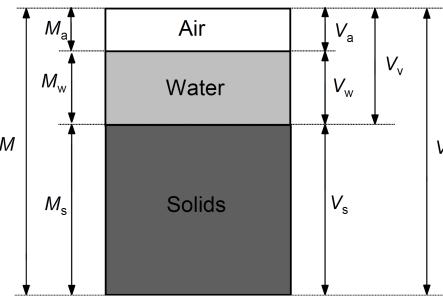


# CIVE50007 SOIL MECHANICS

## 1 PHASE RELATIONS



**Water Content:**  $w = \frac{M_w}{M_s}$

**Specific Gravity:**  $G_s = \frac{\rho_s}{\rho_w}$

**Degree of Saturation:**  $S_r = \frac{V_w}{V_v} = \frac{wG_s}{e}$

**Void Ratio:**  $e = \frac{V_v}{V_s} = \frac{n}{1-n}$

**Porosity:**  $n = \frac{V_v}{V} = \frac{e}{1+e}$

**Specific Volume:**  $v = \frac{V}{V_s} \cdot \frac{1}{n} = \frac{1+e}{e}$

**Bulk Density:**  $\rho = \frac{M}{V} = \frac{\rho_s(1+w)}{1+e} = \frac{\rho_w(G_s+S_re)}{1+e}$

**Bulk Unit Weight:**  $\gamma = \rho g = \frac{Mg}{V}$

**Dry Density:**  $\rho_{dry} = \frac{\rho}{1+w}$

## 2 EFFECTIVE STRESS

### DEFINITIONS

**Phreatic Surface:** Surface with zero pore water pressure

**Artesian:** When phreatic surface is above ground level

### PRINCIPLE OF EFFECTIVE STRESS

$$\sigma' = \sigma - u$$

- $\sigma'$  = effective stress,  $\sigma$  = total stress,  $u$  = pore water pressure

- If there is no flow,  $u = \rho_w gh$  (hydrostatic)

## 3 SEEPAGE

### BERNOULLI'S THEOREM

$$h = \frac{u}{\gamma_w} + z + \frac{v^2}{2g}$$

- $h$  = total head,  $u/\gamma_w$  = pressure head,
- $z$  = elevation head from datum
- $v^2/2g$  = velocity head ( $\approx 0$  in soils)



### DARCY'S LAW

$$q = Aki \Rightarrow v = \frac{q}{A} = ki$$

- $q$  = volumetric flow rate,  $A$  = cross-sectional flow area,  $v$  = discharge/flow velocity
- $k$  = soil permeability [m/s]
  - Typically,  $k_{clay} < 10^{-8} < k_{silt} < 10^{-5} < k_{sand} < 10^{-2} < k_{gravel}$
- $i$  = hydraulic gradient = rate of change of total head  $h$  with distance  $s$  in the flow direction:

$$i = -\frac{\partial h}{\partial s} \quad \text{TYPICALLY } i_{crit} = 1$$

- Quick Conditions:** When  $i_{crit} = \frac{\gamma_s}{\gamma_w} - 1$ , which causes effective stress  $\sigma' = 0$

### FLOW NETS EP ARE LINES WITH EQUAL PIEZOMETRIC HEAD (PRESSURE+ELEVATION)

- If soil is anisotropic, scale the  $x$ -dimension by  $\sqrt{k_y/k_x}$  and use  $k = \sqrt{k_x k_y}$
- Draw equipotential (EP) and flow lines (FL) along all boundary conditions
- Draw FLs such that the  $N_{flow}$  flow channels formed have the same total volume
- Draw EPs with  $N_{drop}$  equal head losses  $\Delta h = \frac{H}{N_{drop}}$ , perpendicular to all FLs, forming curvilinear squares
- Calculate flow  $q = kH \frac{N_{flow}}{N_{drop}}$  [m<sup>2</sup>/s], where  $H$  is the driving head

### PERMEABILITY OF STRATIFIED SOILS

For soil of total depth  $d$  layered into  $n$  distinct beds of thickness  $d_i$  of different permeability  $k_i$ , the overall permeability  $k$  is:

### Horizontal flow, parallel to strata:

$$k = \frac{1}{d} \sum_{i=1}^n k_i d_i \quad k = \frac{d}{\sum k_i d_i}$$

### Vertical flow, perpendicular to strata:

$$k = d \left( \sum_{i=1}^n \frac{d_i}{k_i} \right)^{-1} \quad k = \frac{d}{\sum k_i d_i}$$

## 4 SOIL CONSOLIDATION

### DEFINITIONS

When a saturated soil's load is raised by  $\Delta\sigma$ :

- Drained:** Pore water readily flows out (coarse-grained)  $\Delta u = 0$   
 $\Delta\sigma = \Delta\sigma'$  directly, no  $\Delta u$
- Undrained:** Water slowly flows out (fine-grained)  
 $\Delta u = 0$   
initially  $\Delta\sigma = \Delta\sigma'$ , over time  $\Delta\sigma = \Delta\sigma'$

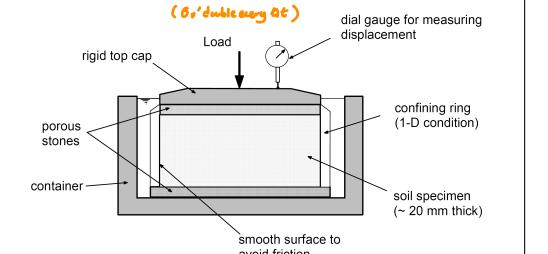
- Excess pore pressure  $\Delta u$  causes pressure gradient, and thus, transient flow until the drained condition is reached
- Rate of  $\Delta u$  dissipation depends on drainage path length (pg 30)

**Consolidation:** Process of soil loading and compression

- Most relevant to fine-grained soils of low permeability
- Particles reorientate, raising inter-particle forces and reducing soil volume
- Assumes the loaded area is much larger than the soil layer's thickness

★ Two main questions: (Two different graph to remember)

- Fixed incremental  $\Delta\sigma'$ , given  $H(t)$  → find  $C_u$
- Not important, given  $H(\Delta\sigma')$  → find  $m_v, C_c$  ( $\Delta\sigma'$  doubles every 24h)

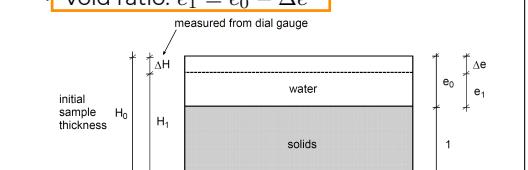


- Set initial pressure at in-situ effective stress ( $u \approx 0, \sigma'_v \approx \Delta\sigma_v$ ) → increase  $\sigma'$  such that  $\Delta\sigma = \Delta\sigma'$  but up to the point where any additional  $\Delta\sigma$  will start to increase  $\Delta u$ .
- Double the loads applied every 24 hours, assuming  $\Delta u$  dissipates completely in between each increment

- Plot the sample's compression/void ratio after each increment against effective stress

• Compression:  $\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$

• Void ratio:  $e_1 = e_0 - \Delta e$



we want to plot  $e$  against  $\sigma'_v$  (we have  $\sigma'_v$  - doubled every day) to find  $e$  for every  $\sigma'_v$ :

- we measure  $H_0$  and  $\Delta H$  for each  $\Delta\sigma'$ , - we calculate  $e_0 = \frac{H_0 G_s}{S_r}$  then we can use formula  $\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$  to get  $\Delta e$ :

## 4. Calculate soil compressibility parameters

### Coefficient of Volume Compressibility:

Volume change per unit volume per increase in effective stress

$$(m_v) = \frac{1}{1+e_0} \left( \frac{\Delta e}{\Delta\sigma'} \right) = \frac{1}{H_0} \left( \frac{H_0 - H_1}{\sigma'_1 - \sigma'_0} \right)$$

need to obtain either from e against  $\sigma'_v$  graph or linear interpolate time!

o Typically,  $m_v = 10^{-3} \text{ m}^2/\text{kN}$  for soft clay and  $10^{-4} \text{ m}^2/\text{kN}$  for stiff clay

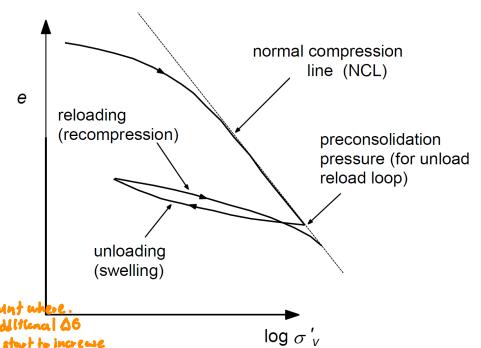
- Long-term Settlement:** after obtaining positive time!  $P_t = U_p m_v$

$$m_v = \frac{1}{H} \frac{\rho_\infty}{\Delta\sigma'_v} \Rightarrow \rho_\infty = m_v H \Delta\sigma'_v$$

- Compression Index:** Gradient of linear part of consolidation curve

$$C_c = \frac{e_0 - e_1}{\log \sigma'_1 - \log \sigma'_0}$$

### CONSOLIDATION CURVE



- Normally-Consolidated (NC):** Never experienced greater pressures than what is being applied

- Over-Consolidated (OC):** Experienced unloading at some point

- Compresses much less than NC clay (much gentler gradient in the loop relative to NCL)

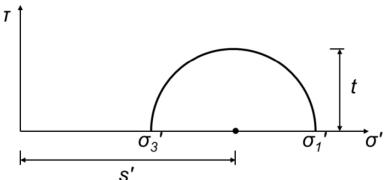
- Exceeding preconsolidation pressure  $\sigma'_c$  causes a steep change in consolidation rate as the curve abruptly follows the NCL



## STRESS INVARIANTS

**Stress Invariants:**  $s = s'$  and  $t = t'$  be the centre and radius of the Mohr circle:

$$t = t' \left\{ \begin{array}{l} t = \frac{1}{2}(\sigma_1 - \sigma_3) \\ t' = \frac{1}{2}(\sigma'_1 - \sigma'_3) \end{array} \right., \quad s = \frac{1}{2}(\sigma_1 + \sigma_3), \quad s' = \frac{1}{2}(\sigma'_1 + \sigma'_3)$$

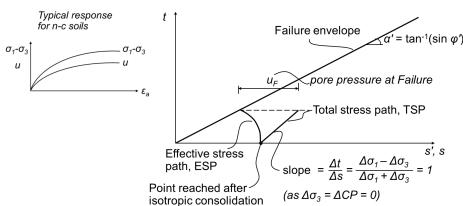


## STRESS PATHS

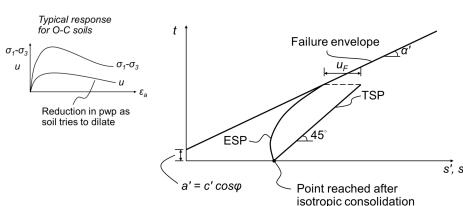
**Stress Path:** The lines that  $\sigma$  and  $\sigma'$  follow as shearing occurs

- $u$  = horizontal distance between effective and total stress paths (ESP and TSP)

### NC Clay (C-U test):



### OC Clay (C-U test)



## 6 GEOTECHNICAL ANALYSIS

### REQUIREMENTS FOR A SOLUTION

**Equilibrium:**  $\sum F_{ext} = \sum F_{int} \Rightarrow \sum F = 0$

$$\begin{aligned} \sum F_x &= 0 \Rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \gamma = 0 \\ \sum F_y &= 0 \Rightarrow \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0 \\ \sum F_z &= 0 \Rightarrow \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \end{aligned}$$

- $\tau_{ij}$  is the stress component parallel to axis  $j$ , acting on a plane perpendicular to axis  $i$
- Self weight  $\gamma$  acts in the  $x$ -direction
- Compressive stresses are positive

**Compatibility:** No overlapping or generation of holes

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

- Compressive strains are positive

**Constitutive Behaviour:** Material's stress-strain behaviour

$$\begin{Bmatrix} \Delta \sigma_x \\ \Delta \sigma_y \\ \Delta \sigma_z \\ \Delta \tau_{xy} \\ \Delta \tau_{xz} \\ \Delta \tau_{zy} \end{Bmatrix} = [D]_{6 \times 6} \begin{Bmatrix} \Delta \varepsilon_x \\ \Delta \varepsilon_y \\ \Delta \varepsilon_z \\ \Delta \gamma_{xy} \\ \Delta \gamma_{xz} \\ \Delta \gamma_{zy} \end{Bmatrix}$$

- Can be expressed in terms of total or effective stresses
- Commonly used to relate increments of stress and strain with  $[D]$  depending on stress history, due to non-linear behaviour of soil

**Plane strain:** Reduction to 2D cross-sections if object is very long along  $z$ -axis

$$\varepsilon_z = \gamma_{yz} = \gamma_{zx} = 0 \Rightarrow \Delta \tau_{xz} = \Delta \tau_{zy} = 0$$

- Non-zero stress changes:  $\Delta \sigma_x, \Delta \sigma_y, \Delta \sigma_z, \Delta \tau_{xy}$

- Axial symmetry:** No displacement in  $\theta$ -direction and displacements in  $r$ - and  $z$ -axes are independent of  $\theta$

$$\varepsilon_r = \frac{\partial u}{\partial r}, \quad \varepsilon_z = \frac{\partial v}{\partial z}, \quad \varepsilon_\theta = \frac{u}{r}$$

$$\gamma_{rz} = \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z}, \quad \gamma_{r\theta} = \gamma_{z\theta} = 0$$

- Non-zero stress changes:  $\Delta \sigma_r, \Delta \sigma_z, \Delta \sigma_\theta, \Delta \tau_{rz}$

**Boundary Conditions:** Force and displacement constraints depending on the situation

## 7 LIMIT EQUILIBRIUM METHOD

- Assume a failure criterion holds everywhere along an arbitrary failure surface
- Treat the failing soil block as a rigid body and apply equilibrium balance
- Derive an expression for the desired quantity
- Optimise to find the critical angle and quantity value

## 8 STRESS FIELD METHOD

### ASSUMPTIONS

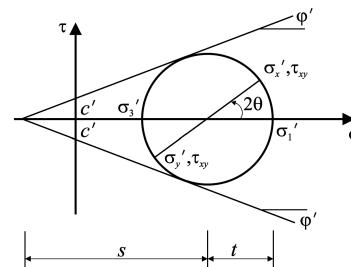
- 2D plane strain conditions

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = \gamma$$

- Mohr-Coulomb failure criterion everywhere in the soil

$$\sigma'_1 - \sigma'_3 = 2c' \cos \phi' + (\sigma'_1 + \sigma'_3) \sin \phi'$$

- Rewriting the criterion in terms of  $s$  and  $t$ :

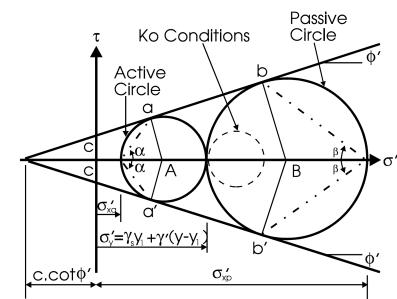


$$s = c' \cot \phi' + \frac{\sigma'_1 + \sigma'_3}{2} = c' \cot \phi' + \frac{\sigma'_x + \sigma'_y}{2}$$

$$t = \frac{\sigma'_1 - \sigma'_3}{2} = \sqrt{\left(\frac{\sigma'_x - \sigma'_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow t = s \sin \phi'$$

## RANKINE STRESS STATES



### Active State (Extension)

- Earth Pressure Coefficient:  $K_a \approx 0.2 - 0.3$
- Earth Pressure:
  - Flat Ground:

$$\sigma'_{xa} = \sigma'_y \tan^2 \left( \frac{\pi}{4} - \frac{\phi'}{2} \right) - 2c \tan \left( \frac{\pi}{4} - \frac{\phi'}{2} \right) = \sigma'_y K_a - 2c K_{ac}$$

- Slope of Inclination  $\beta$ :

$$\sigma'_{xa} = \sigma'_y \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi'}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi'}}$$

- Dotted circle inverts and expands towards the left
- Maximum stress obliquity/Failure planes inclined at  $\pm \alpha = \pm(\frac{\pi}{4} + \frac{\phi'}{2})$  to the direction of  $x$ -axis

### Passive State (Compression)

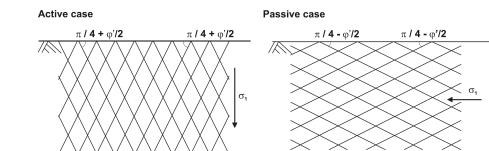
- Earth Pressure Coefficient:  $K_p \approx 3.0$
- Earth Pressure:
  - Flat Ground

$$\sigma'_{xp} = \sigma'_y \tan^2 \left( \frac{\pi}{4} + \frac{\phi'}{2} \right) + 2c \tan \left( \frac{\pi}{4} + \frac{\phi'}{2} \right) = \sigma'_y K_p + 2c K_{pc}$$

- Slope of Inclination  $\beta$ :

$$\sigma'_{xp} = \sigma'_y \cos \beta \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi'}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi'}}$$

- Dotted circle expands towards the right
- Maximum stress obliquity/Failure planes inclined at  $\pm \beta = \pm(\frac{\pi}{4} - \frac{\phi'}{2})$  to the direction of  $x$ -axis



## 9 ENGINEERING GEOLOGY

### DEFINITIONS

**Soil:** Material with unconfined compressive strength (UCS) < 0.6 MPa

**Rock:** Material with UCS > 0.7 MPa

**Rockmass:** Intact rock and its fractures

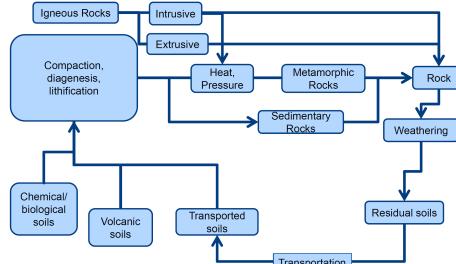
**Igneous (intrusive):** From subsurface magma

**Igneous (extrusive):** From magma ejected above ground

**Sedimentary:** From compaction & lithification of sediment

**Diagenesis:** Aggregation of sediment into sedimentary rock

**Metamorphic:** From intense temperature and pressure



## 10 SITE INVESTIGATION

### AIMS

- Document strength and behaviour of rocks and engineering soils in the area
- Identify potential ground hazards to reduce risk
- Ensure ground is fit for construction

### STAGE 1: INITIAL SCOPING

**Desk Studies:** To interpret existing data and reduce scope of investigation

- Geological & hydrological maps, historical photographs, seismic data, environmental records, aerial photographs, satellite remote sensing data, etc

**Benefits:** Reduces costs, highlights problems early and aids design process

**Site Visits:** To visually assess the site's features

- Slope angles, signs of movement, rockface outcrops (eg. cliffs, quarries), land-use, vegetation changes, groundwater conditions (eg. springs), site access for workers/equipment (eg. overhead cables, road widths), existing structural damage
- Benefits:** Understand locals' problems and build stakeholder relations

### STAGE 2: FIELDWORK

**Geomorphologic Mapping:** To assess surface conditions by field surveys & aerial/satellite photographs

- Distribution & geometry of rock deposits
- Surface features (eg. runoff & drainage patterns, slope angles)
- Extent of permafrost
- Benefits: Cheap

**Geophysical Surveys:** To search for hidden geological features for siting boreholes/pits and interpolate borehole information

- Electromagnetic, electric resistivity, gravitational, magnetic, seismic methods, ground-penetrating radar, ground motion monitoring

**Trial Pits & Trenches:** To assess sub-surface conditions by digging out ground cross-section

- Benefits: Assess 3D nature of rock, easy to extract samples for testing in lab or in-situ

**Boreholes:** To assess subsurface conditions by drilling deep cores

- Spacing
  - Buildings: 10 - 30 m
  - Road Lines: 30 - 300 m
  - Landslides: > 5 m in line for profile
- Depth
  - Generally, 1.5 x foundation width + 10 m control hole and 3 m below rockhead
- Drilling Methods
  - Auger Rotary Drilling
    - Rotating auger progressively digs soil out
    - Completely destroys soil fabric
    - Soils only, shallow depths
  - Cable Percussion
    - Hammers core barrel into the ground
    - Yields disturbed samples
    - Soils & weak rocks, 15 - 40 m depth
    - Cheap and commonly used

**Rotary Drilling Rig**

- Long rotating drill with cooling lubricant to flush chippings to the surface
- Yields intact cores
- Hard rocks, > 100 m depth
- Deep, intact samples, but expensive

**Laboratory Tests:** To characterise rock properties on-site or in laboratory

- Point load tests, index testing, unconfined compressive strength tests, Schmidt Hammer, Brazilian, box & ring shear, triaxial tests

### STAGE 3: REVIEW

- Monitor excavation and construction

### CORE LOGGING

**Material:** Rock properties

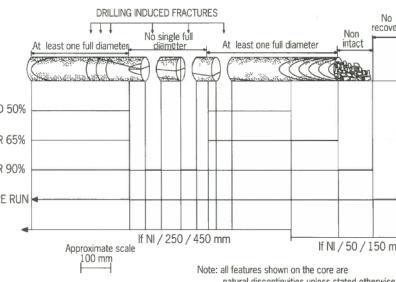
- Unconfined compressive strength (UCS)
- Bedding/layering
- Colours & name

**Discontinuity:** Fractures in the rock

- Fracture spacing, orientation & frequency
- Fracture persistence, size and shape
- Fracture roughness
- Overall fracture sets & block size

**Mass:** Combined rockmass properties

- Total Core Recovery (TCR): Core material recovered over total core run
- Solid Core Recovery (SCR): Intact core material with  $\geq 1$  full diameter over total core run
- Rock Quality Designation (RQD): Total intact core pieces  $> 100$  mm long over total core run
- Fracture Index/Spacing: Number/Spacing of fractures per unit length
- Rock Mass Rating (RMR): Overall summary rating of all rock properties
- Other ratings: Q Index, Geological Strength Index



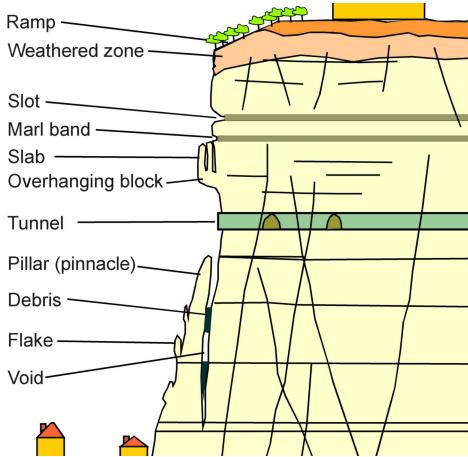
## 11 ROCKMASS INSTABILITY

**Hazard:** Anything that could cause human harm

**Vulnerability:** Chance for human harm

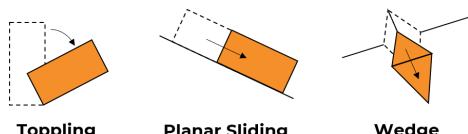
**Risk:** Hazard  $\times$  Vulnerability

### CLIFF ANATOMY



### CLIFF COLLAPSE

#### Failure Types



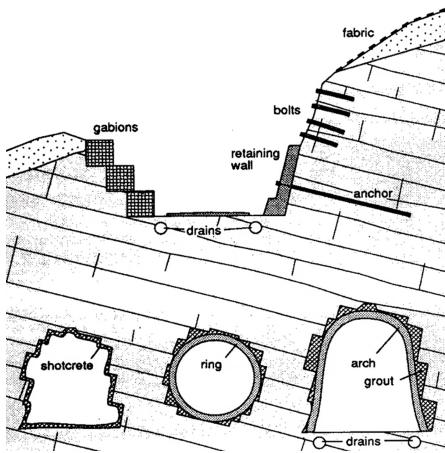
#### Physical Causes

- Low cohesion in unconsolidated material
- Close fracturing (bedding, joints, schistosity) in solid rock
- Impermeable layers that obstruct flow and provide lubricated slip surfaces
- Presence of 'slippery' rocks (eg. clay, coal)
- Downslope dip of potential slip planes

#### Failure Triggers

- External Vibrations: Earthquakes, traffic
- Loading: Buildings, snow, debris, vegetation
- Toe Removal: Loss of underlying support
- Soil Strength Loss: Weathering, porewater changes, old slip surface reactivation

## Excavation Stabilisation Measures



- Prevent toppling by securing slope
  - Netting
  - Rock bolts
- Prevent sliding by obstruction
  - Anchored retaining walls
  - Gabions
- Pore water pressure control
  - Drains at base of excavation
- Tunnel stabilisation
  - Shotcrete
  - Segmented concrete rings/arches
  - Grout filling

## 12 STEREOGRAPHIC PROJECTION

**Stereographic Projection:** Projection of a hemisphere's 3D surface onto its 2D equatorial plane

**Great Circle:** Any circle on a sphere's surface such that they share the same centre

**Pole:** Point of intersection between the hemisphere and a line that passes through the sphere's centre and is perpendicular to the fracture plane

**Dip:** Inclination angle below horizontal plane

**Dip Direction:** 3-digit bearing from North

## REPRESENTATION

### For a dip of $x$ to bearing $y$ :

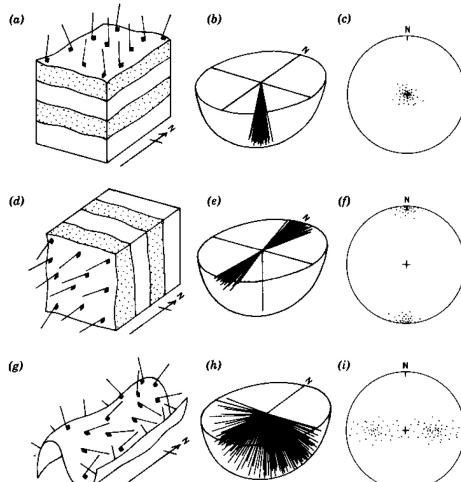
1. Place tracing paper on top of stereonet and pin the centres together

- Latitude and longitude lines are all spaced  $2^\circ$  apart
- 2. Count  $y^\circ$  clockwise from the North pole and mark the perimeter
- 3. Rotate the tracing paper to align this mark with the E-W axis
- 4. Count from the mark towards the stereonet's centre along the E-W axis
  - **Line:** Single point  $x^\circ$  from the mark
  - **Plane:** Entire longitudinal line at  $x^\circ$
  - **Pole:** Single point  $(90 + x)^\circ$  from the mark representing the entire plane
- 5. Rotate the tracing paper back to its original position

Note: Pole projections are typically used to avoid clutter

## POLE PROJECTION PATTERNS

- **Horizontal Bedding:** Clustering near the centre
- **Vertical Bedding:** Clustering near opposite edges
- **Cylindrically-Folded Bedding:** Clustering along a line
  - Line is parallel to the actual folding direction



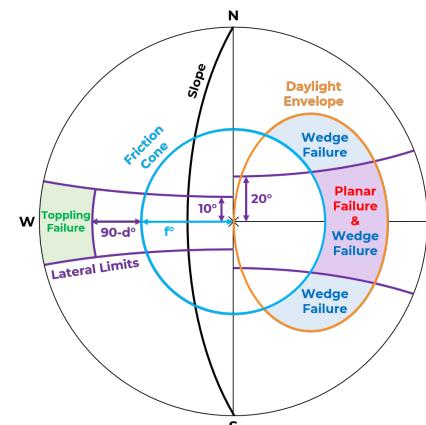
## KINEMATIC ANALYSIS

### Assumptions

- Shear strength of sliding surface is only from friction angle
- Cohesion is zero

## Procedure

1. Set reference axis (eg. road) as the stereonet's N-S axis
2. Draw the great circle of the desired slope with dip  $d^\circ$
3. **Daylight Envelope:** Connect the poles for all points along this great circle
4. **Friction Cone:** Draw a circle of radius  $f^\circ$  in the stereonet centre, where  $f$  is the friction angle of the slope
5. **Lateral Limits:** Draw latitude lines  $10^\circ$  and  $20^\circ$  from either side of the stereonet centre perpendicular to the slope
  - Imposed by adjacent blocks on the slope preventing movement
6. **Planar Failure:** Poles that lie in the daylight envelope between the lateral limits, but outside the friction cone (purple region)
7. **Toppling Failure:** Poles between  $10^\circ$  lateral limits on the convex side of the great circle and  $(90 - d)^\circ$  in that direction from the friction cone (green region)
  - Extra  $(90 - d)^\circ$  is to compensate for part of the slope's great circle 'overlapping' with the friction cone
8. **Wedge Failure:**
  - (a) Group poles that lie on the concave side of the slope's great circle
  - (b) Draw the great circles of these groups' centroids and find their intersection points
  - (c) If the poles of the intersection points of these great circles lie in the blue or purple regions, wedge failure is possible.



## Limitations

- Only tells you if block failure is possible, but not when it will occur
- Does not consider external forces from water & reinforcements (eg. rock bolts)