CIVE50004 WATER RESOURCE & SUPPLY ENGINEERING

1 SURFACE HYDROLOGY

1.1 RIVER FLOW

River Flow/Discharge: Volumetric flux through a given cross-section along a river

- Integrates combined effects of spatial precipitation & evaporation and soil & groundwater routing
- **Runoff:** Discharge normalised by catchment area (How quickly depth accumulate "L/T", not flow velocity!)
- **Hydrograph:** Plot of discharge over time

MEASUREMENT

Direct Methods: Measure flow volume V over time T

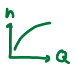
$$Q = \frac{\Delta V}{\Delta T}$$

- Often very impractical to do

Indirect Methods: Integrate flow per unit area v over river cross-sectional area A

$$v(A) = \frac{dQ}{dA} \Rightarrow Q = \int v dA = \overline{v}A$$

- **Measures water velocity directly:**
  - Valeport Propeller Meter: A propeller on a stick for dipping into small streams to measure velocity at various points along A
  - Acoustic Doppler Velocity Profiler: For larger rivers
- **Measures water stage/height:**



  - Automatable with electronic pressure transducers
  - Uses $\underbrace{\text{calibrated stage-discharge relation}}_{\text{called rating curve.}}$ to convert stage to discharge
  - Sensitive (the relation) to external conditions that affect channel shape

rating curve become unstable over time due to...

    - Vegetation cycles, ice cover, bank erosion, etc
    - Major source of uncertainty besides, getting rating curve require manual finding Q beforehand (prone to error too!)
  - Weirs/Flumes can artificially create critical river flow to establish a theoretical reliable theoretical stage-discharge relationship

FLOW INDICES

- Interpolation of instantaneous data to produce continuous graphs
  - Limitation: Undersampling, especially if large sampling interval

Maximum Flow: Plot of peak discharge against drainage area

- Hard to predict exact magnitudes as it is influenced by many factors (eg. heavy rainfall, snowmelt, soil saturation, etc)

Monthly Mean Flow: Estimates of annual river flow patterns

- Influenced by local climate & seasonality
- Requires 20-30 years of flow records to be representative

10th & 90th Percentiles: 90% & 10% chance of exceeding these flow values

Flow Duration Curve: Plot of river flow against exceedance probability

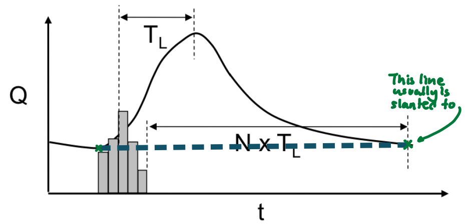
- Steeper curves indicate higher variability in flow

STORMFLOW & BASEFLOW, Q(t)

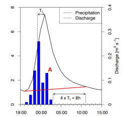
Stormflow/Fast Flow: Overland run-off

Baseflow/Slow Flow: Sub-surface flow

1. Find stormflow start point: Upward inflexion point on the hydrograph
   - Usually with the start of precipitation
   - Could be delayed for larger catchments
2. Find end point: $N \times T_L$, where $N = 4$ usually



3. **Stormflow Volume:** Connect both points linearly and find the area between hydrograph and line
4. **Baseflow Volume:** Rest of the area



1.2 CATCHMENT WATER BALANCE

Catchment Area: Drainage area that contributes flow at a point on a river

Precipitation: Rainfall, snow, sleet, hail

Hyetograph: Plot of rainfall over time

Isohyet: Line of equal precipitation

Evaporation: Loss of water from the land/water surface to the atmosphere

CATCHMENT WATER BALANCE

Conserving all water fluxes through a given catchment:

$$\Delta S \equiv \frac{dS}{dt} = P(t) - E(t) - Q(t) - R(t)$$

- S = internal catchment storage, P = precipitation, E = evaporation, Q = river discharge, R = groundwater recharge convert Q (m^3s^{-1} to $mm\,h^{-1}$) then if P,E,Q,R all $mm\,h^{-1}$: $\int X dt$ or ΣX for $\forall t$ (must convert before equating)
- All units are in mm
- On large time intervals, ΔS will be small relative to the other fluxes $\Rightarrow Q \approx P - E - R$

1.3 PRECIPITATION, P(t)

Humidity: Amount of atmospheric water vapour at a given point

Absolute Humidity: Mass of water vapour per unit volume of air

Relative Humidity: Percentage of the maximum allowable amount of water vapour at a given temperature

$$r = \frac{e_d}{e_a} \times 100\%$$

- e_d = current vapour pressure, e_a = saturation vapour pressure

Dewpoint: Temperature below which condensation occurs

ORIGINS

Atmospheric pressure gradients cause upward movement of air that expand and cool it below dewpoint

- **Convection:** Localised surface heating produces bouyant air parcels
- **Orographic Ascent:** Air forced to flow over obstacles (eg. mountains)
- **Shear Ascent:** Winds differing in height and speed induces turbulence that can cause vertical ascent
- **Frontal Ascent:** Hot and cool air masses meet, forcing warmer air upwards due to density

MEASUREMENT

- **Rain Gauges**
  - Human measurement errors
  - Nearby buildings/vegetation can affect readings based on location
  - Rain gauge shape affects aerodynamic flow of wind and could deflect rain away
- **Remote Sensing Methods** (preferred)
  - Weather Radars: Use Rayleigh scattering of microwaves on raindrops and measure backscattering
  - Satellite measurements

PRECIPITATION PATTERNS

(can interpolate)

Spatial cause precipitation often measured at points

- Need to interpolate across rain stations to get spatial averages
  - Quality depends on station density, rainfall type, topography, timescale
- Advanced statistical properties can generate better predictive rainfall maps now

Temporal (hard to predict)

- Rainfall is highly variable over time (stochastic), so difficult to detect significant trends
- $\underbrace{\text{Intensity}}_{mm h^{-1}}$-$\underbrace{\text{duration}}_{\text{how long for this } mm h^{-1}}$-$\underbrace{\text{frequency}}_{\text{how often}}$ curves represent rainfall via stationary characteristics (eg. mean, standard deviation), but these may be invalidated with climate change

1.4 SOIL WATER STORAGE & PERCOLATION, ΔS

UNSATURATED ZONE

- Soil above the water table, containing air & water in its voids
- Important in distributing precipitation into overland flow, groundwater recharge and evaporation
- Provides water storage capacity
- Predominantly vertical flow due to gravity
  - But, as it recharges the saturated zone and moves the water table, pressure gradients can force horizontal flow

SOIL HYDRAULIC PROPERTIES

different as in time. $\Delta S = \Delta\theta \times depth$.

$\theta(\psi)$ **Moisture Content**

$$\theta = \frac{V_L}{V_T}, \quad \theta_G = \frac{M_L}{M_S} \Rightarrow \theta = \theta_G \frac{\rho_B}{\rho_W}$$

- θ, θ_G = volumetric & gravimetric moisture content
- $\rho_B = \frac{M_S}{V_T}$ = dry bulk density
- Subscripts: L = liquids, S = solids, T = total
- Determination:
  - Removing a soil sample with volume V_T and mass $M_T = M_L + M_S$, oven-drying it until the final mass M_S is constant
  - **Time Domain Reflectrometry:** Use the time taken by an EM wave to propagate between parallel metal rods in the soil to calculate the soil's dielectric constant, and thus, its water content
    - Advantage: Integrates θ over the probe length

known: V_T, M_S, M_L — can find ρ_B — with $\rho_W = 1 g cm^{-3}$ — can find V_L, can find θ
- if we have ρ_S we can find: $V_S \rightarrow V_G = V_T - V_S - V_L$

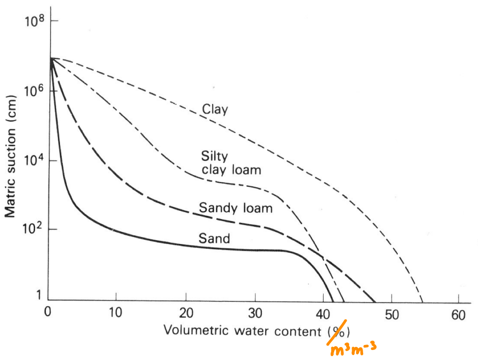
when finding ΔS: $\Delta S = \Delta\theta \times$ length. the TDR's reading gives $\theta \times$ length. Still need two TDR reading at different time! to get ΔS!

ψ **Matric Potential:** $(\psi < 0)$ Potential energy head ψ from capillary retention

if $\psi<0$, matric pot. if $\psi>0$, matric suc. pressure head.

$$\psi = \frac{\phi}{g}$$

- **Soil Water Potential:** Potential energy of soil ϕ depends on:
  - Capillarity: Surface tension between soil, air and water $h = \frac{2\gamma\cos\theta}{\rho g r}$
  - Adsorption: Electrostatic forces between water and soil
  - Osmosis: Diffusion due to solute concentration differences
  - Gravity
- **Matric Suction:** When $\psi > 0$ in the capillary zone
- **Water Retention Curve:** Empirical relation between ψ and θ



- When $\psi = 0$, soil is completely saturated
  - When an external force (eg. gravity) is applied, air gradually replaces water with the largest pores draining first
  - **Saturation Point (0 bar/0 cm):** When all pores are filled and water content is maximum
  - **Field Capacity (0.33 bar/330 cm):** Amount of water when retention forces balance gravity
  - **Wilting Point (~15 bar/15000 cm):** Maximum suction that plants can extract water with

$K(\psi)$
Hydraulic Conductivity: How easily water can flow through the soil

$$v = -K(\theta)\nabla h, \text{ where } h = \psi + z$$

- v = groundwater flow velocity
- ∇h = total hydraulic head gradient

- **Saturated Conductivity:** $K(\theta) = K_s$ in saturation
  - As draining continues, $K(\theta) < K_s$ as water flows through smaller pores
  - $K(\theta)$ is empirical due to complex composition of soil
- **Infiltration Capacity:** The K_s value at the soil surface
- The greater the matric ~~suction~~ potential ψ, the smaller the K (negative)

EQUATIONS

SC: how much water in total can it hold. SMD (how much more water can it hold). AWC (how much water it currently hold)

Given soil of depth d:

Storage Capacity

$$SC = \int_0^d \frac{V_T - V_S}{V_T} dy$$

Actual Water Content

AWC also equal to: "$\theta \times$ depth!"

$$AWC = \int_0^d \frac{V_L}{V_T} dy$$

$\therefore \Delta S = \Delta\theta \times$ depth, $AWC = \theta \times$ depth. $\Delta S = \Delta AWC$

Soil Moisture Deficit

$$SMD = \int_0^d \frac{V_G}{V_T} dy$$

unit is mm! → compare with precipitation (mm) to get more info.
SMD > P → overflow.
AWC < P → not all P go to ground
→ all $\int$ can be written as Σ
eg. $SMD = \underbrace{\Sigma \frac{V_G}{V_T} \Delta y}$
if given M_T, M_S, depth 0.05, 0.15, 0.25
Δy = 0.10 each, total 0.30 depth.
check week 3 ex 5.1-5.3!

- Overland flow occurs if:
  - Precipitation exceeds infiltration capacity
  - $SMD = 0$ (all voids filled up)

1.5 EVAPORATION, $E(t)$

EARTH SURFACE ENERGY BALANCE

$$\lambda E = R_N \pm C \pm V \pm G - P_S$$

- $\lambda \approx 2470$ kJ/kg = latent heat of vapourisation of water
- E = evaporated mass flux [kg m^{-2} s^{-1} $\approx$ mm s^{-1}]
- $R_N = R_I - R_\alpha - R_o$ = net solar radiation
- C = sensible heat transfer
- V = change in energy storage
- G = energy exchange with the ground
- P_S = energy used by photosynthesis
  - Units are all in W/m^2, where unspecified

Bowen Ratio: Energy used for evaporation versus environmental heating

$$\beta = \frac{C}{\lambda E}$$

OPEN SURFACES

- E_o = evaporation from open surfaces (water & soil)
- Influencing factors for water surfaces:
  - Meteorological: Solar radiation, relative humidity, wind speed
  - Physical: Water salinity, depth, surface size
- Soil surfaces are affected similarly, but also by moisture content, capillary characteristics, colour & soil temperature

PLANTS

Precipitation Interception

$$IL = P - TF - SF$$

- **Interception Loss** IL: Water staying on leaves and evaporating, never reaching the ground
  - Factors: Vegetation cover morphology, evaporation rates, duration, intensity & frequency of precipitation
  - Tropical mountain forests can add water by condensation of fog, adding 20% more water instead
- **Throughfall** TF: Excess water that drips off the canopy to the ground
  - Measured by rain gauges under large area of the canopy
- **Stemflow** SF: Water that flows along branches/stems to the ground
  - Measured manually by rain gauges collecting water off tree trunks

Evapotranspiration = evaporation (from soil + water) + transpiration (from plant leaves)

- Depends on plant morphology, time of day, etc
  - Transpiration drops to about zero at night — If given E_p, E_t, or $E_{t,0}$ and a time period like 6am to 6pm, don't need ×2 if need to get per day.
- Types of evapotranspiration are limited by available water and energy
  - **Potential** E_p: When water is sufficient
  - **Actual** E_t: With water stress factored in
  - **Reference** $E_{t,0}$: Baseline when a reference crop (usually grass) is watered sufficiently
    - Estimated by Penman Monteith method

    (~volume) amount of water unit can be L, L3, L/T, L3/T, M. if ask E over a time period, give L or L3 or M (no /T)
  - **Empirical Equations:** $E_t = K_s \underbrace{K_c E_{t,0}}_{E_p}$ (given in data sheet)

$$E_p = K_c E_{t,0}, \quad E_t = K_s E_p = K_s K_c E_{t,0}$$

    - K_c = vegetation coefficient (varies with crop species & growth stage)
    - K_s = water stress coefficient
- Measured by lysimeters
  - Known vegetated soil volume is frequently weighed for water percolation, precipitation or other mass changes
  - Combined with hydrometeorological measurements of the surroundings

* Evapotranspiration, E's unit is usually L/T (mm day^{-1}) but we can convert to (kg) by multiplying with Area, time and density of water: mm day^{-1} × $\underbrace{day}_{T}$ $\underbrace{mm^2}_{A}$ $\underbrace{kg\, mm^{-3}}_{\rho}$ = kg

2 GROUNDWATER HYDROLOGY

2.1 GROUNDWATER

* why groundwater is useful? (important!)

Groundwater: Sub-surface water below a water table in fully saturated soils or geological formations

- Provides baseflow to rivers that sustain ecology, even during droughts
- Typically very pure, useful for industry

Aquifer: A saturated permeable geological unit that stores groundwater and allows its flow under normal conditions

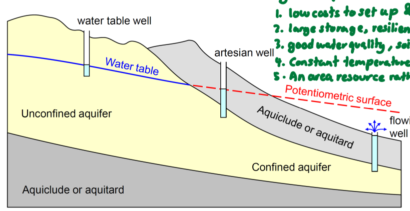
Aquitard: (retard) Less permeable than aquifers

Aquiclude: (exclude) Practically impermeable

Unconfined: With a free water table

Confined: Without a free water table

gw benefits:
1. low costs to set up & run.
2. large storage, resilient to drought.
3. good water quality, soil as filter.
4. Constant temperature throughout the year.
5. An area resource rather than a linear one.



POROSITY

Porosity Types

- All involve volumes of some void type divided by total volume
  - Total n: All voids
  - Effective n_e: Voids accepting water
  - Drainable n_d: Voids drained by gravity ← is also specific yield S_y!
  - Kinematic n_k: Volume of flowing water
- Typically, $n > n_e > n_d \approx n_k$
  - $n > n_e$: Some voids are too small or are isolated, so cannot contain water
  - $n_e > n_d$: Surface tension keeps some water from draining
  - $n_d \approx n_k$: Both related to pore size distribution

Representative Elementary Volume (REV)

- Porosity must be calculated over a volume
  - Large enough to neglect variations between pores
  - Small enough to neglect changes in macrostructure
- Varies between rock types

Porosity Measurement

- Direct: Oven-dry a known volume of soil, get its dry weight, then inject fluid in a vacuum and find the weight & volume of fluid added
- Indirect: Use electric resistivity to estimate porosity

2.2 GROUNDWATER FLOW

HYDRAULIC HEAD

Fluid Potential: Mechanical energy per unit fluid mass

$$\phi = GPE + KE + EE = gz + \frac{u^2}{2} + \int_{P_0}^{P} \frac{dP}{\rho}$$

- GPE = energy to raise fluid
- KE = energy to accelerate fluid
- EE = energy to raise pressure from P_0 to P

Hydraulic Head: Normalised ϕ by g

$$h = z + \frac{u^2}{2g} + \int_{P_0}^{P} \frac{dP}{\rho g}$$

- Assuming $u \to 0$ as velocities are very small in porous media, $\rho \neq f(P)$ as incompressible fluid and $P_0 = 0$ by using gauge pressure:

$$h = z + \frac{P}{\rho g} = z + \psi$$

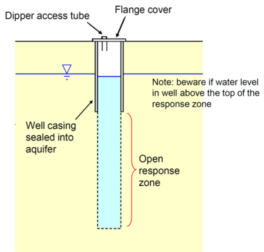
when u is not small!

assumption of $u=0$ breaks down when:
1. in the vicinity of wells being pumped at high rate.
2. in highly permeable formations / region.

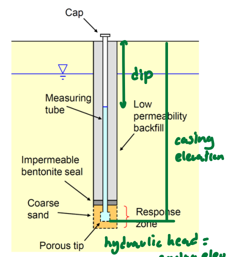
can be solved by using Forchheimer eqn: $q + \frac{Kb^2}{g} q^2 = -K\frac{dh}{dx}$

Hydraulic Head Measurement

- Observation well: Open borehole
  - Water level represents average h over the well width
  - Dip d_w = length of air column
  - If water level is above the response zone, h may be slightly inaccurate due to vertical flow
  - Automatable with pressure transducer
- Piezometer: Measures h at a point



Observation Well



Piezometer hydraulic head = casing elev. - dip.

DARCY'S LAW

$$q = \frac{Q}{A} = -K\frac{dh}{ds} = -\boldsymbol{K}\nabla h$$

dh or δh = $h_{final} - h_{initial}$ (usually negative cause water flow from high h to low)

ds or δs is distance between h_{final} and $h_{initial}$ measured point, always positive!

- q = Darcy flux/velocity or specific discharge
- Q = volumetric flux, A = cross-sectional area
- $\frac{dh}{ds}$ = hydraulic gradient
- s = direction of maximum head gradient
- $\boldsymbol{K}$ = hydraulic conductivity
  - Flow components in 3D space:

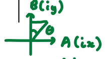
$$q_x = -K_{xx}\frac{dh}{dx}, \quad q_y = -K_{yy}\frac{dh}{dy}, \quad q_z = -K_{zz}\frac{dh}{dz}$$

FLOW DIRECTION

- Always goes from higher h to lower h
- At least 4 piezometers are needed to define flow direction
  - 1 for each axis direction + 1 reference
- **Determination of Flow Direction** This is 2D domain! (x,y)
  1. Assuming that K is constant over a given area and that h varies linearly, $h = Ax + By + C$
  2. Using 3 locations (x_i, y_i) and their head values h_i, solve for A, B and C
  3. Find 2D hydraulic gradient $i = \sqrt{A^2 + B^2}$ with bearing $\theta = \tan^{-1}\left(\frac{A}{B}\right)$
  4. Thus, flow direction $\theta_{\text{flow}} = 180° + \theta$

flow direction and i's direction still require manual judgement

★ A is actually i_x and B is i_y and hence $i = \text{sqrt}(A^2+B^2)$!



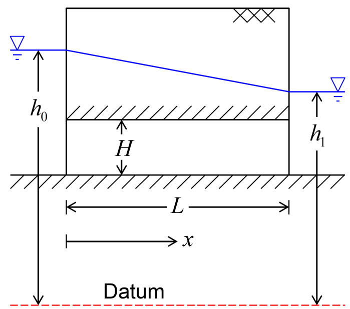
eg. So if $i_x < 0$, $i_y < 0$ θ is Q3! $180 < \theta < 270°$

There is 7 in total

Steady / Unsteady, Radial / Not Radial, Confined / Unconfined.

$2^3 - 1$ (no unsteady, radial unconfined) → we want to solve for either h along x or s along x

STEADY FLOW IN CONFINED AQUIFERS



1. By Darcy's Law, flow per unit width Q':

★ solve: $\boxed{Q' = -HK\frac{dh}{dx}} \Rightarrow Q'x = -HKh + C$

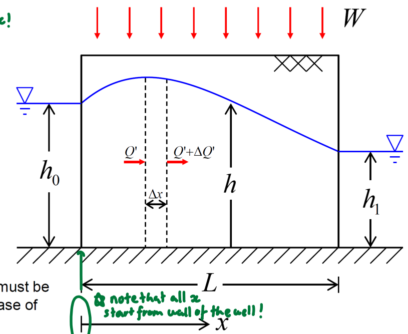
2. When $x = 0$, $h = h_0 \Rightarrow C = HKh_0$
3. When $x = L$, $h = h_1$. Thus:

memorise, change KH to Kh_0 & unconfined, almost equal + no recharge

$$Q' = HK\left(\frac{h_0 - h_1}{L}\right), \quad \boxed{h - h_0 = (h_1 - h_0)\frac{x}{L}}$$

- **Confined Transmissivity:** $T = KH$ [L²/T]

STEADY FLOW IN UNCONFINED AQUIFERS



NB Datum must be moved to base of aquifer

1. By continuity: ★ solve $\boxed{\frac{dQ'}{dx} = W \text{ or } \frac{d}{dx}\left(-hK\frac{dh}{dx}\right) = W}$

$$Q' + W\Delta x = Q' + \Delta Q' \Rightarrow W = \frac{\Delta Q'}{\Delta x} = \frac{dQ'}{dx}$$

$$\Rightarrow Q' = Wx + C_1$$

2. **Dupuit Assumption:** No vertical head gradient, so flow is horizontal: $\frac{dh}{ds} = \frac{dh}{dx} \Rightarrow Q' = -hK\frac{dh}{dx}$

$$\therefore h\frac{dh}{dx} = -\frac{Wx + C_1}{K} = \frac{1}{2}\frac{d(h^2)}{dx}$$

$$\Rightarrow h^2 = -\frac{Wx^2}{K} - \frac{2C_1 x}{K} + C_2$$

3. When $x = 0$, $h = h_0 \Rightarrow C_2 = h_0^2$
4. When $x = L$, $h = h_1 \Rightarrow C_1 = -\frac{WL}{2} - \frac{K}{2L}(h_1^2 - h_0^2)$. Thus:

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}(h_0^2 - h_1^2)$$

$$h^2 - h_0^2 = \frac{W}{K}(Lx - x^2) + (h_1^2 - h_0^2)\frac{x}{L}$$

SPECIAL CASES (there's also unconfined cases with no recharge, $W=0$)

- Almost equal boundary conditions: $|h_1 - h_0| \ll h_0$, and no recharge.

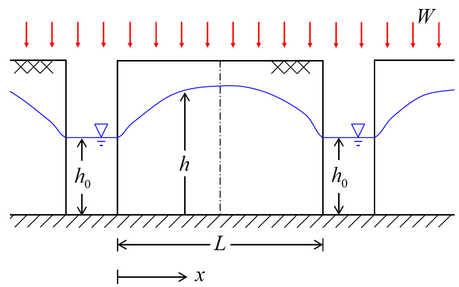
$$Q' \approx \frac{Kh_0}{L}(h_0 - h_1) \approx \frac{T}{L}(h_0 - h_1)$$

$$h - h_0 \approx (h_1 - h_0)\frac{x}{L}$$

  - **Unconfined Transmissivity:** $T = Kh_0$
- Equal boundary conditions: $h_0 = h_1$, with recharge

$$Q' = W\left(x - \frac{L}{2}\right), \quad h^2 - h_0^2 = \frac{W}{K}(Lx - x^2)$$

  - Applicable to finding ideal spacing L between 2 parallel ditches with vertical recharge W, where at $x = 0.5L$, $h_{\text{max}}^2 = h_0^2 + \frac{WL^2}{4K}$

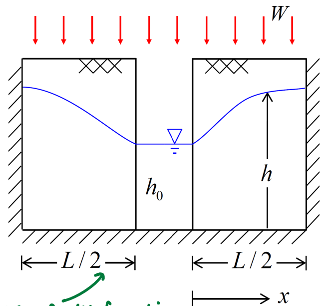


  - Applicable to alluvial aquifers

$-hK\frac{dh}{dx} = Wx + C_1$

$-h\,dh = \frac{1}{K}(Wx + C_1)dx$

$-h^2 = \frac{1}{K}(Wx^2 + C_1 x) + C_2$



in exam for this formation, using either the Q' or $h^2 - h_0^2$ eqns, the $L = L$ not $L/2$!

why? how?
- chaotic nature of rock formation.
- varying pore, fracture sizes and space.

heterogeneous can be treated as homogeneous if the scale > 100m (REV concept) (variability can be averaged out)

when cannot be treated as homogeneous?
- structural change between valleys and interfluves, fault, alluvial deposits.

2.3 HYDRAULIC CONDUCTIVITY

hydraulic property varies with space (especially K)

Heterogeneous: Varies with space

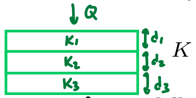
Anisotropic: Varies with space and direction

HYDRAULIC CONDUCTIVITY

- In 3D:

$$\boldsymbol{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

  - If axes of maximum & minimum K are aligned with the Cartesian axes, then all non-main diagonal terms are zero
- For soil of total depth d layered into N homogeneous strata of thickness d_i with different hydraulic conductivities K_i, overall K is:
  - **Vertical flow, perpendicular to strata:**



$$K = d\left[\sum_{i=1}^{N}\frac{d_i}{K_i}\right]^{-1}$$

$\frac{d}{K} = \sum \frac{d_i}{K_i}$ easier to memorise.

  - **Horizontal flow, parallel to strata:**

$$K = \frac{1}{d}\sum_{i=1}^{N} K_i d_i$$

$Kd = \sum K_i d_i$

Mathematical Means

- **Arithmetic:** $K_A = \frac{1}{N}\sum_{i=1}^{N} K_i$
- **Geometric:** $K_G = \exp\left(\frac{1}{N}\sum_{i=1}^{N}\ln K_i\right)$
- **Harmonic:** $K_H = \left(\frac{1}{N}\sum_{i=1}^{N}\frac{1}{K_i}\right)^{-1}$

INTRINSIC PERMEABILITY

1. **Poiseuille's Law:** Steady laminar flow through a circular pipe of radius R (pg 84-85)

$$Q' = -\frac{\pi R^4}{8\mu}\frac{dP}{dx}$$

note that this Q is not $Q = \frac{V}{A}$! It is NQ'! $(Q' = \frac{Q}{N})$

2. For a porous $L \times B \times H$ block with N pipes of radius R, equate total flow $Q = NQ'$ and effective porosity $n_e = \frac{\pi LNR^2}{LHB}$ to get:

$n_e = \frac{\text{volume of void accepting water}}{\text{total volume}}$

$$Q = -\frac{An_eR^2}{8\mu}\frac{dP}{dx}$$

3. Since hydraulic pressure $P = \rho g(h - z) \Rightarrow \frac{dP}{dx} = \rho g\frac{dh}{dx}$, by Darcy's Law:

$$Q = -A\frac{\rho g n_e R^2}{8\mu}\frac{dh}{dx} = -AK\frac{dh}{dx}$$

$$\Rightarrow \boxed{K = \frac{\rho g k}{\mu}} = \frac{\rho g}{\mu}\frac{n_e R^2}{8}$$

- **Intrinsic Permeability:** $k = \frac{n_e R^2}{8}$

hydraulic conductivity and permeability both describe how easy water can pass through a material

Differences: Hydraulic conductivity depends fluid + material; permeability depends material ONLY

MEAN PORE WATER VELOCITY

1. Total flow Q through a porous $L \times B \times H$ block with N pipes of radius R, with mean pore water velocity v:

$$Q = NQ' = N\pi R^2 v = qA$$

   - q = Darcy flux/Flow velocity
2. Since effective porosity $n_e = \frac{\pi NR^2}{A} \Rightarrow N\pi R^2 = n_eA$: ★ memorise!

$$\boxed{v = \frac{q}{n_e}}$$

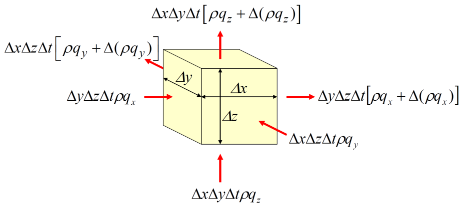
mean pore water velocity

Darcy's velocity (flow rate over an area)

effective porosity

2.4 STORAGE

MASS CONTINUITY



- **3D Mass Continuity:**

$$\frac{\partial m}{\partial t} = -\frac{\partial(\rho q_x)}{\partial x} - \frac{\partial(\rho q_y)}{\partial y} - \frac{\partial(\rho q_z)}{\partial z}$$

- Changes in fluid mass are due to:
  - Increase in fluid density $\Delta\rho$ due to compression from an increase in fluid pressure
  - Increase in porosity Δn due to rock matrix expansion from a decrease in effective stress

1. If n and ρ are uniquely defined in terms of P, by chain rule:

$$\frac{\partial m}{\partial t} \equiv \frac{\partial(n\rho)}{\partial t} = \rho\frac{\partial n}{\partial t} + n\frac{\partial\rho}{\partial t}$$
$$= n\rho\left(\frac{1}{n}\frac{dn}{dP} + \frac{1}{\rho}\frac{d\rho}{dP}\right)\frac{\partial P}{\partial t}$$

2. Since $P = \rho g(h - z) \Rightarrow dP = \rho g\, dh$:

$$\frac{\partial m}{\partial t} = n\rho g\left(\frac{1}{n}\frac{dn}{dP} + \frac{1}{\rho}\frac{d\rho}{dP}\right)\frac{\partial(\rho h)}{\partial t}$$
$$= n\rho g(c_R + c_F)\frac{\partial(\rho h)}{\partial t} = S_s\frac{\partial(\rho h)}{\partial t}$$

   - **Specific Storage Coefficient [L^{-1}]:** $S_s = n\rho g(c_R + c_F)$
   - **Rock Compressibility:** $c_R = \frac{1}{n}\frac{dn}{dP}$
   - **Fluid Compressibility:** $c_F = \frac{1}{\rho}\frac{d\rho}{dP}$
3. Substituting back into the continuity equation, since ρ is effectively constant:

$$S_s\frac{\partial h}{\partial t} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z}$$

CONFINED AQUIFERS

Storativity: Water released per unit area over the entire aquifer thickness H by a unit fall in the potentiometric surface of a confined aquifer

$$S = HS_s$$

- Due to small reduction in porosity and fluid density from the drop in fluid pressure

Transient Flow In Confined Aquifers

1. Substitute Darcy's Law into the continuity equation above to get:

$$S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right)$$

2. Neglecting vertical flow, integrate over the aquifer height with respect to z, assuming K_x, K_y, S_s and h are constant vertically:

$$\boxed{HS_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(HK_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(HK_y\frac{\partial h}{\partial y}\right)}$$

   - **Storativity:** $S = HS_s$
   - **Transmissivity:** $T_x = HK_x, T_y = HK_y$

Effect of Atmospheric Pressure

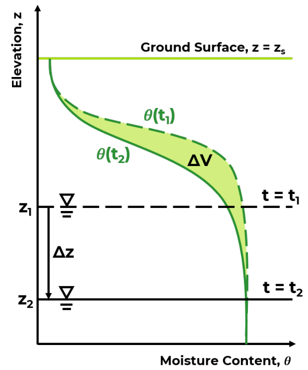
- Atmospheric pressure affects total stress and pressure balance in the piezometer, and thus, piezometer readings
- Hence, barometric pressure must be taken to account for this
- Tides cause similar fluctuations

UNCONFINED AQUIFERS

Specific Yield: Water released per unit area by a unit fall in water table elevation of an unconfined aquifer

$$S_y = \frac{\Delta V}{A\Delta z}, \text{ where } \Delta V = A\int_0^{z_s}\theta(t_1) - \theta(t_2)dz$$

- Due to pore dewatering and reduction in saturation of pores in the capillary fringe



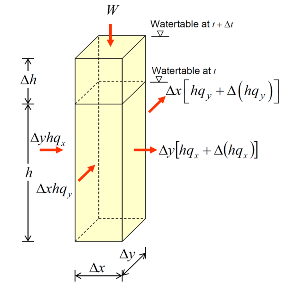
Transient Flow In Unconfined Aquifers

1. Dupuit Assumption: No vertical flow
2. Find mass continuity of a $\Delta x \times \Delta y \times (h + \Delta h)$ column of the aquifer with discharge W:

$$S_y\frac{\partial h}{\partial t} = -\frac{\partial(hq_x)}{\partial x} - \frac{\partial(hq_y)}{\partial y} + W$$

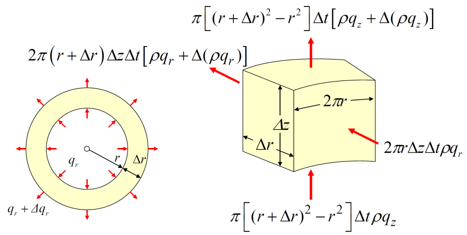
3. **Boussinesq Equation:** Substitute Darcy's Law:

$$\boxed{S_y\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(hK_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(hK_y\frac{\partial h}{\partial y}\right) + W}$$



2.5 RADIAL FLOW

AXIALLY-SYMMETRIC MASS CONTINUITY



1. Storage Change = Mass In - Mass Out

$$\frac{\partial m}{\partial t} = -\frac{\rho q_r}{r} - \frac{\partial(\rho q_r)}{\partial r} - \frac{\partial(\rho q_z)}{\partial z}$$

2. Since $\frac{\partial m}{\partial t} = \rho S_s\frac{\partial h}{\partial t}$ and for a slightly compressible fluid, ρ is practically constant:

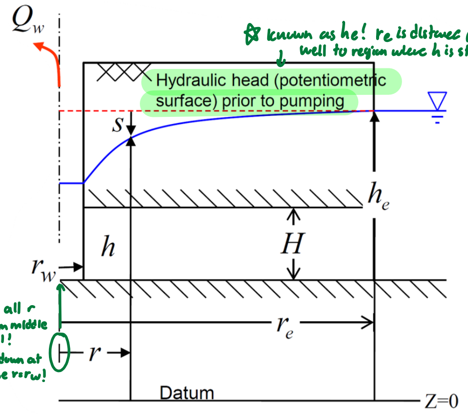
$$S_s\frac{\partial h}{\partial t} = -\frac{q_r}{r} - \frac{\partial q_r}{\partial r} - \frac{\partial q_z}{\partial z}$$

3. Applying Darcy's Law and integrating over thickness H, ignoring vertical flow and assuming homogeneity:

$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{1}{r}\left(\frac{\partial h}{\partial r}\right) + \frac{\partial^2 h}{\partial r^2}$$

- **Storativity:** $S = HS_s$
- **Transmissivity:** $T = HK$

STEADY RADIAL FLOW IN CONFINED AQUIFERS



1. From the origin of an abstraction well, by Darcy's Law, total flow:

$$Q_w = 2\pi r H K \frac{dh}{dr} \Rightarrow h = \frac{Q_w}{2\pi T} \ln r + C$$

   - $Q_w > 0$ when water is being removed
2. Let the radius of influence r_e be the distance that $h = h_e$:

$$\Rightarrow C = h_e - \frac{Q_w}{2\pi T} \ln r_e$$

3. **Theim Equation:** Solve for drawdown s

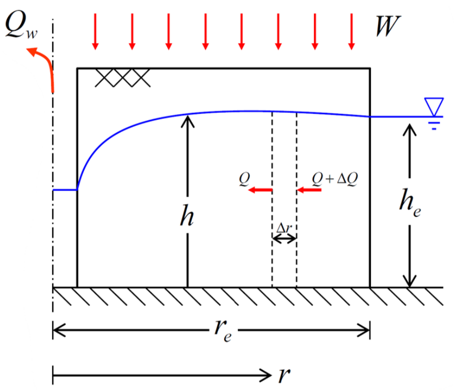
Sw is when: $r_1 = r_w$, $r_2 = r_e$

$$s = h_e - h = \frac{Q_w}{2\pi T} \ln\left(\frac{r_e}{r}\right)$$ given in datasheet

   - Drawdown between 2 radial distances r_1 and r_2:

$$\Delta s = s_1 - s_2 = h_2 - h_1 = \frac{Q_w}{2\pi T} \ln\left(\frac{r_2}{r_1}\right)$$

STEADY RADIAL FLOW IN UNCONFINED AQUIFERS



1. By continuity:

$$Q = Q + \Delta Q + W\pi((r+\Delta r)^2 - r^2)$$
$$\Rightarrow \pi W = -\frac{\Delta Q}{2r\Delta r + \Delta r^2} = -\frac{1}{2r}\frac{dQ}{dr}$$
$$\Rightarrow Q = -\pi W r^2 - C_1$$

2. When $r = 0, Q = Q_w \Rightarrow C_1 = -Q_w$
3. By Dupuit assumption & Darcy's Law, $Q = 2\pi r h K \frac{dh}{dr}$. Equating together:

$$\therefore Q = 2\pi r h K \frac{dh}{dr} = Q_w - \pi W r^2$$
$$\Rightarrow h\frac{dh}{dr} = \frac{1}{2K}\left(\frac{Q_w}{\pi r} - Wr\right) = \frac{1}{2}\frac{d(h^2)}{dr}$$
$$\Rightarrow h^2 = \frac{Q_w}{\pi K}\ln r - \frac{Wr^2}{2K} + C_2$$

4. Let the radius of influence r_e be the distance that $h = h_e$:

$$\Rightarrow C_2 = h_e^2 - \frac{Q_w}{\pi K}\ln r_e + \frac{W r_e^2}{2K}$$

given in data sheets.

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}(r^2 - r_e^2)$$

SPECIAL CASES

- Thick unconfined aquifer: $|h - h_e| \ll h_e$

$$h_e^2 - h^2 = s(h_e + h) \approx 2sh_e$$
$$\Rightarrow s = h_e - h \approx \frac{Q_w}{2\pi h_e K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{4h_e K}(r^2 - r_e^2)$$

METHOD OF IMAGES (recommend go through slides for this part)

- Well flow equations above are derived for infinite aquifers
- But, equipotential boundaries (eg. streams) & impermeable barriers (eg. some hardrock formations) present finite constraints
- Thus, superpose image wells to simulate hydrological boundaries $r_a = \sqrt{(a-x)^2+y^2}$ image: $r_a = \sqrt{(a+x)^2+y^2}$
  - $h=0$ **Rivers:** Add image well with negative drawdown on opposite side of the river
  - $\frac{\partial h}{\partial x} = 0$ **Impermeable Boundaries:** Add identical image well on opposite side of the river
  - **Multiple Boundaries:** Add image wells for all image wells added, as long as the images lie within r_e of the original well

constant head = flip
impermeable = no flip

- Assumptions
  - Constant stream stage
  - Both well and stream fully penetrate the aquifer
  - No sealing layer of fine sediment at riverbed
  - Pseudo-steady state conditions

for method of image, usually it will be independent of r coordinates.

eg. $h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{x+a}{x-a}\right)$, where x-coordinate is defined at mid:

2.6 AQUIFER TESTING → To obtain S and T

compare S (confined), > 5×10⁻⁴ or 10⁻³ (good)
Sy (unconfined), > 0.5 or 1.0 (good)
compare T with 200 m²/day
T > 200 good
T < 200 bad.
with H we can obtain K as well
T = KH or T = Kh₀
(confined) (unconfined)
h₀ = he for pumping test.

UNSTEADY RADIAL FLOW IN CONFINED AQUIFERS

1. Rewrite axially-symmetric mass continuity in terms of drawdown $s = h_e - h$:

$$\frac{S}{T}\frac{\partial s}{\partial t} = \frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}$$

   - When $t = 0$, $s = 0$ for $r > 0$
   - When $t > 0$, $s = 0$ for $r \to \infty$, $Q = Q_w$ and $r\frac{\partial s}{\partial r} = -\frac{Q_w}{2\pi T}$ for $r \to 0$
   - Thus, flow continuity at the well wall:

$$2\pi r H K \frac{\partial h}{\partial r} = -2\pi r T \frac{\partial s}{\partial r} = Q_w$$

2. Transform continuity equation with **Boltzmann variable** $u = \frac{r^2 S}{4Tt}$:

$$\frac{d^2 s}{du^2} + \left(1 + \frac{1}{u}\right)\frac{ds}{du} = 0$$

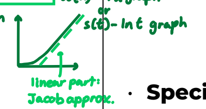
   - For $u \to \infty$, $s = 0$
   - For $u \to 0$, $u\frac{ds}{du} = -\frac{Q_w}{4\pi T}$
3. Integrate in terms of u and use the 2[nd] boundary condition:

$$u\frac{ds}{du} = C_1 e^{-u} \Rightarrow C_1 = -\frac{Q_w}{4\pi T}$$

graph of $W(u_{r,t})$ and $s(t)$ is same use! to find S and T!

$$\therefore \frac{ds}{du} = \frac{Q_w}{4\pi T}\frac{e^{-u}}{u} \Rightarrow \boxed{s = \frac{Q_w}{4\pi T}W(u_{r,t})}$$

   - **Theis Well Function:** The Theis Solution



$$W(u) = \int_u^\infty \frac{e^{-u}}{u}du$$

   - **Exponential Integral:**

$$Ei(x) = -\int_{-x}^\infty \frac{e^{-t}}{t}dt$$

     - Thus, $W(u_{r,t}) = -Ei(-u_{r,t}) = E_1(x)$
   - **Jacob Large Time Approximation:** Valid at late times when $u_{r,t} < 0.1$

$$E_1(x) \approx -0.5772 - \ln(x)$$
$$\Rightarrow \boxed{s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]}$$

given → useful for finding S and T

Assumptions

- Infinite, confined, homogeneous, isotropic aquifer
- Infinitesimal, fully-penetrating well
- Darcy's Law
- Uniform initial conditions
- No recharge
- Constant pumping rate

PUMPING TESTS

→ Constant-rate pumping test (single-step)
- install pump in the test well, one more observation well, r meters away.
- pump at constant discharge Q
- measure and plot s(t)

find S, T and K.

→ Step Drawdown Test } find AQ and BQⁿ

- Monitor aquifer's response to pumping
- Use Theis Solution above to estimate S & T
  - Set $S = S_y$ to use for unconfined aquifers for large times → which can be used as an estimation for he → $T = Kh_e$ (confined: $T = KH$)
- **Advantages:**
  - Done in-situ, so limited disturbance to aquifer
  - Samples a large volume of aquifer, giving an upscaled value of T or K

Step Drawdown Test → To seperate s_w into two component: AQ and BQⁿ

1. Pump well until drawdown reaches quasi-steady state (QSS)
2. Increase flow rate and hold constant until the next QSS is reached $s = \sum_{n=1}^{N}\frac{\Delta Q_n}{4\pi T}W\left(\frac{Sr^2}{4T(t-t_{n})}\right)$
3. Repeat for a few cycles

- Total drawdown s_w is non-linear in flow Q:

$$s_w = AQ + BQ^n$$

  - Formation Loss AQ: From pumping water from the well to the original water level
    - Steady-state flow: $A = \frac{1}{2\pi T}\ln\left(\frac{r_e}{r_w}\right)$ (from Thiem equation)
    - Transient flow: $A = \frac{1}{4\pi T}E_1\left(\frac{r^2 S}{4Tt}\right)$
  - Well Loss BQ^n: From turbulent convergence of streamlines
    - **Jacob's Method:** Set $n = 2$ and plot $\frac{s_w}{Q}$ against Q to find A and B

$$\frac{s_w}{Q} = BQ + A$$

- **Specific Capacity:** Used to characterise discharge capacity of pumped wells

$$\boxed{SC = \frac{Q}{s_w}}$$

- **Well Efficiency:** Specific capacity measured on-site over theoretical capacity Q/AQ

$$\boxed{E = \frac{Q/s_w}{Q/AQ} \times 100\% = \frac{100AQ}{s_w}}$$ $= 100 \times A \times SC$

EX-SITU TESTS

$\frac{Q}{A} = -Ki$, $i = \frac{-h}{L}$ (negative cause final lower than initial)

$Q = AK(h/L)$ $Q = -a\frac{dh}{dt}$ → $h = H_0 e^{-\left(\frac{AK}{aL}\right)t}$ } need to memorise!

- Falling/Constant Head Permeameter
- Particle Size Analysis
- **Advantage:** Useful for estimating K

accurate way to determine K is to T = KH, find T by pumping test, H by borehole log

- **Disadvantages:**
  - May not be representative of the whole aquifer due to heterogeneity
  - Samples are likely disturbed during drilling, affecting validity

3 WATER RESOURCES

3.1 WATER RESOURCE MANAGEMENT

IRRIGATION

amount of water (mm) required for agriculture

- Crops mainly need water for transpiration and some for increasing biomass

Irrigation Amount

total water required = cropwater required = water needed for transpiration = water footprint. all these = Et!

Not all months we need irrigation, I sometimes if P is big, P > Ep+R, we can store excess water instead of requiring I!
if P > Ep+R:
ΔS = P − Ep − Q − R
Q = P − Ep − R
else if P < Ep+R:
I = Ep − P + R

$$I = E_p - P + R$$

★ ONLY SUM POSITIVE I! (if given every month Ep, P and R)

- I = irrigation, P = precipitation, R = recharge
  - Typically in mm/month — but if asked to find I over a period of time, × time period! (check if the time period is weird, eg. ½ month) — I always cut Ep and P (and R) into correct time before finding I
- E_p = potential evapotranspiration

ΔS = P − E − Q − R
if P, Q and R are known, we can find E, but this E is only average or sum!

$$E_p = K_c E_{t,0}$$

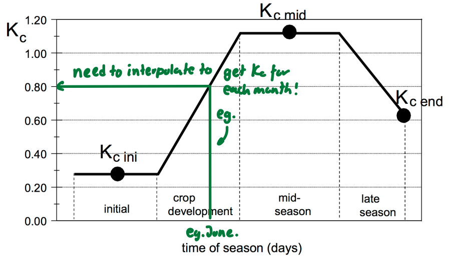
$E_t = K_s \underbrace{K_c E_{t,0}}_{E_p}$ (given in data sheet)
$K_s = 1.0$ for irrigation!

- K_c = crop coefficient, $E_{t,0}$ = grass reference value ★ grass $K_c = 1.0$! $E_p = E_{t,0}$, $E_t = E_p = E_{t,0}$ ($K_s = 1.0$)
  - Assumes no water stress ($K_s = 1$)

* If $K_s < 1$ → water stress act.

Crop Coefficient

- Affected by crop type, wind speed, relative humidity, soil evaporation
- K_c varies over crop growth stages

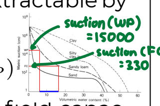


- K_c linearly increases in development phase and decreases in late season

Irrigation Dose

- Affects frequency of irrigation
- **Total Available Water:** Water extractable by plant roots

$$TAW = z_r(\theta_{FC} - \theta_{WP})$$



  - θ_{FC}, θ_{WP} = water contents at field capacity & wilting point
  - z_r = root depth
- **Readily Available Water:** Water accessible without water stress

maximum dose;

$$RAW = pTAW$$

minimum freq. = $\frac{\max(I)}{RAW}$

  - p = depletion fraction before water stress
  - p depends on crop
- Ideally, maximum irrigation dose $\leq RAW$

if moisture content / volumetric water content, $\theta < \underbrace{FC - RAW}_{\text{all no multiplied by depth, } z_r}$ the tree will start experience stress!
else if $\theta > FC$, recharge, R will occur! (R from ΔS = P − Q − E − R)

Irrigation Infrastructure

- Surface irrigation: Continuous flooding, basin, furrow irrigation, etc
- Sprinkler irrigation
- Micro/Drip irrigation: Drastically reduces water use by avoiding surface evaporation
  - Major problem is salinization, as salts are left behind when the water evaporates

ENERGY

- Water used as coolants or for hydropower
- **Runoff Ratio:** Long term yield of a basin

★ if evaporation, E decrease (urbanisation, vegetation decrease). RR will increase!

$$\boxed{RR = \frac{\overline{Q}}{\overline{P}}}$$

need to check unit! usually given in L/T, if want to use $\overline{Q}$ in, have to multiply by A!

  - $\overline{Q}$ = long term average runoff
  - $\overline{P}$ = long term average precipitation
- **Hydropower Equation**

[Q] = m³/s

$$P = \varepsilon_t \varepsilon_g Q \rho_w g H$$

  - P = generated power, Q = flow, H = driving head, ρ_w = density of water, g = gravitational acceleration
  - $\varepsilon_t, \varepsilon_g$ = turbine & generator efficiencies

OTHER USES OF WATER

Domestic: Drinking, cooking, washing

Commercial: Hotels, restaurants, offices, etc

Industrial: Fabrication, processing, washing, cooling, etc

Mining: Quarrying, well operations, etc

Livestock: Watering, feed lots, dairy operations, fish farming, etc

WATER SHORTAGE

- When clean water demand exceeds supply
- Caused by unequal distribution of water and too much being wasted, polluted or unsustainably managed
- In terms of annual country supplies:
  - **Water Stress:** < 1700 m³/person
  - **Water Scarcity:** < 1000 m³/person

VIRTUAL WATER

include precipitation! → Et

- **Water Footprint:** Total amount of water needed to produce something
- Split into different types:
  - **Green:** Evaporated rainwater — Et − I
  - **Blue:** Evaporated surface/groundwater — I!
  - **Grey:** Polluted water after use

if cum. def. I > RAW experience stress check 2015q2!

3.2 ENVIRONMENTAL CHANGE

DRIVERS OF HYDROLOGICAL CHANGE

- **Urbanisation:** Concrete cover reduces surface permeability
- **Agriculture:** Homogenisation of land for farming increases speed of flows, raising flood risks
- **Deforestation:** Reduces soil compaction & water retention
- **Climate Change:** Higher global temperatures makes water cycle more extreme
  - GHGs increase the amount of solar radiation being reflected back to Earth's surface, raising heat absorption

GLOBAL CIRCULATION MODELS

- For testing climate change hypotheses & projecting impacts by GHG emissions
- Global scale, but coarse resolution
- Simulates hydrological processes to represent water and energy exchanges at Earth's surface
  - Different interpretations result in different models
- **Sources of Uncertainty:**
  - Huge scale of the problem exceeding computational power
    - Can reduce scale, but results might not be very useful
  - Reliance on many smaller models to simulate various human and environmental factors
    - Mitigated by parameterisation (eg. empirical K_c values) to reduce complexity

4 FLOODS

4.1 WATER MANAGEMENT & FLOOD RISK

WATER MANAGEMENT

Aims

- Accurately predicting rainfall & surface runoff to enable operation, management & long term planning of urban water systems
- Developing & testing multifunctional, cost-effective, sustainable assessments and adaptation measures for water infrastructure

Challenges

can increase / decrease groundwater recharge:
- increase: leaking water mains under pressure, sewer exfiltration
  ΔS = P − E − Q − R (increase P / decrease E)
- decrease: sewer infiltration
  decrease in direct recharge because of greater surface runoff.

- **Urbanisation:** Makes areas more impervious, increasing run-off with reduced infiltration
  - Greater peak fluxes that drainage systems have to deal with
- **Climate Change:** Higher intense storm frequency
- **Population Growth:** Higher water demand needs greater investment in water infrastructure

Impacts

- **Water Pollution:** Excess wastewater discharge polluting natural water bodies
  - Damage to biologically important sites
- **Flooding:** Lives & homes lost, economic losses
  - Costs estimated by considering floodplain areas
  - Vulnerability assessed based on social groups (eg. schools)

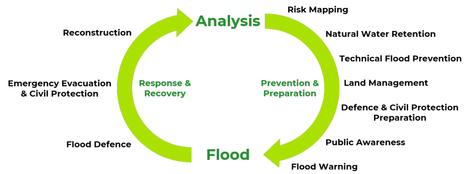
FLOOD TYPES

- **Coastal:** Tidal/Storm/Tsunami surges
- **Fluvial:** Rivers overflowing after heavy rain
  - Relatively slow, so easier to alert people
- **Flash:** Very fast fluvial flood from very intense rain
- **Groundwater:** Water rising from rocks or springs
- **Pluvial:** Urban flood from insufficient drainage capacity
- **Dam/Embankment Failures:** Sudden release of water that they were holding back

FLOOD RISK

$$\textbf{Risk} = \textbf{Flood Probability} \times \textbf{Impacts}$$

- **Risk Types:** Economic, health, social, environmental
- **Flood Risk Management Cycle**



- **Natural Flood Management:** Using natural processes to reduce flood risk & coastal erosion (eg. woodlands, wetlands, shingle beaches)

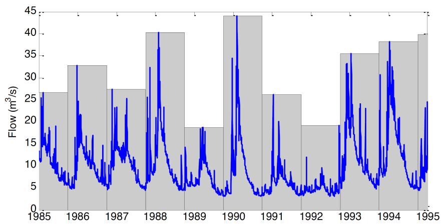
intercept rain, slow down flood — store floodwaters — natural defence against flood.
saltmarshes → absorb wave energy.

4.2 FLOOD FREQUENCY ANALYSIS

- To predict flood frequency
- To estimate flood sizes to design against

EXTREME VALUE SAMPLING

Annual Maxima Series (AMS)

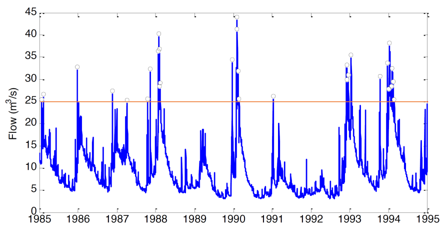


1. Group series by year (grey boxes)
2. Select the highest annual peaks

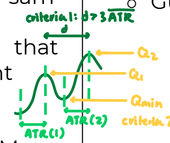
- Pro: Yields more samples
- Con: Need to separate interconnected peaks

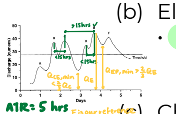
Peaks Over Threshold (POT)

- Used when <14 years of record and this yields the same series length as AMS



1. Select threshold (red line) based on some physical limit or such that there are N samples above it
2. Discard mutually related peaks, so that probability distribution is independent
   - (a) Start with the highest peak flow
   - (b) Eliminate all peaks with $3\times$ ATR
     - **Average Time To Rise (ATR):** Mean time taken for each peak to rise to its highest value in the discharge graph
   - (c) Check that minimum flow between remaining peaks is <2/3 of the ~~highest peak~~ first peak (the earlier one in time!)
   - (d) Repeat procedure with the next highest peak until all peaks are checked





ATR = 5 hrs
1. E highest ✓
2. D × (<3ATR)
3. F × ($Q_{EF,min} > \frac{2}{3}Q_E$)
4. C ✓ (>3ATR, $Q_{CE,min} > \frac{2}{3}Q_E$)
5. B × (<3ATR)

- Pros: Easy to extract data and get $P(q>q_d)$
- Con: Often only a small sample returned

FLOOD FREQUENCY CURVE

Probability of exceedance, $p = P(q>q_d) = \frac{1}{T}$ and $X \sim B(n,p)$ — probability of T years flood occuring in a given year; flood happens X times in n years.

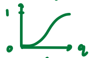
Return Period: 1 in T years probability

mainly we want to find T, i.e. $P(q>q_d)$; hence we find $F(q_d) = 1-P(q<q_d)$ many ways.

$$\boxed{T = \frac{1}{P(q>q_d)}}$$

- If POT data, multiply by correction factor $\frac{n}{t}$, where n = number of years, t = number of selected POT ← number of peaks / number of samples; n usually < t as multiple peaks can be in 1 year.
- **Flood Frequency Curve:** Plot of Q against T

Large Sample Sizes ($N>500$)



- Order the data and directly plot CDF (of q)
- $P(q>q_d) = 1-P(q\le q_d) \approx 1-\frac{m}{N}$
  - m = cumulative frequency, N = sample size

$P(q<q_m) = \frac{m}{N}$

Small Sample Sizes ($N<500$)

- May cause 2 samples to have the same $P(q>q_d)$ value or that $P(q>q_{\max})=0$
- **Plot Position Formulas:** Adjust probabilities to avoid $P(q>q_{m=N})=0$
  1. Arrange AMS in ascending order
  2. Calculate $P_m(q\le q_m)$ for every term q_m
     - **Weibull Formula:**

$$P_m(q\le q_m) = \frac{m}{N+1}$$

     - **Gringorten Formula:**

$$P_m(q\le q_m) = \frac{m-0.44}{N+0.12}$$

note that: both of these give $P(q<q_m)$! If want to find T, $T = \frac{1}{1-P(q<q_m)}$ and might need to $\times\frac{n}{t}$ if from POT!

  - Pros: Simple & usually give reasonable estimates for interpolation
  - Cons: Cannot give continuous curves & unuseful extrapolation

- **Fit Gumbel Distribution:** Avoids both problems
  - Gumbel CDF: $q_m \sim \text{Gumbel}(\alpha,\beta)$

$$F(q_m) = \exp\left(-\exp\left(\frac{\alpha-q_m}{\beta}\right)\right)$$

$$\boxed{\Rightarrow q_m = \beta z + \alpha}$$

need to get $F(q_m)$ with methods above if do linear regression.

  - Gumbel Variate: $z = -\ln(-\ln[F(q_m)])$
  - Estimation of α & β:
    - **Linear Regression:** Plot $q_m = \beta z + \alpha$ and find best-fit line
      - ∗ Need to estimate $F(q_m)$ by using position formulas above
    - **Method of Moments:** Match sample mean $\bar{x}$ and standard deviation s_x

$\bar{x} = \frac{\sum q_d}{N}$, $s_x^2 = \frac{\sum(q_d-\bar{x})^2}{N-1}$

$$\underbrace{\beta = \frac{s_x\sqrt{6}}{\pi}, \quad \alpha = \bar{x} - 0.5772\beta}_{\text{given in data sheet.}}$$

    - **L-moment Matching:** Find L-moments to calculate α & β

$$L_1 = \frac{1}{N}\sum_{m=1}^{N} q_m$$

$$L_2 = \frac{2}{N}\sum_{m=2}^{N}\left(\frac{q_m(m-1)}{N-1}\right) - L_1$$

$$\beta = \frac{L_2}{\ln(2)}, \quad \alpha = L_1 - 0.5772\beta$$

      - ∗ Less influenced by outliers

Tiny Sample Sizes

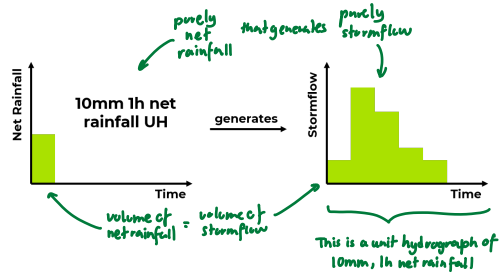
- Usually rare to have flow gauges near sites of interests during floods
- Thus, more samples of Q can be generated by:
  - **Regional Analysis:** Use data from well-gauged sites with similar hydrology
    - **Flood Estimation Handbook:** Uses data from other catchments to model the whole UK
  - **Numerical Models:** Generate flow data theoretically
    - Unit hydrographs

RAINFALL FREQUENCY ANALYSIS

- Same methods can be used for rainfall
- Depends on rain depth & duration
- Results are summarised as depth-duration-frequency curves

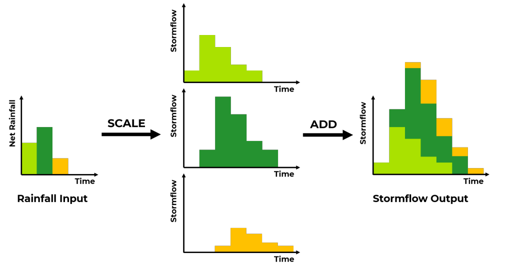
4.3 UNIT HYDROGRAPHS

- Stormflow hydrographs created from a unit depth of constant uniform net rainfall over a given catchment for a unit duration
- Links rainfall to stormflow generated



∗ If given rainfall (not net rainfall) have to get rid of losses first before applying UH!

HYDROGRAPH CONSTRUCTION



∗ volume of rainfall input (mm) = volume of stormflow output (mm³)

1. Scale UH to each net rainfall input given
   - Response is independent of start time
2. Superpose all outputs together to get the final hydrograph
3. Add baseflow if needed

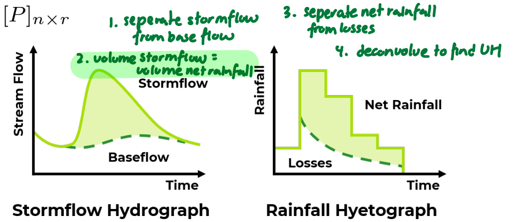
Stormflow Discharge: At time interval t_n

$$Q_n = \sum_{j=1}^{n\le m} P_j U_{n-j+1}$$

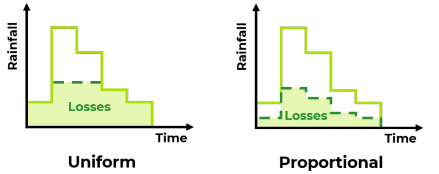
- m = number of rainfall inputs
- n = number of stormflow ordinates
- $r = n-m+1$ = number of UH ordinates

UNIT HYDROGRAPH DERIVATION (if given hydrograph and hyetograph)

1. Allocate losses & determine net rainfall $[P]_{n\times r}$



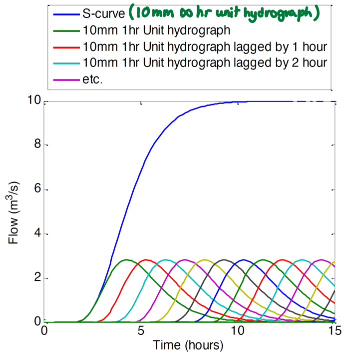
- Rainfall loss types:
  - **Initial Losses:** Interception & depression storage
  - **Continuing Losses:** Infiltration & evapotranspiration
- Modelled as uniform or proportional losses



2. Find stormflow $[Q]_{n\times 1}$ from hydrograph by separating baseflow
3. Use matrix inversion to solve for unit hydrograph $[U]_{r\times 1}$

$$[P]_{n\times r}[U]_{r\times 1} = [Q]_{n\times 1}$$

S-CURVE METHOD → to change ΔT of UH (eg. 10mm 1hr to 10mm 2hr UH)



method 1: this method can use to change ΔT_1 to any ΔT_2 (less than or greater than ΔT_1)

S-Curve: Response when net rain continues indefinitely at uniform intensity

ΔT_1's UH — calculation to find s-curve — MINUS! — ΔT_2's UH

always lag by interval of ΔT_1 to find s-curve!

to find UH, lag by ΔT_2

Time	1mm 1h UH	The 1mm 1h UH delayed 1h	2h	3h	4h	5h	6h	S-curve	S-curve lagged for 0.5h	0.5mm 0.5h UH
h	m³/s	m³/s	m³/s	m³/s	m³/s	m³/s	m³/s	m³/s	m³/s	m³/s
0	0							0		0
0.5	0.04							0.04	0	0.04
1	0.18	0						0.18	0.04	0.14
1.5	0.38	0.04						0.42	0.18	0.24
2	0.53	0.18	0					0.71	0.42	0.29
2.5	0.55	0.38	0.04					0.97	0.71	0.26
3	0.45	0.53	0.18	0				1.16	0.97	0.19
3.5	0.3	0.55	0.38	0.04				1.27	1.16	0.11
4	0.18	0.45	0.53	0.18	0			1.34	1.27	0.07
4.5	0.1	0.3	0.55	0.38	0.04			1.37	1.34	0.03
5	0.03	0.18	0.45	0.53	0.18	0		1.37	1.37	0
5.5	0	0.1	0.3	0.55	0.38	0.04		1.37	1.37	0
6		0.03	0.18	0.45	0.53	0.18	0	1.37	1.37	0
6.5		0	0.1	0.3	0.55	0.38	0.04	1.37	1.37	0

Note: The ΔT_2 UH's depth is $\frac{\Delta T_2}{\Delta T_1}$ of the ΔT_1 UH's depth

$$d_2 = \frac{\Delta T_2}{\Delta T_1} d_1$$

need to do this for both method 1 or 2!

Scale the ΔT_2 UH by $\frac{\Delta T_1}{\Delta T_2}$ to ensure same rainfall depth as ΔT_1 UH

- Scaled ΔT_2 UH should have a higher, earlier peak than the original ΔT_1 UH

method 2:

Special Case: If $\Delta T_2 = k\Delta T_1 > \Delta T_1$, no need to find S-curve to find duration PLUS!

$\Delta T_2 = k\Delta T_1$

→ lag ΔT_1's UH by $(k-1)\cdot\Delta T_1$

→ eg: 1mm 1h UH, find 1mm 2h UH → (lag by 1h, sum with no lag)

eg: If 1mm 0.5h UH, find 3mm 3h UH (lag by 0.5h, 1h, 1.5h, 2h, 2.5h all sum with no lag)

Time	1mm 1h UH	1mm 1h UH lagged for 1h	1mm 2h UH [sum(u)/2]
h	m³/s	m³/s	m³/s
0	0		0
0.5	0.04		0.02
1	0.18	0	0.09
1.5	0.38	0.04	0.21
2	0.53	0.18	0.355
2.5	0.55	0.38	0.465
3	0.45	0.53	0.49
3.5	0.3	0.55	0.425
4	0.18	0.45	0.315
4.5	0.1	0.3	0.2
5	0.03	0.18	0.105
5.5	0	0.1	0.05
6		0.03	0.015
6.5		0	0

already × by $\frac{\Delta T_1}{\Delta T_2}$

EFFECTS OF URBANISATION

- Higher stormflow volume & peak discharge, shorter times to peak
- UHs occur over shorter durations

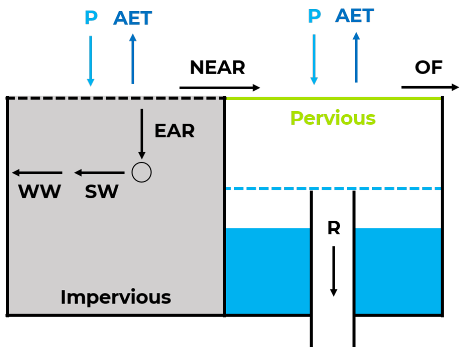
4.4 CONCEPTUAL MODELLING

MODELLING APPROACHES

- **Empirical/Black Box:** Relationships identified without explicit modelling of governing processes
  - Example: Unit Hydrograph
  - Improved by conceptual models by:
    - Not oversimplifying evapotranspiration losses & effective rainfall
    - Continuous streamflow time-series
    - Baseflow simulation with stormflow
    - Responses depend on initial storage
    - Responses can be non-linear

  solved by
- **Conceptual:** Governing processes modelled by relatively simple rules & concepts
  - Example: Stanford Watershed Model
- **Physically-Based:** Explicit equations & physical laws implemented numerically
  - Example: SHE, MIKE11

URBAN AREA SOIL ZONES



- **Overland Flow (OF):** Excess surface runoff
- **Groundwater Recharge (R):** Water added to groundwater stores
- **Wastewater (WW):** Direct water discharge through pipes
- **Actual Evapotranspiration:**

$$AET = \min(PET, IL, P)$$

  - **Potential Evapotranspiration:** Maximum amount of evaporatable water

$$PET = K_e \cdot ET_0$$

    - K_e = evaporation coefficient (0.1 for bare soil), ET_0 = reference evapotranspiration
  - IL = initial losses by depression storage or interception
  - P = precipitation

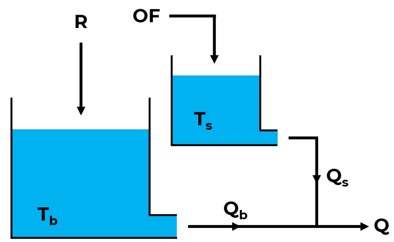
- **Effective Area Runoff:** Water that makes it to the stormwater network (SW)

$$EAR = EA \cdot \max(P - AET, 0)$$

  - $\max(P - AET, 0)$ = total runoff volume
  - EA = percentage of impervious area that is hydrualically connected to the SW
- **Non-Effective Area Runoff:** Water that doesn't make it to the SW

$$NEAR = (1 - EA) \cdot \max(P - AET, 0)$$

FLOW ROUTING FUNCTION



- Functions that keep track of flow variations by storing outflows into theoretical stores and monitoring residence time
- R = recharge, OF = overland flow
- V_b, V_s = water volume in baseflow & stormflow routing store
- T_b, T_s = baseflow & stormflow residence times
- Q_b, Q_s = baseflow & stormflow, where total flow $Q = Q_b + Q_s$

Example Algorithm

Assume volume balance & flows are proportional to stored volumes:

$$\frac{dV_b}{dt} = R_i - Q_{b,i-1}, \quad Q_b = \frac{V_b}{T_b}$$
$$\frac{dV_s}{dt} = OF_i - Q_{s,i-1}, \quad Q_s = \frac{V_s}{T_s}$$
$$\therefore \frac{dQ_b}{dt} = \frac{R_i - Q_{b,i-1}}{T_b}, \quad \frac{dQ_s}{dt} = \frac{OF_i - Q_{s,i-1}}{T_s}$$

CALIBRATION & VALIDATION

Calibration: Adjust parameters until simulated flow matches observed flow

- Typically uses 5 years of test data

Validation: Test parameter estimates on a different test period

- Test period should have some extremer events to fully test the model

Warm-up Period: Interval where results are inaccurate due to errors in initial conditions

4.5 FLOOD FORECASTING

FLOOD WARNING

- Allows timely evacuation, relocation of valuables, infrastructure management
  - Installation of sandbag walls, temporary covers to prevent flooding in subway stations, etc
- Allows activation of flood defences (eg. sluice gates) & emergency services
- **Warning Criteria:** When flood level is predicted to exceed a predefined threshold
  - Usually overbank flow
  - Flood warning grade is dependent on flood severity & prediction confidence
  - **Lead-Time:** Time lag between forecast & predicted time of flood
    - $\geq$ 30 minutes needed for adequate response
    - Environmental Agency aims for $\geq$ 2 hours lead-time

FLOOD FORECASTING

not design activity. not development planning.

Forecasting: 1. Real-time 2. operational activity, with 3. floods that will happen as predicted from 4. real meteorologic data

→ not flood that could happen. → not flood for a given period.

Integrated Flood Forecasting System: Monitor rainfall sources, feed data into flood models for forecasting & sound warnings when needed

Priorities of Forecasting

- Speed & accuracy, flood magnitude (optional)
  - Real-time data of rainfall & streamflow are used
  - If longer lead-time needed, rainfall predictions are used

Forecasting Methods

- **Local Experience** (if-then rule)
- **Linear Regression:** Using upstream flow data to predict if downstream flow will cause floods

$$\hat{x}_{k+T} = a \cdot u_k$$

  - u_k = upstream flow at interval k
  - $\hat{x}_{k+T}$ = modelled downstream flow T intervals later
  - a = parameter to be estimated

- Limitations:
  - If linear relationship is strong, data stations are likely too close & cannot give enough lead-time
  - Does not account for storage & net rainfall between the 2 gauges

(lead time ≤ 2 hours)

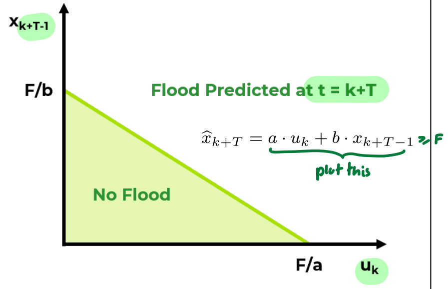
- **Linear Transfer Function:** Accounts for storage of water in the system

$$\widehat{x}_{k+T} = a \cdot u_k + b \cdot x_{k+T-1}$$

  - Usually adds extra term that involve the previous time-step
  - If flow threshold is F, then

$$a \cdot u_k + b \cdot x_{k+T-1} \geq F \Rightarrow x_{k+T-1} \geq \frac{F}{b} - \frac{a}{b} u_k$$

  - Can be plotted graphically



Forecasting Model Updating

- Real-time model adjustments to account for non-linearities & input errors

- **State:** Use real flow data as it becomes available instead of using predicted values
  - Pro: Prevents drifting from true flow due to cumulative errors
  - Con: Data may be inaccurate, can use weights to assign importance to observations and model predictions
- **Errors:** Use errors from past predictions to predict future errors to correct future predictions
- **Parameters:** Iterate model parameters as new data comes or by only using most recent data

$$\widehat{x}_{k+T} = a \cdot u_k + b \cdot x_{k+T-1}$$

parameters to be updated.

(every time we have a new u_k and x_{k+T-1})

RAINFALL FORECASTING

- Forecasting rainfall enables greater lead-time for predicting floods, but introduces more avenues for uncertainty

Statistical Methods: Extrapolate storm movements using radar/satellite data

- Intensity of radar echoes R is related to rainfall rate u, where a & b are location-specific parameters to be calibrated

$$u = aR^b$$

- Up to 1 hour rainfall forecast
- Useful when ground-gauged flow/rainfall does not provide enough lead-time
- High accuracy in short-term only

Numerical Weather Prediction: Process-based climate simulation models

- Up to 12 month climate forecast
- Useful for giving advanced flood indications
- Longer lead-time & larger area covered, but low resolution & very computationally expensive

RP 2021

how to find X and R for solving: $(R^T R)\theta = R^T X$?

$$\widehat{x}_{k+T} = a \cdot u_k + b \cdot x_{k+T-1}$$

$X = \hat{x}_{k+T}$, $R = [u_k \; x_{k+T-1}]$, $\theta = \begin{bmatrix} a \\ b \end{bmatrix}$

eg.

Time (h)	Upstream flow $m^3 s^{-1}$	Downstream flow $m^3 s^{-2}$
1	0.40	0.16
2	2.20	0.20
3	1.20	0.36
4		1.82
5		1.22

if model: $x_{k+2} = a u_k + b x_{k+1}$

$X = \begin{bmatrix} 0.36 \\ 1.82 \\ 1.22 \end{bmatrix}$, $R = \begin{bmatrix} 0.40 & 0.20 \\ 2.20 & 0.36 \\ 1.20 & 1.82 \end{bmatrix}$

$$\widehat{x}_{k+T} = \underline{\underline{a}} \cdot u_k + \underline{\underline{b}} \cdot x_{k+T-1}$$

to find a and b:

$(R^T R)\theta = R^T X$

$$\underbrace{\begin{bmatrix} 0.40 & 2.20 & 1.20 \\ 0.20 & 0.36 & 1.82 \end{bmatrix} \begin{bmatrix} 0.40 & 0.20 \\ 2.20 & 0.36 \\ 1.20 & 1.82 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.40 & 2.20 & 1.20 \\ 0.20 & 0.36 & 1.82 \end{bmatrix} \begin{bmatrix} 0.36 \\ 1.82 \\ 1.22 \end{bmatrix}}_{\text{solve this}}$$

Good Luck in Exams !

ALL THE BEST !