IMPERIAL COLLEGE LONDON

MEng Examination 2023

PART II

This paper is also taken for the relevant examination for the Associateship

CIVE50004: ENVIRONMENTAL ENGINEERING

24 May 2023: 9.30–12.30 British Summer Time (BST)

*This paper contains **THREE** questions.*

*Answer **ALL THREE** questions, each in a **SEPARATE** booklet.*

All questions carry equal marks.

Formulae sheets are provided at the end of the examination paper.

Q1. (Answer ALL parts of this question; total of 40 marks)

An engineering consultant is evaluating the impact of future climate change on a catchment feeding a hydropower plant. The catchment has an area of 15,000 km^2 and is covered by evergreen pine forest. The region has the climate characteristics given in Table 1.1.

Table 1.1 Characteristics of the selected region

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$E_{t,0}$ [mm month^{-1}]	28	47	72	125	150	188	168	140	95	52	33	22
P [mm month^{-1}]	0	230	185	145	120	98	92	74	80	86	108	112

$E_a = K_s K_c E_{t,0}$
$K_s = 1.0$ (irrigation)
E_a

$\bar{Q} = 228\ m^3 s^{-1}$

The average observed inflow in the hydropower reservoir is 228 m^3 s^{-1}. The plant has an operational height of 64 m, a turbine efficiency of 90%, and a generator efficiency of 75%.

(a) Assuming that groundwater recharge is negligible, calculate:

0 (long term $\Delta S = 0$)
$\cancel{\Delta S} = P - E - Q + \cancel{R}^{\,0}$

(i) the average monthly actual evapotranspiration and
(ii) the runoff ratio of the catchment.

$Q = 228 m^3 s^{-1} = 39.4$ mm month^{-1}

(i) $E_a = P - Q$
$\bar{E_a} = \bar{P} - \bar{Q} = 110.83 - 39.4 = 71.43$

(ii) $\bar{P} = \frac{\Sigma P}{n} = 110.83$ mm month^{-1}
$= 641 m^3 s^{-1}$
(times area and change unit)

Runoff Ratio $= \frac{\bar{Q}}{\bar{P}} = \frac{228 m^3 s^{-1}}{641 m^3 s^{-1}} = 0.355$

[4 marks]

(b) A climate model predicts an average decrease in precipitation of 13% and an increase in actual evapotranspiration of 6% because of anthropogenic climate change. Estimate the impact on the theoretical power production of the plant.

$Q = P - E$
$= 0.87P - 1.06E$
$= 0.87(110.83) - 1.06(71.43)$
$= 20.7063$ mm month^{-1} $= 119.83 m^3 s^{-1}$

$P = \varepsilon_t \varepsilon_h h \rho Q g$
before: $P = 0.9 \times 0.7 \times 64 \times 1000 \times 228 \times 9.81$
$= 9.018 \times 10^7 W$

now: $P = 0.9 \times 0.7 \times 64 \times 1000 \times 119.83 \times 9.81$
$= 4.74 \times 10^7 W$

[8 marks]

(c) If you know that the discharge data are obtained from an automatic water level sensor and a stage-discharge curve, list and briefly describe the main sources of error in the calculation of your result for (b).

h-Q

measure h, predict Q.

[6 marks]

(d) Does the vegetation in the catchment experience water stress under the current and future conditions? Motivate your answer.

$E_{t,0}$ in mm year^{-1}
(sum all mm month^{-1})
is greater than E_a in mm year^{-1}
($\bar{E_a} \times 12$)

$E_a = K_s K_c E_{t,0}$
If $K_c > 1$ (large vegetation)
then $E_a < E_{t,0}$ will result in $K_s < 1$ ∴ water stress occurs.

[6 marks]

(e) Explain the potential impacts that deforestation may have on the functioning of the hydropower plant

[8 marks]

(f) The climate model predicts that the impacts on precipitation and evapotranspiration will be strongest during the summer months (June to September). Discuss the implication for the water resources of the basin, and how this trend fits in a global pattern of climate change impacts on the water cycle.

[8 marks]

Q2. (Answer ALL parts of this question; total of 40 marks)

(a) What properties make groundwater preferable as a water resource in comparison with surface water and why are these so?

[8 marks]

(b) Information on four piezometers drilled located screened at the same elevation in a confined sand and gravel aquifer is given in Table 2.1. The aquifer has a uniform thickness of 8.5 metres and a transmissivity of 4.6×10^{-4} $m^2 s^{-1}$. From the information provided, calculate the Darcy velocity in metres per day.

Table 2.1 Well details

Well ID	Easting [m]	Northing [m]	Casing elevation [mAOD]	Dip [m]
OW1	0	0	148.37	23.14
OW2	0	130	150.15	23.58
OW3	225	130	150.72	22.91
OW4	225	0	148.60	N/A

head
125·23
126·57
127·81

(b)(i) $h = Ax + By + C$

$125{\cdot}23 = A(0) + B(0) + C$
$C = 125{\cdot}23$
$126{\cdot}57 = A(0) + B(130) + 125{\cdot}23$
$B = 0{\cdot}01031$
$127{\cdot}81 = A(225) + 0{\cdot}01031(130) + 125{\cdot}23$
$A = 0{\cdot}00551$

$i = \sqrt{A^2 + B^2} = 0{\cdot}01169$

$\theta = \tan^{-1}\left(\frac{A}{B}\right) = 28{\cdot}12$
$\theta_{flow} = 208.12$ (from N)

[7 marks]

$q = -Ki = -5{\cdot}41 \times 10^{-5} \times 0{\cdot}01169 = -6{\cdot}32 \times 10^{-7}$ m/s $= -5{\cdot}46 \times 10^{-2}$ m/day

$T = KH$
$K = \frac{4{\cdot}6 \times 10^{-4}}{8{\cdot}5} = 5{\cdot}41 \times 10^{-5}$

If a dip taken of the fourth well shortly after the other three was 21.13 m, what do you deduce from this result?

$h = 0{\cdot}00551x + 0{\cdot}01031y + 125{\cdot}23$
$= 0{\cdot}00551(225) + 0{\cdot}01031(0) + 125{\cdot}23$
$= 126{\cdot}47$ m

$h = 148{\cdot}60 - 21{\cdot}13$
$= 127{\cdot}47$ m

exactly differs by 1m
error in recording and not measurement.

[3 marks]

(c) A weakly consolidated rock core sample has a diameter and length of 15 cm, an initial mass of 4639 g, and a dry bulk density of 1.7 g cm^{-3}. It required 477 cm^3 of water to bring it to saturation. It was then placed on a wire mesh plate and allowed to drain until the mass was effectively constant. The sample mass was then 4744 g. Calculate two porosity values that can be obtained from these measurements and explain why they are different.

This is NOT dry mass! why?

① $d = l = 15$ cm $\quad V = \frac{\pi}{4} d^2 l = \frac{\pi}{4}(15)^3 = 2650{\cdot}72$ cm^3 (total volume)

total volume × dry bulk density = dry mass $= \underbrace{4506{\cdot}22}_{< 4639 \text{ (initial volume)}}$ cm^3

→ this mean initial value have water!

[6 marks]

we know eventually the two porosity are n_e and n_d

where $n_e = \frac{\text{volume accepting water}}{\text{total volume}}$, $n_d = \frac{\text{volume of water drained}}{\text{total volume}}$

② mass of water (volume) $= 4639 - 4506{\cdot}22 + 477 = 609{\cdot}78$ g $= 609{\cdot}78$ cm^3

③ $n_e = \frac{609{\cdot}78}{2650{\cdot}72} = 0{\cdot}23$

④ to find n_d:
volume of water drained
$= 4639 + 477 - 4744$
(initial) (addition) (final)
$= 372$ cm^3

$n_d = \frac{372}{2650{\cdot}72} = 0{\cdot}14$

reason: not all water held in pore water space can be drained by gravity.
(surface tension holding up some of the water)

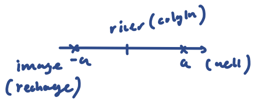
(d) Assuming recharge is negligible, the steady state water table elevation above the base of an unconfined aquifer, with hydraulic conductivity K, in response to a well pumping at a rate Q_w is:

$$h_e^2 - h^2 = \frac{Q_w}{\pi K} \ln\left(\frac{r_e}{r}\right)$$

where h_e is the height of the water table above the aquifer base prior to pumping, r is the radial distance from the well and r_e is the radius of influence.

Show, for a well pumping from such an unconfined aquifer at a distance a from a nearby river, which is in full hydraulic connection with the aquifer, that the water table elevation along an axis x, perpendicular to the direction of the river and with its origin at the river and passing through the well, can be described as:

$$h^2 = h_e^2 - \frac{Q_w}{\pi K}\ln\left(\frac{x+a}{a-x}\right)$$



$s = h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right)$

s_{pump} (well) : $\frac{Q_w}{\pi K}\ln\left(\frac{r_e}{x-a}\right)$

$s_{recharge} = -\frac{Q_w}{\pi K}\ln\left(\frac{r_e}{x+a}\right)$

$\frac{Q_w}{\pi K}\ln\left(\frac{r_e}{x-a}\right) - \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{x+a}\right)$

$= \frac{Q_w}{\pi K}\ln\left(\frac{x+a}{x-a}\right)$

$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{x+a}{x-a}\right)$

$h^2 = h_e^2 - \dots$

[4 marks]

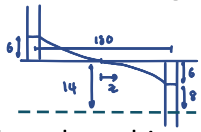
Why is this useful as a water resource?

well draw water from the nearby river via aquifer.
→ good water source (river)
aquifer act as natural filter.

[3 marks]

(e) A step test on a 0.5 m diameter well, sited 90 m from a river hydraulically connected to an unconfined sand aquifer, gave a well loss coefficient of B = 1010 s^2m^{-5}. The sand has a hydraulic conductivity of 2.6×10^{-4} m s^{-1}. The intake of the pump inside the well is at an elevation of 115 m. If the horizontal base of the aquifer is at an elevation of 107 m and the mean river stage during summer is 121 m, calculate the theoretical maximum yield of the well, in megalitres per day, during a prolonged dry period.

$S = AQ + BQ^n$



$\frac{Q_w}{\pi K}\ln\left(\frac{a+x}{a-x}\right) = h_e^2 - h^2$

$Q_w = \frac{\pi K(h_e^2-h^2)}{\ln\left(\frac{a+x}{a-x}\right)} = \frac{\pi(2.6\times10^{-4})(14^2-8^2)}{\ln\left(\frac{90+90-0.25}{90-(90-0.25)}\right)}$

$= 0.0164\ m^3s^{-1}$

$= 1416.21\ m^3\ day^{-1}$

[6 marks]

Why might this yield not be achieved in practice?

[3 marks]

lost $= BQ^n$

unit of $B = 1010\ s^2m^{-5}$

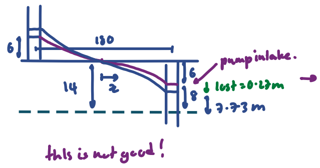
by multiplying Q^2

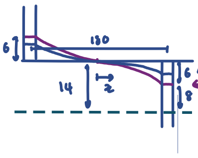
$[S] = m$ ✓

$n = 2$:

lost $= 1010 \times 0.0164^2$

$= 0.27m$





we need to increase the h such that after well loss BQ^2 the water level is still higher than pump intake (at 8m)

$\frac{Q_w}{\pi K}\ln\left(\frac{a+x}{a-x}\right) = h_e^2 - h^2$

$Q_w = \frac{\pi K(h_e^2-h^2)}{\ln\left(\frac{a+x}{a-x}\right)} = \frac{\pi(2.6\times10^{-4})(14^2-8.27^2)}{\ln\left(\frac{90+90-0.25}{90-(90-0.25)}\right)} = 0.0158\ m^3/s$

$s_w = BQ_w^2 = 0.25m$

$\frac{Q_w}{\pi K}\ln\left(\frac{a+x}{a-x}\right) = h_e^2 - h^2$

$Q_w = \frac{\pi K(h_e^2-h^2)}{\ln\left(\frac{a+x}{a-x}\right)} = \frac{\pi(2.6\times10^{-4})(14^2-8.25^2)}{\ln\left(\frac{90+90-0.25}{90-(90-0.25)}\right)} = 0.0159\ m^3/s$

$s_w = BQ_w^2 = 0.25m$

$\therefore Q_w = 0.0159\ m^3/s = 1373\ m^3/day$

<u>Q3.</u> (Answer ALL parts of this question; total of 40 marks)

(a) A catchment has a 2-hour 3mm unit hydrograph (UH) given in Table 3.1. A rainfall event has the intensity shown in Table 3.2.

(b)(i)

* p% is the percentage of effective (net) rainfall out of the total rainfall.

time	2h 3mm UH	2h "2.2×p%" mm	2h "5.4×p%" mm	"3.6p%"	total
0	0	0	0	0	0
1	0.16	0.352p/3	0	0	0.117p
2	1.26	2.772p/3	0	0	0.924p
3	1.93	4.246p/3	0.864p/3	0	1.703p
4	0.71	1.562p/3	6.804p/3	0	2.789p
5	0.14	0.308p/3	10.422p/3	0.576p/3	3.769p
6	0	0	3.834p/3	4.536p/3	2.790p
7	0	0	0.756p/3	6.948p/3	2.568p
8	0	0	0	2.556p/3	0.852p
9				0.504p/3	0.168p
10				0	0

(b)(ii) sub p=0.7

$3.769p = 2.64$

$p = \frac{2.64}{3.769} = 0.7$ (effective)

proportional lost = 30%

Table 3.1 2-hour 3mm unit hydrograph

Time (h)	Flow ($m^3 s^{-1}$)
0	0
1	0.16
2	1.26
3	1.93
4	0.71
5	0.14
6	0

UH can be applied immediately (due to same ΔT) but not the rainfall.

Table 3.2 Rainfall event depths

Time (h)	Total rainfall (mm)
0-2	2.2
2-4	5.4
4-6	3.6

total rainfall ×
we need effective rainfall

Discuss if the information given in Tables 3.1 and 3.2 is sufficient for the UH application to calculate the river flow at the outlet of the catchment.

[3 marks]

(b) If the maximal surface flow for the rainfall event from Table 2.2 is 2.64 $m^3 s^{-1}$, calculate:

(i) The value of the proportional losses in %.

[8 marks]

(ii) Values of the surface flow for all time-steps of the hydrograph and the total duration of the flow event.

[3 marks]

(c) The flow records from peak samples over 10 years have been used to calculate the mean and the variance of the annual maxima (AM) dataset:

- Mean: $\mu = 61.72\ \mathrm{m^3\ s^{-1}}$
- Standard deviation (square root of variance): $\sigma = 19.91\ \mathrm{m^3\ s^{-1}}$

For the same dataset, the return periods for the flow peaks are estimated as shown in Table 3.3.

c(ii)
$F(q_d) = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$
$\alpha - \beta \ln\{-\ln[F(q_d)]\} = q_d$

c(iii)
$F(q_d) = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

Table 3.3 Return period (T) for the observed peaks

Year	T (years)	c(ii) $P(q > q_d) = \frac{1}{T}$	$F(q_d) = 1 - \frac{1}{T}$	$q_d\ [m^3 s^{-1}]$	c(iii) $1.2 q_d\ [m^3 s^{-1}]$	$F(q_d)$
2001	3.1	0.3226	0.6774	67.40	80.880	
2002	13.5	0.0741	0.9259	92.56	111.072	
2003	1.3	0.7692	0.2308	46.82	56.184	
2004	1.7	0.5882	0.4118	54.62	65.544	
2005	1.1	0.9091	0.0909	39.18	47.016	
2006	11.3	0.0885	0.9115	89.69	107.628	
2007	4.8	0.2083	0.7917	75.34	90.408	
2008	1.5	0.6667	0.3333	51.30	61.560	
2009	2.8	0.3571	0.6429	65.44	78.528	
2010	1.03	0.9709	0.0291	33.15	39.780	

Using the information provided:

(i) Calculate parameters of the Gumbel distribution using the Method of moments.

$\alpha = \mu - 0.5772\beta$ $\beta = \frac{\sqrt{6}}{\pi}\sigma = \frac{\sqrt{6}}{\pi}(19.91) = 15.524$
$= 61.72 - 0.5772(15.524)$
$= 52.760$

[2 marks]

(ii) Calculate the values of AM peaks corresponding to the value of the return periods given in Table 3.3.

[6 marks]

(iii) Under climate change, the observed flows calculated under (ii) are predicted to increase 20%, which changes the parameters of the underlying distribution. Calculate the change in the return period due to climate change for the maximal observed flow from (ii).

new $\mu = 73.86$, new $\sigma = 24.23$
$\beta = \frac{\sqrt{6}}{\pi}(24.23) = 18.89$ $\alpha = 73.86 - 0.5772(18.89) = 62.957$

$F(q_d) = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$ find change in the period, we should check the new T of the SAME q_d! cause we can only compare changes to the T for one q_d, with different α and β's Gumbel ...
$F(\overset{92.56}{\cancel{111.072}}) = 0.\cancel{9247}\ 0.8117$, $F(Q > q_d) = 0.1883$ $T = \frac{1}{0.1883} = 5.3$ years.
$P(Q > q_d) = 1 - F(q_d) = 1 - 0.9247 = 0.0753$ $T = \frac{1}{P(Q > q_d)} = 13.277$

[6 marks]

(iv) Comment on the implication of the changed return period of flows for the flood protection and give one example how the system can be upgraded to address the climate change impacts.

big flow (q_d) happens significantly more often.
better flood protection. (check "Senior Environment pg 6. — Flood Risk Management)

[4 marks]

(d) Explain the four differences between the flood forecasting and flood estimation.

[8 marks]

Formulae Sheet

A. Introduction to Hydrology (Prof Wouter Buytaert)

Catchment water balance $\Delta S = P - E - Q - R$

Density of solids $r_S = \frac{M_S}{V_S}$

Dry bulk density $r_B = \frac{M_S}{V_T}$

Total bulk density $r_T = \frac{M_S + M_L}{V_T}$

Porosity $e = \frac{V_L + V_G}{V_T}$

Void ratio $e = \frac{V_L + V_G}{V_S}$

Gravimetric moisture content $q_G = \frac{M_L}{M_S}$

Volumetric moisture content $q = \frac{V_L}{V_T}$

Interrelating formula $q = q_G \frac{q_B}{q_W}$

Penman Monteith $E_a = K_c K_s E_{t,0}$

River flow $Q = \grave{O} v(A)\, dA = \bar{v}A$

Rectangular weir $Q = KbH^{1.5}$

Hydropower equation $P = \varepsilon_t \, \varepsilon_h \, h \, \rho \, Q \, g$

Irrigation $\mathrm{I} = \mathrm{E_p} - \mathrm{P} + \mathrm{R}$

Total available water $\mathrm{TAW} = 1000\,(\theta_{FC} - \theta_{WP})\,\mathrm{Z_r}$

Readily available water $\mathrm{RAW} = \mathrm{p\,TAW}$

B. Groundwater Systems (Prof Adrian Butler)

Darcy's law

$q_i = -K\dfrac{dh}{di}$, where i represents a coordinate direction (e.g. x, y, z)

Water table elevation in an unconfined aquifer

$$h^2 - h_0^2 = \frac{W}{K}\left(Lx - x^2\right) + \left(h_1^2 - h_0^2\right)\frac{x}{L}$$

Total flow per unit width in an unconfined aquifer

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$$

Well pumping confined aquifer without recharge (Thiem equation)

$$s = h_e - h = \frac{Q_w}{2\pi T}\ln\left(\frac{r_e}{r}\right)$$

Well pumping unconfined aquifer with recharge

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}\left(r^2 - r_e^2\right)$$

Theis solution

$s = \dfrac{Q_w}{4\pi T}W(u)$, where $u = \dfrac{r^2 S}{4Tt}$

Jacob's large time approximation

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

C. Flood Risk Estimation and Management (Dr Ana Mijic)

Weibull formula	$P_m(Q \leq q_m) = \frac{\mathrm{m}}{N+1}$
Gringorten formula	$P_m(Q \leq q_m) = \frac{\mathrm{m}-0.44}{N+0.12}$
Gumbel distribution	$F(q_d) = \exp\left[\ -\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$
Gumbel variate	$z = -\ln\{-\ln[F(q_d)]\}$
Probability for sequence of years	$P(x,n) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$
Moment matching	$\alpha = \mu - 0.5772\,\beta$ $\beta = (6^{\frac{1}{2}}/\pi)\sigma$
L-moment matching	$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j$ $L_2 = \frac{2}{n}\sum_{j=2}^{n}\left[\frac{(j-1)q_j}{n-1}\right] - L_1$ $\alpha = L_1 - 0.5772\,\beta$ $\beta = \frac{L_2}{\ln(2)}$
Normal equation for flood forecasting	$(R^T R)\theta = R^T X$