IMPERIAL COLLEGE LONDON

MEng Examination 2020

PART II

This paper is also taken for the relevant examination for the Associateship

CIVE95011: ENVIRONMENTAL ENGINEERING

26 May 2020: 12:00 – 15:00 BST

An extra 30 minutes will be added on to the times shown above in order for you to scan and upload your answers.

*This paper contains **THREE** questions.*

*Answer **ALL THREE** questions. Please start each new question **ON A FRESH PAGE** or **PUT A BLANK PAGE BETWEEN** each question.*

All questions carry equal marks.

Formulae sheets are provided at the end of the examination paper.

Q1. (Answer ALL parts of this question; total of 40 marks)

(a) How is the fluid potential for a unit mass of groundwater is related to hydraulic head and what simplifying assumptions are involved in its derivation?

Describe two situations where these simplifying assumptions might break down?

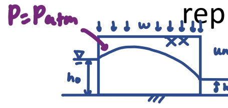
$\phi = zg + \frac{v^2}{2} + \int_{P_0}^{P} \frac{dP}{\rho}$ (fluid potential) $\rightarrow h = \phi/g = z + \frac{v^2}{2g} + \int_{P_0}^{P} \frac{dP}{\rho g}$ (hydraulic head) $\rightarrow h = z + \frac{P}{\rho g}$

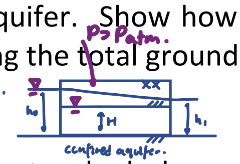
Assumptions:
① $\frac{v^2}{2g} \approx 0$ as groundwater flows slowly in water ② $P_0 = P_{atm} = 0$

assumption ① break down when v is not small:
1. In the vicinity of well that is being pumped at high rate.
2. In highly permeable formation.

[5 marks]

(b) Describe and illustrate diagrammatically how an unconfined aquifer differs from a confined aquifer. Show how this difference is generally handled mathematically when representing the total groundwater flow per unit width in each aquifer type?





confined: $Q' = HK\left(\frac{h_0 - h_1}{L}\right)$

unconfined: $Q' = w\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$

It meant: $Q' = -HK\frac{dh}{dx}$ for confined

$Q' = -hK\frac{dh}{dx}$ ← dupuit approx as no vertical head gradient: $\frac{dh}{ds} \approx \frac{dh}{dx}$

[5 marks]

(c) In groundwater hydrology, what is meant by the term *heterogeneous*. How can heterogeneous geological systems be treated as if they are homogeneous aquifers? Describe two geological features that would prevent an aquifer from being classed as homogeneous.

hydraulic property vary with space!

heterogeneous means vary with space $K(x,y,z) \neq c$ / homogeneous is opposite of heterogeneous, does not vary with space. $K(x,y,z) = c$

when at a big scale ~100m. a heterogeneous geological sys. can be treated as homogeneous.

Geological features:
structural changes between valleys and interfluves.
Faults, Alluvial Deposits...

[5 marks]

(d) The hydraulic conductivities of the Silty Sand and Clays units in Figure 1.1 (below) were obtained from falling head tests and have values of 2.3E-5 m s^{-1} and 3.0E-9 m s^{-1}, respectively. Using the information provided and assuming horizontal flow is negligible, calculate the flux into the underlying aquifer in mm per day and the elevation of the water level in piezometer P3. What uncertainties might affect your calculation?

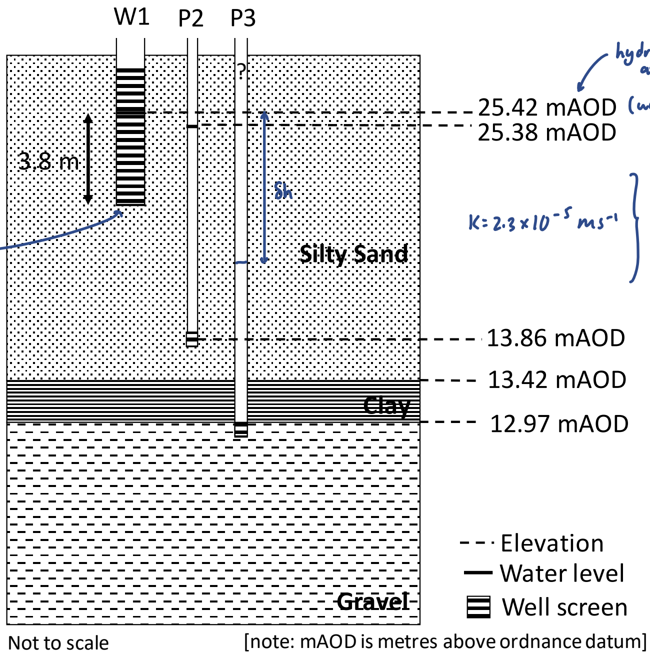
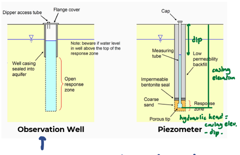


Figure 1.1

hydraulic gradient should start at water table!

(water table should be at this height) observation well.

hydraulic gradient SHOULD NOT START at the bottom of well.



obs well is only to tell us where is the water table!

$K = 2.3 \times 10^{-5}\ ms^{-1}$

we can find q of sand layer:

$q = -K\frac{dh}{dy}$

$= -2.3 \times 10^{-5}\left(\frac{25.38 - 25.42}{25.42 - 13.86}\right)$

$= 7.958 \times 10^{-8}\ ms^{-1}$

$K = 3 \times 10^{-9}\ ms^{-1}$

$\frac{25.42 - 12.97}{K} = \frac{25.42 - 13.42}{K_{silty}\ (2.3 \times 10^{-5})} + \frac{13.42 - 12.97}{K_{clay}\ (3 \times 10^{-9})}$

$K = 8.271 \times 10^{-8}\ ms^{-1}$

since entire groundwater flow is perpendicular to strata flow is in series (imagine like pipe flow)

$q_{silty} = q_{clay} = q_{overall}$.

$\therefore$ flux in the underlying aquifer $= q_{silty} = 7.958 \times 10^{-8}\ ms^{-1} = 6.88\ mm\ day^{-1}$

$q = -K\frac{dh}{dy}$

$\frac{dh}{dy} = \frac{7.958 \times \cancel{10^{-8}\ ms^{-1}}}{8.271 \times \cancel{10^{-8}\ ms^{-1}}} = 0.9622$

[5 marks]

$\delta h = 0.9622\,\delta y$

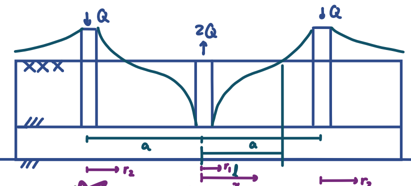
$= 0.9622(25.42 - 12.97)$

$= 11.98\ m$

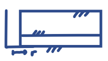
elevation of water level $= h - \delta h$

$= 25.42 - 11.98$

$= 13.44\ mAOD$

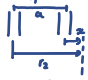


from the datasheets: confined aquifer:
$s = h_e - h = \frac{Q_w}{2\pi T} \ln\left(\frac{r_e}{r}\right)$ ← r-coords sys.



for the pumping well. (left) to define new x coordinate:
- realise that x and r are exactly same, from mid.
- $Q_w = 2Q$
$\therefore\ s = \frac{2Q}{2\pi T} \ln\left(\frac{r_e}{x}\right)$

for the left recharge well: to define new coordinate x:



- $r_2 = a + x$
- $Q_w = -Q$
$\} \ s = \frac{-Q}{2\pi T} \ln\left(\frac{r_e}{a+x}\right)$

for the right recharge well: to define new coord. x:

$x + r_3 = a$
- $r_3 = a - x$
- $Q_w = -Q$
$\} \ s = \frac{-Q}{2\pi T} \ln\left(\frac{r_e}{a-x}\right)$

MUST UNDERSTAND HOW TO DEFINE A NEW COORD SYS! AND WHY!

(e) An infinite, homogeneous and isotropic confined aquifer with a transmissivity T has thee fully penetrating wells positioned along a straight line and spaced a distance a apart. The central well is pumping at a rate of $2Q$ and the two adjacent wells are each recharging at a rate of Q. Assuming that the radius of influence is a same for all three wells, derive an expression for the distance from the central pumping well where the steady state drawdown is zero.

Define a new coordinate system x!

[6 marks]

$\therefore \Sigma s = \frac{Q}{2\pi T}\left[2\ln\left(\frac{r_e}{x}\right) - \ln\left(\frac{r_e}{a+x}\right) - \ln\left(\frac{r_e}{a-x}\right)\right]$
$= \frac{Q}{2\pi T} \ln\left(\frac{a^2 - x^2}{x^2}\right)$

when $s = 0$:
$\ln\left(\frac{a^2 - x^2}{x^2}\right) = 0$
$e^0 = 1$
$\frac{a^2}{x^2} - 1 = 1$
$\frac{a^2}{2} = x^2$
$x = \frac{a}{\sqrt{2}}$

(f) Describe a step-drawdown test. What is its purpose?

[4 marks]

pump well until reach quasi-steady state. measure the drawdown s_w and the Q. then increase till the next QSS. repeat.
} plot $\frac{s}{Q}$ against Q
$s_w = AQ + BQ^2$
$s_w/Q = A + BQ$ $m = B,\ c = A$

- to seperate total drawdown, s_w into two component.
$\underbrace{AQ}_{\text{formation loss}}$ and $\underbrace{BQ^n}_{\text{well loss}}$

A step test on a 0.6 m diameter well, sited 120 m from a river in full hydraulic connection with an unconfined aquifer, gave a well loss coefficient of B = 1010 m/(m^3/s)^2 and a hydraulic conductivity of 1.2E-4 m s^{-1}. The intake of the pump is located at 97.9 mAOD and the pump has a maximum pumping rate of 3 Ml d^{-1}. If the base of the aquifer is 89.5 mAOD and the mean river stage is at 115.8 mAOD, can the pump operate at its maximum rate during the dry season if the water level in the well prior to pumping was the same as the river stage. Given reasons for your answer.

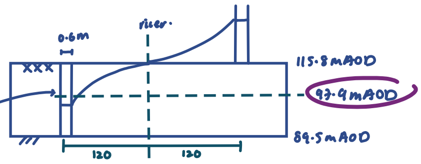
[10 marks]

[Note: mAOD is metres above ordnance datum; Ml d^{-1} is megalitres per day]

indicate BQ^n, n=2!

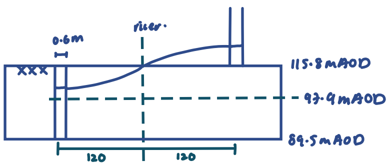
$B = 1010\,m/(m^3/s)^2$
$K = 1.2 \times 10^{-4}\,ms^{-1}$

$s_w = BQ^2$
$= 1010 \times$



not good as pump > water table

we don't actually know 97.9mAOD (the intake of pump) is higher or lower than the water table when $Q = 3Ml\,d^{-1}$



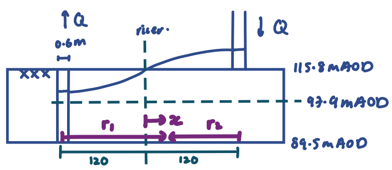
good as pump < water table.

so our aim is to find h when $Q_w = 3Ml\,d^{-1}$

first we use: well pumping, unconfined aquifer with recharge from datasheets

$h_e^2 - h^2 = \frac{Q_w}{\pi K} \ln\left(\frac{r_e}{r}\right) + \cancel{\frac{W}{2K}(r^2 - r_e^2)}$
0 no recharge.

$h_e^2 - h^2 = \frac{Q_w}{\pi K} \ln\left(\frac{r_e}{r}\right)$



$r_1 = 120 + x$, $x = 120 - r_2 \rightarrow r_2 = 120 - x$
$Q_1 = Q$ $Q_2 = -Q$

$h_e^2 - h^2 = \frac{Q}{\pi K} \ln\left(\frac{r_e}{120+x}\right) - \frac{Q}{\pi K} \ln\left(\frac{r_e}{120-x}\right)$
$= \frac{Q}{\pi K} \ln\left(\frac{120-x}{120+x}\right)$

to get the h at the pumping well (left)
$x = \frac{0.6}{2} - 120 = -119.7$
$h^2 = h_e^2 - \frac{Q}{\pi K} \ln\left(\frac{120+119.7}{120-119.7}\right)$; $h_e = 115.8 - 89.5 = 26.3\,m$
$Q = 3Ml\,d^{-1} = 3000\,000\,l\,d^{-1}$
$= 34.72\,l\,s^{-1}$
$= 0.03472\,m^3 s^{-1}$
$\therefore h = \sqrt{76.129} = 8.725\,m.$

lets convert both to drawdown, s_w
$s_w = h_e - h = 26.3 - 8.725 = 17.575$

BUT WAIT! This h is not the final h! we still need to minus well loss! BQ^2

$s_w = BQ^2$
$= 1010 \times 0.03472^2$
$= 1.218\,m$

$\Sigma s_w = 18.793\,m$

total drawdown

$\therefore$ final water table height
$= 115.8 - 18.793$
$= 97.007\,m < 97.9\,mAOD$ (cannot pump this MUCH!)

Q2. (Answer ALL parts of this question; total of 40 marks)

An engineering company builds an irrigation system for a large farm. The soils of the farm have a saturation point of 0.50 m^3 m^{-3}, a field capacity of 0.36 m^3 m^{-3}, and a wilting point of 0.10 m^3 m^{-3}. Water is taken from a nearby river. At the point of abstraction, the river has a catchment area of 1500 km^2. Historic river discharge (Q), precipitation (P), and reference evapotranspiration (ET_0) records are available for the water intake point. From these records, the engineers calculated the long-term monthly averages given in Table 2.1 (below).

-given info: $\theta_{FC} = 0.36\ m^3/m^3$, $\theta_{wp} = 0.10\ m^3/m^{-3}$, $\theta_{sp} = 0.50\ m^3/m^3$, $A = 1500\ km^2$,

Table 2.1. Long term monthly averages for key variable

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Q [10^6 m^3 $month^{-1}$]	31.9	41.7	31.8	24.7	17.1	12.3	4.8	3.1	5.1	11.7	20.1	28.0
P [mm $month^{-1}$]	71	105	78	70	57	43	21	14	32	43	51	68
ET_0 [mm $month^{-1}$]	120	115	119	126	138	144	165	170	155	143	130	125

$E_t = K_s K_c E_{t0}$: 96, 92, 95.2, 100.8, 110.4, 115.2, 132, 136, 124, 114.4, 104, 100

(a) Catchment water balance

Using the information given: when told to sketch "river flow regime" → sketch hydrograph (Q-t) not hyetograph (P-t)!

i. Sketch the river flow regime and discuss briefly its properties.

[6 marks]

hence Q also need to be in mm/month.

ii. Calculate the runoff ratio of the catchment. $RR = \frac{\bar{Q}}{\bar{P}} = \frac{0.0129 \times 10^3}{54.4167} = 0.237$ $\frac{Q\ m^3/month}{1500\ km^2}$

$\bar{P}$ is given in mm/month

[4 marks]

iii. If you know that the discharge data were obtained using the stage-discharge method, discuss the potential sources of error in the data.

vegetation cycles, ice cover, bank erosion. } main error: rating curve become unstable!

[6 marks]

+ stage discharge: stage data convert to discharge with rating curve

rating curve obtained by manually obtaining discharge

prone to error too!

this eqn can only be used when there's excess water, i.e. P > Ep

(b)(i)

after having E, P, Q and knowing long term ΔS=0 look like we can find R?

$\Delta S = P - E - Q - R$

$I = Ep - P + R$

can find how much I is required, need to find Ep first.

(b) Water resources

NO. Irrigation: R=0!

i. The farm will produce mangos, which is a tree crop with a constant K_c value of 0.8. If local regulations allow the farm to abstract 30% of the river discharge, and assuming sufficient water storage capacity on the farm, what is the maximum area of mango that can be irrigated? Assume that all rainfall can be consumed by the crop.

- for irrigation purposes, assume $K_s = 1.0$

$I = Ep - P = 1320$ mm/year $- 653$ mm/year
$= 667$ mm/year.

$Q = 232.3 \times 10^6\ m^3/year$
$= 154.867\ mm^3/year.$

30% of Q = 46.46.

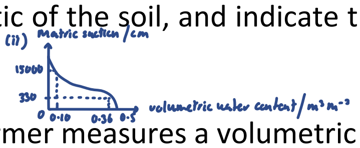
[8 marks]

only $46.46/667 \approx 0.06966$
$= 6.97\%$
of crop can be irrigated!

to find area of crop irrigated:
$6.97\% \times 1500\ km^2 = 104.5\ km^2$

ii. Sketch the soil moisture characteristic of the soil, and indicate the most important water retention points on your sketch.

(ii)



[4 marks]

iii. At a certain moment in time, the farmer measures a volumetric soil moisture content of 0.20 m^3 m^{-3} on her field. If you know that mangos have a depletion factor of 0.6, do the trees suffer water stress at that moment?

$TAW = 0.36 - 0.10 = 0.26\ m^3 m^{-3}$ } missing depth is fine.

$RAW = pTAW = 0.156\ m^3 m^{-3}$

experience stress when θ less than FC − RAW
= 0.20 = 0.36 − 0.156
= 0.204
0.20 < 0.204 → Experience water stress!

[4 marks]

(c) Environmental change

i. The current vegetation in the catchment is forest. Discuss what impacts you expect on the water supply for the irrigation farm if the catchment is deforested.

[8 marks]

Q3. (Answer ALL parts of this question; total of 40 marks)

(a) Flood hydrograph simulation

- hydrograph is 2h, but rainfall event (hyetograph) is given in units of 1h.
→ hence need to change. $\Delta T_1 = 2h$, $\Delta T_2 = 1h$.
$\Delta T_2 \neq k\Delta T_1$ where $k \in \mathbb{Z}^+$ hence: S-curve ONLY!

i. A catchment has a 2-hour 10mm unit hydrograph given in Table 3.1 (below). An $\boxed{\text{observed rainfall event has the intensity of 8 and 14 mm}}$ per hour, respectively. If the proportional losses are 35%, calculate the flood hydrograph and the peak flow produced by the storm.

These are total rainfall,
to find net rainfall:
$(1-0.35)\times 8 = 5.2mm$, $(1-0.35)\times 14 = 9.1mm$

[12 marks]

ii. What are the main limitations of the unit hydrograph approach?

[3 marks]

to get s-curve, delay every UH by 2hrs!

Table 3.1 2-hour 10mm unit hydrograph

Time (h)	Flow ($m^3 s^{-1}$)
0	0
1	0.8
2	2.3
3	1.8
4	1.0
5	0.7
6	0

2h 10mm UH delayed by:

10mm

2h 10mm UH	2h	4h	6h	S-curve	S-curve delay by 1h	1h 5mm UH
0	0	0	0	0	0	0
0.8	0	0	0	0.8	0	0.8
2.3	0	0	0	2.3	0.8	1.5
1.8	0.8	0	0	2.6	2.3	0.3
1.0	2.3	0	0	3.3	2.6	0.7
0.7	1.8	0.8	0	3.3	3.3	0
0	1.0	2.3	0	3.3	3.3	
0	0.7	1.8	0.8	3.3		
0	0	1.0	2.3	3.3		
		0.7	⋮	⋮		
		0				

don't actually need this
only need up to repeat...

1h 1mm UH	1h 5.2mm UH	1h 9.1mm UH	Stormflow, Q[mm]
0	0	0	0
0.16	0.832	0	0.832
0.30	1.560	1.456	3.016
0.06	0.312	2.730	3.042 ← max
0.14	0.728	0.546	1.274
0	0	1.274	1.274
0	0	0	0

(b) Flood frequency analysis

Table 3.2 (below) lists a total of 10 annual maxima daily river flows that were measured during the period 2000-2009.

Table 3.2 Annual maxima observations for daily flows

Year	Max. daily flow ($m^3 s^{-1}$)
2000	52
2001	66
2002	109
2003	40
2004	86
2005	87
2006	122
2007	97
2008	78
2009	51

i. Explain what are the two main problems that arise from using smaller numbers of flood peak samples to construct the flood frequency curve.

may have two sample with the same $P(q>q_d)$

$P(q>q_{max})$ might be $= 0$ (which shouldn't be)

[2 marks]

ii. Fit a Gumbel distribution to the annual maxima series given in Table 3.2 using the method of moments, and using the fitted distribution calculate the magnitude of the 100-year flood.

Method of moments equations are:

$$\alpha = \mu - 0.5772\,\beta \qquad \beta = \left(\frac{\sqrt{6}}{\pi}\right)\sigma$$

$\bar{x} = 78.8, \; s = 26.645$

$\beta = \frac{\sqrt{6}}{\pi} \times 26.645 = 20.775$

$\alpha = 78.8 - 0.5772\,(20.775)$
$= 66.80867$

$T = 100$ years

$P(q>q_d) = \frac{1}{T}$

$P(q>q_d) = \frac{1}{100}$

$F(q_d) = 1 - \frac{1}{100} = \frac{99}{100}$

$F(q_d) = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

$\frac{99}{100} = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

$q_d = 162.38\ m^3 s^{-1}$

[6 marks]

iii. Describe two methods that can be used to generate sufficient flood peak data when local flow measurements are either not available or there is a low level of confidence in their reliability.

use data from well gauge site with similar hydrology.

use conceptual or numerical modelling to generate data

[2 marks]

(c) Flood warning and forecasting

i. A real time flow forecasting model can be described by the following transfer function model:

$$\hat{x}_{k+2} = a u_k + b x_k$$

Where $\hat{x}_{k+2}$ is the downstream flow forecast with a lead time of 2 hours. For the data given in the Table 3.3 (below) calculate values of parameters *a* and *b* in the linear transfer function model by defining matrix *R* and vector *X* defined by the normal equation (see Formulae sheet).

[8 marks]

Table 3.3 Observed flow values for flood foresting modelling

Time (hours)	Upstream flow ($m^3\ s^{-1}$)	Downstream flow ($m^3\ s^{-1}$)
1	0.24	0.14
2	0.35	0.36
3	0.73	0.56
4	0.54	1.43
5	-	2.13
6	-	2.45

$X = [\hat{x}_{k+2}] = \begin{bmatrix} 0.56 \\ 1.43 \\ 2.13 \\ 2.45 \end{bmatrix}$

$R = [u_k \; x_k] = \begin{bmatrix} 0.24 & 0.14 \\ 0.35 & 0.36 \\ 0.73 & 0.56 \\ 0.54 & 1.43 \end{bmatrix} \qquad R^T = \begin{bmatrix} 0.24 & 0.35 & 0.73 & 0.54 \\ 0.14 & 0.36 & 0.56 & 1.43 \end{bmatrix}$

$(R^T R)\theta = R^T X, \; \theta = \begin{bmatrix} a \\ b \end{bmatrix}$

$\begin{bmatrix} 1.0046 & 1.3406 \\ 1.3406 & 2.5077 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3.5128 \\ 5.2895 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{pmatrix} 3.4731 & -1.8567 \\ -1.8567 & 1.3913 \end{pmatrix}\begin{bmatrix} 3.5128 \\ 5.2895 \end{bmatrix}$

$a = 2.379, \; b = 0.84$

ii. Using the fitted transfer function model calculate the value of flood threshold *F* assuming that the flood event that occurs at *t=4* lies on the flood-no flood boundary line.

$\hat{x}_4$ is already flood.

$\hat{x}_4 = au_2 + bx_2$
$= 1.13$ ← note that this predicted $\hat{x}_4$ (1.13) ≠ real $\hat{x}_4$ (1.43) ~0.3 error

$au_k + bx_k > 1.13$ means flood } hence flood boundary should follow estimation's one

[1 mark]

(d) Flood management and cities

i. Explain the main advantages and disadvantages of 1D overland flow simulation models for urban flood modelling.

[4 marks]

ii. Explain what is meant by ‘Robust’ and ‘Adaptive’ water infrastructure planning if their design applies the concept of Decision Making Under Deep Uncertainty.

[2 marks]

Formulae Sheet

A. Introduction to Hydrology (Dr Wouter Buytaert)

Catchment water balance	$\Delta S = P - E - Q - R$
Density of solids	$\rho_S = \frac{M_S}{V_S}$
Dry bulk density	$\rho_B = \frac{M_S}{V_T}$
Total bulk density	$\rho_T = \frac{M_S + M_L}{V_T}$
Porosity	$\varepsilon = \frac{V_L + V_G}{V_T}$
Void ratio	$e = \frac{V_L + V_G}{V_S}$
Gravimetric moisture content	$\theta_G = \frac{M_L}{M_S}$
Volumetric moisture content	$\theta = \frac{V_L}{V_T}$
Interrelating formula	$\theta = \theta_G \frac{\theta_B}{\theta_W}$
Penman Monteith	$E_a = K_c K_s E_{t,0}$
River flow	$Q = \int v(A)\, dA = \bar{v}A$
Rectangular weir	$Q = KbH^{1.5}$
Hydropower equation	$P = \varepsilon_t\, \varepsilon_h\, h\, \rho\, Q\, g$

Irrigation $\mathrm{I} = \mathrm{E_p} - \mathrm{P} + \mathrm{R}$

Total available water $\mathrm{TAW} = 1000\,(\theta_{FC} - \theta_{WP})\,Z_r$

Readily available water $\mathrm{RAW} = \mathrm{p\,TAW}$

B. Groundwater Systems (Prof Adrian Butler)

Darcy's law

$q_i = -K\dfrac{dh}{di}$, where i represents a coordinate direction (e.g. x, y, z)

Water table elevation in an unconfined aquifer

$$h^2 - h_0^2 = \frac{W}{K}\left(Lx - x^2\right) + \left(h_1^2 - h_0^2\right)\frac{x}{L}$$

Total flow per unit width in an unconfined aquifer

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$$

Well pumping confined aquifer without recharge (Thiem equation)

$$s = h_e - h = \frac{Q_W}{2\pi T}\ln\left(\frac{r_e}{r}\right)$$

Well pumping unconfined aquifer with recharge

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}\left(r^2 - r_e^2\right)$$

Theis solution

$$s = \frac{Q_w}{4\pi T}W(u), \text{ where } u = \frac{r^2 S}{4Tt}$$

Jacob's large time approximation

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

C. Flood Risk Estimation and Management (Dr Ana Mijic)

Weibull formula $P_m(Q \le q_m) = \frac{\mathrm{m}}{N+1}$

Gringorten formula	$P_m(Q \leq q_m) = \frac{\mathrm{m}-0.44}{N+0.12}$
Gumbel distribution	$F(q_d) = \exp\left[\ -\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$
Gumbel variate	$z = -\ln\{-\ln[F(q_d)]\}$
Probability for sequence of years	$P(x,n) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$
Moment matching	$\alpha = \mu - 0.5772\,\beta$ $\beta = (6^{\frac{1}{2}}/\pi)\sigma$
L-moment matching	$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j$ $L_2 = \frac{2}{n}\sum_{j=2}^{n}\left[\frac{(j-1)q_j}{n-1}\right] - L_1$ $\alpha = L_1 - 0.5772\,\beta$ $\beta = \frac{L_2}{\ln(2)}$
Normal equation for flood forecasting	$(R^T R)\theta = R^T X$