PASSIVE SCALAR

Diffusion Equation ($\vec{q}=-D\nabla C$)

Mass continuity (derive!)

$$\frac{\partial c}{\partial t}=-\nabla\cdot\vec{q}$$

Fick's law (derive!)

$\vec{q}=-D\nabla C$ (mass flux by diffusion)

$$\underbrace{\frac{\partial c}{\partial t}=D\nabla^2 C \;:\; \frac{\partial c}{\partial t}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)}$$

anisotropic:
$$\frac{\partial c}{\partial t}=D_x\frac{\partial^2 c}{\partial x^2}+D_y\frac{\partial^2 c}{\partial y^2}+\ldots$$
heterogeneous:
$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(D\frac{\partial c}{\partial y}\right)+\ldots$$
both:
$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_y\frac{\partial c}{\partial y}\right)$$

↓ Solution (1D, 2D, 3D)

1D: $C(x,t)=\frac{M}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}}$ where $\sigma=\sqrt{2Dt}$ (spread width) $\frac{d\sigma}{dt}=\frac{D}{\sigma}$ (rate of spread)

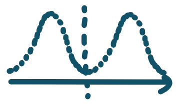
2D: $C(x,y,t)=\frac{M}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}$

3D: $C(x,y,z,t)=\frac{M}{(2\pi)^{3/2}\sigma^3}e^{-\frac{x^2+y^2+z^2}{2\sigma^2}}$

↓ C decays faster as no. of dimension increases:
- 1D: $C\sim\sigma^{-1}\sim t^{-1/2}$
- 2D: $C\sim\sigma^{-2}\sim t^{-1}$
- 3D: $C\sim\sigma^{-3}\sim t^{-3/2}$

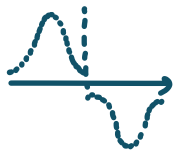
Boundary Condition.

Reflector at $(x=h)$: $q_x(x=h)=-D\frac{\partial c}{\partial x}\Big|_{x=h}=0$



$$C(x,t)=\frac{M}{\sqrt{2\pi}\sigma}e^{-\frac{(x-0)^2}{2\sigma^2}}+\frac{M}{\sqrt{2\pi}\sigma}e^{-\frac{(x-2h)^2}{2\sigma^2}}$$

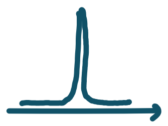
Absorber at $(x=h)$: $C(x=h)=0$



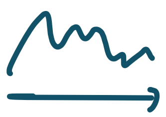
$$C(x,t)=\frac{M}{\sqrt{2\pi}\sigma}e^{-\frac{(x-0)^2}{2\sigma^2}}-\frac{M}{\sqrt{2\pi}\sigma}e^{-\frac{(x-2h)^2}{2\sigma^2}}$$

Initial Condition: $C(t=0)$

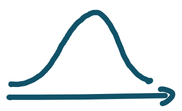
1. $C(t=0)=M\delta(x)$ — dirac's delta function
   - C: kg m^{-3}; $M\delta$: kg m^{-2} m^{-1}
   - unit: $1/x$
   - eg. $[\delta(x,y,z)]=m^{-3}$



2. or if arbitrary:
   $C(t=0)=C_0(x)\rightarrow M(x')=C_0(x')\delta x'$
   sol. becomes:
   $$C(x,t)=\int_{-\infty}^{\infty}\frac{C_0(x',t=0)}{\sqrt{2\pi}\sigma}e^{-\frac{(x-x')^2}{2\sigma^2}}dx'$$



3. or if initially gaussian:
   $C(t=0)=C_0(x)=\frac{M}{\sqrt{2\pi}\sigma_0}e^{-\frac{x^2}{2\sigma_0^2}}$
   sol. becomes:
   $C(x,t)=\frac{M}{\sqrt{2\pi}\sigma_0}e^{-\frac{x^2}{2\sigma^2}}$, but $\sigma^2=\sigma_0^2+2Dt$



Advection + Diffusion ($\vec{q}=\vec{u}C$)

Advective mass flux

$$\vec{q}=\underbrace{\vec{u}C}_{\text{advection}}-\underbrace{D\nabla C}_{\text{diffusion}}$$

(sub into $\frac{\partial c}{\partial t}=-\nabla\vec{q}$)

$$\frac{\partial c}{\partial t}+\underbrace{\vec{u}\cdot\nabla C}_{\text{advection}}=\underbrace{D\nabla^2 C}_{\text{diffusion}}:\ \frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)$$

↓ Solution : $X=x-ut,\ Y=y-vt,\ Z=z-wt$

eg. $\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=0$, $C_0(x,y)=\frac{M}{2\pi\sigma_0^2}e^{-\frac{x^2+y^2}{2\sigma_0^2}}$ (σ_0 constant)

sol. $C(x,y,t)=C_0(X,Y,t)$
$$=\frac{M}{2\pi\sigma_0^2}e^{-(X^2+Y^2)/2\sigma_0^2}$$
$$=\frac{M}{2\pi\sigma_0^2}e^{-\frac{[(x-ut)^2+(y-vt)^2]}{2\sigma^2}}$$

this example is pure advection!
if advection-diffusion:
- solve diffusion first
- then replace $X=x-ut,\ Y=y-vt,\ Z=z-wt$

Péclet Number:

$Pe=\frac{\text{advection}}{\text{diffusion}}=\frac{U_0L_0}{D}$,
- if Pe is big: advection dominate (can ignore diffusion)
- if Pe is small: diffusion dominate (can ignore advection)

Continuous Release.

$$\frac{\partial c}{\partial t}+\vec{u}\nabla C=D\nabla^2 C+\dot{M}\delta(x,y,z)$$

↓ Solution

Diffusion only:

3D: $C(x,y,z,t)=\int_0^t\frac{\dot{M}}{(2\pi)^{3/2}\sigma^3}e^{-\frac{r^2}{2\sigma^2}}dt'=\frac{\dot{M}}{4\pi Dr}\mathrm{erfc}\left(\frac{r}{\sqrt{4Dt}}\right)$

$M(t')=\dot{M}\delta t'$; $\sigma=\sqrt{2D(t-t')}$

3D steady ($t\to\infty$): $C(x,y,z)=\frac{\dot{M}}{4\pi Dr}$ (steady: not time dependent)

Diffusion-advection:

3D steady ($t\to\infty$): $C(x,y,z)=\frac{\dot{M}}{4\pi Dr}e^{-\frac{U}{2D}(r-x)}$

solved from full eqn:
$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)+\dot{M}\delta(x,y,z)$$

3D steady + far field ($t\to\infty$, $x\to\infty$): $C(x,y,z)=\frac{\dot{M}}{2\pi U\sigma^2}e^{-\frac{y^2+z^2}{2\sigma^2}}$, $\sigma^2=\frac{2Dx}{U}$

(derived from 2D instantaneous release. sub $x=Ut$)

solved from simplified eqn:
$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=D\left(\frac{\partial^2 c}{\partial y^2}+\frac{\partial^2 c}{\partial z^2}\right)+\dot{M}\delta(x,y,z)$$
- advection only in x
- diffusion only in y,z

2D steady + far field ($t\to\infty$, $x\to\infty$): $C(x,y)=\frac{\dot{M}}{\sqrt{2\pi}hU\sigma}e^{-\frac{y^2}{2\sigma^2}}$, $\sigma^2=\frac{2Dx}{U}$

(far field means only advection in x, no diffusion)

Turbulence Dispersion ($\vec{q}=\overline{u'c'}$)

from advection + diffusion:

$$\vec{q}=\underbrace{\vec{u}C}_{\text{advection}}-\underbrace{D\nabla C}_{\text{diffusion}}$$

↓ Reynold's averaging the $\vec{u}$ and c (from advection)

$u(t)=\bar{u}+u'(t)$; $c(t)=\bar{c}+c'(t)$

$$\overline{uc}=\overline{(\bar{u}+u')(\bar{c}+c')}=\overline{\bar{u}\bar{c}}+\cancel{\overline{u'\bar{c}}}+\cancel{\overline{\bar{u}c'}}+\overline{u'c'}$$
$$=\bar{u}\bar{c}+\overline{u'c'}$$

$$\vec{q}=\underbrace{\bar{u}\bar{c}}_{\text{advection}}+\underbrace{\overline{u'c'}}_{\text{turb.}}-\underbrace{D\nabla\bar{C}}_{\text{diffusion.}}$$

(sub $\vec{q}$ into $\frac{\partial\bar{c}}{\partial t}=-\nabla\vec{q}$)

$\vec{q}_{turb}=\overline{u'c'}=-ul\frac{\partial\bar{c}}{\partial x}=-D_T\frac{\partial\bar{c}}{\partial x}$ (ul is eddy diffusivity) (derivation pls memorise pg 13)

$$\frac{\partial\bar{c}}{\partial t}+\underbrace{\vec{\bar{u}}\cdot\nabla C}_{\text{advection}}=\underbrace{\cancel{D\nabla^2 C}}_{0\text{ as }D_T\gg D}+\underbrace{D_T\nabla^2 C}_{\text{turbulence.}}$$

$\vec{\bar{u}}=\begin{pmatrix}\bar{u}\\ \bar{v}\\ \bar{w}\end{pmatrix}$

(basically if turbulent, replace u with $\bar{u}$, c with $\bar{c}$ and D with D_T)

↓ solution : replace D with D_T

$$\bar{c}(x,y,z,t)=\frac{M}{(2\pi)^{3/2}\sigma^3}e^{-\frac{x^2+y^2+z^2}{2\sigma^2}},\ \sigma=\sqrt{2D_Tt}$$

if turbulence anisotropic:
$D_T=[D_{Tx},D_{Ty},D_{Tz}]$
$$\frac{\partial\bar{c}}{\partial t}+\vec{\bar{u}}\cdot\nabla C=D_{Tx}\frac{\partial^2 c}{\partial x^2}+D_{Ty}\frac{\partial^2 c}{\partial y^2}+D_{Tz}\frac{\partial^2 c}{\partial z^2}$$
$$\bar{c}(x,y,z,t)=\frac{M}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}e^{-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}+\frac{z^2}{2\sigma_z^2}\right)}$$

Shear Dispersion ($\vec{q}=\langle\tilde{u}\tilde{c}\rangle$)

from advection + diffusion:

$$\vec{q}=\underbrace{\vec{u}C}_{\text{advection}}-\underbrace{D\nabla C}_{\text{diffusion}}$$

↓ Reynold's and Depth averaging the u and C:

$u=U+\tilde{u}+u'$

$C=\langle\bar{C}\rangle+\tilde{c}+c'$

$$\vec{q}=\underbrace{U\langle C\rangle}_{\text{adv.}}+\underbrace{\langle\tilde{u}\tilde{c}\rangle}_{\text{shear}}+\underbrace{\langle\overline{u'c'}\rangle}_{\text{turb.}}-\underbrace{D\frac{\partial\langle\bar{c}\rangle}{\partial x}}_{\text{dif}}$$

(sub $\vec{q}$ into $\frac{\partial\langle\bar{c}\rangle}{\partial t}=-\nabla\vec{q}$)

$\vec{q}_{shear}=\langle\tilde{u}\tilde{c}\rangle=-D_s\frac{\partial\langle\bar{c}\rangle}{\partial x}$

$$\frac{\partial\langle\bar{c}\rangle}{\partial t}+U\nabla\langle\bar{c}\rangle=\cancel{D\nabla^2 C}+\cancel{D_T\nabla^2 C}+D_s\nabla^2 C$$

0 as $D_s\gg D_T\gg D$

↓ solution: replace D with D_s

$$\langle\bar{c}\rangle(x,y,z,t)=\frac{M}{(2\pi)^{3/2}\sigma^3}e^{-\frac{x^2+y^2+z^2}{2\sigma^2}},\ \sigma=\sqrt{2D_st}$$

The Boussineq Approximation

Definition of buoyancy: Buoyancy is the reduced gravity acceleration acting on a fluid parcel.

$$b = \frac{\rho_h - \rho}{\rho_0} g \approx \frac{T - T_h}{T_0} g$$ (note that T_0 must be expressed in Kelvin!)

$\rho(x)$ is density of the parcel of fluid at location x; $\rho_h(z)$ is the density of surrounding at height z;
ρ_0 is a suitable reference density (eg. $\rho_0 = \rho_h(z=0)$ is often used)

ACTIVE SCALAR

Emptying Boxes (Simple case)

replenish source modelled as F (if there's any), no stratification

Part 1: What Drives a flow?

1. Temperature Difference creates Density Differences (Boussineq)

According to the eqn of state (or known as The Boussineq Approx.), fluid density depends on temperature:

$$b = \frac{\rho_0 - \rho}{\rho_0} g \approx \frac{T - T_0}{T_0} g$$ (taking reference ρ and T as equal to surrounding)

$$\rho = \rho_0 - \rho_0\left(\frac{T - T_0}{T_0}\right) \rightarrow \rho = \rho_0(1 - \beta(T - T_0)) \rightarrow$$ hotter fluid ($T > T_0$) is lighter ($\rho < \rho_0$)

2. Density Differences create Hydrostatic Pressure Differences. (Hydrostatic)

When the fluid is at rest (but not necessary in equilibrium), pressure is determined by the weight of fluid column above it.

$$\frac{dp}{dz} = -\rho g \quad \begin{cases} \text{high density: pressure decreases rapidly with height} \\ \text{low density: pressure decreases slowly with height.} \end{cases}$$

(if question starts by asking how temp. dif. drives flow?)

$$\frac{dp}{dz} = -\rho_0 g + \frac{\rho_0}{T_0}(T - T_0) g$$ → pressure changes is proportional to $T - T_0$

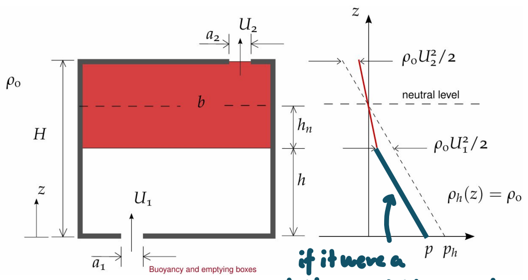
Because pressure drops at different rates outside and inside (or left and right), a pressure difference (Δp) develops between it at any given height (except at neutral level)

3. Pressure differences drives flow. (Bernoulli)

Fluids move from areas of high pressure to area of low pressure. The magnitude of this pressure difference determines the velocity of the air through the opening.

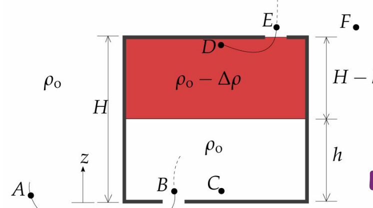
$\Delta p = \frac{1}{2}\rho U^2$ → potential energy (pressure) converts into kinetic energy (velocity)

Part 2: Sketch the Pressure Distribution.



if it were a left room right room qtn, this line's gradient (ρ) can be different from surrounding (ρ_0)

Part 3: Calculate Flow Rate (Q)



this mean like $H_A = H_B$

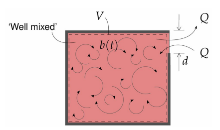
Bernoulli's: $\underbrace{\rho_0 \frac{U^2}{2}}_{\text{kinetic}} + \underbrace{p}_{\text{pressure}} + \underbrace{\int_0^z \rho(z) g\,dz}_{\text{potential}} = \text{constant}$

1. A-B, D-E: $\rho_0 \frac{U^2}{2} + p = \text{constant}$ ($\rho g z$ constant → same height)
2. C-D: $p + \int_0^z \rho(z) g\,dz = \text{constant}$ ($\frac{1}{2}\rho U^2$ constant → $U \approx 0$ in the box)
3. B-C doesn't follow bernoulli's as there is energy lost (due to vena contracta). Hence:
   B-C: $p = \text{constant}$ (pressure balance assumption)

By solving all these equations, we get the formula:

$$Q = A^*\sqrt{2b(H-h)}, \quad A^* = \sqrt{\frac{a_1^2 a_2^2}{a_1^2 + a_2^2}}$$

F (buoyancy flux) = Qb



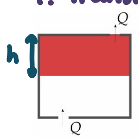
for one opening (well-mixed box):
$$Q = \frac{k}{3} w \sqrt{b d^3} \begin{cases} k \approx 0.5 \text{ vertical opening} \\ k \approx 0.25 \text{ horizontal opening.} \end{cases}$$

Part 4: Transient vs Steady State.

Transient: no sources replenishing the 'heated' layer
Steady State: there's F replenishing source of buoyancy. $F_{replenish} = F_{out} = Qb$

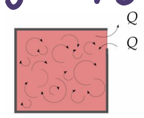
$$\frac{dV}{dt} = Q_{plume}(h) - Q_{vent}$$
$$\frac{d(bV)}{dt} = F_{source} - Q_{vent} \cdot b$$

1. Transient: How long to empty the box?



$$\frac{dh}{dt} = \frac{Q(t)}{S}, \quad Q = A^*\sqrt{2b(H-h)}$$

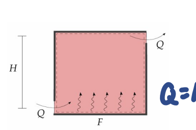
solution:
$$\frac{h}{H} = 1 - \left(1 - \frac{t}{t_e}\right)^2, \quad t_e = \left(\frac{2S}{A^*}\right)\left(\frac{H}{2b}\right)^{1/2}$$



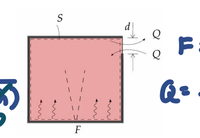
$$\frac{db}{dt} = -\frac{b(t) \cdot Q(t)}{V}, \quad Q = \frac{k}{3} w\sqrt{bd^3}$$

solution:
$$\frac{b}{b_0} = \left(1 + \frac{t}{\tau}\right)^{-2}, \quad \tau = \frac{6V}{kw}(b_0 d)^{-1/2}$$

2. Steady: $F_{replenish} = F_{out} = Qb$ (F, Q, b given 1, find the other 2)



$F = Qb$
$Q = A^*\sqrt{2b(H - \cancel{h})}$, with $h = 0$

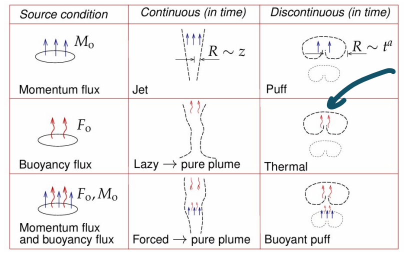


$F = Qb$
$Q = \frac{k}{3} w\sqrt{bd^3}$ } solve simultaneous equation.

eg. if given F, solve for Q and b: (treat F as a constant)

$$Q = (2A^{*2} F H)^{1/3}, \quad b = \left(\frac{F^2}{2A^{*2} H}\right)^{1/3}$$

$$b = \left(\frac{3F}{kw d^{3/2}}\right)^{2/3}, \quad Q = \frac{k}{3} w\sqrt{bd^3}$$



Introduction to Dimensional Analysis.

using thermal as example (cause it is simple)

Every phenomenon has a driver and obey conservation laws.
For thermal, the driver is the discontinuous source of buoyancy flux, F_0
And total buoyancy, $B = bV$ is conserved.
(even though $b(t)$ and $V(t)$ changes with time)

Question: How $R(t)$, $b(t)$, and $z(t)$ changes with time?

$[R] = L$, $[b] = LT^{-2}$, $[B] = L^4T^{-2}$, $[t] = T$

→ $R(t) \sim B^{1/4} t^{1/2}$, $b(t) \sim B^{1/4} t^{-3/2}$, $z \sim B^{1/4} t^{1/2}$ (and if wind blow horizontally: $x \sim Ut$, $z \sim \sqrt{x}$)

Turbulent Jets vs Plumes

Part 1: Turbulent Jets vs. Turbulent Plumes.

* Characteristic quantity:

$$w_m = \frac{M}{Q}, \quad r_m = \frac{Q}{\sqrt{M}}, \quad b_m = \frac{F}{Q} \text{ (plumes only)}$$

Turbulent Jets

1. Driver
   A continuous source of momentum (M_0) with zero buoyancy ($b = 0$)
2. Physics
   Driven by inertia (initial kick)
   Scale with Momentum flux (M)
3. Conservation Laws
   Volume flux INCREASED ($\frac{dQ}{dz} > 0$) due to entrainment.
   Momentum flux CONSERVED ($\frac{dM}{dz} = 0$). No external forces acting on it.
   Buoyancy Flux (F) is zero. No buoyancy ($b = 0$)

Turbulent Plumes

1. Driver
   A continuous source of Buoyancy flux (F_0) (eg. heat source, fire, fresh water salt water)
2. Physics
   Driven by gravity acting on density differences ($b = \frac{\rho_h - \rho}{\rho_0}$)
   Fluid accelerates as it rises.
3. Conservation Laws
   Volume flux INCREASED ($\frac{dQ}{dz} > 0$) due to entrainment.
   Momentum flux INCREASED ($\frac{dM}{dz} > 0$). The buoyancy force creates acceleration, constantly adding momentum to the flow.
   Buoyancy flux CONSERVED ($\frac{dF}{dz} = 0$). Heat is neither created nor destroyed, just spread out.

(everything that is not conserved can be find as a function of time by using dimensional analysis)

Part 2: Dimensional Analysis

far field behaviour, supposedly:
$Q = f(z, M_0, \cancel{Q_0})$, $Q = f(z, \cancel{M_0}, \cancel{Q_0}, F_0)$
The momentum flux $M(z)$ is much greater than the source momentum flux M_o. (plume)
The volume flux $Q(z)$ is much greater than the source volume flux Q_o (jet, plume)

Turbulent Jets cause M_0 is constant

The only quantity that are known: $[M] = L^4T^{-2}$, $[z] = L$

We can hence solve for:
1. Volume flux, Q: $[Q] = [M]^{1/2}[z]^1 \rightarrow Q \propto M_0^{1/2} z$
2. Velocity, w_m: $[w] = [M]^{1/2}[z]^{-1} \rightarrow w_m \propto M_0^{1/2} z^{-1}$
3. Radius, r_m: $[r] = [Q][M]^{-1/2} \rightarrow r_m \propto z$

Turbulent Plumes cause F_0 is constant

The only quantity that are known: $[F] = L^4T^{-3}$, $[z] = L$

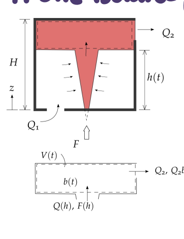
We can hence solve for:
1. Volume flux: $[Q] = [F]^{1/3}[z]^{5/3} \rightarrow Q \propto F_0^{1/3} z^{5/3}$
2. Momentum flux: $[M] = [F]^{2/3}[z]^{4/3} \rightarrow M \propto F_0^{2/3} z^{4/3}$
3. Velocity, w_m: $[w] = [F]^{1/3}[z]^{-1/3} \rightarrow w_m \propto F_0^{1/3} z^{-1/3}$
4. Buoyancy, b_m: $[b] = [F]^{2/3}[z]^{-5/3} \rightarrow b_m \propto F_0^{2/3} z^{-5/3}$

(although w_m, r_m, b_m can be found using dimensional analysis, in exam it is more pratical to calculate Q (and M for plume) and then use characteristic eqn.)

(Note that if it is planar jets/plume: Q, M, F are all per unit length, eg. $[Q] = L^2T^{-1}$)

Part 3: Emptying Filling Box (turbulent plume sources)

1. One isolated point source of buoyancy (Turbulent Plume)



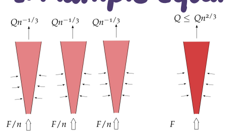
$$\frac{dV}{dt} = \underbrace{cF^{1/3}h^{5/3}}_{Q_{plume}(h)} - \underbrace{A^*\sqrt{2b(H-h)}}_{Q_{vent}} = 0 \quad \text{STEADY STATE}$$

$$\frac{d(bV)}{dt} = \underbrace{F}_{F_{source}} - \underbrace{bA^*\sqrt{2b(H-h)}}_{Q_{vent} \cdot b} = 0$$

Trying to solve for: What h gives steady-state?
Unknown: F, b, A^*, h, H (A^* and H are constants)
→ 3 unknowns, 2 eqn...
→ remember characteristic eqn: $b = \frac{F}{Q_{plume}} = \frac{F}{cF^{1/3}h^{5/3}} = \frac{F^{2/3}}{ch^{5/3}}$

solution: $\frac{\sqrt{2}A^*}{H^2 c^{3/2}} = \left(\frac{\xi^5}{1-\xi}\right)^{1/2}$ (use numerical method) ($A^* \uparrow, \xi \uparrow$)

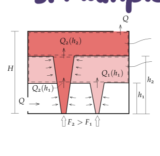
2. Multiple equal isolated point source



Single turbulent plume: $Q \propto F^{1/3}$
n small turbulent plume: $Q \propto n\left(\frac{F}{n}\right)^{1/3}$ (total buoyancy flux still equal to F)
$Q \propto n^{2/3} F^{1/3}$

solution: $\frac{\sqrt{2}A^*}{nH^2c^{3/2}} = \left(\frac{\xi^5}{1-\xi}\right)^{1/2}$ it's weird that there is just n without any power that's because $Q_{vent} \propto (A^*)^{2/3}$ $\therefore n^{2/3} \propto (A^*)^{2/3}$

3. Multiple unequal isolated point source



$Q = A^*\sqrt{2b_1(h_2 - h_1) + 2b_2(H - h_2)}$
$F_1 = Q_1(h_1) \cdot b_1$
$F_2 = Q_2(h_2) \cdot b_2 - Q_1(h_1) \cdot b_1$

can be used even if no turbulent plume point source eg. transient state emptying box replace Q in $\frac{dh}{dt} = \frac{Q}{S}$ if it is multiple layer.

* If stratified instead of layers:
$Q = A^*\left(2\int_0^H b(z)\,dz\right)^{1/2}$