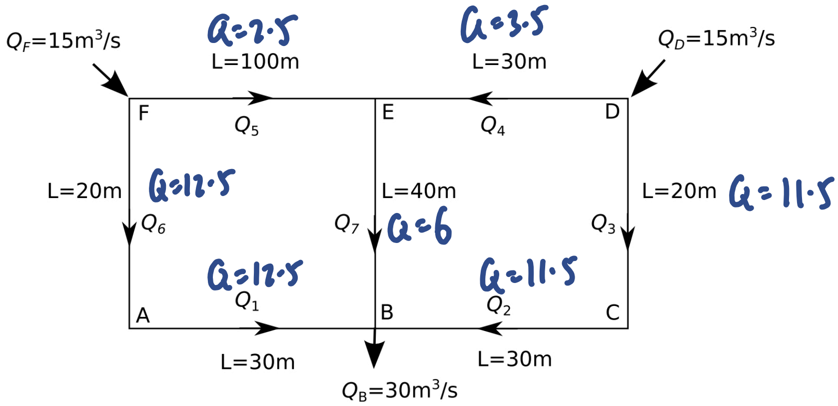
$= -2.69$ (Q_7, Q_5 too big)

$$K_7 Q_7^2 - K_2 Q_2^2 - K_3 Q_3^2 + K_4 Q_4^2 = 0$$

$\lambda[40 \times 12^2 - 30 \times 8^2 - 20 \times 8^2 + 30 \times 7^2]$

$= 3.32878$ (Q_4, Q_7 too big)

looking at both results:
we need to reduce the top part's Q.



$K_6Q_6^2 + K_1Q_1^2 - K_7Q_7^2 - K_5Q_5^2 = 0$

$8.26\times10^{-4}[20\times12.5^2 + 30\times12.5^2 - 40\times6^2 - 100\times2.5^2]$

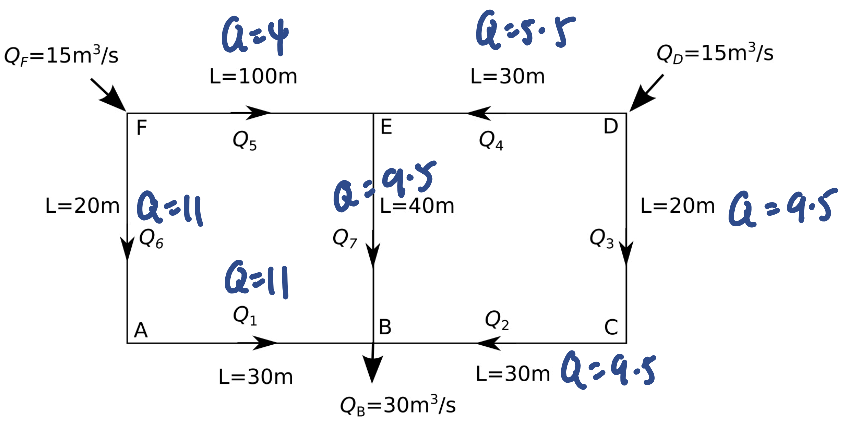
$= 4.7474$ (new Q_5, Q_7 is too small)

$K_7Q_7^2 - K_2Q_2^2 - K_3Q_3^2 + K_4Q_4^2 = 0$

$= -3.97$

(new Q_5, Q_7 is too small)

last time was closer



final guess