Imperial College
London

4 Open Channel Flow

Open Channel Flow

Uniform Flow

- Introduction of new term like:
  water Surface Slope, dH/dx
  Energy Slope, $-S_f$
  Bed Slope, $S_0 = -dz/dx$
- Models: (find U and Q)
  Darcy-Weisbach: $U = \left(\frac{8g}{f}\right)^{1/2} R^{1/2} S^{1/2}$ derived from $\tau_0 = \rho g R S = \frac{f}{8}\rho U^2$; $\frac{1}{f} \approx -2\log\left(\frac{k_s/R}{14.84}\right)$
  Chezy's: $U = CR^{1/2}S^{1/2}$ $f = \frac{8g}{C^2}$
  Manning: $U = \frac{1}{n}R^{2/3}S^{1/2}$ $f = 8gn^2/R^{1/3}$ — given h find Q (easy)
  * All the S are S_f but also: $S_0 = S$
- How to find Normal Depth (for all flows, inc. non-uniform flow) — given Q find h (hard)

→ Non-rectangular, wide channel:
Solve iteratively, $F(h_n) = \frac{Q}{A} - \left(\frac{8g}{f}\right)^{1/2}\left(\frac{A}{P}\right)^{1/2} S^{1/2}$ for h_n

→ If rectangular, wide channel:
$h_n = \left(\frac{fq^2}{8gS_0}\right)^{1/3}$, where $q = Q/b$ $[q] = m^2/s$

Gradually Varying Flow

- How to find critical depth.
  $dE/dh = 0$ or $Fr = 1$ (If wide-rectangular channel: $h_c = \left(\frac{q_c}{g}\right)^{1/3}$)
  $E(h) = h + \frac{q^2}{2gh^2}$
  aim: want to find $h(x)$!
  $H = h + z + \frac{U^2}{2g}$
  known: $\frac{dh}{dx} = S_0(1 - S_f/S_0)(1 - Fr^2)^{-1}$
  { wide rectangular channel }
  $$\frac{dh}{dx} = S_0 \frac{1 - (h_n/h)^3}{1 - (h_c/h)^3}$$
  how to solve? Two methods!

Method A: Direct Integration.
→ solve $\frac{dh}{dx} = S_0(1 - S_f/S_0)(1 - Fr^2)^{-1}$ too hard!
→ solve $\frac{dh}{dx} = S_0 \frac{1 - (h_n/h)^3}{1 - (h_c/h)^3}$: answer given in data sheets

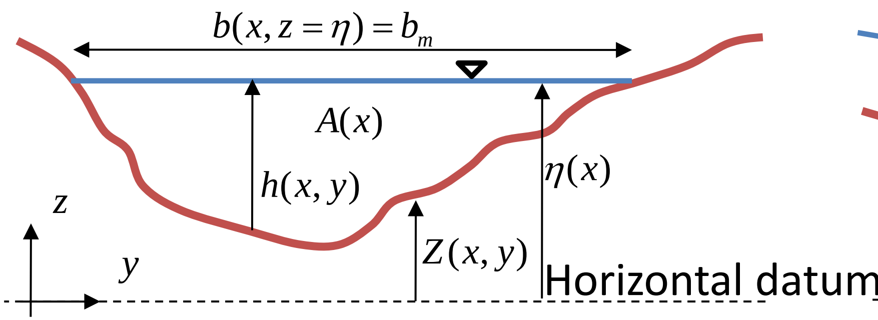
method 2: Direct-step method.
$$x_2 = x_1 + \frac{E_2 - E_1}{S_0 - \frac{1}{2}(S_{f,1} + S_{f,2})}$$
← find for all x one step by one step! $E = h + \frac{U^2}{2g}$!

- 12 types of curves

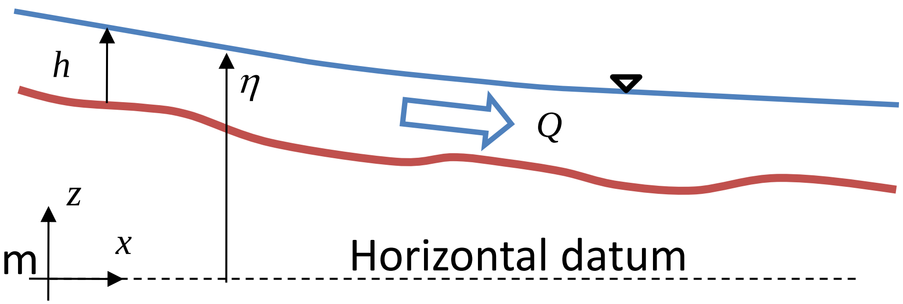
Introduction

Area	Wetted perimeter
$A=\int_0^{b_m} h\ \mathrm{d}y=\int_0^{h_m} b\ \mathrm{d}h,$	$P=\int_0^{b_m}\sqrt{1+\left(\frac{\mathrm{d}h}{\mathrm{d}y}\right)^2}\ \mathrm{d}y=\int_0^{h_m}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}h}\right)^2}\ \mathrm{d}h.$

Natural river/stream



Channel cross-section



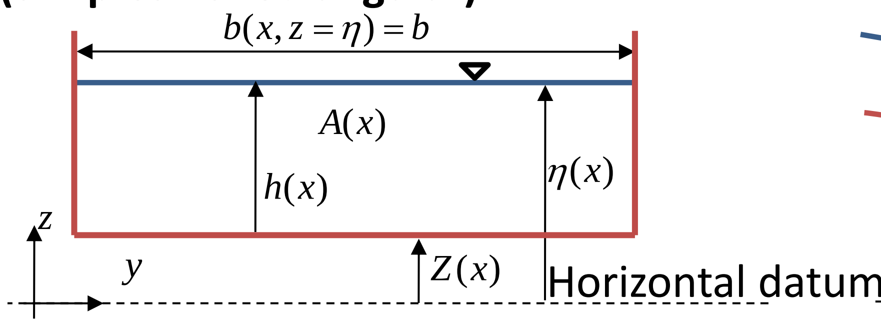
Longitudinal profile

- Flow where the uppermost boundary is a water-air interface, ‘free surface’
- Includes natural streams and rivers, but also man-made canals, sluiceways, partially filled enclosed conduits (pipes), etc…
- **Natural rivers/streams irregular geometry, *A*(*x*) & *P*(*x*)**
- Man-made canals, sluiceways often more regular
- Free-surface adds addition degree of freedom, more challenging than pipe flow
- Free surface subjected to atmospheric pressure (constant)
- Nearly horizontal flow, but gravity drives the flow

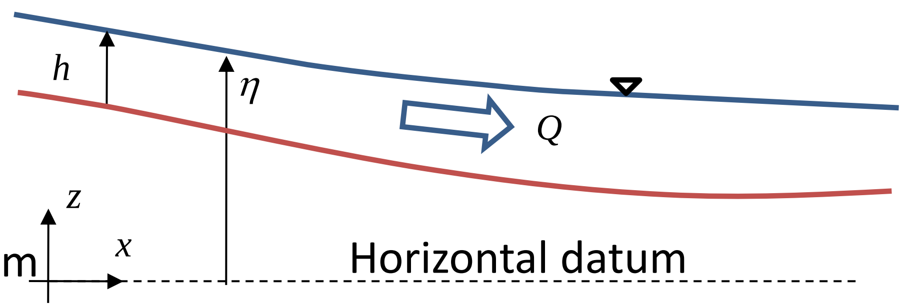
Introduction

Area	Wetted perimeter
$A = \int_0^b h \ \mathrm{d}y = \int_0^h b \ \mathrm{d}h = b\,h,$	$P = b + 2h.$

<u>Man-made canal/sluiceway</u>
(Simplest is rectangular)



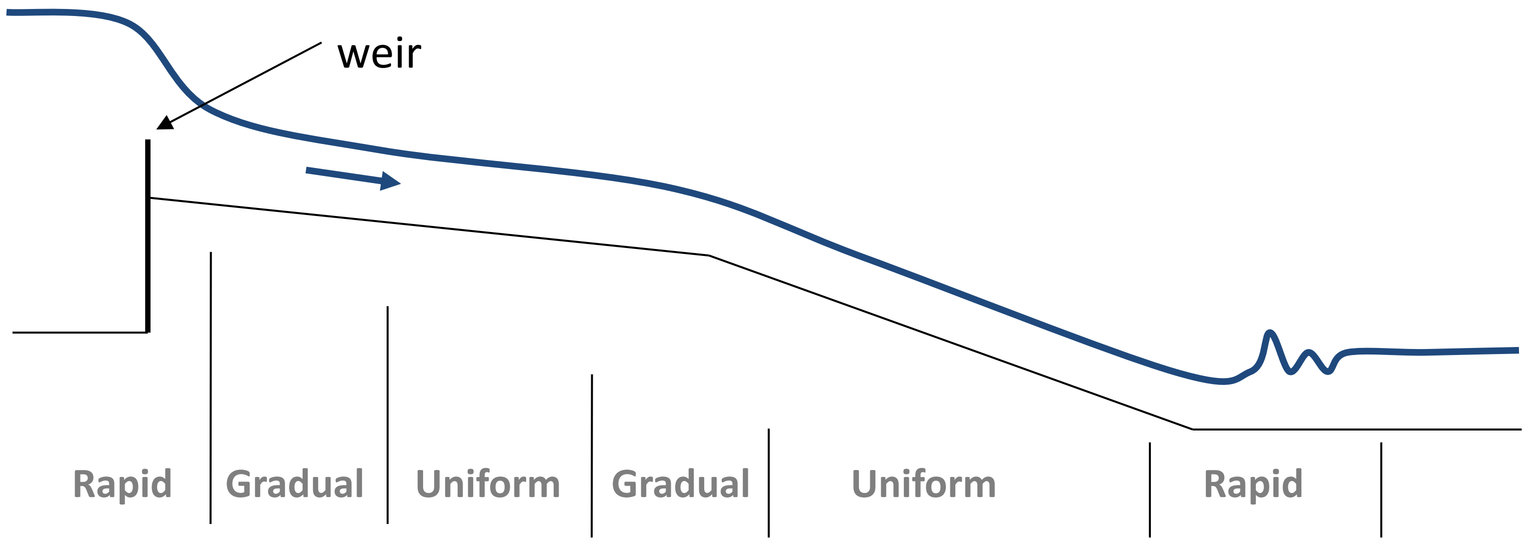
Channel cross-section



Longitudinal profile

- Flow where the uppermost boundary is a liquid-air interface, ‘free surface’
- Includes natural streams and rivers, but also man-made canals, sluiceways, partially filled enclosed conduits (pipes), etc…
- Natural rivers/streams irregular geometry, $A(x)$ & $P(x)$
- **Man-made canals, sluiceways often more regular constant for uniform flow**
- Free-surface adds addition degree of freedom, more challenging than pipe flow
- Free surface subjected to atmospheric pressure (constant)
- Nearly horizontal flow, but gravity drives the flow

Classification



Steady Flow [t] – No variation with time

- The surface profile is a function of space only, i.e. $h = h(x)$ and $\eta = \eta(x)$
- Also, $A(x)$, $P(x)$, $b(x,z=\eta)$, $Z(x)$
- AND volume flow rate Q = constant

Spatial Variations [x]

- Uniform flow - No variation with x
- Rapidly Varied Flow (RVF) - Very steep profile (theoretically a step)
- Gradually Varied Flow (GVF) - h is slowly varying with x

Fundamentals of Open Channel Flow

- Complex flow and/or geometries => utilise properties characteristic of cross-sectional-averaged quantities.

dz/dx dh/dx

- Gradients in: bed height, $\mathrm{d}Z/\mathrm{d}x$, and flow depth, $\mathrm{d}h/\mathrm{d}x$, result in horizontal gradients of pressure (at heights above datum), $p_1(z=a) \neq p_2(z=a)$, which drive the flow.
- Total energy, $H = Z + h + U^2/2g$, Energy (friction) slope $\frac{\mathrm{d}H}{dx} = -S_f$

Energy slope by dimensional analysis:

h = [L]– flow depth,
R_H = [L]– hydraulic radius,
k_s = [L]– charac. roughness,

Describes the channel

U = [L T^{-1}] – flow velocity,
ρ = [ML^{-3}]– density,
ν = [L^{2}T^{-1}] – viscosity

Describe the fluid/flow

g = [L T^{-2}] – gravity,

Forcing

- 7 independent variables (n), in
- 3 independent physical units (k), =>
- 4 (n-k) Non-Dimensional parameters.

- Inertia, $\rho U^2 A$, is induced by gravitational force, $\rho g h A$, consider the force balance: $\frac{\rho U^2 A}{\rho g h A} = \boxed{\frac{U^2}{gh} = Fr^2}$

- Friction (shear) at the walls/bed -> make use of the considered thinking done for pipe flow, e.g. slide 51, and so characterise with friction factor:

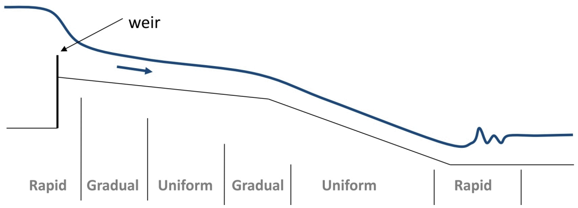
$$\boxed{f = \mathrm{f}\left(\mathrm{Re} = \frac{U 4R_H}{\nu}, \frac{k_s}{4R_H} \right)}$$

Note that the friction factor determined by 2 Non-Dimensional parameters.

- Shape of channel, think wetted perimeter per channel width: $\frac{P}{A/h} = h\frac{P}{A} = \boxed{\frac{h}{R_H}}$

4.1 Uniform Flow (Review)

Uniform and **steady open channel flow** is characterised by



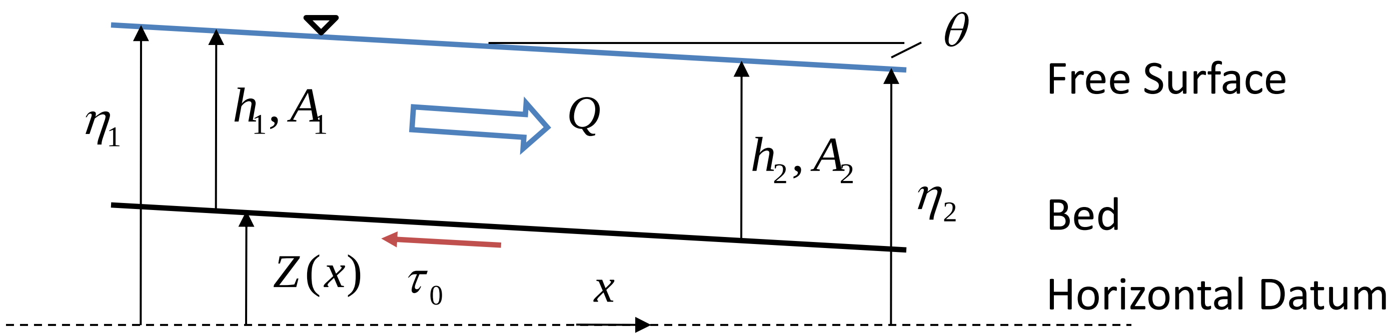
$dt=0$ $dx=0$

- **Velocity** of liquid does **not change** with **time** or **space**

steady uniform flow

- No change in cross sectional area
- The **surface** must be **parallel** to the channel **bed**
- **Nearly horizontal flow**
- Water surface **slope** ($d\eta/dx$) is **very small** ($\approx 10^{-3}$ or 1:1000), but this small **slope drives** the flow
- Generally the flow is at **high Reynolds** number where viscous effects are independent of Reynolds number (remember the RH end of the Moody diagram) and so $h_f \sim U^2$, where the constant of proportionality reflects the channel roughness
- **Forces** due to gravity **MUST be in a balance** with drag/shear forces (at the walls and bed), otherwise there would be an acceleration

Surface and Bed Slope – Uniform flow

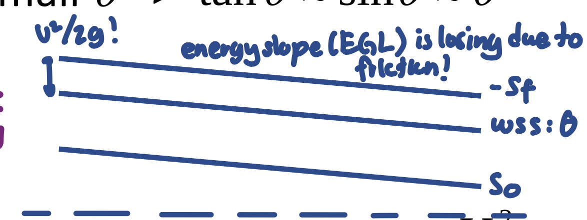


- Note the definition of η with respect to the datum (not the bed)
- Uniform flow has $h_1 = h_2 = h$ and $A_1 = A_2 = A$

Water surface slope $= -\dfrac{d\eta}{dx} = -\dfrac{\eta_2 - \eta_1}{\Delta x} = \tan\theta$, for small θ => $\tan\theta \approx \sin\theta \approx \theta$

Bed slope $\boxed{= S_0 = -\dfrac{dZ}{dx}}$

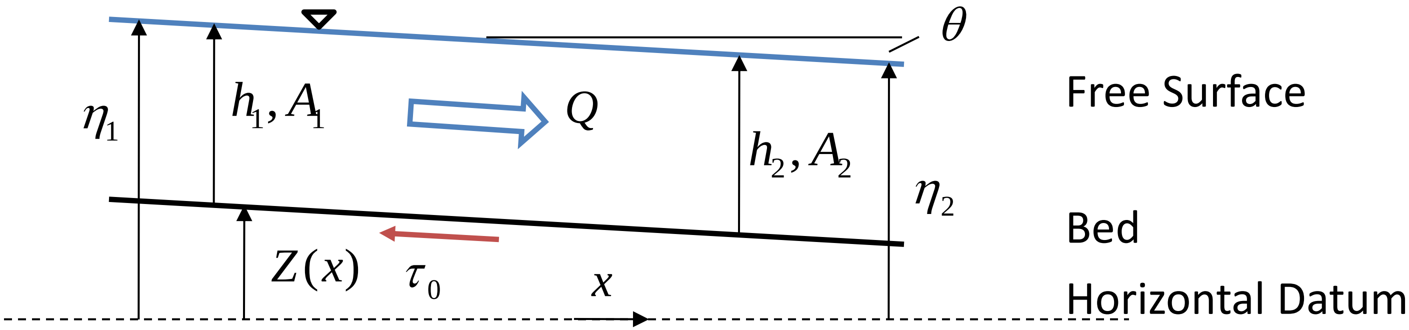
since constant $\frac{U^2}{2g}$ for uniform flow:



Energy slope $\boxed{= \dfrac{dH}{dx} = -S_f}$ where the total energy head is $H = Z + h + \dfrac{U^2}{2g}$

For uniform flow: Water surface slope = Bed slope = Energy slope

Momentum Balance at Uniform Flow



Momentum conservation gives

$$\sum F = \frac{d}{dt}(mu)$$

Since the acceleration of the flow is zero (steady flow)

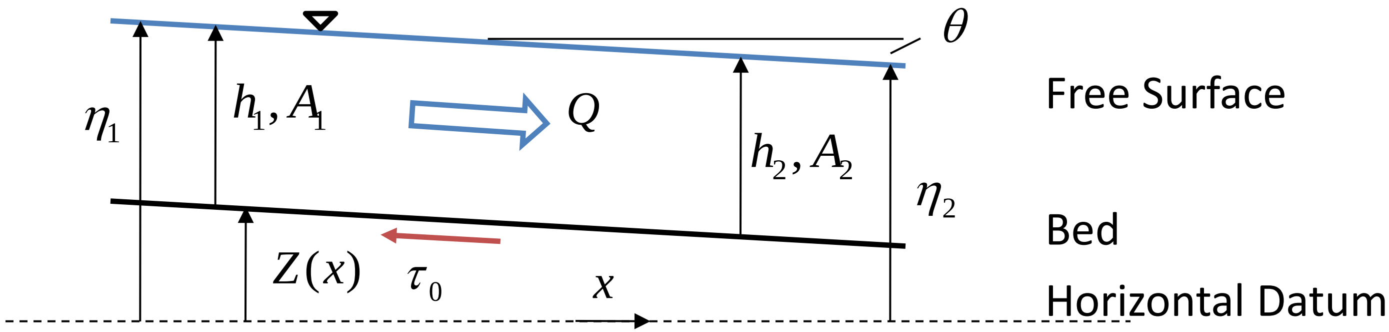
$$\sum F = 0$$

The force balance for uniform flow gives (Q = constant)

$$\left(pA\right)_1 + \rho g A\,\Delta x\,\tan\theta - \tau_0 P \Delta x / \cos\theta - \left(pA\right)_2 = 0, \qquad \text{for small } \theta,\ \cos\theta \approx 1,\ \tan\theta \approx \theta$$

where P is the wetted perimeter. Note that the wetted perimeter does not include the free surface.

Momentum Balance at Uniform Flow



Since $A_1 = A_2$ and the <u>pressure</u> is <u>hydrostatic</u> (streamlines are straight and parallel), $(pA)_1 = (pA)_2$ so the **shear force** and the **gravity force** must **balance**

$$\rho g A\,\Delta x \tan\theta - \tau_0 P \Delta x = 0$$

The boundary shear is generally written as

$$\tau_0 = \rho g R S_f$$

- $R = A/P$ is the hydraulic radius
- For uniform flow, the energy or friction slope is $S_f = S_0 = S = \tan\theta \approx \theta$

Uniform Flow – physical considerations

(i) The **Darcy-Weisbach** boundary shear (see pipe flow, e.g. slide 65) is

$\tau_0 = \frac{f}{8}\rho U^2$ **[**so $\frac{f}{8} = \frac{\tau_0}{\rho U^2}$ is the ratio of the boundary shear to the momentum flux**]**

Combining this expression with the Open Channel boundary shear: $\tau_0 = \rho g R S_f$

gives:

$$\frac{f}{8}\rho U^2 = \rho g R S_f$$

use to derive Darcy-Weisbach Model (to find U, one out of 3 methods)

uses f to find U.

both of these are given in data sheets!

Hence the water surface slope

$$S_f = \frac{f}{8}\frac{h}{R}\frac{U^2}{gh} = \frac{f}{8}\frac{h}{R}Fr^2$$

Note these are all non-dimensional variables, compare with slide 112

Ratio of inertial forces to gravitational forces

Ratio of boundary shear to mom. flux

Channel shape, wetted-perimeter per channel width

Uniform Flow models

(i) The **Darcy-Weisbach** boundary shear (see steady pipe flow, slide 54) is

$$\tau_0 = \frac{f}{8}\rho U^2$$

Combining this expression with the Open Channel boundary shear $\tau_0 = \rho g R S_f$ gives:

$$\frac{f}{8}U^2 = gRS_f \qquad \text{or:} \qquad \boxed{U = \frac{Q}{A} = \left(\frac{8g}{f}\right)^{1/2} R^{1/2} S^{1/2}}$$

new method !

uses f from Moody Diagram

with *f* taken from the Moody diagram

(ii) **Chezy** (1768) introduced a uniform flow model as

$$\boxed{U = CR^{1/2}S^{1/2}}$$

where [C] = [$m^{1/2}/s$], typically $C \approx 50 m^{1/2}/s$

(iii) **Manning** (1889) uniform flow model

$$\boxed{U = \frac{1}{n}R^{2/3}S^{1/2}}$$

where [n] = [$s/m^{1/3}$], typically $n \approx 0.02$

Open Channel models – a comparison

All three models are **empirical**. Let's examine the implications for the friction factor *f* of each model in order to make some comparison:

Colebrook-White (**Re** in the order of 10^6 for open channel flow, **turbulent**)

$$\frac{1}{f^{1/2}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\, f^{1/2}} + \frac{k_s/(4R)}{3.71}\right) \approx -2\log_{10}\left(\frac{k_s/R}{14.84}\right) = \log_{10}\left(\frac{220.2R^2}{k_s^2}\right)$$

basically f from Moody's Diagram's Turbulent flow

or $$f/8 = (1/8)\left[\log_{10}\left(\frac{220.2R^2}{k_s^2}\right)\right]^{-2}$$ [With $k_s/R \approx 0.02$, $f \approx 0.03$]

Chezy (*C* from table) implies that:

$$f/8 = g/C^2$$ [With $C \approx 50\mathrm{m}^{1/2}/\mathrm{s}$, $f \approx 0.03$]

Manning (*n* from table) implies that:

$$f/8 = gn^2/R^{1/3}$$

- Best friction factor model for open channel flows depends on a number of parameters
- A detailed investigation of friction losses is outside the scope of this course
- See Henderson (1966) and Webber (1971) for reference

Manning’s model

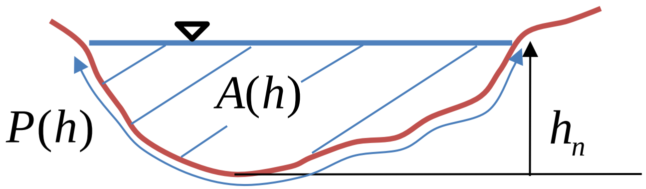
- In practice, it is often difficult to appropriately characterise the relative wall roughness for channels –making Colebrook White slightly impractical
- Manning’s model has proved popular, here are some approximate values for *n* to use in the equation:

$$U = \frac{1}{n} R^{2/3} S^{1/2}$$

Channel type	Surface material and form	Manning’s ‘*n*’
River	Earth, straight	0.02-0.025
	Earth, meandering	0.03-0.05
	Gravel, straight	0.03-0.04
	Gravel, meandering	0.04-0.08
Unlined canal	Earth, straight	0.018-0.025
	Rock, straight	0.025-0.045
Lined canal	Concrete	0.012-0.017
Lab models	Mortar	0.011-0.013
	Perspex	0.009

The depth the flow tends to in a prismatic channel, given constant Q

Normal Depth – useful concept beyond uniform flow



Natural Channel

Area	Wetted perimeter
$A=\int_0^{b_m} h\ \mathrm{d}y=\int_0^{h_n} b\ \mathrm{d}h,$	$P=\int_0^{b_m}\sqrt{1+\left(\frac{\mathrm{d}h}{\mathrm{d}y}\right)^2}\ \mathrm{d}y=\int_0^{h_n}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}h}\right)^2}\ \mathrm{d}h.$

- The normal (or uniform) depth, h_n, is the characteristic depth of the flow if it is in uniform flow, i.e. the depth if the **gravity force** is in **balance** with the **drag/shear force**
- Use an open channel flow model:

e.g. Darcy-Weisbach

$$U=\frac{Q}{A}=\left(\frac{8g}{f}\right)^{1/2}R^{1/2}S^{1/2}$$

or Manning

$$U=\frac{Q}{A}=\frac{1}{n}R^{2/3}S^{1/2}$$

Balance =>

$$\boxed{F(h_n)=0\Rightarrow Q=\left(\frac{8g}{f}\right)^{1/2}\left(\frac{A^{3/2}}{P^{1/2}}\right)S^{1/2}}$$

given in formula booklet, used to find h_n

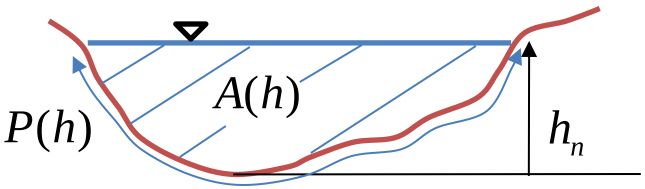
$$Q=\frac{1}{n}\left(\frac{A^{5/3}}{P^{2/3}}\right)S^{1/2}$$

basically h_n is the only unknown: For a given Q find h_n

- Note that $A=A(h)$ and $P=P(h)$
- To determine normal (uniform) height, h_n, a numerical solution required...

the reason why so complicated: A, P, S all depends on h_n!

Normal (Uniform) Depth – some physics



Natural Channel

Area	Wetted perimeter
$A = \int_0^{b_m} h \; \mathrm{d}y = \int_0^{h_n} b \; \mathrm{d}h,$	$P = \int_0^{b_m} \sqrt{1+\left(\frac{\mathrm{d}h}{\mathrm{d}y}\right)^2} \; \mathrm{d}y = \int_0^{h_n} \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}h}\right)^2} \; \mathrm{d}h.$

- The normal (or uniform) depth, h_n, is the characteristic depth of the flow if it is in uniform flow, i.e. the depth if the **gravity force** is in **balance** with the **drag/shear force**
- Consider Darcy-Weisbach, for any channel in uniform flow the depth must satisfy:

$$U = \left(\frac{8g}{f}\right)^{1/2} R^{1/2} S^{1/2} \quad \text{so} \quad U^2 = \frac{8}{f} g \, R \, S \quad \text{and} \quad U^2 = \frac{8}{f} g \, h_n \frac{R}{h_n} S$$

Rearranging gives:

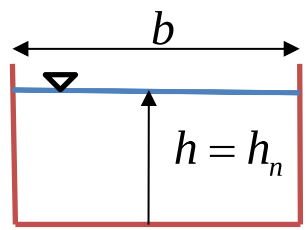
$$S = \frac{f}{8} \frac{h_n}{R} \frac{U^2}{g \, h_n} \quad \text{or} \quad Fr_n^{\;2} = S \frac{8}{f} \frac{R}{h}$$

Remember for uniform flow, bed is parallel to water surface, i.e.

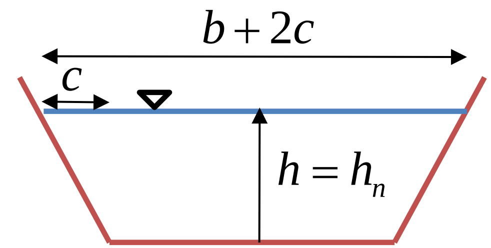
bed slope S_0 = water surface slope $\mathrm{d}h/\mathrm{d}x$ = energy slope S_f = S

You can now complete: Tutorial 6

Normal Depth - simple man-made cases



Rectangular Channel



Trapezium channel

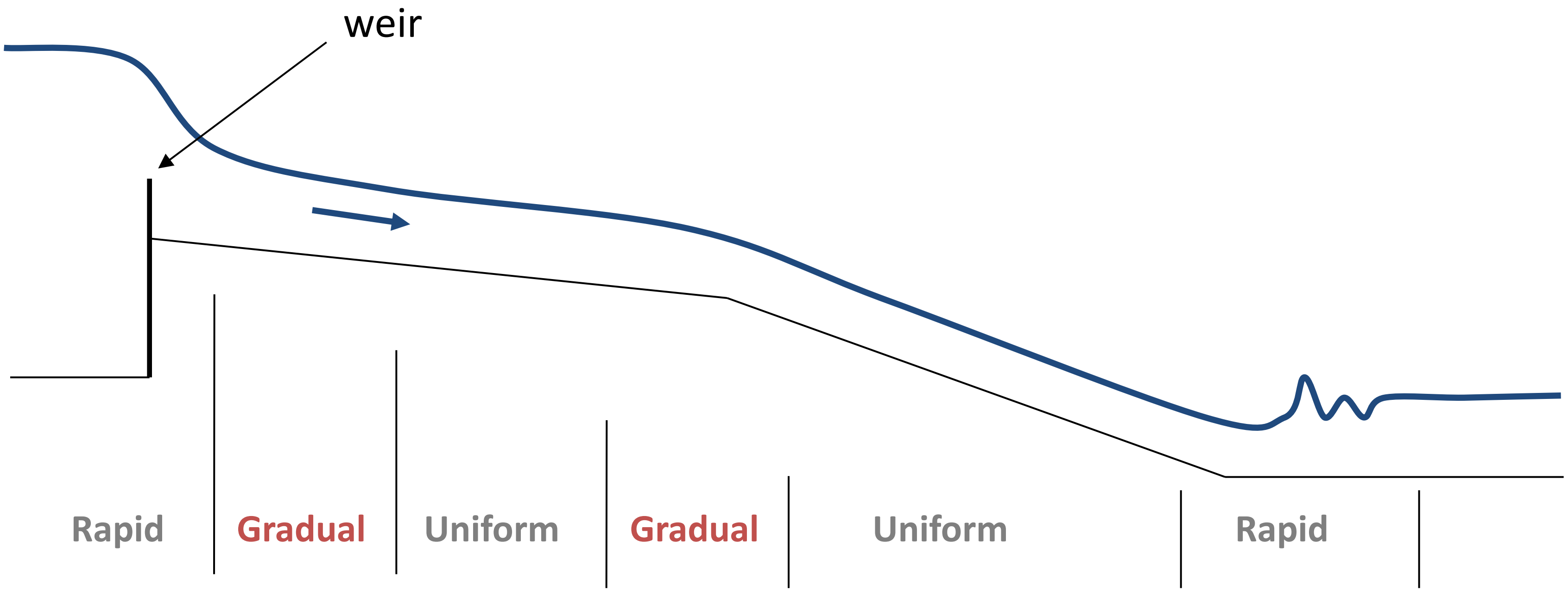
- The normal (or uniform) depth, h_n, is the characteristic depth of the flow if it is in uniform flow, i.e. the depth if the **gravity force** is in **balance** with the **drag/shear force**
- Use, for example, Manning: $U = \frac{1}{n} R^{2/3} S^{1/2}$ where (slide 119) $f/8g = n^2 / R^{1/3}$
- Consider the rectangular channel: $A = b\,h_{\mathrm{n}},\ P = b + 2h_{\mathrm{n}},$

$$F(h_n) = 0 \Rightarrow Q = \frac{1}{n}\left(\frac{(b\,h_n)^{5/3}}{(b+2h_n)^{2/3}}\right) S^{1/2} = \left(\frac{8g}{f}\right)^{1/2} \left(\frac{(b\,h_n)^{3/2}}{(b+2h_n)^{1/2}}\right) S^{1/2}$$

$$F(h_n) = 0 \Rightarrow Fr_n = \frac{Q/(bh_n)}{\sqrt{g\,h_n}} = \frac{1}{n}\frac{b^{2/3} h_n^{1/6}}{g^{1/2}(b+2h_n)^{2/3}}\sqrt{S} = \sqrt{\frac{8}{f}\frac{b}{(b+2h_n)}S}$$

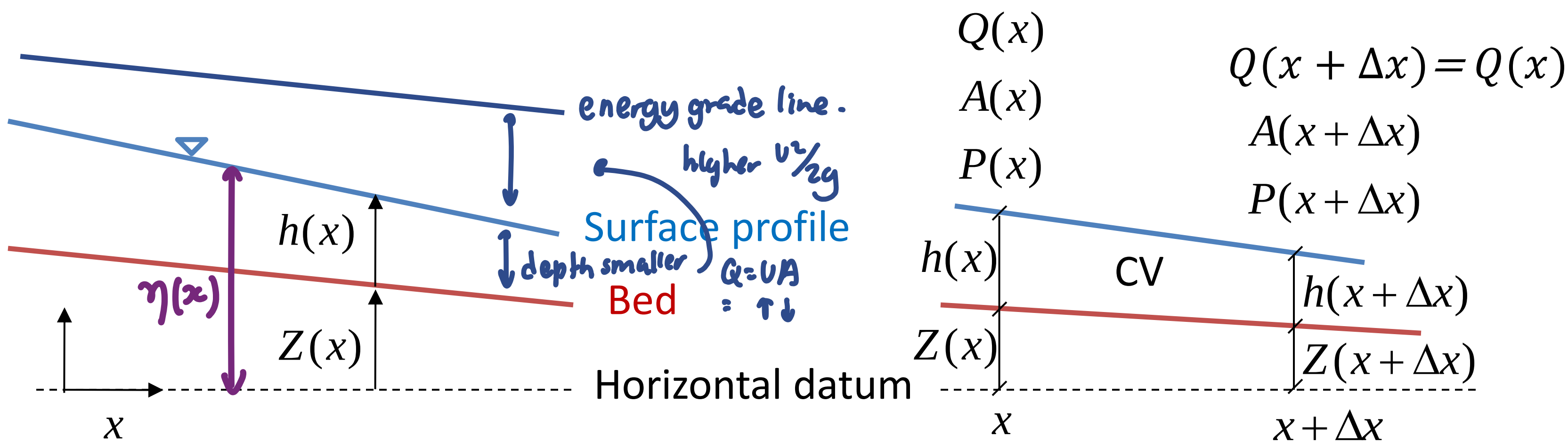
- **Implicit** algebraic equation
- Solution for h_n by trial-and-error, Newton-Raphson ...

4.2 Introduction to Gradually Varied Flow



- Completely uniform flow is rarely observed in nature
- In many channels, the shape and size of the cross section, and slope of the bed, vary considerably
- Changes in depth and velocity occur over relatively long lengths of channel
- Typical control volume lengths are more than 100 channel depths
- Channel resistance and slope play dominant roles and must be considered

Introduction



- The flow is no longer uniform
- The water surface profile, $h(x)$, is **not parallel** to the bottom profile, $Z(x)$
- The dependent variable $h(x)$ is **slowly varying** with x
- Also the area (A) and the wetted perimeter (P) are slowly varying with x
- However, assuming **steady flow conditions**
  - gives **no** accumulation of mass within the control volume
  - so the **discharge** Q is **constant**!

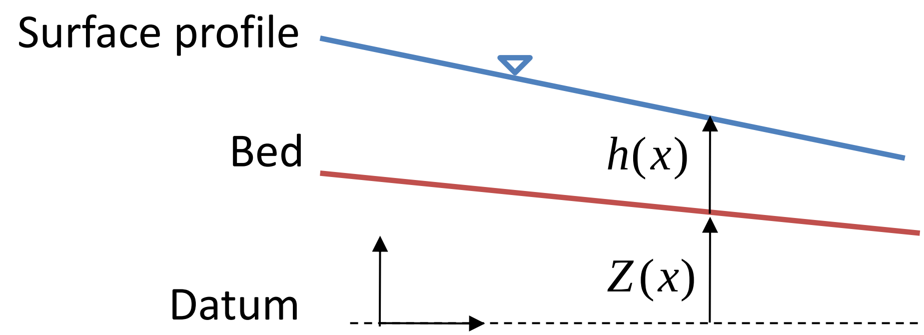
$$h = h(x) \qquad A = A(x) \qquad P = P(x) \qquad Q = \text{constant}$$

☆ OUR AIM IS TO FIND $\frac{dh}{dx}$, knowing gradient + one coordinate allows us to find coordinate of all points

Gradually Varied Flow (GVF) Equations

The total energy head is

$$H = Z + h + \frac{U^2}{2g}$$



Differentiating the total head with respect to the horizontal coordinate *x*

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{U^2}{2g}\right)$$

Using earlier definitions (slide 98): energy (or friction) slope is: $-S_f = \frac{dH}{dx}$

and bed slope is $-S_0 = \frac{dZ}{dx}$

Energy slope

$$-S_f = -S_0 + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{U^2}{2g}\right)$$

Bed slope

rate of change of water depth.

Gradually Varied Flow Equations 2

Our interest lies in finding an expression for the surface profile $h(x)$

$$\frac{dh}{dx} = S_0 - S_f - \frac{d}{dx}\left(\frac{U^2}{2g}\right)$$

We can determine $h(x)$ by knowing $h(x_0)$ (at any point x_0) **and knowing $\frac{dh}{dx}$**

The velocity head term may be rewritten as follows

$$\frac{d}{dx}\left(\frac{U^2}{2g}\right) = \frac{d}{dx}\left(\frac{1}{2g}\frac{Q^2}{A^2}\right) = \frac{dA}{dx}\frac{d}{dA}\left(\frac{1}{2g}\frac{Q^2}{A^2}\right) = -\frac{dA}{dx}\frac{Q^2}{gA^3} = -\frac{dA}{dh}\frac{dh}{dx}\frac{Q^2}{gA^3}$$

dA/*dh* is rate of change of flow area with depth, i.e. the flow width. Note that these are depth averaged Equations so we are interested in cross-sectional averaged values (slide 98) of *dA*/*dh*, i.e. 'mean' width, denote *B*, so *B* = *dA*/*dh*. Hence:

$$-\frac{dA}{dh}\frac{dh}{dx}\frac{Q^2}{gA^3} = -B\frac{dh}{dx}\frac{Q^2}{gA^3} \boxed{= -\frac{dh}{dx}\frac{BQ^2}{gA^3}}$$

$$\frac{d}{dx}\left(\frac{U^2}{2g}\right) = -\frac{dh}{dx}\frac{BQ^2}{gA^3} = -\frac{dh}{dx}\frac{A}{h}\frac{Q^2}{gA^3}\text{; note } U = \frac{Q}{A} \quad \text{so now} \quad \boxed{\frac{d}{dx}\left(\frac{U^2}{2g}\right) = -\frac{dh}{dx}\frac{U^2}{gh} = -\frac{dh}{dx}Fr^2}$$

Aside: Strictly $B = \frac{1}{h}\int_0^h b \; \mathrm{d}h$, and remember (slide 109) $A = \int_0^h b \; \mathrm{d}h$,

so for rectangular channels: $A = B\,h = b\,h$

Gradually Varied Flow Equations 3

Our interest lies in finding an expression for the surface profile $h(x)$

$$\frac{dh}{dx} = S_0 - S_f - \frac{d}{dx}\left(\frac{U^2}{2g}\right) = S_0 - S_f + \frac{dh}{dx}\left(\frac{Q^2 B}{gA^3}\right) \quad \textbf{or} \quad \frac{dh}{dx} = S_0 - S_f + \frac{dh}{dx} Fr^2$$

So

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Q^2 B / gA^3} \quad \textbf{or} \quad \frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$

Write as

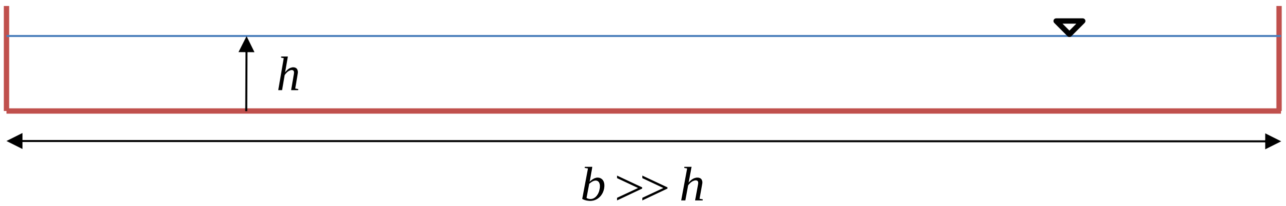
$$\frac{dh}{dx} = S_0 \frac{1 - S_f / S_0}{1 - Q^2 B / gA^3} \quad \textbf{or} \quad \frac{dh}{dx} = S_0 \frac{1 - S_f / S_0}{1 - Fr^2}$$

Can be useful to think

$$\frac{dh}{dx} = S_0 \left(1 - S_f / S_0\right)\left(1 - Q^2 B / gA^3\right)^{-1} \quad \textbf{or} \quad \frac{dh}{dx} = S_0 \left(1 - S_f / S_0\right)\left(1 - Fr^2\right)^{-1}$$

This equation is valid for an arbitrary channel geometry, solution of which requires certain procedures depending on the flow / channel conditions (considered in next part).

Defining the Critical Depth: Use Example of Wide Rectangular Channel



$$A = bh$$
$$P = b + 2h$$
$$R = A/P$$
$$= bh/(b+2h) \approx h$$

- Volume flow rate **through** section / unit width $= q = Q/b = U\ h$
- Mass flux **through** section / unit width $= \rho\, q$
- $U = q/h$

•**Specific energy**

$$E(h) = h + \frac{q^2}{2gh^2}$$

(gravitational head)

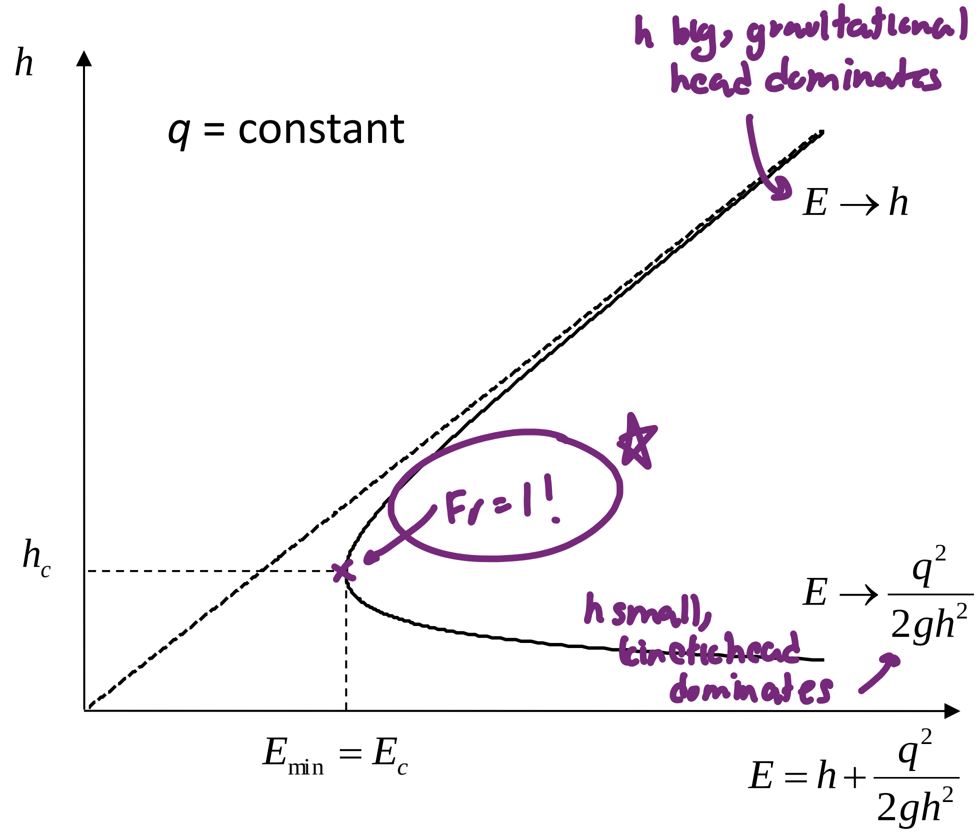
piezometric head
(no $P/\rho g$ for open chanel flow)

kinetic head .

h usually $= \frac{p}{\rho g} + z$
but for open channel flow,
$P/\rho g = 0$ and hence $h = z$ only.

Specific Energy Function and Critical Depth

- The resulting **cubic equation** may have **two positive roots**
- As a result, the solution for *h* is **not unique** and additional, problem specific information, is required
- Consider *E* as a function of *h*, with *q* specified as a constant
- Since *E* has **two positive asymptotes**, *E* must have a **minimum** for some value of *h*
- The value of *h* that makes *E*(*h*) a minimum is called **critical depth**



(derivation not important)

$$\frac{dE}{dh} = 0 = 1 - \frac{q^2}{gh_c^3}, \quad \boxed{h_c = \left(\frac{q^2}{g}\right)^{1/3}} \Rightarrow \frac{(q/h_c)^2}{gh_c} = \frac{U^2}{gh_c} = 1 = Fr_c^2$$

formula for RECTANGULAR channel only! (wide or not wide)

$$\boxed{Fr \equiv \frac{U}{(gh)^{1/2}} \equiv \frac{q/h}{(gh)^{1/2}} \quad \text{and for } h = h_c, \; Fr_c = 1}$$

Towards classifying water surface slope

- Remember our two important flow depths:

- Normal (or uniform) depth h_n – the depth of the flow if gravitational forces balance drag/shear forces
- Critical depth h_c – the depth of the flow which results in the minimum energy state for the flow, i.e. $Fr = 1$.

1. if $S_f \to S_0$, $\frac{dh}{dx} = 0$ (uniform flow!)
2. if $Fr = 1$, $\frac{dh}{dx} \to \infty$ (hydraulic jump)

- Expression for WS gradients $\frac{dh}{dx} = S_0 (1 - S_f / S_0)(1 - Fr^2)^{-1}$

Water Surface (WS) Slope Classification

- Introduce a system of **logical classifications** on the different types of water surface (WS) profiles
- Classification system valid for all channel types, not limited to wide rectangular channel
- **Primary** classification according to the **slope of the bed**
- Slope can be:
  - Mild (M) – slope less than critical slope
  - Critical (C) – slope such that uniform flow is critical
  - Steep (S) – slope greater than critical slope
  - Horizontal (H) – slope is horizontal, or
  - Adverse (A) – slope is negative, i.e. up hill

$$\frac{dh}{dx} = S_0\left(1 - S_f / S_0\right)\left(1 - Fr^2\right)^{-1}$$

Water Surface (WS) Slope Classification

- Introduce a system of **logical classifications** on the different types of water surface (WS) profiles
- Further **classification** according to the **depth of the flow:**

i. Depth **greater** than both the **normal** and the **critical** depth $\rightarrow$ Type 1

ii. Depth **between normal** and **critical** depth $\rightarrow$ Type 2

iii. Depth **less** than both the **normal** and the **critical** depth $\rightarrow$ Type 3

$$\frac{dh}{dx} = S_0\left(1 - S_f / S_0\right)\left(1 - Fr^2\right)^{-1}$$

Water Surface (WS) Profile Types

- Given:
    - five classes of slope (letters) and
    - three types of flow (numbered)

    => 15 possible classifications
- But… only **12** of those are realistic
- Normal/uniform flow not possible on adverse or zero slope
- For critical slope, $h_0 = h_c$, and C_2 cannot occur

- To obtain water surface profile, considerer the flow domain in distinct parts then join to solve over the required domain

Note: The following slides give **representative examples**, but the list is by no means complete. All **slopes** in the figures that follow are **greatly exaggerated**!

NDL - Normal Depth Line
CDL - Critical Depth Line

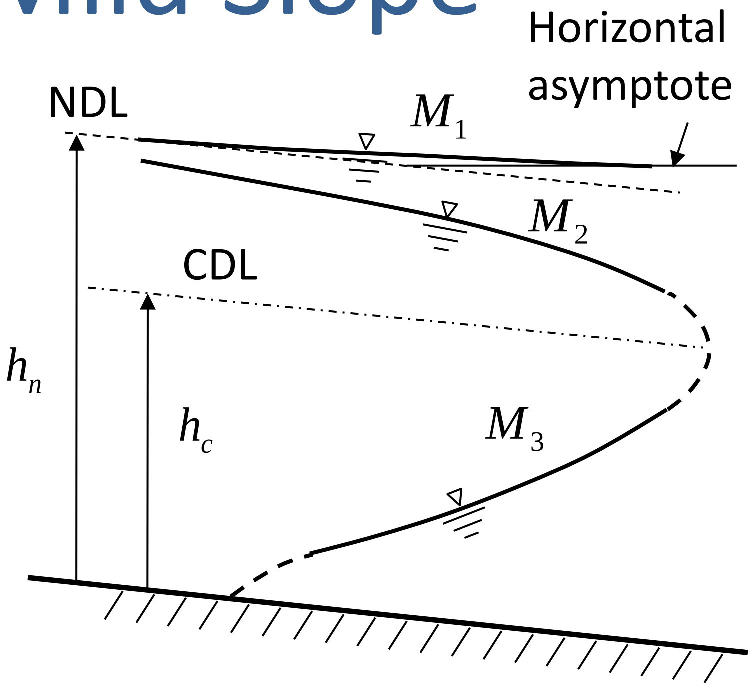
super important things to note:
- type 1 depth always increases downstream, type 2 decreases and type 3 increases.
- type 1 and type 3 different is type 1 is subcritical and type 3 is supercritical!!

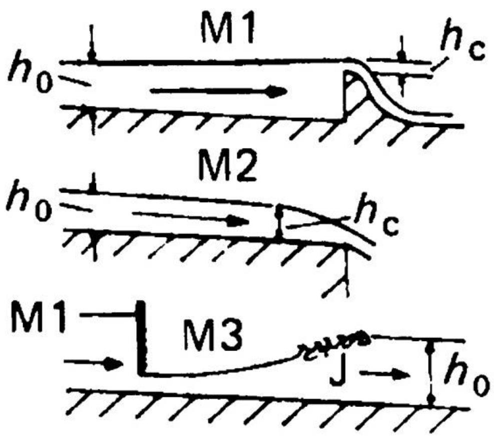
Water Surface (WS) Slope Classification

Slope Type	Slope Notation	Froude Number	Surface Shape
$S_0 < S_{0c}$	Mild (M)	Fr < 1	M_1
$h_n > h_c$	subcritical →	Fr < 1	M_2
		Fr > 1	M_3
$S_0 = S_{0c}$	Critical (C)	Fr < 1	C_1
		Fr > 1	C_3
$S_0 > S_{0c}$	Steep (S)	Fr < 1	S_1
$h_c < h_n$	supercritical →	Fr > 1	S_2
		Fr > 1	S_3
$S_0 = 0$	Horizontal (H)	Fr < 1	H_2
		Fr > 1	H_3
$S_0 < 0$	Adverse (A)	Fr < 1	A_2
		Fr > 1	A_3

Mild Slope

∴ normal depth > critical depth when uniform flow occurs subcritically

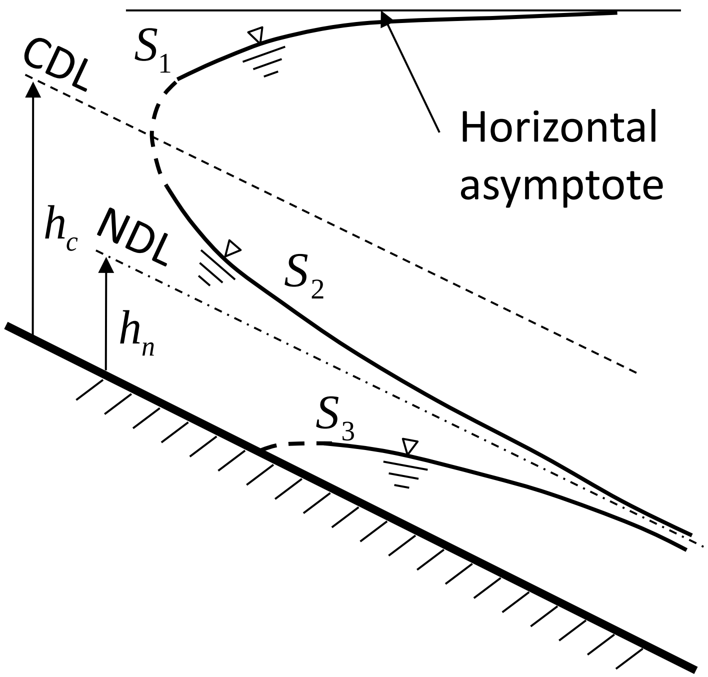


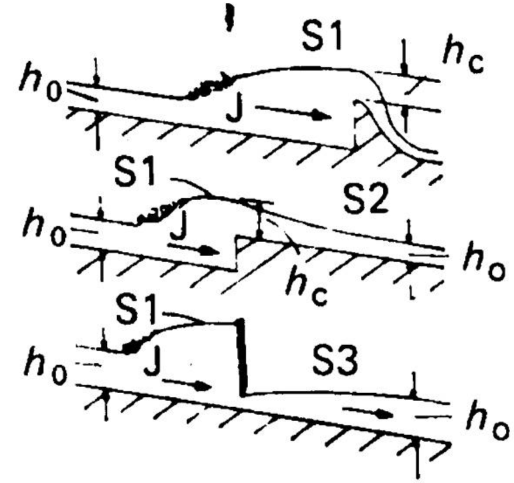


- M_1 profile is extremely common
- Control structures (weirs and sluices) and natural features (narrows and bends) often produce a backwater effect reaching for several km upstream (M_1)
- M_2 profile occurs when depth is reduced (enlargement, free outfall)
- M_3 profile is found downstream of a change in slope from steep to mild, or at the exit of a sluice

Steep Slope

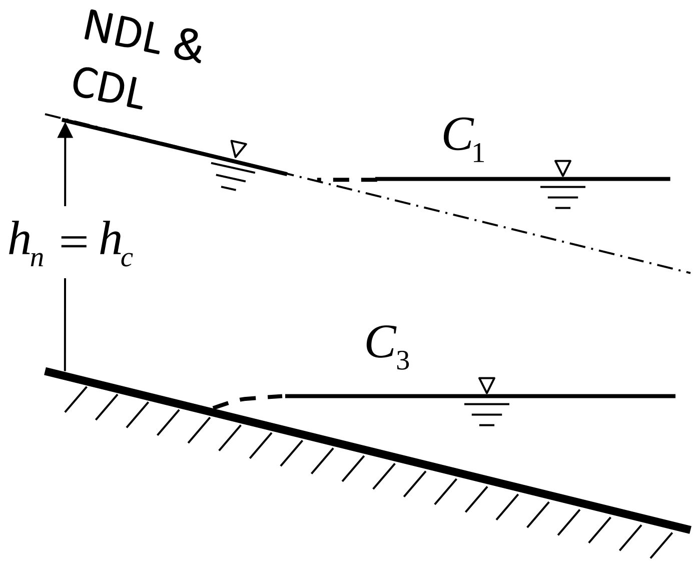
⇒ normal depth < critical depth
when uniform flow occurs supercritically.

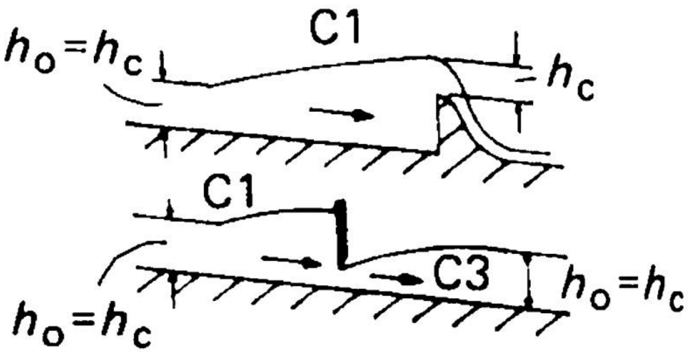




- S_1 profile is produced by control structures (dam or sluice) on a steep slope
- S_1 commences with a hydraulic jump
- S_2 found at the entrance to a steep slope, or a change of slope from mild to steep
- S_3 profile is produced downstream of a sluice gate on a steep slope

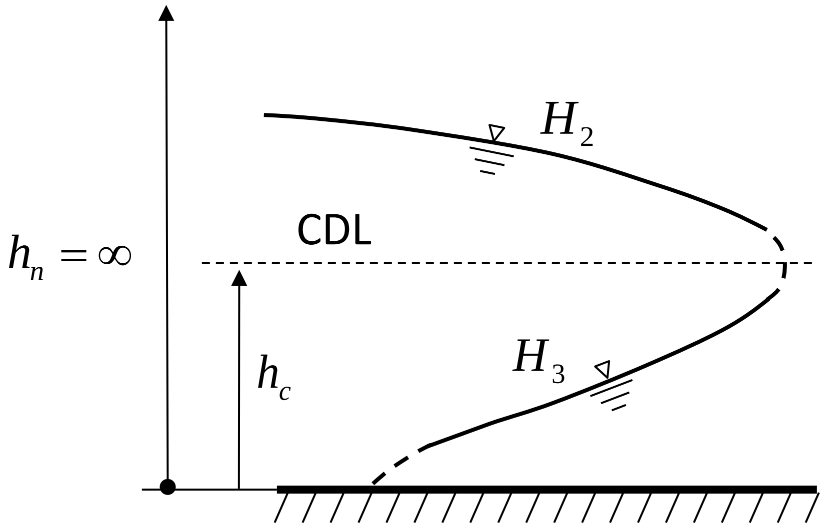
Critical Slope

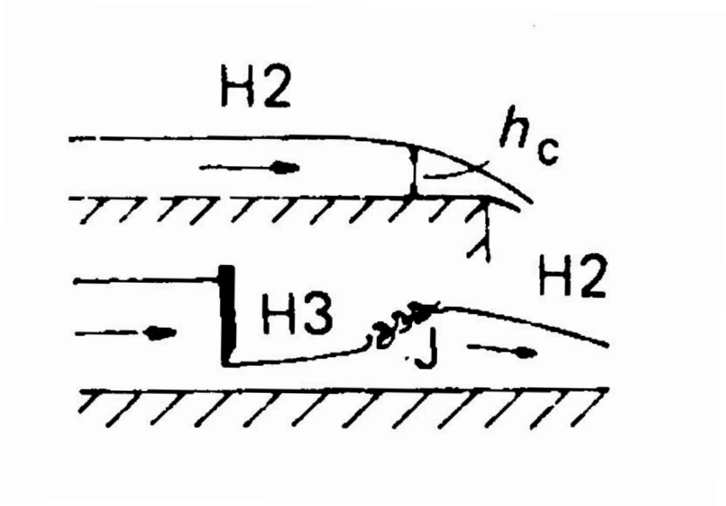




- Normal and critical depth coincide, C_2 does not exist
- Instability of critical depth condition leads to surface undulation

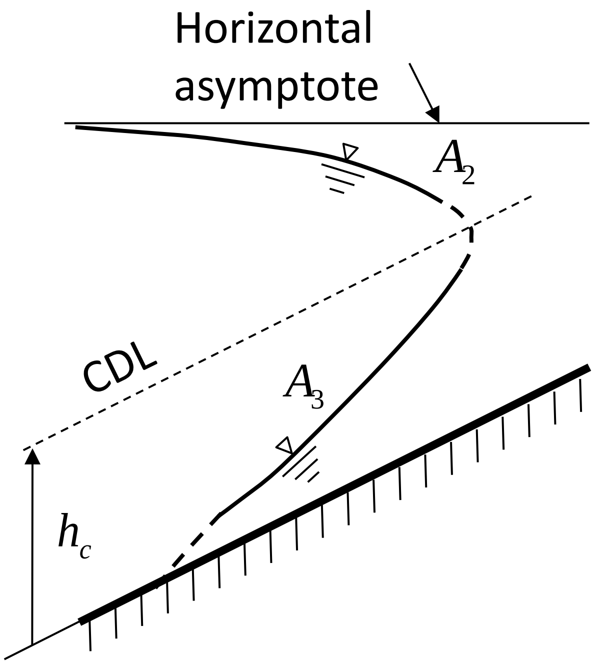
Horizontal Bed

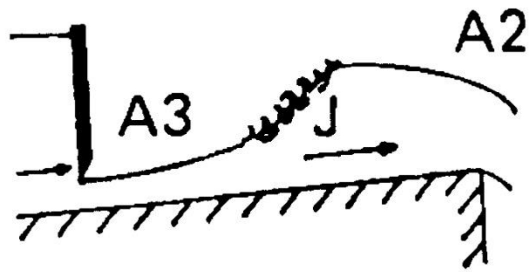




- Horizontal bed is the lower limit of the mild slope
- For uniform flow, bed shear and gravity must exactly balance, but the component of gravity that drives the flow is zero
- The normal depth is infinity, so that H_1 does not exist

Adverse Slope

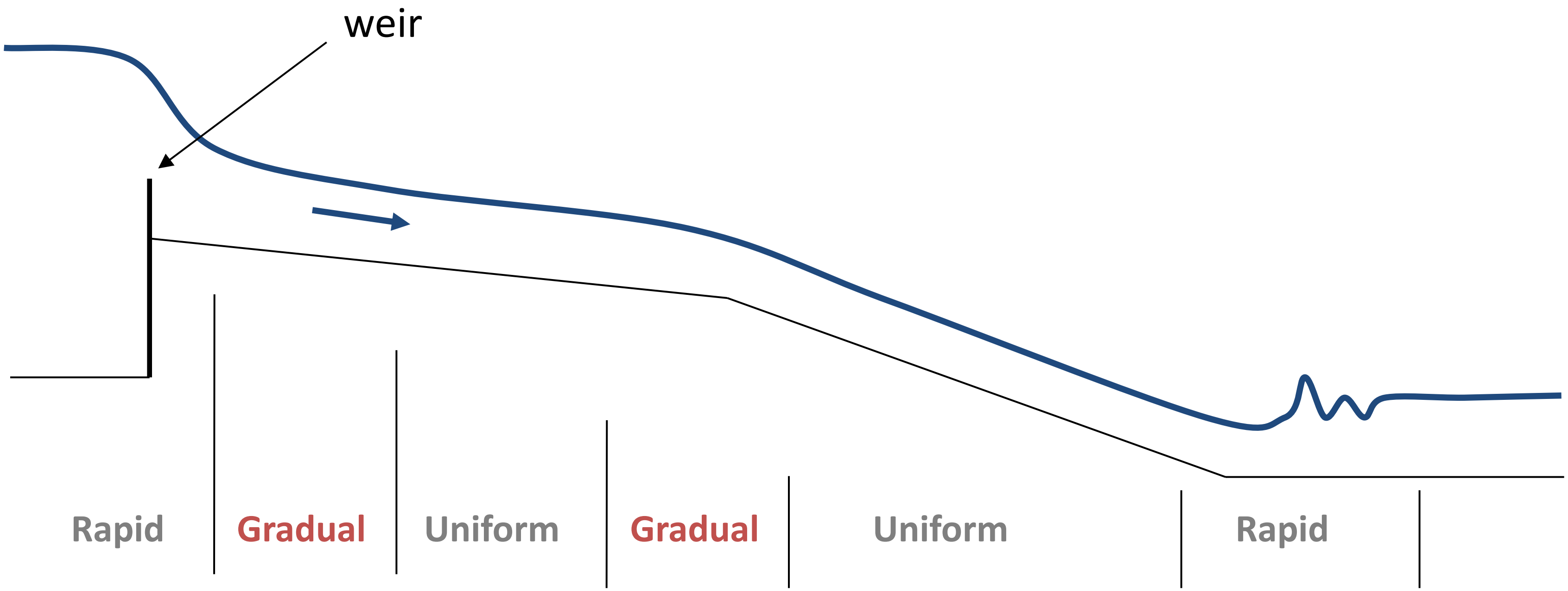




- Normal (uniform) flow cannot establish
- Adverse bed slopes are very rare
- Profiles are extremely short

basically solving for h(x) from having $\frac{dh}{dx}$ (differential equation)

4.3 Calculation Approaches for GVF



- Various calculation approaches exist
- Two approaches will be introduced here, but a number of others also exist
- Approaches ranging from direct integration to trial-and-error

one downside compared to approach B is if S_0 changed, you have to break into segments and do multiple time Approach A.

Summary of method A:
- can be used for any channel, but the integration is too hard → only possible one is wide rectangular cause simplification can be made.

Approach A: Direct Integration

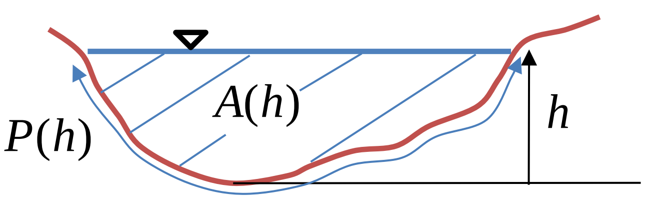
B and A are function of h

$$\underbrace{\frac{dh}{dx} = S_0\left(1 - S_f / S_0\right)\left(1 - Q^2 B / gA^3\right)^{-1}}_{\text{too hard to solve for } h!} \quad \textbf{or} \quad \frac{dh}{dx} = S_0\left(1 - S_f / S_0\right)\left(1 - Fr^2\right)^{-1}$$

simplification →

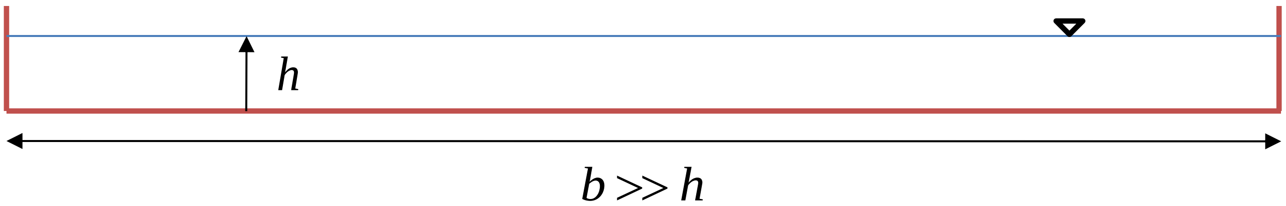
$$\boxed{\frac{dh}{dx} = S_0 \frac{1-\left(h_n / h\right)^3}{1-\left(h_c / h\right)^3} = S_0 \frac{1-\left(Fr^2 / Fr_n^2\right)}{1-\left(Fr^2 / Fr_c^2\right)}}$$

- Integrating dh/dx may seem trivial
- However, is implicit since both B and A depend on h
- Direct analytical integration not commonly possible, unless some further simplifications are made only applicable for wide rectangular channel!
- Leads to Bresse integration for rectangular channel. Theory presented here, with example in Tutorial sheet.



Natural Channel

Integration for Special Case: Wide Rectangular Channel



$$A = bh$$
$$P = b + 2h$$
$$R = A/P$$
$$= bh/(b+2h) \approx h$$

The **variation** in "flow cross-sectional area" for a **constant width** rectangular channel is

$$B = \frac{dA}{dh} = \frac{d}{dh}(bh) = b$$

For **normal (uniform) depth**, i.e. uniform flow (e.g. slide 117/8), **bed slope S_0 = water surface slope S_f** for a rectangular channel geometry (slide 121):

$$F(h_n) = 0 \Rightarrow Q = \left(\frac{8g}{f}\right)^{1/2} \left(\frac{(b\,h_n)^{3/2}}{(b+2h_n)^{1/2}}\right) S_0^{1/2} \quad \text{or} \quad Fr_n = \frac{Q/(b\,h_n)}{\sqrt{g\,h_n}} = \sqrt{\frac{8}{f}\frac{b}{b+2h_n} S_0}$$

For a **wide rectangular channel**, this expression simplifies to

$$\boxed{h_n = \left(\frac{fq^2}{8gS_0}\right)^{1/3}} \qquad \text{or} \quad Fr_n = \sqrt{\frac{8}{f} S_0}$$

not given in data sheets!

Remember that the **normal depth** is the **flow depth** for **uniform flow**!

Wide Rectangular Channel - Surface Slope

As derived earlier (slide 130) , the **critical depth** ($Fr = Fr_c = 1$) for a wide rectangular channel is:

$$\boxed{h_c = \left(\frac{q^2}{g}\right)^{1/3}}$$

works for all rectangular channel (non-wide too)

Note that: $$Fr_c^2 = \frac{q^2}{gh_c^3} = 1$$

Introducing the normal depth and the critical depth, the GVF equation becomes (see Tutorial, which we can go through on the board during the tutorial if you like)

$$\boxed{\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3} = S_0 \frac{1-\left(Fr^2/Fr_n^2\right)}{1-\left(Fr^2/Fr_c^2\right)}}$$

given in data sheets in exam!
just have to remember this only works for wide rectangular channel!

- Surface profile with a variety of forms, depending on control structures and bed slopes
- Depending on the magnitude of the quantities involved, *dh/dx* may be positive or negative
- Ordinary differential equation (ODE) that can be solved for with known initial conditions

Water Surface Profile Prediction

Analytic integration of the GVF momentum equation for a **wide rectangular channel** with $S_0 \neq 0$ gives (NOTE: $\gamma \equiv \dfrac{h}{h_n}$ is a dummy variable of integration)

$$\int_{x_1}^{x_2} S_0 dx = \int_{h_1}^{h_2} \frac{1-(h_c/h)^3}{1-(h_n/h)^3}\,\mathrm{d}h$$

or

$$\int_{x_1}^{x_2} \frac{S_0}{h_n} dx = \int_{\gamma_1}^{\gamma_2} \left\{1-\left[1-\left(\frac{h_c}{h_n}\right)^3\right]\frac{1}{1-\gamma^3}\right\} d\gamma \qquad \text{where: } \gamma \equiv \frac{h}{h_n}$$

which evaluates to

$$\frac{S_0}{h_n}(x_2-x_1) = (\gamma_2-\gamma_1) - \left[1-\left(\frac{h_c}{h_n}\right)^3\right]\left(\Phi(\gamma_2)-\Phi(\gamma_1)\right)$$

after solving $\frac{dh}{dx} = S_0 \frac{1-(h_c/h)^3}{1-(h_n/h)^3}$

given in data sheets in exam.

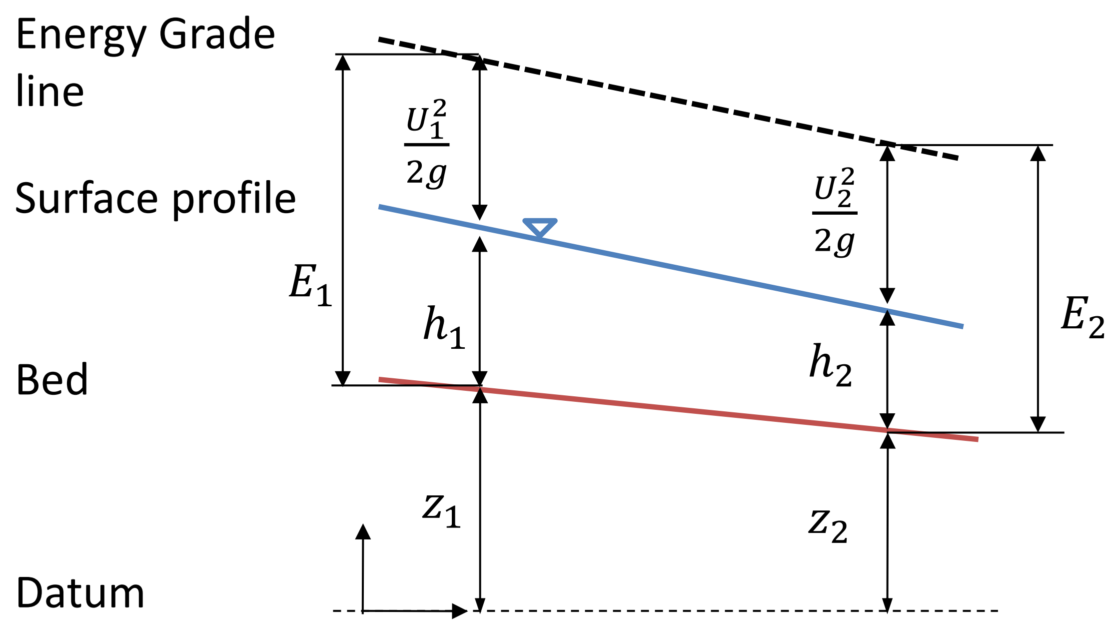
where the Bresse function is

$$\Phi(\gamma) = \frac{1}{6}\ln\frac{\gamma^2+\gamma+1}{(\gamma-1)^2} + \frac{1}{\sqrt{3}}\arctan\frac{2\gamma+1}{\sqrt{3}}$$

Some numbers to check your coding: $\Phi(0.2) = 0.5027$ $\quad \Phi(1.2) = 1.3867$

Approach B: Direct-Step Method

find Δx such that h increase or decrease each step by Δh



Bernoulli gives:

$$H_1 = \boxed{z_1 + h_1 + \frac{U_1^2}{2g}} = H_2 + h_f = \boxed{z_2 + h_2 + \frac{U_2^2}{2g} + h_f}$$

With S_0 as the channel slope : $z_2 = z_1 - S_0(x_2 - x_1)$

$$S_0(x_2 - x_1) + h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} + h_f$$

Approach B: Direct-Step Method

$$S_0(x_2 - x_1) + h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} + h_f$$

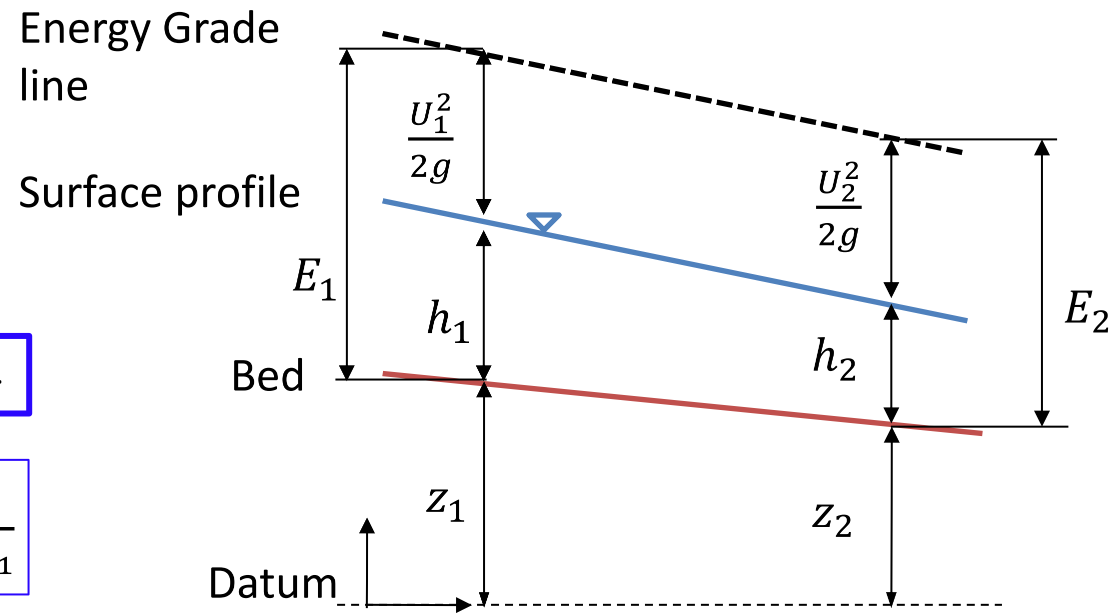
Furthermore, the specific Energy E is

$$E_1 = h_1 + \frac{U_1^2}{2g}$$

$$E_2 = h_2 + \frac{U_2^2}{2g}$$

From slide 114

$$\boxed{\frac{dH}{dx} = \frac{H_2 - H_1}{x_2 - x_1} = -S_f = \frac{-h_f}{x_2 - x_1}}$$



$$S_0(x_2 - x_1) + E_1 = E_2 + S_f(x_2 - x_1) \rightarrow x_2 = x_1 + \frac{E_2 - E_1}{S_0 - S_f}$$

What is S_f?

Approach B: Direct-Step Method

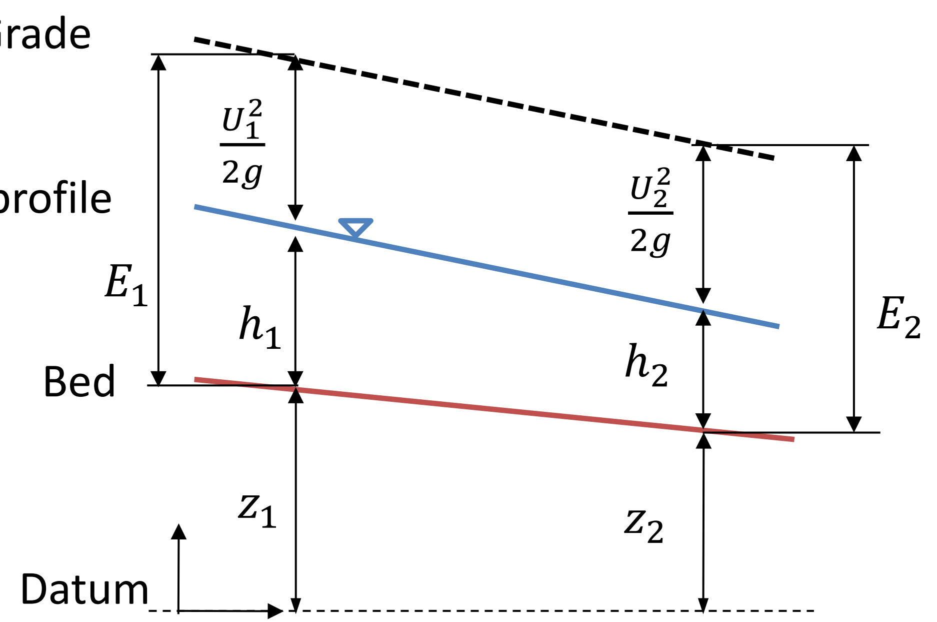
$$E_1 = h_1 + \frac{U_1^2}{2g}$$

$$E_2 = h_2 + \frac{U_2^2}{2g}$$

$$\boxed{\frac{dH}{dx} = -S_f = \frac{H_2 - H_1}{x_2 - x_1} = \frac{-h_f}{x_2 - x_1}}$$

$$\boxed{\frac{dz}{dx} = -S_0 = \frac{z_2 - z_1}{x_2 - x_1}}$$

$$x_2 = x_1 + \frac{E_2 - E_1}{S_0 - S_f}$$



What is S_f?

- ***S*** S_f the slope of the energy grade line in **gradually varied flow**
- **Estimated based on** the corresponding formulas in **uniform form** that give S_{fn}
  - – this approximation is only applied to the variable S_f
- Errors due to this approximation typically small. **NB: we are not assuming $S_f = S_0$**
- Since flow depth varies with distance, such that the friction slope S_f also varies

Representative Friction Slope

Need to define representative friction slope between 1-2, and a number of alternative formulations for S_f have been proposed in the past

Average (arithmetic mean) friction slope

$$\bar{S_f} = \frac{1}{2}(S_{fn,1} + S_{fn,2})$$

Geometric mean friction slope

$$\bar{S_f} = \sqrt{S_{fn,1} S_{fn,2}}$$

Harmonic mean friction slope

$$\bar{S_f} = \frac{2(S_{fn,1})(S_{fn,2})}{S_{fn,1} + S_{fn,2}}$$

- It can be shown that these three formulations are identical if terms of order

$$\left(\frac{\Delta S^2}{S_{fn,1}^2}\right) = \left(\frac{S_{fn,2} - S_{fn,1}}{S_{fn,1}}\right)^2$$

and higher order are neglected.

- The average slope (first expression) generally gives the lowest maximum error if the distance between sections 1-2 is short or the flow depth z_1 and z_2 are similar

Direct Step Calculation

Using the average friction slope expression, we can write the head loss as

$$h_f = \frac{1}{2}(S_{fn,1} + S_{fn,2})(x_2 - x_1)$$

and the Bernoulli balances gives

$$z_1 + E_1 = z_2 + E_2 + \frac{1}{2}(S_{fn,1} + S_{fn,2})(x_2 - x_1)$$

Substituting the earlier result that: $z_2 = z_1 - S_0(x_2 - x_1)$

eliminates both z_2 and z_1 and yields

not given in data sheets !

$$\boxed{x_2 = x_1 + \frac{E_2 - E_1}{S_0 - \frac{1}{2}(S_{fn,1} + S_{fn,2})}}$$

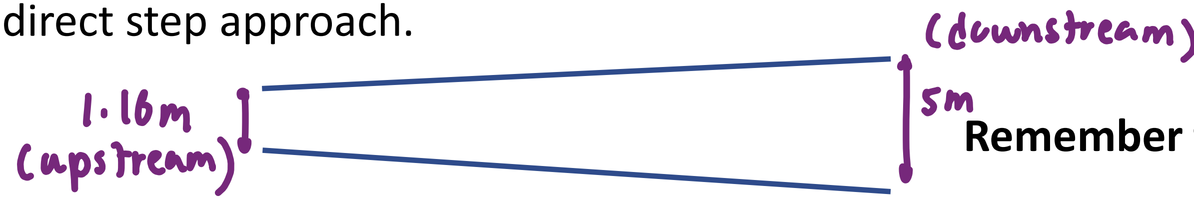
where $E = h + \frac{v^2}{2g}$

one upside of Approach B is since it is a step method, if S_0 changed, we can set z_2 at the point where S_0 change and the next z_1 to start from there !

This enables calculation of the value x_2 based on x_1, which will in turn serve as the starting value for the next step, i.e. it enables an iterative scheme to be deployed.

Direct Step Calculation: Example

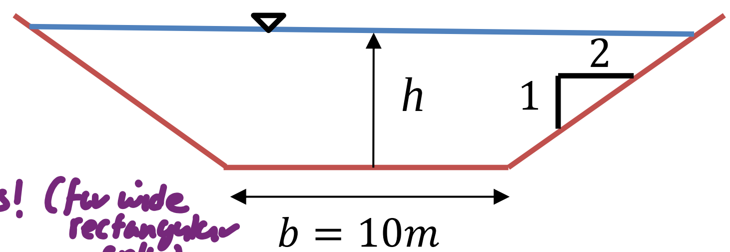
A trapezoidal channel with a bottom slope of S_0=0.001 is carrying a flow rate of 30 m^3/s. The bottom width is 10m, and the slide slopes are behaving as 2:1 (see figure). A control structure is built downstream which raises the water depth at the downstream end to 5m. The Manning coefficient for all surfaces is n=0.013. Compute the water surface profile using the direct step approach.



Remember for uniform flow

$$Q^2 = \frac{8g}{f}\frac{A^3}{P}S_{fn}, \quad \text{(slide 121)}$$

$$\frac{f}{8} = gn^2\frac{P^{1/3}}{A^{1/3}} \quad \text{(slide 119)}$$



$A = bh + 2h^2$

$P = b + 2\sqrt{5}h$

CANNOT USE $h_n = \left(\frac{fq^2}{8gS_0}\right)^{1/3}$ from data sheets! (for wide rectangular only)

should use $\frac{Q}{A} - \left(\frac{8g}{f}\right)^{1/2}\left(\frac{A}{P}\right)^{1/2} S^{1/2} = 0$ to solve for h_n

The normal depth for this channel can be found as h_n=1.16m. Calculations are hence undertaken from 5m (downstream end) to 1.25m (close to normal depth) in steps of 0.25m.

Direct Step Calculation: Example

The results of the calculation are given on slide 152. The columns are calculated as follows:

(1) This is the flow depth h which is taken from 5 m to 1.25m in 0.25m steps

(2) The flow area A for the depth of column (1)

(3) The wetted perimeter P for a trapezoidal channel of flow depth in column (1)

(4) The hydraulic radius obtained as $R = A / P$ (column (2) / column (3))

(5) The flow velocity U obtained by dividing Q over A

question specific, 3 ways to find S_f : Darcy-Weisbach, Chezy, or Manning.

(6) The friction slope S_f obtained as $\boxed{S_{fn} = n^2 U^2 / R^{(4/3)}}$ (slide 118 and rearrange)

(7) The average of S_{fn} between the current depth and the previous depth. This is left blank in the first row, since there is no previous depth. The first entry between rows 1 and 2 indicates the average between these two friction slopes

Direct Step Calculation: Example

(8) The result of substrating S_{fa} (column 7) from the given S_0 = 0.001.

(9) The specific energy $\boxed{E = h + U^2 / 2g}$

(10) The change in specific energy between the current and the previous depth. Again, this is left blank for the first row, where no previous depth exists. The difference between two depths is entered in between two rows.

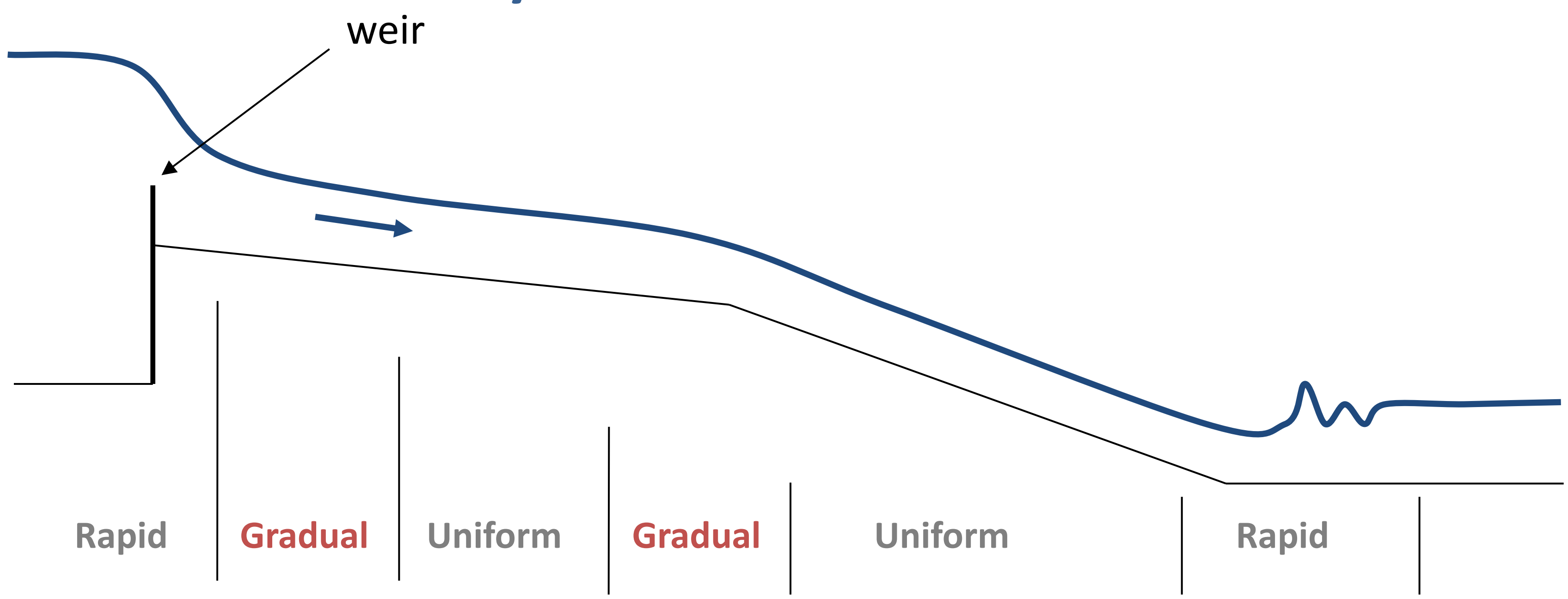
(11) Calculated from $\boxed{\Delta x = \dfrac{E_2 - E_1}{S_0 - \frac{1}{2}(S_{fn,1} + S_{fn,2})}}$ (Column 10 divided by column 8)

$\Delta x = \dfrac{\Delta E}{S_0 - S_{fa}}$ memorise!

(12) The new location x_2 calculated as $x_1 + \Delta x$.

				Q/A (Q fixed)		$\frac{1}{2}(S_{fn,1}+S_{fn,2})$			E_2-E_1	$\frac{(10)}{(8)}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
h	A	P	R	U	S_{fn}	S_{fa}	$S_0 - S_{fa}$	E	ΔE	Δx	x_2
5.00	100	32.36	3.09	0.30	0.000003	-	-	5.00459	-	-	0
						0.000004	0.000996		-0.24924	-250.19	
4.75	92.625	31.24	2.96	0.32	0.000004			4.75535			-250.19
						0.000005	0.000995		-0.24907	-250.24	
4.50	85.5	30.12	2.84	0.35	0.000005			4.50627			-500.43
						0.000006	0.000994		-0.24885	-250.32	
4.25	78.625	29.01	2.71	0.38	0.000007			4.25742			-750.75
						0.000007	0.000993		-0.24857	-250.43	
4.00	72	27.89	2.58	0.42	0.000008			4.00885			-1001.18
						0.000010	0.000990		-0.24820	-250.58	
3.75	65.625	26.77	2.45	0.46	0.000011			3.76065			-1251.77
						0.000012	0.000988		-0.24769	-250.80	
3.50	59.5	25.65	2.32	0.50	0.000014			3.51296			-1502.56
						0.000016	0.000984		-0.24701	-251.11	
3.25	53.625	24.53	2.19	0.56	0.000019			3.26595			-1753.68
						0.000022	0.000978		-0.24604	-251.59	
3.00	48	23.42	2.05	0.63	0.000025			3.01991			-2005.27
						0.000030	0.000970		-0.24466	-252.33	
2.75	42.625	22.30	1.91	0.70	0.000035			2.77525			-2257.60
						0.000043	0.000957		-0.24263	-253.52	
2.50	37.5	21.18	1.77	0.80	0.000051			2.53262			-2511.12
						0.000063	0.000937		-0.23952	-255.55	
2.25	32.625	20.06	1.63	0.92	0.000075			2.29310			-2766.67
						0.000095	0.000905		-0.23459	-259.24	
2.00	28	18.94	1.48	1.07	0.000115			2.05851			-3025.92
						0.000151	0.000849		-0.22632	-266.70	
1.75	23.625	17.83	1.33	1.27	0.000187			1.83219			-3292.61
						0.000257	0.000743		-0.21155	-284.55	
1.50	19.5	16.71	1.17	1.54	0.000326			1.62064			-3577.16
						0.000473	0.000527		-0.18275	-347.05	
1.25	15.625	15.59	1.00	1.92	0.000621			1.43789			-3924.21

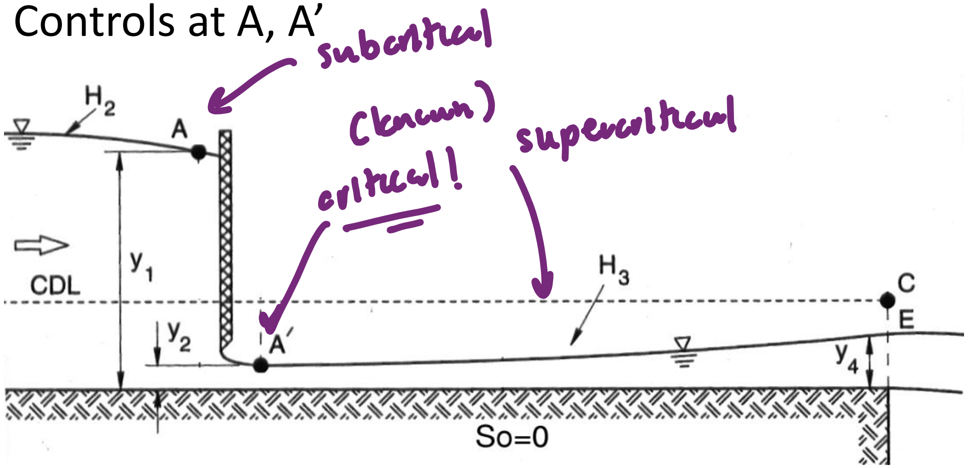
4.4 Profile Synthesis in GVF



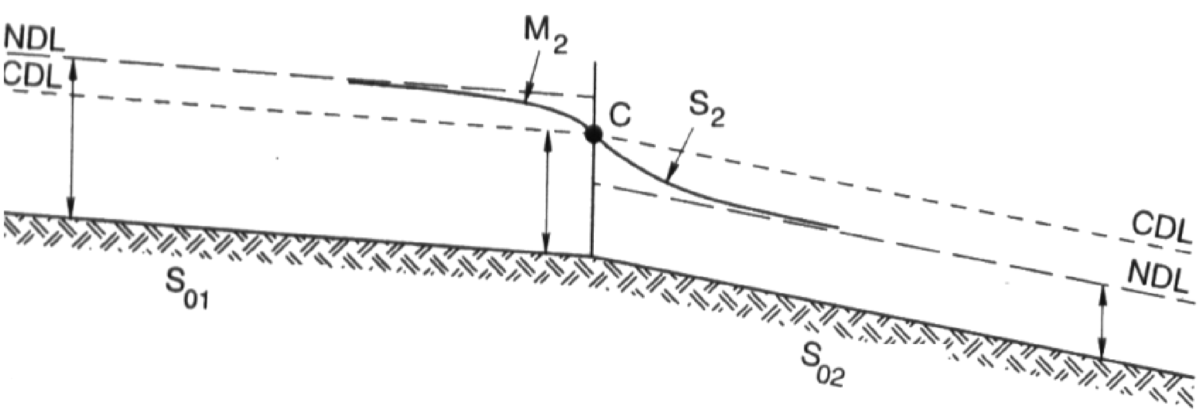
- The challenge that remains is where to start the profile computation?
- Requires x_0 and $h(x_0)$
- Initial step is water surface (WS) profile synthesis
- Control points are points with a definite relation between discharge and flow depth
- Look for open channel controls, where WS elevation h_0 is fixed by flow Q at particular position x_0

Water Surface Profile Synthesis

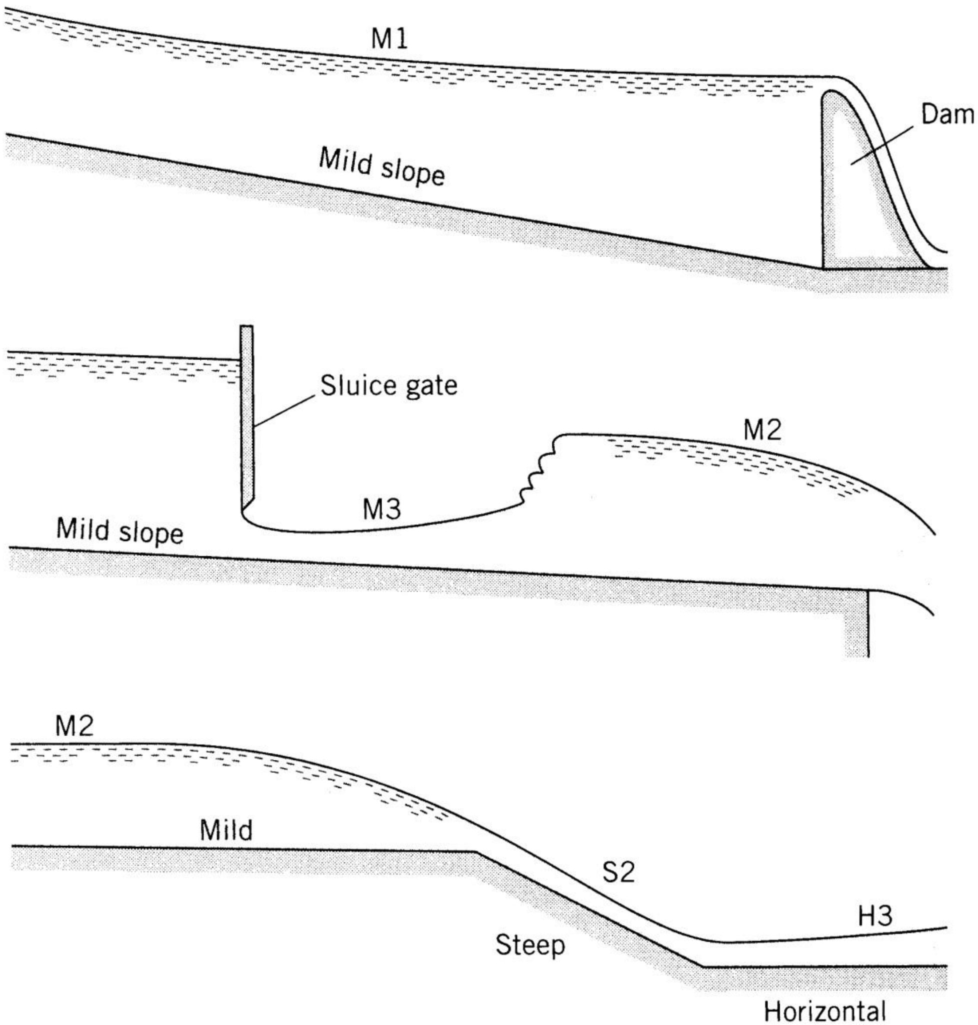
Example 1: Controls at A, A'



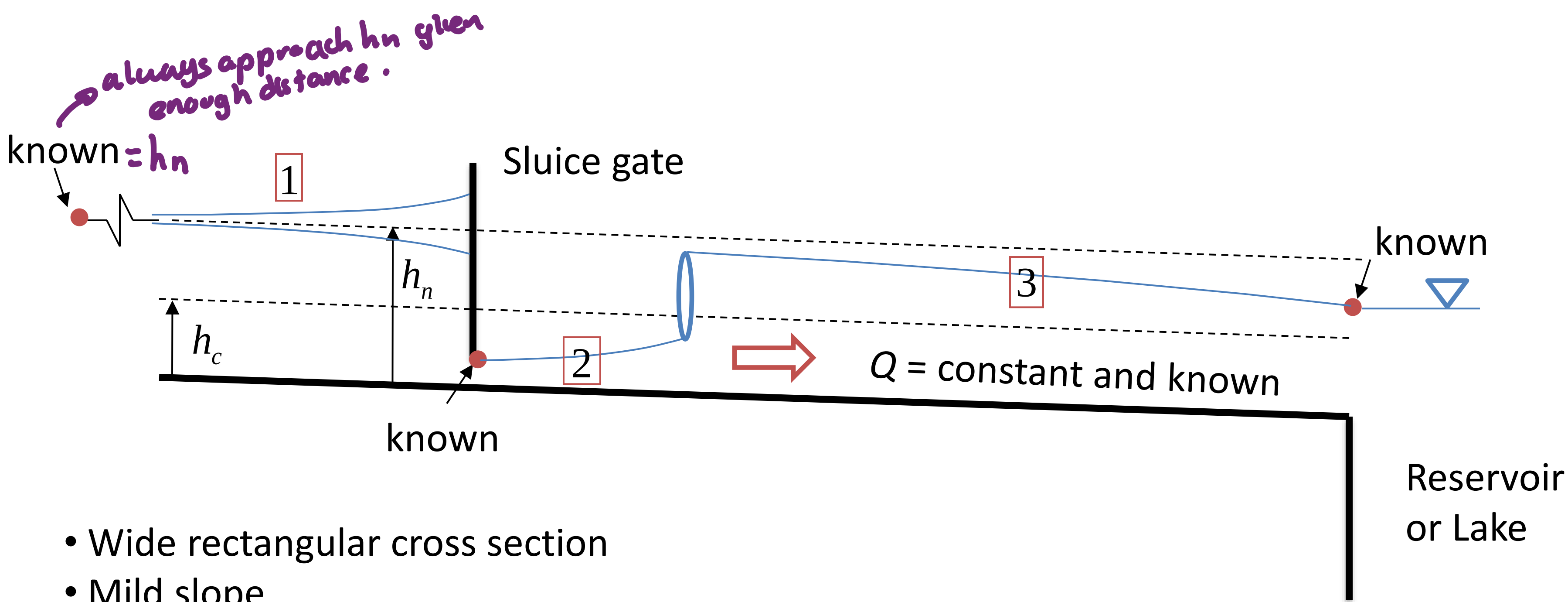
Example 2: Control at C



Water Surface Profile Examples



Wide Rectangular Channel: Example



- Wide rectangular cross section
- Mild slope

i. WS profile synthesis
ii. WS profile computation
iii. Analysis of results

Example: WS Profile Synthesis

Synthesis considerations:

- For $h_c < h_n \rightarrow$ M profiles (a result of the mild slope)
- Far upstream, no influence of the sluice gate and $h \rightarrow h_n$ asymptotically
- Section 1
  - $h > h_n \rightarrow M_1$ profile
  - **or** $h_c < h < h_n \rightarrow M_2$ profile
  - depending on alternate depth across sluice gate (see Year 1)
- Section 2
  - $h < h_c \rightarrow M_3$ profile **CONTROL**
- Section 3
  - $h_c < h < h_n$ at reservoir $\rightarrow M_2$ profile **CONTROL**
- Jump
  - M_2 to M_3 must cross h_c **RVF**
- Location of Jump
  - Force balance, $J_{us} = J_{ds} \rightarrow$
  - Specific momentum flux + specific pressure force, must balance

    $\rightarrow \quad m_{us} + \mathrm{pf}_{us} = m_{ds} + \mathrm{pf}_{ds}$

Example: WS Profile Computation

Assume the following parameters:

- Sluice gate at $x = 0$, $h_0 = 0.25$m
- Reservoir at $x = L = 500$m, $h_L = 1$m
- $q = 2.5\text{m}^2/\text{s}$, $S_0 = 0.001$, $f = 0.025$

$$h_n = \left(\frac{fq^2}{8gS_0}\right)^{1/3} = 1.258\text{m}$$

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = 0.86\text{m}$$

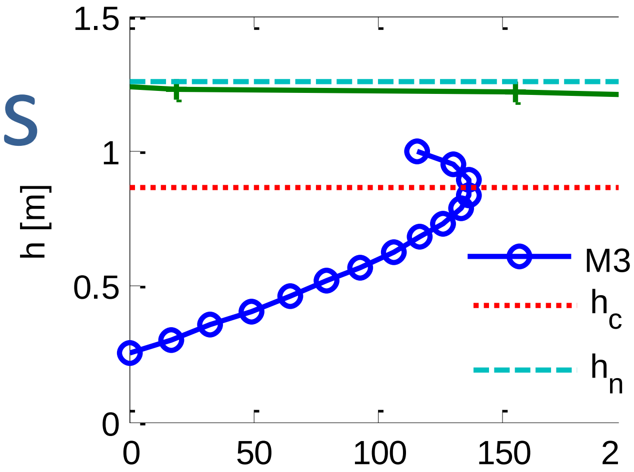
$$x_2 = x_1 + \frac{h_n}{S_0}\left\{\gamma_2 - \gamma_1 - \left[1 - \left(\frac{h_c}{h_n}\right)^3\right](\Phi(\gamma_2) - \Phi(\gamma_1))\right\}$$

Force balance: consider a vertical coordinate, z, pointing down from the free surface at $z = 0$. Now:

$$J = \int_0^h u^2\,\mathrm{d}z + \int_0^h gz\,\mathrm{d}z = u^2 h + \frac{1}{2}gh^2 = \frac{q^2}{h} + \frac{1}{2}gh^2$$

$$J_{us} = J_{ds} \rightarrow \frac{q^2}{h_{us}} + \frac{1}{2}gh_{us}^2 = \frac{q^2}{h_{ds}} + \frac{1}{2}gh_{ds}^2$$

Example: WS Profile Results



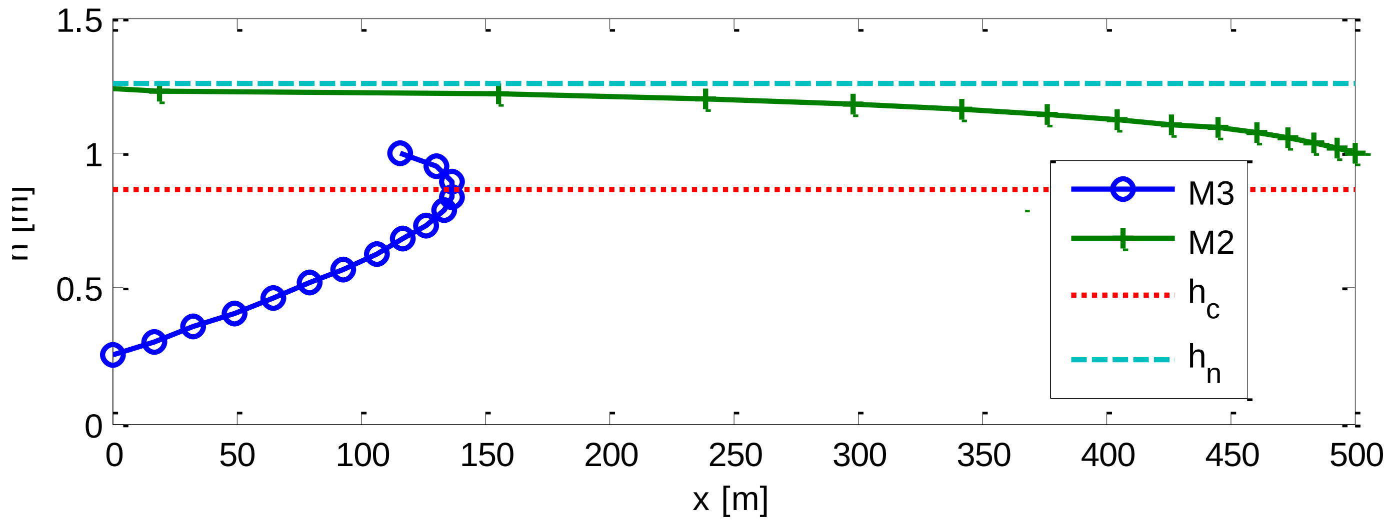
i) M_3 GVF profile in section 2

h [m]	x [m]	$\gamma = h/h_n$	$\Phi(\gamma)$	J/ρ [m^3/s^2]
0.25	0.0	0.1987	0.5014	25.307
0.3	15.6	0.2385	0.5416	21.275
0.4	46.1	0.3180	0.6229	16.410
0.5	74.8	0.3975	0.7062	13.726
0.6	100.5	0.4769	0.7930	12.182
0.7	121.5	0.5564	0.8854	11.332
0.8	135.1	0.6359	0.9865	10.952
0.9	136.4	0.7154	1.1018	10.917
1.0	116.1	0.7949	1.2424	11.155

Example: WS Profile Results

ii) M_2 GVF profile in section 3

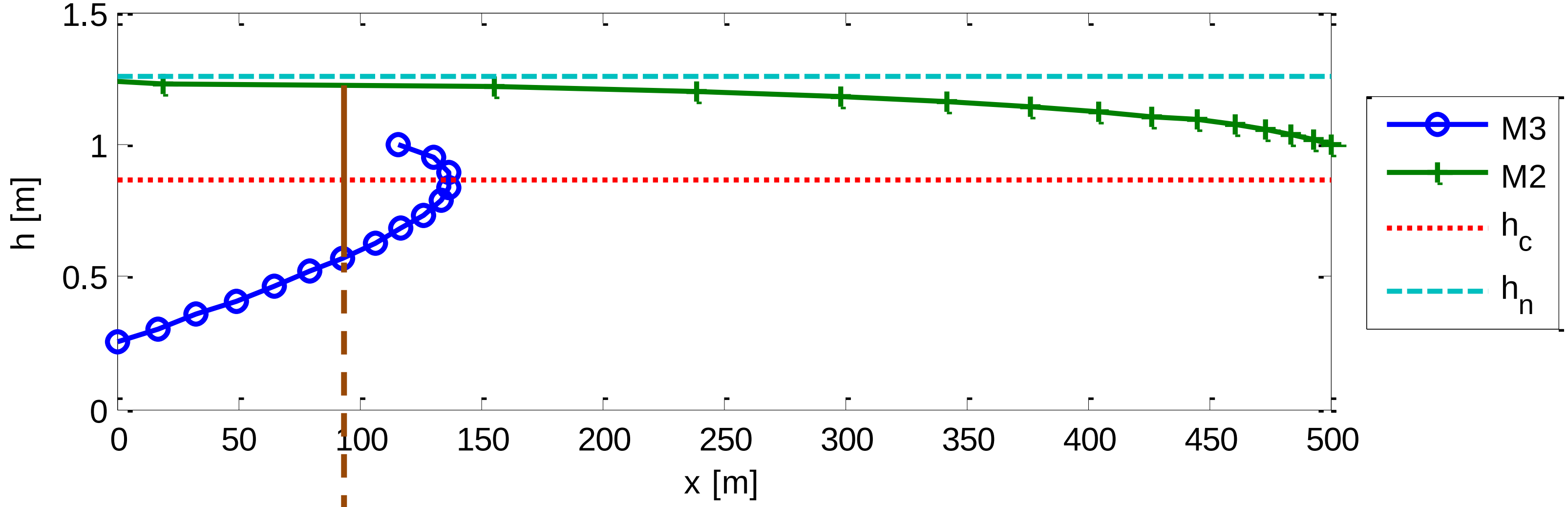
h [m]	x [m]	$\gamma = h/h_n$	$\Phi(\gamma)$	J/ρ [m³/s²]
1.00	500.0	0.7949	1.2424	11.155
1.05	475.7	0.8346	1.3293	11.360
1.10	434.8	0.8744	1.4355	11.617
1.15	364.1	0.9141	1.5766	11.922
1.20	225.1	0.9539	1.7976	12.272
1.25	-300.8	0.9936	2.4708	12.664

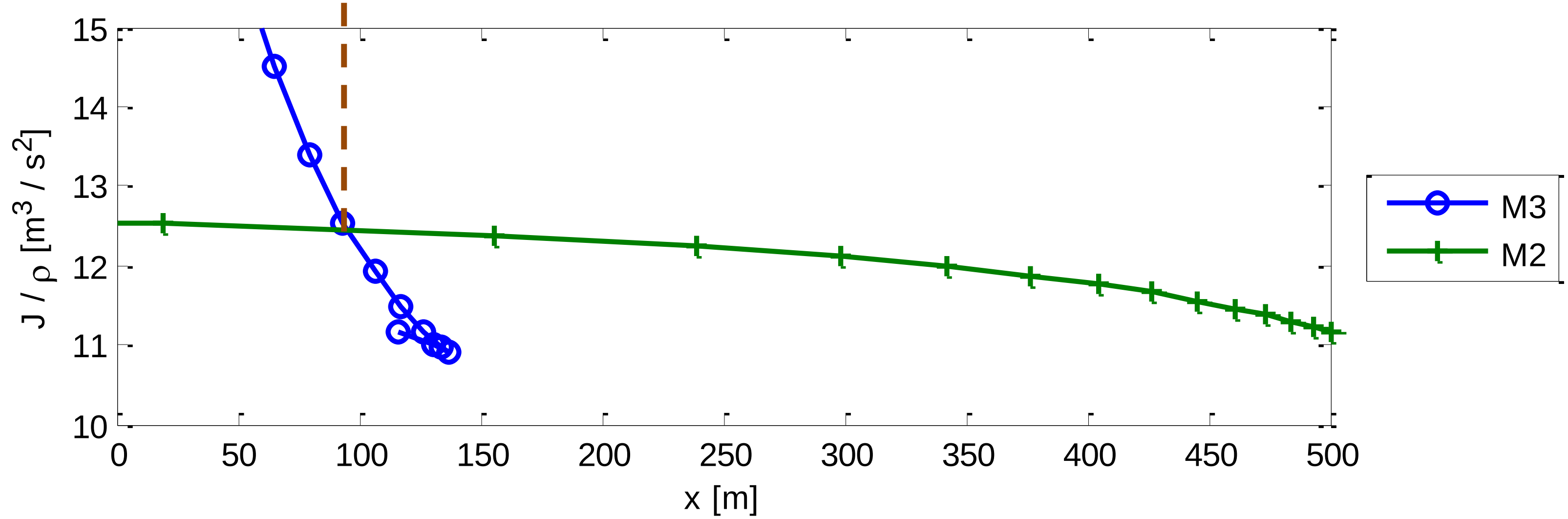


iii) Jump location at $J_{M3} = J_{M2}$

You can now complete: Tutorial 7

Example: WS Profile Illustration





Thank you for your interest and attention!

I hope that you feel you’ve learnt something.

I’ve really enjoyed it,

I hope that some of you have too!