Tutorial 2: Real Fluids

Question 1

The dynamic viscosity μ of a fluid may be determined by measuring the constant velocity of a sphere sinking within the fluid. This experiment is based on the assumption that the flow around the sphere is laminar (use $Re_{max} = 0.5$). We would like to prove this using oil ($\mu = 0.8kg/ms$, $\rho = 900kg/m^3$) and a steel sphere ($\rho = 7700kg/m^3$).

i. Summarise the forces acting on the sphere.

force balance + Reynold's number.

ii. Determine the maximum sphere diameter so that the flow remains laminar.

iii. Determine the maximum sphere diameter if the medium under consideration is water at 20°C. Is this a practical result? same as (ii), replace μ with 1.005×10^{-3} kg/ms

iv. Calculate the constant sphere velocity for both cases.

just take the founded "D", sub-back in either of the eqn (force balance / Re)

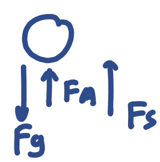
[Ans: ii) 4.6 mm, iii) 0.0515 mm, iv) for oil 0.096 m/s, for water 0.00977 m/s]
Note: The volume of a sphere is given by $V = (\pi D^3)/6$.

i.

(drag force) Stokes law : $F_S = 3\pi\mu u D$

(buoyant force) Archimedes Force : $F_A = V\rho_{fluid} g$

(weight) Gravity Force : $F_g = V\rho_{sphere} g$



ii. $Re_{max} = 0.5$

1. find Re=0.5 → requires U

$\frac{\rho U L}{\mu} = 0.5$

(L, length scale in this case use D, diameter)

$D = \frac{0.5 \times 0.8}{900 \times U}$

need to find U first: we can get it from force balance!

2. find U from force balance.

$D = \frac{0.5 \times 0.8}{\frac{900}{18\mu} D^2 g(\rho_s - \rho_f)}$

$F_S + F_A = F_g$

$3\pi\mu u D + V\rho_f g = V\rho_s g$

$u = \frac{Vg(\rho_s - \rho_f)}{3\pi\mu D}$

$= \frac{\frac{1}{6}\pi D^{\cancel{3}\,2} g(\rho_s - \rho_f)}{3\pi\mu \cancel{D}}$

$= D^2 g(\rho_s - \rho_f)/18\mu$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$

$= \frac{1}{6}\pi D^3$

$D^3 = \frac{0.5 \times 0.8}{\frac{900}{18 \times 0.8} \times 9.81 \times (7700 - 900)}$

$D = 4.6mm.$

Question 2

non-slip condition

$\Sigma F = 0,\ a = 0$!

A cubic block of mass $m = 5kg$ and dimensions $120mm \times 120mm \times 120mm$ slides steadily down an oil coated incline, Figure 1. The incline is at $10°$, the oil layer is $d = 0.2mm$ thick and the oil viscosity is $\mu = 0.1kg/ms$. Ignore end and edge effects and assume a **linear velocity profile** in the thin oil layer.

i. Estimate the speed at which the block sides down the incline.

ii. How does the slope affect the velocity? increase

iii. How does the viscosity affect the velocity? decrease.

[Ans: i) 1.18 m/s]

$R = 5 \times 9.81 \cos 10°$

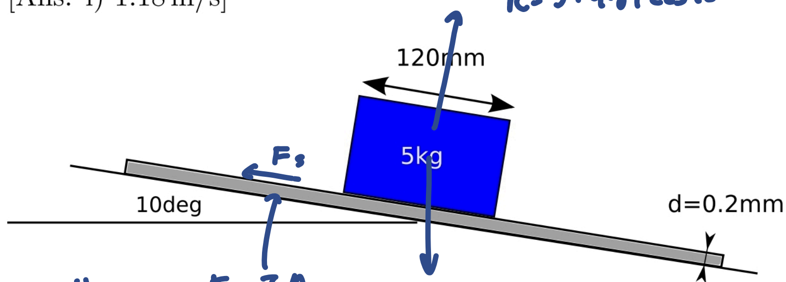
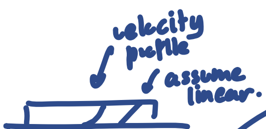
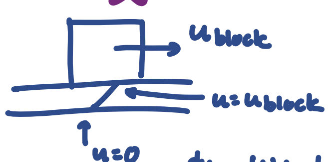


Figure 1: Sliding block on a thin oil layer.



need τ !



i. $\tau(y) = \mu \dfrac{du}{dy}$

$\tau(y = 0.2mm) = \mu\left(\dfrac{u_b}{d}\right)$

$\dfrac{du}{dy} = \dfrac{u_{block}}{d}$ ← and since it is linear

notice that the u of oil at surface, is equal to the u of block sliding (due to no-slip)

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{u_{block}}{d}$

$\Sigma F = ma$

$$5 \times 9.81 \sin 10° - 0.1\left(\frac{u_b}{0.2 \times 10^{-3}}\right) \times \underbrace{120 \times 10^{-3} \times 120 \times 10^{-3}} = 0$$

Area is the area where shear stress acted, i.e. base of the block.

$u_b = 1.183\ m/s$

Question 3

There are many fluids that exhibit non-Newtonian behaviour (i.e. shear stress is not linearly proportional to strain, as expected by Newton's Law of viscosity). For a given fluid, the distinction between Newtonian and non-Newtonian behaviour is usually based on measurements of shear stress τ and rate of shearing strain $\frac{du}{dy}$. This relationship is tested in a suitable viscometer for a blood sample. The tabulated results are

$\tau[N/m^2]$	0.04	0.06	0.12	0.18	0.30	0.52	1.12	2.10
$\frac{du}{dy}[s^{-1}]$	2.25	4.50	11.25	22.5	45.0	90.0	225	450
$\mu = \tau / \frac{du}{dy} =$	0.0178	0.0133	0.0107	0.008	0.0067	0.0058	0.0050	0.0047

Determine if the blood sample is a Newtonian or non-Newtonian fluid.

not constant → non-newtonian.

if newtonian: $\tau = \mu \frac{du}{dy}$, μ = constant.

Question 4

The velocity profile for laminar steady flow of oil ($\rho = 910kg/m^3$, $v = 4.10^{-4}m^2/s$) between two plates is given by

$$u(y) = U_{max}\left(1 - \frac{y^2}{W^2}\right)$$

$\frac{du}{dy} = U_{max}\left(-\frac{2y}{W^2}\right)$

$W = 25mm$

where $U_{max} = 0.20m/s$ and the distance between the two plates is $50mm$. Figure 2 illustrates this profile.

i. What is the wall shear stress [Pa] at the bottom plate? $y = -W$

ii. What is the wall shear stress [Pa] at the top plate? negative of (i)

iii. What is the shear stress [Pa] at the centreline? we know it's zero

$\tau = \mu \frac{du}{dy}$ $\mu = \nu\rho$

$= \nu\rho U_{max}\left(\frac{-2(-W)}{W^2}\right)$

$= +5.8Pa$

[Ans: i) +5.8 Pa, ii) -5.8 Pa, iii) 0 Pa]

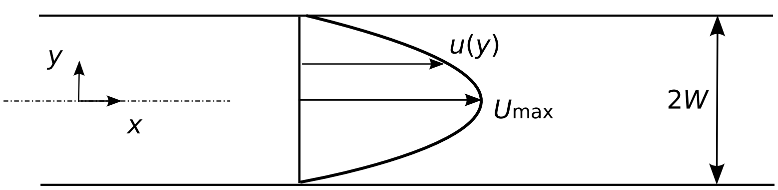


Figure 2: Velocity profile between two plates.

Question 5

Air ($\rho = 1.2kg/m^3$) flows in the pipe system illustrated in Figure 3. The diameter decreases in two stages from $50mm$ to $25mm$ to $10mm$. Each section is $10m$ long.

$Re = \frac{UD}{\nu}$ kinda hard to see cause D↑ U↓; D↓ U↑ but U is more significant, so section 3.

i. As the flow is increased, which section will become turbulent first?

ii. Determine the volume flow rate at which the flow in this section becomes turbulent.

for section 3 $\left\{ \begin{array}{l} Re = Re_{crit} \\ \frac{\rho UL}{\mu} = 2300 \\ U = 3.45 m/s \end{array} \right.$

$Q = UA = Q = U(\frac{\pi}{4}D^2) = 2.7 \times 10^{-4} m/s$

iii. Determine the volume flow rate at which the flow in all three sections becomes turbulent.

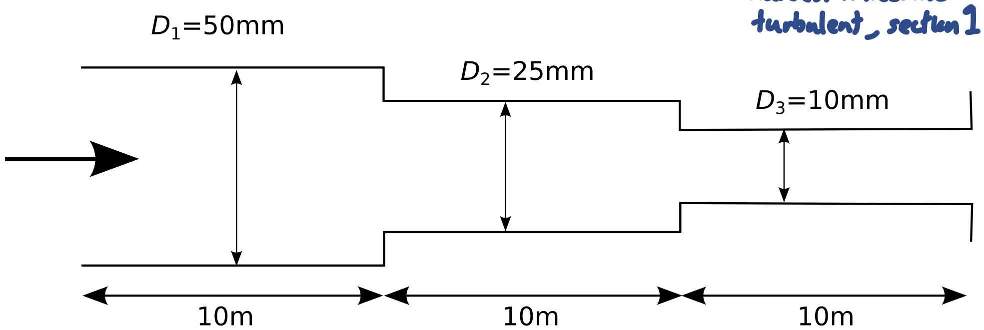
so just calculate the hardest to become turbulent, section 1:

[Ans: ii) $2.7\times10^{-4}\,\text{m}^3/\text{s}$, iii) $1.35\times10^{-3}\,\text{m}^3/\text{s}$]

$\frac{\rho UL}{\upsilon} = 2300$

$u_1 = 0.69 m/s$

$Q = u_1 A_1 = 1.35 \times 10^{-3}$



although u to become turbulent, is smaller than section 3, but remember we compare Q cause all section have same volume flow rate, and section 1 require higher volume flow rate !

Figure 3: Pipe system with three pipes.