c1. Introduction

1. Direction Conventions

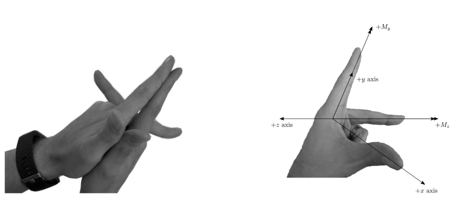


Figure 1.7: Left hand used to represent the action of positive bending moments M_y and M_z in a right hand axis system.

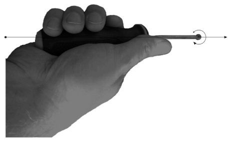


Figure 1.3: Right hand screw rule.

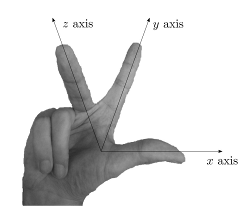


Figure 1.2: Right hand axis system.

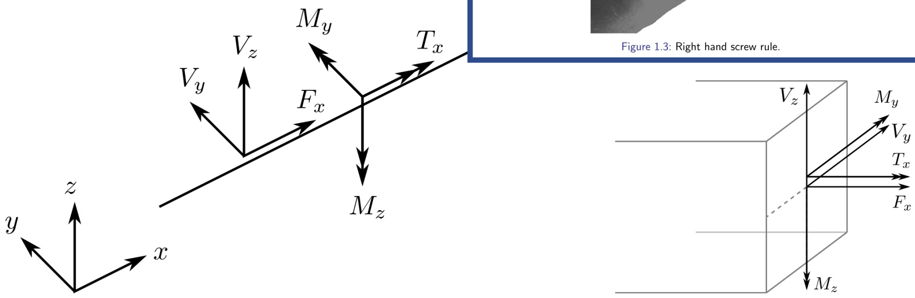


Figure 1.11: Forces and moments acting on a beam in 3D space. V_z and V_y are shear forces, F_x is an axial force, M_y and M_z are bending moments, and T_x is a twisting moment. The forces and moments are also shown on the face of a rectangular section.

2. Quick summary on next four chapters

We can agree that each of the loading modes results in one of two types of stress:

- Normal stress $\begin{cases} \text{due to axial force} \\ \text{due to bending moments} \end{cases}$ $\sigma_{xx} = \frac{F_x}{A} + \frac{M_y z}{I_{yy}} + \frac{M_z y}{I_{zz}}$
- Shear stress $\begin{cases} \text{due to shear forces} \\ \text{due to torsion moment} \end{cases}$ $\tau = \frac{T_x r}{J} + \frac{V_z Q_y}{I_{yy} b} + \frac{V_y Q_z}{I_{zz} b}$

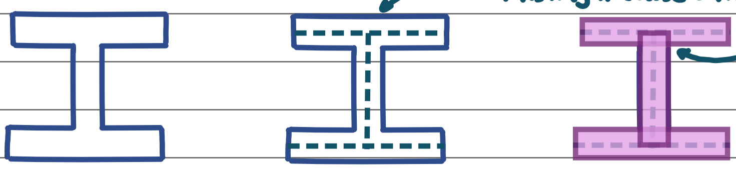
c2 - axial force, c3 - bending moment, c4 - twisting moment, c5 - shear forces.

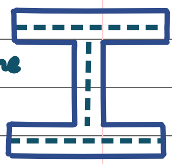
3. Centreline Approximation.

Centreline approximation is useful when finding something that involves A, for example: I, J,...

the centerline must be "connected"! not:

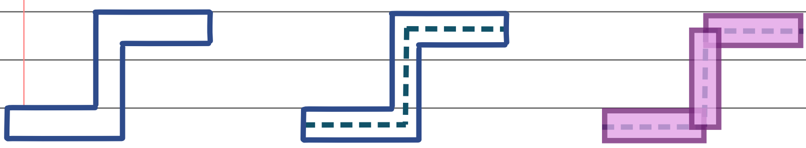
this might cause some overlap but it is fine





draw centerline.

treat each of the line as the centerline of a new rectangle, that rectangle is the "Area" of that centerline.



C2. Axial Force

$$\boxed{\sigma_{xx} = \frac{F_x}{A}} \quad (1.1)$$

where A is the area of the cross-section.

The material yield load is calculated:

not really necessary to remember (1.2):

$\sigma_{xx} = \frac{F_x}{A}$, if $\sigma_{xx} = f_y$:

$$F_y = Af_y \quad (1.2)$$

$F_x = F_y = Af_y$

where y is used as a subscript to indicate yield and f_y is the yield stress.

In the elastic regime the change in length of the structural element due to axial force is given as:

$\rightarrow \sigma_{xx} = \frac{F_x}{A}$

$$\boxed{\delta_x = \frac{F_x L}{EA} = \frac{L\sigma_{xx}}{E} = L\varepsilon_{xx}} \quad (1.3)$$

$E = \frac{\sigma_{xx}}{\varepsilon_{xx}}$

where E is the Young's modulus (around 200 to 210 GPa for steel and around 35 GPa for concrete), and L is the length of the structural element.

The Euler buckling load for a column is calculated:

$$\boxed{P_{\mathrm{cr}} = \frac{\pi^2 EA}{\lambda^2}} \quad (1.4)$$

$\lambda = \frac{Le}{r_g}$, $r_g = \sqrt{\frac{I_{min}}{A}}$

given in formula sheet as:

$p_c = \frac{\pi^2 E I_{min}}{Le^2}$

where P_{cr} is the compression load at which the column will buckle, L_e is the effective length of the column ($L_e = L$ for a pin-ended column), $I_{\min}$ is the minimum principal second moment of area, r_g is the radius of gyration, and λ is the slenderness ratio.

To assess whether a column is likely to buckle prior to material yield we can calculate the critical stress as:

not really important as:

$$\sigma_{\mathrm{cr}} = \frac{\pi^2 E}{\lambda^2}$$

$\sigma = \frac{F}{A} \rightarrow \sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E \cancel{A}}{\cancel{A} \lambda^2} = \frac{\pi^2 E}{\lambda^2}$

where the *cr* subscript is used to indicate that this is the normal stress throughout the cross-section at the critical buckling load. After buckling the normal stress will no longer have a constant value throughout the cross-section.

The normalised slenderness ratio can be given as:

can remember but basically what it says:

if $F_y < P_{cr}$, material yield first

if $P_{cr} < F_y$, material buckle first.

$$\boxed{\bar{\lambda} = \frac{\lambda}{\lambda_1} = \sqrt{\frac{F_y}{P_{\mathrm{cr}}}}} \quad (1.8)$$

where λ_1 is the slenderness ratio that would be required to allow material yield and Euler buckling to occur at the same time ($\sigma_{\mathrm{cr}} = f_y$).

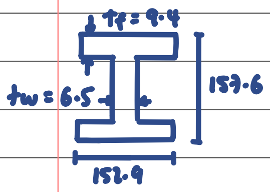
- When $\bar{\lambda} < 1$ this implies the column will undergo material yielding.
- When $\bar{\lambda} \geq 1$ this implies the column will undergo buckling prior to material yielding.

Worked Examples

a For a 152 × 152 × 30 UKC column section (see the supplementary section tables) calculate the material yield load if the section is used as a column subject to axial load. You may assume the structural element is manufactured from grade S355 steel. ← f_y!

$F_y = Af_y$

1. determine what is asked, F_y.
2. determine what is required to solve for it, A and f_y



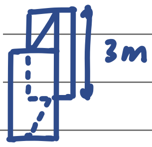
From UKC-section table,

$A = 38.3\ cm^2 = 3830\ mm^2$

$F_y = Af_y = 3830\ mm^2 \times 355\ N/mm^2 = 1360\ kN$ //

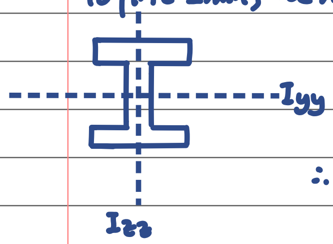
P_{cr}

b Calculate the Euler buckling load if the same section is used as a 3m long pin-ended column. You may assume a Young's modulus of 205 GPa. State whether the column would fail due to material yield or buckling. E



$P_{cr} = \frac{\pi^2 EA}{\lambda^2}$, where $\lambda = \frac{L_e}{r_g}$, and $r_g = \sqrt{\frac{I_{min}}{A}}$

To find I_{min}, we need to compare both I_{yy} and I_{zz}:



$I_{yy} = 12.5 \times 10^6\ mm^4$
$I_{zz} = 5.6 \times 10^6\ mm^4$ } from UKC section table.

$\therefore I_{min} = I_{zz} = 5.6 \times 10^6$

$r_g = \sqrt{\frac{5.6 \times 10^6}{3830}} = 38.2380\ mm$, $\lambda = \frac{L_e}{r_g} = \frac{3 \times 10^3}{38.2380} = 78.4561$

$P_{cr} = \frac{\pi^2 \times 205 \times 10^3 \times 3830}{78.4561^2} = 1259\ kN$

$\therefore$ Since $P_{cr} < F_y$, column will fail in buckling (not yield)

can also find $\bar{\lambda} = \sqrt{\frac{F_y}{P_{cr}}} = 1.04 \geq 1$ (fail in buckling)

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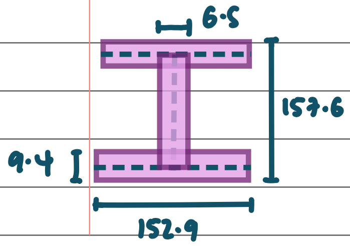
c For the section use simplified calculations to check the values of A, I_{yy} and I_{zz} given in the section tables. Note that although the sections tables give values in cm^2 and cm^4 it is usual to use units of mm^2 and mm^4 when calculating area and second moments of area by hand. This reduces the risk of making unit errors in other calculations.

} just an example (revision) on how to find I of a cross section (year 1) → revise parallel axis theorem.

using centerline approx.:



$A = (157.6 - 9.4) \times 6.5 + 2 \times 152.9 \times 9.4 = 3837.82\ mm^2$

$I_{yy} = \frac{6.5 \times (157.6 - 9.4)^3}{12} + 2\left(\frac{152.9 \times 9.4^3}{12} + 152.9 \times 9.4 \times \left(\frac{157.6 - 9.4}{2}\right)^2\right)$

$= 17.5677 \times 10^6\ mm^4$

$\underbrace{\frac{bh^3}{12}}_{I_{yy}\text{ of rectangle.}} + \underbrace{Ay^2}_{}$ parallel axis theorem.

$$I_{zz} = \frac{(157.6 - 9.4) \times 6.5^3}{12} + 2\left(\frac{9.4 \times 152.9^3}{12}\right)$$

$$= 5.6035 \times 10^6 \ mm^4$$

c3. Bending Moment

3.1 Biaxial Bending.

- The major bending axis is associated with the highest value of second moment of area and is the axis about which it is most difficult to bend the section.
- The minor bending axis is associated with the lowest value of second moment of area and is the axis about which it is easiest to bend the section.

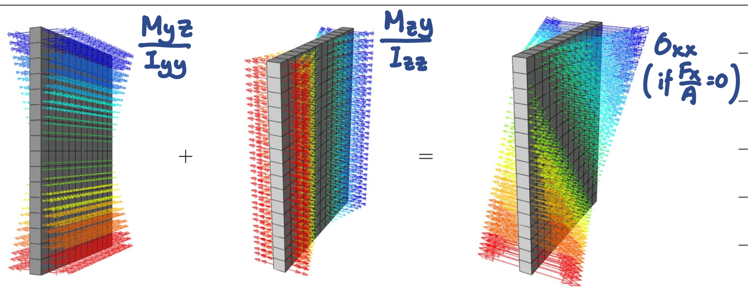
c2. Axial Force

$$\sigma_{xx} = \frac{F_x}{A} + \boxed{\frac{M_y z}{I_{yy}} + \frac{M_z y}{I_{zz}}}$$

c3. Bending Moment

This applies when the y and z axes are the major and minor bending axes.

(only work if $I_{yz} = 0$; Doesn't work if asymmetric section OR applied load is not aligned with principal axes)



Linear variation of σ_{xx} due to M_y (left) and M_z (middle) and combined (right) (here the bending moments were both negative).

3.0 Uniaxial (Only M_y)

For uniaxial (about one axis) bending of a symmetric cross-section the elastic moment capacity of a cross-section can be calculated:

$$M_{el,y} = W_{el,y}\ f_y = \frac{I_{yy}\ f_y}{z} \tag{2.3}$$

where $M_{el,y}$ is the moment M_y at which yielding will begin to take place in the top and bottom fibres of the cross-section, f_y is the yield stress, $W_{el,y}$ is the elastic modulus about the y axis (given in the section tables), and z is the distance from the y axis to the extreme top and bottom fibres of the cross-section.

Also for uniaxial bending the plastic moment capacity of a cross-section can be calculated:

$$M_{pl,y} = W_{pl,y}\ f_y = (A_t z_t + A_c z_c) f_y \tag{2.4}$$

where $M_{pl,y}$ is the moment M_y at which yielding will have taken place throughout the material in the top and bottom parts of the cross-section allowing a plastic hinge to form, $W_{pl,y}$ is the plastic modulus about the y axis (given in the section tables), A_t and A_c are the areas of the cross-section in tension and compression, and z_t and z_c are distances in the z direction to the centroids of the areas of the cross-section in tension and compression, respectively.

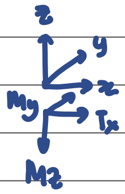
we can still find $M_{el,y}$ or $M_{pl,y}$ (or even $M_{el,z}$ and $M_{pl,z}$) but these moment at which material yields (or fail plasticly) is only refering to UNIAXIAL LOADING only !

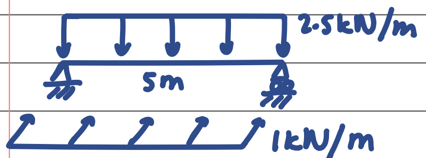
Worked Example

a For a 305 × 102 × 33 UKB beam section (an I-sections) in the section tables calculate and sketch the normal stress (σ_{xx}) distribution at midspan if the section is used as a 5 m long simply supported beam, with a distributed load of 2.5 kN/m acting downwards (in the negative z direction) and a distributed load of 1 kN/m acting on the left hand side (in the positive y direction) of the beam. You may assume the structural element is manufactured from grade S355 steel with a Young's modulus of 205 GPa.

$E = 205$ GPa, $f_y = 355$ N/mm

1. so first we try to find M_y and M_z at midspan.
2. With that, we find σ_{xx} at different point in the cross-section then plot the distribution.





Step 1. Find M_y and M_z at midspan

$\Sigma F_z = 0$

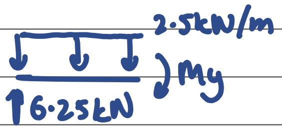
$-2.5k(5) + 2R_z = 0$

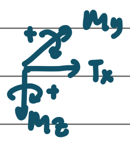
$R_z = 6.25$ kN

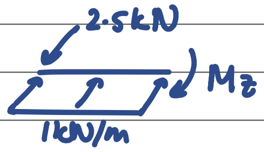
$\Sigma F_y = 0$

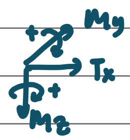
$+1k(5) + 2R_y = 0$

$R_y = -2.5$ kN









$\Sigma M_y = 0$

$M_y + 6.25(2.5) - 2.5\left(\frac{2.5^2}{2}\right) = 0$

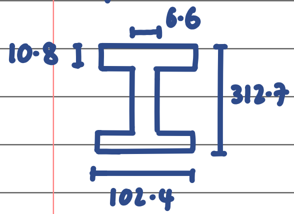
$M_y = -7.8125$ kNm

$\Sigma M_z = 0$

$M_z - 2.5(2.5) + 1\left(\frac{2.5^2}{2}\right) = 0$

$M_z = 3.125$ kNm

Step 2. Obtain I_{yy} and I_{zz} from section table (and geometry too)

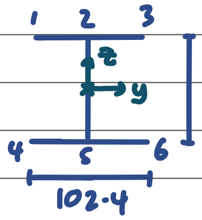


$A = 4180\,mm^2$

$I_{yy} = 65 \times 10^6\,mm^4$

$I_{zz} = 1.94 \times 10^6\,mm^4$

Step 3. Find σ_{xx} at specific point of the I-section



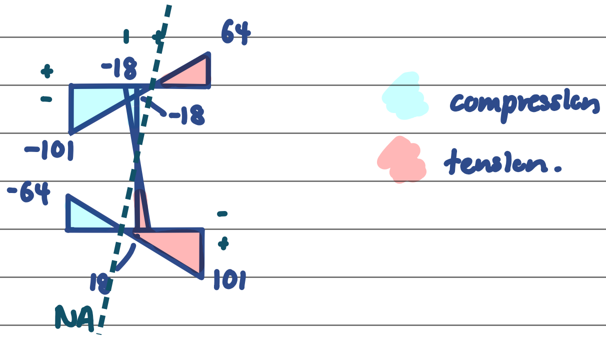
$312.7 - 2\left(\frac{10.8}{2}\right) = 301.9$

$$\sigma_{xx} = \frac{M_y z}{I_{yy}} + \frac{M_z y}{I_{zz}}$$

$$= -\frac{7.8125 \times 10^6\, z}{65 \times 10^6} + \frac{3.125 \times 10^6\, y}{1.94 \times 106}$$

$$= 1.6108y - 0.1202z$$

Node	y (mm)	z (mm)	σ_{xx} (N/mm^2)
1	-51.2	150.95	-101
2	0	150.95	-18
3	51.2	150.95	64
4	-51.2	-150.95	-64
5	0	-150.95	18
6	51.2	-150.98	101

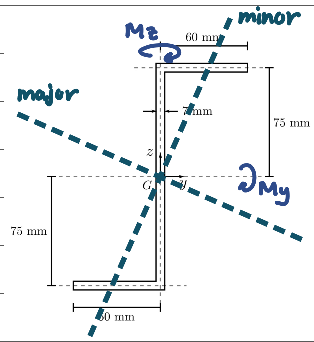


b State if any material in the beam will yield under the loading. Explain your answer.

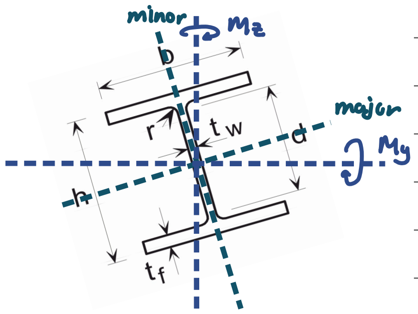
M_y and M_z is maximum at midspan. Hence σ_{xx} maximum found in this cross-section is maximum throughout the entire span of the beam. And since σ_{xx} max is only $101\,N/mm^2$ and is less than $f_y = 355\,N/mm^2$, the beam will not yield.

3.2 Asymmetric Bending

Asymmetric bending is the situation where a moment is applied about an axis which is not one of the principal (major or minor) bending axes.



case 1. Asymmetric section



Case 2. Symmetric but rotated section

$I_{yz} = 0$ IF there is a symmetry (or two) about the shape's centroid's y-axis or z-axis. eg: symmetrical → $I_{yz} = 0$

given in formula booklet

Product second moment of area I_{yz}

$$I_{yz} = -\int_A yz \, dA \tag{2.13}$$

$$I_{yy} = \int_A z^2 \, dA$$

$$I_{zz} = \int_A y^2 \, dA$$

recall that second moment of area about y-axis and z-axis we never care about the sign, as there is a power 2!

y and z need to be careful of sign!
→ check with right hand rule!

given in formula booklet

Generalised bending formula

$$\sigma_{xx} = \left[\frac{M_y I_{zz} + M_z I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right] z + \left[\frac{M_z I_{yy} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right] y \tag{2.20}$$

If we calculate the second moments of area of a section by dividing it into rectangular areas, the definitions can be written as:

$$\begin{aligned} I_{yy}^G &= \Sigma I_{yy}^A + \Sigma A\bar{z}^2 \\ I_{zz}^G &= \Sigma I_{zz}^A + \Sigma A\bar{y}^2 \\ I_{yz}^G &= \Sigma I_{yz}^A - \Sigma A\bar{y}\bar{z} \end{aligned}$$

parallel axis theorem:

parallel axis theorem $I_{yy} = \int z^2 dA$

$= A_1 \; \bar{z}_1 + A_2 \; \bar{z}_2 + \frac{bh^3}{12} + \frac{bh^3}{12}$

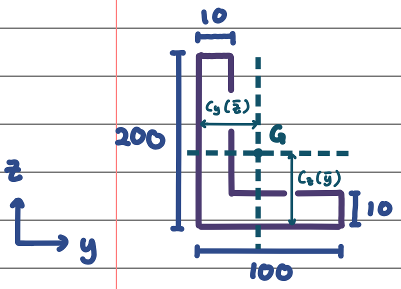
First term I_{yz}^A is usually 0, eg:
A_1, A_2: $I_{yz}^{A_1}$ and $I_{yz}^{A_2} = 0$ because:
symmetry (z1)
symmetry (y1)
, but note that second term, $\Sigma A\bar{y}\bar{z} \neq 0$!

where the superscript G indicates a value calculated at the centroid of the entire section, and the superscript A indicates a value calculated for one of the rectangular areas of the cross-section, $\bar{z}$ and $\bar{y}$ are the distance in the z and y directions respectively, going from the centroid of the entire section to the centroid of the rectangular areas.

Worked Example 1

a For a $200 \times 100 \times 10$ unequal angle section use simplifications to calculate the values of I_{yy}, I_{zz} and I_{yz} based on the orientation of the section given by the $y-y$ and $z-z$ axes in the section tables.

Step 1: Obtain required values from section table.



From section table,

$C_y(\bar{y}) = 6.93\,cm$, $C_z(\bar{z}) = 2.01\,cm$, $A = 29.2\,cm^2$

$I_{yy} = 1220\,cm^4$, $I_{zz} = 210\,cm^4$

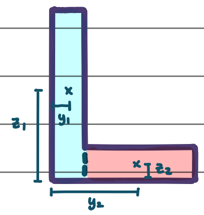
$I_{max} = 1290\,cm^4$ (shown in table as I_{uu})

$I_{min} = 135\,cm^4$ (shown in table as I_{vv})

$\tan\alpha = 0.263$ ($\alpha = 14.74°$) ← useful for final answer checking!

Step 2: Find centroid's coordinate

$$
\left.
\begin{aligned}
I_{yy}^G &= \Sigma I_{yy}^A + \Sigma A\bar{z}^2 && (2.21)\\
I_{zz}^G &= \Sigma I_{zz}^A + \Sigma A\bar{y}^2 && (2.22)\\
I_{yz}^G &= \Sigma I_{yz}^A - \Sigma A\bar{y}\bar{z} && (2.23)
\end{aligned}
\right\} \text{need to find } \bar{y} \text{ and } \bar{z} \text{ first}
$$



$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{200\times10\times5 + 90\times10\times(10 + 90/2)}{200\times10 + 90\times10} = 20.52\,mm$$

← shown as C_z in section table (not C_y)

$$\bar{z} = \frac{\Sigma A_i \bar{z}_i}{\Sigma A_i} = \frac{200\times10\times100 + 90\times10\times5}{200\times10 + 90\times10} = 70.52\,mm$$

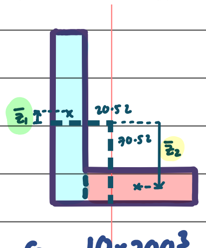
← shown as C_y in section table (not C_z!)

Step 3. Find I_{yy}, I_{zz} and I_{yz}

☆ $\bar{y}$ and $\bar{z}$ is always FROM centroid TO reference area center!

$$I_{yy}^G = \Sigma I_{yy}^A + \Sigma A\bar{z}^2$$

this $\bar{z}$ is not the same $\bar{z}$ as before, this is the vertical distance between centroid's z-coordinate and reference's Area's centroid z-coordinate.

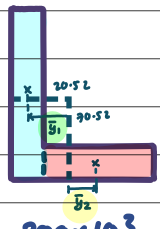


$$I_{yy}^G = \frac{10\times200^3}{12} + \frac{90\times10^3}{12} + 200\times10\times(-70.52+100)^2 + 90\times10\times(-70.52+5)^2$$

$$= 12.2759\times10^6\,mm^4$$

$$I_{zz}^G = \Sigma I_{zz}^A + \Sigma A\bar{y}^2$$

this $\bar{y}$ is also the distance between centroid's y-coordinate and reference area's center's y-coordinate.

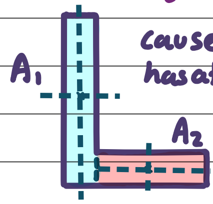


$$I_{zz}^G = \frac{200\times10^3}{12} + \frac{10\times90^3}{12} + 200\times10\times(-20.52+5)^2 + 90\times10\times(-20.52+10+90/2)^2$$

$$= 2.1759\times10^6\,mm^4$$

$$I_{yz}^G = \Sigma I_{yz}^A - \Sigma A\bar{y}\bar{z}$$

☆ $I_{yz}^{A_1}$ and $I_{yz}^{A_2}$ in this question (and most other question) is 0! why?:



cause both A_1 and A_2 has at least one symmetry about their own center!

(as long as there is at least 1 symmetry, I_{yz}^A of that area = 0)

$$I_{yz} = 0 - \left\{200\times10\times(-20.52+5)(-70.52+100) + 90\times10\times\left(-20.52+10+\frac{90}{2}\right)(-70.52+5)\right\}$$

$$= 2.9483\times10^6\,mm^4$$

not required to find in this question: but good to know.

$$\left(\tan\alpha = \frac{I_{yz}}{I_{max}-I_{zz}} = 0.2703 \qquad \alpha = 15.14° \right)$$

b Using Mohr's circle check the values of I_{max} and I_{min} and the orientation of the major ($u-u$) and minor ($v-v$) bending axes given in the section tables, using the second moment of area values you found in part (a).

Step 1. Find center, C and radius, r of the Mohr's Circle

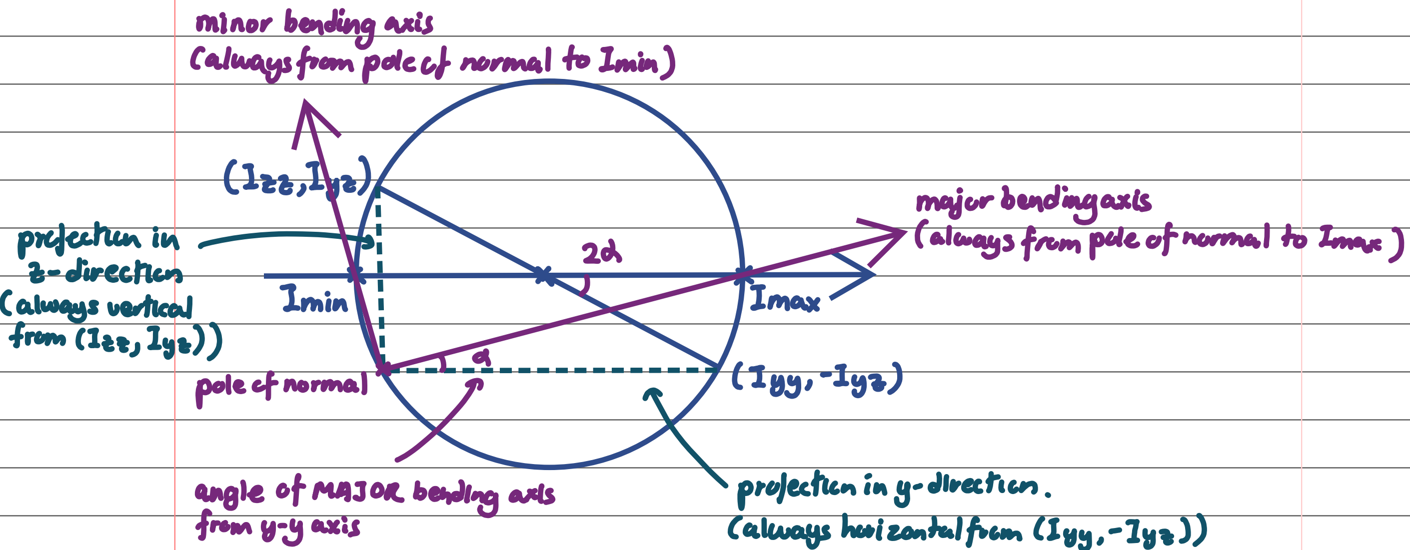
$$c = \frac{I_{yy}+I_{zz}}{2} = \frac{12.2759\times10^6 + 2.1769\times10^6}{2} = 7.2259\times10^6$$

$$r = \sqrt{\left(\frac{I_{yy}-I_{zz}}{2}\right)^2 + I_{yz}^2} = 5.8476\times10^6$$

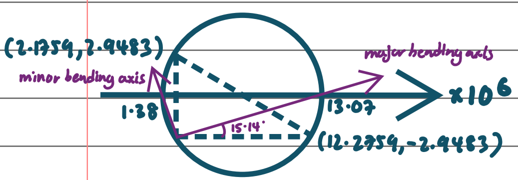
$\therefore I_{max} = c + r = 13.07\times10^6 \ mm^4$

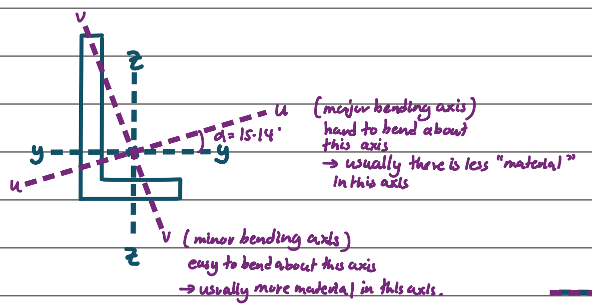
$\therefore I_{min} = c - r = 1.38\times10^6 \ mm^4$

how to remember all these formula? easy, remember how the graph looks like.

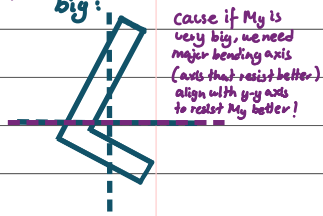


eg. for this question:

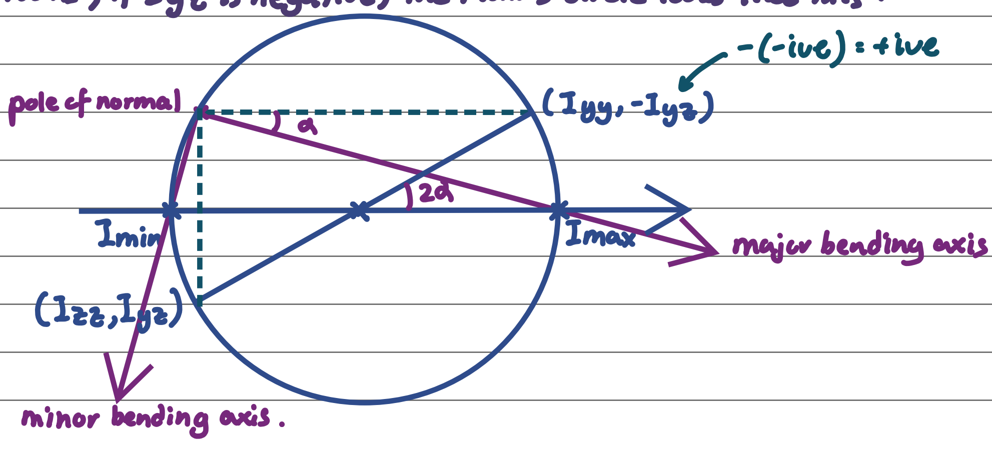




Knowing where major bending axis is useful. We can rotate our cross-section, if we know for example My is very big:

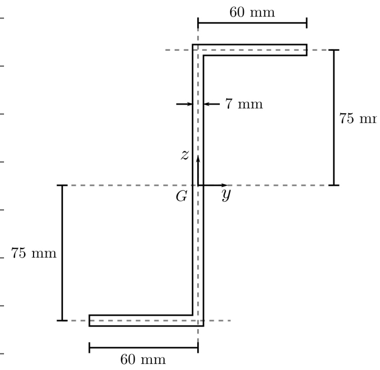


NOTE, if Iyz is negative, the Mohr's Circle looks like this:



Worked Example 2.

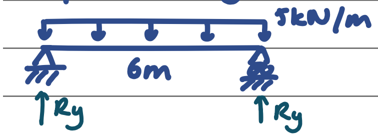
a For the Z-section shown calculate the normal stress (σ_{xx}) distribution at midspan if the section in used as a 6 metre long simply supported beam, with a distributed load of 5 kN/m acting downwards (in the negative z direction).



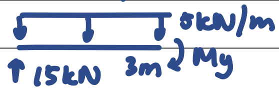
Generalised bending formula

$$\sigma_{xx} = \left[\frac{M_y I_{zz} + M_z I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right] z + \left[\frac{M_z I_{yy} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right] y \qquad (2.20)$$

Step 1. Find M_y and M_z (if any)



At midspan,



$\sum F_y = 0$

$-5k(6) + 2R_y = 0$

$R_y = 15\,kN$

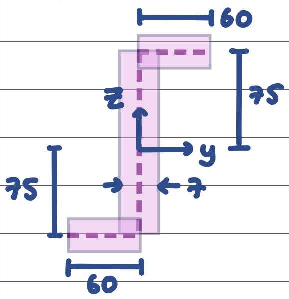
$\sum M_{mid} = 0$

$15k(3) - 5\left(\frac{3^2}{2}\right) + M_y = 0$

$M_y = -22.5\,kNm$

(no M_z cause only vertical loading)

Step 2. Find I_{yy}, I_{zz} and I_{yz} (using centerline approx.)



$$I_{yy} = \sum I_{yy}^{A} + \underbrace{\sum A\bar{z}^2}_{\text{parallel axis theorem's term}}$$

$$= \frac{7\times150^3}{12} + 2\left(\frac{60\times7^3}{12}\right) + 2\left(60\times7\times75^2\right)$$

$$= 6.6972\times10^6\ mm^4$$

parallel axis theorem's term. since the middle area's center coincide with centroid, so only account for the top and bottom term.

$$I_{zz} = \sum I_{zz}^{A} + \sum A\bar{y}^2$$

$$= \frac{150\times7^3}{12} + 2\left(\frac{7\times60^3}{12}\right) + 2\left(60\times7\times30^2\right)$$

$$= 1.0123\times10^6\ mm^4$$

$$I_{yz} = \cancel{\sum I_{yz}^{A}} - \sum A\bar{y}\bar{z}$$

0 because all 3 rectangle have at least one symmetry.

$$= -\left[(60\times70)(30)(75) + (60\times7)(-30)(-75)\right]$$

$$= -1.89\times10^6\ mm^4$$

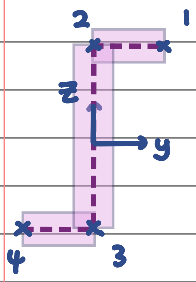
Step 3. Find σ_{xx}

Generalised bending formula

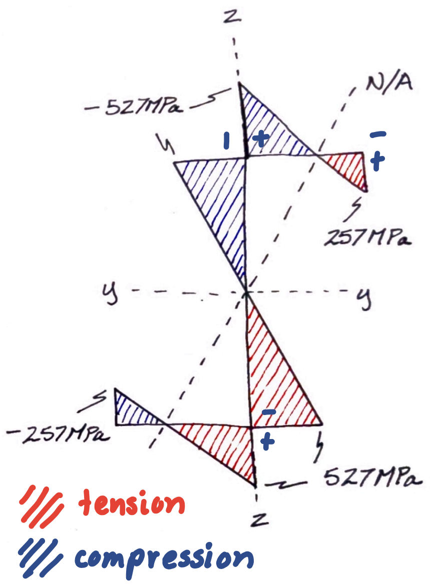
$$\sigma_{xx} = \left[\frac{M_y I_{zz} + \cancel{M_z I_{yz}}^{\,0}}{I_{yy} I_{zz} - I_{yz}^2}\right] z + \left[\frac{\cancel{M_z I_{yy}}^{\,0} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2}\right] y \qquad (2.20)$$

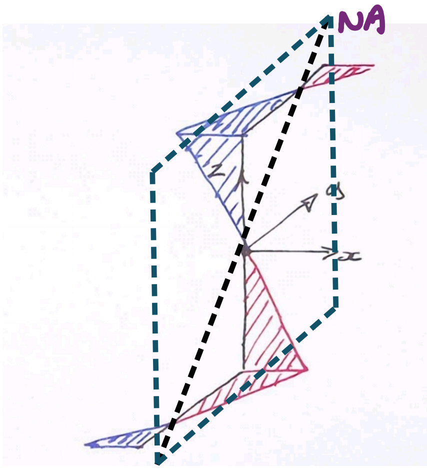
$= \ldots$ (substitute M_y, I_{yy}, I_{zz} and I_{yz})

$= -7.02z + 13.06y$



Node	z (mm)	y (mm)	σ_{xx} (N/mm²)
1	75	60	257
2	75	0	-527
3	-75	0	527
4	-75	-60	-257





note that NA is parallel to the cross section, as shown more clearly in the second figure.

b Comment on a potential issue that might arise if the Z-section is used in the orientation shown and suggest how this might be mitigated.

Maximum σ_{xx} = 527 MPa which is bigger than usual f_y = 355 MPa for S355 steel. So we might need to use higher strength of steel for this case.

c Assuming the Z-section has been designed to maximise its bending resistance when used as a purlin in a roof cladding system, calculate the expected pitch of the roof.

find α

$$c = \frac{I_{yy} + I_{zz}}{2} = 3.8612 \times 10^6$$

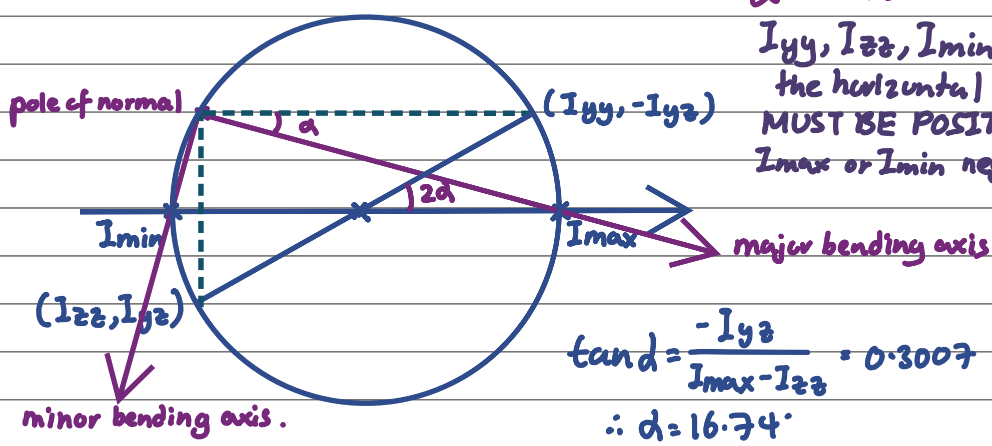
$$r = \sqrt{\left(\frac{I_{yy} - I_{zz}}{2}\right)^2 + I_{yz}^2} = 3.4152 \times 10^6$$

$$I_{max} = c + r = 7.2764 \times 10^6 \text{ mm}^4$$

$$I_{min} = c - r = 0.4460 \times 10^6 \text{ mm}^4$$

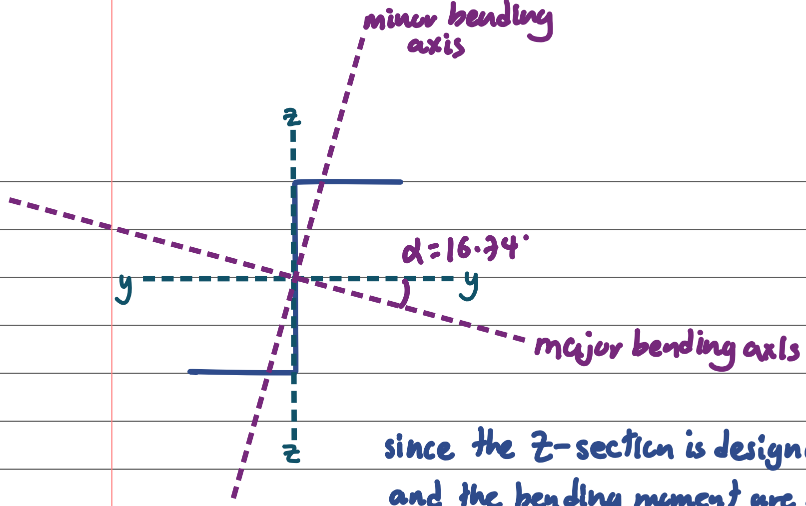
VERY IMPORTANT!!!

I_{yy}, I_{zz}, I_{min} and I_{max}, i.e. the horizontal axis of Mohr's circle MUST BE POSITIVE! If you calculate I_{max} or I_{min} negative, there's mistake!

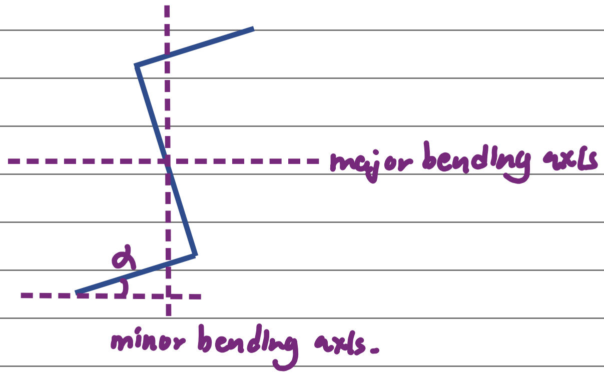


$$\tan\alpha = \frac{-I_{yz}}{I_{max} - I_{zz}} = 0.3007$$

$$\therefore \alpha = 16.74°$$

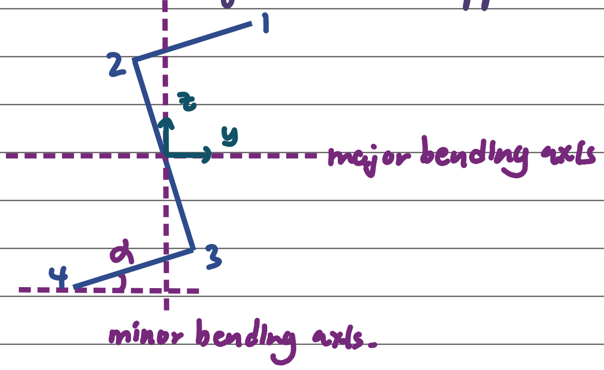


since the z-section is designed to maximise bending resistance, and the bending moment are all about y-y axis (only M_y) hence we rotate major bending axis by $\alpha = 16.74°$ to align with y-y axis :



d For the same setup as in (a) calculate the normal stress (σ_{xx}) distribution at midspan if the section is used in the orientation that maximises its bending resistance.

☆ Asymmetric bending is when I_{uu} (major bending axis) and I_{vv} (minor bending axis) doesn't align with the load applied. However, since we rotate it such that:



Now it is no longer asymmetrical bending !

this mean :

1. $I_{yy} = I_{max}$
2. $I_{zz} = I_{min}$
3. $I_{yz} = 0$

Hence we can use the formula :

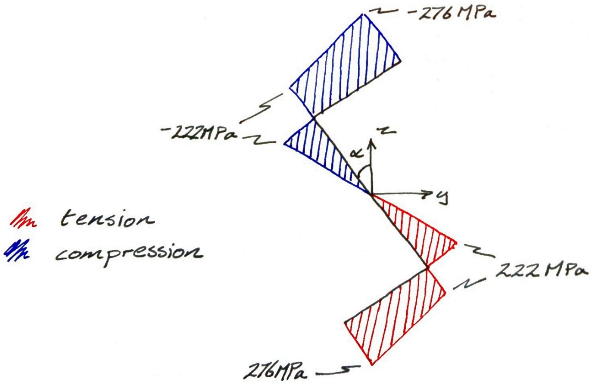
Biaxial bending formula (y and z must be the major and minor bending axes)

$$\sigma_{xx} = \frac{M_y z}{I_{yy}} + \frac{M_z y}{I_{zz}} \quad (2.5)$$

$$\sigma_{xx} = \frac{M_y z}{I_{yy}} = -3.0922\, z$$

Node	z (mm)	σ_{xx} (N/mm²)
1	$71.82 + 60 \sin\alpha = 89.10$	-276
2	$75 \cos\alpha = 71.82$	-222
3	-71.82	222
4	-89.10	276

notice that by just rotating the z-section by $\alpha = 16.74°$, the σ_{xx} of the cross section all are reduced by a lot ! (all < 355)



A method to final check answer :

$$M_y = \int \sigma z \, dA$$

$$= 2\times 222\times 75\times 7\times \frac{1}{2}\times\frac{2}{3}\times 75\cos\alpha$$
$$+ 2\times 222\times 60\times 7\times(75\cos\alpha + 30\sin\alpha)$$
$$+ 2\times 54\times 60\times 7\times\frac{1}{2}\times(75\cos\alpha+40\sin\alpha)$$
$$= 22.5\,\text{kNm}$$ (same as M_y calculated in part (a))

C4. Twisting

4.1 Introduction

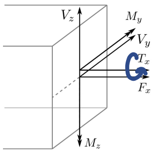
We can agree that each of the loading modes results in one of two types of stress:

$$\sigma_{xx} = \frac{F_x}{A} + \frac{M_y z}{I_{yy}} + \frac{M_z y}{I_{zz}}$$

- Normal stress $\begin{cases} \text{due to axial force} \\ \text{due to bending moments} \end{cases}$
- Shear stress $\begin{cases} \text{due to shear forces} \\ \text{due to torsion moment} \end{cases}$

$$\tau = \boxed{\frac{T_x r}{J}} + \frac{V_z Q_y}{I_{yy} b} + \frac{V_y Q_z}{I_{zz} b}$$

Twisting rotation is caused by twisting moment, in a similar way that curvature is caused by bending moment. Twisting moment T_x results in shear stress τ in the yz plane. We will consider the distribution of shear stress in two types of cross-section commonly used in steel construction:



- Thin-walled closed cross-sections
- Thin-walled open cross-sections

The relationship between the rate of twist θ_x, twisting moment T_x, torsion constant J (a cross-sectional property) and shear modulus G (a material property), for twisting about the x axis running along the length of the beam is given as:

similar to second moment of area, I
- both are cross-section property.
- both can be calculated, or refer from section table!

This formula works for all type of section including:
- thin-walled closed section
- thin-walled open section

Twisting moment relationship

(given in formula booklet)

$$\theta_x = \frac{T_x}{GJ} \qquad (3.2)$$

Torsional Stiffness, GJ

similar to Axial Stiffness, EA
Bending Stiffness, EI

$GJ: \frac{d\theta}{dx} = \frac{T(x)}{GJ}$

$EA: \frac{du}{dx} = \frac{N(x)}{EA}$

$EI: \frac{d^2u}{dx^2} = \frac{M(x)}{EI}$

The shear modulus for isotropic materials such as steel is given as:

(given in formula booklet)

$$G = \frac{E}{2(1+\nu)} \qquad (3.3)$$

In a similar way that the Young's modulus describes the relationship between normal stress and normal strain:

$$E = \frac{\sigma_n}{\varepsilon_n} \qquad (3.4)$$

the shear modulus describes the relationship between shear stress and shear strain:

$$G = \frac{\sigma_s}{2\varepsilon_s} = \frac{\tau}{\gamma} \qquad (3.5)$$

where ε_s is mathematical shear strain and γ is engineering shear strain.

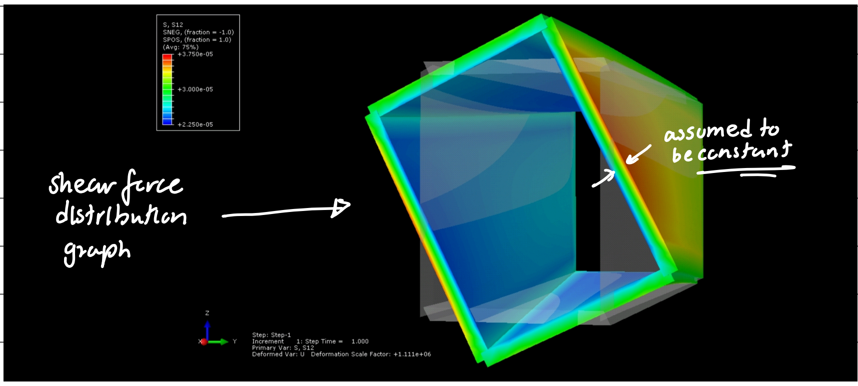
As for normal stress σ_n we are interested in finding shear stress τ to assess the risk of material yield in shear. For steel the shear yield stress is given as:

this shows $f_{s,y} < f_y$: material is weaker to shear than to normal stress.

$$f_{s,y} = \frac{f_y}{\sqrt{3}} \qquad (3.6)$$

this formula only apply under pure shear stress.
If there is normal stress:
$\sigma_{vm} = \sqrt{\sigma_n^2 + 3\sigma_s^2}$
(von mises yield stress)

4.2 Thin-walled CLOSED sections.



Main difference between CLOSED and OPEN thin-walled sections is:

For CLOSED, the shear stress is assumed to be CONSTANT across the thickness of the closed section.

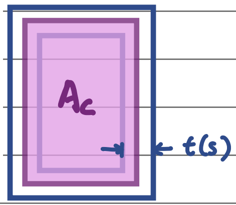
but for OPEN, it is linear variation.

given in formula booklet

Shear stress in thin-walled closed sections

$$\tau = \frac{T_x}{2A_c t(s)} \tag{3.9}$$

where A_c is the area enclosed within the perimeter of the closed section calculated from the centre of the wall thickness, and $t(s)$ is thickness as a function of the perimeter position s, from an arbitrary starting point.



At this point we can introduce the concept of **shear flow** as:

Shear flow definition

$$q = \tau t \tag{3.10}$$

$q = \tau t(s)$
$= \frac{T_x}{2A_c \cancel{t(s)}} \times \cancel{t(s)}$
$= \frac{T_x}{2A_c}$ q doesn't depends on $t(s)$! that's why it is constant throughout the section! Even if thickness vary throughout the section!

For a closed thin-walled section the shear flow q due to a twisting moment T_x will be constant throughout the section, whereas the shear stress τ will vary depending on the wall thickness t. Although the shear stress can vary around the perimeter at any location the shear stress is assumed to be constant across the wall thickness.

(for closed thin-wall section only!)

Hence we can find the warping displacement at a position s around the perimeter, from an arbitrary starting point as:

1. $w(p) - w(0) = 0$ for ALL CLOSED thin walled section

For a closed thin-walled section if we integrate around the entire perimeter p then w must be equal to zero at the start and end points as otherwise we would have a split or discontinuity in the section. Hence for the hollow circular section we can state:

$$w(s = p) - w(s = 0) = \theta_x r \oint \mathrm{d}s + \frac{\tau}{G} \oint \mathrm{d}s = 0 \tag{3.14}$$

2. $w(s) - w(0) = 0$ for hollow and solid circular section

Warping occurs in all sections with the exception of hollow circular and solid circular sections, explaining their preferential use where twisting moments are expected to occur.

given in formula booklet

Warping formula

$$w(s) - w(0) = \theta_x \int_0^s r \, \mathrm{d}s + \int_0^s \frac{\tau}{G} \, \mathrm{d}s \tag{3.13}$$

The torsion constant for all thin-walled closed sections including those with varying wall thickness is given as:

Torsion constant for thin-walled closed sections

$$J = \frac{4A_c^2}{\oint \frac{1}{t(s)} \, \mathrm{d}s} \tag{3.19}$$

Worked Example

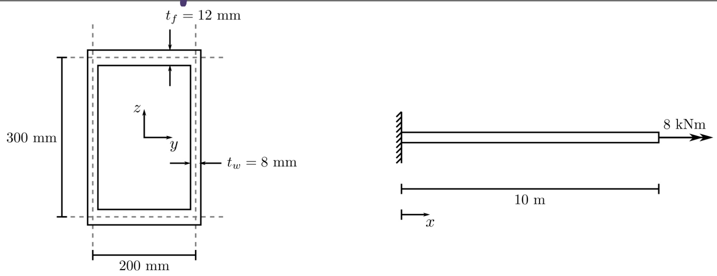


Figure 3.5: Box section twisting worked example.

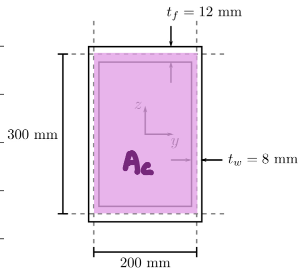
1 The cantilever shown in Figure 3.5 is constructed from a steel box section. Material properties for steel may be taken as $E = 205$ GPa and $\nu = 0.3$. For the applied loading shown determine:

- a The torsion constant of the box section.
- b The shear stress due to twisting in the web and flange of the section.
- c The angle of twist at the free end of the cantilever.

(a)

Torsion constant for thin-walled closed sections $\oint \frac{1}{t(s)}\,ds \approx \sum_{bi} \frac{s_i}{t_i}$

$$J = \frac{4A_c^2}{\oint \frac{1}{t(s)}\,\mathrm{d}s} \qquad (3.19)$$



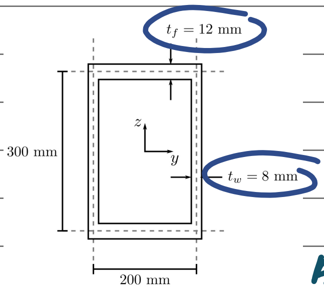
$$J = \frac{4(200\times300)^2}{2\left(\frac{300}{8}\right) + 2\left(\frac{200}{12}\right)} = 132.9231\times10^6\ \text{mm}^4$$

(b)

Shear stress in thin-walled closed sections

$$\tau = \frac{T_x}{2A_c t(s)} \qquad (3.9)$$

Note that $\tau(s)$ depends on $t(s)$.
In this section there is two t: t_f and t_w



$$\tau_w = \frac{T_x}{2A_c t_w} = \frac{8\times10^6}{2(200\times300)(8)} = 8.33\ \text{N/mm}^2$$

$$\tau_f = \frac{T_x}{2A_c t_f} = \frac{8\times10^6}{2(200\times300)(12)} = 5.56\ \text{N/mm}^2$$

Although these (τ_w and τ_f) are what question asked...
I want to show a statement:
Remember we said that $q = \tau t$ is fixed no matter what is the t?

$$\left.\begin{aligned} q_w &= \tau_w t_w = 8.53(8) = 66.64\ \text{N/mm} \\ q_f &= \tau_f t_f = 5.56(12) = 66.72\ \text{N/mm} \end{aligned}\right\} q \approx 66.7 \text{ throughout the section!}$$

(c)

Twisting moment relationship

$$\theta_x = \frac{T_x}{GJ} \tag{3.2}$$

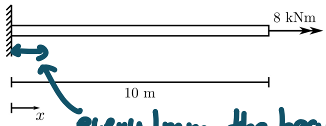
we have T_x, J but missing G:

$$G = \frac{E}{2(1+\nu)} = \frac{205\times10^3}{2(1+0.3)} = 78846 \text{ MPa}$$

$$\theta_x = \frac{8\times10^6}{78846\times132.9231\times10^6} = 7.6332\times10^{-7} \text{ rad/mm}$$

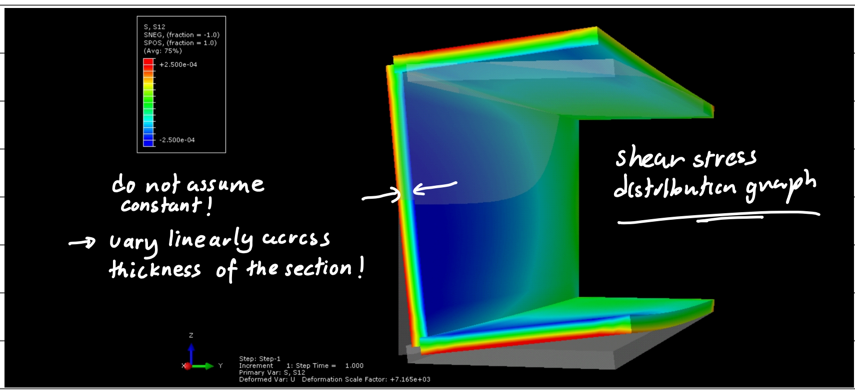
$$\phi = 7.6332\times10^{-7} \times \underbrace{10000}_{\text{10m length in mm}} \times \frac{180}{\pi} = 0.44^\circ$$

$\theta_x = \frac{d\theta}{dx}$ is the twist per x (in mm)



every 1mm, the beam is twisted by 7.6332×10^{-7} rad.
so the total angle of twist $= \theta_x \times$ length of the beam!

4.3 Thin-walled OPEN Section



given in formula booklet

Shear stress in thin-walled open sections

$$\tau_{max}(s) = \frac{T_x t(s)}{J} \tag{3.24}$$

I modified this formula so its a bit different from lecture slide. I believe this is easier to understand.

τ_{max} at position s around the perimeter, depends on the thickness, $t(s)$ at that position!
Why are we only finding τ_{max}?

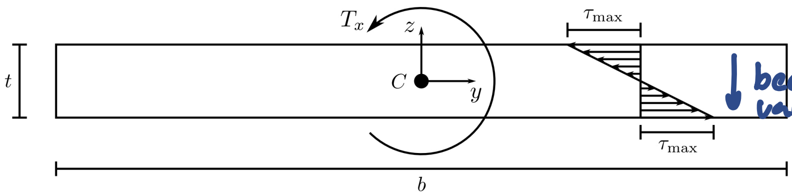


Figure 3.6: Twisting moment in a thin-walled open section.

because we know its linear varying across the thickness! so by knowing τ_{max} we essentially know all τ at a single position s!

given in formula booklet

Torsion constant for thin-walled open sections

$$J \approx \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{3.28}$$

Where n is the number of plates comprising the cross-section.

Worked Example

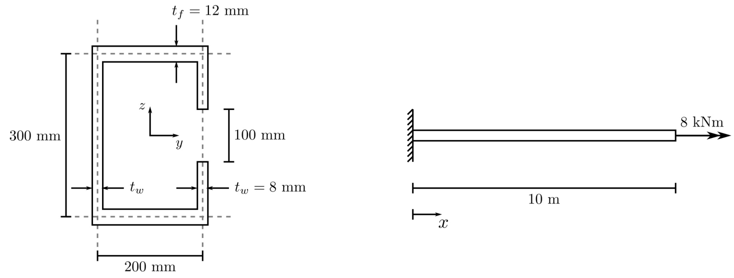


Figure 3.8: Channel section twisting worked example.

1 The cantilever shown is constructed from a steel channel section. Material properties for steel may be taken as $E = 205$ GPa and $\nu = 0.3$. For the applied loading shown determine:

- a The torsion constant of the channel section.
- b The maximum shear stress in the web and flange of the section.
- c The angle of twist at the free end of the cantilever.

(a)

Torsion constant for thin-walled open sections

$$J \approx \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \qquad (3.28)$$

Where n is the number of plates comprising the cross-section.

$$J = \frac{1}{3}\sum bt^3 = \frac{1}{3}\left[2(200)(12)^3 + (300+2\times100)(8)^3\right]$$
$$= 0.3157\times10^6 \text{ mm}^4$$

(b)

Shear stress in thin-walled open sections

$$\tau_{\text{max}}(s) = \frac{T_x t(s)}{J} \qquad (3.24)$$

$$\tau_{max,w} = \frac{T_x t_w}{J} = \frac{8\times10^6\times8}{0.1357\times10^6} = 203 \text{ N/mm}^2$$

$$\tau_{max,f} = \frac{T_x t_f}{J} = \frac{8\times10^6\times12}{0.1357\times10^6} = 304 \text{ N/mm}^2$$

Not asked in the question but...
(assumed that there is only pure shear stress, no normal stress)

$$f_{y,s} = \frac{f_y}{\sqrt{3}} = \frac{355}{\sqrt{3}} \text{ (for S355 steel)}$$
$$= 204.96 \text{ N/mm}^2$$

$\tau_{max,f}$ actually exceed $f_{y,s}$ and hence material will yield under stress if use S355 steel.

(c)

Twisting moment relationship

$$\theta_x = \frac{T_x}{GJ} \qquad (3.2)$$

we have T_x, J but missing G:

$$G = \frac{E}{2(1+\nu)} = \frac{205\times10^3}{2(1+0.3)} = 78846 \text{ MPa}$$

$$\theta_x = \frac{8\times10^6}{78846\times0.3157\times10^6} = 3.2139\times10^{-4} \text{ rad/mm}$$

$$\phi = \theta_x \times \text{total length} = 3.2139\times10^{-4}\times10\times10^3\times\frac{180}{\pi} = 184^\circ$$ (184°! will certainly fail!)

C5. Shear Force

5.1 Introduction

We can agree that each of the loading modes results in one of two types of stress:

$\sigma_{xx} = \frac{F_x}{A} + \frac{M_y z}{I_{yy}} + \frac{M_z y}{I_{zz}}$

- Normal stress { due to axial force / due to bending moments
- Shear stress { due to shear forces / due to torsion moment

$\tau = \frac{T_x r}{J} + \boxed{\frac{V_z Q_y}{I_{yy} b} + \frac{V_y Q_z}{I_{zz} b}}$ C5.

As we did for twisting, we will consider the distribution of shear stress in two types of cross-section commonly used in steel construction:

- Thin-walled open cross-sections
- Thin-walled closed cross-sections

open, is easier than close
but the main difference is for close we gonna 'cut' it open too.

Shear flow definition

$$q = \tau t \tag{3.10}$$

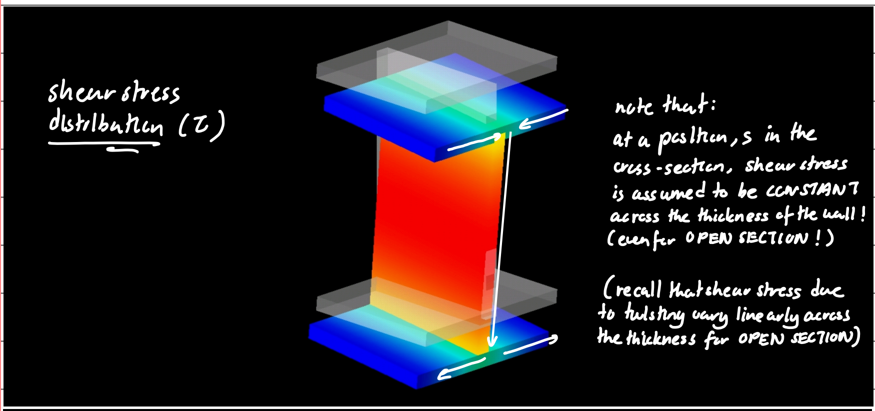
what is the difference between shear flow, q and shear stress τ?
In year 1, we always introduce τ as $\frac{VQ}{It}$ and now q as $\frac{VQ}{I}$
τ and q are similar in the sense that τ account for thickness and q does not.
you can think like q is amount of shear stress, τ per unit thickness.

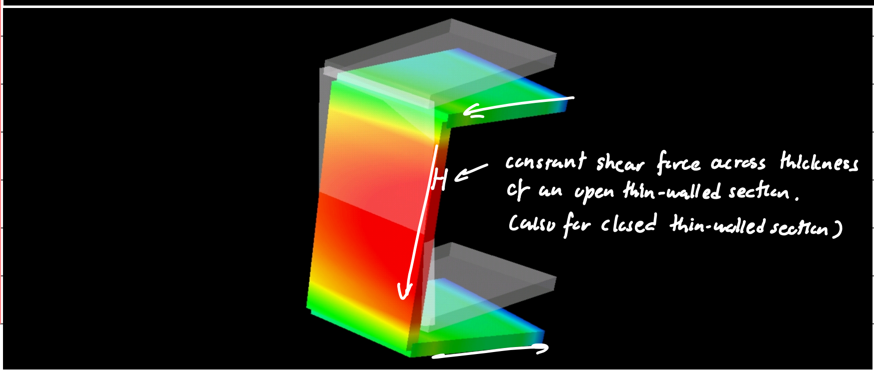
5.2 Thin-walled OPEN cross-sections

Through the principle of superposition the shear flow due to shear forces perpendicular to the major and minor bending axes, at a particular position in the cross-section s is given as:

Shear flow due to shear forces perpendicular to the major and minor bending axes

$$q(s) = \frac{V_z Q_y(s)}{I_{yy}} + \frac{V_y Q_z(s)}{I_{zz}} \tag{4.12}$$





b Check that the integration of the shear stress returns the expected shear force:

$$V_z = \int_A \tau \, dA$$

c State if the shear stress distribution will vary at other locations along the length of the cantilever. Explain your answer.

d State if any part of the beam will yield in shear under the loading. Explain your answer.

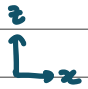
Worked Example

a For a 356 × 127 × 33 beam section calculate the shear flow q distribution at the mid-point if the section is used as a 5 metre long cantilever, with a 10 kN point load acting downwards (in the negative z direction) at the free end. You may assume the structural element is manufactured from grade S355 steel with a Young's modulus of 205 GPa. You should calculate the value of I_{yy} that you use based on simplifications.

Shear flow due to shear forces perpendicular to the major and minor bending axes

$$q(s) = \frac{V_z Q_y(s)}{I_{yy}} + \frac{V_y Q_z(s)}{I_{zz}} \qquad (4.12)$$

Step 1. Find V_z and V_y (if any)



$\sum F_z = 0$
$R_z = 10kN$

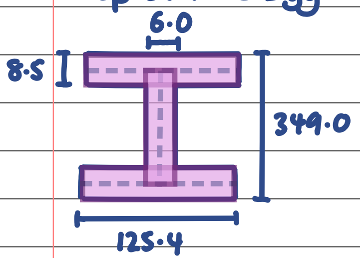
At midspan:

$\sum F_z = 0$
$V_z = -10kN$

This shows that shear force is downward and hence shear flow will flow downward:

we can now proceed and drop the sign of V_z when doing intermediate calculation.
Just remember to draw the arrow in the correct direction.

Step 2. Find I_{yy} and I_{zz} (if any)



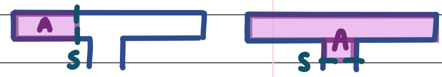
$$I_{yy} = \sum I_{yy} + \sum A \bar{z}^2$$
$$= \frac{6(349-8.5)^3}{12} + 2\left(\frac{125.4(8.5)^3}{12}\right) + 2\left((125.4 \times 8.5)\left(\frac{349}{2} - \frac{8.5}{2}\right)^2\right)$$
$$= 81.5420 \times 10^6 \ mm^4$$

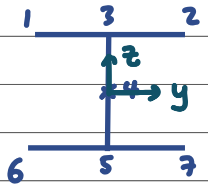
since $q(s) = \underbrace{\frac{V_z}{I_{yy}}}_{\text{only depends on section (constant)}} \overbrace{Q_y(s)}^{\text{vary depends on position, s.}}$, $q(s) = \frac{10 \times 10^3}{81.5420 \times 10^6} Q_y(s) = 1.2264 \times 10^{-4} \ Q_y(s)$

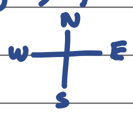
Step 3. Find $Q_y(s)$ for specific position, s

$Q_y(s) = \int y \, dA$ where A is:

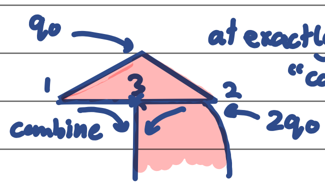
eg.







if I say 3^W
it means just to the left of 3, but not at 3



at exactly 3, the q will "combined"

At node 1 and node 2, $Q_y = 0 \ mm^3$
At node 3^W and 3^E,

$$Q_y(s) = 8.5\left(\frac{125.4}{2}\right)\left(\frac{349-8.5}{2}\right) = 90735 \ mm^3$$

At node 3^S

$$Q_y(s) = 2 \times 90735 = 181470$$

At node 4,

$$Q_y(s) = \underbrace{181470}_{\text{top flange}} + \underbrace{6\left(\frac{349-8.5}{2}\right)\left(\frac{349-8.5}{4}\right)}_{\text{upper half of the web.}} = 268425 \text{ mm}^3$$

At node 5^N,

z is negative! cause it's below center.

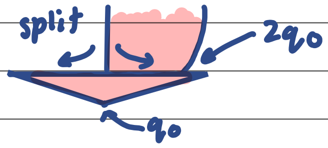
$$Q_y(s) = 268425 + 6\left(\frac{349-8.5}{2}\right)\left(-\frac{349-8.5}{2}\right)$$

$= 181470$ (notice it is the same as node 3^S)

In exam, don't need to do these as it is symmetry.

At node 5^E and 5^W,

$$Q_y(s) = \frac{181470}{2} = 90735$$

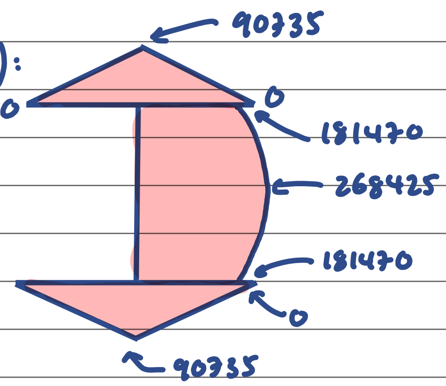


At node 6 and 7,

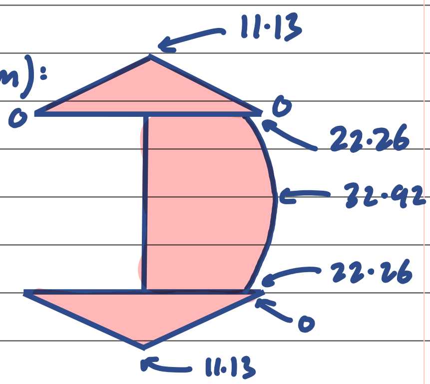
$$Q_y(s) = 90735 + 8.5\left(\frac{125.4}{2}\right)\left(-\frac{349-8.5}{2}\right) = 0 \text{ (as expected)}$$

Node	$Q_y(s)$ (mm³)	$q_y(s)$ (N/mm)
1,2	0	0
$3^E, 3^W$	90735	11.13
3^S	181470	22.26
4	268425	32.92

$Q_y(s)$ (mm³):

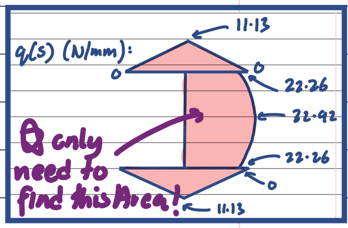


$q(s)$ (N/mm):



b Check that the integration of the shear stress returns the expected shear force:

$$V_z = \int_A \tau \, dA$$

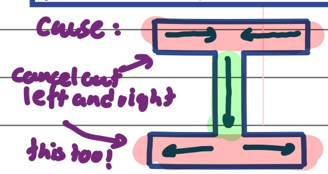


$V_z = \int \tau \, dA = \int q \, ds$

$= 22.26 \times 340.5 + \frac{2}{3}(32.92 - 22.26) \times 340.5$

$= 10 \text{ kN}$ ✓

Cause:

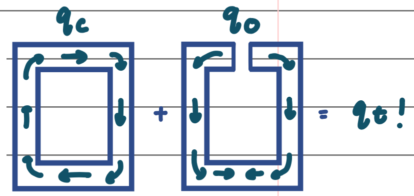


c State if the shear stress distribution will vary at other locations along the length of the cantilever. Explain your answer. No. V_z constant across the length of beam!

d State if any part of the beam will yield in shear under the loading. Explain your answer. No. $f_{y,s} = f_y/\sqrt{3}$ and $\tau \ll f_{y,s}$ (no yield)

5.3 Thin-walled CLOSED cross section

We can express the shear flow in a closed section due to shear as:



$$q_t = q_c + q_o \tag{4.14}$$

where q_t is the total shear flow, q_c is the circulating shear flow, and q_o is the open shear flow that would be calculated if we placed a split in the perimeter of the section and treated it as an open section.

Noting that q_c is a constant we can rearrange to give:

$$\boxed{q_c = \frac{-\oint \frac{q_o}{t(s)}\,\mathrm{d}s}{\oint \frac{1}{t(s)}\,\mathrm{d}s}} \tag{4.16}$$

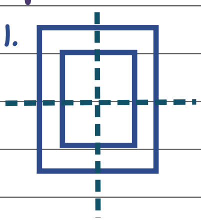
cutting at appropriate point can save us time by not finding q_c as $q_c=0$, but how?

In practice we can ensure that $q_c = 0$ by choosing an appropriate point in the perimeter of a closed cross section to place the split to allow us to treat it as open section. The point chosen depends on the direction of the shear force.

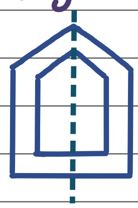
IMPORTANT: How to find where to cut, such that $q_c = 0$?

Step 1: Find axis of symmetry

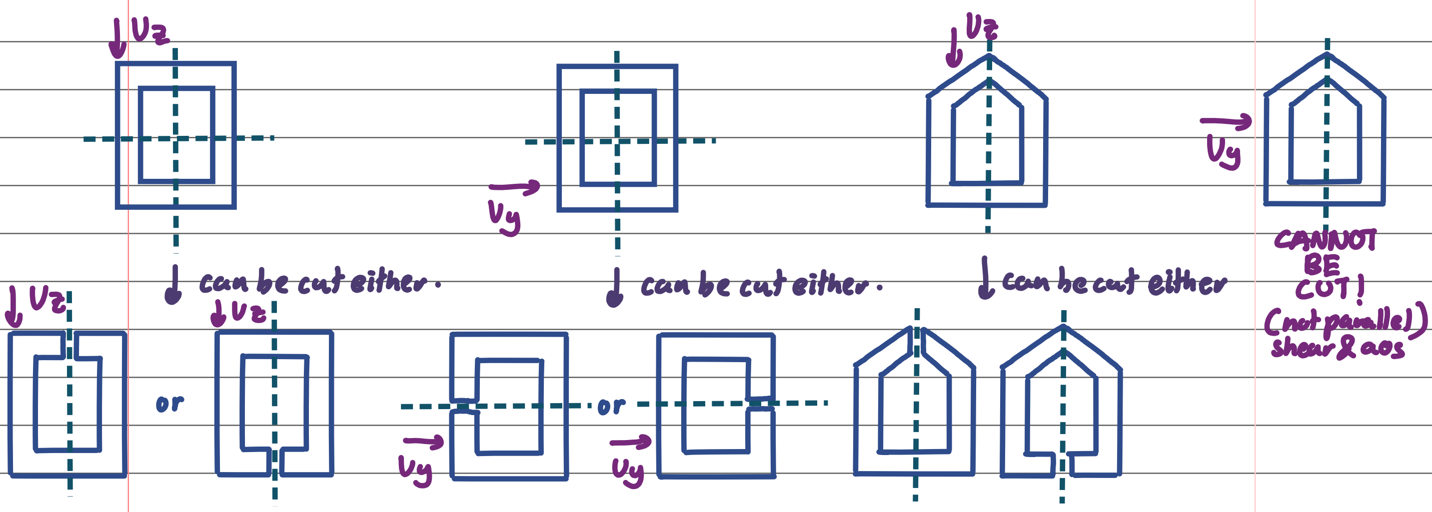
eg1.



eg2.



Step 2: Check if the load applied is PARALLEL to any axis of symmetry.
If yes, cut at THAT Parallel axis of symmetry. (any point)



MUST BE ON THE AXIS OF SYMMETRY
NOT THE LINE OF ACTION OF LOAD!

b Check that the integration of the shear stress returns the expected shear force.

c State if the shear stress distribution will vary at other locations along the length of the beam. Explain your answer.

d State if any part of the beam will yield in shear under the loading. Explain your answer.

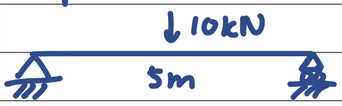
Worked Example

a For a 300 × 150 × 10 box section calculate the shear flow distribution (q) at the quarter-point if the section is used as a 5 metre simply supported beam, with a 10 kN point load acting downwards (in the negative z direction) at the mid-point. You may assume the structural element is manufactured from grade S355 steel with a Young's modulus of 205 GPa. You should calculate the value of I_{yy} that you use based on simplifications.

Shear flow due to shear forces perpendicular to the major and minor bending axes

$$q(s) = \frac{V_z Q_y(s)}{I_{yy}} + \frac{V_y Q_z(s)}{I_{zz}} \quad (4.12)$$

Step 1. Find V_z and V_y (if any)



$\Sigma F_z = 0$

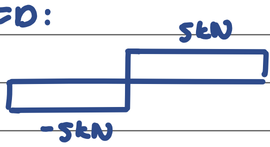
$2R_z - 10k = 0$

$R_z = 5kN$

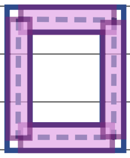
At midspan (just before 10kN load)

$V_z = -5kN$

SFD:



Step 2. Find I_{yy} and I_{zz} (if any)



$$I_{yy} = \Sigma I_{yy}^{A} + \Sigma A\bar{z}^2$$

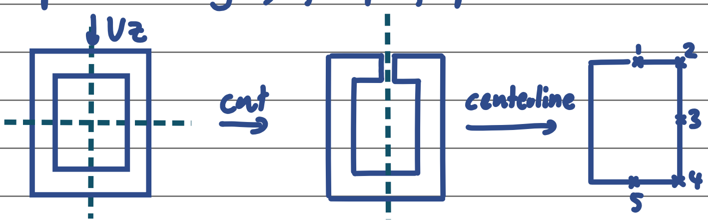
$$= 2\left(\frac{10(300-10)^3}{12}\right) + 2\left(\frac{(150-10)(10)^3}{12}\right) + 2\left[10(150-10)\times\left(\frac{300}{2}-\frac{10}{2}\right)^2\right]$$

$$= 99.5417\times10^6 \text{ mm}^4$$

since $q(s) = \underbrace{\frac{V_z}{I_{yy}}}_{\text{only depends on section (constant)}} \overbrace{Q_y(s)}^{\text{vary depends on position, } s.}$, $q(s) = \frac{-5\times10^3}{99.5417\times10^6} Q_y(s) = -5.0230\times10^{-5} Q_y(s)$

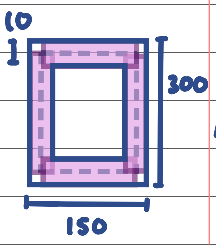
we can drop the negative sign after this...

Step 3. Find $Q_y(s)$ for specific position, s



At node 1. (open end → no surface to shear with)

$Q_y(s) = 0 \text{ mm}^3$



At node 2,

$$Q_y(s) = 10\left(\frac{150-10}{2}\right)\left(\frac{300-10}{2}\right) = 101500 \text{ mm}^3$$

At node 3,

$$Q_y(s) = 101500 + 10\left(\frac{300-10}{2}\right)\left(\frac{300-10}{4}\right) = 206\,625 \text{ mm}^3$$

during exam can skip after symmetry.

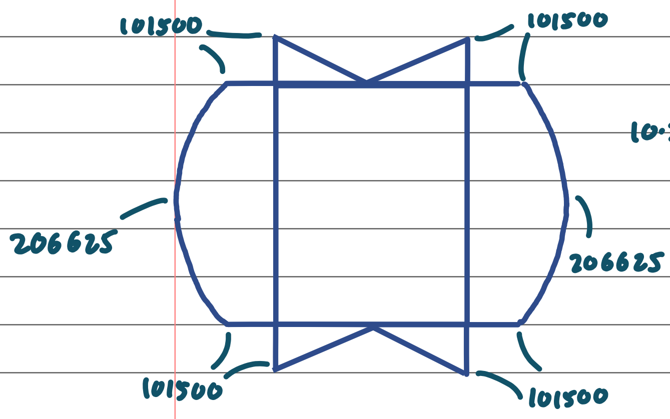
At node 4,

$$Q_y(s) = 206\,625 + 10\left(\frac{300-10}{2}\right)\left(-\frac{300-10}{4}\right) = 101\,500 \text{ mm}^3$$

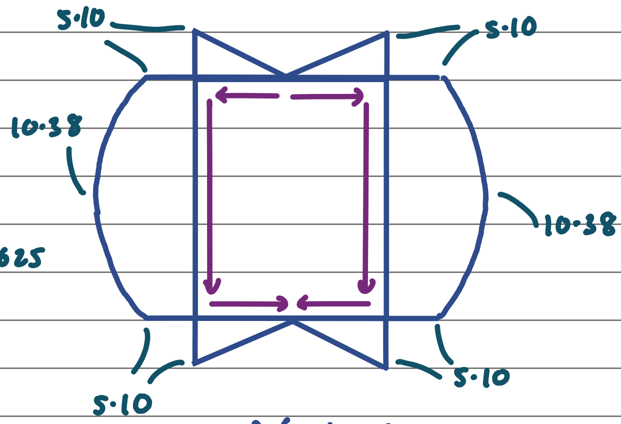
At node 5,

$$Q_y(s) = 101500 + 10\left(\frac{150-10}{2}\right)\left(-\frac{300-10}{2}\right) = 0 \text{ mm}^3$$

Node	$Q(s)$ (mm³)	$q(s)$ (N/mm)
1	0	0
2	101500	5.10
3	206625	10.38
4	101500	5.10
5	0	0



$Q_y(s)$ (mm³)

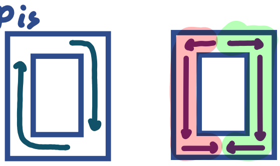


$q(s)$ (N/mm)

Remember we said that the way we cut it, $q_c = 0$? recall that

$$q_c = \frac{-\oint \frac{q_o}{t(s)}\,\mathrm{d}s}{\oint \frac{1}{t(s)}\,\mathrm{d}s} = \frac{-\oint q_o \, ds}{P}$$

P is



the right side is in the direction of P but left side is all opposite direction of P. Hence cancel out.

5.4 Centre of Twist

The **shear centre** is coincident with the **centre of twist**. It represents the point through which shear force would need to act in order to prevent the introduction of additional shear stress due to the introduction of a twisting moment, and the point about which twisting will occur if a twisting moment is introduced.

The centre of twist C is a section property, although it is not routinely reported in section tables.

The centre of twist can be established once the shear flow due to shear force has been found, as the position about which the forces produced by integrating the shear stress in the individual walls of the cross-section result in zero twisting moment. The twisting moment T_x produced by a shear force acting away from the centre of twist is given as:

$$\boxed{T_x = V_z e} \tag{4.17}$$

where e is the eccentricity of the shear force V_z.

Worked Example

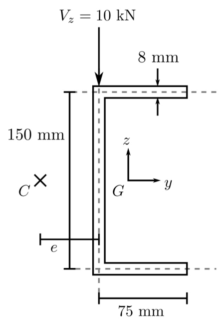
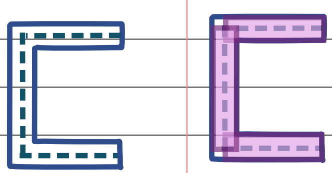


Figure 4.6: Shear force resulting in a twisting moment.

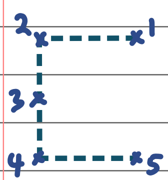
For the channel section shown find the:

a Shear flow distribution if the shear force could pass through the centre of twist.



$$I_{yy} = \frac{8\times150^3}{12} + 2\left(\frac{75\times8^3}{12}\right) + 2\left(8\times75\times\left(\frac{150}{2}\right)^2\right) \quad ; \quad q = \frac{V_z Q_y}{I_{yy}} = 1.1103\times10^{-3}\, Q_y$$

$$= 9.0064\times10^6 \text{ mm}^4$$



At node 2,

$Q_y = 8\times75\times{}^{150}/_{2} = 45000 \text{ mm}^3$

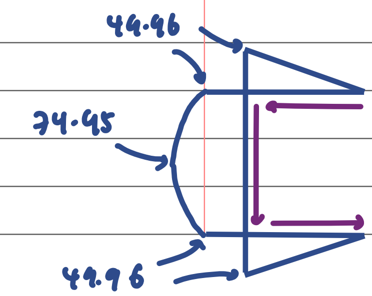
$q = 45000 \times 1.1103\times10^{-3} = 44.96$ N/mm

At node 3,

$Q_y = 45000 + 8\times{}^{150}/_{2}\times{}^{150}/_{4} = 67500 \text{ mm}^3$

$q = 67500 \times 1.1103\times10^{-3} = 74.95$ N/mm

q (N/mm):



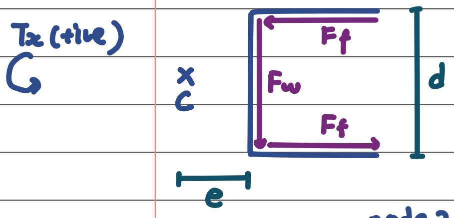
quick check:

$V_z = \int \tau \, dA = \int q \, ds = 44.96\times150 + \frac{2}{3}(74.95-44.96)\times150$

$= 9.993$ kN (close to 10 kN) ✓

b Position of the centre of twist.

1. we found q, we can convert q to force by $V=\int q\,ds$
2. to find centre of twist $\sum M_c = 0$



$$\sum M_c = 0$$

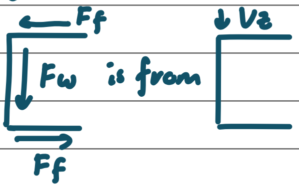
$$F_f\left(\frac{d}{2}\right) + F_f\left(\frac{d}{2}\right) - F_w e = 0$$

$$F_f d - F_w e = 0 \qquad \text{(need to find } F_f \text{ and } F_w\text{)}$$

$$F_f = \int_{\text{node 1}}^{\text{node 2}} q\,ds = \frac{1}{2} \times 49.96 \times 75 = 1873.5\ \text{N}$$

$$F_w = \int_{\text{node 2}}^{\text{node 4}} q\,ds = 49.96 \times 150 + \frac{2}{3}(74.95 - 49.96) \times 150 = 9993\ \text{kN}$$

(for F_w there is a shortcut: since



is from , hence $F_w = V_z$ (cause no other force in z-direction))

$$F_f d - F_w e = 0$$

$$e = \frac{F_f d}{F_w} = \frac{1873.5 \times 150}{10 \times 10^3} = 28.1025\ \text{mm}$$

c Twisting moment due to the eccentricity of the shear force.

$$T_x = V_z e \tag{4.17}$$

$$T_x = V_z e = -10 \times 10^3 \times 28.1025 = -281025\ \text{Nmm}$$

d Maximum shear stress due to shear and twisting.

c5 c4

1. It is an open thin-walled section, shear stress due to twisting vary linearly across thickness
2. For shear stress due to shear (c5) we already found q, which we just need to:
   $q = \tau t \rightarrow \tau = q/t$

Step 1: Find shear stress due to twisting.

Shear stress in thin-walled open sections

$$\tau_{\max}(s) = \frac{T_x t(s)}{J} \tag{3.24}$$

← depends on thickness, since $t_w = t_f$ in this question, $\tau_{\max,f} = \tau_{\max,w}$

Torsion constant for thin-walled open sections

$$J \approx \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \tag{3.28}$$

Where n is the number of plates comprising the cross-section.

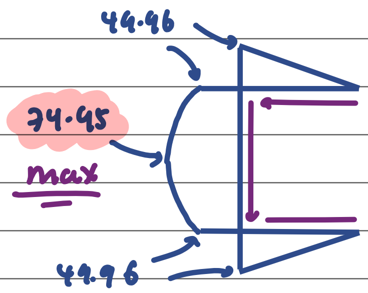
$$J \approx \frac{1}{3}\sum bt^3 = \frac{1}{3}[2\times75\times8^3 + 150\times8^3] = 51200\,\text{mm}^4$$

$$\tau_{max,w} = \tau_{max,f} = \frac{T_x t_w}{J} = \frac{281025\times8}{51200} = 43.91\,\text{N/mm}^2$$

Step 2. Find shear stress due to shear

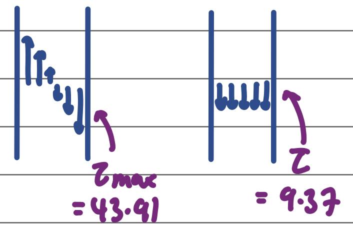
$$q = \tau t \rightarrow \tau = \frac{q}{t} \qquad \tau_{max\,(shear)} = \frac{74.95}{8} = 9.37\,\text{N/mm}^2$$

q (N/mm): from part (a)



Step 3. Finally, sum shear stress due to twisting and shear.

$$\tau_{twist} + \tau_{shear} = 43.91 + 9.37 = 53.28\,\text{N/mm}^2$$



e If the section was used as a 5 metre cantilever calculate the angle of twist at the free end due to the twisting moment produced by the shear force, assuming the shear force is constant along the length of the cantilever.

Twisting moment relationship

$$\theta_x = \frac{T_x}{GJ} \tag{3.2}$$

$$G = \frac{E}{2(1+\upsilon)} = \frac{205\times10^3}{2(1+0.3)} = 78846\,\text{N/mm}^2$$

$$\theta_x = \frac{-281025}{78846\times51200} = -6.9614\times10^{-5}\,\text{rad/mm}$$

$$\phi_x = -6.9614\times10^{-5}\times\underbrace{5\times10^3}_{\text{length of beam}}\times\overbrace{\frac{180}{\pi}}^{\text{conversion to degree}} = -19.94^\circ$$