IMPERIAL COLLEGE LONDON

MEng Examination 2017

PART II

This paper is also taken for the relevant examination for the Associateship

CI2.260: ENVIRONMENTAL ENGINEERING

30 May 2017: 14.00 – 17.00

This paper contains ***THREE*** *questions.*

Answer ***ALL THREE*** *questions.*

All questions carry equal marks.

Formulae sheets are provided at the end of the examination paper.

Please use separate answer book for each of the three questions.

DO NOT OPEN THIS EXAMINATION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

how to find Kc of all months
remember to ÷2 if half month
both (a) and (b) are tricky
must find RAW separately
min freq. 's max(I)
sometimes need to check reference time!

Q1 (Answer ALL parts of this question; total of 40 marks)

A farmer in northern India grows rice and wheat in rotation on the same field. The growing period of wheat starts 15 December, and that of rice 1 May. The fields have clay soils, with a wilting point of 0.32 $m^3 m^{-3}$ and a field capacity of 0.48 $m^3 m^{-3}$. The depletion fractions for wheat and rice are resp. 55% and 20%, and their average root depths are resp. 0.8 m and 0.5 m.

The crop properties are:

Growing stage	Crop coefficient	Length (days)	Crop coefficient	Length (days)
	Rice	Rice	Wheat	Wheat
Initial	1.05	30	0.3	15
Development	N/A	30	N/A	30
Mid	1.2	80	1.15	45
Late	0.6	40	0.4	30

Reference evapotranspiration (ET0) and precipitation (P) for the region in units of mm $month^{-1}$ are:

☆ divide two because we only looking at half month. ☆ divide by two, same reason!

(a) step 2.

$I = E_p - P + R$
$R = 0.15P$
$I = E_p - P + 0.15P$
$= E_p - 0.85P$

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
ET0	89.5	113.1	132.1	~~152.2~~ 76.1	163.3	146.8	156.7	163.7	147.5	114.8	86.8	~~65.6~~ 32.8
P	22	18	14	~~8~~ 4	11	55	194	194	123	22	3	~~8~~ 4
Kc:	0.725	1.15	1.056	0.588	1.05	1.125	1.2	1.2	1.175	0.825	-	0.3
Et:	64.8875	130.065	139.4976	44.7468	171.465	165.15	188.04	196.44	173.3125	94.71	-	9.84
I	46.1875	114.765	127.5976	41.3468	162.115	118.4	23.14	31.54	68.7625	76.01	-	6.44

Based on the given information answer the following questions:

we know: $E_t = K_s K_c E_{t,0}$
for irrigation, $K_s = 1.0$

☆ if $E_p - P < 0$ we don't need to irrigate!

(just an extra info:
in other past paper have been tested,
this extra water can be stored by using the eqn: $\cancel{\Delta S}^{0\,(\text{long term})} = P - Q - E - \cancel{R}^{0}$ $\underbrace{Q = P - E}_{\text{excess water that can be stored}}$)

a)

Sketch the evolution of the crop coefficient over the year for both crops, and calculate the crop coefficient for each month. For simplification, assume that all months are 30 days long. Calculate the total crop water and irrigation requirement for both crops in [mm]. Assume that 15% of the rainfall percolates through the soil as groundwater recharge;

(a) step 3:
total irrigation = 816.3044 mm.
total crop water required = E_t = 1378.1544 mm.

[16 marks]

b)

(b)(i) TAW (wheat) = (0.48 − 0.32) × 0.8 m = 128 mm.
RAW (wheat) = 0.55 × 128 = 70.4 mm.
TAW (rice) = (0.48 − 0.32) × 0.5 m = 80 mm
RAW (rice) = 0.20 × 80 mm = 16 mm.
RAW = pTAW

☆ (ii) max. period (wheat) = $\frac{70.4}{127.5976}$ = 0.5517 month = 16.55 days
max period (rice) = $\frac{16.0}{162.115}$ = 0.0987 month = 2.96 days. wrong max(I)

i. What is the maximum dose of irrigation water (in mm) that can be applied with minimal losses?

minimum freq. = $\frac{\max(I)}{RAW}$ → max period = $\frac{RAW}{\max(I)}$

ii. What is the maximum time between two irrigation doses during the mid-stage of the crop development?

have to use mid stage
May is not mid stage!
mid is Jul, Aug, Sep based on the Kc!

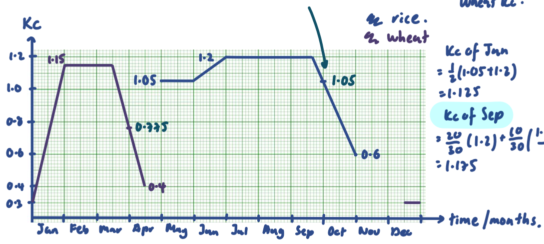
max period (rice) = $\frac{16}{67.4}$ = 0.237 month = 7.12 days

[4 marks]

(a) step 1:

how to find this 1.05?
$1.2 - \frac{10\text{ days}}{40\text{ days}} \times (1.2 - 0.6) = 1.05$

	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec ☆	Jan	Feb	Mar	Apr ☆
Rice Kc:	1.05	1.125	1.2	1.2	1.175	0.825	-					
Wheat Kc:							-	0.3	0.725	1.15	1.056	0.588



Kc of Jan
$= \frac{1}{2}(1.05 + 1.2)$
$= 1.125$

Kc of Sep
$= \frac{20}{30}(1.2) + \frac{10}{30}\left(\frac{1.2 + 1.05}{2}\right)$
$= 1.175$

Kc of Oct
$= \frac{1}{2}(1.05 + 0.6)$
$= 0.825$

Kc of Mar
$= \frac{15}{30}(1.15) + \frac{15}{30}\left(\frac{1.15 + 0.775}{2}\right)$
$= 1.05625$

Kc of Apr
$= \frac{1}{2}(0.775 + 0.4)$
$= 0.5875$

c)

The rainfall data in the table above are obtained from rain gauges. What are two alternative methods that can be used to measure rainfall? Describe briefly their main characteristics.

[8 marks]

d)

Northern India is a region with very intensive environmental change, with 3 major drivers of change: population growth, increasing urbanisation, and intensification of agriculture. Describe for each of them how they affect water resources. Use bullet points to list the main processes and explain briefly each of them.

[12 marks]

<u>Q2</u> (Answer ALL parts of this question; total of 40 marks)

a) You are required to calculate the velocity of contamination in groundwater in an unconfined aquifer affected by leachate from a landfill. Explain, **in detail**, what infrastructure and measurements you would need and how these would be used.

$v = \frac{q}{n_e}$
- need q and ne
- $q = -Ki$
- need K and i
- three observation well
- measure hydraulic head and solve ✓i_x ✓i_y
  $h = Ax + By + C$ for A and B
- find $i = \sqrt{A^2+B^2}$ and $\theta = \tan^{-1}(A/B)$
- to find K, can use constant-rate pumping test to obtain S_y and T
  or we can use ex-situ test like falling head permeameter
  S if confined, S_y if unconfined.
- $T = Kh_0$, where h_0 is initial saturated thickness.
- ne can be estimated with S_y

[8 marks]

b) A pond in southern Bangladesh is used as a source of drinking water. There are concerns that water is being lost from the pond through seepage into the underlying sediments during the dry season. A stilling well to measure pond water level and a piezometer screened at a depth of 3 metres below the base of the pond were installed. Measurements of water levels in these are given in Table 1. In addition, a falling head permeameter test, with a 0.2 cm diameter upper tube, was undertaken on a cored sediment sample, 30 cm length and 10 cm diameter. The results are shown in Table 2. If measurements of water loss from the pond give a mean value of 7.5 mm per day, estimate how much of this might be due to seepage. What is the other reason for the water loss?

this is what a stilling well looks like
don't have to draw this, draw water table is enough!

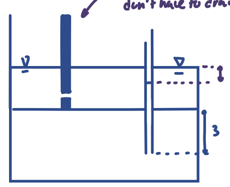
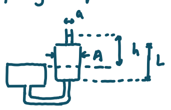


Table 1. Water level measurements

Device	Water level (m above datum)
Pond stilling well	3.2
Piezometer screened at 3 m below base of pond	1.9

observation well is just to let us know where is the water table!

a quick exercise on how to deduce.
falling head permeameter test eqn :



$\frac{Q}{A} = -Ki$ $\quad Q = -a\frac{dh}{dt}$

$i = -\frac{h}{L}$

$Q = AK\frac{h}{L}$ — $AK\frac{h}{L} = -a\frac{dh}{dt}$

$-\frac{AK}{aL}dt = \frac{1}{h}dh$

$-\frac{AK}{aL}t = \ln(h) + c$

$e^{-\frac{AK}{aL}t} \times e^c = h$

when $t = 0$, $h = H_0$
$\therefore e^c = H_0$

$h = H_0 e^{-\lambda t}$, $\lambda = \frac{AK}{aL}$

Table 2. Falling head test measurements

Time (min)	Head (cm) h	$\ln h$
0	100.0	4.605
10	81.8	4.404
20	67.0	4.205
30	54.8	4.004
40	44.9	3.804
50	36.7	3.603
60	30.1	3.405

given parameter : $d = 0.2$ cm, $L = 30$ cm, $D = 10$ cm, $\frac{dh}{dt} = 7.5$ mm/day

$a = \frac{\pi}{4}d^2 = 0.03142$ cm^2

$A = \frac{\pi}{4}D^2 = 78.5398$ cm^2

- I don't know what are we trying to find, but I know I can find K from this test.

$h = H_0 e^{-\lambda t}$

$\frac{h}{H_0} = e^{-\lambda t}$

$\ln\left(\frac{h}{H_0}\right) = -\lambda t$

$\ln(h) - \ln(H_0) = -\lambda t$

$\ln(h) = -\lambda t + \ln H_0$

plotting ln(h) against t,
gradient $= -\lambda$
by doing linear regression,
gradient $= -0.02$

$0.02 = \frac{78.5398 \times K}{0.03142 \times 30}$

$\therefore K = 2.4 \times 10^{-4}$ cm/min

- After having K let us re-evaluate what do we have in our hands right now..
- we have $K = 2.4 \times 10^{-4}$ cm/min,
  $\frac{dh}{dt} = 7.5$ mm/day
- we can find $i = \frac{dh}{dl} = -\frac{1.3}{3} = -0.4\overline{3}$
- having K of the soil and i we can find darcy's vel.
  $q = -Ki = -2.4 \times 10^{-4}$ cm/min $\times -0.4\overline{3}$
  $= 1.04 \times 10^{-4}$ cm/min
  convert to mm/day ↓
  $q = 1.4976$ mm/day is lost due to seapage.

$\frac{1.4976}{7.5} \approx$ <u>20% lost to seapage</u>

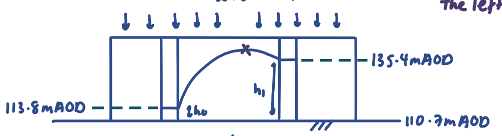
Note: Falling head permeameter test equation is: $h = H_0 e^{-\lambda t}$, where $\lambda = \frac{AK}{aL}$

[8 marks]

c) A homogeneous and isotropic unconfined aquifer, with a horizontal base at 110.7 mAOD and a hydraulic conductivity of 1.7×10^{-4} m.s^{-1}, is bounded to the west and east by river valleys, which are 10 km apart. The west and east rivers are in full hydraulic connection with the aquifer and have stages of 113.8 mAOD and 135.4 mAOD, respectively. If the mean annual recharge is 800 mm, calculate the location of the groundwater divide in terms of its distance from the west river.

→ $W = 800$ mm/year
$= 2.536 \times 10^{-8}$ m/s

we want to find "x" marked on the left figure.



h₀ is always the one on the left!

use formula sheets :
why I chose this? 1. unconfined aquifer 2. <u>no well pumping</u>!
why I didn't modify into other form? like $h_0 = h_1$ or etc. cause no need. (W also ≠ 0)

$h^2 - h_0^2 = \frac{W}{K}(Lx - x^2) + (h_1^2 - h_0^2)\frac{x}{L}$

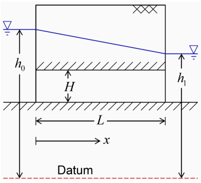
$h^2 - (113.8 - 110.7)^2 = \frac{2.536 \times 10^{-8}}{1.7 \times 10^{-4}}(10 \times 10^3 x - x^2) + \left[(135.4 - 110.7)^2 - (113.8 - 110.7)^2\right]\frac{x}{10 \times 10^3}$

$h^2 = 9.61 + 1.4918x - 0.0001492x^2 + 0.060048x$
$= 9.61 + 1.55185x - 0.0001492x^2$

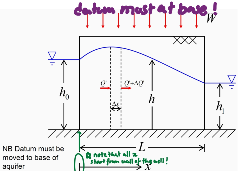
$2h\frac{dh}{dx} = 1.55185 - 0.0002984x \rightarrow x = 5200$ m. ✓

0

datum doesn't matter



datum must at base!



for some reason it doesn't yield the same result as marking scheme that uses another approach:

At the point where groundwater divide, $Q'=0$.

MARKING SCHEME TAKES h_0 as 113.8 and not (113.8 − 160.7) and h_1 as 135.4 and not (135.4 − 160.7) } wrong because unconfined aquifer MUST move the datum to the base!

$Q'=0$

$W\left(x-\frac{L}{2}\right)+\frac{K}{2L}\left(h_0^2-h_1^2\right)=0$ $\quad x=5200\text{ m}!$ ✓

$x=\frac{L}{2}-\frac{K}{2WL}\left(h_0^2-h_1^2\right)$

$W=(800\pm160)\text{mm}$, $K=(1.7\pm0.34)\times10^{-4}\text{ ms}^{-1}$

$x=\frac{L}{2}-\frac{K}{2WL}\left(h_0^2-h_1^2\right)$

big K small W gives small x — ①
small K big W gives big x — ②
$x=(①, ②)$

If there is an uncertainty of around 20% in the hydraulic conductivity and recharge values, what does this mean for the expected location of the groundwater divide?

[6 marks]

d) It is proposed to use the Theis equation to analyse drawdown, s at an observation well, which is at a distance r from a well being pumped at a constant rate Q_w in an unconfined aquifer. The Theis equation is given as:

$$s=\frac{Q_w}{4\pi T}E_1\left(u\right), \quad \text{where } u=\frac{r^2S}{4Tt} \text{ and } E_1 \text{ is the exponential}$$

integral.

What condition is required to justify using this equation for an unconfined aquifer?

Set $S=S_y$; time, t must be large.

not accurate.

ans: s is small (<10% saturated thickness, h) (drawdown); elastic storage effect is small.

[2 marks]

The exponential integral can represented by the following expression:

$$E_1\left(u\right)=-0.5772-\ln\left(u\right)+\left(\sum_{k=1}^{\infty}\frac{\left(-1\right)^{n+1}u^n}{n.n!}\right) \text{ approx. } 0$$

$s=\frac{Q_w}{4\pi T}\left(-\ln\left(\frac{r^2S}{4Tt}\right)-0.5772\right)$

$=\frac{Q_w}{4\pi T}\left(\ln\left(\frac{4T}{r^2S}\right)+\ln t-0.5772\right)$

plot s against $\ln t$

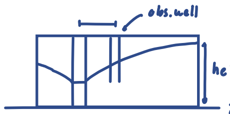
gradient $=\frac{Q_w}{4\pi T}$

y-intercept $=\frac{Q_w}{4\pi T}\left(\ln\left(\frac{4T}{r^2 \cancel{S}\, S_y}\right)-0.5772\right)$

you forget $S=S_y$!

Using this expression, show how drawdown can be written so that the aquifer's hydraulic properties can be obtained using linear regression. After what point in the test is this expression applicable? when $u<0.1$ $\quad \frac{r^2S}{4Tt}<0.1$ $\quad t>\frac{r^2S}{0.4T}$ ✓ [6 marks]

e) A constant rate pumping test was undertaken on a fully penetrating well sited in a homogeneous and isotropic unconfined aquifer. The horizontal base of the aquifer is at an elevation of 25mAOD. The elevation of the water table prior to pumping is 52mAOD. Dips, at times following the start of pumping, in an observation well located 20 m from the abstraction well are given in Table 3 below. If the pumping rate is 1000 m^3day^{-1} calculate the hydraulic conductivity and specific yield of the aquifer.

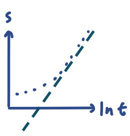


[8 marks]

Table 3. Time drawdown data for pumping test

	Time (min)	ln t	Dip (cm)
x	5	1.61	0.0
x	10	2.30	0.3
x	20	3.00	1.9
x	30	3.40	3.9
x	60	4.09	9.3
✓	120	4.78	16.8
	240	5.48	25.5
	360	5.89	30.9
	720	6.58	40.6
	1440	7.27	50.5
	2880	7.97	60.6
	4320	8.37	66.6

this is drawdown, s



first, plot, then judge where the linear part start...

6 to 12 looks linear ← I looked at marking scheme.

using linear regression on calculator...

$m=13.99\approx14$

$c=-50.95\approx-51$

$\frac{Q_w}{4\pi T}=10.561$

convert Q_w to $cm^3\,min^{-1}$:

$Q_w=1000\text{ m}^3\text{day}^{-1}$

$=\frac{1000\text{ m}^3}{\text{day}}$

$=\frac{1000\text{ cm}^3}{10^{-6}\times24\times60\text{ min}}$

$=694444\text{ cm}^3\text{min}^{-1}$

$T=\frac{Q_w}{4\pi(14)}=3947.3$ [cm²/min]

$T=Kh_0$ (where $h_0=h_e$)

$K=\frac{T}{h_0}=\frac{3947.3\ [\text{cm}^2/\text{min}]}{27\times10^2\ [\text{cm}]}$

$K=1.46\text{ cm min}^{-1}$

convert back to m day⁻¹

$K=\frac{1.46\text{ cm}}{1\text{ min}}=\frac{1.46\times10^{-2}\text{ m}}{1\text{ day}}\times24\times60=21\text{ m/day}$

to find S_y:

$\frac{Q_w}{4\pi T}\left[\ln\left(\frac{4T}{r^2S}\right)-0.5772\right]=-51$

$r=20\text{ m}=2000\text{ cm}$

$\therefore S=0.08466$

Comment on the aquifer's suitability as a water resource.

$T=3947\text{ cm}^2/\text{min}$

$=\frac{3947\times10^{-4}\text{ m}^2}{1\text{ day}}\times24\times60$

$=568.368\text{ m}^2/\text{day}$.

$T>200\text{ m}^2/\text{day}$ (good water resource)

$S_y=0.085>0.05$ (good storativity)

[2 marks]

Q3 (Answer ALL parts of this question; total of 40 marks)

a) Provide a short definition of the following terms in the context of flooding (you can use equations and sketches where helpful):

i. Pluvial flooding
ii. Plotting position
iii. Empirical models
iv. Baseflow
v. Regionalisation

[10 marks]

b) The 5mm 1hour triangular unit hydrograph for a catchment (area 25 km^2) is given below (t_1=2 hours and t_2=6 hours).

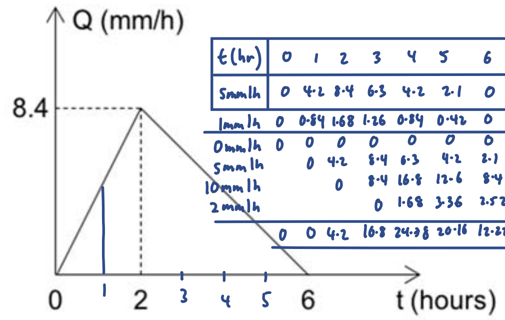
i. Using the rainfall data below, calculate the stream flow at the end of each hourly interval in units m^{3}s^{-1}. Use the ϕ-index method of estimating losses with ϕ= 3 mm hour^{-1}. Assume a constant baseflow of 1.2 mm hour^{-1}.

ii. After a period of urbanisation, t_1 and t_2 decrease by 1 hour, and the baseflow decreases by 0.3 mm hour^{-1}. Calculate and sketch the new 5mm 1hour unit hydrograph, and, for the same rainfall input, estimate the impacts of urbanisation on the stream flow peak and time to peak.

[18 marks]

	Time (h)	0-1	1-2	2-3	3-4
total	Rainfall (mm/h)	3	8	13	5

effective rainfall = total rainfall $-\underbrace{\text{losses}}_{3\text{mm h}^{-1}}$

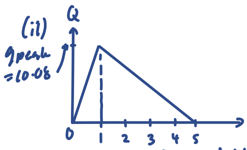


t(hr)	0	1	2	3	4	5	6			
5mm/h	0	4.2	8.4	6.3	4.2	2.1	0			
1mm/h	0	0.84	1.68	1.26	0.84	0.42	0			
0mm/h	0	0	0	0	0	0	0			
5mm/h		0	4.2	8.4	6.3	4.2	2.1	0		
10mm/h			0	8.4	16.8	12.6	8.4	4.2	0	
2mm/h				0	1.68	3.36	2.52	1.68	0.84	0
	0	0	4.2	16.8	24.78	20.16	12.22	5.88	0.84	6

this is stormflow

to find stream flow (total flow)

= stormflow + $\underbrace{\text{baseflow}}_{1.2\text{ mmh}^{-1}}$



how to find new Qpeak of UH?

old total Q for 5mm 1h UH:

$Q_{total} = \frac{1}{2} \times 6 \times 8.4 = 25.2$

new total Q = old total Q

$\frac{1}{2} q_{peak} \times 5 = 25.2$

$q_{peak} = 10.08$

c) Define elements of the Flood risk management cycle and list three catchment scale flood management measures.

[6 marks]

d) The sample of annual maxima flood flow data is given below. Using the L-moments method estimate the parameters for the Gumbel distribution, and:

i. Estimate the T=100 year flood flow.

ii. The probability of this flood being exceeded in at least one year within a 100 year period.

[6 marks]

Q (m3/s)	35	38	47	53	57
Q (m3/s)	63	71	74	84	122

(i)

$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j = \frac{644}{10} = 64.4$

$L_2 = \frac{2}{n}\sum_{j=2}^{n} \frac{(j-1)q_j}{n-1} - L_1 \quad ; n=10$

$= \frac{2}{10 \times 9}\left(1\times38 + 2\times47 + 3\times53 + 4\times57 + 5\times63 + 6\times71 + 7\times74 + 8\times84 + 9\times122\right) - 64.4$

$= 14.\bar{4}$

$\beta = \frac{L_2}{\ln 2} = 20.839$

$\alpha = L_1 - 0.5772\beta = 52.37$

$F(q_d) = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

$\frac{99}{100} = \exp\left[-\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

$q_d = 148.23\ m^3/s$ ✓

marking scheme ($q_d = 101.6\ m^3/s$)
is definitely wrong.
cause $q(100)$ must be greater than
any of the 10 q_d shown in table, by a lot.

$T = \frac{1}{P(q > q_d)}$

$\frac{1}{100} = P(q > q_d)$

$1 - F(q_d) = \frac{1}{100}$

$F(q_d) = \frac{99}{100}$

let $p = P(q > q_d) = \frac{1}{100}$

$X \sim B(100, \frac{1}{100})$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - \frac{99}{100}^{100}$

$= 0.634 \approx 63.4\%$

Formulae Sheet

A. Introduction to Hydrology (Dr Wouter Buytaert)

Catchment water balance $\Delta S = P - E - Q - R$

Density of solids $\rho_S = \dfrac{M_S}{V_S}$

Dry bulk density $\rho_B = \dfrac{M_S}{V_T}$

Total bulk density $\rho_T = \dfrac{M_S + M_L}{V_T}$

Porosity $\varepsilon = \dfrac{V_L + V_G}{V_T}$

Void ratio $e = \dfrac{V_L + V_G}{V_S}$

Gravimetric moisture content $\theta_G = \dfrac{M_L}{M_S}$

Volumetric moisture content $\theta = \dfrac{V_L}{V_T}$

Interrelating formula $\theta = \theta_G \dfrac{\theta_B}{\theta_W}$

Penman Monteith $ET_a = K_c K_s ET_0$

River flow $Q = \int v(A)\,dA = \overline{v}A$

Rectangular weir $Q = KbH^{1.5}$

Hydropower equation $P = \varepsilon_t \varepsilon_h h \rho Q g$

Irrigation $\mathrm{I} = \mathrm{ET_c} - \mathrm{P} + \mathrm{R}$

Total available water $\mathrm{TAW} = 1000\,(\theta_{\mathrm{FC}} - \theta_{\mathrm{WP}})\,\mathrm{Z_r}$

Readily available water $\mathrm{RAW} = \mathrm{p\,TAW}$

B. Groundwater Systems (Dr Adrian Butler)

Darcy's law

$q_i = -K\frac{dh}{di}$, where i represents a coordinate direction (e.g. x, y, z)

Water table elevation in an unconfined aquifer

$$h^2 - h_0^2 = \frac{W}{K}\left(Lx - x^2\right) + \left(h_1^2 - h_0^2\right)\frac{x}{L}$$

Total flow per unit width in an unconfined aquifer

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$$

Well pumping confined aquifer without recharge (Thiem equation)

$$s = h_e - h = \frac{Q_w}{2\pi T}\ln\left(\frac{r_e}{r}\right)$$

Well pumping unconfined aquifer with recharge

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}\left(r^2 - r_e^2\right)$$

Theis solution

$s = \frac{Q_w}{4\pi T}W\left(u\right)$, where $u = \frac{r^2 S}{4Tt}$

Jacob's large time approximation

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

C. Flood Risk Estimation and Management (Dr Ana Mijic)

Weibull formula $P_m(q \le q_m) = \frac{\mathrm{m}}{N+1}$

Gringorten formula $P_m(q \leq q_m) = \frac{\mathrm{m}-0.44}{N+0.12}$

Gumbel distribution $F(q_d) = \exp\left[-\exp\left(\frac{\alpha-q_d}{\beta}\right)\right]$

Gumbel variate $z = -\ln\{-\ln[F(q_d)]\}$

Probability for sequence of years $P(x,n) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$

Moment matching

$$\alpha = \mu - 0.5772\,\beta$$
$$\beta = (6^{\frac{1}{2}}/\pi)\sigma$$

L-moment matching

$$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j$$
$$L_2 = \frac{2}{n}\sum_{j=2}^{n}\left[\frac{(j-1)q_j}{n-1}\right] - L_1$$
$$\alpha = L_1 - 0.5772\,\beta$$
$$\beta = \frac{L_2}{\ln(2)}$$

Conceptual rainfall-runoff model

$$PSMD(i) = SMD(i-1) + PE(i) - P(i)$$
$$Q_b(i) = Q_b(i-1) + [(1-w)EP(i) - Q_b(i-1)]/T_b$$
$$Q_s(i) = Q_s(i-1) + [wEP(i) - Q_s(i-1)]/T_s$$