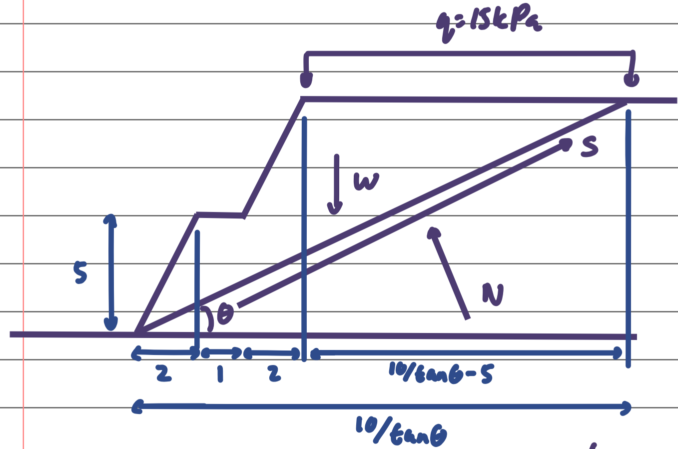
2020. What dependent variable is critical, and what is that independent variable.

this question: S_u depends on critical $\theta \rightarrow \partial S_u / \partial \theta = 0$!



If tresca condition (i.e. $\tau = S_u$)
N (or you may say σ') is
independent of S! (or τ)

$$Area = \frac{1}{2}(2\times5) + 5\times1 + \frac{1}{2}(5+10)(2) + \left(\frac{10}{\tan\theta} - 5\right)\times10 - \frac{1}{2}(10)\left(\frac{10}{\tan\theta}\right)$$

$$= \frac{100}{\tan\theta} - 25 - \frac{50}{\tan\theta}$$

$$= \frac{50}{\tan\theta} - 25$$

$$W = Area \times \gamma_w \quad [m^2]\times[kN\,m^{-3}]$$

$$= 20\left(\frac{50}{\tan\theta} - 25\right)$$

$$= \frac{1000}{\tan\theta} - 500 \quad kN\,m^{-1}$$

$$Q: 15kN\,m^{-2} \times \left(\frac{10}{\tan\theta} - 5\right) m$$

$$= \frac{150}{\tan\theta} - 75 \quad kN\,m^{-1}$$

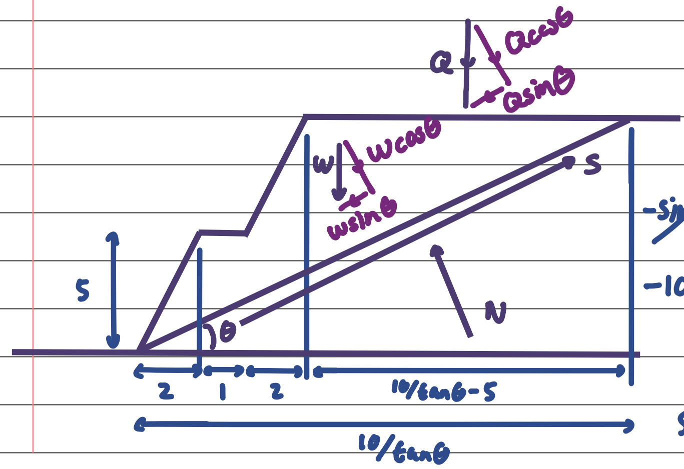
$$S = S_u \cdot L \quad [kN\,m^{-2}]\times[m]$$

$$= S_u\left(\frac{10}{\sin\theta}\right)$$

$$= \frac{10S_u}{\sin\theta} \quad kN\,m^{-1}$$

$$\Sigma F_{(N\text{-direction})} = 0$$

$$-W\cos\theta - Q\cos\theta + N = 0$$



$$\Sigma F_{(S\text{-direction})} = 0$$

$$-W\sin\theta - Q\sin\theta + S = 0$$

$$-\cancel{\sin\theta}\left(1000\frac{\cos\theta}{\cancel{\sin\theta}} - 500\right) - \sin\theta\left(150\frac{\cos\theta}{\sin\theta} - 75\right) + \frac{10S_u}{\sin\theta} = 0$$

$$-1000\cos\theta + 500\sin\theta - 150\cos\theta + 75\sin\theta + \frac{10S_u}{\sin\theta} = 0$$

$$-1150\cos\theta + 575\sin\theta + \frac{10S_u}{\sin\theta} = 0$$

$$S_u = 115\sin\theta\cos\theta - 57.5\sin^2\theta$$

$$\frac{\partial S_n}{\partial \theta} = -115\sin^2\theta + 115\cos^2\theta - 115\sin\theta\cos\theta = 0$$

$$\left(\sin^2\theta - \cos^2\theta + \sin\theta\cos\theta = 0\right) \div \cos^2\theta$$

$$\tan^2\theta - 1 + \tan\theta = 0$$

$$x = \tan\theta: \quad x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)}}{2} = 0.618, -1.618$$

$$\tan\theta = 0.618, -1.618$$

basic $\theta = 31.72, -58.28$. (choose +ive)

$$\theta = 31.72$$

$$S_n = 115\sin\theta\cos\theta - 57.5\sin^2\theta$$

$$= 115\sin(31.72)\cos(31.72) - 57.5\sin^2(31.72)$$

$$= 35.54$$

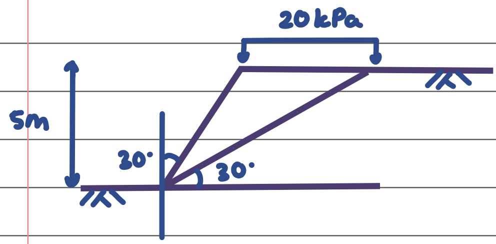
find N with $\quad \Sigma F_{(N\text{-direction})} = 0$

$$-\underline{\underline{W}}\cos\theta - \underline{\underline{Q}}\cos\theta + N = 0$$

so find W and Q first: $W = \frac{1000}{\tan\theta} - 500$, $Q = \frac{150}{\tan\theta} - 75$

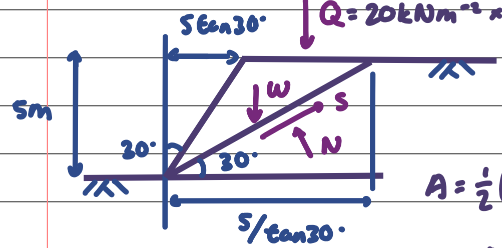
$$= 1117.87 \qquad\qquad = 167.68$$

2024. Mohr's Columb Failure always can $\tau = c' + n' \tan\varphi'$ to get one extra eqn.



$\gamma = 20 kN/m^3$
$c' = 8 kPa$

$Q = 20 kNm^{-2} \times (5/\tan 30° - 5\tan 30°) = 115.47 kNm^{-1}$



$\gamma = 20 kN/m^3$
$c' = 8 kPa$

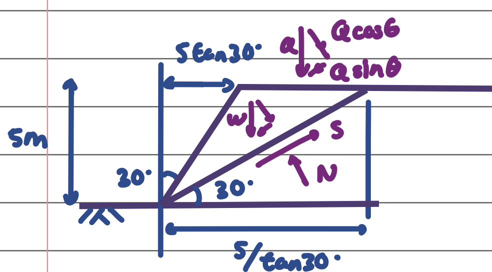
$A = \frac{1}{2}\left(\frac{5}{\tan 30°} \times 2 - 5\tan 30°\right)(5) - \frac{1}{2}(5)\left(\frac{5}{\tan 30°}\right) = 14.43$

$W = A\gamma = 288.68 \ kNm^{-1}$

so find S, and N through equilibrium to find τ and σ' → to find φ'

$N = \sigma' l \ ; \ S = \tau l \qquad l = \sqrt{5^2 + (5/\tan 30°)^2} = 10$

$\underbrace{N = 10\sigma' \ ; \ S = 10\tau}_{\tau = c' + \sigma' \tan\varphi'}$



$\Sigma F_S = 0$
$-Q\sin 30° - W\sin 30° + S = 0$
$S = 202.075 \ kNm^{-1}$

$\Sigma F_N = 0$
$-Q\cos 30° - W\cos 30° + N = 0$
$N = 350 \ kNm^{-1}$

$\underbrace{\tau = \frac{S}{10} = 20.2075 \qquad \sigma' = \frac{N}{10} = 35}$

$20.2075 = 8 + 35\tan\varphi'$
$\varphi' = 19.23°$

2021.

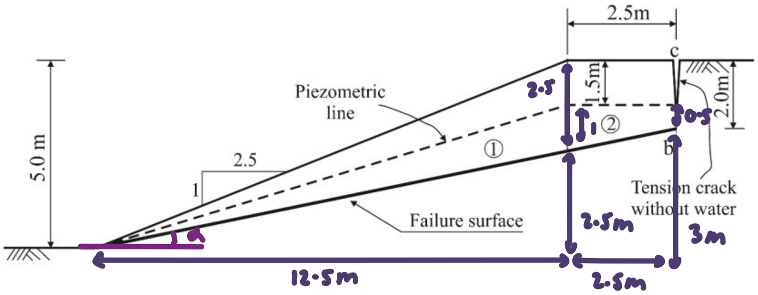
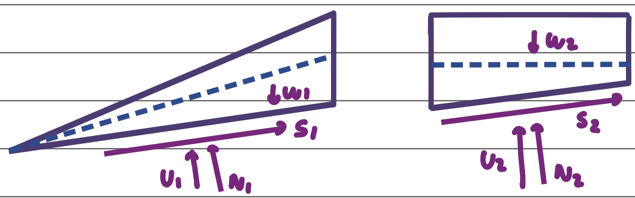


Figure Q4

$\gamma = 19$, $c' = 0$, find φ'



$$f_s = \frac{c'l + N\tan\varphi'}{S}$$

$W_1 = 19 \times \left\{\frac{1}{2}(12.5 \times 5) - \frac{1}{2}(12.5 \times 2.5)\right\}$
$= 296.875 \text{ kN m}^{-1}$

$W_2 = 19 \times \left\{\frac{1}{2}(2.5+2)(2.5)\right\}$
$= 106.875 \text{ kN m}^{-1}$

$U_1 = 10 \times \left\{\frac{1}{2}(12.5)(1+2.5) - \frac{1}{2}(12.5)(2.5)\right\}$
$= 62.5 \text{ kN m}^{-1}$

$U_2 = 10 \times \left\{\frac{1}{2}(1+0.5)(2.5)\right\}$
$= 18.75 \text{ kN m}^{-1}$

I JUST REALISED THIS IS A SHORTCUT THAT DOESN'T WORK ALL THE TIME. BETTER USE 2017'S METHOD ?

(c) find φ', given $F_s = \dfrac{\Sigma(c'l + (W\cos\alpha - U)\tan\varphi')}{\Sigma(W\sin\alpha)}$

this is to hint that:
$N = W\cos\alpha - U$, $S = W\sin\alpha$
so if you don't know F_s, when do $\Sigma F_s = 0$ and $\Sigma F_N = 0$, you will get the same expression !

$F_s = 1$ for critical failure:

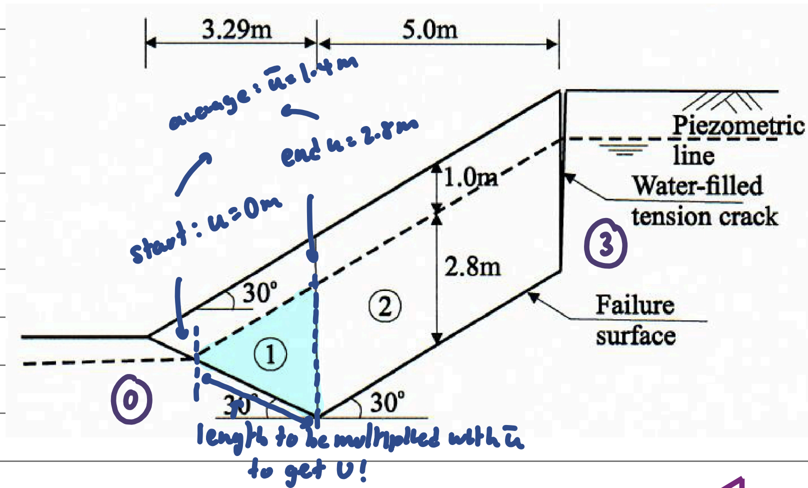
$\Sigma W\sin\alpha = \Sigma\left(\cancelto{0}{c'l} + (W\cos\alpha - U)\tan\varphi'\right)$

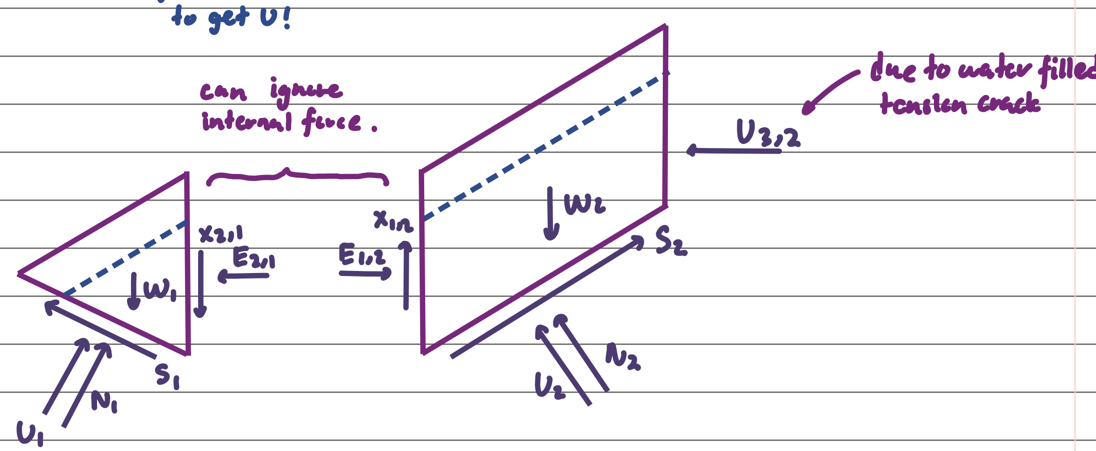
$296.875\sin(11.3) + 106.875\sin(11.3) = \left(296.875\cos(11.3) - 62.5 + 106.875\cos(11.3) - 18.75\right)\tan\varphi'$

$\tan\varphi' = 0.2516$

$\varphi' = 14.12°$

2017.

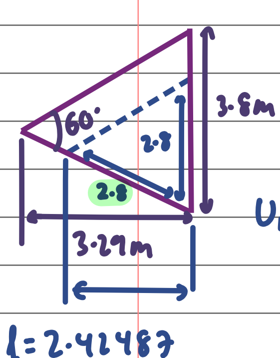




(a) W and U are found by W: Area of slice × γ_{sat} ; U: Average pore water pressure × length.

$$W_1 = \underset{[m]}{\tfrac{1}{2}(3.8)}\underset{[m]}{(3.29)} \times \underset{[kNm^{-3}]}{20} = 125.02\ kNm^{-1}$$

$W_2 = 3.8 \times 5 \times 20$
$= 380\ kNm^{-1}$

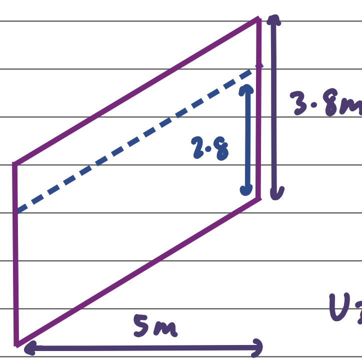


$l = \frac{1.4}{\tan 30^\circ} = 2.42487$

$l = 2.42487$

$\bar{u}_1 = \frac{1}{2}(0+2.8) = 1.4m$

$U_1 = 1.4m \times 2.8m \times 9.81\ kNm^{-3}$
$= 39.4552\ kNm^{-1}$



$\bar{u}_2 = \frac{1}{2}(2.8+2.8) = 2.8\ m$

$U_2 = 2.8m \times \frac{5}{\cos 30^\circ} m \times 9.81\ kNm^{-3}$
$= 158.5866\ kNm^{-1}$

$U_{3,2} = \frac{2.8 \times 9.81}{2} \times 2.8$
$= 38.4552$

$\Sigma F_{(N\text{-direction})} = 0$
$N_1 + U_1 - W_1\cos 30^\circ = 0$
$N_1 = -39.4552 + 125.02\cos 30^\circ$
$= 69.81\ kNm^{-1}$

$\Sigma F_{(N\text{-direction})} = 0$
$N_2 + U_2 - W_2\cos 30^\circ + U_{3,2}\sin 30^\circ = 0$
$N_2 = -158.5866 + 380\cos 30 - 38.4552\sin 30^\circ$
$= 151.28\ kNm^{-1}$

$\Sigma F(s\text{-direction}) = 0$

$W_1 \sin 30^\circ - S_1 = 0$

$S_1 = 125.02 \sin 30^\circ$

$= 62.50\ kNm^{-1}$ (to the left)

$\Sigma F(s\text{-direction}) = 0$

$S_2 - W_2 \sin 30^\circ - U_{3,2} \cos 30 = 0$

$S_2 = 380 \sin 30^\circ + 38.4552 \cos 30^\circ$

$= 223.30\ kNm^{-1}$ (to the right)

(b)

$$F_S = \frac{c'\Sigma l + \tan\varphi'\Sigma N}{\Sigma S}$$

$1.1\Sigma S = c'\Sigma l + \tan\varphi'\Sigma N$ (c' = 0 is more critical, we don't know value of c' so put = 0 !)

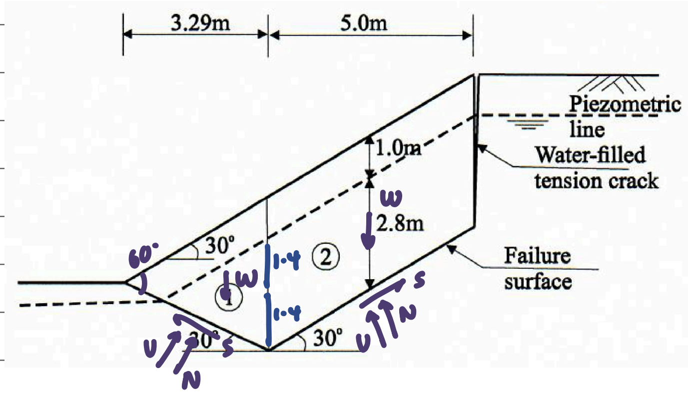
$1.1(-62.50 + 223.30) = \tan\varphi'(69.81 + 151.28)$

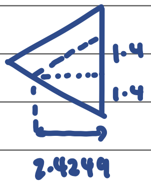
$\tan\varphi' = 0.8$

$\varphi' = 38.66^\circ$

(c)

$$F_S = \frac{\tan 32^\circ\ \Sigma N}{\Sigma S} = 0.859 < 1 \text{ (fail)}$$





$33.9486 + N - W\cos 30° = 0$

$N =$