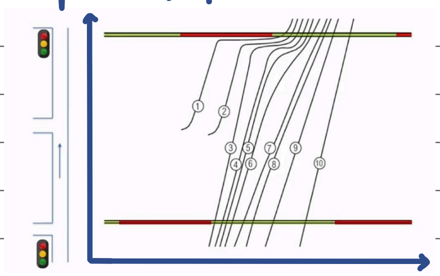
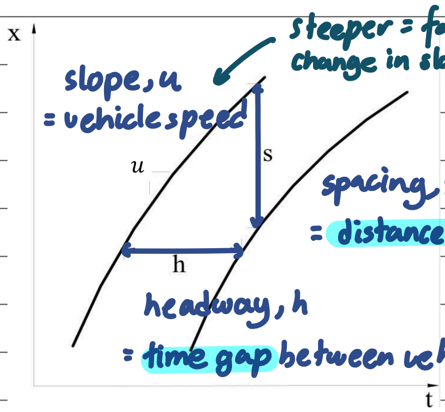
c1. Traffic Stream Variable

1.1 Vehicle Trajectories



Trajectory is graph of position vs time (showing vehicle movement)



steeper = faster

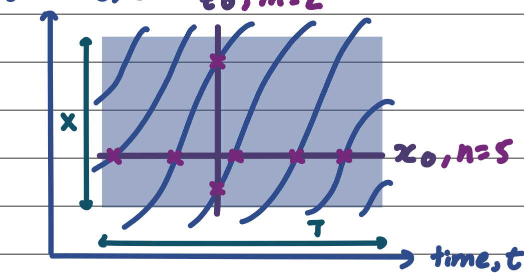
change in slope (u') = acceleration / deceleration.

slope, u = vehicle speed

spacing, s = distance gap between vehicle at a time.

headway, h = time gap between vehicle at a point

these 3 are microscopic variables. (i.e. individual vehicle measurements)



mean headway, $\bar{h} = \frac{\sum h}{n} = \frac{T}{n}$ how many n over T?

(at specific x_0, from t_0 to t_1 for period T)

space-mean headway, $\bar{s} = \frac{\sum s}{m} = \frac{X}{m}$

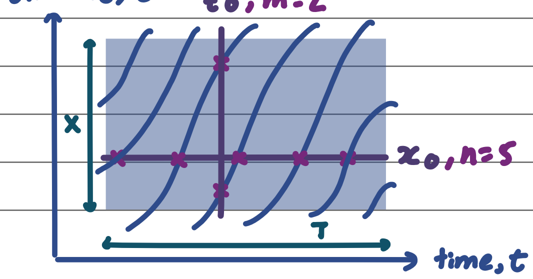
(at specific t_0, from x_0 to x_1 for length X)

by knowing s and h we could find all 5 traffic stream vars. q, k, u, s, h

1.2 Traffic Stream Variables. (Macroscopic)

Macroscopic variables are aggregated traffic properties.

→ describe overall state of the traffic stream. (as a whole! not individual)

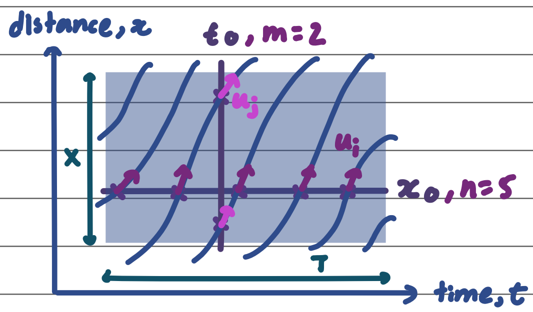


flow, $q = \frac{n}{T} = \frac{1}{\bar{h}}$

(number of vehicles passing through x_0 from t_0 to t_1 for period T)

density, $k = \frac{m}{X} = \frac{1}{\bar{s}}$

(number of vehicles present at t_0 from x_0 to x_1 for distance X)



time mean speed, $u_t = \frac{\sum u_i}{n}$ — speed over time — how many n over T

(average speed of individual vehicle at x_0 for T)

space mean speed, $u_s = \frac{\sum u_j}{m}$ — speed over space.

(average speed of vehicles at t_0 over X)

✿ you can get u_s from u_i too!

$$u_s = \frac{1}{\frac{1}{n}\sum \frac{1}{u_i}}$$

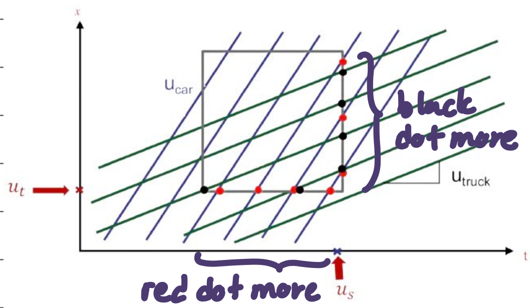
u_t can be expressed in term of u_s where:

✿ $u_t = u_s + \frac{\sigma_s^2}{u_s}$

$\sigma_t^2 = \frac{\sum (u_i - u_t)^2}{n-1}$ variance of speed over time

$\sigma_s^2 = u_s u_t - u_s^2$ variance of speed over space.

✿EXAM✿ Proof that $u_t \geq u_s$



✿ $u_t \geq u_s$
fast car pass observation point more often.

1.3 $q = ku$ (relationship between macroscopic var.)

$q = k \times u_s$
(flow) (density) (space-mean speed)

✿ flow ~ freq. when used to find u →

proof it's u_s not u_t:

$q = \sum q_j = \sum k_j u_j = \sum k \cdot \frac{k_j}{k} u_j = k \underbrace{\sum \frac{k_j u_j}{k}}_{u_s} = k u_s$

note: $u_s = \frac{\sum k_j u_j}{k}$, $u_t = \frac{\sum q_j u_j}{q}$

eg. ($t_1 q_3$) 3. The data shows the outcome of a spot speed study. Compute the time-mean speed and space mean-speed. Verify the relationship between them.

Speed group (mph)	Frequency
0-10	6
10-20	16
20-30	24
30-40	25
40-50	17

since $u_t = \frac{\sum q_j u_j}{q}$ and $q = \frac{n}{T}$ (freq.)

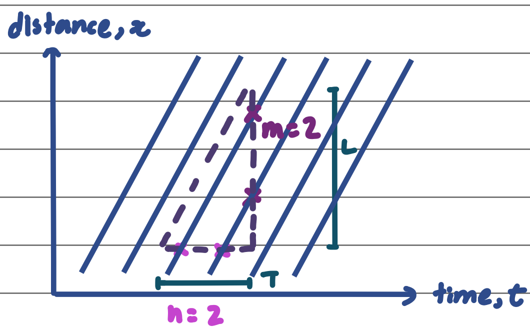
freq, n ~ flow, q (can treat n as q as $\frac{\sum q_i u_i}{q}$, T will cancel out!)

Speed group (mph)	Frequency (f=qi)	ui	qi*ui	ki=qi/ui	k*ui_sq	ki.ui=qi
0-10	6	5	30	1.2	30	6
10-20	16	15	240	1.0667	240	16
20-30	24	25	600	0.96	600	24
30-40	25	35	875	0.7143	875	25
40-50	17	45	765	0.3778	765	17
	88		2510	4.3187	2510	88

$$u_t = \frac{2510}{88} = 28.52\ mph$$

$$u_s = \frac{88}{4.3187} = 20.38\ mph$$

EXAM Derivation of $q = ku$



method 1:

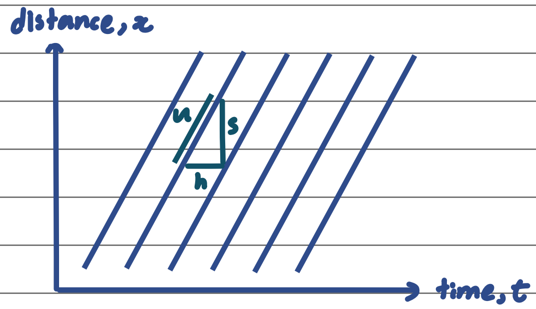
for steady and uniform traffic condition ($q = ku$ only valid when it is),

$n = m$

$qT = kL$ } $u = \frac{L}{T}$ (gradient)

$qT = kTu$

$q = ku$



method 2:

$u = \frac{s}{h}$ (spacing) / (headway) } only valid for steady, uniform traffic condition.

$s = uh$

$\frac{1}{k} = u\frac{1}{q}$

$q = ku$

1.4 Speed Studies

- to estimate speed distribution of vehicle speed in a stream of traffic at a point on a highway.
- use $\underbrace{\text{calculated mean speed}}_{\text{sample (depend on sample size!)}}$ to represent $\underbrace{\text{true mean speed}}_{\text{population}}$

EXAM 1.4.1 Finding minimum sample size, n

idea: $CI = \bar{x} \pm \underbrace{z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}_{\text{measurement of error}}$; $CI = \bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ ← we set ourselves the max of this, d

$d = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or $d = t_{\alpha/2}\frac{s}{\sqrt{n}}$

note that it is $\alpha/2$ two tailed! { $n = \left(\frac{z_{\alpha/2}\sigma}{d}\right)^2$ $n = \left(\frac{t_{\alpha/2, n-1}\, s}{d}\right)^2$

(use this if population s.d. is known) (use this if population s.d. is unknown) more common.

usual step to find min n:

1. estimate s.d., s
2. estimate initial value of n, n_1
3. find corresponding t with the degree of freedom = n − 1
4. calculate new n, n_2 — round up not n_2!
5. if $n_1 > n_2$, n_1 is our final answer
   else recalculate t with dof = $n_2 - 1$

eg 1. (t1q5)
z-test:

5. A cohort of students must determine the sample size needed to ascertain the mean speed on a segment of a motorway. It is known that the standard deviation of speeds on motorways is 5 miles per hour. With a desired confidence level of 95% and an acceptable margin of error set at 1.5 miles per hour, calculate the required sample size.

Standard deviation of the population (i.e. all motorways) is given so we could employ the Z-test

$\sigma = 5\ mph$
$CI = 95\%$
$MOE = \pm 1.5\ mph$
$Two\ tailed\ test$

Confidence Level (%)	Constant Z
68.3	1.00
86.6	1.50
90.0	1.64
95.0	1.96
95.5	2.00
98.8	2.50
99.0	2.58
99.7	3.00

$$n = \left(\frac{Z * SD}{d}\right)^2 = \left(\frac{1.96 * 5}{1.5}\right)^2 = 42.68 \sim 43$$

eg 2 (t1q6)
t-test:

6. A manufacturer has put forward a new radar-based device for measuring speed and its performance needs to be assessed to determine if it meets the clients' performance target. The stated performance target is that the mean error should lie between ±2kmh at the 95% confidence level at speeds of 50, 90 and 120 kmh. The performance will be assessed at an off-road test track by driving a vehicle equipped with automatic cruise control past the device at each of the design speeds. How many times should the vehicle be driven at each speed in order to determine whether or not the performance target can be met.

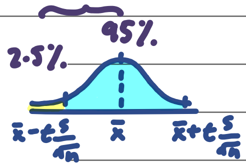
±2kmh
↑
hint at this is two tailed

step 1: estimate s.d.
let $s = 5$

step 2: estimate initial value of n, n_1
let $n_1 = 10$

step 3: find corresponding value of t, with $dof = n_1 - 1$
$dof = 9$

97.5% (t-table read to the left)



STUDENT'S t Table

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144

$\therefore t = 2.262$

step 4: calculate new n, n_2

$t_{\alpha/2} \frac{s}{\sqrt{n}} = d$

$n = \left(\frac{t \cdot s}{d}\right)^2 = \left(\frac{2.262 \times 5}{2}\right)^2 = 31.97 \approx 32$ (round up) $n_2 = 32 > n_1 = 10$ (continue!)

$n = 32$, $dof = 32 - 1 = 31$

STUDENT'S t Table

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144
11	0.260	0.443	0.697	0.876	1.214	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.442	0.695	0.873	1.209	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.441	0.694	0.870	1.204	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.440	0.692	0.868	1.200	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.439	0.691	0.866	1.197	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.439	0.690	0.865	1.194	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.438	0.689	0.863	1.191	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.438	0.688	0.862	1.189	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.438	0.688	0.861	1.187	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.437	0.687	0.860	1.185	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.437	0.686	0.859	1.183	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.437	0.686	0.858	1.182	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.436	0.685	0.858	1.180	1.319	1.714	2.069	2.500	2.807	3.485
24	0.256	0.436	0.685	0.857	1.179	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.436	0.684	0.856	1.178	1.316	1.708	2.060	2.485	2.787	3.450
26	0.256	0.436	0.684	0.856	1.177	1.315	1.706	2.056	2.479	2.779	3.435
27	0.256	0.435	0.684	0.855	1.176	1.314	1.703	2.052	2.473	2.771	3.421
28	0.256	0.435	0.683	0.855	1.175	1.313	1.701	2.048	2.467	2.763	3.408
29	0.256	0.435	0.683	0.854	1.174	1.311	1.699	2.045	2.462	2.756	3.396
30	0.256	0.435	0.683	0.854	1.173	1.310	1.697	2.042	2.457	2.750	3.385
35	0.255	0.434	0.682	0.852	1.170	1.306	1.690	2.030	2.438	2.724	3.340

$t = 2.042 - \frac{1}{5}(2.042 - 2.030)$
$= 2.0396$
≈ 2.04

$n = \left(\frac{2.04 \times 5}{2}\right)^2 = 26.01 \approx 27$ (roundup) $\underbrace{n_1 = 32}$ $> n_2 = 27$

final answer: if $s = 5$, $n = 32$

1.4.2 Comparison of Mean Speed

step 1: set up H_0 and H_1 (eg. $H_0: \mu_A = \mu_B$; $H_1: \mu_A \neq \mu_B$)

step 2: calculate test statistic ($t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$) } this is a two-sample (asymmetric) t-test. supposedly we need to do pool variance but for some reason they ignored it in this module.

step 3: find the critical t-value.

step 4: test if test stat lies in critical region. If yes, reject H_0

recommend check "Statistic Year 2.pdf" c8.2 (pg52)

1.5 Volume Studies.

Average Daily Traffic (ADT)

is the average of 24-hour counts collected over a number of days greater than one but less than a year:

- planning of highway activities
- measurement of current demand
- evaluation of existing traffic flow

Average Annual Daily Traffic (AADT)

is the average of 24-hour counts collected every day of the year:

- safety analysis
- traffic volume trends
- economic feasibility of highway projects

measure traffic volume

Peak Hour Volume (PHV)

focus on peak congestion period.

is the max # of vehicles that pass a point on a highway during a period of 60 consecutive minutes:

- functional classification of highways
- design of geometric characteristics
- capacity analysis

Vehicle Classification (VC)

break traffic down by vehicle types for analysis

records volume with respect to the type of vehicles, for example, passenger cars, light duty vehicles (LDVs), heavy goods vehicles (HGV):

- design of geometric characteristics
- capacity analysis
- structural design of highway bridges

Vehicle Miles of Travel (VMT)

is a measure of travel along a section of road. It is the product of the traffic volume and the length of roadway miles. This is used as the primary measure of traffic exposure.

quantify total travel exposures

M25 Motorway:
ADT = 145,000
Length = 117 mile
VMT = 16.965 million

1.5.1 Finding minimum sample size, n (no. of counting station)

minimum sample size, $n = \dfrac{t_{\alpha,N-1}^2\left(\frac{s^2}{d^2}\right)}{1+\left(\frac{1}{N}\right)t_{\alpha,N-1}^2\left(\frac{s^2}{d^2}\right)}$

note that it is α, one tailed! different from speed studies.

($t_{\alpha,N-1}^2$ is one tailed t-value, s is s.d., d is allowable error, N is no. of link)

eg. (ls1 pg50)

To determine a representative value for the ADT on 100 motorway links that have similar volume characteristics, it was decided to collect 24-hour volume counts on a sample of these links. Estimates of mean and standard deviation of the link volumes for the type of highways in which these links are located are 32,500 and 5,500 respectively. Determine the minimum number of stations at which volume counts should be taken if a 95% confidence interval is required with a 10% allowable error.

$N = 100$, $s = 5500$,

$d = 0.10 \times 32500 = 3250$

$t_{\alpha,N-1}$ is complicated, we still don't know dof $(N-1)$ is what. It's not $(100-1=99)$ as dof is the actual size of our sample.

iterate: dof $= 10-1 = 9$, $t_{0.95,9} = 1.833$

$$n = \frac{1.833^2\left(\frac{5500^2}{3250^2}\right)}{1+\frac{1}{100}\times 1.833^2\left(\frac{5500^2}{3250^2}\right)} = 8.78$$

iterate : $dof = 9-1 = 8$, $t_{0.95,8} = 1.860$, $n = 9.01$
iterate : $dof = 10-1 = 9$, $t_{0.95,9} = 1.833$, $n = 8.78$

stuck in infinite iteration, pick higher n to be safe

$\therefore n = 10$

C2. Traffic Flow Theory

2.0 Introduction

Traffic flow arises from driver behaviour and interactions
(driving, route choice, departure time)

Traffic flow theory is used to :
- design road with better level of service
- optimize traffic operations
- predict system response to changes (eg. land changes)

2.1 Greenshields Model (1934)

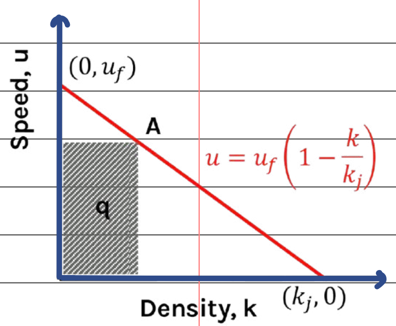
Idea : When there's few vehicles on the road, speed of vehicles are high
(free flow speed, u_f) ;
When there's a lot of car on the road (road is at max capacity),
speed of vehicles are zero.

Greenshields Model (1931) assume speed is linear proportional to density.

$$u = \underbrace{u_f}_{\text{free flow speed}}\left(1 - \frac{k}{\underbrace{k_j}_{\text{jam density}}}\right)$$

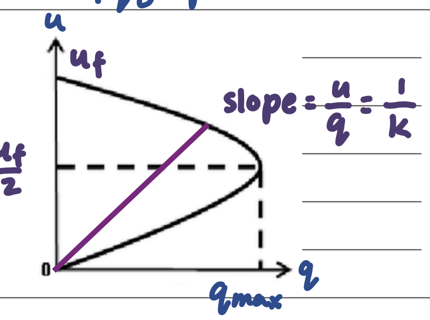
Fundamental Diagram
- u/k graph
- u/q graph
- q/k graph

1. u/k graph. (Model)



when $k = k_j$, $u = 0$ (jam density)
when $k = 0$, $u = u_f$ (free flow speed)

2. u/q graph.

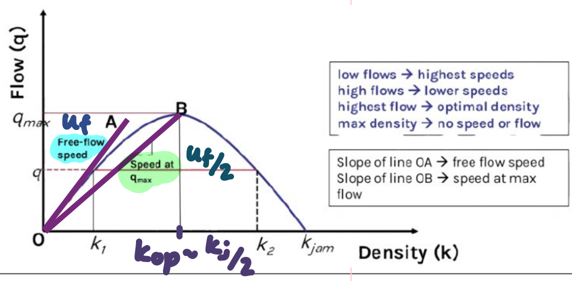


(capacity of road)

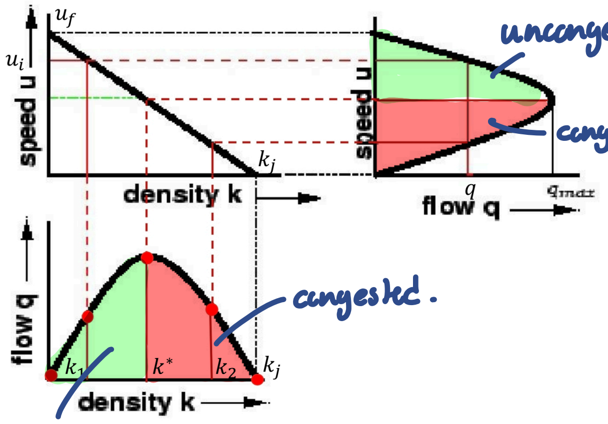
$q = 0$ can mean $\underbrace{u = u_f}_{\text{no car, drive fast}}$ and $\underbrace{u = 0}_{\text{too much car, can't move.}}$

* max flow, q_{max} occur at $u = u_f/2$

3. q/k graph



one q has two k value.
* q_{max} occur at ~ $k_j/2$



THESE THREE ARE IMPORTANT!

1. $u = u_f : q = 0, k = 0$
2. $u = 0 : q = 0, k = k_j$ } Boundary Condition (ALL MODELS)
3. $u = u_f/2 : q = q_{max}, k \sim k_j/2$

special identity for greenshield.

EXAM

Capacity, q_{max}

we know that $q = q_{max}$ when $u = u_f/2$ and $k = k_j/2$

proof: $u = u_f\left(1 - \frac{k}{k_j}\right)$

$q = ku$

$q = ku_f\left(1 - \frac{k}{k_j}\right)$

to find k when $q = q_{max}$, $\frac{dq}{dk} = 0$

$\frac{dq}{dk} = u_f - 2u_f\frac{k}{k_j} = 0$

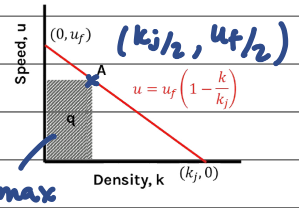
$1 - \frac{2k}{k_j} = 0$

$\therefore k = \frac{k_j}{2}$

$u = u_f\left(1 - \frac{k_j/2}{k_j}\right)$

$\therefore u = \frac{u_f}{2}$

proven that $q = q_{max}$ when $u = u_f/2$, $k = k_j/2$



$\therefore q = ku = k_j/2 \times u_f/2$

$= \frac{k_j u_f}{4}$

if we want to find q_{max} but it is not greenshield model, we can still dq/dk to find k that yields q_{max}, then find q_{max} by $q = ku$ where $u = f(k)$

2.2 Other Speed-Density Models

Greenberg (1959): $u = c \ln \frac{k_j}{k}$ ← violates BC: $k = 0, u = u_f$

(problem: not defined at $k \to 0$)

identity: $u_{opt} = c$, $k = k_j/e$, $q = q_{max}$

Underwood: $u = u_f e^{-k/k_o}$

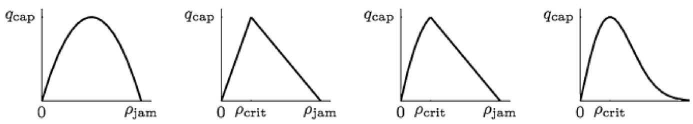
(problem: less accurate at $k \to k_j$)

identity: $u_{opt} = u_f/e$, $k = k_o$, $q = q_{max}$

different model has different fundamental diagram! u/k, u/q, and q/k graph.

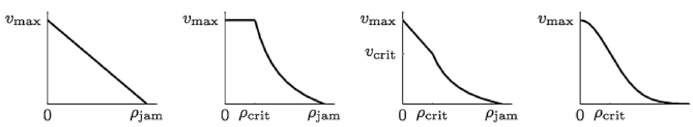
$q = ku$: found by $dq/dk = 0$ find k

like greenshield is $u = u_f/2$, $k = k_j/2$, $q = q_{max}$. (check example 2 and 3)



(a) Greenshields (parabolic). (b) Daganzo (bi-linear). (c) Smulders (parabolic-linear). (d) Drake.

} q/k graph. get k_{crit} and q_{max} by $dq/dk = 0$ if possible.



(e) Greenshields. (f) Daganzo. (g) Smulders. (h) Drake.

} u/k graph.

2.3 Macroscopic Fundamental Diagram (just understand is enough)

Unlike the FD that we just learnt, which is for one segment of road ... we look at relationship between q, k, u

macroscopic FD is for entire network / area

we look at relationship between average / total flow vs average / total density.

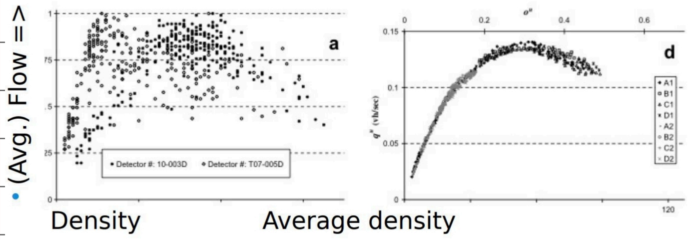
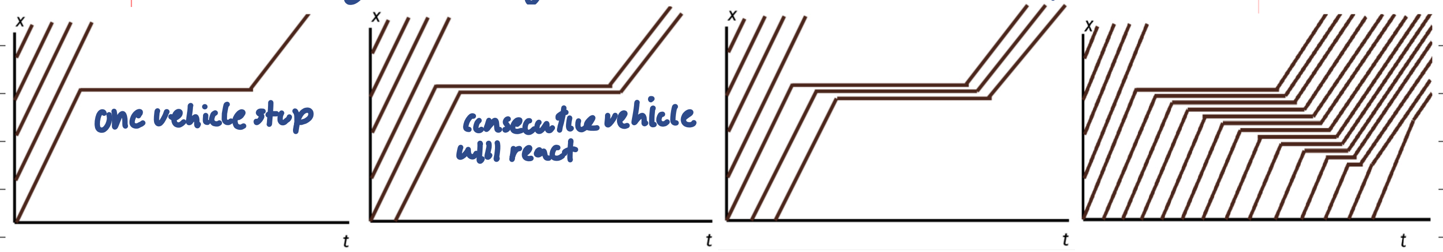


Fig: (Geroliminis and Daganzo)

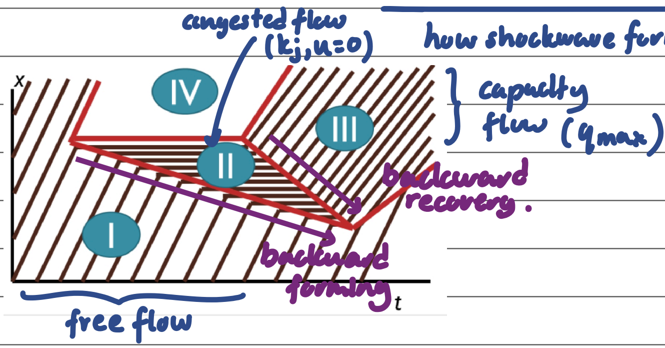
2.4 Shockwaves

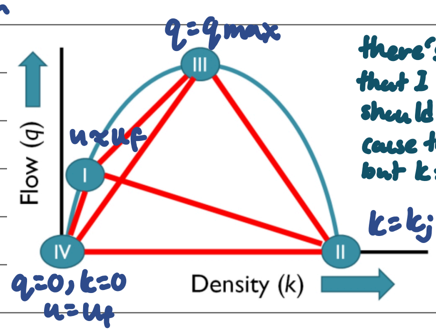
Shockwave is a transition between two steady state traffic condition

Can be caused by traffic congestion (sudden decrease in road capacity)



how shockwave form



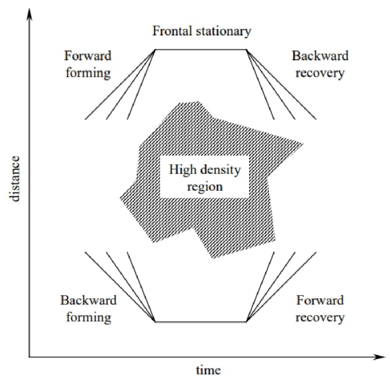


there's this misconception that I (free flow state) should be at IV. cause that is where u is max but k=0! so I is at a pt close to IV and u_f is close to theoretical maximum in real world. But in theory u_f is still max.

there's ${}^4C_2 = 6$ shockwave

4 is no. of state.

2 is fixed (state of shockwave)



2.5 LWR Theory (Lighthill, Whitam and Richard Theory)

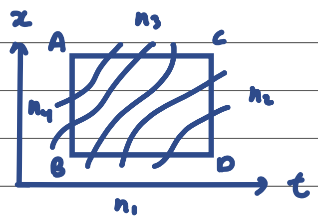
LWR Theory is a dynamic model that describe traffic conditions change over time and space.
used to analyse formation of traffic jam and movement of shockwaves.

It is very different from the speed density model we learnt before like Greenshield, which can only be applied on steady-state traffic condition.
Whereas LWR Theory can be applied on dynamic traffic condition. $q = f(k)$ ☆

It's derived from the principle of conservation of vehicle (like conservation of mass) i.e. "flow" cannot be lost.

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0$$

✿EXAM✿ derivation:



no. of vehicle entering = no. of vehicles exiting.

$$n_1 + n_4 = n_2 + n_3$$

$$q_{BD}\Delta T + k_{AB}\Delta X = k_{CD}\Delta X + q_{AC}\Delta T$$

from c1: $q = \frac{n}{T}$, $k = \frac{m}{X}$

$$(q_{AC} - q_{BD})\Delta T = (k_{CD} - k_{AB})\Delta L$$

$$\frac{\Delta k}{\Delta T} + \frac{\Delta q}{\Delta X} = 0$$

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0$$

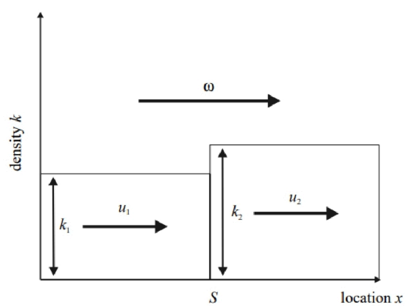
☆ since $q = f(k)$ we can:

$$\frac{\partial k(x,t)}{\partial t} + \underbrace{\frac{\partial q}{\partial k}}_{c(k) = f'(x)}\frac{\partial k(x,t)}{\partial x} = 0$$

2.5.1 Shockwave Speed and Kinematic Wave Speed.

Shockwave: major transition in state of traffic flow

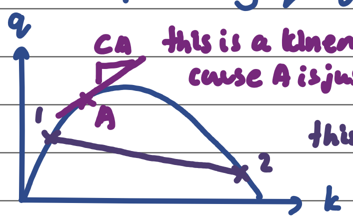
Kinematic Wave: minor disturbance. in state of traffic flow



$$q_{enter} = k_1(u_1 - w)$$
$$q_{exit} = k_2(u_2 - w)$$
$$w = \frac{q_1 - q_2}{k_1 - k_2}$$

derivation of shockwaves speed.

or we can find using q-k graph: disturbance moving forward.



this is a kinematic wave, $c_A > 0$ cause A is just a point.

Shockwave moving backwards.

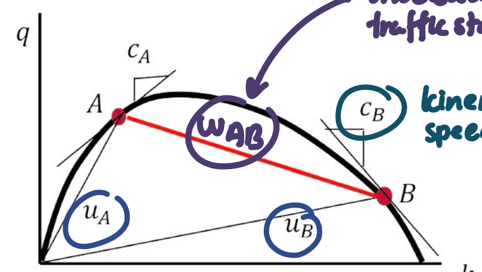
this is a shockwave, $w < 0$ cause state 1 and state 2 are huge difference.

usually backward in heavy traffic, forward in light traffic.

disturbance/wave travel same direction as the traffic flow

shockwave speed from traffic state A to B.

kinematic wave speed at traf



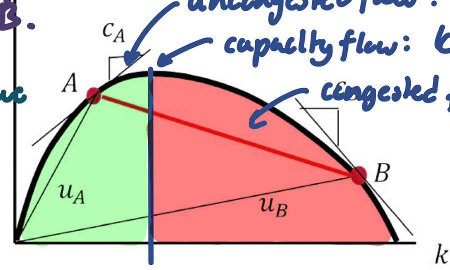
average traffic speed at traffic state A.

average traffic speed at traffic state B

uncongested flow : $k < k_c$, $c > 0$

capacity flow: $k = k_c$, $c = 0$

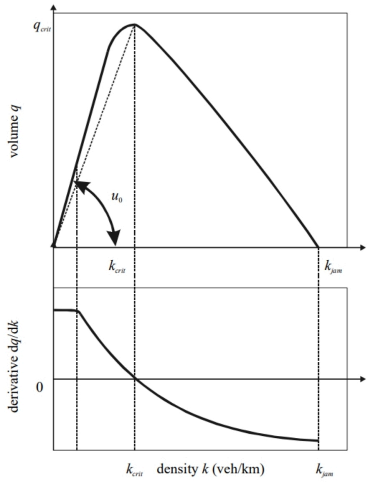
congested flow: $k > k_c$, $c < 0$



disturbance opposite dir. as flow.

k_c (capacity density/ critical density)

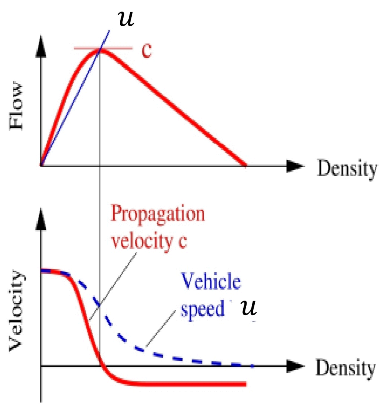
2.5.2 Principles of LWR Theory.



1. when k is small, $c\left(\frac{dq}{dk}\right) \approx u$

2. as k increases, difference between u and c increases. (it holds up to k_j; even if $u=0$, cause c is very negative)

3. congested region : $c<0$ implying disturbance is propagating upstream (backwards)



4. u is always greater than c, implying traffic information can never travel faster than the traffic carrying it.

EXAM

2.6 Criticism of LWR Theory.

- The LWR model is only a **coarse representation** of reality
- It **does not** capture:
  - Driver differences
  - Traffic instability
- Then, for example, the model is **inaccurate** when there is a significant amount of lane-changing manoeuvres.
- The model also assumes (**erroneously**) that people can change speeds (through a shock wave) instantaneously (i.e. with infinite decelerations or accelerations)
- Lastly, the model is always stable even when the world is **not** (e.g., it cannot explain stop and go traffic)

Examples

1. **Following data are collected on a highway segment:** (shows how to get model from data)

Speed km/h	90	80	65	54	35
Density veh/km	7	27	38	50	65

Assuming that the relationship between speed and density can be explained by the Greenshields model, find:

can use / derive this or just follow my working,

(i) Free flow speed u_f $(k=0)$
(ii) Jam density k_j $(u=0)$
(iii) Capacity of the segment q_{max} $(k = k_j/2, u = u_f/2)$

$u = u_f\left(1-\frac{k}{k_j}\right)$ but we need to get $u=f(k)$ where $f(k)$ is a linear function

→ Do linear regression (y2 stats c9)

$u = a + bk$

$b = \frac{cov_{u,k}}{var_k} = -0.963854188$

$a = \bar{u} - b\bar{k} = 64.8 - (-0.9639)(37.4) = 100.8481466$

↑ can be done with calculator linear regression!

$\therefore \underbrace{u = 100.8481 - 0.9639k \qquad u = u_f - \frac{u_f}{k_j}k}$

$u_f = 100.8481, \quad k_j = \frac{100.8481}{0.9639} = 104.63$

$q = ku = k(100.8481 - 0.9639k)$

$\frac{dq}{dk} = 100.8481 - 1.9278k = 0$

$k = 52.3125$

$q_{max} = 100.8481(52.3125) - 0.9639(52.3125)^2 = 2638 \text{ veh/hr}$

alt method is $q_{max} = \frac{k_j u_f}{4}$ cause we know is Greenshield

2. Greenberg model of traffic flow theory can be explained as follows: (how to get u_{opt}, k_{opt} by $dq/dk = 0$)

$$u = C\ln\left(\frac{k_j}{k}\right)$$

Show that:

(i) $u = C$ when flow is maximised

(ii) $k = \frac{k_j}{e}$

What is the primary problem with this model?

$q = ku = kC\ln\left(\frac{k_j}{k}\right)$

$\frac{dq}{dk} = 0$

$\cancel{k}C\left(-\frac{1}{\cancel{k}}\right) + C\ln\left(\frac{k_j}{k}\right) = 0$

$-C + C\ln\left(\frac{k_j}{k}\right) = 0$

$\ln\left(\frac{k_j}{k}\right) = 1$

$u = C\ln\left(\frac{k_j}{k}\right) = C(1) = C$

$\ln\left(\frac{k_j}{k}\right) = 1$

$\frac{k_j}{k} = e$

$k = k_j/e$

not defined at $k \to 0$

3. Underwood model of traffic flow theory can be explained as follows: (how to get u_{opt}, k_{opt} by $dq/dk=0$)

$$u = u_f \, . \, e^{-\frac{k}{k_0}}$$

Show that:

$$(i)\ k = k_0$$
$$(ii)\ u_{max} = \frac{u_f}{e}$$

What is the problem with this model?

$$q = ku = ku_f e^{-k/k_0}$$

$$\frac{dq}{dk} = k\left(-\frac{1}{k_0}\right)u_f e^{-k/k_0} + u_f e^{-k/k_0} = 0$$

$$-\frac{k}{k_0} + 1 = 0$$

$$k_c = k_0 \quad \text{(critical k)}$$

$$u_{max} = u_f e^{-k_0/k_0}$$
$$= u_f/e$$

less accurate at $k \to k_j$

★ (find stop-wave speed)

4. Suppose $u = 90e^{-0.5\left(\frac{k}{120}\right)^2}$, where *v* is the space mean speed (km/h) and *k* is the traffic density (veh/km). Calculate:

a. (u_f: $k=0$)
b. ($dq/dk=0$)

a. Free-flow speed.
b. Speed, flow and density at maximum flow.
c. The speed of the stop wave (shockwave to jam) if traffic is approaching at maximum flow and assuming that jam density is three times density at maximum flow.
d. The jam density that would yield a stop wave speed of -18 km/h.
e. Discuss problems with this curve.

cannot use $q = \frac{u_f k_j}{4}$ only valid for greenshield's !

★ stop wave speed is defined as shockwave speed from q_{max} to $q=0$ (jam)

(a) u_f happens at $k=0$

$$u_f = 90e^{-0.5(0/120)^2} = 90$$

(b) $q = ku = 90ke^{-0.5(k/120)^2}$

$$\frac{dq}{dk} = \cancel{90}k\left[-\frac{1}{14400}k\cancel{e^{-0.5(k/120)^2}}\right] + \cancel{90}\cancel{e^{-0.5(k/120)^2}} = 0$$

$$\frac{1}{14400}k^2 = 1$$

$$k^2 = 14400$$

$$k = 120 \text{ veh/km}$$

$$u = 90e^{-0.5(120/120)^2} = 54.59 \text{ km/hr}$$

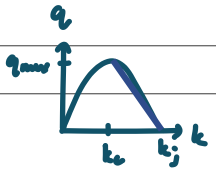
$$q_{max} = 120 \times 54.59 = 6550.53 \text{ veh/hr}$$

(c) $w = \dfrac{q_1 - q_2}{k_1 - k_2}$

from max flow to jam : $q_1 = q_{max}$, $q_2 = 0$
$k_1 = k_c$, $k_2 = k_j = 3k_c$

$$= \frac{6550.53 - 0}{120 - 3(120)}$$

$$= -27.29 \text{ km/hr}$$



IMPORTANT.

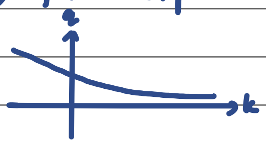
(d) $w = -18$

$$\frac{q_{max} - 0}{k_c - k_j} = -18$$

$$\frac{6550.3}{120 - k_j} = -18$$

$k_j = 483.91$ veh/hr

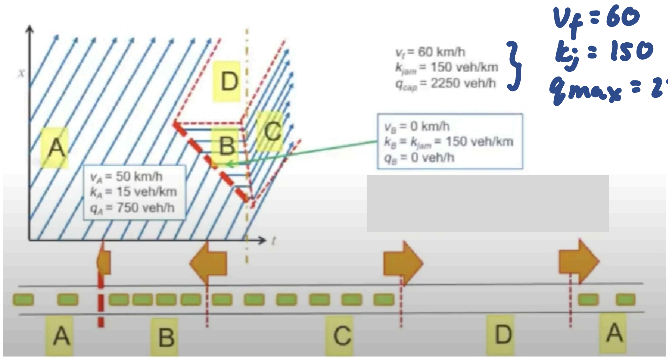
(e) for all exponential curve model,



curve will be asymptotic to k-axis.
Therefore k_j cannot be derived as k_j is k when $q = 0$ but q can never be 0!

5. The following diagram shows a time-space diagram of vehicle trajectories at a traffic signal. Different traffic states (A, B, C and D) are also shown in the diagram. Estimate the shockwave speeds:
   (i) at the back of the queue.
   (ii) at the front of the queue.
   Comments on the calculated values of the shockwave speed.

(how to read and get data from x-t graph with shockwave)



$V_f = 60$
$k_j = 150$
$q_{max} = 2250$

to find u_c, k_c, q_c
$q_c = q_{max} = 2250$.
assume greenshield's:
$u = u_f/2 = 60/2 = 30$
$k = k_j/2 = 150/2 = 75$

(i) back of the queue's shockwave is formed between state A and B,

$$w = \frac{q_A - q_B}{k_A - k_B} = \frac{750 - 0}{15 - 150} = -5.56 \text{ km/h}$$ (upstream cause of -ive sign)

(ii) front of the queue's shockwave is formed between state B and C,

$$w = \frac{q_B - q_C}{k_B - k_C} = \frac{0 - 2250}{150 - 75} = -30 \text{ km/h}$$

★ (how to find queue length) and max queue length)

6. Approaching a stop light up ahead on a road whose flow is 1000 vph and average speed is 50 mph. Jam density on the road is 150 veh/mile. For a red phase =30 seconds, find: (i) velocity of the backward forming shockwave, (ii) queue at the end of the red.

How about the unlit taillights as traffic leaves the red light? How fast is that travelling? Assume that the flow rate leaving that light is at a maximum flow rate, 1800 vph and density is 75 veh/mile (critical density). How far back from the red light does this wave meet the previous wave? It is at this point where the queue is at the maximum.

(i) →

car approaching
$q = 1000$
$u = 50$
$k = q/u = 20$

stop light
$q = 0, u = 0, k = k_j = 150$

$$w = \frac{1000 - 0}{20 - 150} = -7.69 \text{ mph}$$

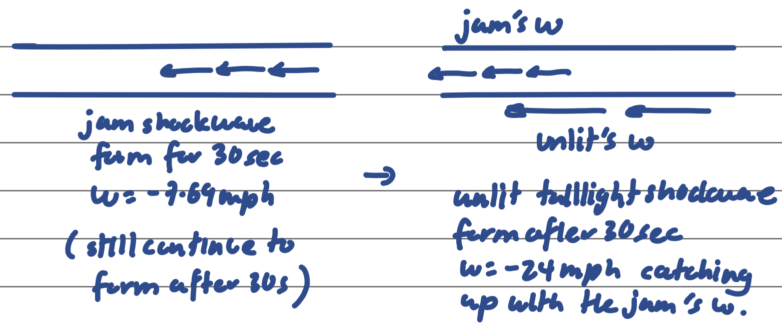
(ii) queue's unit is length (mile)
we have shockwave speed (mile/hr)

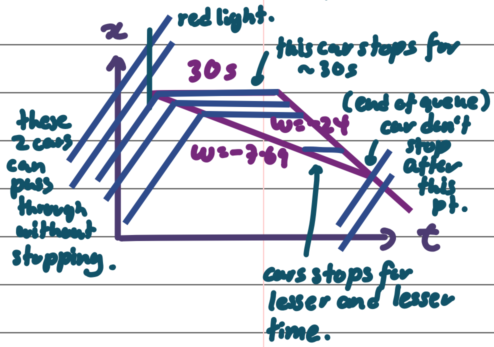
queue length = w × red phase
(mile) (mile/hr) (hr)

$= 7.69 \times 30/3600$
$= 0.0641$ mile
≈ 338.46 ft.

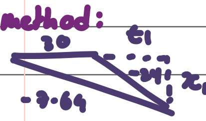
(iii) unlit taillight's shockwave speed.
(from jam to capacity state, q_{max}) ← different from jam forming:
(from normal state, q to jam $q=0, k_j$)

$$w = \frac{q_{max} - 0}{k_{opt} - k_j} = \frac{1800 - 0}{75 - 150} = -24 \text{ mph}$$





alt method:



$-7.69 = \frac{x_1}{30 + t_1}$

$-24 = \frac{x_1}{t_1}$

$7.69(30 + t) = 24t$
$230.7 + 7.69t = 24t$
$230.7 = 16.31t$
$t = 14.14s$

$14.14 \times 24 = 339.47$ mph × 1s
$= 0.0943$ mile
≈ 497.89 ft.

7. A slow-moving truck drives along the roadway at 10mph. The existing conditions on the roadway before the truck enters are shown at point 1 below: 40 mph, flow of 1000 vehicles per hour, and density of 25 vehicles per mile. The truck enters the roadway and causes a queue of vehicles to build, giving the characteristics of point 2 below: flow of 1200 vehicles per hour and a density of 120 vehicles per mile. Using the information provided below, find the velocity of the shockwave at the front and back of the platoon.

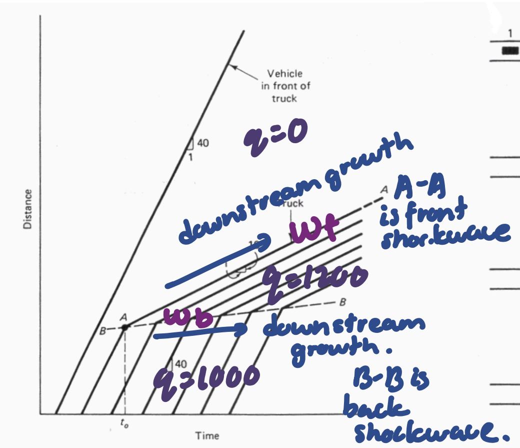


Figure 3.6.3 Time–distance diagram of platoon formation.

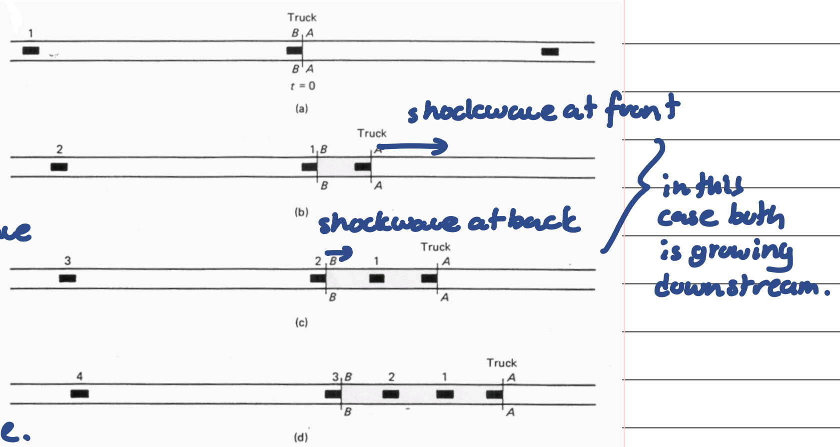
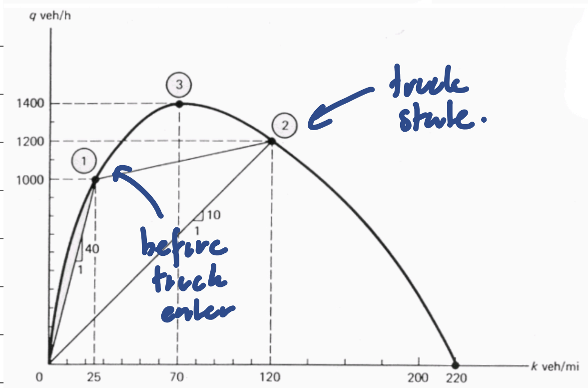


Figure 3.6.2 Platoon formation.



front of the truck no vehicle: $q=0, k=0, u=u_f$
(i.e. origin of q/k graph)

$$w_{front} = \frac{q_2 - 0}{k_2 - 0} = \frac{1200 - 0}{120 - 0} = 10 \text{ mph}$$

(basically the speed of truck)

both +ive

$$w_{back} = \frac{q_2 - q_1}{k_2 - k_1} = \frac{1200 - 1000}{120 - 25} = 2.1 \text{ mph.}$$

platoon growth speed: $10 - 2.1 = 7.9$ mph.

c3 Queueing Theory

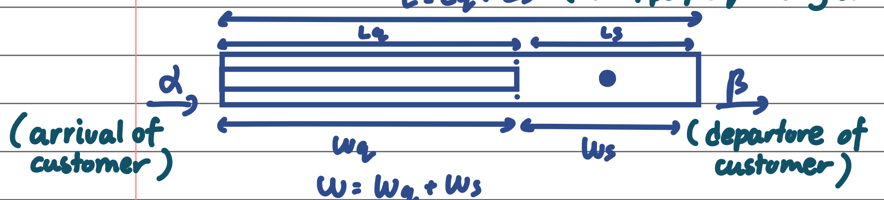
3.0 Introduction.

Queueing Theory is the study of movement of people, objects or information through a line or a queue

A basic queue has four components :

Demands : Customer / Vehicle arriving (at random times)
Queue : Storage / Waiting line / Capacity
System : Resource / Server that processes the demands (toll booth, traffic signal etc.)
Output : Customer / Vehicle departing

$L = L_q + L_s$ (no. of people queueing and no. of people being served)



$W = W_q + W_s$

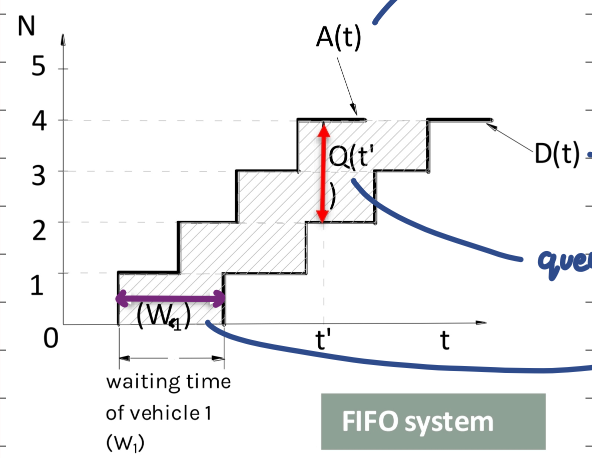
(average waiting time in queue and average waiting time being served)

$L, L_q, L_s, W, W_q, W_s, \alpha, \beta$

all are average values!

3.1 Cummulative Plot

CDF of no. of arrival



D(t) — CDF of no. of departure.

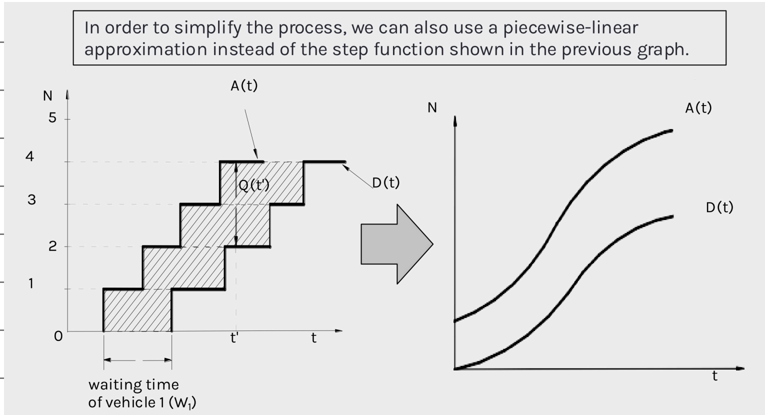
queue length (no. of vehicle accumulated at a specific time varies from time to time)

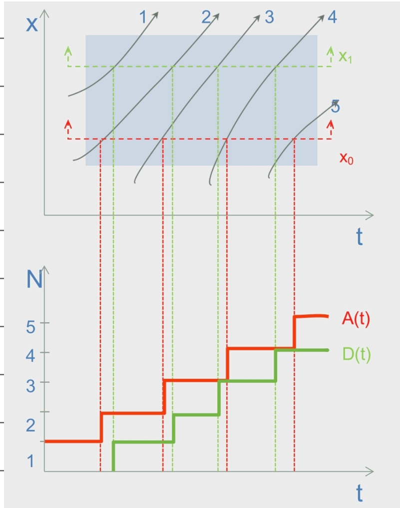
waiting time (for a specific customer eg. W_1 for vehicle 1, W_2 for 2 etc. varies between customer)

shaded area = total wait time in queue (for all vehicle total)

$\frac{\text{shaded area}}{\text{no. of vehicle}, N}$ = average wait time in queue (for each vehicle)

$\frac{\text{shaded area}}{\text{total time}, t}$ = average no. of vehicle in queue.





To obtain cummulative plot from time-space plot:

1. take horizontal slices at arrival, x_0 and departure, x_1
2. construct $A(t) = N(t, x_0)$ and $D(t) = N(t, x_1)$

3.2 Arrival and Departure Process

1. Arrival Process

Average arrival rate, α (no. of unit arriving per unit time, eg. 10 vehicles/hr)

Average interarrival time, $1/\alpha$ (average time between consecutive arrival, eg. 0.1 hr / vehicle)
= 6min / vehicle

2. Service / Departure Process

Average service rate, β (no. of unit the system can serve per unit time, eg. 12 vehicles / hr)

Average service time, $1/\beta$ (average time to serve one unit, eg. $1/12$ hr / vehicle)
= 5 min / vehicle

Stochastic (Random) Process.

In traffic, arrivals are often random, α and $1/\alpha$ are all "average", hence :

continuous → $X \sim Exp(\lambda = \alpha)$; $X \sim Po(\mu = \alpha t)$ ← discrete.

(X is dist. of interarrival time) (X is no. of arrival in an interval of time t)

remember that in " Statistic Year 2, c4.1 (pg15) and c4.2 (pg16),

$X \sim Exp(\lambda = \alpha)$: λ is no. of occurence per unit time and, eg. 10 veh / hr

$X \sim Po(\mu = \lambda t)$: μ is no. of occurence in a $\underbrace{\text{reference time}}_{t}$ eg. 10 veh in an hour

units

eg (t3q1)

Q1. *Vehicles arrive at an incident location on a motorway site at a rate of 1,000 vehicles per hour.*
(a) *What is the probability that the headway between successive vehicles will be less than 5 seconds?*
(b) *What is the probability that the headway will be between 5 seconds and 8 seconds? Assume that: inter-arrival time is exponentially distributed.*
(c) what is the prob. of more than 3000 vhc in 2hr

(a) $\lambda = 1000$ vhc/hr, find $P(X \le 5sec)$, where X is interarrival time

Convert λ to vhc/sec

$\lambda = 1000$ vhc/hr $= \dfrac{1000}{60 \times 60} = \frac{5}{18}$ vhc/sec

$X \sim Exp(5/18$ vhc/sec$)$

$f(x) = \lambda e^{-\lambda x}$

$F(5) = \int_0^5 \frac{5}{18} e^{-\frac{5}{18}x} dx = \left[-\cancel{\frac{18}{5}} \times \cancel{\frac{5}{18}} e^{-\frac{5}{18}x} \right]_0^5$

$= -e^{-\frac{5}{18}(5)} + e^{-\frac{5}{18}(0)}$

$= 0.7506 \quad (75.06\%)$

how do i know to use $X \sim Exp(\lambda)$ not $X \sim Po(\mu)$?

hint 1: we are finding interarrival time, which are time taken till next independent event occurs. (ONE event)

If we were told to find in a reference time, how many independent event occurs (MULTIPLE event) it would be $X \sim Po(\mu = \lambda t)$

(b) $F(8) - F(5) = 1 - e^{-\frac{5}{18}(8)} - 0.7506$

$= 0.1410 \quad (14.10\%)$

(c) more than 3000 vhcs in $\underbrace{2hrs}$
reference time $\rightarrow X \sim Po(\mu)$
$\mu = \lambda t$

$\mu = 1000$ vhc/hr $\times$ 2hr
$= 2000$ vhc (in 2hrs, matches with required reference time)

$X \sim Po(2000)$

$P(X=k) = \dfrac{e^{-\mu}\mu^k}{k!}$

$P(X > 3000) = 1 - P(X \le 3000)$

$= 1 - P(X=0) - P(X=1) - P(X=2) - \dots - P(X=3000)$

$= 1 - \dfrac{e^{-2000} \cdot 2000^0}{0!} - \dfrac{e^{-2000} \cdot 2000^1}{1!} - \dots$

$= ?$

Service design

Degree of saturation (traffic intensity), $\rho = \alpha / \beta$ ($\beta \sim 0.8$ is optimum)

When $\rho < 1$, arrivals rate is smaller than service rate, conditions are undersaturated, and average queue remains stable over time (stationary queue)	$\alpha > \beta$ (good)
As ρ approaches 1, conditions become saturated, and average queue approaches infinity (not instantly but over time → queue keeps increasing over time, see later: $L = \rho/(1-\rho)$)	$\alpha \approx \beta$ (bad)
If $\rho > 1$, arrivals rate is larger than service rate, conditions are oversaturated, and average queue tends to increase over time (non-stationary queue)	$\alpha < \beta$ (very bad)

3.3 Queueing Model (to find L_q, L → to calculate W_q, W)

with little's formula.

arrival / departure / no. of service channel.

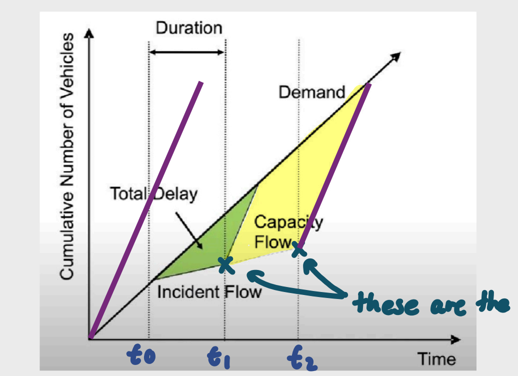
D/D/1
- D stands for deterministic
- this model can find:
  - total delay caused by an incident (veh min)
  - no. of vehicle affected by an incident.

D/D/1	M/M/1
- Deterministic arrivals - Deterministic departures - 1 channel departures - Graphical solution	- Exponential arrivals - Exponential departures - 1 channel departures - Mathematical
M/D/1 - Exponential arrivals - Deterministic departures - 1 channel departures - Mathematical solution	**M/M/N** - Deterministic arrivals - Deterministic departures - 1 channel departures - Simulation

M/M/1
- M stands for Markovian (random dist. → Exp, Po)
- to derive and find P_j (prob. having j customers)
- to derive and find L and L_q
- use little formula to find W and W_q

these two are not in syllabus.

3.3.1 D/D/1 Model

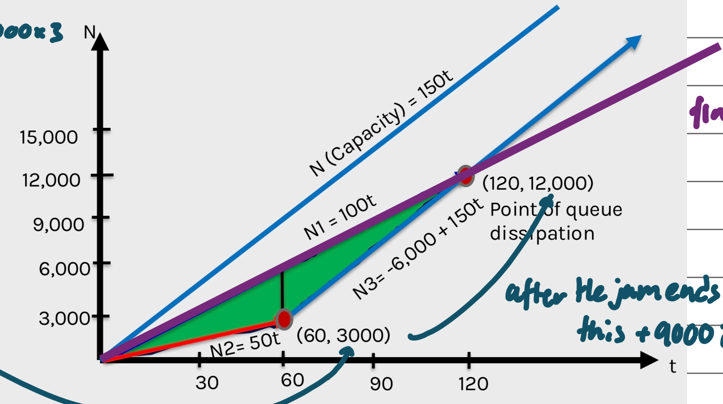


Reduce duration by 50%, reduce delay by 75%

if the traffic jam last from t_0 to t_2
→ the total delay would be Area: (green) + (yellow)
but if the traffic jam is cut down to t_0 to t_1
→ the total delay would be Area: (green) only.

3-Lane Motorway segment (per lane)
Capacity: 3,000 vphpl = 9,000 vph ← 3000×3
Arrival/demand: 2,000 vphpl
= 6,000 vph ← 2000×3
Incident duration: 1 hr
Incident flow: 3,000 vph

1hr jam: 3000 vph × 1h
= 3000 v



- Queue duration: 120 min
- # of vehicles affected: 12,000
- Total delay = 180,000 veh-min

Area of green.

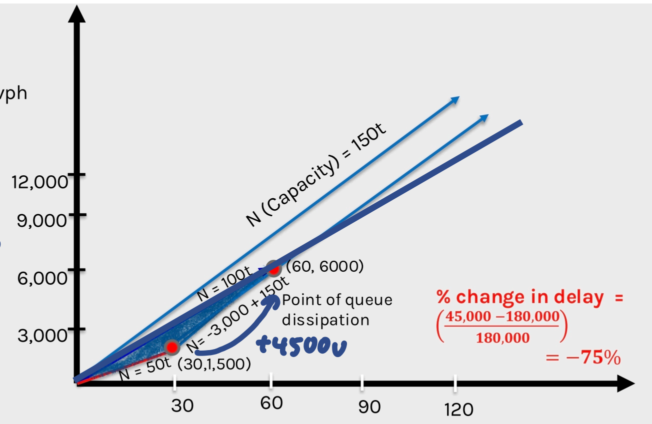
after the jam ends 3000 → 12000
this +9000 is from capacity.

final: free flow, capacity flow, congested flow

3-Lane Motorway segment
Capacity: 3,000 vphpl = 9,000 vph
Arrival/demand: 2,000 vphpl = 6,000 vph
Incident duration: 30 mins
Incident flow: 3,000 vph

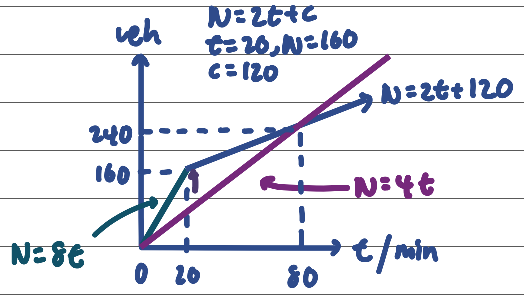
capacity = 9000 vph
for 0.5h : 9000 × 0.5 = 4500v

- Queue duration: 60 min
- # of vehicles affected: 6,000
- Total delay = 45,000 veh-min



eg. (t3q2) ☆ (very interesting qtn, previous example is on traffic flow : jam → capacity → normal) this question is on queueing : arrival and service rate)

Q2: Vehicles arrive at a toll station on an approach to a bridge with a single gate where an operator takes the payment. The bridge opens at 8am at which the vehicles arrive at a rate of 480 vehicles per hour. After 20 minutes, the arrival flow rate declines to 120 vehicles per hour and continues at that level for the reminder of the day. If the time required for the operator to serve a vehicle is 15 seconds, and assuming the D/D/1 queueing model, calculate the operational characteristics of the queue system.



$d = 480\,veh/hr = \frac{480}{60}\,veh/min = 8\,veh/min.$

$d_j = 120\,veh/hr = \frac{120}{60}\,veh/min = 2\,veh/min$

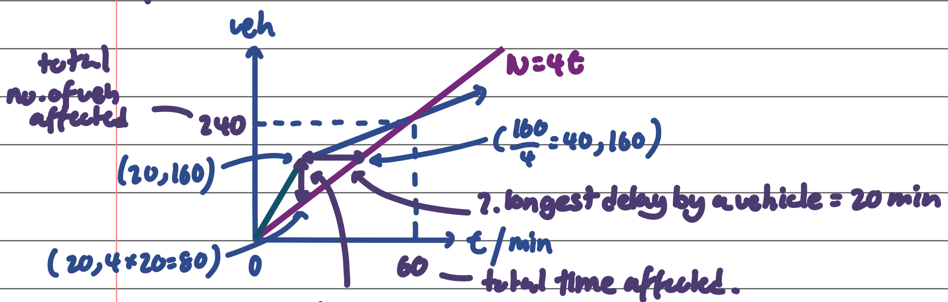
$\beta = \frac{1}{15}\,veh/sec = \frac{1}{15} \times 60\,veh/min = 4\,veh/min$

$2t + 120 = 4t$

$2t = 120$

$t = 60 \quad N = 4(60) = 240$

Operational Characteristics (3)



1. longest veh queue = 80veh.

3. Area bounded by N=8t, N=2t+120, N=4t (total delay)

$A = \frac{1}{2}\begin{vmatrix} 0 & 60 & 20 & 0 \\ 0 & 240 & 160 & 0 \end{vmatrix}$

$= \frac{1}{2}\left|60 \times 160 - 240 \times 20\right| = 2400\text{ veh-min}$

4. Average vehicle delay

$= \frac{2400\text{ veh min}}{240\text{ veh}}$ ← total no. of vehicle affected.

$= 10\text{ min}$

5. Average queue length

$= \frac{2400\text{ veh min}}{60\text{ min}}$ ← total time affected.

$= 40\text{ veh}$

3.3.2 M/M/1 Model (lots of derivation: P_j, L, L_q)

All formula to be derived:

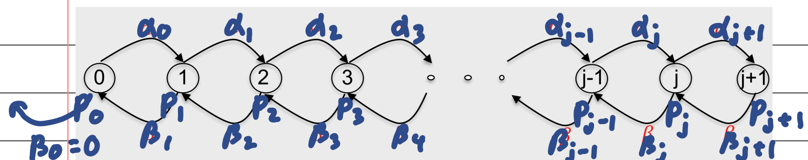
$P_j = \rho^j(1-\rho)$, where $\rho = \alpha/\beta$ (probability that there is j customers in the system)

$L = \frac{\rho}{1-\rho} = \frac{\alpha}{\beta-\alpha}$ (no. of customers in the system) ← service + queue

$L_q = \frac{\rho^2}{1-\rho} = \frac{\alpha^2}{\beta(\beta-\alpha)}$ (no. of customers in the queue)

all are average values.

1. Derivation of $P_j = \rho^j(1-\rho)$



Markov chain model for M/M/1

Steady state: expected no. of departure = expected no. of arrival

Assuming α and β doesn't depends on no. of customers in the system,

State 0: $\alpha_0 \times P_1 + \underbrace{\beta_0 \times P_0}_{0} = \beta_1 \times P_1$

(departure) (arrival)

$$P_1 = P_0\left(\frac{\alpha_0}{\beta_1}\right)$$

State 1: $\alpha_1 P_1 + \beta_1 P_1 = \alpha_0 P_0 + \beta_2 P_2$

$$P_2 = P_0\left(\frac{\alpha_0\alpha_1}{\beta_1\beta_2}\right)$$

hence: $$P_j = P_0\left(\frac{\alpha_0\alpha_1\alpha_2\ldots\alpha_{j-1}}{\beta_1\beta_2\beta_3\ldots\beta_j}\right)$$

and since: $\alpha_0 = \alpha_1 = \alpha_2 = \ldots = \alpha$ and $\beta_0 = \beta_1 = \beta_2 = \ldots = \beta$,

$$P_j = P_0\left(\frac{\alpha^j}{\beta^j}\right) = P_0\left(\frac{\alpha}{\beta}\right)^j = P_0\rho^j$$

to further simplify,

$P_0 = P_0\rho^0$

$P_1 = P_0\rho^1$

$P_2 = P_0\rho^2$

$\vdots$

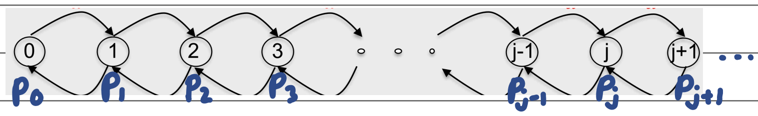
$P_j = P_0\rho^j$

$\vdots$

$$\sum_{j=0}^{\infty} P_j = P_0(\rho^0 + \rho^1 + \rho^2 + \ldots \rho^j + \ldots)$$

$$1 = P_0\left(\frac{1}{1-\rho}\right) \rightarrow P_0 = 1-\rho \qquad \therefore P_j = (1-\rho)\rho^j$$

2. Derivation of $L = \frac{\rho}{1-\rho}$



$$L = 0 \times P_0 + 1 \times P_1 + 2 \times P_2 + 3P_3 + \ldots$$

$$= \sum_{j=0}^{\infty} jP_j \quad \text{, where } P_j = (1-\rho)\rho^j$$

$$= \sum_{j=0}^{\infty} j(1-\rho)\rho^j$$

$$= (1-\rho)\underbrace{\sum_{j=0}^{\infty} j\rho^j}$$

$$\sum_{j=0}^{\infty} j\rho^j = 1\rho + 2\rho^2 + 3\rho^3 + \ldots$$

$$(-) \quad \rho\sum_{j=0}^{\infty} j\rho^j = \rho^2 + 2\rho^3 + 3\rho^4 + \ldots$$

$$\sum_{j=0}^{\infty} j\rho^j \times (1-\rho) = \rho + \rho^2 + \rho^3 + \ldots$$

$$= \rho(1 + \rho + \rho^2 + \ldots)$$

$$= \rho\left(\frac{1}{1-\rho}\right)$$

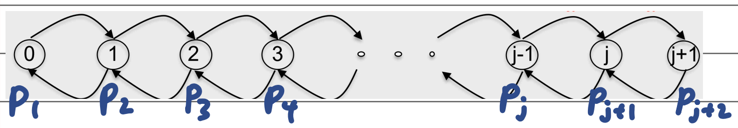
$$\sum_{j=0}^{\infty} j\rho^j = \frac{\rho}{(1-\rho)^2}$$

$$\therefore L = (1-\rho) \times \frac{\rho}{(1-\rho)^2} = \frac{\rho}{1-\rho}$$

to further simplify, $\rho = \alpha/\beta$

$$L = \frac{\alpha/\beta}{1-\alpha/\beta} = \frac{\alpha}{\beta-\alpha}$$

3. Derivation of $L_q = \frac{\rho^2}{1-\rho}$



☆ why for $L_q = 0$ (no. of customer in queue) the $P_j = P_1$?

Cause P_j is prob. having no. of customer in SYSTEM.

i.e. if $L_q = 0$, $\underbrace{\text{there's } L_s = 1}_{\text{no. of customer in service always } = 1 \text{ in M/M/1 model}}$, $\underbrace{\text{hence } L = 1}_{\text{relate to } P_j}$

$$L_q = 0 \times P_1 + 1 \times P_2 + 2 \times P_3 + \ldots$$

$$= \sum_{j=1}^{\infty} (j-1)P_j$$

$$= \underbrace{\sum_{j=1}^{\infty} jP_j}_{L} - \underbrace{\sum_{j=1}^{\infty} P_j}_{P_1 + P_2 + P_3 + \ldots = 1 - P_0}$$

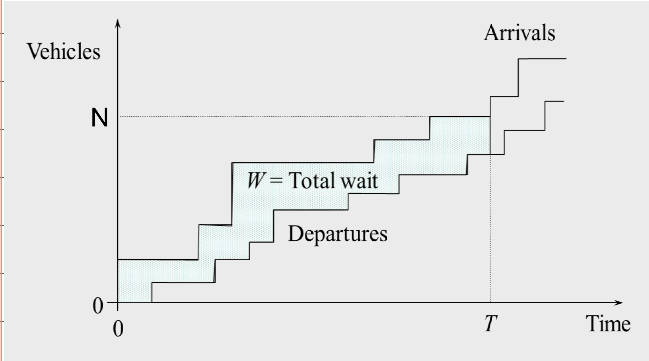
$$\therefore L_q = L - (1 - P_0) \quad \text{; } P_0 = 1-\rho \text{ from (1)}$$

$$= \frac{\rho}{1-\rho} - (1-(1-\rho))$$

$$= \frac{\rho}{1-\rho} - \rho = \frac{\rho - \rho(1-\rho)}{1-\rho} = \frac{\rho^2}{1-\rho} \quad //$$

Little's Formula (still for c3.3.2 M/M/1 Model)

→ to derive W and W_q from L and L_q



remember that in c3.1 we mentioned:

total area shaded = total wait time in queue.

$\frac{\text{total area shaded}}{\text{no. of vehicle, } N}$ = average wait time per vehicle in queue, W_q

$\frac{\text{total area shaded}}{\text{total time, } T}$ = average no. of vehicle in queue, L_q

equating total area shaded: $W_q \times N = L_q \times T$

note that this is not waiting time!

$$L_q = W_q \left(\frac{N}{T}\right)$$

$\frac{N}{T}$ is also α, arrival rate!

$\therefore L_q = W_q \alpha$ (average queue length = average waiting time per vehicle × average arrival rate)

we can also infer that:

$$L = W\alpha, \quad L_q = W_q \alpha, \quad L_s = W_s \alpha$$

also by substituting $L = \frac{\rho}{1-\rho}$ and $L_q = \frac{\rho^2}{1-\rho}$, and $\rho = \frac{\alpha}{\beta}$:

$$W = \frac{1}{\beta - \alpha}, \quad W_q = \frac{\alpha}{\beta(\beta - \alpha)}$$

and finally, by $L = L_s + L_q$ and $W = W_s + W_q$,

$$L_s = \rho, \quad W_s = \frac{1}{\beta}$$

all are important!

eg. (E3q3)

Q3: *A petrol station has one pump which can serve 15 vehicles per hour. Vehicles arrive at the station at a rate of 10 vehicles per hour. Both arrival and service rates are Poisson distributed. Estimate operational characteristics of the systems @service design.*
What is the probability that there are three vehicles in the queueing system?

$\left.\begin{array}{l}\beta = 15\text{ veh/hr}\\ \alpha = 10\text{ veh/hr}\end{array}\right\} \rho = \frac{\alpha}{\beta} = \frac{10}{15} = 0.67$

$P_j = \rho^j(1-\rho)$

$\therefore P_3 = 0.67^3(1-0.67) = 0.099$

(extras: we can find L, L_q, W, W_q (also L_s, W_s)

$L = \frac{\rho}{1-\rho} = \frac{0.67}{1-0.67} = 2$ vehicles

$L_q = \frac{\rho^2}{1-\rho} = 1.36$; $L = \alpha W$ $\quad$ $L_q = \alpha W_q$

$W = \frac{2}{10} = 0.2$ hrs $\quad$ $W_q = \frac{1.36}{10} = 0.136$ hrs)

c4. Microsimulation Models

- → Car following model (longitudinal)
- → Land changing model (lateral)

4.0 Introduction.

Simulation is a mathematical representation of a real process, solved using computational.

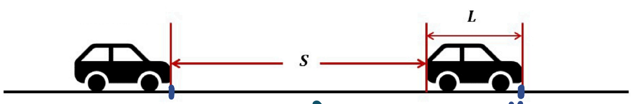
Microsimulation is a type of simulation that represent individual elements (eg. single vehicle, single pedestrians) and their interaction. It often includes stochastic (random) behaviours to model real world variability.

4.1 Car-following Model

- → Reuschel and Pipe Model
- → General Motor / GMR Model
- → Cellular Automata Model
- → Gipps' Model

Model how a vehicle follows the vehicle directly in front of it.

vehicle j+1 (following)   vehicle j (leading)



τ (reaction time)

x_{j+1} (position)	x_j
$\dot{x}_{j+1}$ (speed)	$\dot{x}_j$
$\ddot{x}_{j+1}$ (acceleration)	$\ddot{x}_j$

space headway $= x_j - x_{j+1}$

net space headway $= s = x_j - x_{j+1} - L$

4.1.1 Reuschel and Pipe's Model (1950s)

Concept: Minimum safe/net space headway increases linearly with speed.

$$x_j - x_{j+1} - L = k\dot{x}_{j+1}$$

$$\frac{d}{dt}\left(x_j - x_{j+1} - L = k\dot{x}_{j+1}\right)$$

$$\left.\begin{aligned}\dot{x}_j - \dot{x}_{j+1} &= k\ddot{x}_{j+1}\\ \dot{x}_j - \dot{x}_{j+1} &= \frac{1}{\lambda}\ddot{x}_{j+1}\end{aligned}\right\} \quad k = \frac{1}{\lambda},$$

where λ is a sensitivity factor, representing driver's responsiveness. (greater λ, greater $\ddot{x}_{j+1}$, make sense!)

$$\ddot{x}_{j+1} = \lambda(\dot{x}_j - \dot{x}_{j+1})$$

Pipe's Model: Follower's acceleration ($\ddot{x}_{j+1}$) is a reaction to difference in speed ($\dot{x}_j - \dot{x}_{j+1}$)

↓ add a reaction time lag, τ

$$\boxed{\ddot{x}_{j+1}(t+\tau) = \lambda[\dot{x}_j(t) - \dot{x}_{j+1}(t)]}$$

↓ generalise such that follower reacts to several preceding / ahead vehicles.

N=1

$$\ddot{x}_n(t+\tau) = \sum_{i=1}^{N} \lambda_i[\dot{x}_{n-1}(t) - \dot{x}_n(t)]$$

Reuschel and Pipe's Model is classify as a "Stimulus-Response Model"

$$\underbrace{\ddot{X}_n(t+\tau)}_{\text{Response}} = \sum_{i=1}^{N} \underbrace{\lambda_i}_{\text{Sensitivity}} \underbrace{[\dot{X}_{n-1}(t) - \dot{X}_n(t)]}_{\text{Stimulus.}}$$

Response = Sensitivity × Stimulus

What are the limitation of Reuschel and Pipe's Model?

if $\dot{X}_j - \dot{X}_{j+1} = 0$ (no difference in speed between leading and following vehicle),
$\ddot{X}_{j+1} = 0$ (leading vehicle won't decelerate)

This is true if X_j and X_{j+1} has a big gap. Imagine you and the car in front of you are travelling at the same speed. But there is 50 m between you and him. You won't brake! But if it is 1m gap, you would decelerate!

4.1.2 General Motor's (GM) / Gazis-Herman Rothery (GHR) Model

(more generalised stimulus-response model)
(upgraded Pipe's Model)

recall pipe's model:

$$\ddot{X}_{j+1}(t+\tau) = \lambda[\dot{X}_j(t) - \dot{X}_{j+1}(t)]$$

generalise λ:

$$\lambda = \frac{\lambda_0 \dot{X}_{j+1}(t+\tau)^m}{[X_j(t) - X_{j+1}(t)]^l}$$

$$\boxed{\ddot{X}_{j+1}(t+\tau) = \frac{\lambda_0 \dot{X}_{j+1}(t+\tau)^m}{[X_j(t) - X_{j+1}(t)]^l} \times [\dot{X}_j(t) - \dot{X}_{j+1}(t)]}$$

It is still a stimulus response model! "response = sensitivity × stimulus"

GM/GHR Model: Sensitivity now depends on follower's speed (exponent m) and the gap distance (exponent l)

Special case:

$m=0, l=0$: Pipe's Model

$m=0, l\neq 0$: Sensitivity inversely proportional to gap (drivers are more sensitive when closer)

$m\neq 0, l=0$: Sensitivity proportional to own speed (drivers are more sensitive when drives faster)

$m\neq 0, l\neq 0$: Sensitivity depends on both gap and own speed.

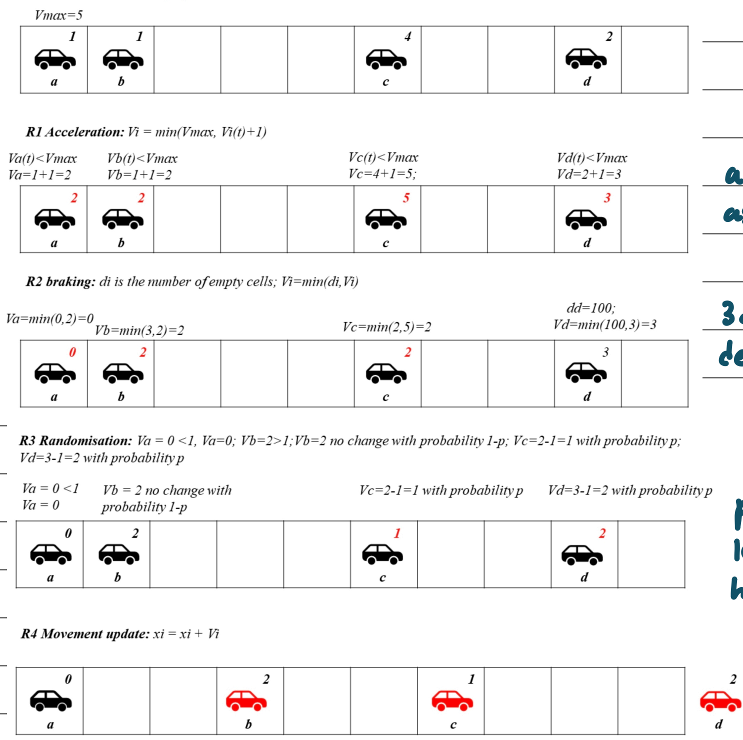
4.1.3 Cellular Automata (CA) —— Nagel & Schreckenberg (N-SCH) Model

Concept of a CA Model: Simplified Model that discretise space, time, and velocity (divide road into cells)

N-SCH Model's Rules

1. Accelerating : $v \to \min(v_{max}, v+1)$ (if not at max speed, speed up)
2. Braking : $v \to \min(d, v)$ (d = no. of empty cells ahead) (if $v > d$, reduce v to d)
3. Randomisation : $v \xrightarrow{p} \max(v-1, 0)$ (reduce v randomly by chance, but never $v < 0$)
4. Movement update : $x \to x + v$ (vehicle move forward by v cells)

eg.



all vehicle's v increased by 1 as none of it reaches v_{max} yet.

3 out of the 4 vehicles had to decrease v to d as $v > d$.

probability p is available, let's say in this case $p = 0.5$ half of the car randomly chose to "$v-1$"

update movement for each vehicle.

4.1.4 Gipp's Model

Concept: A "safety distance" or "collision avoidance" model. Each driver plans their speed to ensure they can safely stop if the vehicle in front brakes suddenly.

Speed for next time step, $v_n(t+\Delta t)$ is calculated as the minimum of two component:

(acceleration component) v_n^a : the speed based on the driver's desire to reach their preferred speed (V_n), within their maximum acceleration capabilities (a_n)

(braking component) v_n^b : the maximum safe speed that allows the vehicle to stop if the leader ($n-1$) brakes, considering the leader's position (x_{n-1}) and speed (v_{n-1}) and maximum deceleration (b_{n-1})

v_n^a and v_n^b formula:

$$v_n^a = v_n(t) + 2.5 a_n \Delta t (1 - v_n(t)/V_n)\sqrt{0.025 + v_n(t)/V_n}$$

$$v_n^b = b_n \Delta t + \{(b_n \Delta t)^2 - b_n[2(x_{n-1}(t) - s_{n-1}(t) - x_n(t)) - v_n(t)\Delta(t) - (v_{n-1}(t))^2/\hat{b}]\}^{1/2}$$

(b_n is follower's maximum deceleration)

$$v_n(t+\Delta t) = \min(v_n^a, v_n^b)$$

$$\boxed{x_n(t+\Delta t) = x_n(t) + 0.5\big(v_n(t) + v_n(t+\Delta t)\big) \times \Delta t}$$ Gipp's Model

dictionaries and some facts for Gipp's Model:

* a_n : maximum acceleration that the diver of vehicle n wishes to undertake
* b_n : maximum braking (deceleration) and $b_n < 0$
* s_n : minimum spacing (effective size of vehicle n, i.e. the physical length plus a margin into which the following vehicle is not willing to intrude)
* Δt: reaction time of drivers and common to all vehicles
* x_n: the location of the front of vehicle n at t
* V_n is the speed at which the driver of vehicle n wishes to travel
* $\hat{b}$ is the value of b_{n-1} estimated by the driver of vehicle n who cannot know this value from direct observation

a_n is sampled from a normal distribution, $N(1.7, 0.3^2)$ m/sec^2

$b_n = -2.0a_n$

s_n and V_n are sampled from normal populations, $N(6.5, 0.3^2)$ m and $N(20.0, 3.2^2)$ m/sec

$\hat{b} = Min(-3.0, 0.5(b_n - 3.0))$

4.2 Lane-changing Models

→ Process
→ Component
→ Models → MOBIL

4.2.1 Lane-changing Process / Decision.

1. Gipp's (1986) Framework
   - Is it possible to change lane? (Is there physical space?)
   - Is it necessary to change lane? (to exit or current lane is blocked)
   - Is it desirable to change lane? (to go faster)

2. Ahmed et al. (1996) Framework
   - Decision to consider a lane change.
   - Selection of target to lane change.
   - Gap acceptance in the target lane.

   } our focus

4.2.2 Key components in a Land-change Model. (Ahmed et al. Framework)

1. Decision to change.
   - can be triggered by car-following model.
   - eg: If car-following model calculates a negative acceleration ($\ddot{x}_{j+1} < 0$), it means following vehicle have to slow done, triggering a desire to lane change.

2. Land selection

- often modelled as a utility maximisation problem.
- driver perceives a "utility" (attractiveness) U_i for each lane i
- utility can be a function of speed, density, gap, etc.
- the probability of choosing lane i is given by a $\underbrace{\text{logit model}}$:

$$p(i) = \frac{e^{U_i}}{\sum_{i=1}^{n} e^{U_i}}$$

what is a logit model?
human are stupid. we can't just correctly predict each lane's utility score, U_i
what a logit model do is instead of saying:
$U_1 = 10$, $U_2 = 20$, $U_3 = 5$, we certainly choose lane-2;
instead:
we turn this into probabilities where choosing lane-2 is most probable but not certain (as a human)

eg. if $U_1 = 10, U_2 = 20, U_3 = 5$

$$p(1) = \frac{e^{10}}{e^{10}+e^{20}+e^{5}}$$

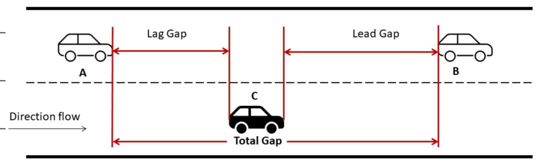
$$p(2) = \frac{e^{20}}{e^{10}+e^{20}+e^{5}}$$

$$p(3) = \frac{e^{5}}{e^{10}+e^{20}+e^{5}}$$

$p(2) > p(1) > p(3)$
lane-2 most probable to be chosen!

3. Gap acceptance

- after making the decision to change lane (1) and make the selection of lane (2), driver still have to find an acceptable gap in the selected target lane.
- this involve a lead gap and lag gap
- lead gap needs to be greater than its critical lead gap, and lag gap needs to be greater than its critical lag gap (and yet even if both are, is just a prob. that we will accept it!)



(btw, when we say gap we mean time gap! unit: s and not space gap!)

- probability of accepting a gap can be modelled as a function:

$$p(t) = \begin{cases} 1 - e^{-\delta(t-T)}, & \text{if } t > T \\ 0 & \text{otherwise.} \end{cases}$$

δ is a coefficient
T is the critical time gap
$t = SD/\dot{X}$ is the actual time gap
SD is the safe distance

notice that if t is just slightly greater than T, $t \approx T$
$p(t) \approx 1 - e^{-\delta(T-T)}$
$\approx 1 - e^{0}$
$\approx 1 - 1$
≈ 0
i.e. if the gap is small we still probably not accept it!

- hence, total probability of accepting a gap:

$P = P_{lead} \times P_{gap}$

4.2.3 MOBIL Lane-changing Model.

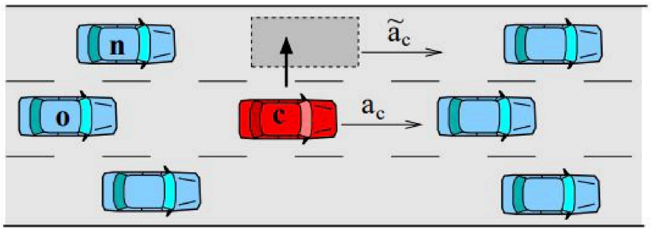
MOBIL stands for "Minimising Overall Braking Induced by Land Change"

Concept: Driver will change land if it provides an acceleration advantage and does not excessively inconvenience other drivers.

Decision rule: A land change will be made if

$$\underbrace{(\tilde{a}_c - a_c)}_{\text{my advantage}} + \underbrace{p \times (\tilde{a}_n - a_n + \tilde{a}_o - a_o)}_{\text{impact on others}} > \Delta a_{th}$$

- a_c = my current acceleration
- $\tilde{a}_c$: my acceleration in new lane
- a_n, a_o : neighbours' acceleration before I change lane
- $\tilde{a}_n, \tilde{a}_o$ = neighbours' acceleration after I change lane
- p = politeness factor ($p=0$: driver is extremely selfish; $p=1$: driver is extremely considerate)
- Δa_{th} = advantages threshold (driver only change if the advantages big and passes this threshold)



C5. Traffic Signal Control

5.0 Introduction (Not important)

Traffic control helps achieve planning objectives
→ minimise delay and cost
→ improve safety
→ protect environment
→ support policy goals.

- signs
- marking
- roundabouts
- priority junction
- traffic signals

Traffic signal is preferred when:
- Serious safety problems or strong conflict between users
- Gaps in major flow are too short for side road (cannot just junction)
- There's need to manage access / give priority (eg. bus/taxi priority)
- Policy aims to give more space / time to pedestrians / cyclist.

Signal Control Strategy

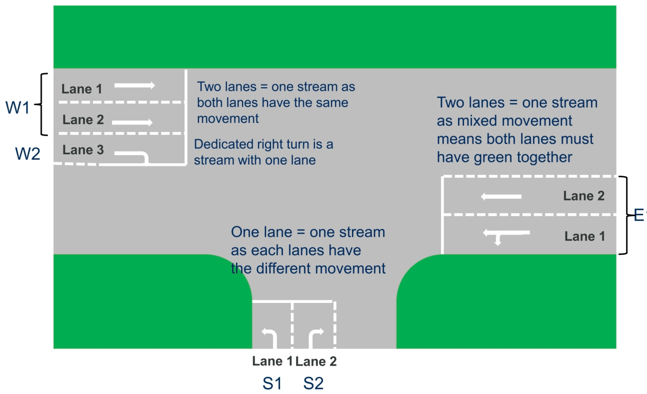
Fixed time (Offline)
- cycle time, c and effective green time, g_e calculated off-site, based on arrival rate, q and saturation flow, s

Traffic Response (Online)
- Vehicle actuated: timing respond to dectors at stop line
- Dynamic controlled / area wide: timing computed centrally depending on flows, densities, speeds, time and date.

5.1 Fixed Time (Offline)

5.1.1 Basic Concept and Terminology

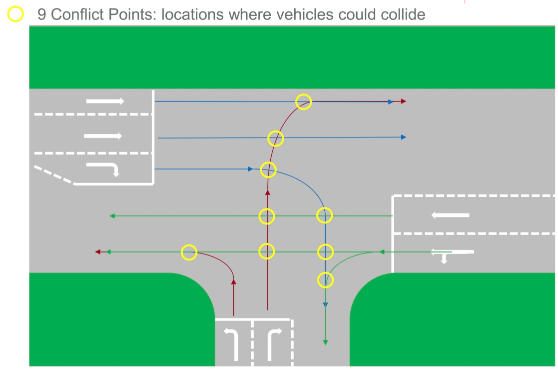
- **Movement**: a specific path through the junction (eg right turn from west to south).
- **Lane**: one physical lane, may serve one or more movements.
- **Stream**: the smallest unit of traffic for control, a queue of one or more lanes with the same movements.
- **Phase (group)**: set of streams that get the same signal display at the same time.
- **Stage**: period when the signal aspects are steady and a set of phases have green. (>2, <4)
- **Conflict**: when two phases cannot safely show green together.



W_1, W_2, S_1, S_2, E_1 all of these are streams.

eg for phase:
phase A: W_1, W_2
phase B: E_1
phase C: S_1, S_2

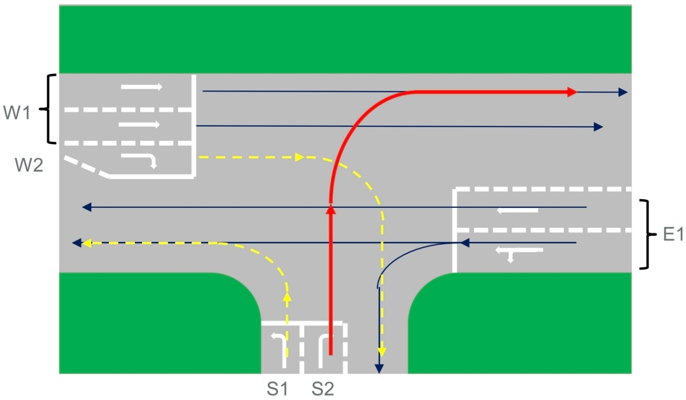
eg for stage:
stage 1: phase A, B
stage 2: phase C



these are conflict points.

Good junction layout should:

- Be safe, efficient and fair for all users.
- Use lane markings, stop lines and arrows to define movements.
- Provide enough queue storage.
- Ensure good visibility with primary and secondary signal heads.



- Traffic streams can be:

Compatible: No conflict area exists, can be released at the same time and be included in the same phase

Conflicting: Conflict area exists, cannot be included in the same phase

Semi-compatible: Conflict area exists, but can be included in the same phase subject to priority rules

semi-compatible is usually still been used as it can reduce the no. of stages in a plan. as long as it's safe

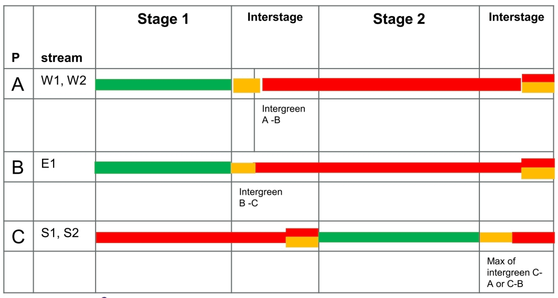
Phase	Option 1	Option 2	Option 3
A			
B			
C			Option 3 requires only 2 phases but has a semi-compatible movement.

Phase: A set of streams that receive identical signal indications at the same time (also called **group**).

although even if I want to choose plan 3...
It's better to split the phase for each segment then combine them in stages.

eg:

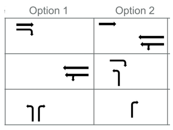
3 phases



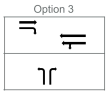
2 stages

why?
cause if we have multiple phase (eg. 3 phase), we have option to choose 2 stages or 3 stages.
(free choice)

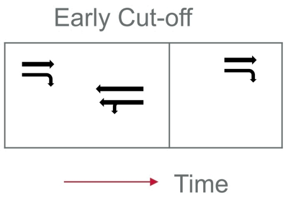
- **Right-turning streams** have many conflicts with other streams and require special treatment:
  - **Secured**: all semi-compatible streams are cut-off – Options 1 and 2



- **Not secured**: released together with semi-compatible streams if their intensity is low such that no congestion occurs in the junction: Option 3



- **Temporally secured**: release temporally shifted with respect to opposing semi-compatible streams through an early **cut-off** or a **late release**



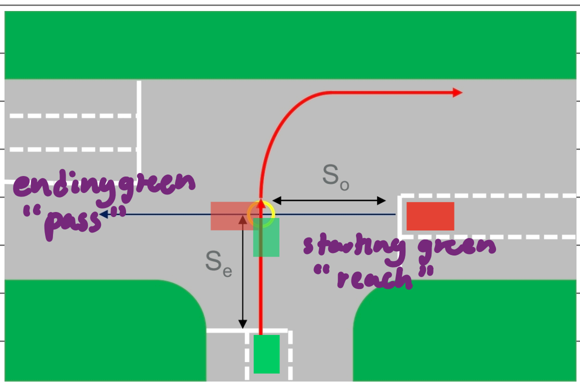
5.1.2 Clearance Time, t_z

Time gap between end of green (of clearing phase) and start of green (of next phase)

Purpose of clearance time: To ensure when one incompatible stream loses green and another gain green, vehicle won't collide.

$t_z = t_u + t_r - t_e$

- t_u: extra time for reactions (usually 3s)
- t_r: time for last vehicle in the clearing system to pass the conflict point.
- t_e: time from green onset till the first vehicle reaches the conflict point.



ending green "pass"

ending green (clearing stream)

starting green "reach"

starting green (next stream)

eg. $u = 10\,m/s$, $t_u = 3s$

$S_o = 13m$, $S_e = 11m$, length of vehicle, $l = 6m$

$u = \frac{s}{t} \rightarrow t = \frac{s}{u}$

$t_r = \underbrace{\frac{13+6}{10}}_{\text{add } l \text{ as need to 'pass' critical pt.}} = 1.9s$; $t_e = \frac{11}{10} = 1.1s$

$\therefore t_z = 3 + 1.9 - 1.1 = 3.8s$

these t_z can be filled into a spreadsheet known as clearance time matrix.

Phase Intergreen Times

FROM \ TO	A	B	C	D	E	F	G	H	I	J	K	L
A				6								
B			8	8								
C		7		6								
D	7	6	7									
E												
F												
G												
H												
I												
J												
K												
L												

Clearance time matrix

t_z can be used to find L, total time lost (which is used to find cycle time, c)

$L = \sum t_{z,crit}$

An important thing to note is if stage 1 contain phase A, phase B and stage 2 contain phase C, to find L we need to find t_z from stage 1 to 2 and stage 2 to 1

However for stage 1 to 2 there's $\underbrace{A-C}_{\text{has a } t_z}$, $\underbrace{B-C}_{\text{has a } t_z}$; stage 2 to 1 there's $\underbrace{C-A}_{\text{has a } t_z}$, $\underbrace{C-B}_{\text{has a } t_z}$

t_z for stage 1→2 is the bigger value (critical) amongst these two

t_z for stage 2→1 is the bigger value (critical) amongst these two

5.1.3 Key Variables of Signal Control

→ q, s
→ p_i
→ Y, L → c
→ g_k

Arrival rate, q : Demand flow of a stream (veh/hr or pcu/hr)

differences from veh/hr is pcu/hr take into acc vehicle types using equiv. factor (eg. bus=2, car=1)

Saturation flow, s : Maximum theoretical departure rate across stop line for a stream, during hypothetical hour of continuous green.

factors that can affect s:
- direction of traffic (eg. straight ahead, turn left, mixed)
- external factors (eg. land width, turning radius, slope)

Saturation flow estimated at **1800veh/hr** or calculated using the method in **Road Note 34**:

A histogram of the number of departures (profile) constructed using initial interval of 8 sec, then 6-sec intervals, and final interval of up to 9 sec;

Saturation flow is the average height of histogram excluding first and last interval

Road Note 34 also enables calculation of so-called **start lag** and **end lag**

Start lag: $l_f = 8 - n_f / s$

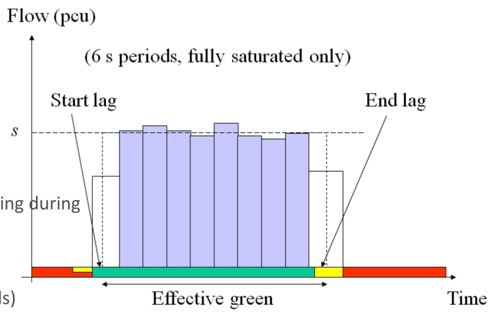
End lag: $l_l = n_l / s - (t_l - a)$

n_f, n_l: number of vehicles departing during first and last interval

t_l: duration of last interval

s: saturation flow

a: amber time (typically 3 seconds)



- Kimber et al (1986) formula:

$$s = \frac{2080 - 140\delta_n - 140\delta_G G + 100(w - 3.25)}{1 + 1.5\frac{f}{r}}$$

where
$\delta_n =$ 1 if nearside, 0 otherwise
$\delta_G =$ 1 if uphill, 0 otherwise
$G =$ gradient (%)
$w =$ lane width (m)
$f =$ proportion of turning traffic
$r =$ radius of turn (m)

effective green = actual green − start lag + end lag

Degree of Saturation

for a specific stream i, $p_i = \frac{c \cdot q_i}{g_i \cdot s_i}$ (c = cycle time, q_i = flow rate, g_i = effective green time, s_i = saturation flow)

degree of saturation, of a stage k: (stage, not phase)
take the maximum p among its stream (critical degree of saturation)

design aim:
- $p < 1$ for all stages (otherwise queue grows without bound)
- but because of randomness, aim for $p < 0.9$ in practise

} if any stage's $p_k > 1$, we can combine more phases into one stage.

Cycle time (using Webster's method)

Define Y
- for each stream, find q/s ratio.
- for each stage k, find the critical / highest $(q/s)_{k,crit}$
- sum over all stages: $Y = \sum_k (q/s)_{k,crit}$

Define L
- sum of all interstage lost time
- $L = \sum(\underbrace{\text{max intergreen}}_{\text{max } t_z \text{ of phase pair}} + \underbrace{\text{lag time}}_{\sim 2s} - \underbrace{\text{amber time}}_{\sim 3s})$

  ($t_z = t_u + t_r - t_e$)

(eg. stage 1: phase A, B
stage 2: phase C,

A-C → $t_{z,AC}$
B-C → $t_{z,BC}$ } max intergreen is max of these two)

} BOTH DEPENDS ON STAGE DESIGN

Finally after defining Y and L, find cycle time, c:

Minimum feasible cycle time	Best (optimum) cycle time	Maximum cycle time (for $p=0.9$)
$c_{min} = \frac{L}{1-Y}$	$c = \frac{1.5L + 5}{1-Y}$	$c_{0.9} = \frac{0.9L}{0.9-Y}$

If possible choose from the optimum case
typically: $60 < c < 90s$, limits: $25 < c < 120s$

the only time we need to find c_{min}, $c_{0.9}$ is when $c_{opt} < 25$ or $c_{opt} > 120$
then we have to adjust our c based on the limit $25 < c < 120$.
however we need to check if the new c ($c = 25$ or $c = 120$) is between $c_{min} < c < c_{0.9}$

Effective Green Time

for a specific stage k,

Minimum feasible effective green time	Best (Optimum) effective green time	Maximum effective green time (for $p = 0.9$)
$g_k = c\left(\frac{q}{s}\right)_{k,crit}$	$g_k = \frac{c-L}{Y} \cdot \left(\frac{q}{s}\right)_{k,crit}$	$g_k = \frac{c}{0.9} \cdot \left(\frac{q}{s}\right)_{k,crit}$

if possible choose from the optimum case,

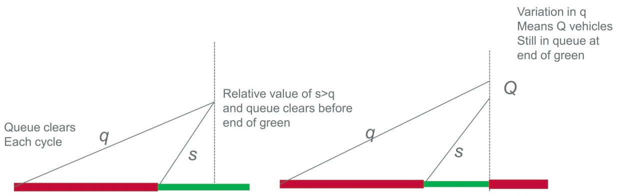
typically: for normal traffic stage, $g_k \geq 7s$

Average Delay

per vehicle for a specific stream i, $w_i = \underbrace{\frac{0.45 s_i (c - g_i)^2}{c(s_i - q_i)}}_{\text{uniform}} + \underbrace{\frac{Q_i}{q_i}}_{\text{random}}$.

where residual queue at end of the green,

$$Q_i = \frac{0.5 \rho_i^2}{1 - \rho_i}$$



$p < 1$

- note that this w_i is per vehicle that's why even if $q < s$, there might be chances that there's a residual queue. Yes, on average as long as $q < s$ there shouldn't be any queue but if per vehicle, there's chance.

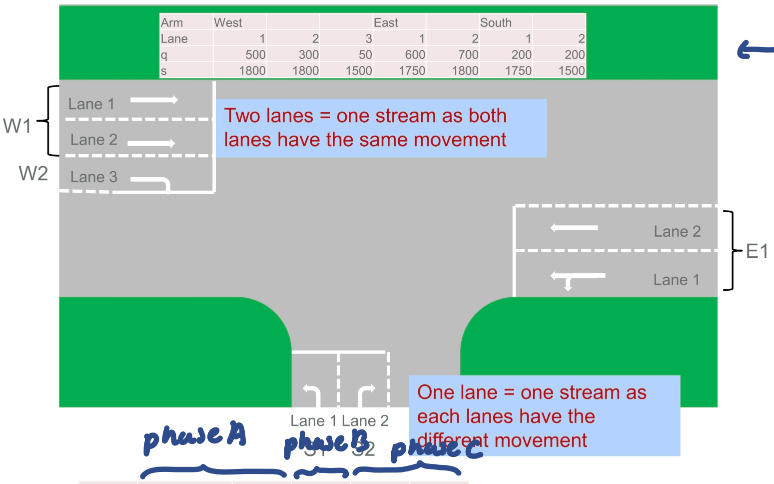
Designing Fixed Time Signal.

5.1.1 Basic Concept (Design)
5.1.2 Clearance Time
5.1.3 Key variables

1. For each lane, consider q (flow) and s (saturation flow)
2. Define streams, by grouping lanes of the same direction
3. For each **stream**, calculate the ratio q/s
4. Define signal stages, a stage being a group of streams obtaining green at the same time
5. For each stage k, find the critical stream (**stream with the maximum q/s ratio**) and store the corresponding q/s value - $(q/s)_{k,crit}$
6. Calculate $Y = \Sigma_k (q/s)_{k,crit}$
7. If $Y \geq 1$, go back to 4 and re-define stages.
   If $Y < 1$, proceed to 8.

   calc. L
8. Calculate the cycle time using Webster's formula: $c = (1.5L + 5) / (1 - Y)$
9. For each stage k, calculate the green time: $g_k = (c - L)\,(q/s)_{k,crit} / Y$
10. For each stage k, calculate the degree of saturation: $\rho_k = c\,(q/s)_{k,crit} / g_k$
11. If $\rho_k \geq 1$, go back to 4 and re-define stages

Worked Example.



Arm	West			East		South	
Lane	1	2	3	1	2	1	2
q	500	300	50	600	700	200	200
s	1800	1800	1500	1750	1800	1750	1500

$s = 1800$ (high, if only straight)
$s = 1750$ (mid, if mix)
$s = 1500$ (low, if turn)

phase A phase B phase C

S1 S2

Stream	W1	W2	E1	S1	S2
q	800	50	1300	200	200
s	3600	1500	3550	1750	1500
q/s	0.222	0.033	0.366	0.114	0.133

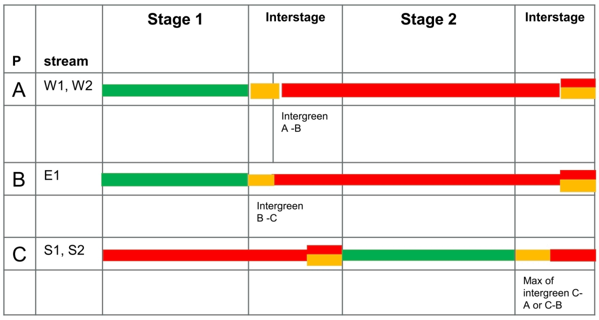
stage 1 stage 2

Critical value Stage 2
Critical value Stage 1

Y = 0.366 + 0.133 = 0.499

Option 3 phasing with 2 stages

Time



Y = 0.366 + 0.133 = 0.499
This is less than 1 so we can proceed to estimate cycle time.

Clearance time matrix

		Starting Green		
		A	B	C
Ending green	A		7s	9s
	B	0s		6s
	C	15s	4s	

L = 15+9 + 2*(2-3) = 22s

$c = (1.5L + 5) / (1 - Y) = (1.5*22+5)/(1-0.499) = 38/0.501 = 75.9 = 76s$

	crit q/s	g	Deg Sat
Stage 1	0.366	40	0.70
Stage 2	0.133	14	0.70

$(g_k ? 7) \; (\rho < 0.9)$

$g_k = (c - L)\,(q/s)_{k,crit} / Y$

$\rho_k = c\,(q/s)_{k,crit} / g_k$

✱ to find L we need $t_{z,crit}$ between all stages

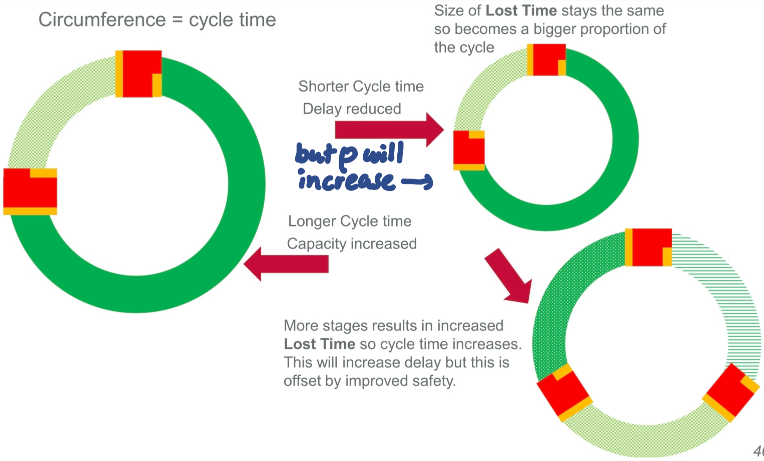
stage 1 → stage 2:
A → C ($t_z = 9$)
B → C ($t_z = 6$)
$t_{z,crit} = 9$

stage 2 → stage 1:
C → A ($t_z = 15$)
C → B ($t_z = 4$)
$t_{z,crit} = 15$

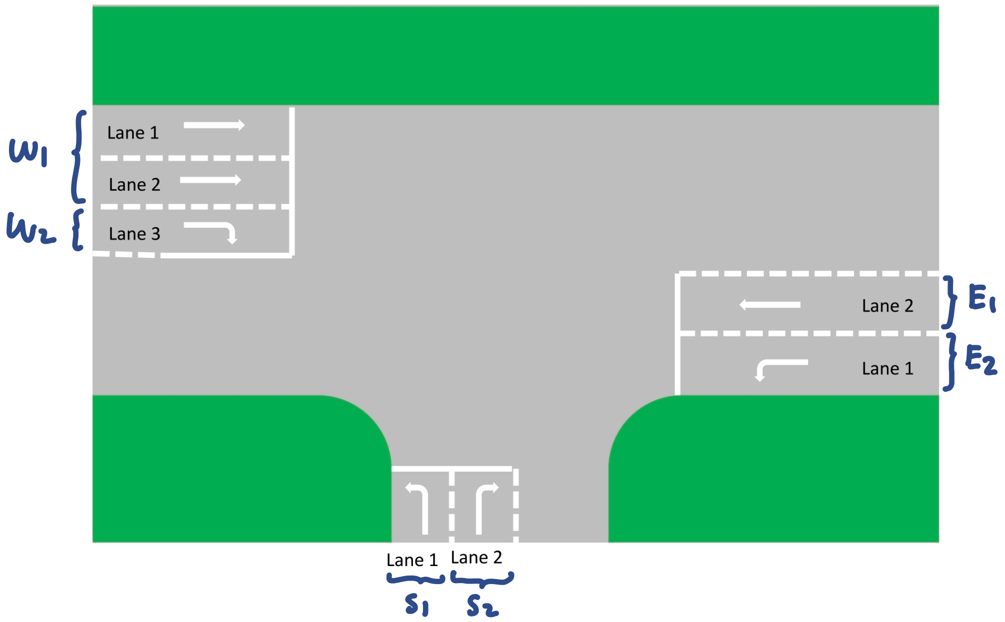
$L = 15 + 9 - 1 - 1 = 22s$

extra knowledge:

Circumference = cycle time



eg (ESq1) a) Identify the streams



b) Given

Arm	West			East		South	
Lane	1	2	3	1	2	1	2
q	500	300	50	600	700	200	200
s	1800	1800	1500	1750	1800	1750	1500

Total Lost time = 22 seconds (L)

The signal plan allows for semi-compatible turns and has two stages:

Stage 1 : All West and East streams
Stage 2 : All South streams

Questions:

i. Calculate minimum feasible cycle time
ii. Calculate 90% saturation cycle time
iii. Calculate Websters Best cycle time

(b) to find c we need Y and L (given) first:

$$Y = \sum_{\forall k} \left(\frac{q}{s}\right)_{k,crit}, \text{ where } k \text{ are all stages.}$$

Arm	West			East		South	
Lane	1	2	3	1	2	1	2
q	500	300	50	600	700	200	200
s	1800	1800	1500	1750	1800	1750	1500

$$\frac{q}{s}: \underbrace{\frac{800}{3600}, \frac{50}{1500}, \frac{600}{1750}, \frac{700}{1800}}_{\text{stage 1}}, \underbrace{\frac{200}{1750}, \frac{200}{1500}}_{\text{stage 2}}$$

$$\frac{q}{s_{crit}}: \quad \frac{700}{1800} \approx 0.389 \quad \frac{200}{1500} \approx 0.133$$

$$Y = \sum \left(\frac{q}{s}\right)_{k,crit} = 0.389 + 0.133 = 0.522 < 1 \text{ (good)}$$

i. $C_{min} = \frac{L}{1-Y}$

$= \frac{22}{1-0.522}$

$= 46\,s$

ii. $C_{0.9} = \frac{0.9L}{0.9-Y}$

$= \frac{0.9(22)}{0.9-0.522}$

$= 52.4\,s$

iii. $C_{opt} = \frac{1.5L+5}{1-Y}$

$= \frac{1.5(22)+5}{1-0.522}$

$= 79.5\,s$

c) Calculate the Delay per vehicle for each Stream for the Webster Best cycle time and confirm the timings are feasible.

$$w_i = \frac{0.45 s_i (c-g_i)^2}{c(s_i - q_i)} + \frac{Q_i}{q_i} \quad \text{where} \quad Q_i = \frac{0.5\rho_i^2}{1-\rho_i}, \quad \rho_i = \frac{c \cdot q_i}{g_i \cdot s_i}$$

we already have s_i, q_i, c, we just need to find g_i

$$g_k = \frac{c-L}{Y} \cdot \left(\frac{q}{s}\right)_{k,crit} \quad \text{(use the optimum's version)}$$

Arm	West			East		South	
Lane	1	2	3	1	2	1	2
q	500	300	50	600	700	200	200
s	1800	1800	1500	1750	1800	1750	1500

$$g_k = \frac{79.5-22}{0.522}(0.389) = 42.85 \qquad g_k = \frac{79.5-22}{0.522}(0.133) = 14.75$$

Arm	West			East		South	
Lane	1	2	3	1	2	1	2
q	500	300	50	600	700	200	200
s	1800	1800	1500	1750	1800	1750	1500
ρ_i	0.412		0.062	0.635	0.92	0.617	0.72
Q_i	0.144		0.002	0.553	0.928	0.498	0.928
w_i	10.4		3.4	9.0	11.0	22.1	28.2

(all $\rho < 0.9$ good!)

TIPS:
ANY PARAMETER THAT ENDS WITH "i" IS STREAM'S AND WITH "k" IS STAGE'S

eg. q_i, s_i is something each stream are different from other stream.
But when combined to $\left(\frac{q}{s}\right)_{k,crit}$ one stage has only one of that.
so when finding g_k, each stage has one g_k.
BUT, when using g_k to find ρ_i, Q_i and w_i, all these parameter are stream's variable!

d) A planned new development has been modelled to potentially increase in traffic from West:

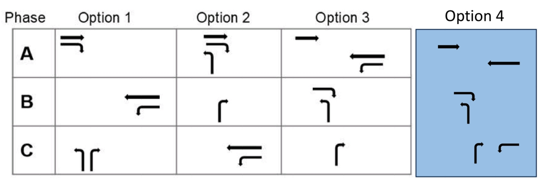
Lane 1 900 vph

Lane 2 900 vph

Lane 3 400 vph

At the same time the Highway Authority is implementing a road safety policy that means that semi-compatible turns are not allowed.

Propose some alternative signal plans and determine whether or not they are feasible.



objective: find if Y for each option < 1

step 1: find $\frac{q}{s}$ for each stream

step 2: find $Y = \Sigma\left(\frac{q}{s}\right)_{k,crit}$ and check if $Y < 1$

	Arm	West			East		South		
	Lane	1	2	3	1	2	1	2	
Lanes	q	900	900	400	600	700	200	200	
	s	1800	1800	1500	1750	1800	1750	1500	
Streams		W1		W2	E1	E2	S1	S2	
	q	1800		400	600	700	200	200	
	s	3600		1500	1750	1800	1750	1500	
	q/s	0.500		0.267	0.343	0.389	0.114	0.133	
									Y
(W)+ (E) +(S)	Crit		0.500		0.389		0.133		**1.022**
(W&S1) +(S2) +(E)	Crit	0.500				0.389		0.133	**1.022**
(W1&E) +(W2&S1) +(S2)	Crit	0.500		0.267				0.133	**0.900**

Third option is feasible (just). Because East is combined with a stream (W1) that has a higher degree of saturation this makes the overall value of Y lower as East is not running alone.

Extra point (not asked for)
However, assuming Lost Time does not increase, cycle time calcs shows 220-380s cycle time which is not acceptable.

c5.2: Traffic Signal Control 2

1. Vehicle Actuated (VA) Signals

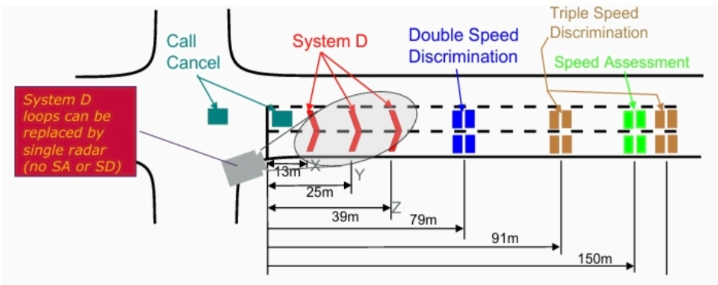
Standard control for isolated junctions.

A. Fundamental Concept (EXAM)

- **Principle:** Signals adjust green times based on real-time vehicle detection.
- **Limits:** Operation is constrained between pre-set **Minimum Green** and **Maximum Green** times.
- **Logic:**
  - **No Traffic:** Junction runs at Minimum Green.
  - **High Traffic:** Junction extends to Maximum Green.
  - **Gap Out:** If a phase is green and no vehicle is detected for a specific duration (e.g., >2s), the signal "gaps out" (terminates green) to serve waiting traffic on other arms.

~~B.~~ Detection Layouts

- **System D (Standard):** Uses 3 loops per lane (X, Y, Z) located at the stop line, 12m, and 25m upstream.
  - Registers demand and extends green.
- **High-Speed Roads (>35mph / 56km/h):**
  - **Problem:** Drivers approaching a green light at speed face a "Dilemma Zone" (too close to stop safely, too far to clear intersection before red).
  - **Solution:** Add **Speed Discrimination** loops upstream.
  - **Double Speed Discrimination:** Uses a pair of loops further back to detect fast vehicles and extend the green time specifically to allow them to clear the stop line safely.



2. MOVA (Microprocessor Optimised Vehicle Actuation)

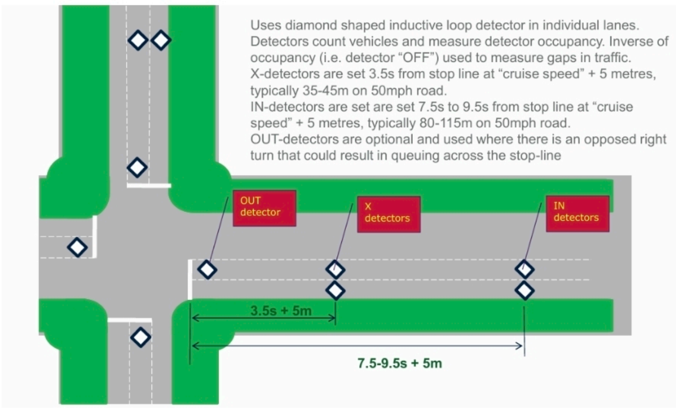
Advanced isolated control. Frequent exam topic for sketches and mode comparisons.

~~A.~~ Detector Layout

You must be able to draw a single lane approach with the following specific distances:

1. **IN-Detectors: * Location: 80m – 115m** upstream (approx. 8 seconds travel time at cruise speed).
   - **Function:** Counts vehicle inputs and measures cruise speed.
2. **X-Detectors:**
   - **Location: 35m – 45m** upstream (approx. 3.5 seconds travel time).
   - **Function:** Tracks the end of the queue and identifies gaps.

3. **Lane Data:** Detectors are typically diamond-shaped and lane-specific.



B. The Two Modes of Operation (EXAM)

MOVA is "unstable" (in a good way)—it switches modes based on saturation.

Mode 1: Delay Minimisation (Under-Saturated)

- **Condition:** Queue clears every cycle.
- **Logic:** Calculates a **Benefit vs. Disbenefit** equation.
  - *Benefit:* Reduced delay/stops for moving vehicles on the current green.
  - *Disbenefit:* Increased delay for vehicles waiting at red lights on conflicting approaches.
- **Decision:** If Benefit > Disbenefit ⟶ Extend Green. Else ⟶ Change Stage.

Mode 2: Capacity Maximisation (Over-Saturated)

- **Condition:** Queue extends back to the IN-detector (does not clear).
- **Goal:** Ignore delay; maximize throughput.
- **Logic:** Runs the green as long as traffic maintains **Saturation Flow**.
- **Decision:** Measures "flow efficiency" every 0.5s. If efficiency drops (gaps appear or speed drops), it changes stage immediately to use the lost capacity on another arm.

~~C.~~ Where is MOVA Beneficial?

Memorise this list for 10-mark questions:

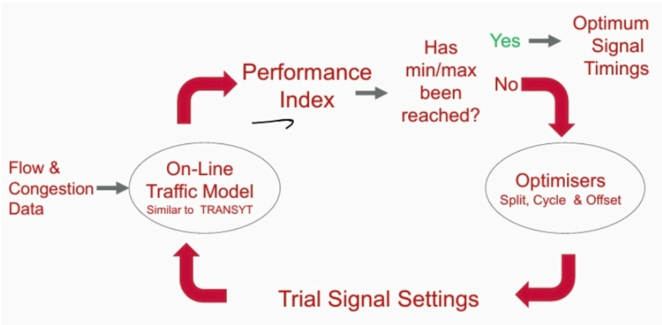
1. **Variable/Seasonal Flow:** Sites where traffic patterns fluctuate wildly (e.g., holiday routes, motorway diversions) where fixed plans fail.
2. **Capacity Issues:** Junctions that are overloaded/congested under standard VA control.
3. **High-Speed Roads:** Sites >35mph where safety and red-light compliance are issues (MOVA manages the dilemma zone better than VA).

3. SCOOT (Split Cycle Offset Optimisation Technique) (EXAM)

Adaptive Network Control. Focus on the flow of data and the three optimisers.

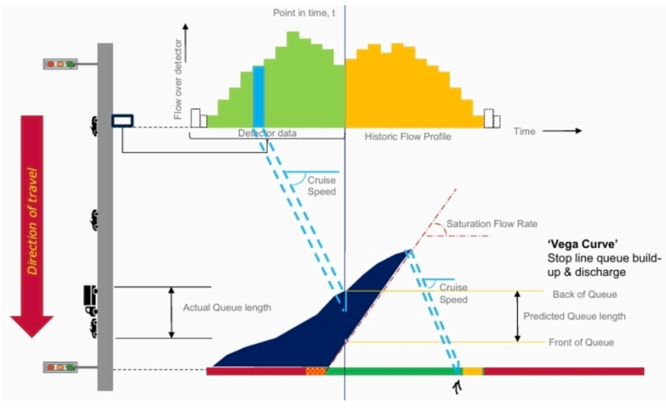
A. Architecture & Data Flow

- **Acronym:** Split, Cycle, Offset Optimisation Technique.
- Flow Hierarchy:
  1. Detectors (Input real-time data)
  2. UTC Computer (Runs the Traffic Model)
  3. Optimisers (Calculate adjustments)
  4. Signal Timings (Output to street)



B. The Traffic Model (Vega Diagram)

- **Concept:** SCOOT cannot "see" the queue; it predicts it.
- **Cyclic Flow Profile (CFP):** Data from upstream detectors (vehicles arriving at cruise speed).
- **Queue Prediction:** Combines real-time CFP with "Historic Data" to predict:
  - **Back of Queue:** Where the queue ends.
  - **Front of Queue:** Vehicles discharging at saturation flow.
- **Visual:** This forms the "Vega Diagram" (a time-space diagram).



C. The Three Optimisers (Memorise the Table)

Optimiser	Scope	Frequency	Objective & Method
Split Optimiser	**Node** (Single Junction)	Once per **stage**	**Obj:** Minimize congestion/delay at that specific junction. **Method:** Balances the **degree of saturation** across all arms. **Action:** Adjusts green time by **±4s** (often dampened to ±1s for stability). **Constraints:** Cannot violate minimum green (safety) or max cycle time.
Offset Optimiser (EXAM)	**Link** (Between Junctions)	Once per **cycle**	**Obj:** Minimize stops and interruptions. **Method:** Adjusts the start time of green relative to the upstream junction. **Action:** Creates a **"Green Wave"** so platoons arrive at green.
Cycle Optimiser (EXAM)	**Region** (Whole Area)	Every **2.5 - 5 mins**	**Obj:** Minimize region-wide delay. **Method:** Identifies the **"Critical Node"** (the most saturated junction). **Action:** Sets a common cycle time for the region to keep that node at **90% saturation** (leaving 10% reserve capacity).

D. Comparison Summary

- **SCOOT** = Network Control (Coordinating many junctions).
- **MOVA** = Isolated Control (Optimising one junction perfectly).

c6: Motorway Management Systems (EXAM)

1. Ramp Metering

Regulating flow from the slip road (ramp) onto the main carriageway.

A. Purpose & Benefits

- **Primary Goal:** Maximise throughput on the main carriageway by keeping flow just below the breakdown density.
- **Mechanism:** Breaks up platoons of vehicles entering from the slip road.
- **Recovery:** Helps traffic recover faster if flow breakdown has already occurred.

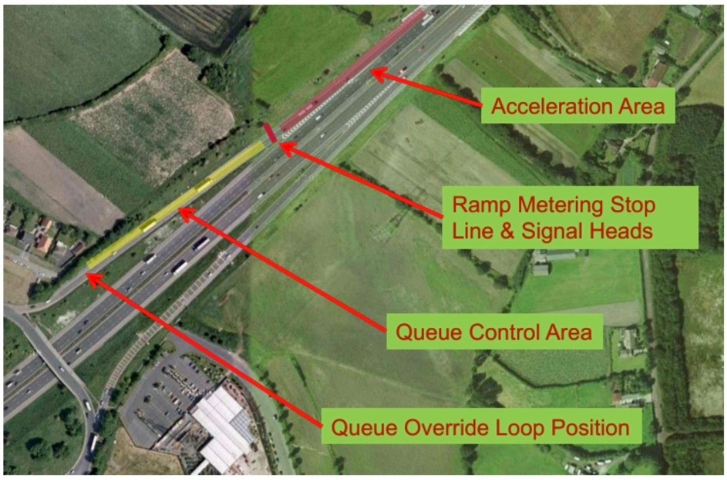
B. Operation Styles

- **US Style:** Fast metering (1-2 vehicles per green).
- **European/UK Style:** Platoon metering (releases small groups based on ramp queue length).

C. The Layout

You should be able to describe or sketch the UK layout:

1. **Main Carriageway Loops:** Upstream and Downstream detection (MIDAS).
2. **Ramp Signals:** Standard traffic lights at the bottom of the ramp.
3. **Stop Line:** Where vehicles wait.
4. **Queue Override Loop:** Located at the top of the slip road. **Crucial Safety Feature:** If the queue backs up to the junction entrance, metering is released to prevent blocking the local road network.



D. Algorithms

- **Demand-Capacity (Feed-Forward):** Measures upstream flow and calculates remaining capacity. Simple but can be inaccurate.
- **ALINEA (Feedback):** The gold standard.
  - Uses **Downstream Occupancy** (feedback) to adjust the metering rate.
  - If downstream occupancy > target (critical) occupancy $\rightarrow$ Reduce ramp flow.

2. Queue & Congestion Management

Two distinct systems often used together: HIOCC for queues (safety) and MIDAS for congestion (smoothing).

A. Queue Protection (HIOCC Algorithm)

- **Objective:** Prevent rear-end collisions by warning of stopped traffic ahead.
- **Detection (The "2 Seconds" Rule):**
  - Monitors inductive loops every 0.1s.
  - **Trigger:** If a detector has **100% occupancy for 2 consecutive seconds** (indicating a stationary or very slow vehicle), an Alarm is raised.
- **Response:**
  - Sets **40 mph** on the signal immediately upstream.
  - Sets **60 mph** on signals further upstream.
  - Displays "QUEUE AHEAD".
- **Removal:** Uses "Smoothed Occupancy". The alarm is only cleared when the smoothed occupancy drops below a threshold (preventing flickering on/off).

B. Congestion Management (MIDAS)

- **Objective:** Variable Mandatory Speed Limits (VMSL) to stabilise flow and delay breakdown.
- **Logic (The "Hysteresis" Rule):**
  - **To Lower Speed Limit:** Triggered if Flow increases **OR** Speed decreases (Fast reaction).
  - **To Raise Speed Limit:** Triggered ONLY if Flow decreases **AND** Speed increases (Slow reaction).
  - *Why?* To prevent "oscillation" (speeds jumping up and down) which causes stop-start waves.

3. Dynamic Hard Shoulder (DHS) Running

The first generation of "Smart Motorways".

A. Concept of Operations

- The Hard Shoulder is opened as a running lane **temporarily** during peak times or congestion.
- **Safety Requirement:** Requires **100% CCTV coverage** of the Hard Shoulder. An operator must visually scan the entire length to ensure no vehicles are broken down before opening the lane.

B. Infrastructure (Heavy)

- **Gantries:** Full-span gantries are required frequently to display lane arrows/Red X over the hard shoulder.
- **Process:**
  1. Detect high flow.
  2. Lower speed limit (e.g., 60mph).
  3. Operator checks CCTV.
  4. Open Hard Shoulder (Speed limit applies to all lanes).

4. All Lane Running (ALR)

The current standard for Smart Motorways.

A. Key Differences from DHS

- **Permanent:** The Hard Shoulder is **permanently** converted into a running lane (Lane 1).
- **Infrastructure (Light):**
  - Uses **Cantilever signs** (side-mounted) instead of full gantries (cheaper).
  - **Emergency Refuge Areas (ERAs):** Orange-surfaced lay-bys spaced every 1.5 miles (ideally closer, e.g., 0.75 miles in new standards).
- **Detection:** Relies heavily on **Radar** (Side-fire radar) rather than loops for vehicle detection.

B. Pros & Cons

- **Pros:** Cheaper to build, provides constant capacity increase.
- **Cons:** No continuous safe place to stop. Relies on "Red X" compliance for safety if a vehicle breaks down in a live lane.

5. Issues with Smart Motorway Operations (SVD)

This is the "Critical Analysis" part of the exam.

A. Stopped Vehicle Detection (SVD)

- **The Problem:** In ALR, if a vehicle breaks down in Lane 1, there is no hard shoulder. It sits in live traffic.
- **The Solution:** SVD (Radar systems) designed to automatically detect a stationary object in a live lane.
- **The Risk:** If SVD fails or is slow, it takes time for the Control Centre to set a **Red X**. The time gap between breakdown and Red X is the period of highest danger.

B. Operational Issues

1. **Red X Compliance:** Drivers ignoring the Red X lane closure signal (now a criminal offence).
2. **Resources:** DHS requires significant operator time to open/close lanes; ALR reduces this but increases reliance on automated detection.
3. **Maintenance:** High technology dependence. If the radar/CCTV fails, the safety case for the motorway is compromised.

c7: Road Safety (EXAM)

1. Safe Systems Concept

The modern, proactive approach to road safety: moving from blaming users to designing safer systems.

A. Core Philosophy (The 4 Principles)

1. **Human Fallibility:** People make mistakes. Errors are inevitable and the system should account for them.
2. **Human Vulnerability:** The human body is fragile. It has a limited tolerance to crash forces (kinetic energy).
3. **Shared Responsibility:** Safety is not just the driver's job. It is shared between road users and system designers (engineers, vehicle manufacturers, policymakers).
4. **Zero Harm Goal:** The only acceptable ethical goal is zero fatalities and serious injuries.

B. The 5 Pillars (The "Swiss Cheese" Model)

These layers work together. If one fails, the others provide protection.

1. **Safe Roads & Roadsides:** Forgiving design (e.g., barriers, clear zones) that minimizes severity if a vehicle leaves the road.
2. **Safe Speeds:** Limits appropriate for the road environment and potential conflicts.
3. **Safe Vehicles:** Technology that prevents crashes (e.g., Autonomous Emergency Braking) or protects occupants (e.g., crumple zones).
4. **Safe Road Users:** Education, licensing, and enforcement to encourage compliance and alertness.
5. **Post-Crash Response:** Fast emergency response and high-quality trauma care to save lives *after* a crash.

2. Elements of Road Safety

Practical measures and infrastructure used to improve safety for different road users.

A. Hierarchy of Road Users

- **Concept:** Prioritise the most vulnerable.
- **Order: Pedestrians** $\rightarrow$ **Cyclists** $\rightarrow$ **Horse Riders** $\rightarrow$ **Motorcyclists** $\rightarrow$ **Cars/HGVs**.
- **Rule:** Those who can do the greatest harm (large vehicles) bear the greatest responsibility.

B. Pedestrian Facilities

- **Physical Separation:** Dedicated footpaths (removes conflict but expensive).
- **Zebra Crossings:** Markings and beacons. Relies on driver compliance. Effective at lower flows.
- **Signal Controlled Crossings:**
  - **Pelican:** Traditional push-button. Uses *flashing amber* for vehicles (ambiguous priority).
  - **Puffin:** Smart crossing with detectors.
    - *Kerbside Detector:* Cancels demand if pedestrian walks away.
    - *On-Crossing Detector:* Extends red light for slower pedestrians.
    - *No Flashing Amber:* Removes ambiguity for drivers.
  - **Toucan:** "Two-can" cross. Shared crossing for pedestrians and cyclists (wider, dedicated signals).

C. Cyclist Facilities

- **Physical Segregation:** Dedicated cycle tracks separated from traffic (gold standard).
- **Advanced Stop Lines (ASL):** Boxes at traffic lights allowing cyclists to wait *ahead* of cars (improves visibility).
- **Sparrow Crossing:** Parallel signalised crossing that keeps cyclists separate from pedestrians.

D. Speed Management

- **Limit Selection:** Based on crash history, road geometry, and function.
- **Engineering Measures:**
  - *Vertical:* Speed humps, cushions.
  - *Horizontal:* Chicanes, road narrowing.
  - *Psychological:* Visual clues, removing centre lines, changing surface colour.
- **20mph Zones:** Often self-enforcing through physical traffic calming measures.

E. Low Traffic Neighbourhoods (LTNs)

- **Concept:** Stop "rat-running" (through-traffic) in residential areas.
- **Mechanism:** Use **modal filters** (bollards, planters, cameras).
- **Result:** Residents maintain access, but driving *through* the area as a shortcut is restricted. Creates safer streets for walking/cycling.

F. Road Safety Audits

- **Definition:** A formal evaluation of a highway improvement scheme.
- **Stages:**
  1. **Preliminary Design:** Before planning permission.
  2. **Detailed Design:** Before tender.
  3. **Construction Completion:** Prior to opening.
  4. **Post-Opening:** Monitoring (12 months after opening).

c9: Safety Modelling

~~1.~~ Development of the Model: Poisson Regression

Why use it? Traffic accidents are "count data" (non-negative integers: 0, 1, 2...). Standard linear regression (OLS) is unsuitable because it can predict negative accidents and assumes constant variance.

A. The Poisson Distribution

- **Assumption:** The number of accidents (Y) occurring in a fixed time/space follows a Poisson distribution.
- Key Characteristic (Equidispersion): The Mean (μ) is equal to the Variance (σ^2).

$$E(Y) = Var(Y) = \mu$$

- Probability Mass Function (PMF):
  The probability of observing exactly y accidents given a mean rate μ is:

$$P(Y = y|\mu) = \frac{e^{-\mu} \cdot \mu^y}{y!}$$

B. The Regression Model

- We link the expected number of accidents (μ) to explanatory variables (traffic flow, speed, geometry, etc.) using a **logarithmic link function** to ensure predictions are always positive.
- Equation:

$$\ln(\mu_i) = b_0 + b_1 X_{i1} + b_2 X_{i2} + \cdots$$

or

$$\mu_i = \exp(b_0 + b_1 X_{i1} + b_2 X_{i2} + \cdots)$$

C. Estimation: Maximum Likelihood Estimation (MLE)

- **Goal:** Find the values of the coefficients (b_0, b_1 ...) that make the observed data (y) most probable.
- Likelihood Function (L): The product of the probabilities of all individual observations.

$$L(\mu|y) = \prod \frac{e^{-\mu_i} \cdot \mu_i^{y_i}}{y_i!}$$

- Log-Likelihood (LL): We take the natural log of L (turning products into sums) to make the maths easier to differentiate.

$$\ln(LL) = \sum \{-\mu_i + y_i \ln(\mu_i) - \ln(y_i!)\}$$

- **Maximisation:** We take the derivative of the Log-Likelihood with respect to b, set it to 0, and solve for b.

2. Interpretation of the Model (EXAM)

*You must know how to calculate the **Slope** and **Elasticity** for different model forms.*

A. Definitions

- **Slope ($\frac{dy}{dx}$):** The absolute change in accidents (Y) for a 1-unit change in the variable (X).
- **Elasticity (E):** The percentage change in accidents for a 1% change in the variable.

$$E = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \text{Slope} \cdot \frac{x}{y}$$

B. The Four Functional Forms (Memorise this Table)

In the exam, check if the variables (x and y) have been logged ($\ln$) or not.

Model Form	Equation	Slope ($\frac{dy}{dx}$)	Elasticity (E)	Interpretation Shortcut
Linear (Lin-Lin)	$y = b_0 + b_1 x$	b_1	$b_1 \cdot \left(\frac{x}{y}\right)$	b_1 is just the slope.
Log-Log	$\ln(y) = b_0 + b_1 \ln(x)$	$b_1 \cdot \left(\frac{y}{x}\right)$	b_1	b_1 **is the elasticity.** (1% change in X = b_1% change in Y)
Log-Lin	$\ln(y) = b_0 + b_1 x$	$b_1 \cdot y$	$b_1 \cdot x$	1 unit change in X = $b_1 \cdot 100\%$ change in Y.
Lin-Log	$y = b_0 + b_1 \ln(x)$	$b_1 \cdot \left(\frac{1}{x}\right)$	$b_1 \cdot \left(\frac{1}{y}\right)$	1% change in X = $b_1/100$ unit change in Y.

※ IF THERE IS A DUMMY VAR. (BOOLEAN)

eg. $\ln(y) = b_0 + b_1 x + b_2 \ln x + b_3 D$, where $D = \{0,1\}$

the interpretation for D specifically is NOT slope or elasticity but,

Indicator = $(e^b - 1) \times 100\%$.

(if $y = b_0 + b_1 x + b_2 \ln x + b_3 D$

indicator = $b \times 100\%$.)

eg.

$$Y_i \sim Poisson(\mu_i) = \frac{e^{-\mu_i}\mu_i^{y_i}}{y_i!}$$

$$\ln(\mathrm{E(Y)}) = \ln(\mu_i) = b_0 + b_1X_{i1} + b_2X_{i2} + \cdots \ldots \ldots . + b_kX_{ik}$$

$\ln(E(Y) = \ln(\mu)$ = -14.657 – 1.139ln(Speed) + 0.798ln(AADT) + 0.853 ln(Length) + 0.04 ln(Radii) + 1.199 ln(SpeedLimit) – 0.395 Lane 2 + 0.088 Lane 4 + 0.055 Max Gradient + 0.065 Year 2004 + 0.023 Year 2005 - 0.094 Year 2006 – 0.125 Year 2007

Variable	Model coefficient (b)	Mean (Observed data; without ln)	Elasticity	Slope
Accident		8.96 ($\bar{y}$)		
ln(Speed)	-1.139	84.86 ($\bar{x}_1$)	-1.14 (b)	-0.12 $= b\left(\frac{\bar{y}}{\bar{x}}\right)$
ln(AADT)	0.799	45,801 ($\bar{x}_2$)	0.79 (b)	1.5x10^{-4}
ln(Length)	0.853	5,029m ($\bar{x}_3$)	0.85 (b)	1.2x10^{-3}
ln(Min radius)	0.04	672.9m ($\bar{x}_4$)	0.04 (b)	1.3x10^{-2}
ln(Speed limit)	1.199	109.58 ($\bar{x}_5$)	1.199 ~ 2.0 (b)	0.082
Max gradient	0.055	3.189 ($\bar{x}_6$)	0.493 $= b * \bar{y}$	0.175 $= b * \bar{x}$

log-log: $E = b$, $\frac{dy}{dx} = b\frac{\bar{y}}{\bar{x}}$

log-lin: $E = b\bar{y}$, $\frac{dy}{dx} = b\bar{x}$

$\bar{x}$ and $\bar{y}$ will be given in exam

Year 2004's indicator

$= (e^b - 1) \times 100\%$

$= (e^{0.065} - 1) \times 100\%$

$= 6.75\%$

Year 2006's indicator

$= (e^b - 1) \times 100\%$

$= (e^{-0.094} - 1) \times 100\%$

$= -8.97\%$

what does these value indicate?

the poisson regression model is based on year 2003 ...
and for year 2007 indicator is -8.97%.
that means year 2007 has 8.97% less accidents compared to year 2003.

c10: Performance Measures

1. Rationale Behind Performance Measures

Why do we need them? They are essential for the **Evaluation Cycle** of any transport project.

- **Pre-implementation:** To identify problems (e.g., congestion), select strategies, and define project objectives.
- **Post-implementation:** To monitor if the deployed system works as intended and meets user needs.
- **Feedback:** To refine operations and guide future investments.
- **Key Function:** To quantify performance against policy goals (e.g., reducing emissions, improving safety).

2. The 4-Dimensional Framework (Memorise these 4 pillars)

Dimension	Focus
1. Traffic Efficiency	**Mobility:** Ability to reach destinations (Supply vs Demand). **Reliability:** Consistency of travel time. **Operational Efficiency:** Resource usage (e.g., mode share). **System Condition:** Physical state of infrastructure.
2. Safety	Reducing accidents and conflicts. Includes direct impacts (safety schemes) and indirect impacts (traffic management).
3. Land Use	**Accessibility:** How easily people can reach key services (jobs, schools). **Equity:** Access for special groups/demographics.
4. Emission	Environmental impact from motor vehicles (GHG, fuel type) and electric vehicles.

3. Calculation of Typical Indicators

You must be able to calculate these specific metrics.

A. Mobility Indicators

- **Volume to Capacity (V/C) Ratio:** $V/C = \frac{\text{Traffic Volume}}{\text{Road Capacity}}$ *(If V/C > 1.0, the facility is congested).*
- **Vehicle-Miles of Travel (VMT):** $VMT = \text{AADT} \times \text{Segment Length} \times 365$
  (Total distance travelled by all vehicles in a year).
- **Average Delay (SRN):** $\text{Delay} = \frac{\sum(\text{Observed Time} - \text{Speed Limit Time}) \times \text{Flow}}{\sum \text{Flow}}$

B. Reliability Indicators (Crucial)

- **Travel Time Index (TTI):** $TTI = \frac{\text{Travel Time}}{\text{Freeflow Travel Time}}$
  (e.g., TTI of 1.5 means a 20-min trip takes 30 mins in peak).
- **Planning Time Index:** $\text{PTI} = \frac{\text{95th Percentile Travel Time}}{\text{Freeflow Travel Time}}$
  (Measures the total time a traveller must allow to ensure on-time arrival 95% of the time).
- **Buffer Index:** $\text{Buffer Index} = \frac{\text{95th Percentile Time} - \text{Average Travel Time}}{\text{Average Travel Time}}$
  *(Measures the **extra** time buffer needed due to uncertainty).*

4. Data Used to Measure Performance

- **Aggregate Data:**
  - **Characteristics:** Easier to collect, provides a "big picture" system view.
  - **Examples:** Traffic counts (loops), speed measurements (radar), car park occupancy, environmental sensors.
- **Disaggregate Data:**
  - **Characteristics:** Harder/more expensive to collect, provides detailed insight into individual user behaviour.
  - **Examples:** On-board public transport surveys, intercept surveys, travel diaries, stated preference surveys.

c11: Emerging Trends

1. Signal Priority for Buses

A. Why is it needed?

- **Economic Benefit:** Buses transport more people per unit of road space than cars. Reducing bus delay has a higher total value of time benefit.
- **Journey Time:** Reducing journey time makes buses more competitive, increasing ridership.
- **Efficiency:** Faster running times mean fewer buses are needed to run the same frequency service (cost saving).
- **Regularity:** Irregularity (bunching) increases passenger waiting time significantly. Priority helps maintain headways.

B. Calculating Performance (Weighted Delay)

- **Standard Delay:** $D = \sum(q \times d)$
  - Where q is flow and d is delay. This treats every vehicle equally.
- **Weighted Delay (Bus Priority):** To reflect the value of buses, we weight the delay by passenger occupancy.
  - **New Formula:** $D = \sum(w \times q \times d)$
  - Weight (w): The ratio of occupants in that vehicle class compared to a car.

$$w = \frac{\text{Mean Occupancy of Vehicle Class}}{\text{Mean Occupancy of Car}}$$

    (e.g., If a bus holds 50 people and a car holds 1.5, the bus has a much higher weight).

C. Active Priority Methods

- **Extension:** If a bus arrives at the end of a green phase, the green is extended to let it through. (Low disruption).
- **Recall:** If a bus arrives on red, the other stages are truncated (shortened) to bring the bus stage forward sooner. (High disruption to other traffic).

2. Car Following Models for AVs/CAVs

For the exam, you need to "Comment on models only" (no calculation) and "Discuss differences".

Model	Type	Key Characteristic / Difference
Intelligent Driver Model (IDM)	**Desired Measures**	Calculates acceleration based on the difference between current speed/gap and *desired* speed/gap. It is continuous and realistic for AVs.
ACC (Adaptive Cruise Control)	**Sensor Based**	Uses Radar/Lidar to measure range and rate of the vehicle ahead. Maintains a set time gap. Reactive (cannot see beyond the lead vehicle).
CACC (Cooperative ACC)	**Communication Based**	Uses **V2V (Vehicle-to-Vehicle)** communication to receive acceleration data directly from the lead vehicle. Allows for *much* smaller gaps (platooning) because reaction time is near zero.
Gipps	**Safe Distance**	A "safety-first" model. Calculates speed based on the safe braking distance required to avoid a collision if the leader brakes hard. Conservative.
CA (Cellular Automata)	**Rule Based**	Discrete model (grid-based). Simple rules update position/speed. computationally efficient but less realistic for smooth AV dynamics compared to IDM.

3. Capacity Management (RoW Reallocation)

How to manage mixed traffic (AVs + Human Driven Vehicles).

- **The Problem:** At low penetration rates, AVs might be conservative, potentially reducing capacity.
- **The Solution: Right of Way (RoW) Reallocation** (Dedicated Lanes).
  - **Strategy:** Segregate AVs into dedicated lanes.
  - **Benefit:** Allows AVs to use CACC (platooning) effectively, significantly increasing capacity in that lane.
  - **Result:** Theoretical capacity increases **convexly** with AV penetration rate. Reallocating lanes is beneficial even at medium penetration rates.

4. Machine Learning in Transport

Why is it appropriate?

1. **Data Nature:** Transport produces massive amounts of quantitative (loops, GPS) and qualitative (surveys) data that ML handles well.
2. **Complexity:** Interactions between system components (drivers, signals, network) are complex and not fully understood physically. ML can model these relationships without needing an analytical equation.
3. **Uncertainty:** Human behaviour is stochastic (unpredictable). ML is good at finding patterns in noisy, uncertain data.
4. **Intractability:** Some optimisation problems (like real-time dynamic traffic assignment) are too slow to solve mathematically. ML (e.g., Neural Networks) can approximate solutions instantly.