IMPERIAL COLLEGE LONDON

MEng Examination 2021

PART II

This paper is also taken for the relevant examination for the Associateship

CIVE50004: ENVIRONMENTAL ENGINEERING

25 May 2021: 12:00 – 15:00 BST

An extra 30 minutes will be added on to the times shown above in order for you to scan and upload your answers.

*This paper contains **THREE** questions.*

*Answer **ALL THREE** questions.*

All questions carry equal marks.

Formulae sheets are provided at the end of the examination paper.

Q1. (Answer ALL parts of this question; total of 40 marks)

(a) Describe, in detail, the installation and use of any relevant infrastructure and the measurements needed, along with any underlying assumptions and calculations, in order to obtain the horizontal pore water velocity at a given location in an unconfined aquifer?

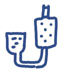
homogeneous and isotropic!

mean pore water velocity, $v = \frac{q \text{ (darcy's velocity)}}{n_e \text{ (effective porosity)}}$

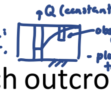
to measure q (darcy velocity):

$q = -Ki$

to measure K: constant-rate pumping test.



can also use ex-situ test like falling head permeameter.



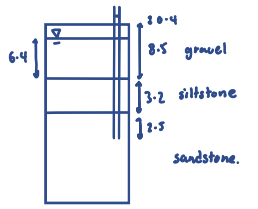
- plot s(t) and use Theis' solution to estimate S and T; T = h₀K

to measure i:
- 3 observation well.
- obtain h (hydraulic head) at each of the well (x, y)
- solve $h = Ax + By + C$ for A (ix) and B (iy)
- $i = \sqrt{A^2+B^2}$, $\theta = \tan^{-1}\left(\frac{A}{B}\right)$

to find h_0 for the $T = Kh_0$:
- drill borehole log to determine the base of the aquifer.

[10 marks]

(b) An 8.5 m thick gravel unit, which outcrops at the surface, overlies a fractured sandstone. The gravel and sandstone layers are separated by a horizontally uniform, 3.2 m thick, siltstone. In the gravel aquifer there is a water table, which is at a height of 6.4 m above its base with the siltstone layer. A piezometer, with a narrow response zone of 20 cm, is completed in the sandstone aquifer with the centre of the response zone at a depth of 2.5 m below the boundary between the sandstone and siltstone units. The water level in the piezometer is 14.6 m above the centre of the piezometer response zone.



water level in piezometer is above the ground surface.

Draw a diagram illustrating the problem. What is particularly noticeable about this?

[4 marks]

If the hydraulic conductivities of the gravel, the siltstone and the sandstone units are 15 m d^{-1}, 0.02 m d^{-1} and 3 m d^{-1}, respectively, what does this mean if you were conducting a water balance calculation for the gravel aquifer?

this is $\Delta S = P - E - Q - R$.

$Q = -AKi$

this K should take the K of the entire strata and not each layer separately:

$\frac{12.1}{K} = \frac{6.4}{15} + \frac{3.2}{0.02} + \frac{2.5}{3}$

$K = 0.075 \text{ m d}^{-1}$

$i = \frac{2.5}{12.1} = 0.2066$

$\frac{Q}{A} = Ki = 0.0155 \text{ m d}^{-1}$ (upwards)

→ this Q is an input/influx rather than output.

$\Delta S = P - E - Q - R$ — not this, this is runoff! (river discharge)

$\Delta S = P - E - Q - R + \underline{\text{aquifer input}}$

[6 marks]

(c) "All aquifers are heterogeneous"

Describe, using two examples, what is meant by this statement. If this is correct, explain why many aquifers are treated as if they're homogeneous. What implications does this have when measuring $\underbrace{\text{aquifer properties}}_{S,\,T,\,K}$?

heterogeneous mean ← hydraulic properties vary with space (especially K)

- chaotic nature of rock formation (varying pore/space/fracture size)

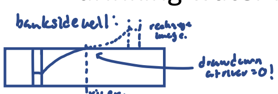
at a larger scale (> 100 m) variability can be averaged out and treated as homogeneous.

when measuring this (constant rate pumping test) have to do it at a big scale.
OR small scale with multiple measurement and get the average!

[5 marks]

(d) What is a bankside well? Give detailed reasons why these are often used as sources of drinking water in alluvial valleys?

bankside well:



- river recharges the well.
- the aquifer act as a nature filter to the source of water — clean!

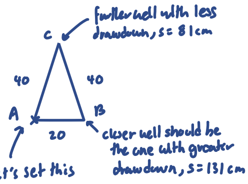
[5 marks]

(e) Three 0.4 m diameter abstraction wells fully penetrate an unconfined sand aquifer. Two wells are 20 m apart from each other and are 40 m from the third well, forming an isosceles triangle. The horizontal rest level of the water table is 12.4 m above a horizontal impermeable clay base.

kinda new question. usually we have multiple well we want to find h at a specific coordinate.
→ we superpose:
$h_e^2 - h^2 = \frac{Q}{2K}\ln\left(\frac{r_e}{\right.}$

When one of the wells at the base of the triangle was pumped at a rate of 100,000 litres per day the steady state drawdowns in the other two wells were 131 and 81 centimetres, respectively. Making any assumptions necessary, calculate the hydraulic conductivity of the sand aquifer and the radius of influence of the pumping well. Would you recommend this aquifer as suitable water resource for a large city with one million residents?

[10 marks]



at $Q = 100\,000 \text{ l day}^{-1} = 100 \text{ m}^3\text{day}^{-1}$

assumptions: hydraulic properties especially hydraulic conductivity is homogeneous and isotropic.

from data sheets, well pumping unconfined aquifer (with recharge)

$h_e^2 - h^2 = \frac{Q_w}{2K}\ln\left(\frac{r_e}{r}\right) + \cancel{\frac{W}{2K}(r^2 - r_e^2)}$ assuming no recharge, W.

$h_e^2 - h^2 = \frac{Q_w}{2K}\ln\left(\frac{r_e}{r}\right)$

marking scheme is wrong! this is "simultaneous eqn" not "superposition"!

we know at well B:
$h = 12.4 - 1.31 = 11.09\text{ m}$,
$r = 20\text{ m}$.

fixed for well A. $\begin{cases} Q_w = 100 \text{ m}^3/\text{day} \text{ (this one is } Q_w \text{ of A!)} \\ h_e = 12.4\text{ m} \end{cases}$

all these are used with reference of well A not B!

$\underbrace{12.4^2 - 11.09^2 = \frac{100}{2K}\ln\left(\frac{r_e}{20}\right)}_{\text{eqn.1}}$

we know at well C:
$h = 12.4 - 0.81 = 11.59\text{ m}$,
$r = 40\text{ m}$.

fixed for well A. $\begin{cases} Q_w = 100 \text{ m}^3/\text{day} \text{ (this one is } Q_w \text{ of A!)} \\ h_e = 12.4\text{ m} \end{cases}$

$\underbrace{12.4^2 - 11.59^2 = \frac{100}{2K}\ln\left(\frac{r_e}{40}\right)}_{\text{eqn.2}}$

eqn.1 − eqn.2:
$11.34 = \frac{100}{2K}\ln(2)$ $\therefore K = 1.946 \text{ m/day}$

Subbing K into eqn 1 (or 2):
re = 131.2 m

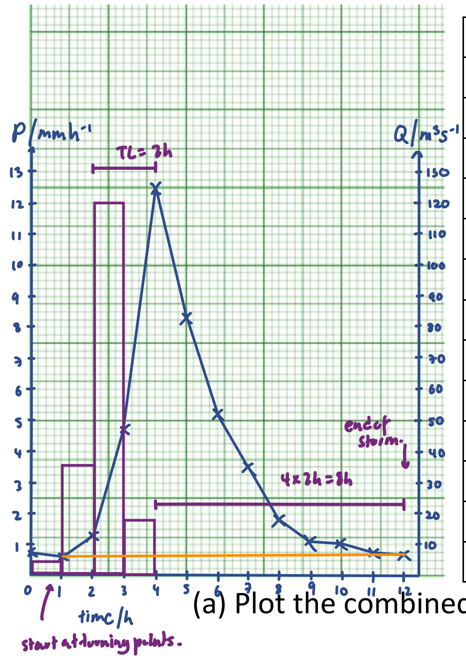
suitable or not compare:
1. T = Kh₀ → $T = Kh_0$
(h₀=he) T = 24 m²/day < 200 m²/day (unsuitable)

Q2. (Answer ALL parts of this question; total of 40 marks)

A catchment of 220 km^2 has sandy soils with an average depth of 0.4m. Their volumetric moisture content is 0.38 at saturation, 0.12 at field capacity, and 0.04 at wilting point. The following precipitation and river discharge data are recorded:

P is measured for the duration of Δt so P=0.4 means from t=0 to t=1

Q is measured instantaneous at time t! so Q=8.5 is exactly at t=0



Time (h)	P (mm h^{-1})	Q (m^3 s^{-1})	Q [mm h^{-1}]
0	0	8.5	0.139
1	0.4	6.7	0.109
2	3.6	13.2	0.196
3	12.1	47.8	0.782
4	1.8	123.5	2.021
5	0	83.4	1.365
6	0	51.5	0.843
7	0	35.2	0.576
8	0	17.1	0.280
9	0	11.1	0.182
10	0	9.8	0.160
11	0	7.4	0.121
12	0	6.9	0.113

$\bar{Q}$ [mm h^{-1}] $= \frac{6.887}{13} = 0.5298$ $\quad \bar{P} = \frac{17.9}{13} = 1.377$

(a) Plot the combined hydrograph and hyetograph of this event.

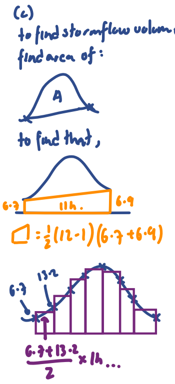
[5 marks]

(b) Calculate the runoff ratio of this event.

convert Q to mm h⁻¹ to match unit of P.

$RR = \frac{\bar{Q}}{\bar{P}} = \left(\frac{Q \times 10^9}{A \times 10^{12} \times 60^2} \right) = 0.385$

[4 marks]



(c) Calculate the stormflow volume (in m^3) of this event.

[6 marks]

(d) If you know that the precipitation data are obtained from an automatic rain gauge at the outlet of the catchment, list and describe briefly the main sources of error in the calculation of the runoff ratio.

- vegetation and nearby building might affect the collection of precipitation.
- human measurement errors.
- the shape of rain gauge might affects the aerodynamic of wind and cause rain to be blown away.

precipitation varies spatial and temporally. have to interpolate across multiple rain gauge to get P across catchment area. } might not be accurate.

[6 marks]

(e) The average soil moisture in the catchment increased from 0.18 to 0.20 during the event.

(i) Calculate the volume (m³) of precipitation that was stored in the soil.

$(0.20-0.18) \times \text{depth} = 0.008\text{m}$. × catchment area.
$= 1.76 \times 10^6 \text{m}^3$

[3 marks]

(ii) Is the value you obtained for soil storage in (i) compatible with the catchment water balance equation? Briefly explain your answer.

$\Sigma P = 3.938 \times 10^6 \text{m}^3$ $\quad$ $\Delta S = P - E - Q - R$. $\quad P = \Delta S + E + Q + R$

we don't know if there's E or R yet but we can conclude: $P > \Delta S + Q$
$3.938 \times 10^6 > 1.76 \times 10^6 + 1.520 \times 10^6$
yes true!

[m³s⁻¹] $Q = \Sigma Q \times 3600$

[3 marks]

(iii) Did groundwater recharge happen during this event? Motivate briefly your answer.

if moisture content / volumetric water content. θ < FC − RAW
the tree will start experience stress! all no multiplied by depth, Z_r
elseif θ > FC, recharge, R will occur! (R from ΔS = P − Q − E − R)

formulas
∴ yes, will occur.

[3 marks]

(f) Explain how anthropogenic climate change is expected to impact precipitation patterns and how this impact can be quantified. Discuss the limitations of the latter.

[10 marks]

Q3. (Answer ALL parts of this question; total of 40 marks)

(a) Flood hydrograph simulation

this is not net rainfall!

A catchment has a 1-hour 2mm unit hydrograph (UH) given below. An observed rainfall event has the intensity of 3 and 7 mm per hour, respectively. If the total overland flow resulting from the UH is 22.75 m3 s-1, calculate the rainfall losses and the peak flow assuming that they can be assessed using the proportional losses method.

[10 marks]

1-hour 2mm UH

Time (h)	UH ($m^3\ s^{-1}$)	1h 1mm UH	1h 3mm UH	1h 7mm UH	Stormflow (before taking away rainfall losses)	after
0	0	0	0	0	0	0
1	0.6	0.3	0.9	0	0.9	0.63
2	2.1	1.05	3.15	2.1	5.25	⋮
3	1.8	0.9	2.7	7.35	10.05	→ 7.035 peak flow
4	1.4	0.7	2.1	6.3	8.4	
5	0.6	0.3	0.9	4.9	5.8	
6	0	0	0	2.1	2.1	
				0	0	

total: 32.5 m^3s^{-1}

$32.5 - 22.75 = 9.75$ (lost)

$\underbrace{\frac{9.75}{32.5} = 0.3 = 30\%}$

after finding the lost... we can find stormflow by multiplying. stormflow before lose by $1-p = 1-0.3 = 70\%$

(b) Flood frequency analysis

(i) If the original unit hydrograph (UH) is derived for 12mm 3h rainfall, which method would you use to derive the following UHs and why:

$\Delta T_1 = 3h$

1. 4mm 1h UH
2. 12mm 6h UH
3. 12mm 4h UH

(1) $\Delta T_2 = 1h$, $\Delta T_2 \neq k\Delta T_1$, $k \in \mathbb{Z}^+$
→ s-curve, can't use shortcut method.

(2) $\Delta T_2 = 6h$, $\Delta T_2 = 2\Delta T_1$
→ use shortcut method: delay 12mm3h by 3h and sum with original 12mm 3h

(3) $\Delta T_2 = 4h$, $\Delta T_2 \neq k\Delta T_1$, $k \in \mathbb{Z}^+$
→ s-curve, can't use shortcut method.

remember all to times $\frac{\Delta T_1}{\Delta T_2}$ to obtain the wanted UH as changing ΔT_1 to ΔT_2's UH will alter the magnitude of UH.

[3 marks]

(ii) Using the peaks over threshold (POT) dataset given in table below:

1. Extract the annual maxima flows and estimate parameter of the Gumbel distribution using the methods of moments (see Formulae sheet for equations).
2. Apply the Gumbel distribution to estimate the return period of Q=40 $m^3\ s^{-1}$.
3. Calculate what is the probability that the 100-yr flood will not occur in 5 consecutive years.
4. Calculate the value of the flow, q, for a 100-yr flood event.

[10 marks]

POT dataset from 2001-2006

Year	Q ($m^3 s^{-1}$)	Year	Q ($m^3 s^{-1}$)
2001	27	2004	22
2001	16	2004	11
2001	12	2005	18
2002	17	2005	13
2002	28	2005	25
2003	27	2005	25
2003	16	2006	32
2003	21	2006	18
2003	36	2006	27
2004	12	2006	12

27 28 36 22 25 32

1. $S_x^2 = 25.067$

$\bar{x} = 28.333$

$\beta = \frac{\sqrt{6}}{\pi} \times \sqrt{25.067} = 3.90$

$\alpha = \bar{x} - 0.5772\beta = 26.08$

2. $F(40) = \exp\left[-\exp\left(\frac{\alpha - 40}{\beta}\right)\right]$

$= 0.9772$

$T = \frac{1}{P(q > q_d)} = \frac{1}{1 - F(q_d)} = \frac{1}{1 - 0.9772} = 35.99$ years

3. $P(q > q_d) = \frac{1}{T} = \frac{1}{100}$

$X \sim B\left(5, \frac{1}{100}\right)$

$P(X=0) = {}^5C_0 \times \frac{1}{100}^0 \times \frac{99}{100}^5 = \frac{99}{100}^5 = 0.951$

4. $F(q_d) = 1 - P(q > q_d) = \frac{99}{100}$

$\frac{99}{100} = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

$q_d = 44.02$

(c) Flood warning and forecasting

A real time flow forecasting model can be described by the following transfer function model:

$$\hat{x}_{k+2} = au_k + bx_k$$

$au_k + bx_k \geq F$

$x_k \geq -\frac{a}{b}u_k + \frac{F}{b}$

y-intercept $= \frac{F}{b} = 2.82$

$\hookrightarrow b = \frac{F}{2.82} = \frac{2.3}{2.82} = 0.816$

gradient $= -\frac{a}{b} = -\frac{2.82}{0.97} \rightarrow a = 2.372$

where $\hat{x}_{k+2}$ is the downstream flow forecast with a lead time of 2 hours. For the observed flow data a graphical support for flood warning has been developed and shown in figure below. Using the information presented in the graph:

(i) Calculate the values of parameters *a* and *b* in the linear transfer function model if the flood threshold is equal to *F* = 2.3 $m^3 s^{-1}$.

[4 marks]

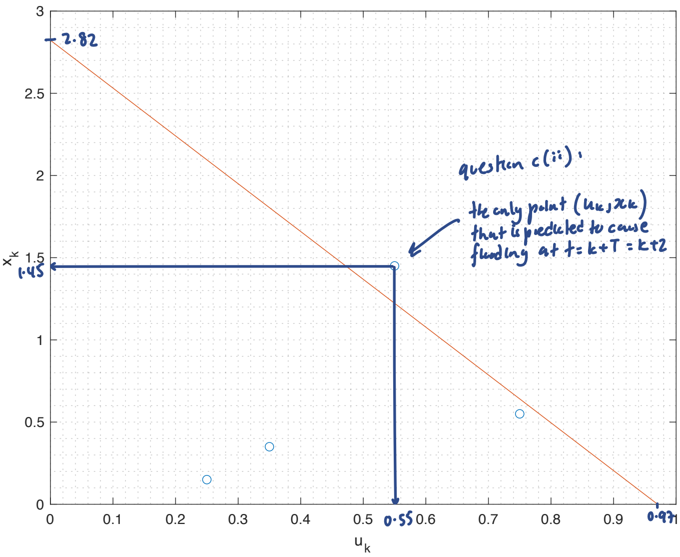
(ii) Use the fitted linear transfer function model to calculate the downstream flow at t=6 hours which causes the flooding. Explain any assumptions made for the calculation.

[4 marks]

$\hat{x}_6 = au_4 + bx_4$

$= 2.372(0.55) + 0.816(1.45)$

$= 2.4878$



Graphical method to support flood warning

(d) Flood management and cities

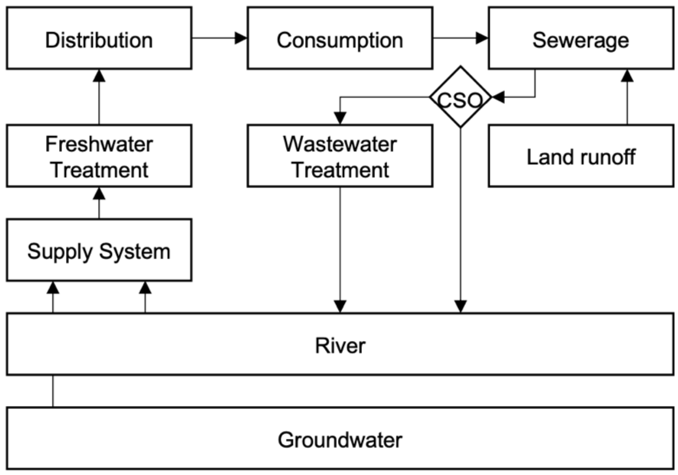
Urban water system can be represented using the concept diagram shown below.

(i) Explain three ways how urban water system interacts with the water environment.

[3 marks]

(ii) In your role of an urban water manager, you are asked to propose three ways of managing negative impacts of water abstraction and discharges on the river flow and water quality. Taking the examples from the lectures as your starting point, explain briefly alternative management options that you would propose taking into account that you can manage and alter any component of the urban water system shown below. Briefly explain why the measure was chosen and how you would expect it to improve the river water flow and quality. Note that the Land runoff component is used to represent the surface overland flow in the system.

[6 marks]



Concept diagram of urban water system

Formulae Sheet

A. Introduction to Hydrology (Prof Wouter Buytaert)

Catchment water balance	$\Delta S = P - E - Q - R$
Density of solids	$\rho_S = \frac{M_S}{V_S}$
Dry bulk density	$\rho_B = \frac{M_S}{V_T}$
Total bulk density	$\rho_T = \frac{M_S + M_L}{V_T}$
Porosity	$\varepsilon = \frac{V_L + V_G}{V_T}$
Void ratio	$e = \frac{V_L + V_G}{V_S}$
Gravimetric moisture content	$\theta_G = \frac{M_L}{M_S}$
Volumetric moisture content	$\theta = \frac{V_L}{V_T}$
Interrelating formula	$\theta = \theta_G \frac{\theta_B}{\theta_W}$
Penman Monteith	$E_a = K_c K_s E_{t,0}$
River flow	$Q = \int v(A)\,dA = \overline{v}A$
Rectangular weir	$Q = KbH^{1.5}$
Hydropower equation	$P = \varepsilon_t \, \varepsilon_h \, h \, \rho \, Q \, g$

Irrigation $\mathrm{I} = \mathrm{E_p} - \mathrm{P} + \mathrm{R}$

Total available water $\mathrm{TAW} = 1000\,(\theta_{\mathrm{FC}} - \theta_{\mathrm{WP}})\,\mathrm{Z_r}$

Readily available water $\mathrm{RAW} = \mathrm{p\,TAW}$

B. Groundwater Systems (Prof Adrian Butler)

Darcy's law

$q_i = -K\dfrac{dh}{di}$, where i represents a coordinate direction (e.g. x, y, z)

Water table elevation in an unconfined aquifer

$$h^2 - h_0^2 = \frac{W}{K}\left(Lx - x^2\right) + \left(h_1^2 - h_0^2\right)\frac{x}{L}$$

Total flow per unit width in an unconfined aquifer

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$$

Well pumping confined aquifer without recharge (Thiem equation)

$$s = h_e - h = \frac{Q_W}{2\pi T}\ln\left(\frac{r_e}{r}\right)$$

Well pumping unconfined aquifer with recharge

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}\left(r^2 - r_e^2\right)$$

Theis solution

$s = \dfrac{Q_w}{4\pi T}W(u)$, where $u = \dfrac{r^2 S}{4Tt}$

Jacob's large time approximation

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

C. Flood Risk Estimation and Management (Dr Ana Mijic)

Weibull formula $P_m(Q \le q_m) = \dfrac{\mathrm{m}}{N+1}$

Gringorten formula	$P_m(Q \leq q_m) = \frac{\mathrm{m}-0.44}{N+0.12}$
Gumbel distribution	$F(q_d) = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$
Gumbel variate	$z = -\ln\{-\ln[F(q_d)]\}$
Probability for sequence of years	$P(x,n) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$
Moment matching	$\alpha = \mu - 0.5772\,\beta$ $\beta = (6^{\frac{1}{2}}/\pi)\sigma$
L-moment matching	$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j$ $L_2 = \frac{2}{n}\sum_{j=2}^{n}\left[\frac{(j-1)q_j}{n-1}\right] - L_1$ $\alpha = L_1 - 0.5772\,\beta$ $\beta = \frac{L_2}{\ln(2)}$
Normal equation for flood forecasting	$(R^T R)\theta = R^T X$