IMPERIAL COLLEGE LONDON

MEng Examination 2024

PART II

This paper is also taken for the relevant examination for the Associateship

CIVE50004: ENVIRONMENTAL ENGINEERING

13 May 2024: 9.30–12.30 British Summer Time (BST)

*This paper contains **THREE** questions.*

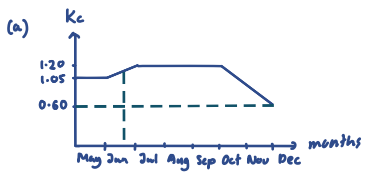
*Answer **ALL THREE** questions, each in a **SEPARATE** booklet.*

All questions carry equal marks.

Formulae sheets are provided at the end of the examination paper.

Q1. (Answer ALL parts of this question; total of 40 marks)

A farmer in South Asia grows rice on a field of 2.5 ha (25,000 m^2). The growing period starts on 1 May. The fields have clay soils, with a wilting point of 0.32 $m^3\ m^{-3}$ and a field capacity of 0.48 $m^3\ m^{-3}$. The depletion fraction for rice is 20%, and the average root depth is 0.5 m. The crop properties are given in Table 1.1.



Kc takes mid month !

K_c (June) = (1.05+1.20) ÷ 2 = 1.125

K_c (Oct) = $1.2 - \frac{1}{4}(1.2-0.6) = 1.05$

K_c (Nov) = $1.2 - \frac{3}{4}(1.2-0.6) = 0.75$

Table 1.1 Crop properties of rice

Growing stage	Crop coefficient	Length (days)
Initial	1.05	30
Development	N/A	30
Mid	1.2	90
Late	0.6	60

Reference evapotranspiration ($E_{t,0}$) and precipitation (P) for the region are (in units of mm month^{-1}) given in Table 1.2.

(b)

(find all Kc)

(find $E_t = K_s K_c E_{t,0}$)

since $K_s = 1.0$,

$E_p = E_t$

$I = E_p - P + R$

$R = 0.15P \rightarrow I = E_p - 0.85P$

$K_s = 1.0$

$E_t = K_s K_c E_{t,0}$

Table 1.2 Reference evapotranspiration and precipitation for the region (in mm month^{-1})

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$E_{t,0}$	89.5	113.1	132.1	152.2	163.3	146.8	156.7	163.7	147.5	114.8	86.8	65.6
P	22	18	14	8	11	55	194	194	123	22	3	8
Kc	-	-	-	-	1.05	1.125	1.20	1.20	1.20	1.05	0.75	-
Et	-	-	-	-	171.5	165.2	188.0	196.4	177.0	120.5	65.1	-
I	-	-	-	-	162.2	118.5	23.1	31.5	72.5	101.8	62.6	-

ΣE_t = 1083.7 mm × 25000 m^2 = 27092.5 m^3

ΣI = 572.2 mm × 25000 m^2 = 14305 m^3

Using the information provided:

(a) Sketch graphically the evolution of the crop coefficient over the year and calculate the crop coefficient for each month. For simplicity, assume that all months are 30 days long.

[6 marks]

(b) Calculate the total crop water (Ea or Et) and irrigation requirements (I ?) for the crop (in m^3). Assume that 15% of the rainfall percolates through the soil as groundwater recharge.

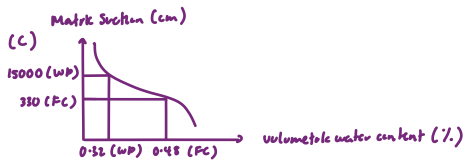
[8 marks]

(c) Sketch the soil moisture characteristic and indicate the points of specific interest

[6 marks]

(d) What is the maximum dose of irrigation water (in mm) (RAW) that can be applied to minimize losses?

[2 marks]



$TAW = (\theta_{FC} - \theta_{WP}) Z_r$

$= (0.48 - 0.32) \times 0.5$

$= 0.08\,m$ (= 80 mm)

$RAW = pTAW = 20\% \times 80\,mm = 16\,mm$

trying to find maximum time between doses
actually mean we should check how frequent.
highest demand (Refer to I not Et!) : May (162.1 mm/month)
Irrigation required
total water required

(e) What is the maximum time between two irrigation doses?

$freq = \frac{I}{RAW} = \frac{162.1 \text{ mm month}^{-1}}{16 \text{ mm}}$

$= 10.13 \text{ month}^{-1}$

$period = \frac{1}{freq} = \frac{1}{10.13} \approx 3 \text{ days}$

[2 marks]

maximum time means less freq } $f = \frac{1}{T}$
also means less I (irrigation requirement)
least I is during July : 23.1 mm

$freq. = \frac{I}{RAW}$
$= \frac{23.1 \text{ mm month}^{-1}}{16 \text{ mm}}$
$= 1.44 \text{ month}^{-1}$

$period = \frac{1}{freq} = 0.693 \text{ month} \approx 20 \text{ days}$

(f) Explain the concept of the virtual water footprint and its relevance for water resources management.

[8 marks]

(g) If you know that the average yield of rice is 5,000 kg ha^{-1}, calculate:

(i) The water footprint of a kg of rice produced by the farm (in litres).

Et

$27092 \text{ m}^3 \div (5000 \text{ kg } \cancel{ha^{-1}} \times 2.5 \cancel{ha}) = 2.16736 \text{ m}^3 \text{ kg}^{-1}$
$= 2167.36 \text{ l kg}^{-1}$

[4 marks]

(ii) The blue water share of the water footprint (in litres)

I

[2 marks]

(iii) The green water share of the water footprint (in litres)

Et-I

[2 marks]

max 32/40

Q2. (Answer ALL parts of this question; total of 40 marks)

(a) Provide five (5) characteristics of groundwater that make it a useful resource.

[5 marks]

(b) What equations, infrastructure and measurements would you need to determine the pore water velocity in an unconfined aquifer?

$v = \frac{q}{n_e}$ $q = -K\nabla h$ * to find k : T = kH * $\nabla_{xy}(h)$ can be obtained with at least 2+1 observation wells.

* to find T we can use pumping test.

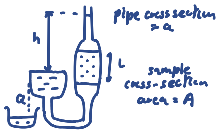
* to find H = h − Zb, Zb can be obtained by borehole logs * K can be obtained with permeameter test.

* h can be obtained with observation well.

[8 marks]

(c) Provide a diagram of a falling head permeameter and, using the principle of continuity and Darcys law, show the time varying head (h) in the permeameter is given as:

$$h = H_0 e^{-\left(\frac{AK}{aL}\right)t}$$



$Q = -a\frac{dh}{dt}$ $Q = AK\frac{h}{L}$

$-a\frac{dh}{dt} = AK\frac{h}{L}$

$\int \frac{1}{h}dh = \int -\frac{AK}{aL}dt$

$\ln h = -\left(\frac{AK}{aL}\right)t + c$

$h = e^{-\left(\frac{AK}{aL}\right)t} \times e^{c}$

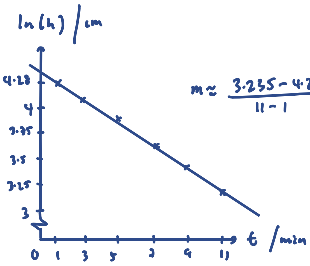
$h = H_0 e^{-\left(\frac{AK}{aL}\right)t}$

[5 marks]

(d) (i) A 20 cm long, 20 cm diameter circular core of silty sand was placed in a permeameter with a circular upper tube of diameter 3 cm. Table 2.1 gives the time dependent height of the water level in the upper tube, which has an initial height of 80 cm above the outflow.

Table 2.1 Falling head permeameter test values.

Time (min)	Height (cm)	ln h
1.0	72.5	4.284
3.0	58.7	4.072
5.0	47.5	3.861
7.0	38.5	3.651
9.0	31.2	3.440
11.0	25.4	3.235



$m \approx \frac{3.235 - 4.284}{11 - 1} = -0.1049$ cm/min

What is the hydraulic conductivity of the sample in metres per second?

$L = 20$, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20)^2 = 314.159$, $a = \frac{\pi}{4}(3)^2 = 7.069$, $H_0 = 80$

$h = H_0 e^{-\left(\frac{AK}{aL}\right)t}$

$h = 80e^{-2.222Kt}$

$\ln(h) = \ln 80 \underbrace{-2.222K}_{m}t$

$K = \frac{-0.105 \text{ cm/min}}{-2.222} = 0.0472 \frac{\text{cm}}{\text{min}} = 7.87 \times 10^{-6}$ m/s

[5 marks]

(ii) A particle size analysis of the sample in part (i) gave a median value of 0.2 mm. The dry mass of the sample is 9,425 g and after wetting to saturation the sample mass was 10,681 g. Using the Kozeny-Carman equation, calculate the hydraulic conductivity of the sample. Compare this answer with that from part d(i) and comment their use in a groundwater model.

Kozeny-Carman equation: $k = \frac{D_{50}^2}{180}\frac{n_e^2}{(1-n_e)^2}$ [m^2]

$D_{50} = 0.2$ mm

$n_e = \frac{\text{volume of water added}}{\text{volume.}}$

Dynamic viscosity of water is 10^{-3} Pa.s

[5 marks]

volume = A × L
= 314.59 × 20
= 6291.8 cm^3

volume of water added
= 10681 − 9425 = 1256 g = 1256 cm3
(mass of water = volume due to density being 1 g cm^{-3})

$n_e = \frac{1256}{6292} = 0.2$

$\therefore k = \frac{(0.2 \times 10^{-3})^2}{180} \times \frac{0.2^2}{(1-0.2)^2} = 2.78 \times 10^{-12}$ m^2

$K = \frac{k\rho g}{\mu} = 2.73 \times 10^{-5}$

(more inaccurate) (more accurate)
KC method > permeameter
(by ~3.5 times)

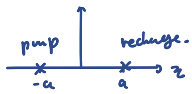
(e) (i) Assuming recharge is negligible, the steady state water table elevation above the base of an unconfined aquifer, with hydraulic conductivity K, in response to a well pumping at a rate Q_w is:

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right)$$

where h_e is the height of the water table above the aquifer base prior to pumping, r is the radial distance from the well and r_e is the radius of influence.

A well is being pumped at a rate of Q_w. A second well, at a distance $2a$ from the pumping well, is being recharged at a rate of Q_w. Show, that the steady-state water table elevation at a location x on a straight line between the two wells, with the origin at the centre point between the wells and x positive towards the recharge well, can be described as:

$$h^2 = h_e^2 - \frac{Q_w}{\pi K}\ln\left(\frac{a-x}{a+x}\right)$$



$S_{net} = \frac{Q_w}{\pi k}\ln\left(\frac{\frac{r_e}{x+a}}{\frac{r_e}{x-a}}\right)$

$= \frac{Q_w}{\pi k}\ln\left(\frac{x-a}{x+a}\right) \times -1 \quad \times -1$

$h^2 = h_e^2 - \frac{Q_w}{\pi k}\ln\left(\frac{a-x}{a+x}\right)$

$s = h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right)$

$h_e^2 - h^2 = \frac{Q_w}{\pi k}\ln\left(\frac{a-x}{a+x}\right)$

[4 marks]

$S_{pump} = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{x+a}\right)$ $S_{recharge} = -\frac{Q_w}{\pi k}\ln\left(\frac{r_e}{x-a}\right)$

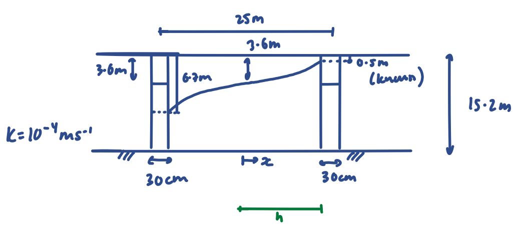
(ii) The manager of an office building wishes to use groundwater in a heat exchanger for heating. The building, which is built on a large, covered area, overlies a coarse sand aquifer, which has a hydraulic conductivity of 10^{-4} m s^{-1} and a horizontal impermeable base at 15.2 m below ground level (bgl). A pumping well and a recharge well, both with diameters of 30 cm and fully penetrate the aquifer, are to be installed at a distance of 25 m apart. Prior to pumping, the rest water level was 3.6 m bgl. When the wells are in operation, the depth to the water level in the recharge well should be maintained at level which is 0.5 m bgl. If the temperature of the water from the pumping well is 11°C and the water going to the recharge well is 9°C then, assuming no well and heat losses, calculate the energy in Watts that is available for heating the building.

[Note: the specific heat capacity of water is 4.2 kJ kg^{-1}] notice that if find Q [m^{3}s^{-1}]

$\times \rho$ [kg m^{-3}] = kg s^{-1}

$\times$ = kJ s^{-1} = Watt!

[8 marks]



take h till well. $h = 15.2 - 0.5 = 14.7$m

h_e always, for superposition taken at mid $(x=0)$!

find Q_w

$h^2 = h_e^2 - \frac{Q_w}{\pi k}\ln\left(\frac{a-x}{a+x}\right)$

$\frac{Q_w}{\pi K}\ln\left(\frac{a-x}{a+x}\right) = h_e^2 - h^2$

$Q_w = \frac{\pi k(h_e^2 - h^2)}{\ln\left(\frac{a-x}{a+x}\right)}$

$= \frac{\pi(10^{-4})(11.6^2 - 14.7^2)}{\ln\left(\frac{12.5-(12.5-0.15)}{12.5+(12.5-0.15)}\right)}$

$= 0.005$ m^3/s

$P = 0.005\,\cancel{m^3}s^{-1} \times 1000\,\cancel{kg}\,\cancel{m^{-3}} \times 4.2\,kJ\,\cancel{kg^{-1}}\,\cancel{°C^{-1}} \times 2\cancel{°C}$

$= 42$ kJ s^{-1}

$= 42$ kW

Q3. (Answer ALL parts of this question; total of 40 marks)

(a) A unit hydrograph (UH) method is used to estimate overland flow at a catchment outlet. The UH is defined by the duration and depth of rainfall that generates a unit flow. Then, the flow for any rainfall event in the catchment can be calculated. Explain which condition needs to be satisfied for the UH to be applied to a rainfall event with a duration of 1 hour.

The UH needs to have the same duration, and to be scaled later accordingly.

[2 marks]

(b) A catchment has a 1-hour 5 mm unit hydrograph given in Table 3.1. Depths during a rainfall event over a period of 6 hours are shown in Table 3.2.

Table 3.1 1-hour 5 mm unit hydrograph

Time (h)	Flow ($m^3 s^{-1}$) 1 hour 5mm	1 hour 5mm lagged by 1 hour	2 hour 10 mm UH:
0	0	0	0
1	0.16	0	0.16
2	1.26	0.16	1.42
3	1.93	1.26	3.19
4	0.71	1.93	2.64
5	0.14	0.71	0.85
6	0	0.14	0.14
7	0	0	0

2h 2.5mm	2h 5.2mm	2h 1.3mm
0	0	0
0.0400	0.0832	0.0208
0.3550	0.7384	0.1846
0.7975	1.6588	0.4147
0.6600	1.3728	0.3432
0.2125	0.4420	0.1105
0.0350	0.0728	0.0182
0	0	0

Table 3.2 Rainfall event depths

Time (h)	Effective rainfall (mm)
0-2	2.5
2-4	5.2
4-6	1.3

(i) Considering the condition for the application of the UH, comment on the application of the UH given in Table 3.1 to the recorded rainfall event (Table 3.2). If the UH cannot be applied directly, select the appropriate rescaling method, and justify your selection.

UH is 1 hour duration (ΔT_1)
and the rainfall event is 2 hour duration (ΔT_2)
since $\Delta T_2 = k\Delta T_1$, $k \in \mathbb{Z}^+$, we can use the shortcut method!

[2 marks]

(ii) Using the information given and considering your answer for b(i) calculate the peak flow and the time to peak.

[8 marks]

	2h 2.5mm	2h 5.2mm	2h 1.3mm
0th hour :	0	0	0
1st hour :	0.0400	0.0832	0.0208
	0.3550	0.7384	0.1846
	0.7975	1.6588	0.4147
	0.6600	1.3728	0.3432
	0.2125	0.4420	0.1105
	0.0350	0.0728	0.0182
	0	0	0

→

0			0
0.0400			0.04
0.3550	0		0.36
0.7975	0.0832		0.88
0.6600	0.7384	0	1.40
0.2125	1.6588	0.0208	1.89
0.0350	1.3728	0.1846	1.59
0	0.4420	0.4147	0.86
	0.0728	0.3432	0.42
	0	0.1105	0.11
		0.0182	0.02
		0	0

5 hours to peak

(c) A sample of 10 annual maxima daily river flows is given in Table 3.3.

(i) Calculate the parameters of the Gumbel distribution using the Method of moments.

Table 3.3 Annual maxima flows (in $m^3 s^{-1}$) for the period 2000-2009

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Q ($m^3 s^{-1}$)	52	66	109	60	86	87	122	97	78	51

from data sheets : $\alpha = \mu - 0.5772\beta$ $\quad \beta = \frac{6^{1/2}}{\pi}\sigma$

$\bar{x} = \frac{\Sigma Q}{n} = 80.8$

$S_x^2 = \frac{\Sigma(Q-\bar{x})^2}{n-1} = 577.51$

$S_x = 24.0315$

$\therefore \beta = \frac{6^{1/2}}{\pi} \times 24.0315 = 18.7373$

$\alpha = 80.8 - 0.5772(18.7373) = 69.9848$

[4 marks]

(ii) Calculate the return period of a 100 $m^3 s^{-1}$ flood.

$F(q_d) = \exp[-\exp(\frac{\alpha - q_d}{\beta})]$

subbing $q_d = 100$: $F(100) = 0.8175$

$T = \frac{1}{P(q > q_d)} = \frac{1}{1-F(q_d)} = \frac{1}{1-0.8175} = 5.48$ years.

[2 marks]

(iii) Climate change will increase the flows shown in Table 3.3, which changes the parameters of the underlying distribution. Calculate the percentage increase in flows if the new values of Gumbel distribution parameters under climate change are:

$$\alpha = 84\, m\, s^{-1}$$

$$\beta = 22.5\, m\, s^{-1}$$

new $\bar{x} = \alpha + 0.5772\beta$

$= 96.987$

old $\bar{x} = 80.8$ $\quad$ increase $= \frac{96.987}{80.8} = 1.2$

20% increase

[6 marks]

(d)

(i) Explain the difference between empirical and conceptual rainfall-runoff models.

[2 marks]

(ii) List four limitations of the unit hydrograph method and explain how they can be overcome using conceptual modelling.

[4 marks]

(e) The following flow forecasting model can be used for the data given in Table 3.4:

$$x_{k+2} = au_k + bx_{k+1}$$

where u is the upstream and x is the downstream flow (in $m^3 s^{-1}$).

Table 3.4 Observed data for the flood forecasting

Time (h)	Upstream flow $m^3 s^{-1}$	Downstream flow $m^3 s^{-2}$
1	0.40	0.16
2	2.20	0.20
3	1.20	0.36
4		1.82
5		1.22

(i) Write the linear transfer function model in a form that can be used for a matrix inversion by defining vector *X* and matrix *R*.

$x_{k+2} = au_k + bx_{k+1}$ $\quad X = \begin{Bmatrix} 0.36 \\ 1.82 \\ 1.22 \end{Bmatrix} \quad R = \begin{Bmatrix} 0.40 & 0.20 \\ 2.20 & 0.36 \\ 1.20 & 1.82 \end{Bmatrix}$

[6 marks]

(ii) For the above model the parameters values have been estimated as:

$$a = 0.805 \text{ and } b = 0.140$$

and the value of the flood threshold is F = 1 $m^3 s^{-1}$. Sketch the graphical representation to support a flood warning and determine which combination of upstream and downstream flows will cause flooding.

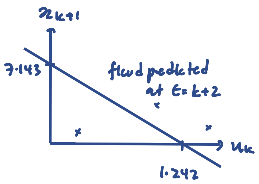
$x_{k+2} = 0.805\,u_k + 0.140\,x_{k+1} \geq 1$

[4 marks]

$0.140\,x_{k+1} \geq -0.805\,u_k + 1$

$x_{k+1} \geq -\frac{0.805}{0.140}u_k + \frac{1}{0.140}$

$x_{k+1} \geq -5.75\,u_k + 7.143$



flood is predicted at $t = 2+2$ and $t = 3+2$
$= 4$ hrs $\quad = 5$ hrs

Formulae Sheet

A. Introduction to Hydrology (Prof Wouter Buytaert)

Catchment water balance	$\Delta S = P - E - Q - R$
Density of solids	$r_S = \frac{M_S}{V_S}$
Dry bulk density	$r_B = \frac{M_S}{V_T}$
Total bulk density	$r_T = \frac{M_S + M_L}{V_T}$
Porosity	$e = \frac{V_L + V_G}{V_T}$
Void ratio	$e = \frac{V_L + V_G}{V_S}$
Gravimetric moisture content	$q_G = \frac{M_L}{M_S}$
Volumetric moisture content	$q = \frac{V_L}{V_T}$
Interrelating formula	$q = q_G \frac{q_B}{q_W}$
Penman Monteith	$E_a = K_c K_s E_{t,0}$
River flow	$Q = \grave{o} v(A)\, dA = \bar{v} A$
Rectangular weir	$Q = KbH^{1.5}$
Hydropower equation	$P = \varepsilon_t \varepsilon_h h \rho Q g$

Irrigation $I = E_p - P + R$

Total available water $TAW = 1000\,(\theta_{FC} - \theta_{WP})\,Z_r$

Readily available water $RAW = p\,TAW$

B. Groundwater Systems (Prof Adrian Butler)

Darcy's law

$q_i = -K\dfrac{dh}{di}$, where i represents a coordinate direction (e.g. x, y, z)

Water table elevation in an unconfined aquifer

$$h^2 - h_0^2 = \frac{W}{K}\left(Lx - x^2\right) + \left(h_1^2 - h_0^2\right)\frac{x}{L}$$

Total flow per unit width in an unconfined aquifer

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$$

Well pumping confined aquifer without recharge (Thiem equation)

$$s = h_e - h = \frac{Q_w}{2\pi T}\ln\left(\frac{r_e}{r}\right)$$

Well pumping unconfined aquifer with recharge

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}\left(r^2 - r_e^2\right)$$

Theis solution

$s = \dfrac{Q_w}{4\pi T}W(u)$, where $u = \dfrac{r^2 S}{4Tt}$

Jacob's large time approximation

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

C. Flood Risk Estimation and Management (Dr Ana Mijic)

Weibull formula	$P_m(Q \le q_m) = \frac{\mathrm{m}}{N+1}$
Gringorten formula	$P_m(Q \le q_m) = \frac{\mathrm{m}-0.44}{N+0.12}$
Gumbel distribution	$F(q_d) = \exp\left[\ -\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$
Gumbel variate	$z = -\ln\{-\ln[F(q_d)]\}$
Probability for sequence of years	$P(x,n) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$
Moment matching	$\alpha = \mu - 0.5772\,\beta$ $\beta = (6^{\frac{1}{2}}/\pi)\sigma$
L-moment matching	$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j$ $L_2 = \frac{2}{n}\sum_{j=2}^{n}\left[\frac{(j-1)q_j}{n-1}\right] - L_1$ $\alpha = L_1 - 0.5772\,\beta$ $\beta = \frac{L_2}{\ln(2)}$
Normal equation for flood forecasting	$(R^T R)\theta = R^T X$