Tutorial 7: Gradually varied open channel flow

Question 1

In the context of gradually varied open channel flow, the surface slope for an arbitrary channel geometry is given by

$$\frac{dh}{dx} = S_0 \frac{1 - S_f/S_0}{1 - Q^2B/(gA^3)}$$

where B is defined as $B = dA/dh$. Assuming a **wide rectangular channel**, show that the surface slope may be expressed as

$$\frac{dh}{dx} = S_0 \frac{1 - (h_n/h)^3}{1 - (h_c/h)^3} = S_0 \frac{1 - (Fr/Fr_n)^2}{1 - (Fr/Fr_c)^2}$$

Question 2

Using your lecture notes **and** a textbook (e.g. Massey), familiarise yourself with the surface profile characteristics in gradually varied open channel flow. In particular:

(i) Given that there are five types of bed slope (M, S, C, H and A), and three number categories (1, 2 and 3), one would expect 15 different classifications. Why do only 12 categories exist?

(ii) Derive the sign of the surface slope for each of the 12 categories.

(i) Type 1 Flow: " $\underbrace{\text{Gravity} = \text{Friction}}$ " for uniform flow

cannot occur on horizontal slope and adverse slope → no normal depth.

(H_1) ← doesn't exist → (A_1)

Type 2 Flow: normal depth < Depth for type 2 flow < critical depth.

however for critical slope, normal depth = critical depth.

→ C_2 doesn't exist.

(ii) assume wide rectangular section (for simplification)

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3}$$

is considered to be +ive.

is considered to be −ive.

(M_1) $\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3}$ $h_n < h$, $h_c < h$ (overall $\frac{dh}{dx}$ +ive)

(M_2) $\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3}$ $h_n > h$

Question 3

A long length of open channel at bed slope $S_0 = 0.0005$ delivers a steady flow of $3\text{m}^2/\text{s}$ (per unit width). The channel is wide and rectangular. The channel friction, using the Darcy-Weisbach model, changes from $f_1 = 0.020$ upstream of location G to $f_2 = 0.002$ downstream of location G (at $x = 0$), Figure 1.

(i) Sketch the water surface profile throughout the long channel and identify the open channel control(s).

(ii) Is a flow depth of 0.5 m possible anywhere in this channel? If so, predict the x location (upstream negative or downstream positive) of this flow depth relative to G.

(iii) Is a flow depth of 1.5 m possible anywhere in this channel? If so, predict the x location (upstream negative or downstream positive) of this flow depth relative to G.

[Ans: iii) -570 m]

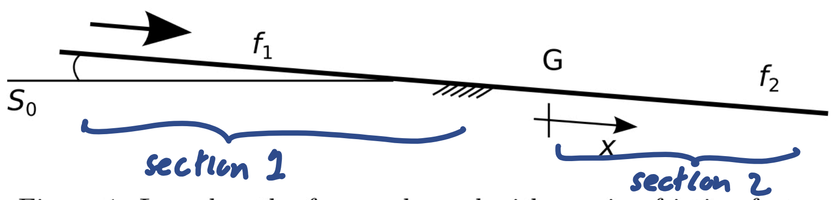


Figure 1: Long length of open channel with varying friction factor

(i) Step 1: Need to know what kind of slope this is, find h_n and h_c

for wide rectangular section, we can use the simplified formula:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.972\,\text{m}$$

$$h_n = \left(\frac{f q^2}{8 g S_0}\right)^{1/3}$$

depends on friction factor, f.

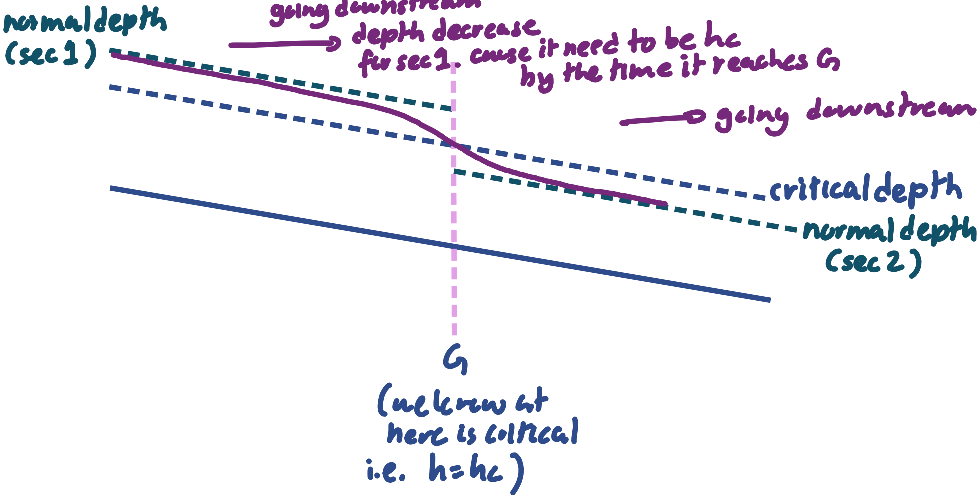
$= 1.662\,\text{m}$ (for section 1)

$= 0.771\,\text{m}$ (for section 2)

$h_{n,1} > h_c$ (mild slope for section 1)

$h_{n,2} < h_c$ (steep slope for section 2)

hints



why is it not:

cause need to be asymptote with hn!

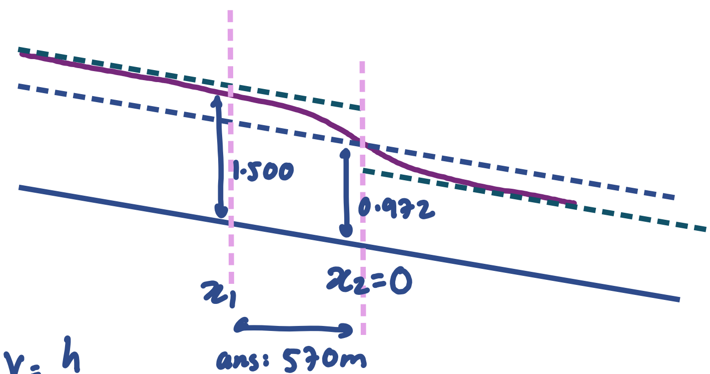
(ii) not possible.

(iii) possible, in section 1 (M_2 curve)

from data sheets:

$$\frac{S_0}{h_n}(x_2-x_1) = (\gamma_2-\gamma_1) - \left[1-\left(\frac{h_c}{h_n}\right)^3\right]\left(\Phi(\gamma_2)-\Phi(\gamma_1)\right)$$

let $x_2 = x_G = 0$, so we want to find $\underline{\underline{x_1}}$



$$\gamma = \frac{h}{h_n}$$

$$\gamma_1 = \frac{1.5}{1.662} = 0.9025 \qquad \gamma_2 = \frac{0.972}{1.662} = 0.585$$

use $h_{n,1}$ (sec 1)!
cause we are looking in section 1 only!
(one single h function represent one section only!)

$$\Phi(\gamma_1) = \frac{1}{6}\ln\frac{\gamma_1^2+\gamma_1+1}{(\gamma_1-1)^2} + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\gamma_1+1}{\sqrt{3}}\right)$$

$$= 1.532$$

$$\Phi(\gamma_2) = 0.920$$

$$x_1 = x_2 + \frac{h_{n,1}}{S_0}\left\{(\gamma_1-\gamma_2) - \left[1-\left(\frac{h_c}{h_{n,1}}\right)^3\right]\left(\Phi(\gamma_1)-\Phi(\gamma_2)\right)\right\} = \underline{\underline{-570\text{m}}}$$

Question 4

A steady water flow of $7\text{m}^2/\text{s}$ (per unit width) is established in a very long and wide rectangular channel. The constant bed slope is 0.006 and the Darcy-Weisbach friction factor is 0.015. At position P in the channel, the flow passes over a sharp-crested weir (see Figure 2). The water surface at the weir is 5 m above the channel bed.

(i) Sketch and name the water surface profile(s) upstream from the weir. Clearly mark any control sections.

(ii) Predict and plot the water surface profile depths [m] upstream from the weir. Use engineering judgement to select locations that uniquely identify the profile.

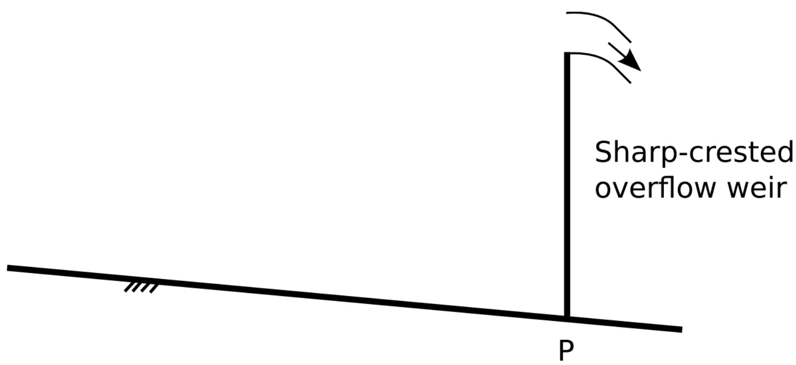
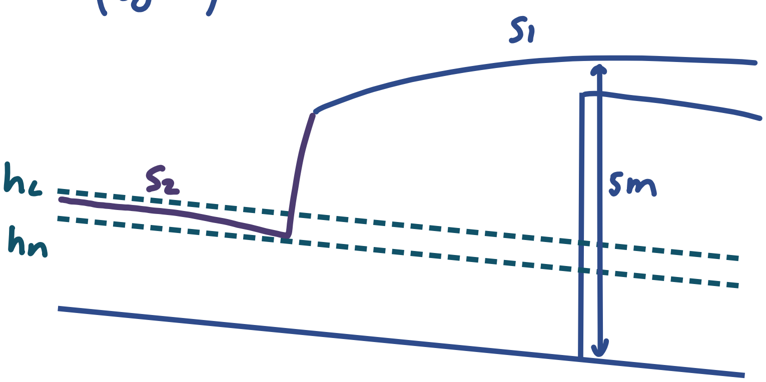


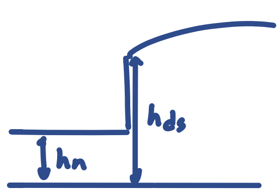
Figure 2: Rectangular channel with sharp-crested weir

$$\left.\begin{aligned} h_c &= \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{7^2}{9.81}\right)^{1/3} = 1.709 \\ h_n &= \left(\frac{fq^2}{8gS_0}\right)^{1/3} = 1.160 \end{aligned}\right\} \quad h_c > h_n \text{ (steep slope)}$$



To determine x-coordinate of hydraulic jump:

Step 1: Force balance - (find $h_{downstream}$)



$$\frac{1}{2}gh_n^2 + \frac{q^2}{h_n} = \frac{1}{2}gh_{ds}^2 + \frac{q^2}{h_{ds}}$$

$$48.8415 = 4.905h_{ds}^2 + \frac{49}{h_{ds}}$$

solve this iteratively to get
$h_{ds} = 2.411$

Step 2: Find x when $h = h_{ds} = 2.411$

since it is wide rectangular channel, we can use Method A. Direct Integration.

$$\frac{S_0}{h_n}(x_2 - x_1) = (\gamma_2 - \gamma_1) - \left[1 - \left(\frac{h_c}{h_n}\right)^3\right](\Phi(\gamma_2) - \Phi(\gamma_1))$$

$$\Phi(\gamma) = \frac{1}{6}\ln\frac{\gamma^2+\gamma+1}{(\gamma-1)^2} + \frac{1}{\sqrt{3}}\arctan\frac{2\gamma+1}{\sqrt{3}}$$

one known coordinate:

$h=5, x=0$

$\gamma = \frac{h}{h_n}$ ...

for $h = 2.41$ $x = -391.4$