IMPERIAL COLLEGE LONDON

MEng Examination 2023

PART II

This paper is also taken for the relevant examination for the Associateship.

CIVE 50010: STRUCTURAL MECHANICS 2

Tuesday 16 May 2023, 09:30 to 12:30 BST

*This paper contains **FOUR** questions.*

*Answer **ALL FOUR** questions.*

Questions 1, 3 and 4 carry 20 marks.
Question 2 carries 10 marks.

Formula Sheets are provided.

1. A Z-section with the centreline dimensions shown in Figure Q1 is subject to the loading also shown in the figure.

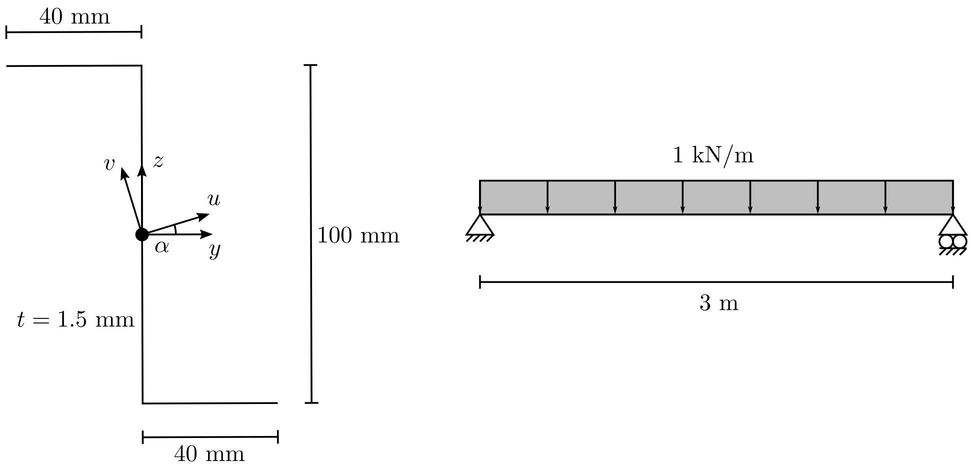


Figure Q1

a) Calculate the second moments of area I_{yy}, I_{zz}, I_{yz}, $I_{\max} = I_{uu}$ and $I_{\min} = I_{vv}$, and the angle α. Your solution should include a clear sketch of Mohr's circle showing the position of the pole of normals and indicating the major and minor bending axes.

[12 marks]

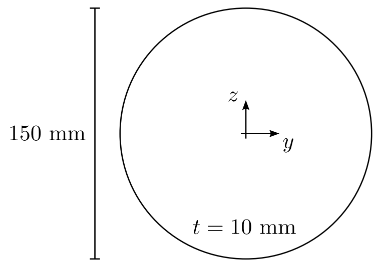
b) Sketch the shear force and bending moment diagrams.

[2 marks]

c) If the Z-section is used in the orientation that maximises the flexural rigidity calculate the normal stress distribution due to bending at the location along the beam where the maximum stresses due to bending will occur and sketch this clearly indicating any significant values. Your solutions should include a clear sketch to show the orientation of the cross-section indicating the major bending axis.

[6 marks]

2. A hollow circular section with the centreline dimensions shown in Figure Q2 is subject to the loading also shown in the figure.



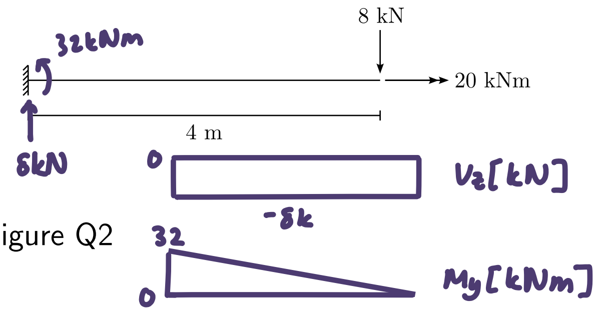


Figure Q2

a) Calculate the normal stress distribution due to bending at the location along the beam where the maximum stress due to bending will occur and sketch this clearly indicating any significant values. You should calculate the value of the second moment of area of a hollow circular section as:

$$\sigma_{xx} = \frac{M_y}{I_{yy}} z + \cancel{\frac{M_z}{I_{zz}} y} = \frac{32\times10^6}{1.3254\times10^7} z$$

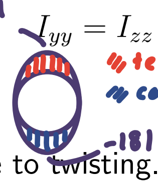
0

at $z = 75$, $\sigma_{xx} = 181$ N/mm²

$$I_{yy} = I_{zz} \approx \pi r^3 t$$

$I_{yy} = \pi(75)^3(10)$

$= 1.3254\times10^7$ mm⁴



tension

compression.

[3 marks]

b) Calculate the shear stress due to twisting.

$\tau = \frac{T}{2A_c t}$ (for closed section), since $t = 10$ constant: $\tau = \frac{20\times10^6}{2(\pi\times75^2)\times10} = 56.59$ N/mm²

[2 marks]

c) Calculate the maximum value of von Mises stress and state where this occurs in the beam. You may assume that the shear stress due to shear force is negligible. State if the section will yield under the applied loading, assuming it is manufactured from grade S355 steel.

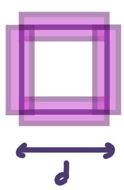
$T(s)$ is constant, hence σ_{vm} max depends on σ_{xx}.

$\sigma_{xx,max} = 181$

$\therefore \sigma_{vm,max} = \sqrt{181^2 + 3(56.59)^2} = 205.84 < f_y$

[2 marks]

d) A colleague suggests using a hollow square section rather than a hollow circular section. Using reasonable approximations find the side length of the hollow square section required to achieve the same second moment of area, if the thickness is kept the same. State potential advantages and disadvantages that using a hollow square section could introduce.



$$I_{yy} = \overbrace{\frac{d\times10^3}{12}}^{\text{flange } I_{yy} \text{ negligible}} \times 2 + \frac{10\times d^3}{12} \times 2 + (10d)\times\frac{d}{2}^2 \times 2$$

[3 marks]

I_{yy} (part a) $= 1.3254\times10^7$

$\frac{35}{3} d^3 = 1.3254\times10^7$

$d = 104.34$ mm

3. The three-span beam ABMCD is shown in Figure Q3. It is pinned at A and supported through rollers at B, C, and D. A uniform distributed load of 10 kN/m is applied on the second span BMC. The flexural rigidity of all the members is $EI=10000\ kNm^2$

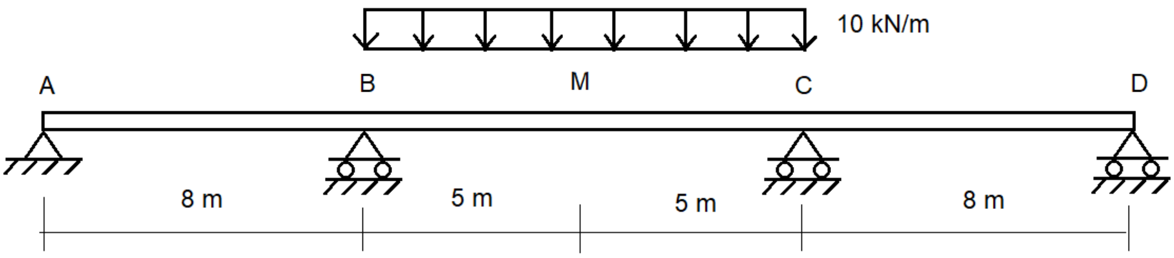


Figure Q3

(a) Calculate the redundancy of the structure.

[1 mark]

Given the symmetry of the structure and the loading, you can assume the identical values of the vertical reactions at A and D ($R_A=R_D$), or the bending moments at B and C ($M_B=M_C$). By using one of these two assumptions, answer to the following questions:

(b) Describe how you would obtain the vertical reaction at A using the flexibility method. State very clearly which statically determinate structure, load cases, and compatibility equation need to be considered, and how to obtain the corresponding flexibility coefficients. State the magnitude that each of the flexibility coefficients is representing

[6 marks]

(c) Obtain the bending moment diagram in each of the load cases stated in your response to point b).

[4 marks]

(d) Calculate the flexibility coefficients stated in your response to point b).

[4 marks]

(e) Calculate the vertical reaction at A.

[1 mark]

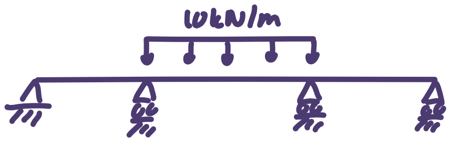
(f) Calculate and sketch the bending moment diagram and annotate the values of the bending moments at A, B, M, C, and D.

[2 marks]

(g) Calculate the vertical displacement of the beam at point M.

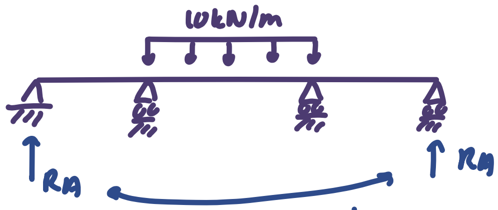
[2 marks]

(a)

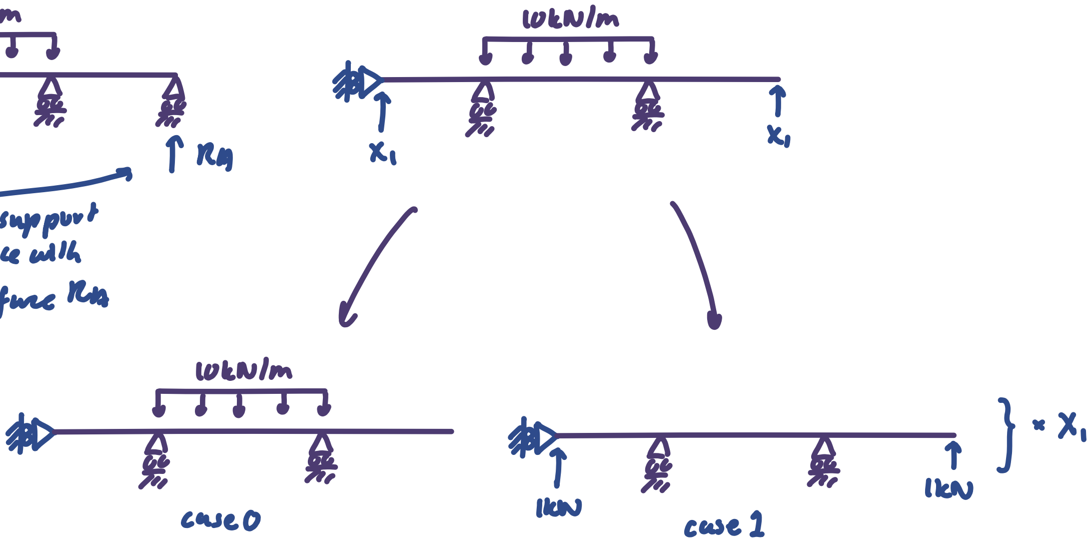


redundancy = 5 − 3 = 2

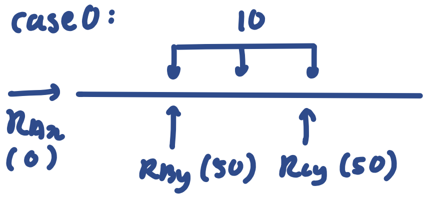
(b)



remove support replace with external force RA



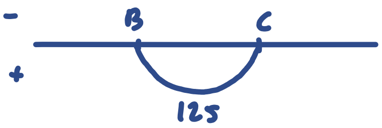
case 0:



$\sum M_{mid} = 0$

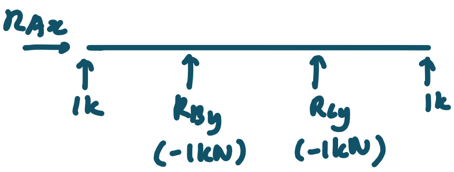
$50k(5) - 10k\left(\frac{5^2}{2}\right) - M = 0$

$M = 125\ kNm$



M [kNm]

case 1:



$\sum M_B = 0$

$1k(8) - R_{cy}(10) - 1k(18) = 0$

$R_{cy} = -1k$

$\sum F_y = 0$

$1k + 1k - 1k + R_{By} = 0$

$R_{By} = -1k$

at B: $\sum M_B = 0$,

$1k(8) - M = 0$

$M = 8\ kNm$

at C: $\sum M_c = 0$,

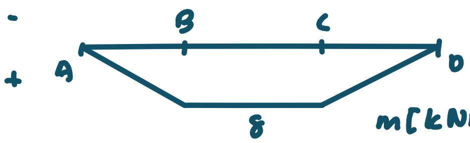
$1k(18) - 1k(10) - M = 0$

$M = 8\ kNm$

at mid, $\sum M_{mid} = 0$,

$1k(13) - 1k(5) - M = 0$

$M = 8\ kNm$.



m [kNm]

$f_{10} = \int m \frac{M}{EI} dx$

$= \frac{2}{3}(125)(8)(10)$

$= \frac{20000}{3}\left(\frac{1}{EI}\right)$

$f_{11} = \int m \frac{m}{EI}$

$= \frac{8 \times 8}{6}(2 \times 8) \times 2 + 8(8)(10)$

$= \frac{2944}{3}\left(\frac{1}{EI}\right)$

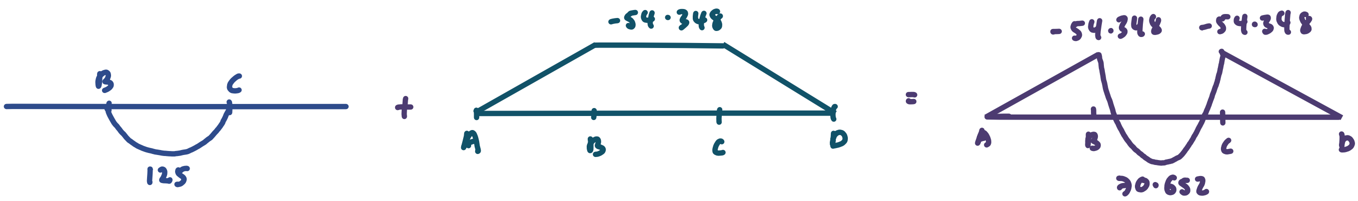
$f_{10} + X_1 f_{11} = 0$

$X_1 = -6.793$

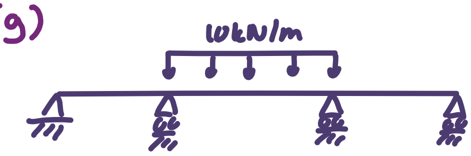
(f)

case 0:

case 1 × $X_1(-6.793)$:

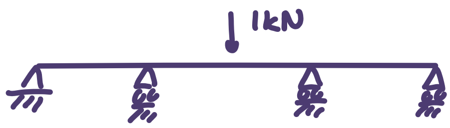


(g)

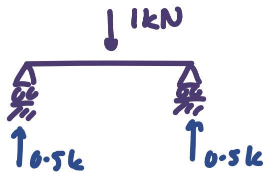


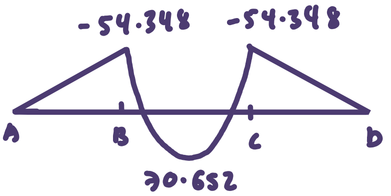
to find δ at M :

virtual case, with unit load at M.



simplify it.





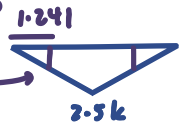
M[kNm]

this happen when case 0 from part (b), M=+54.348 :

$50x - 10\left(\frac{x^2}{2}\right) - 54.348 = 0$

$x = 1.241$

we need to find what length when M=0!



$\frac{1.241}{5} \times 2.5k = 0.6205$

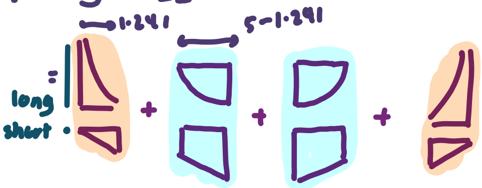
m [kNm]

$\Sigma M_m = 0$

$0.5k(5) - M = 0$

$M = 2.5\text{kNm}.$

$f_M = \int m \frac{M}{EI} dx$



$f_i \setminus f_j$	a, L	a_1, a_2, L	αL, βL, a, L	a, L †	a, L long	a, L †
b, L	abL	$\frac{bL}{2}(a_1+a_2)$	$\frac{1}{2}abL$	$\frac{2}{3}abL$	$\frac{1}{3}abL$	$\frac{2}{3}abL$
~~b, L~~ long	$\frac{1}{2}abL$	$\frac{bL}{6}(a_1+2a_2)$	$\frac{abL}{6}(1+\alpha)$	$\frac{1}{3}abL$	$\frac{1}{4}abL$	$\frac{5}{12}abL$
b, L short	$\frac{1}{2}abL$	$\frac{bL}{6}(2a_1+a_2)$	$\frac{abL}{6}(1+\beta)$	$\frac{1}{3}abL$	$\frac{1}{12}abL$	$\frac{1}{4}abL$
b_1, b_2, L	$\frac{aL}{2}(b_1+b_2)$	$\frac{L}{6}[2a_1b_1+a_1b_2+a_2b_1+2a_2b_2]$	$\frac{aL}{6}[(1+\beta)b_1+(1+\alpha)b_2]$	$\frac{aL}{3}(b_1+b_2)$	$\frac{aL}{12}(b_1+3b_2)$	$\frac{aL}{12}(3b_1+5b_2)$
αL, βL, b, L	$\frac{1}{2}abL$	$\frac{bL}{6}[(1+\beta)a_1+(1+\alpha)a_2]$	$\frac{1}{3}abL$ ‡	$\frac{abL}{3}(1+\alpha\beta)$	$\frac{abL}{12}(1+\alpha+\alpha^2)$	$\frac{abL}{12}(5-\beta-\beta^2)$

5-1.241

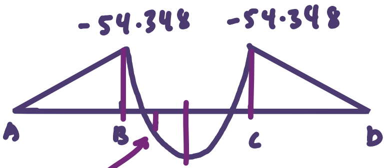
$= \frac{1}{12} \times (-54.348) \times 0.6205 \times 1.241 \times 2 + \frac{125 \times 3.759}{12}(3 \times 0.6205 + 5 \times 2.5) \times 2$

$= \frac{1117.71}{10000} = 0.11177\text{ m} = 111.77\text{ mm}$

something is wrong I can't find it, check alternative method which I got the correct answer.

---

alternative method: simpson's rule.



need to find this mid length's $M \to f\left(\frac{a+b}{2}\right)$

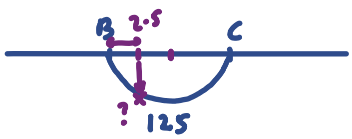
M[kNm]

cut into two parts cause not continuous here.

m [kNm]

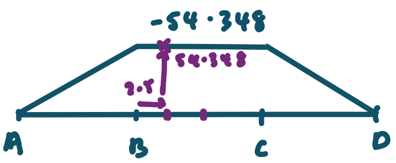
recall that:

case 0:

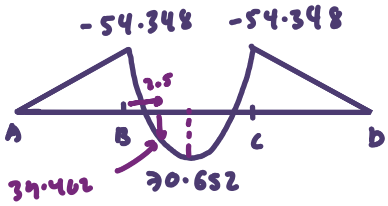


+

case 1 × X_1 (-6.793) :



=

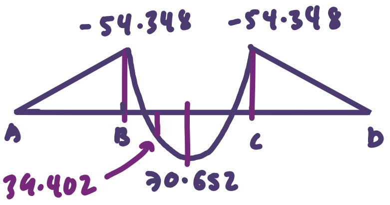


$\Sigma M_{x=2.5} = 0$

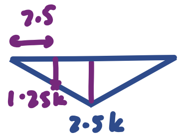
$50(2.5) - 10\left(\frac{2.5^2}{2}\right) - M = 0$

$M = 93.75$

$\Sigma M = 93.75 - 54.348 = 39.402$



M[kNm]



m[kNm]

$$f_m = \int m\frac{M}{EI}dx = \frac{5}{6}\Big[\underbrace{-54.348\times 0}_{f(a)} + \underbrace{4\times 34.402\times 1.25}_{4f\left(\frac{a+b}{2}\right)} + \underbrace{70.652\times 2.5}_{f(b)}\Big] \underline{\times 2}$$

symmetry.

$$= \frac{622.733}{10000}$$

$$= 0.06227\,m = 62.3\,mm$$

4. (a) Figure Q4(a) shows a continuous beam ABCDE with three spans of flexural rigidity EI_1 = 7 MNm^2, EI_2 = 5 MNm^2 and EI_3 = 10 MNm^2. The beam has a pinned support at A, a fixed support at E and is continuous over simple supports at C and D. Span AC is subjected to a point load V = 15 kN at B, while span DE is subjected to a uniformly distributed load q = 8 kN/m. Considering the moment distribution method:

   i. Calculate the distribution factors.

   [2 marks]

   ii. Calculate the fixed end moments.

   [2 marks]

   iii. Determine the bending moments at the supports A, C, D and E.

   [6 marks]

(b) The plane frame ABCD shown in Figure Q4(b) is to be modelled using three inextensible beam elements AB, BC and CD. It has fixed supports at A and D and the element lengths are shown in the figure. Members AB and CD have flexural rigidity EI_1 = 3.5 MNm^2, while member BC has a flexural rigidity EI_2 = 8 MNm^2. The structure is subjected to a horizontal point load H = 35 kN at B and a vertical load q = 9 kN/m uniformly distributed along member BC.

   i. Identify the kinematic indeterminacy.

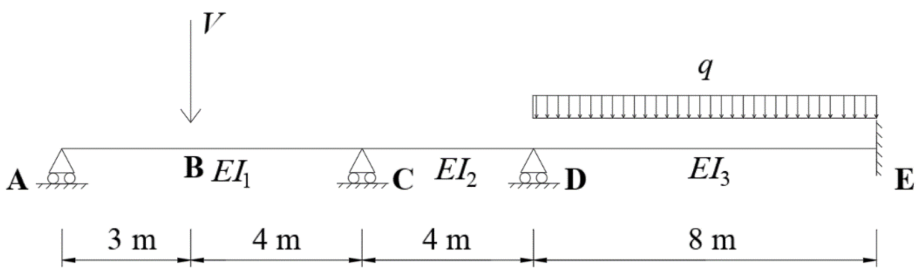
   [1 mark]

   ii. Calculate the vector of the equivalent nodal forces.

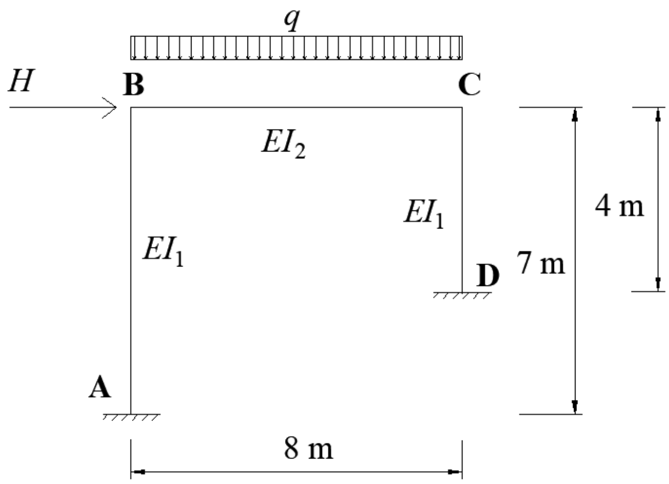
   [3 marks]

   iii. Determine the system stiffness matrix.

   [6 marks]



(a)



(b)

Figure Q4

EXAMINATION SOLUTIONS
2022/2023Session

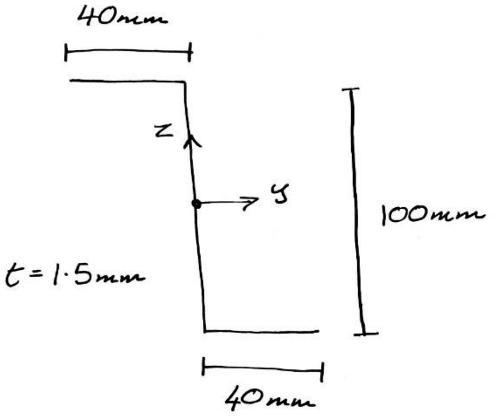
Title of Paper: Structural Mechanics 2

Paper Set By: Dr Andrew Phillips (Q1 and Q2), Dr Ana Ruiz-Teran (Q3), Dr Fiona Walport (Q4)

Please write on this side only, legibly and neatly, between the margins

Question Number: 1 — **Marks Allocated:**

a)



$$I_{yy} = \frac{1{\cdot}5 \times 100^3}{12} + \frac{2 \times 40 \times 1{\cdot}5^3}{12} + 2 \times 40 \times 1{\cdot}5 \times 50^2$$

$$= \underline{0{\cdot}4250 \times 10^6}\ mm^4$$ **2**

$$I_{zz} = \frac{2 \times 1{\cdot}5 \times 40^3}{12} + \frac{100 \times 1{\cdot}5^3}{12} + 2 \times 40 \times 1{\cdot}5 \times 20^2$$

$$= \underline{0{\cdot}0640} \times 10^6\ mm^4$$ **2**

$$I_{yz} = -\left[40 \times 1{\cdot}5 \times -20 \times 50 + 40 \times 1{\cdot}5 \times 20 \times -50\right]$$

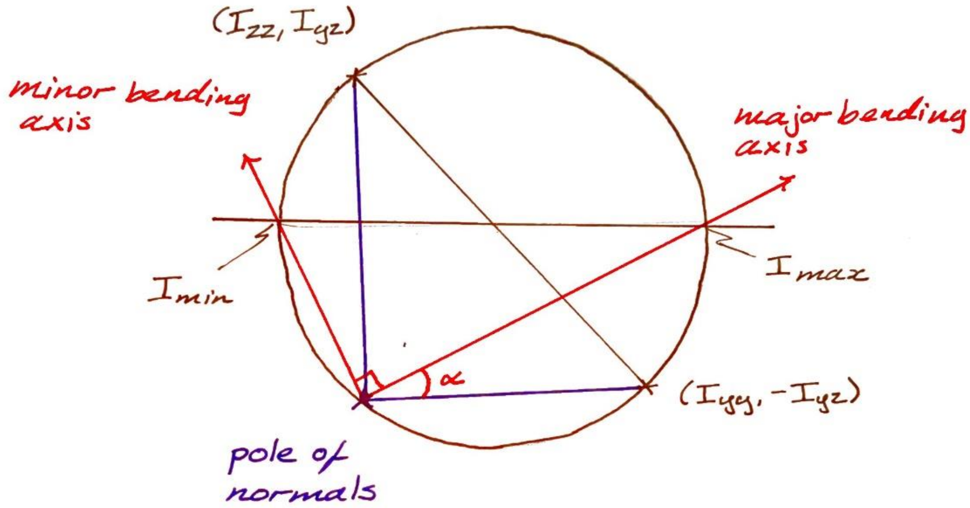
$$= \underline{0{\cdot}12 \times 10^6}\ mm^4$$ **2**

$$c = \frac{I_{yy} + I_{zz}}{2} = \underline{0{\cdot}2445 \times 10^6}\ mm^4$$

$$r = \sqrt{\left[\left(\frac{I_{yy} - I_{zz}}{2}\right)^2 + I_{yz}^2\right]} = 0{\cdot}2167 \times 10^6\ mm^4$$

$I_{max} = I_{uu} = c + r = 0.4612 \times 10^6 \text{ mm}^4$

$I_{min} = I_{vv} = c - r = 0.0278 \times 10^6 \text{ mm}^4$ **2**

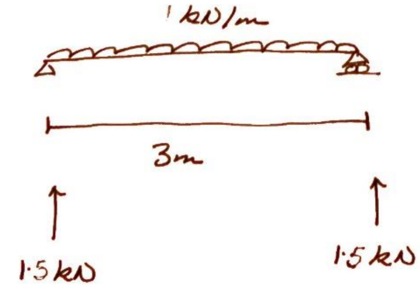


3

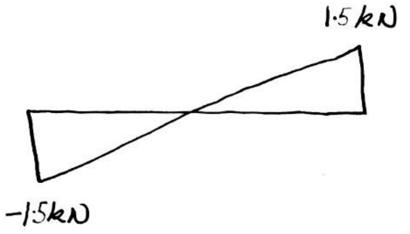
$$\alpha = \tan^{-1} \frac{I_{yz}}{I_{max} - I_{zz}} = \tan^{-1} 0.3021$$

$\therefore \ \alpha = 16.81^\circ$ **1**

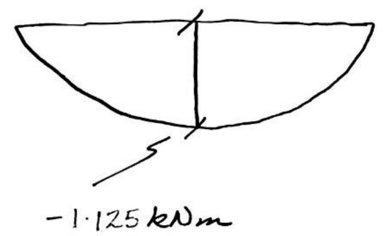
b)



SFD

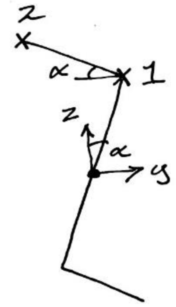


BMD



2

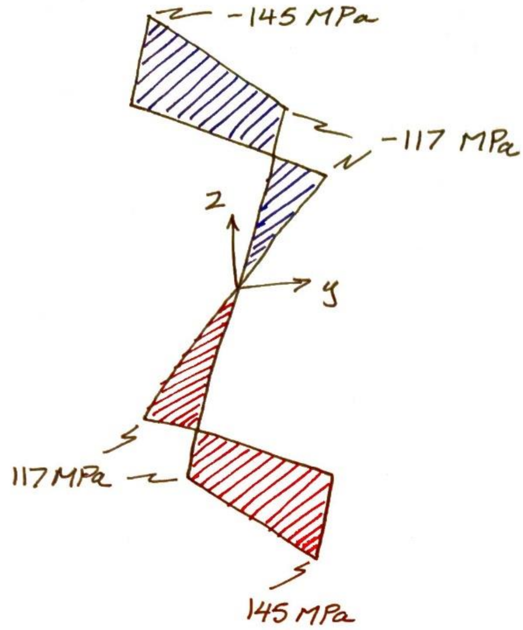
c) $\sigma_{xx} = \dfrac{M_y z}{I_{max}}$



$\sigma_{xx} = \dfrac{-1.125 \times 10^6}{0.4612 \times 10^6} z = -2.4393 z$

	z (mm)	σ_{xx} (N/mm^2)
1	$50 \cos\alpha = 47.86$	-117
2	$47.86 + 40 \sin\alpha = 59.43$	-145

4



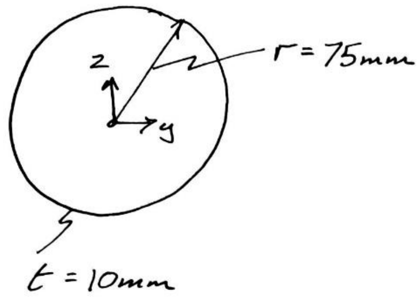
compression

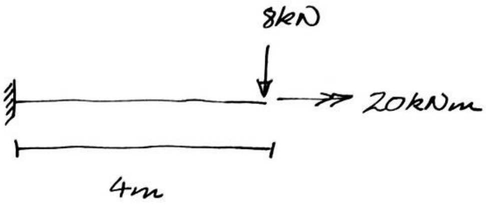
tension

$\alpha = 16{\cdot}81^{\circ}$

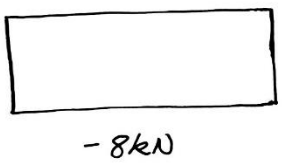
2

Question Number: 2

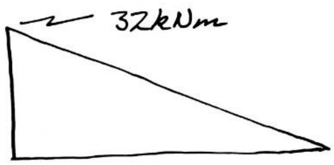




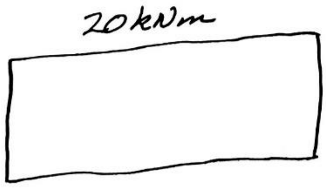
SFD



BMD

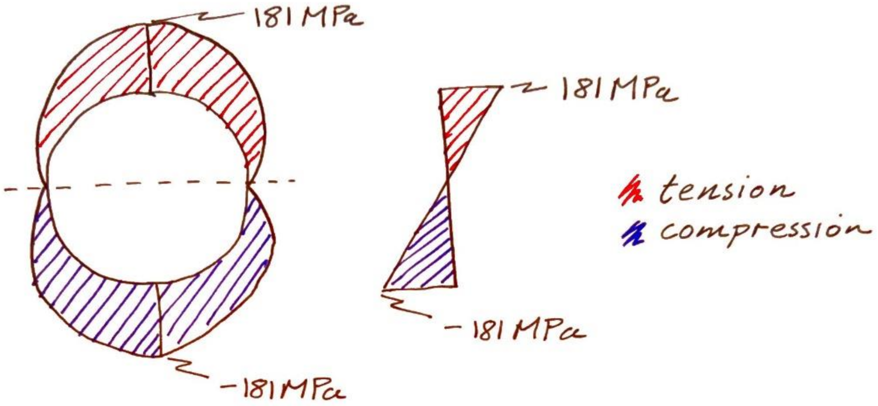


TMD



a) $I_{yy} = \pi \times 75^3 \times 10 = 13.2536 \times 10^6 \ mm^4$

$$\sigma_{xx} = \frac{M_y z}{I_{yy}} = \frac{32 \times 10^6 \times 75}{13.2536 \times 10^6} = 181 \ MPa$$



any reasonable diagram would be accepted | 3

b) $$\tau = \frac{T}{2A_c t} = \frac{20 \times 10^6}{2 \times \pi \times 75^2 \times 10} = \underline{56.59} \ MPa$$ | 2

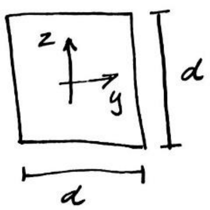
c) $\sigma_{vm(max)}$ will occur at the fixed end at the top and bottom of the cross-section

$$q = \sqrt{[\sigma_{xx}^2 + 3\tau^2]}$$

$$= \sqrt{[181^2 + 3 \times 57^2]} = \underline{206\ MPa}$$

The section will not yield as this is below the yield stress of 355 MPa 2

d)



$t = 10mm$

$$I_{yy} \approx \frac{2td^3}{12} + 2td\left(\frac{d}{2}\right)^2$$

$$I_{yy} \approx \frac{td^3}{6} + \frac{2td^3}{4} = \frac{td^3}{6} + \frac{td^3}{2} = \frac{2td^3}{3}$$

to find d:

$$\frac{2td^3}{3} = \pi r^3 t$$

$$d^3 = \frac{3\pi r^3}{2}$$

$$d = \sqrt[3]{\frac{3\pi r^3}{2}} = \sqrt[3]{\frac{3\pi \times 75^3}{2}} = \underline{126} \text{ mm}$$

2

using a hollow square box section could introduce warping

1

σ_{xx} maximum would be decreased as $\frac{d}{2} < r$

$J_{\square} < J_{O}$ so θ and ϕ would be increased

$A_{c\square} < A_{cO}$ so τ would be increased

EXAMINATION SOLUTIONS
2022/2023Session

Title of Paper: Structural Mechanics

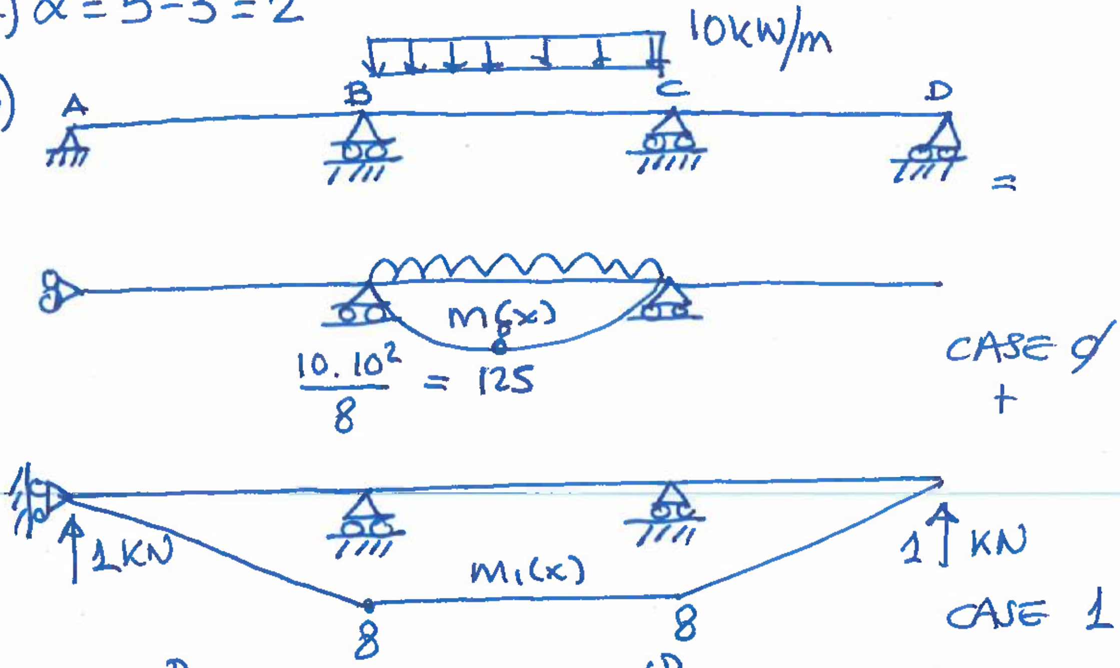
Paper Set By: Dr Ana M. Ruiz Teran

Please write on this side only, legibly and neatly, between the margins

Question Number: 3 | **Marks Allocated:**

a) $\alpha = 5-3 = 2$

b)



$f_{10} = \int_A^D \frac{M_0(x) \cdot M_1(x)}{EI}$ $\quad f_{11} = \int_A^D \frac{m_1(x) \cdot m_1(x)}{EI} \cdot dx$ — 4

f_{10} = 2 times the vertical displacement at A or D in case ∅

f_{11} = 2 times the vertical displacement at A or D in case 1 — 2

c) As above — 4

d) $f_{10} = \frac{2}{3}\, 10 \times 125 \times 8 \times 10^{-4} = 0{,}667 \text{ kN.m}$

$f_{11} = \left(\frac{1}{3}\, 8\times8\times8 + 8\times10\times8 + \frac{1}{3}\, 8\times8\times8\right) \cdot 10^{-4} =$

$= 0.0981 \text{ kN.m}$ — 4

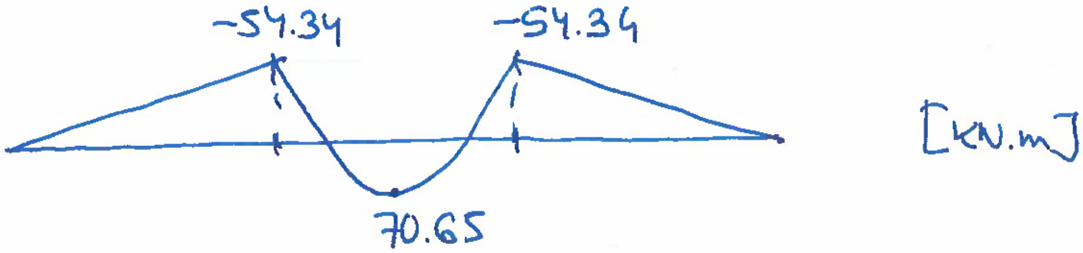
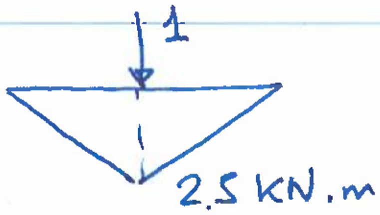
e) $R_A = -\, 0.667/0.0981 = -\,6.79 \text{ kN}$ — 1

EXAMINATION SOLUTIONS
2022/2023Session

Title of Paper: Structural Mechanics

Paper Set By: Dr Ana M Ruiz Teran

Please write on this side only, legibly and neatly, between the margins

Question Number: 3	Marks Allocated:
f) $M_B = -6.79 \times 8 = -54.34$ kN.m	
	
$M_M = 125 - 54.34 = 70.65$ kN.m	2
g) 	
$f = \frac{1}{24} \cdot 10 \times 2.5 \left(-54.34 \times 2 + 70.65 \times 10\right) \cdot 10^{-4}$ $= 0.062\,m = 62\,mm$	2

EXAMINATION SOLUTIONS
2022/2023 Session

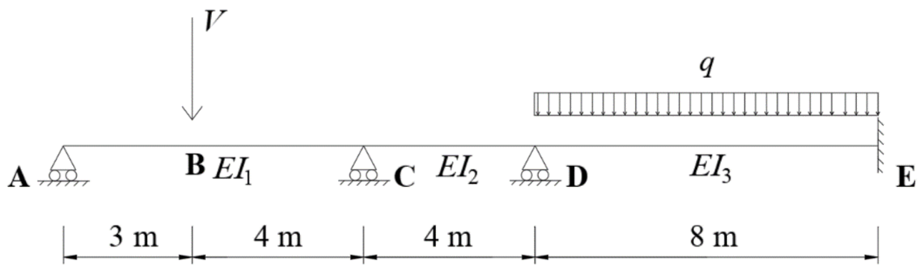
Title of Paper: Structural Mechanics 2

Question Set By: Dr Fiona Walport

Please write on this side only, legibly and neatly, between the margins

Question Number: Q4 | **Marks Allocated:**

(a)



EI_1 = 7 MNm^2, EI_2 = 5 MNm^2, EI_3 = 10 MNm^2, V = 15 kN, q = 8 kN/m

i. Distribution factors:

At C:

$$\mathbf{DF_{CA}} = \frac{3EI_1/L_1}{3EI_1/L_1 + 4EI_2/L_2} = \frac{3(7)/7}{3(7)/7 + 4(5)/4} = \frac{3}{8} = 0.375$$

$$\mathbf{DF_{CD}} = \frac{4EI_2/L_2}{3EI_1/L_1 + 4EI_2/L_2} = \frac{4(5)/4}{3(7)/7 + 4(5)/4} = \frac{5}{8} = 0.625$$

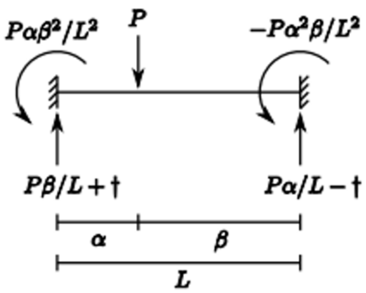
(2)

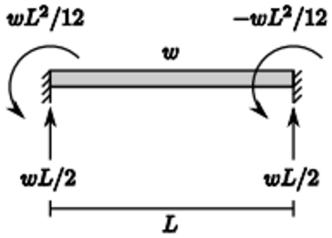
At D:

$$\mathbf{DF_{DC}} = \frac{4EI_2/L_2}{4EI_2/L_2 + 4EI_3/L_3} = \frac{4(5)/4}{4(5)/4 + 4(10)/8} = 0.5$$

$$\mathbf{DF_{DE}} = \frac{4EI_3/L_3}{4EI_2/L_2 + 4EI_3/L_3} = \frac{4(10)/8}{4(5)/4 + 4(10)/8} = 0.5$$

ii. Fixed end moments (FEMs):





Member AC:

$P = V = 15$ kN, $\alpha = 3$ m, $\beta = 4$ m, $L = 7$ m.

$$\boldsymbol{M^{\mathrm{F}}_{\mathrm{AC}} = -\frac{P\alpha\beta^2}{L^2} = -\frac{15(3)(4)^2}{7^2} = -14.69\ \mathrm{kNm}}$$

$$\boldsymbol{M^{\mathrm{F}}_{\mathrm{CA}} = \frac{P\alpha^2\beta}{L^2} = \frac{15(3)^2(4)}{7^2} = 11.02\ \mathrm{kNm}}$$

Member DE:

$w = q = 8$ kN/m, $L = 8$ m. **(2)**

$$\boldsymbol{M^{\mathrm{F}}_{\mathrm{DE}} = -\frac{wL^2}{12} = -\frac{8(8)^2}{12} = -42.67\ \mathrm{kNm}}$$

$$\boldsymbol{M^{\mathrm{F}}_{\mathrm{ED}} = \frac{wL^2}{12} = \frac{8(8)^2}{12} = 42.67\ \mathrm{kNm}}$$

Release at A to account for pinned joint:

$$\boldsymbol{M^{\mathrm{P}}_{\mathrm{AC}} = 0}$$

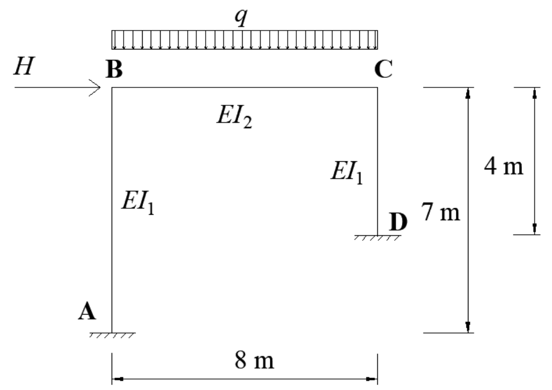
$$\boldsymbol{M^{\mathrm{P}}_{\mathrm{CA}} = M^{\mathrm{F}}_{\mathrm{CA}} - M^{\mathrm{F}}_{\mathrm{AC}}/2 = 11.02 - 0.5(-14.69) = 18.37\ \mathrm{kNm}}$$

iii. Moment distribution table:

	A	C		D		E
DFs		0.375	0.625	0.5	0.5	
FEMs	*-14.69* *+14.69*	*11.02* 18.37			 -42.67	 42.67
Balance at C and D		-6.89	-11.48	21.34	21.34	
Carry over			10.67	-5.74		10.67
Balance at C and D		-4.00	-6.67	2.87	2.87	
Carry over			1.43	-3.33		1.43
Balance at C and D		-0.54	-0.90	1.67	1.67	
Carry over			0.83	-0.45		0.83
Balance at C and D		-0.31	-0.52	0.22	0.22	
Carry over			0.11	-0.26		0.11
Balance at C and D		-0.04	-0.07	0.13	0.13	
Carry over			0.07	-0.04		0.07
Balance at C and D		-0.02	-0.04	0.02	0.02	
End moments (kNm)	0	6.56	-6.56	16.53	-16.43	55.78

(6)

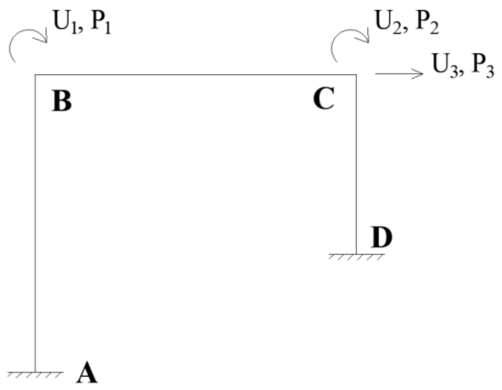
(b)



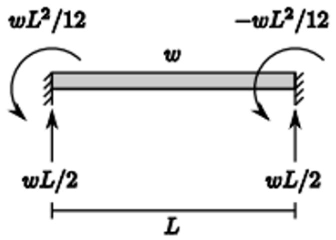
$EI_1 = 3.5$ MNm^2, $EI_2 = 8$ MNm^2, $H = 35$ kN, $q = 9$ kN/m

i) Kinematic indeterminacy:

Considering the number of joints (A, B, C and D: J=4), the support conditions (R=6) and the inextensibility constraint for the three elements AB, BC and CE (c=3), the kinematic indeterminacy (nodal degrees of freedom) in the proposed modelling for the frame is β=3·J-R-c=3: rotation U_1 at joint B, rotation U_2 at joint C and horizontal displacement of the top beam U_3. **(1)**



ii) Vector of equivalent nodal forces:



FEM:

$$\boldsymbol{M_{\mathbf{BC}}^{\mathbf{F}} = -\frac{wL^2}{12} = -\frac{9(8)^2}{12} = -48\ \mathbf{kNm}}$$

$$\boldsymbol{M_{\mathbf{CB}}^{\mathbf{F}} = \frac{wL^2}{12} = \frac{9(8)^2}{12} = 48\ \mathbf{kNm}}$$

Equivalent nodal forces are equal and opposite to fixed-end forces: **(3)**

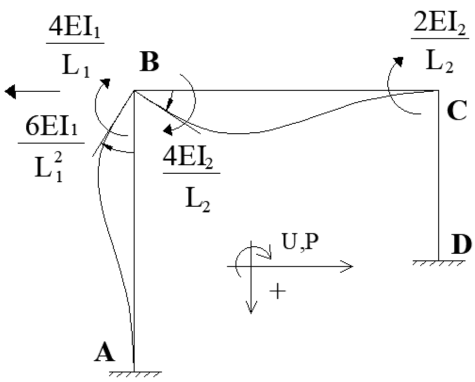
$$\boldsymbol{P_1} = -\boldsymbol{M^{\mathrm{F}}_{\mathrm{BC}}} = \mathbf{48\ kNm}$$
$$\boldsymbol{P_2} = -\boldsymbol{M^{\mathrm{F}}_{\mathrm{CB}}} = \mathbf{-48\ kNm}$$
$$\boldsymbol{P_3} = \boldsymbol{H} = \mathbf{35\ kN}$$

iii) System stiffness matrix:

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K_{1,1}} & \boldsymbol{K_{1,2}} & \boldsymbol{K_{1,3}} \\ \boldsymbol{K_{2,1}} & \boldsymbol{K_{2,2}} & \boldsymbol{K_{2,3}} \\ \boldsymbol{K_{3,1}} & \boldsymbol{K_{3.2}} & \boldsymbol{K_{3,3}} \end{bmatrix}$$

Mode 1:

$U_1=1$ $U_2=0$ $U_3=0$

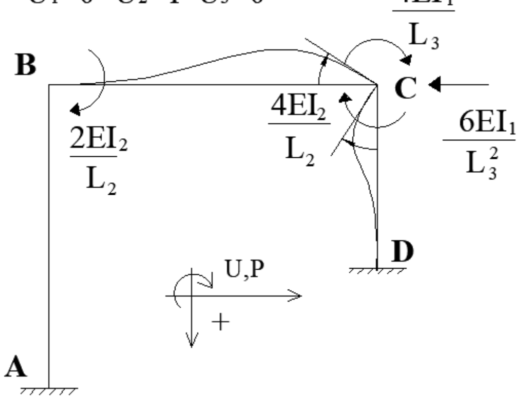


(2)

$$\boldsymbol{K_{1,1}} = \frac{\mathbf{4(3.5)}}{\mathbf{7}} + \frac{\mathbf{4(8)}}{\mathbf{8}} = \mathbf{6\ MNm}$$
$$\boldsymbol{K_{2,1}} = \frac{\mathbf{2(8)}}{\mathbf{8}} = \mathbf{2\ MNm}$$
$$\boldsymbol{K_{3,1}} = -\frac{\mathbf{6(3.5)}}{\mathbf{7^2}} = \mathbf{-0.43\ MN}$$

Mode 2:

$U_1=0$ $U_2=1$ $U_3=0$

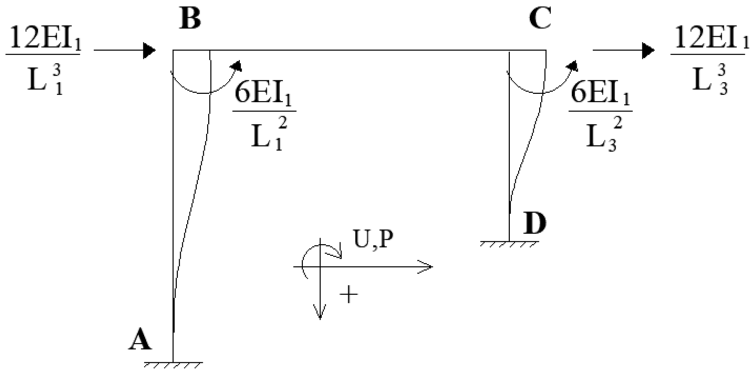


$$\boldsymbol{K_{1,2}} = \frac{\mathbf{2(8)}}{\mathbf{8}} = \mathbf{2\ MNm}$$
$$\boldsymbol{K_{2,2}} = \frac{\mathbf{4(8)}}{\mathbf{8}} + \frac{\mathbf{4(3.5)}}{\mathbf{4}} = \mathbf{7.5\ MNm}$$
$$\boldsymbol{K_{3,2}} = -\frac{\mathbf{6(3.5)}}{\mathbf{4^2}} = \mathbf{-1.31\ MN}$$

(2)

Mode 3:

$U_1=0 \quad U_2=0 \quad U_3=1$



$$K_{1,3} = -\frac{6(3.5)}{7^2} = -0.43\ \mathrm{MN}$$

$$K_{2,3} = -\frac{6(3.5)}{4^2} = -1.31\ \mathrm{MN}$$

$$K_{3,3} = \frac{12(3.5)}{7^3} + \frac{12(3.5)}{4^3} = 0.78\ \mathrm{MN/m}$$

$$K = \begin{bmatrix} 6 & 2 & -0.43 \\ 2 & 7.5 & -1.31 \\ -0.43 & -1.31 & 0.78 \end{bmatrix}$$

(2)

Σ20