Imperial College London

3 Pipe Flow

Fluid Mechanics 2 (CIVE50005)

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Pipes are everywhere

Pipes

- **Energy Lost**
  - Major (friction)
    - ★ Find f:
      - Moody Diagram
      - → Laminar
        - Re
      - → Turbulent
        - k_s/D
      - → How to find f for non circular?
    - find f is to find h_f!
  - Minor (enlargement, contraction, etc)
- **Velocity Profile and Shear Stress Profile.**
  - Laminar
    - $u(r) = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1 - \frac{r^2}{R^2}\right)$ (for all r)
    - $\tau(r) = \mu \frac{du(r)}{dr}$
  - Turbulent
    - $u(r)$ divided into three region.
    - $\tau(r) = -\rho\overline{u'v'} + \overline{\tau}(r)$
    - $= \underbrace{-\rho\overline{u'v'}}_{\text{reynold stress}} + \underbrace{\mu\frac{d\overline{u(r)}}{dr}}_{\text{viscous stress}}$
    - reynold stress $\approx \tau_0 \frac{r}{R}$ if core region; viscous stress ≈ 0 if core region
  - having an alternative way to determine $\tau(r)$
    - → another way to determine τ_0
    - → another way to find h_f!
- **Pipe System.**
  - (When doing Bernoulli, the energy lost, $h = KQ^2$ instead of $\xi\frac{U^2}{2g}$)
  - basically any type of question just mass conservation and energy conservation. (both can have multiple eqns)

TWO METHOD TO FIND h_f!

- method 1: find f is to find h_f!
- method 2: another way to find h_f!

h_f and τ_0 are interchangeable:

$$h_f = \frac{\tau_0 P L}{\rho g A} = \underbrace{\frac{4\tau_0 L}{\rho g D}}_{\text{circular pipe}}$$

this mean there's two ways to find τ_0 too!

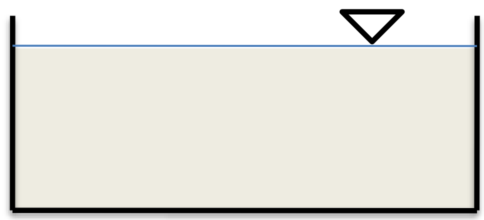
(2) one is find $\tau(r)$, one is

(1) $\tau_0 = \frac{f}{8}\rho U^2$

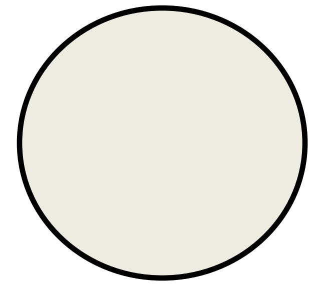
important notes / trick:

- smooth pipe doesn't mean no friction, but ideal fluid does mean no friction

3.1 Steady Pipe Flow



Channel Flow



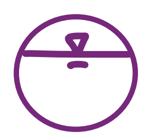
Pipe Flow

Channel Flow

- liquid only (big density difference maintains concept of ‘free surface’)
- atmospheric pressure at free surface
- elevation of free surface, i.e. water depth and surface profile respond to flow
- cross-sectional area of flow determined by channel shape and flow conditions

Pipe Flow

- liquid or gas
- fluid fills entire pipe (in all our examples)

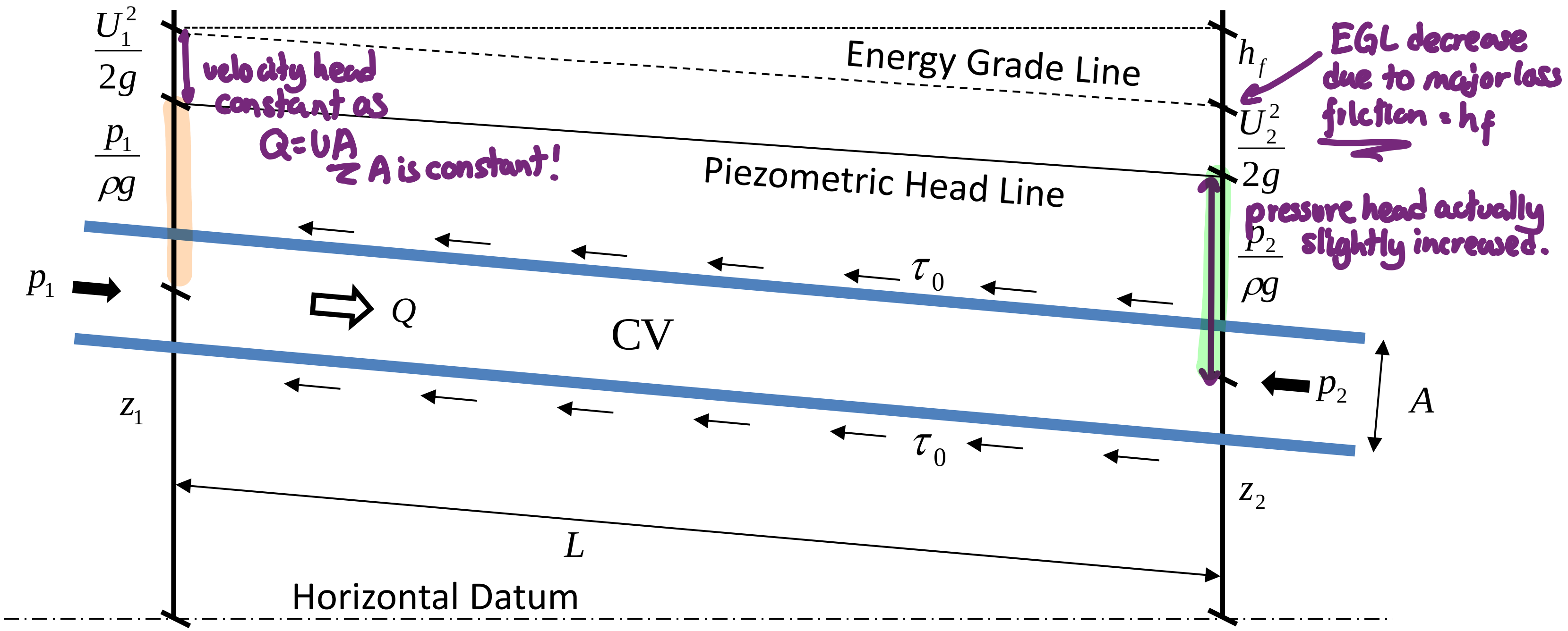


we don't look at cases like this

- cross-sectional area of flow determined entirely by pipe geometry
- pressures can vary considerably, above and below atmospheric pressure

eg.

Steady Flow Analysis (CI140)



Area (CV) $A = (\pi/4)D^2$

Volume (Mass) conservation (CV) ***- This must be true for <u>all</u> flows (lam/turb) and <u>all</u> vel profiles***

Q=UA if only U is constant for ∀r

$Q\,[m^3/s] = \text{constant} = 2\pi\int_0^{D/2} u(r)\cdot r\ \mathrm{d}r$, for uniform flow ONLY $u(r) = U$ then $Q = (\pi/4)D^2U$

or else ΣF= dM/dt

Momentum conservation(CV) ***- Mathematically true ONLY if velocity profile remains unchanged 1->2(slide 63+)***
- Physically true if no other net forces act on the Control Volume, think Δp and τ

$M\,[m^4/s^2] = 2\pi\int_0^{D/2} u^2(r)\cdot r\ \mathrm{d}r$, for uniform flow ONLY $u(r) = U$ then $M = (\pi/4)D^2U^2 = QU$

M=U²A only if U is constant for ∀r

same as $Q=\int u\,dA$; $M=\int \rho u^2 dA$ a different form (Φ: ∫uX dA, year 1)

Energy and Hydraulic Grade Line

The functions of **total energy** and **piezometric head** are referred to as **the energy grade line (*H*=EGL)** and **hydraulic grade line (*h*=HGL)**, respectively.

- The energy and hydraulic grade lines are a **graphical representation of the energy state of the fluid** along a streamline
- The difference between **the total energy head** (*H*) [Pressure E. + P.E. + K.E.]

and the **piezometric head** (*h*) [Pressure E. + P.E.] is the **velocity head**

- The **velocity head** is associated with the **kinetic energy** [K.E.] of the flow
- **Energy losses** (h_f) translate to a **decrease of the energy head** *H*
- **EGL = Press E. + P.E. + K.E. ;   HGL = Press E. + P.E.**

weight of fluid in the dir. of pipe

(wetted perimeter × total length)

Area of fluid contact with pipe

Force balance down the pipe

force balance.

$$p_1A - p_2A + \rho g A z_1 - \rho g A z_2 - \tau_0 PL = 0$$

← force balance. $\Sigma F = 0$

or

energy balance

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2 + \frac{\tau_0 PL}{\rho g A} \Rightarrow (\mathrm{HGL})_1 = (\mathrm{HGL})_2 + \mathrm{Losses}(h_f)_{1\to 2}$$

energy balance. $\Sigma E_{in} = \Sigma E_{out} + \Sigma E_{loss}$

both of these are when there is no change in momentum i.e. $\frac{v^2}{2g}$ is fixed $\Sigma F = 0$ or else $\Sigma F = ma$!

A – pipe cross section

P – wetted perimeter

Bernoulli equation

Steady flow Bernoulli equation for **ideal** fluid is

$$\frac{p_1}{\rho g}+z_1+\frac{U_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{U_2^2}{2g} \quad \Rightarrow (\mathrm{EGL})_1=(\mathrm{EGL})_2$$

Real fluid, we could try to solve N.S. = bad idea, tough. Instead parameterise losses based on force balance. The head loss 1 to 2, in this case due to friction (shear stress) at the wall, is given by:

$$h_f=\frac{\tau_0 PL}{\rho g A}=\frac{\tau_0 \pi D L}{\rho g \pi D^2/4}=\frac{4\tau_0 L}{\rho g D}$$

$h_f = f\frac{L}{D}\frac{U^2}{2g}$ (in year 1's fluid)

better version of $f\frac{L}{D}\frac{U^2}{2g}$
(but need to find τ_0)

Now for **Real** fluid:

$$\frac{p_1}{\rho g}+z_1+\frac{U_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{U_2^2}{2g}+h_f \quad \Rightarrow (\mathrm{EGL})_1=(\mathrm{EGL})_2+\mathrm{Losses}(h_f)_{1\to 2}$$

How is shear stress modelled? What might the shear stress, τ_0, depend on?

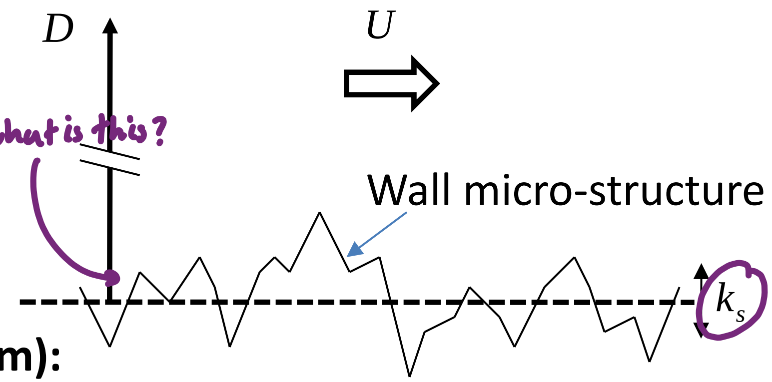
so how to find τ_0 ?

Shear Stress at Pipe Wall

Thinking and/or observation suggests that wall-shear might depend on:

$$\tau_0 = \tau_0(U, D, \rho, \nu, k_s) = [\mathrm{M \cdot L^{-1} \cdot T^{-2}}] = [\mathrm{kg / ms^2}] = [\mathrm{N / m^2}]$$

- U = [L T^{-1}] = [m s^{-1}] – mean fluid speed = Q/A
- D = [L] = [m]– pipe diameter
- k_s = [L] = [m] – charac. wall roughness 'height' } what is this?
- ρ = [ML^{-3}] = [kg m^{-3}] – density
- ν = [L^{2}T^{-1}] = [m^2 s^{-1}] – kinematic viscosity



more smooth, smaller ks

Dimensional Analysis (see Buckingham Pi theorem):

- **5 independent variables (n), in**
- **3 independent physical units (k), =>** (mass, length and time)
- **2 (n-k) Non-Dimensional parameters, physically sensible to choose:**

  - Re = *pipe* Reynolds number
  - k_s /D = relative roughness

$$\mathrm{Re} = \frac{UD}{\nu}$$

IMPORTANT:

$$\tau_0 = \frac{f}{8}\rho U^2$$

where

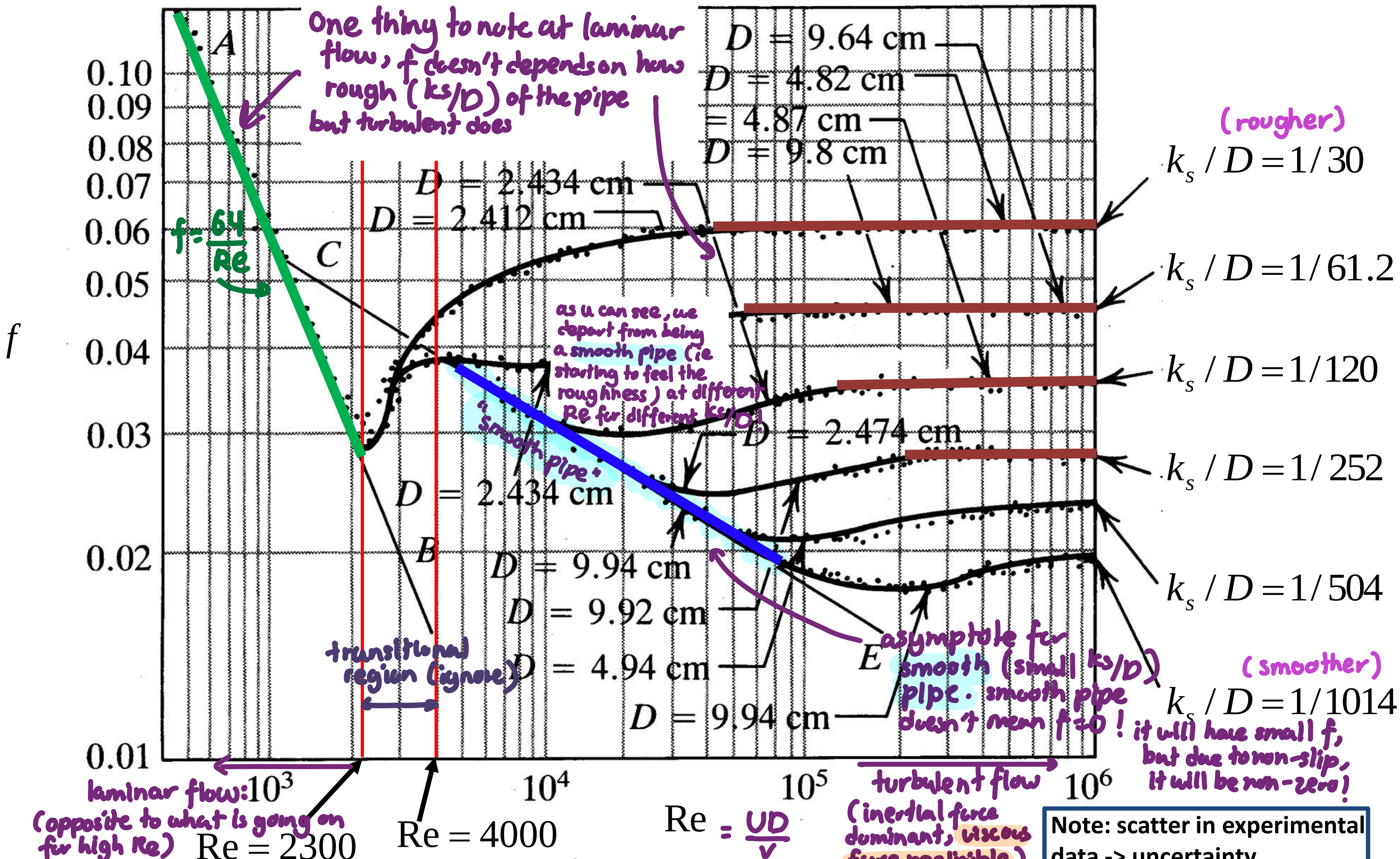
$$f = \mathrm{f}\left(\mathrm{Re}, \frac{k_s}{D}\right)$$

f depends on Re and ks/D

Why the factor (1/8) in τ_0? See 3 slides on, slide 68[?]...

important: f is very dependent on Re, almost independent on ks/D for laminar (small Re)
f is very dependent on ks/D and almost independent on Re for turbulent (large Re)

Nikuradse's Sand-Roughened Experiments



$= \frac{8\tau_0}{\rho U^2}$

laminar flow: (opposite to what is going on for high Re)
→ f very dependent on Re. ($f = 64/Re$ laminar)
$f \propto \frac{1}{Re}$ relationship
(slope = -1 on log scale) ← MATH

$Re = \frac{UD}{\nu}$

turbulent flow (inertial force dominant, viscous force negligible)
(higher Re — turbulent)
f independent of Re → horizontal line (depends on ks/D only!)

Note: scatter in experimental data -> uncertainty

$f = \frac{c}{Re}$ $\log f = \log c \underset{=}{-} \log Re$

$m = -1$

Experimental Data Fit

- From the **theory** of slow viscous flow (confirmed by experiments):

Laminar flow – Hagen-Poiseuille flow (Line A-B)

$$f = \frac{64}{\mathrm{Re}} \quad \text{for } \mathrm{Re} < 2300$$

- From the **experimental data**, the following relationships were established for

Turbulent flow in **smooth** or **rough pipes** – Colebrook-White equation

$$\boxed{\frac{1}{f^{1/2}} = -2\log_{10}\left[\frac{k_s / D}{3.71} + \frac{2.51}{\mathrm{Re}\, f^{1/2}}\right] \quad \text{for Re} > 4000. \qquad \text{-(CW)}}$$

- The equation for turbulent flow is **implicit in *f***
  - Solution by: (i) trial and error, (ii) Newton-Raphson ... or (iii) **graphical**
- Colebrook-White equation forms basis for Moody Diagram

Note for later, as: $\mathrm{Re} \to \infty$ then (CW) becomes: $\underbrace{\frac{1}{f^{1/2}} = -2\log_{10}\left[\frac{k_s / D}{3.71}\right]}$

which is a horizontal line eqn of $f = c_1$

Turbulent flow in **smooth pipes** (Line C-E)

$k_s / D \to 0$ then (CW) becomes:

$$\frac{1}{f^{1/2}} = 2\log_{10}\left[\mathrm{Re}\, f^{1/2}\right] \text{ -0.8} \quad \text{for Re} \geq 4000$$

Moody Diagram – plot of Hagen-Poiseuille flow and Colebrook-White equation

General case $h_f = \dfrac{\tau_0 PL}{\rho g A}$

Pipes $\tau_0 = \dfrac{f}{8}\rho U^2$

and $P = \pi D; A = \pi D^2 / 4$

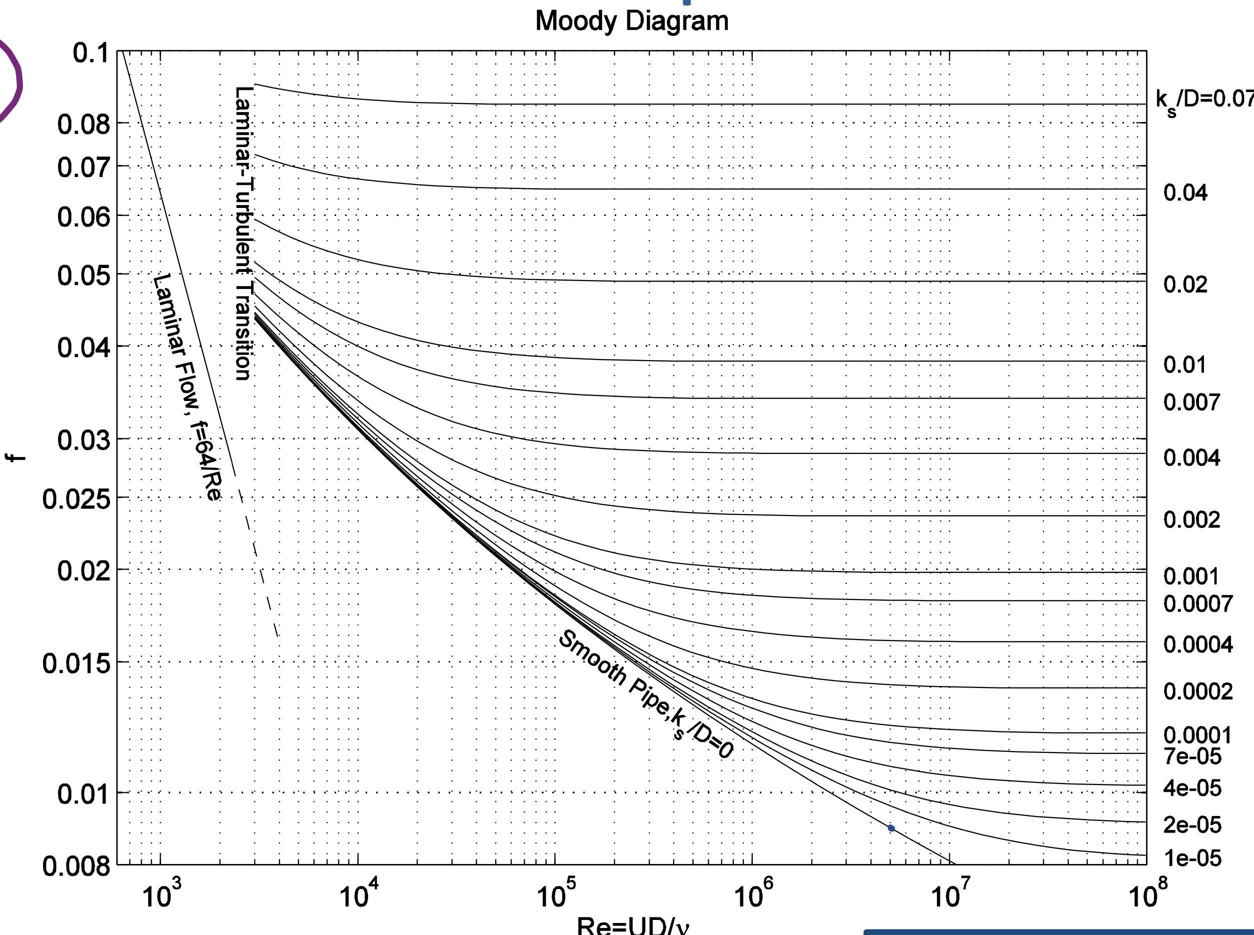
make hf a factor of kinetic head.

So for pipes

$$h_f = f\frac{L}{D}\frac{U^2}{2g}$$

Moody Diagram

$$f = \mathrm{f}\left(\mathrm{Re}, \frac{k_s}{D}\right)$$



- Here f is the dimensionless **Darcy-Weisbach** friction coefficient
- [Some textbooks define: *friction coefficient = f/4]*

Note from 3 Slides back: The factor (1/8) there, results in (1/2) in the velocity head here…

Discussion of Moody Diagram

In the general case, the **Darcy-Weisbach** friction coefficient is a function of both Reynolds number and relative wall roughness.

Laminar flow (Re < 2300)

- The friction coefficient is **independent** of the roughness of the pipe walls
- Experimental results confirm that this is valid unless the roughness becomes so great that it affects the diameter of the pipe, i.e. $D - 2k_s << D$

Turbulent flow (Re > 4000)

- The friction coefficient is indeed a function of Reynolds number **and** relative wall roughness
- Each curve in the diagram represents the friction coefficient for a particular value of k_s/D
- **Moderate** values of k_s/D: curves coincide with smooth pipe line for **intermediate** values of Re – *'Smooth zone of flow'*
- For **very large** Reynolds numbers each curve flattens out and f is hence **independent** of Re, and: $\frac{1}{f^{1/2}} = -2\log_{10}\left[\frac{k_s/D}{3.71}\right]$ or $f = \left[-2\log_{10}\left(\frac{k_s/D}{3.71}\right)\right]^{-2} = \left[\log_{10}\left(\frac{13.76D^2}{k_s^2}\right)\right]^{-2}$

Understanding Pipe Losses

- **Laminar pipe flow** is governed by the Hagen-Poiseuille solution. This is the exact **analytical solution**.
- **MOST pipe flows are turbulent** and, as a result, highly complex. It is outside the scope of this course to introduce any analysis of the turbulence itself – **we will** parameterise the effects of turbulence and **gain physical insight into turbulence in pipes** (slide 75&76).
- In turbulent flows, an **irregular fluctuation** is superimposed on the main flow – remember Reynolds decomposition & averaging (slide 44).
- The expressions for turbulent pipe losses (Colebrook-White, Moody diagram) are based on **experimental data**, **NOT** theoretical results.
- For very **large Re** numbers (fully turbulent flow) simple viscous effects (which are responsible for the losses in laminar flow) no longer vary with Reynolds number.
- In highly turbulent flows (very **large Re**) the energy is lost due to **turbulent mixing** where losses are proportional to the square of the mean velocity. For a given relative roughness, the friction factor f must thus be independent of the Re number and is hence **constant** in this regime – flat curves @ RH end of Moody diagram.

Steady Pipe Flow Problem 1

Assume water (at 20°C v=1.0x10^{-6} m^2/s) flowing in a pipe with diameter D = 75 mm, length L = 300 m and wall roughness k_s = 0.15 mm.

Given a volume flow rate of $Q = 3x10^{-3}$ m^3/s, what is head loss h_f?

Solution:

$$h_f = f\frac{L}{D}\frac{U^2}{2g} \qquad f = f\left(\mathrm{Re}, \frac{k_s}{D}\right) \qquad \mathrm{Re} = \frac{UD}{\nu} = 5.09\cdot 10^4 \qquad \frac{k_s}{D} = 0.002$$

i. Determine the mean velocity:

$$U = \frac{Q}{A} = \frac{Q}{(\pi/4)D^2} = 0.68\,\mathrm{m/s}$$

ii. Use the Moody diagram to obtain the Darcy-Weisbach coefficient

$$f \xrightarrow{\text{MoodyDiagram}} 0.027$$

iii. Obtain the head loss

$$\boxed{h_f = 2.54\,\mathrm{m}}$$

Steady Pipe Flow Problem 2

Assume water (at 20°C ν=1.0x10^{-6} m^2/s) flowing in a pipe with diameter D = 75 mm, length L = 300 m and wall roughness k_s = 0.15 mm.

Given a head loss of h_f = 2 m, what is the volume flow rate Q?

Solution: $h_f = f \frac{L}{D}\frac{U^2}{2g}$ $\qquad f = f\left(\mathrm{Re}, \frac{k_s}{D}\right) = f\left(\frac{UD}{\nu}, \frac{k_s}{D}\right)$

- Implicit equation in U, trial and error solution

U [m/s]	Re	f	h_f [m]
1	$7.50 \cdot 10^4$	0.026	5.3
0.5	$3.75 \cdot 10^4$	0.027	1.38
0.7	$5.25 \cdot 10^4$	0.0265	2.65
0.6	$4.50 \cdot 10^4$	0.0265	1.94
0.609	$4.57 \cdot 10^4$	0.0265	2.00

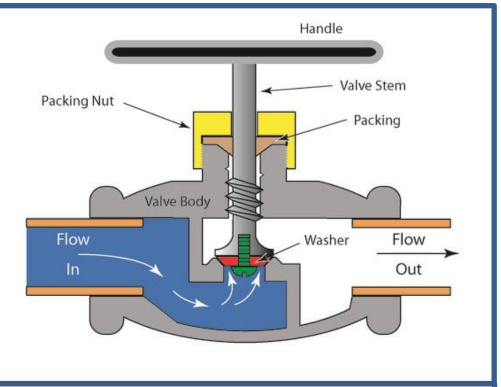
$$\boxed{Q = UA = U(\pi/4)D^2 = 2.69 \cdot 10^{-3}\ \mathrm{m^3/s}}$$

Minor Losses

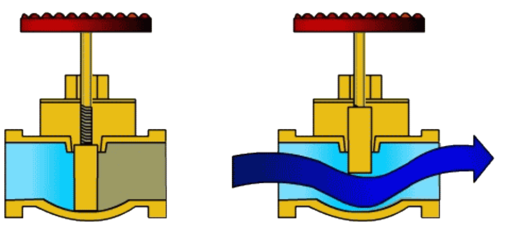
Why minor? Because major loss, h_f, is often bigger: often $f \approx 10^{-2}$, $10^{-1}\text{m} \lesssim D \lesssim 10^{0}\text{m}$ so for $10\text{m} \lesssim L \lesssim 100\text{m}$ then pre-factor in the 'major' losses, h_f, is $f\,L/D \gg 1$

minor loss sometimes can be greater than major loss

Globe valve



Gate valve



Including minor losses, the steady flow Bernoulli equation (Bernoulli balance) is:

$$\frac{p_1}{\rho g} + z_1 + \frac{U_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{U_2^2}{2g} + h_f + h_L$$

The loss factor is defined as (1st year)

$$h_L = \xi \frac{U^2}{2g}$$

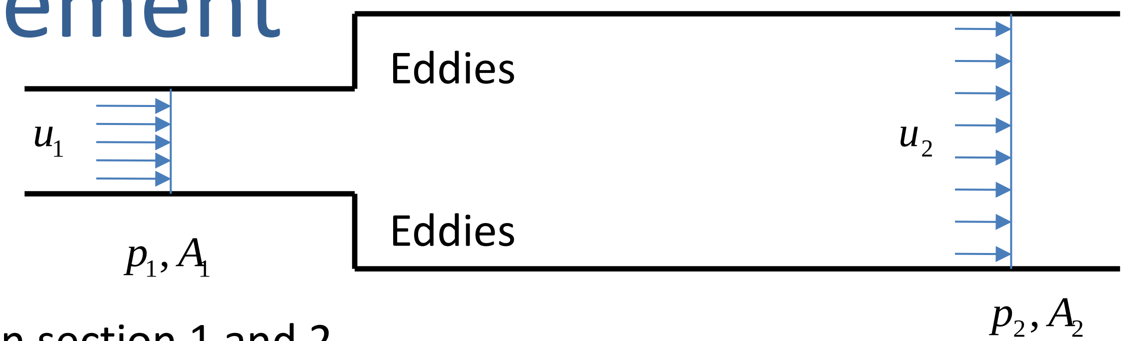
no matter $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right)$ or $h_\ell = \xi\left(\frac{U^2}{2g}\right)$

the factor is not bounded by 1. (you can lose more than KE head)

Approximate Loss Coefficients ξ for Commercial Pipe Fittings

Type	Screwed	Flanged
Valve, wide open (Globe)	10	5
Valve, wide open (Gate)	0.2	0.1
Return bend	1.5	0.2
Elbow, 90 regular	1.5	0.3
Elbow, 90 long radius	0.7	0.2
Tee, line flow	0.9	0.2
Tee, branch flow	2	1

Sudden Enlargement



- Flow is assumed to be steady in section 1 and 2
- Sudden expansion typically causes turbulent eddies in the corners, hard to deal with analytically -> parameterise
- Additional dissipation of kinetic energy (via inconsequential heating)
- Additional dissipation is turbulent, remember the Reynolds stress terms (slide 45)

Assuming that at the expansion the pressure is approximately equal to p_1 (streamlines approximately parallel and experiments support this idea). Then a force balance gives:
$(p_1 - p_2)A_2 = \rho Q(U_2 - U_1)$ and this into Bernoulli gives:

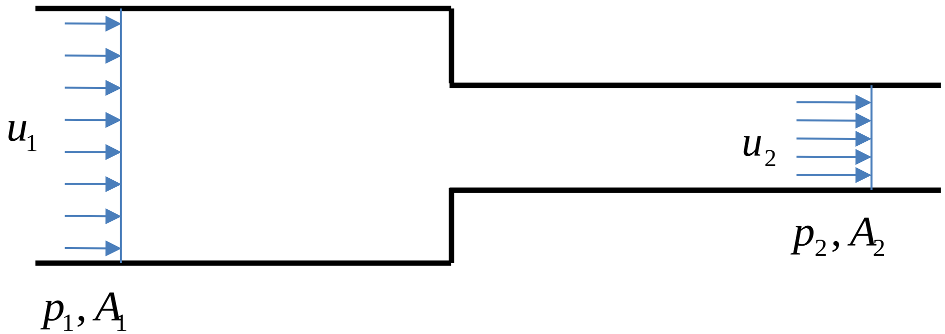
$$\boxed{h_L = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{U_1^2}{2g} = \left(\frac{A_2}{A_1} - 1\right)^2 \frac{U_2^2}{2g}}$$

Defining the head loss factor **relative to** the **highest velocity** gives $\xi \frac{U_1^2}{2g}$

$$\xi = \left(1 - \frac{A_1}{A_2}\right)^2$$

A_1/A_2	0 (exit)	0.1	0.25	0.5	0.75	1
ξ	1	0.81	0.5625	0.25	0.0625	0

Sudden Contraction



- Sudden contraction causes a *vena contracta* (the point where the diameter of the fluid stream is minimal) immediately after the junction
- Eddies are formed between this vena contracta and the pipe walls
- Difficult to parametrise, losses depend on the 'min' width felt by the flow during the *vena contracta* analytically
- The 'min' width itself is flow dependent and may be unsteady
- Empirical data has been developed by experimental engineers

For coaxial circular pipes and fairly high Reynolds numbers, Massey (1998) suggests:

A_2/A_1	0 (entrance)	0.2	0.4	0.6	0.8	1.0
ξ	0.5	0.45	0.38	0.28	0.14	0

$\xi \frac{U_2^2}{2g}$ ← higher velocity how to find $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right)$ if pipe is non circular?

You can now complete: Tutorial 3

Non circular pipes - Hydraulic Radius

For circular pipes:

$$\frac{p_1}{\rho g} + z_1 + \frac{U_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{U_2^2}{2g} + h_f$$

and

Darcy-Weisbach equation

$$h_f = \frac{\tau_0 PL}{\rho g A} = f\frac{L}{D}\frac{U^2}{2g} \quad \text{(slide 54)}$$

These equations can remain valid for **non-circular, prismatic** ducts. Introduce the **Hydraulic Radius**

$$R_H = \frac{A}{P} = \frac{\text{Flow Area}}{\text{Wetted Perimeter}}$$

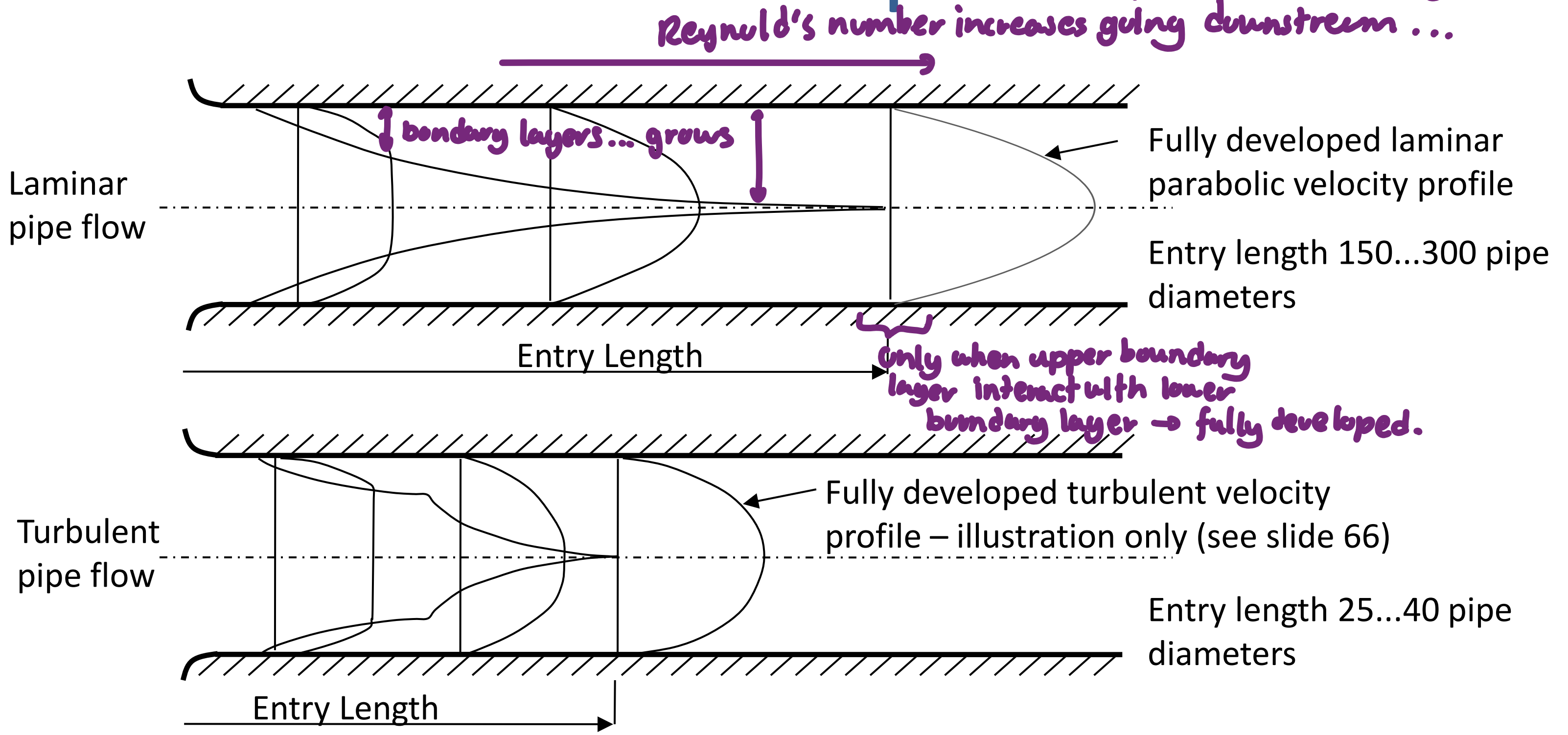
- *a x b* rectangular duct: $$R_H = \frac{ab}{2(a+b)}$$
- circular pipe: $$R_H = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

Must be consistent for pipes between using physical pipe diameter *D* and hydraulic radius R_H , **so replace *D* with 4R_H everywhere** Darcy-Weisbach equation becomes

$$h_f = f\frac{L}{4R_H}\frac{U^2}{2g} \qquad f = \mathrm{f}\left(\frac{U4R_H}{\nu}, \frac{k_s}{4R_H}\right)$$

3.2 Flow Profiles in Pipes

$Re = \frac{\text{Inertia}}{\text{Viscosity}}$ → U increases



- At pipe entry, velocity profile is typically near uniform
- Boundary layer (BL) development from pipe wall
- Region where Boundary layer (BL) developing termed the '**entry length**'
- **Fully developed flow** when boundary layers converges to pipe centreline >>>
    - Flow profile ceases to vary with axial coordinate so u(x,r) = u(r) velocity profile stop changing after fully developed
- Possible laminar-turbulent transition in boundary layer

Laminar Flow Profiles in Pipes

★ EVERYTHING WE TALK ABOUT AFTER THIS POINT (LAMINAR / TURBULENT) IS GOING TO BE FULLY-DEVELOPED!

The **exact solution** of the Navier-Stokes equations for **steady (laminar)** flow in a pipe is referred to as '**Hagen-Poiseuille**' **flow**. The derivation will be omitted here but even Wikipedia has a reasonable derivation, note velocity *u* is not a function of time *u(r)*.

The velocity profile is parabolic (*cf*. steady flow between plates) and given by

$$u(r) = -\frac{R^2}{4\mu}\frac{dp}{dx}\left(1-\frac{r^2}{R^2}\right) \quad \text{and} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{dp}{dx}r$$

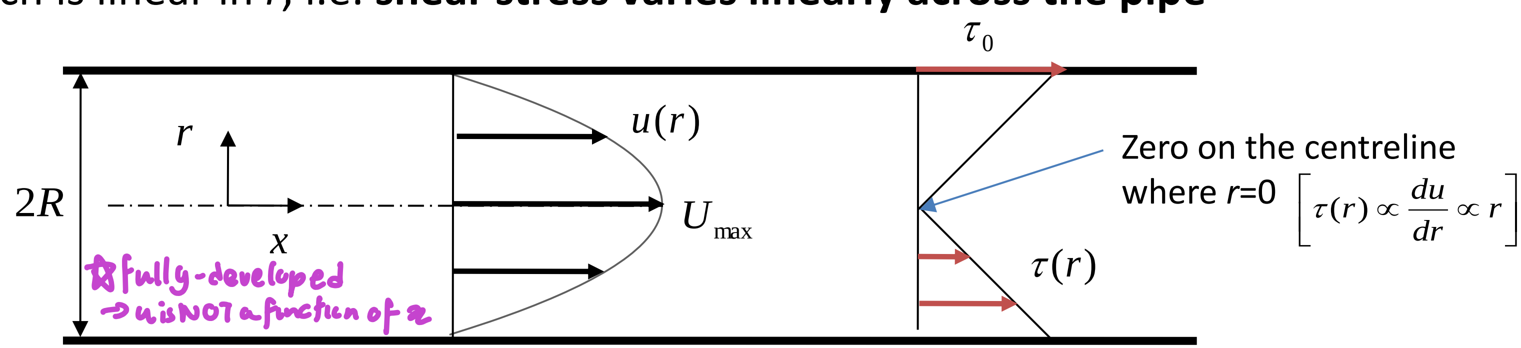
quite similar to the solution of navier-stoke equation for steady, laminar, fully-developed case.

$$u(r) = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

For a **Newtonian fluid**, the shear stress distribution is

$$\tau(r) = \mu\frac{du}{dr} \quad \text{so here} \quad \tau(r) = \mu\frac{du}{dr} = \frac{1}{2}\frac{dp}{dx}r$$

which is linear in *r*, i.e. **shear stress varies linearly across the pipe**



★ fully-developed
→ u is NOT a function of x

steady
→ u is NOT a function of t

Turbulent Flow Profiles, some notation

- For a turbulent flow velocity is a function of space (position, 3 dimensions) and time

- Turbulent flow in circular pipes natural axis of symmetry so: $u = u(x, r, t)$

- For fully developed flow, by definition: $u = u(r, \boxed{t})$

  the difference between laminar and turbulent is: for turbulent, u is a function of t

- Time averaged velocities, often very meaningful: $\overline{u(r)} = \frac{1}{T}\int_0^T u(r,t)\ \mathrm{d}t$

- Mean (cross-sectional average) velocity (slide 48): $U = \overline{\langle u(r) \rangle} = \frac{Q}{A} = \frac{1}{\pi R^2}\int_0^R 2\pi r\ \overline{u(r)}\ \mathrm{d}r$

- Time-averaged velocity on centreline is maximum: $u_{max} = \overline{u(r=0)} = \frac{1}{T}\int_0^T u(0,t)\,\mathrm{d}t$

- We're thinking about profiles, typically $\dfrac{\overline{u(r)}}{u_{\max}}$ or $\dfrac{\overline{u(r)}}{U}$

  we usually normalise time averaged velocity with umax remember, as mentioned previously, smooth pipes doesn't mean no friction! it just mean small k_s/D

Turbulent Flow Profiles in Smooth Pipes

Based on **experimental evidence**, the radial distribution of the (time-averaged) axial velocity in **smooth pipes** may be expressed by

$$\frac{\overline{u(r)}}{u_{\max}} = \left(\frac{R-r}{R}\right)^{\frac{1}{n}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$$

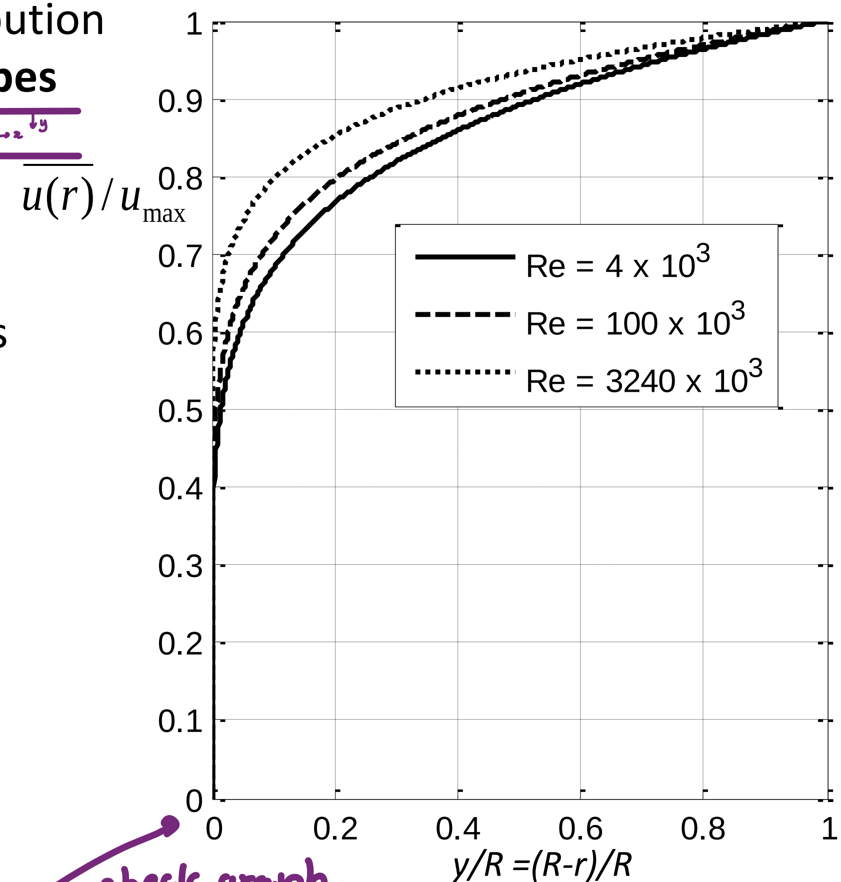
where the exponent n varies slightly with Reynolds number.

The ratio of mean, U, to maximum velocity , u, is

$$\frac{U}{u_{max}} \approx \frac{2n^2}{(n+1)(2n+1)} = \frac{2n^2}{2n^2+3n+1}$$

So what is n? Empirical and approximate…

Re	$4 \cdot 10^3$	$100 \cdot 10^3$	$3240 \cdot 10^3$
n	6	7	10
U/u_{max}	0.791	0.837	0.865

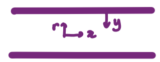


check graph.

- Velocity profile ‘fuller’ (flatter, i.e. closer to uniform velocity) for larger Re number
- **Results Differ** for **rough pipes -** validity close to the wall is not clear
- **Empirical** *laws of friction* or *laws of resistance* (pressure gradient and rate of flow)

Introduction to Boundary Layers (1)

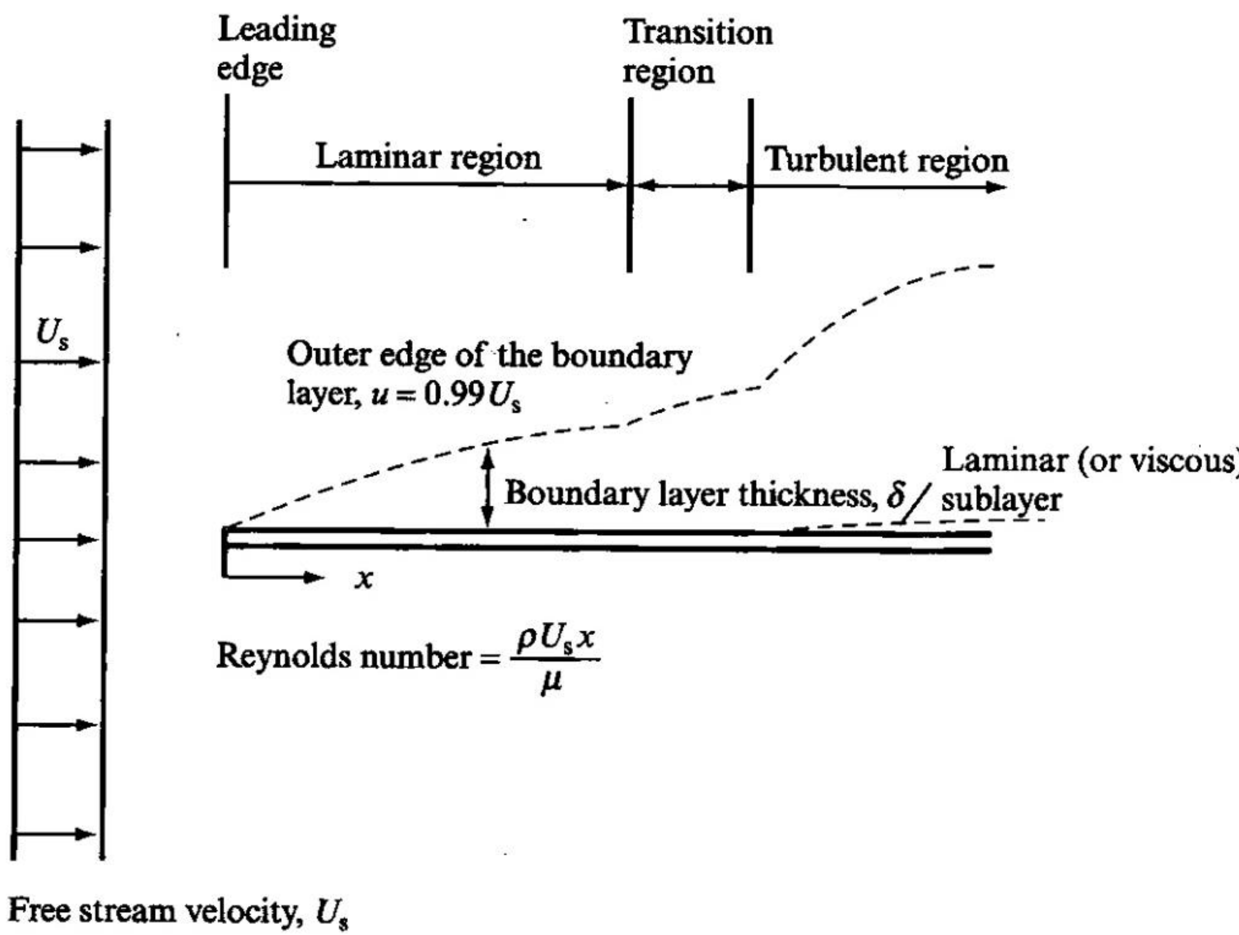
For wall-bounded flows cross-stream coordinate is typically denoted *y* with *y=0* at the wall, EXCEPTION: pipes *r* is typical with *r=0* on the centreline, i.e. *y = R - r*.



- Boundary layer is the region of fluid close to an immersed surface

where velocity changes significantly

- Consider example of a flat plate
- Fluid in contact with plate has zero velocity (no slip)
- Gradient in velocity between plate surface and free stream velocity



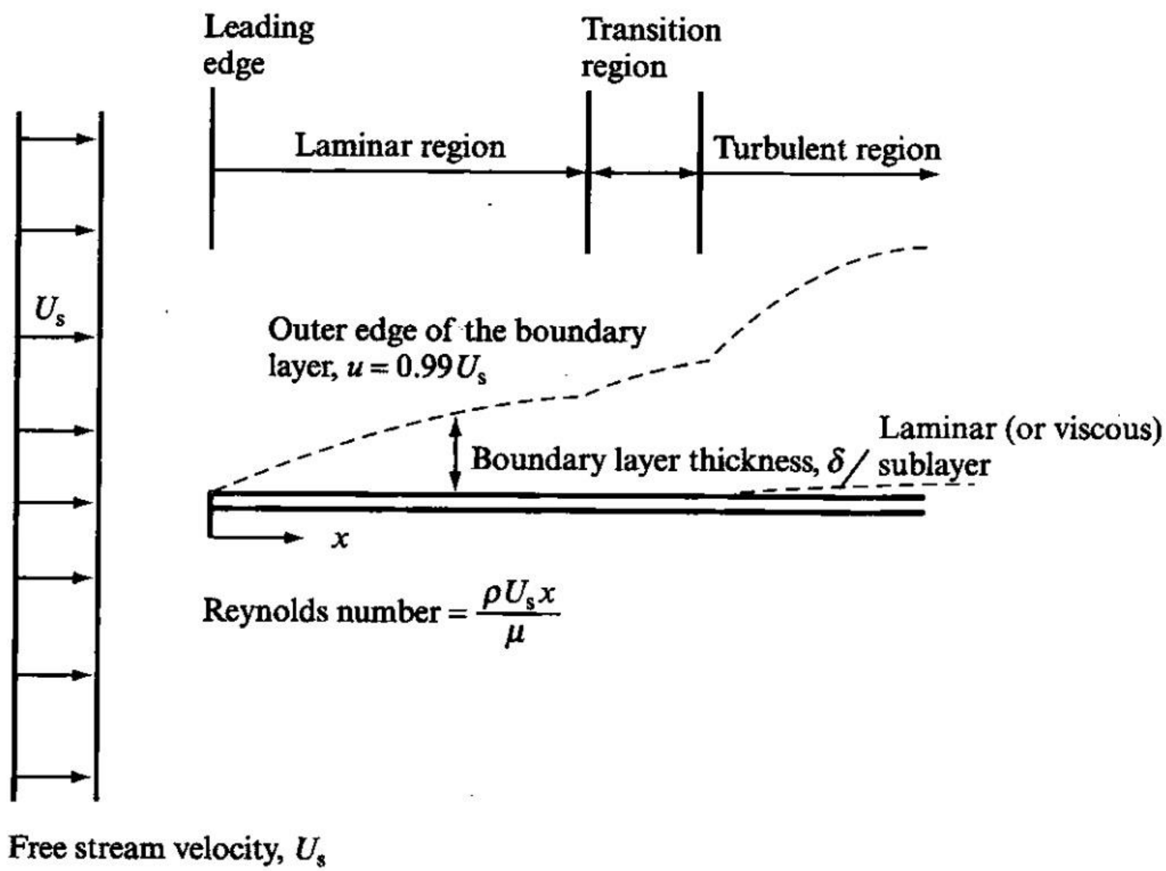
Introduction to Boundary Layers (2)

- Shear stress $\tau = \mu \dfrac{du}{dy}$

acting at the plate sets up a shear force that opposes the fluid motion

- Effect of shear force increases along plate, increasing the thickness of fluid layer that is affected

- Reynolds number typically increases along plate, leading to turbulent flow conditions



Boundary Layer Development (1)

- Very close to the plate surface the flow remains viscously dominated (laminar-like)

Velocity gradient is governed by the fluid viscosity and the wall-shear stress, τ_0

$$\frac{du}{dy} = \tau_0 / \mu$$

valid primary when viscous stress dominates (laminar / viscous sublayer)

Integrating both sides

$$\int du = \int \frac{1}{\mu}\tau_0 \, dy$$

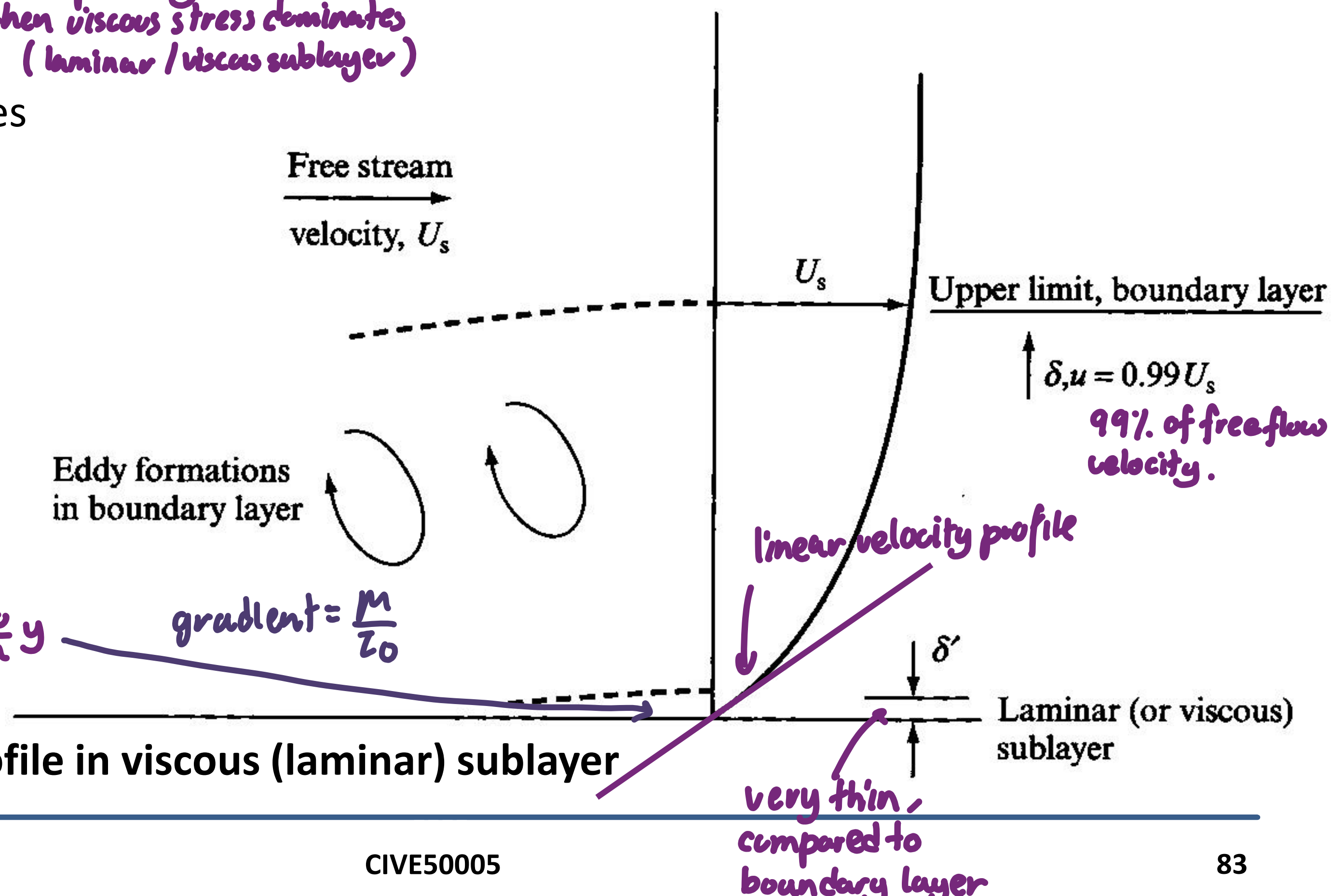
$$u = \tau_0 \frac{y}{\mu} + C$$

$$u(y=0) = 0$$

$$\Rightarrow C = 0$$

$$u = \tau_0 \frac{y}{\mu}$$

$u = \frac{\tau_0}{\mu} y$ gradient $= \frac{\mu}{\tau_0}$



- **Linear velocity profile in viscous (laminar) sublayer**

Boundary Layer Development (2)

The velocity in the viscous sublayer is (using $\mu \equiv \rho \nu$)

$$u = \tau_0 \frac{y}{\mu} = \frac{\tau_0}{\rho} \frac{y}{\nu} \qquad \text{-(A)}$$

rewrite $\mu = \rho\nu$

kinematic viscosity

We want to write a wall-shear velocity, $u_* = \mathrm{f}(\tau_0)$, i.e. the velocity evident from the wall-shear stress [M L^{-1} T^{-2}].

Dimensionally wall-shear velocity [L T^{-1}] cannot depend on mass and so

$$u_* = \mathrm{f}\left(\frac{\tau_0}{\rho}\right) \quad \text{and} \quad \frac{\tau_0}{\rho} = \left[\frac{\mathrm{ML^{-1}T^{-2}}}{\mathrm{ML^{-3}}}\right] = \left[\frac{\mathrm{T^{-2}}}{\mathrm{L^{-2}}}\right] = \left[\frac{\mathrm{L^2}}{\mathrm{T^2}}\right]$$

So the wall-shear velocity is:

$$u_* = \sqrt{\tau_0 / \rho}$$

what is u_*?

- a characteristic velocity scale.

(i.e. not an actual velocity at any specific point y)

Now eq (A) can be written as: $u = \frac{u_*^2}{\nu} y$, or more intuitively: $\boxed{\frac{u}{u_*} = \frac{u_*}{\nu} y}$

important length scale

why length?

cause u/u_* is dimensionless

if $[y] = m$, $[u_*/\nu] = m^{-1}$

Length scale ν / u_* known as 'wall units'

Experimentally is has been shown that the viscous sub-layer thickness is $\delta' = (3-5)\nu / u_*$

"3 to 5" not "3 minus 5"

<u>We want to exploit this thinking for BLs and apply it to pipes</u>

Consider two regions: a) Wall region 'near' the wall, b) Core region towards the pipe centre

Turbulent Flow Profiles in Pipes (Wall)

a) Wall region $0 < y < (70-200)\dfrac{\nu}{u_*}$

Viscous sub layer (within a few wall units of the wall)

$$U^+ = \frac{\overline{u(y)}}{u_*} = \frac{y}{\nu / u_*} = y^+$$

Beyond but still near the wall – the law of the wall!

In general

$$\frac{\overline{u(y)}}{u_*} = C_1 \ln \frac{y}{\nu / u_*} + C_2 \quad \text{or} \quad U^+ = C_1 \ln y^+ + C_2$$

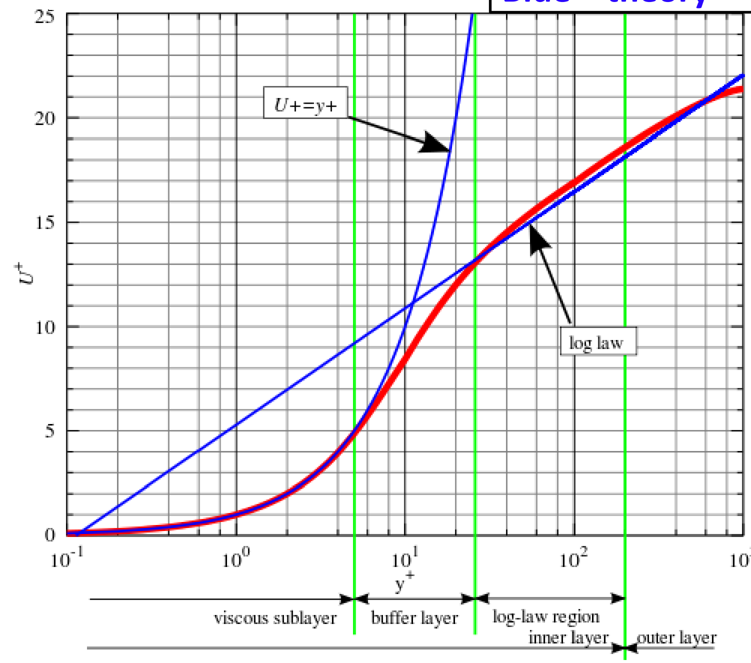
it's a log curve!

Typical to scale the:

- Cross-stream coordinate by wall units $y^+ = \dfrac{y}{\nu / u_*}$
- Velocity by wall-shear velocity $U^+ = \dfrac{u(y)}{u_*}$

Red = data

Blue = theory



For very **large Reynolds** numbers

a) $0 < \dfrac{yu_*}{\nu} < 3\sim5$ — **purely laminar friction** — Viscous sub layer

$10\sim40 < \dfrac{yu_*}{\nu} < 70\sim200$ — **laminar-turbulent friction** — Law of the wall

Consider two regions: a) Wall region 'near' the wall, b) Core region towards the pipe centre

Turbulent Flow Profiles in Pipes (Wall)

Typical to scale the:
- Cross-stream coordinate by wall units $y^+ = \dfrac{y}{\nu / u_*}$
- Velocity by wall-shear velocity $U^+ = \dfrac{u(y)}{u_*}$

a) Wall region $0 < y < (70-200)\dfrac{\nu}{u_*}$

Viscous sub layer (within a few wall units of the wall)

$$\boxed{U^+ = \frac{\overline{u(y)}}{u_*} = \frac{y}{\nu / u_*} = y^+}$$

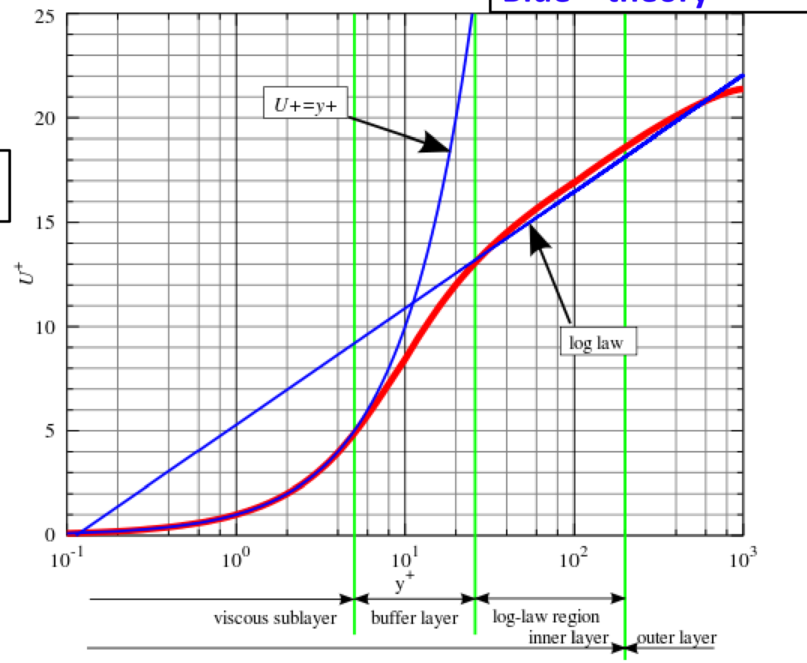
Beyond but still near the wall – the law of the wall!

For rough pipes $\boxed{U^+ = \dfrac{\overline{u(y)}}{u_*} = \dfrac{1}{\kappa}\ln\dfrac{y}{k_s} + 8.5}$ y scaled by roughness

$\dfrac{1}{\kappa} \approx \dfrac{1}{0.4} = 2.5;\ \kappa$ known as von Karman's constant

Red = data

Blue = theory



For very **large Reynolds** numbers

a) $0 < \dfrac{yu_*}{\nu} < 3\sim5$ linear — **purely laminar friction** Viscous sub layer

$10\sim40 < \dfrac{yu_*}{\nu} < 70\sim200$ log — **laminar-turbulent friction** Law of the wall

b) $\dfrac{yu_*}{\nu} > 70\sim200$ log — **purely turbulent friction** velocity defect law

Consider two regions: a) Wall region 'near' the wall, b) Core region towards the pipe centre

Turbulent Flow Profiles in Pipes (Core)

b) Core Region $(70-200)\frac{\nu}{u_*} < y \le R$

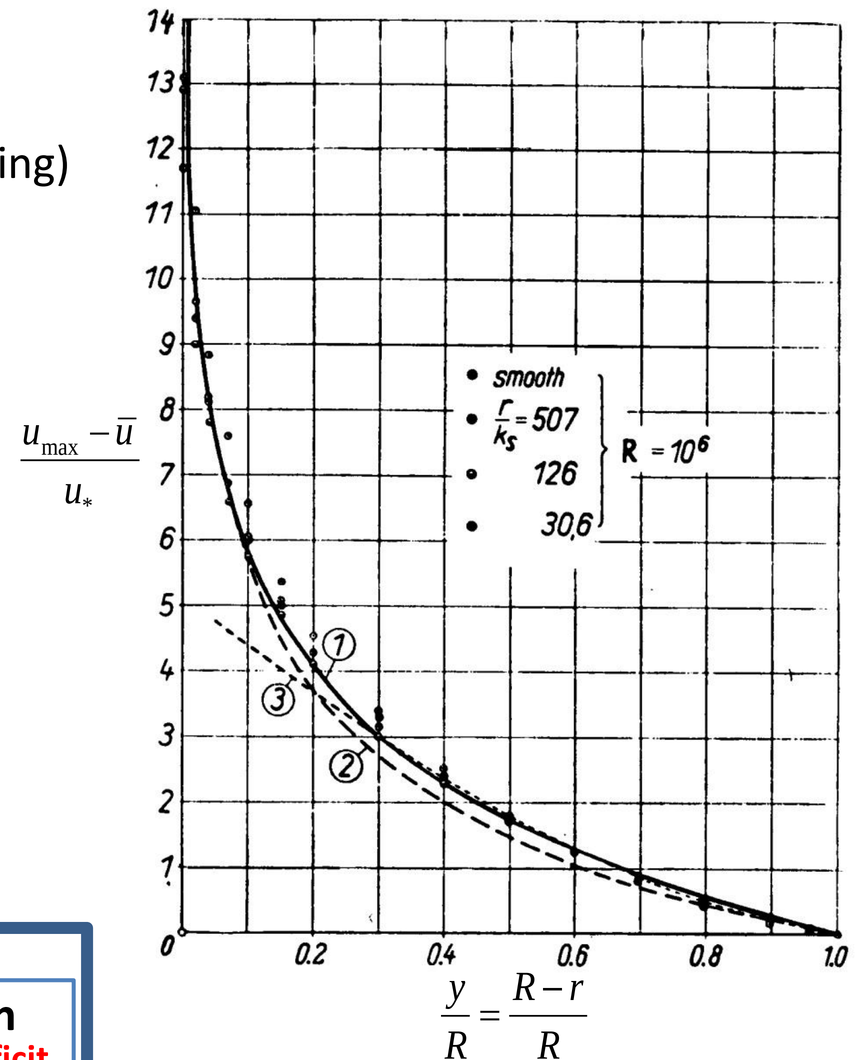
Velocity Defect Law after Prandtl (see Schlichting)

$$\frac{u_{max} - \overline{u(y)}}{u_*} = -\frac{1}{\kappa}\ln\frac{y}{R}$$

or

$$\boxed{\overline{u(y)} = \frac{u_*}{\kappa}\ln\frac{y}{R} + u_{max}}$$

- u_{max} is the centreline velocity
- Velocity deficit law for radial distribution gives good fit to experimental data
- Curve (1) is Velocity Defect Law



b) $\frac{yu_*}{\nu} > 70 - 200$ **purely turbulent friction**

Core – Velocity deficit

Shear Stress in Turbulent Pipe Flow

We developed the shear velocity (slide 69)

$$u_* = \sqrt{\tau_0 / \rho}$$

We defined the wall-shear (slide 54)

$$\tau_0 = \frac{f}{8}\rho U^2$$

where U is the cross-section-averaged velocity.
Equating τ_0 in both gives:

$$\tau_0 = \rho u_*^2 = \frac{f}{8}\rho U^2 \qquad \text{hence} \qquad u_* = \sqrt{\frac{f}{8}}U$$

$$Q = 2\pi\int_0^R r\,\overline{u}(r)\ \mathrm{d}r$$

$$M = 2\pi\int_0^R r\,\overline{u}^2(r)\ \mathrm{d}r$$

The cross-section-averaged velocity is given by (slides 79)

$$U = \frac{Q}{A} = \frac{1}{\pi R^2}2\pi\int_0^R r\,\overline{u(r)}\ \mathrm{d}r$$

Substituting the velocity defect law gives

$$U = \frac{1}{\pi R^2}\int_0^R 2\pi r\left[u_{\max} + \frac{u_*}{\kappa}\ln\left(1-\frac{r}{R}\right)\right]dr = u_{\max} - \frac{3}{2}\frac{u_*}{\kappa}$$

$$\frac{1}{\kappa} \approx \frac{1}{0.4} = 2.5;$$

κ known as von Karman's constant

This slides is to find τ_0 (shear stress at wall)

Turbulent Pipe Flow – what we can do

From:

Easily measured by a pitot tube on the centreline, see your upcoming lab

$$U = u_{max} - \frac{3}{2}\frac{u_*}{\kappa}$$

find this easily

von Karman's constant ≈ 0.4

Often known or prescribed by a pump, $U = Q/A$

then able to find τ_0

We can infer an estimate of the wall shear velocity $u_* = \sqrt{\tau_0 / \rho}$ and hence estimate the wall shear τ_0 and the size of the wall unit ν / u_* .
We could then estimate the thickness of the viscous sub-layer and it's profile and so too for the 'law of the wall' region

Also, remember (slide 64), $h_f = \dfrac{4\tau_0 L}{\rho g D}$ so: $h_f = \dfrac{4\tau_0 L}{\rho g D} = \dfrac{2\tau_0 L}{\rho g R} = \dfrac{2u_*^2}{g}\dfrac{L}{R}$

We could compare this estimate of h_f to other empirical estimates,
e.g. from Darcy Weisbach equation (slide 54) $h_f = f\dfrac{L}{D}\dfrac{U^2}{2g}$ where f is determined empirically (Moody diagram)

This is significant progress in understanding pipe flows, well done!

This slide is to find $\tau(r)$ (shear stress distribution)

You can now complete: Tutorial 4

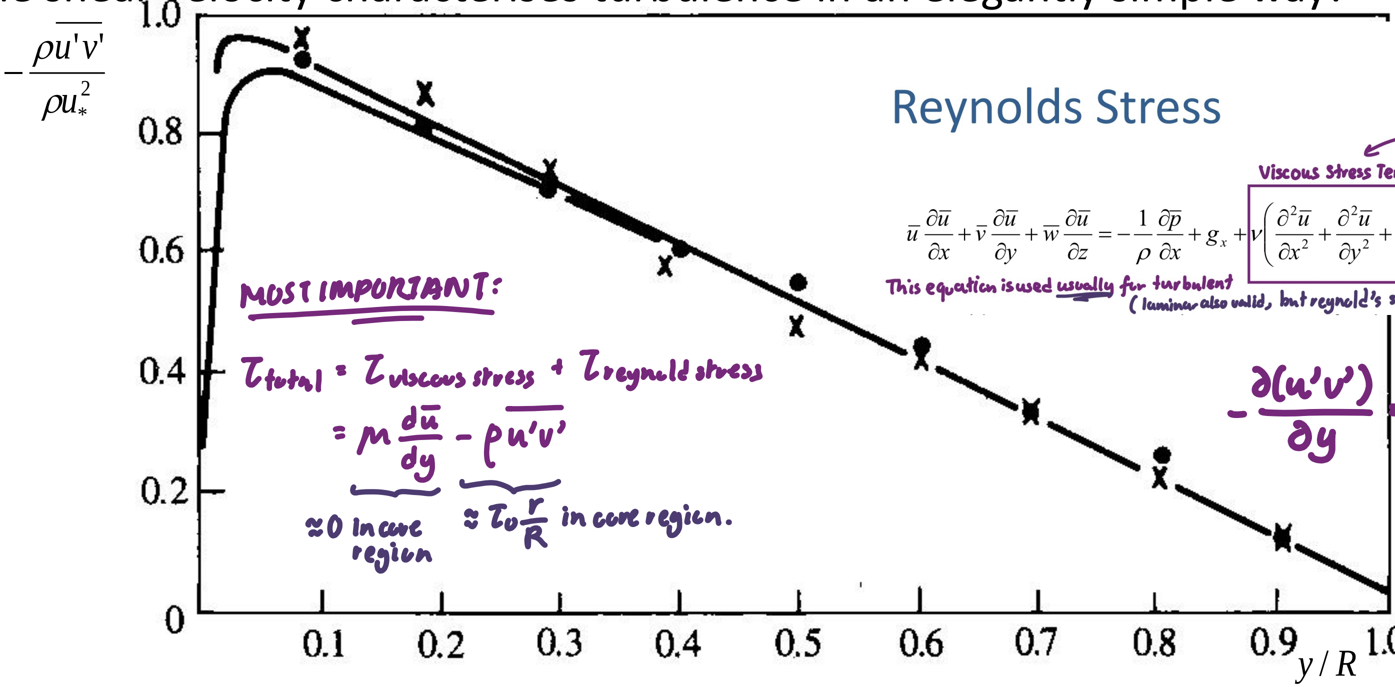
Shear Stress in Turbulent Pipe Flow

In the **core region** the Reynolds stress ($\overline{\rho u'v'}$ see slide 59) profile is found to be approximately **linear**

$$-\frac{\rho\overline{u'v'}}{\tau_0} = -\frac{\rho\overline{u'v'}}{\rho u_*^2} \approx 1-\frac{y}{R} = \frac{r}{R}$$

Note this flow is statistically axisymmetric and so only $\overline{\rho u'v'}$ terms remain

The shear velocity characterises turbulence in an elegantly simple way!



MOST IMPORTANT:

$$\tau_{total} = \tau_{viscous\ stress} + \tau_{reynold\ stress} = \underbrace{\mu\frac{d\bar{u}}{dy}}_{\approx 0 \text{ in core region}} - \underbrace{\rho\overline{u'v'}}_{\approx \tau_0 \frac{r}{R} \text{ in core region.}}$$

due to turbulent transport of momentum from high momentum regions to low momentum region.

Viscous Stress Terms — usually alot bigger than — Reynolds stress terms!

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + g_x + \nu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}\right) - \left(\cancel{\frac{\partial \overline{u'^2}}{\partial x}} + \frac{\partial \overline{u'v'}}{\partial y} + \cancel{\frac{\partial \overline{u'w'}}{\partial z}}\right)$$

0 0

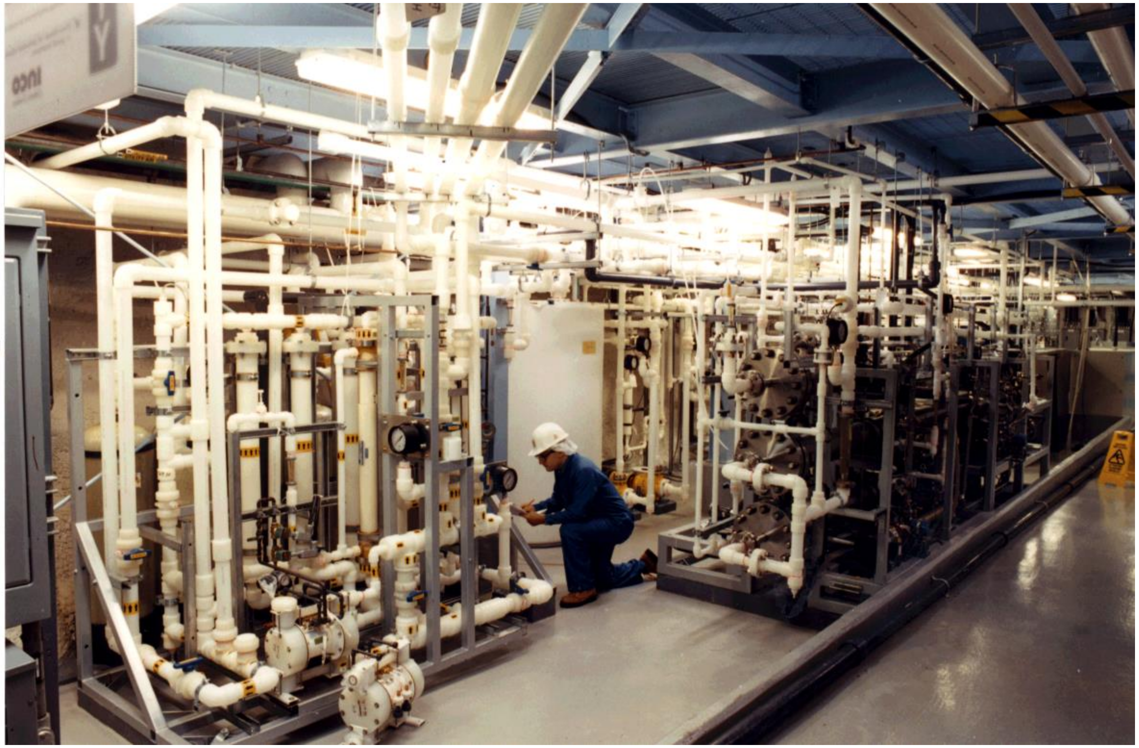
This equation is used usually for turbulent (laminar also valid, but reynold's stress terms ≈ 0)

$$-\frac{\partial(\overline{u'v'})}{\partial y} = -\frac{\partial}{\partial y}\left(u_*^2\left(1-\frac{y}{R}\right)\right) = \frac{u_*^2}{R} = \underbrace{\boxed{\frac{1}{\rho}\frac{\tau_0}{R}}}_{\text{reynold's stress term for Navier's Stokes Eqn form}}$$

We can use our estimates of the wall shear stress via the shear velocity to estimate the Reynolds stress – we stated (slide 59) that characterising the Reynolds stress was extremely challenging – you've made real progress in pipes, well done!

3.3 Pipe Systems

rarely able to be drive by gravity.
Else we'll just use open channel flow



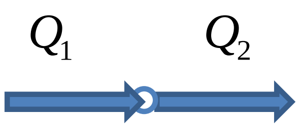
- Pipe **networks** such as in domestic water supply can be **very large**
- In addition to pipes, networks contain pumps, valves, etc…
- Real pipes networks usually lead to a **large set of coupled (simultaneous) equations,** some of which are nonlinear. Hence an **iterative scheme** is usually required, and is frequently the simplest route, to find a solution

Mass Conservation

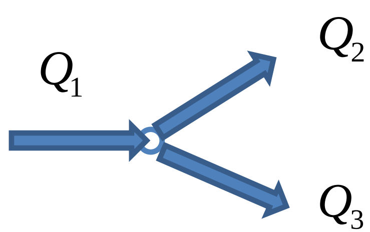
In the scope of this course, analysis is in the context of:

- **Steady** flow - $\partial/\partial t = 0$
- **Incompressible** flow, ρ = constant
- Work with volume (mass) flux as flow variable, i.e. Q rather than U

Mass conservation at a junction



Pipes in series: $Q_1 = Q_2$

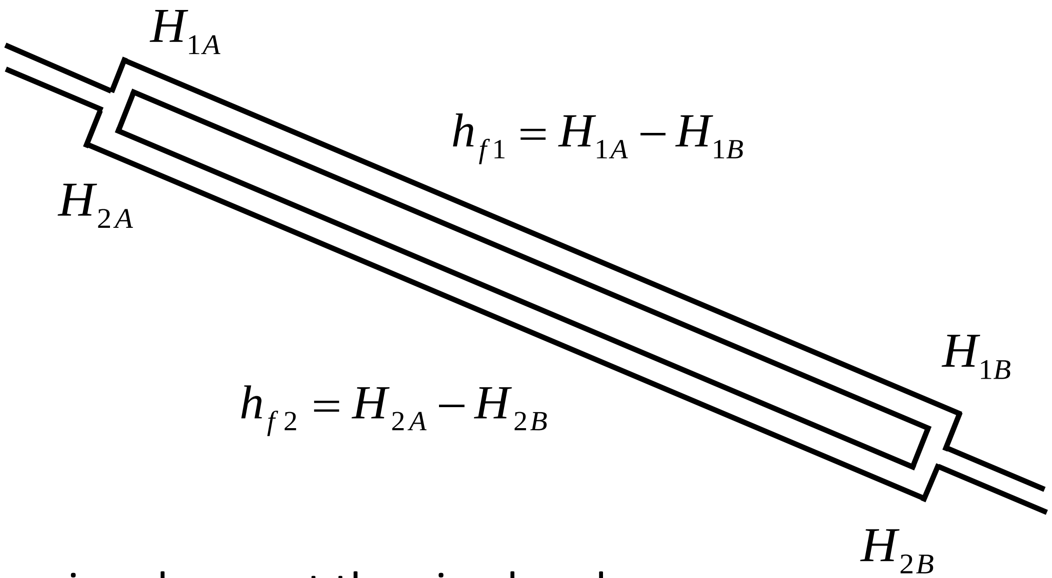


like unknown force, you can always assume the direction of volume flux by drawing arrows. If the assumed direction is wrong, Q will be negative

Pipes in parallel: $Q_1 = Q_2 + Q_3$

Momentum Conservation

Head loss across two pipe segments in parallel



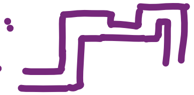
Neglecting minor losses at the pipe bends

$$H_{1A} = H_{2A} \qquad H_{1B} = H_{2B}$$

Hence

$$\boxed{h_{f1} = h_{f2}}$$

pipe segment is not last page

it is: eg. 

Head Loss in a Pipe Segment

From 1st **and** 2nd year we know that the **total head loss** is given by

$$h_f + h_L = \left(f\frac{L}{D} + \sum_j \xi_j \frac{A^2}{A_j^2} \right) \frac{U^2}{2g}$$

we want to write in term of Q instead of U

Introducing the flow variable Q instead of U yields

$$h_f + h_L = \left(f\frac{L}{D} + \sum_j \xi_j \frac{A^2}{A_j^2} \right) \frac{Q^2}{2gA^2} = \left(f\frac{L}{D} + \sum_j \xi_j \frac{D^4}{D_j^4} \right) \frac{Q^2}{2g(\pi/4)^2 D^4} = KQ^2$$

where

$$K = \frac{8}{g\pi^2 D^4} \left(\overbrace{f\frac{L}{D}}^{\text{major}} + \overbrace{\underbrace{\sum_j \xi_j \frac{D^4}{D_j^4}}_{\text{neglect}}}^{\text{minor}} \right) = \left[\frac{T^2}{L^5} \right]$$

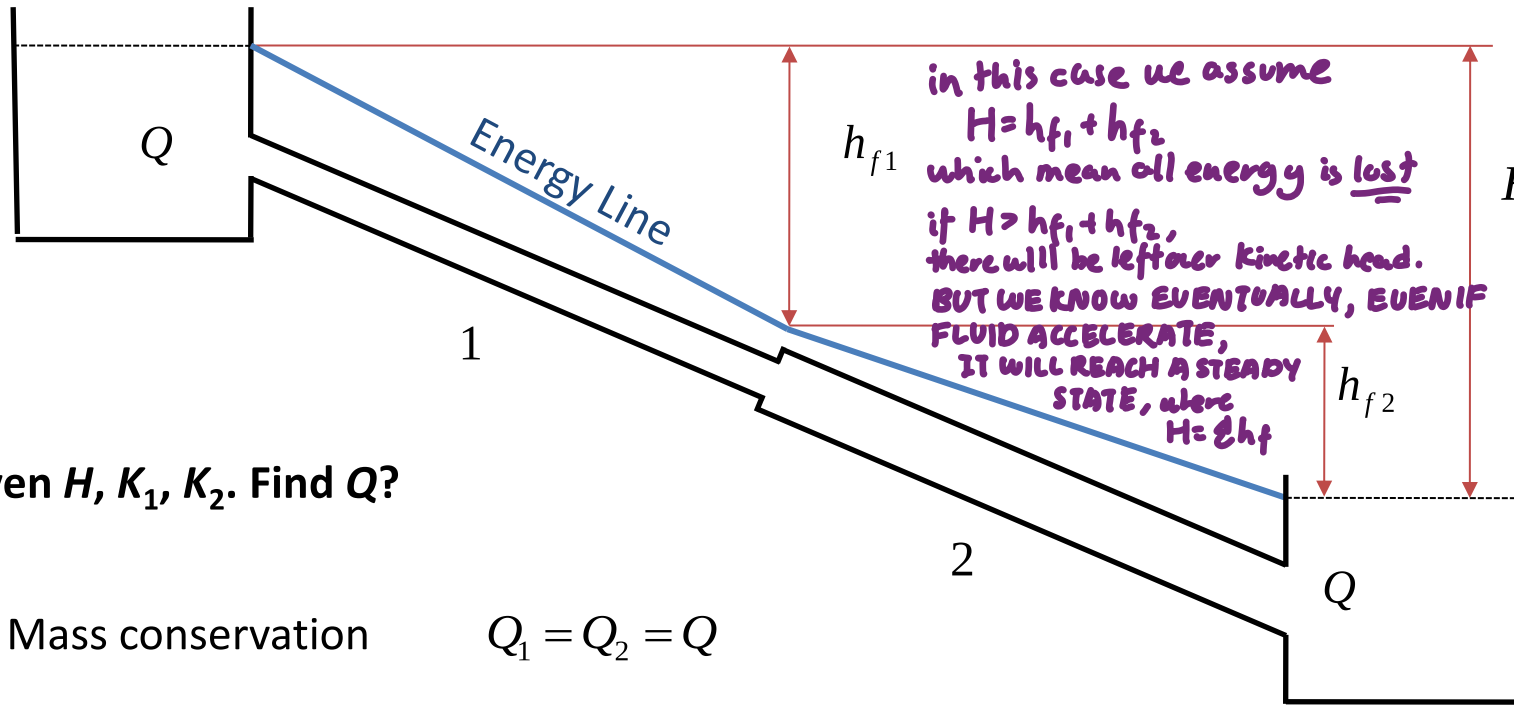
Neglecting minor losses: $K \approx \dfrac{8fL}{g\pi^2 D^5}$

To deal with larger systems, K is mostly assumed to be constant:

- requiring f = constant
- use **Re** $\rightarrow \infty$ limit of Colebrook-White Equation → constant f for specific ks/D (even if you change U of Re = $\frac{UD}{\nu}$)
- called "rough turbulent" approximation

Horizontal asymptote of Moody diagram (slide 55) $\quad f = \left[\log_{10}\left(\dfrac{13.76D^2}{k_s^2} \right) \right]^{-2}$

Pipes in Series



in this case we assume
$H = h_{f1} + h_{f2}$
which mean all energy is <u>lost</u>
if $H > h_{f1} + h_{f2}$,
there will be leftover kinetic head.
BUT WE KNOW EVENTUALLY, EVEN IF FLUID ACCELERATE,
IT WILL REACH A STEADY STATE, where
$H = \sum h_f$

Given *H*, K_1, K_2. Find *Q*?

(i) Mass conservation $\quad Q_1 = Q_2 = Q$

(ii) Steady => Bernoulli balance so: losses = work done, here work is done only by gravitational head *H*, so:

$$H = h_{f1} + h_{f2} = K_1 Q_1^2 + K_2 Q_2^2$$

what we did in y1:

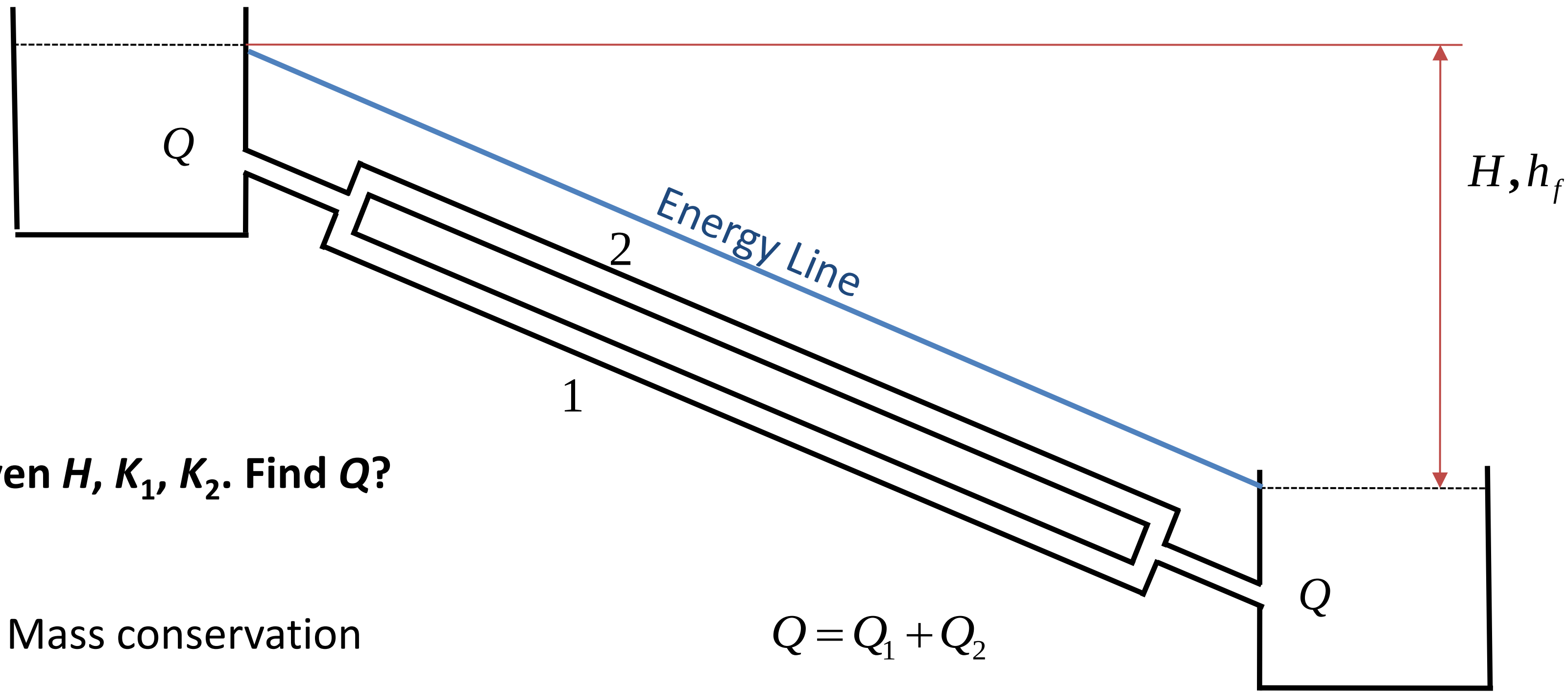
if energy before = energy after...

$$h_1 + \cancel{\frac{u_1^2}{2g}}_{0} = h_2 + \cancel{\frac{u_2^2}{2g}}_{0} + h_{f1} + h_{f2}$$

$$h_1 - h_2 = h_{f1} + h_{f2} \rightarrow H = h_{f1} + h_{f2}$$

Solution is direct: (i) in (ii) $\quad Q = \sqrt{\dfrac{H}{K_1 + K_2}}$

Pipes in Parallel



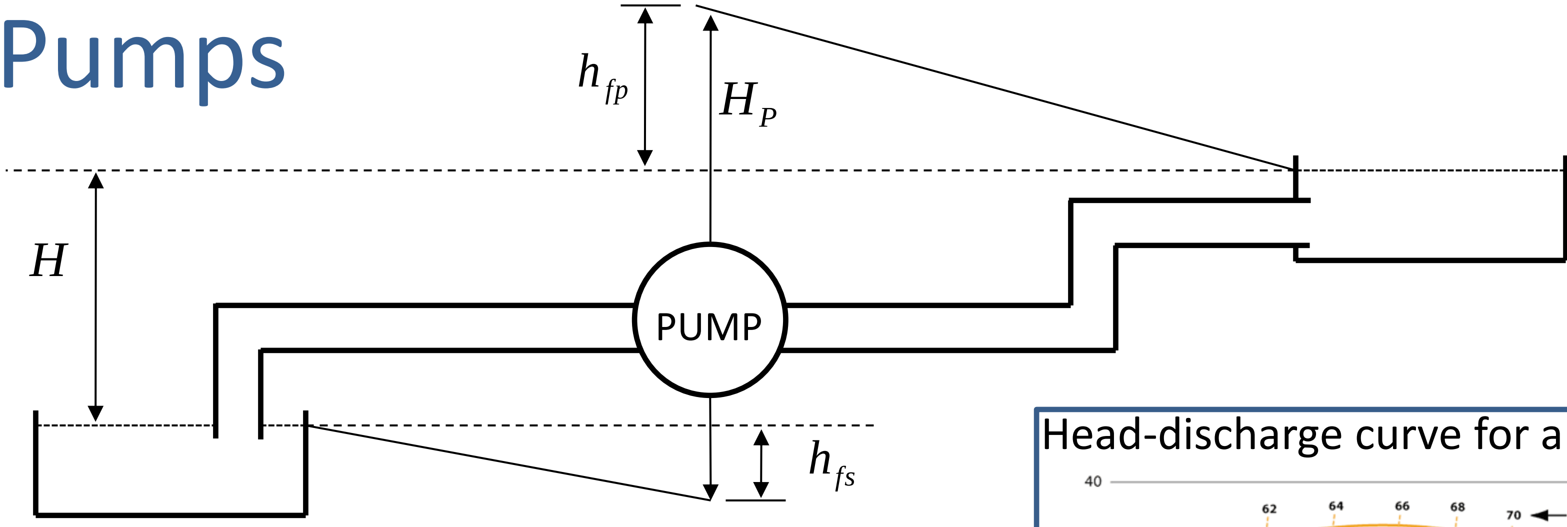
Given *H*, K_1, K_2. Find *Q*?

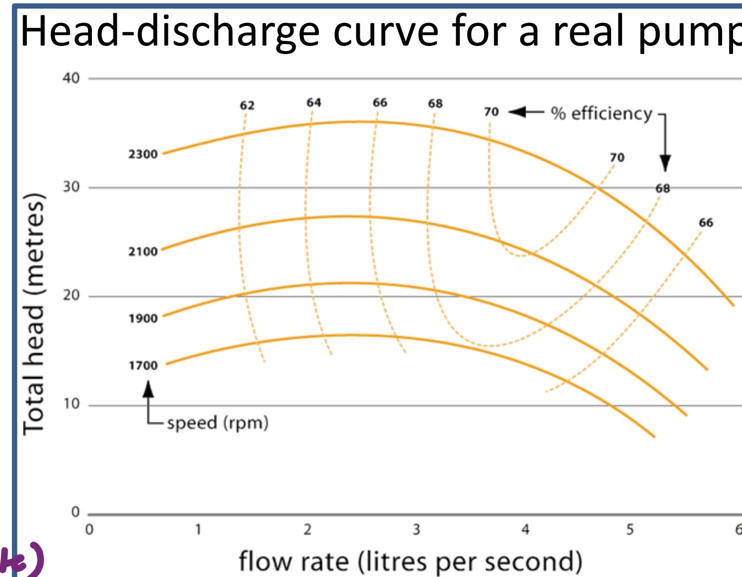
(i) Mass conservation $$Q = Q_1 + Q_2$$

(ii) Steady balance (as before): $$H = h_{f1} = h_{f2} = K_1 Q_1^2 = K_2 Q_2^2$$

Solution is direct: $$Q_1 = \sqrt{\frac{H}{K_1}} \qquad Q_2 = \sqrt{\frac{H}{K_2}}$$

Pumps





Given *H*, *$H_p(Q)$*, *K_s*, *K_p*. Find *Q*?

- $H_p = H_p(Q)$ is head-discharge rating curve for pump
- Mass conservation, Q = constant
- Steady, Bernoulli balance => losses = work done

$$h_{fs} + h_{fp} = H_P - H$$

or energy gain = energy lost

$H_p = H + h_{fs} + h_{fp}$

if $H_p \neq H + h_{fs} + h_{fp}$ (non steady state) there will be $u^2/2g$!

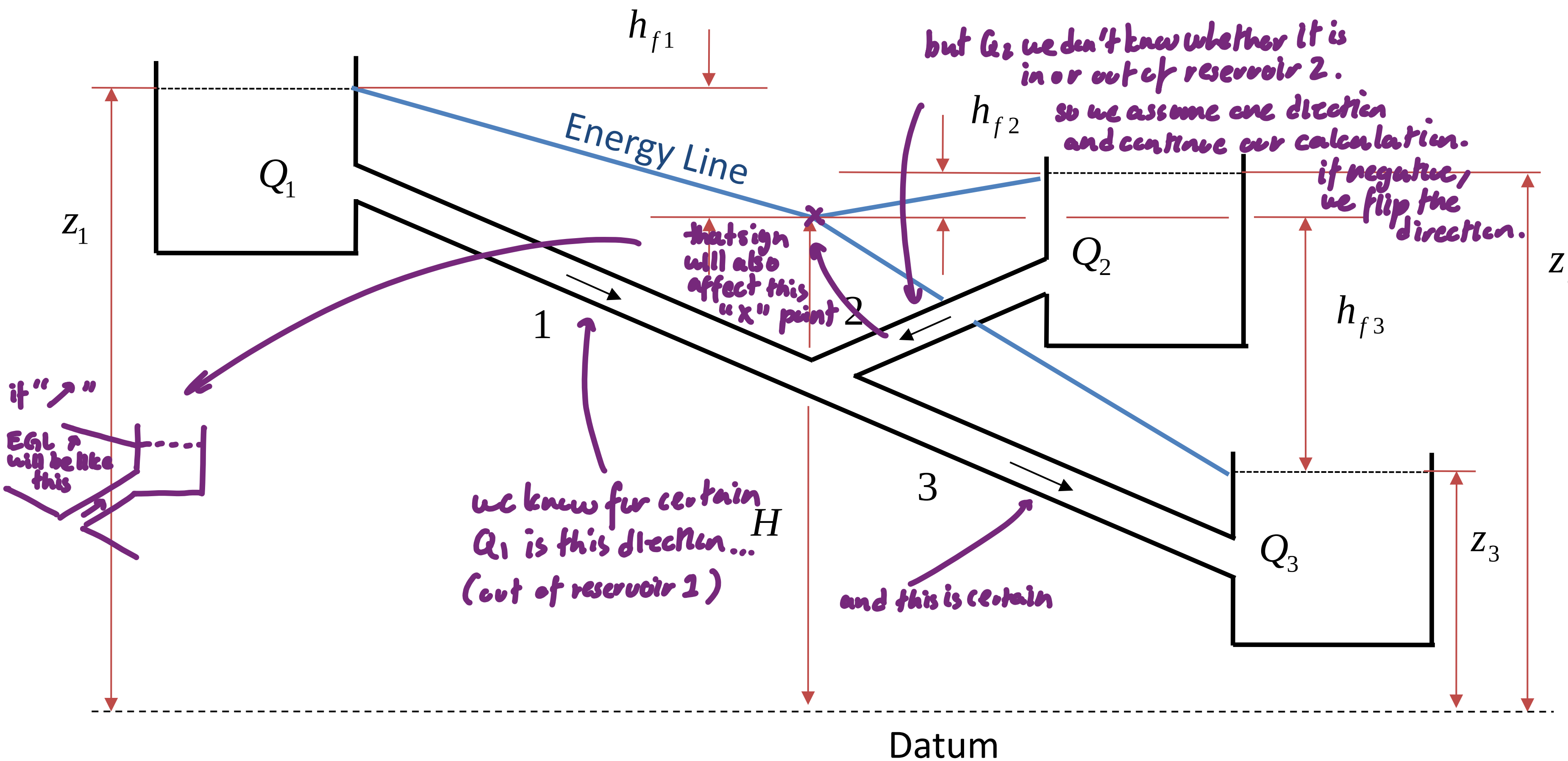
Iterative Solution

I. Guess Q

II. Bernoulli balance so, $0 = H_p(Q) - H - (K_s+K_p)\,Q^2$

III. If Bernoulli balance is satisfied (i.e. sums to zero), then Q is the required flow, otherwise iterate…

Branched Pipes – 3 Reservoir Problem



Branched Pipes – 3 Reservoir Solution

Given z_1, K_1, z_2, K_2, z_3, K_3. Find Q_1, Q_2, Q_3?

calling that center point as C

$E_{before} = E_{after} + E_{lost}$.

$\rightarrow E_1 = E_c + E_{lost,1}$

$\rightarrow E_2 = E_c + E_{lost,2}$

$\rightarrow E_c = E_3 + E_{lost,3}$

E_1 and E_2 only z no $\frac{v^2}{2g}$ or $\frac{p}{\rho g}$

E_c has all three so name it H_c

$z_1 = H_c + K_1|Q_1|Q_1$

$\rightarrow z_2 = H_c + K_2|Q_2|Q_2$

$H_c = z_3 + K_3|Q_3|Q_3$

(i) Assign positive flow direction (arbitrary)

(ii) Mass conservation $\quad Q_1 + Q_2 = Q_3$

(iii) Steady flow so apply Bernoulli balance

this is modulus

$$
\begin{aligned}
H &= z_1 - h_{f1} = z_1 - K_1|Q_1|Q_1 \\
&= z_2 - h_{f2} = z_2 - K_2|Q_2|Q_2 \\
&= z_3 + h_{f3} = z_3 + K_3|Q_3|Q_3
\end{aligned}
$$

$$
K_1|Q_1|Q_1 = \begin{cases} K_1 Q_1^2 & \text{if } Q_1 > 0 \\ -K_1 Q_1^2 & \text{if } Q_1 < 0 \end{cases}
$$

Solution:

(1) Guess H

(2) Use Bernoulli balance

$$
\left.
\begin{aligned}
|Q_1|Q_1 &= (z_1 - H)/K_1 \rightarrow Q_1 \\
|Q_2|Q_2 &= (z_2 - H)/K_2 \rightarrow Q_2 \\
|Q_3|Q_3 &= (H - z_3)/K_3 \rightarrow Q_3
\end{aligned}
\right\}
$$

If $\quad Q_1 + Q_2 = Q_3 \quad$ then valid solution. Otherwise, return to 1

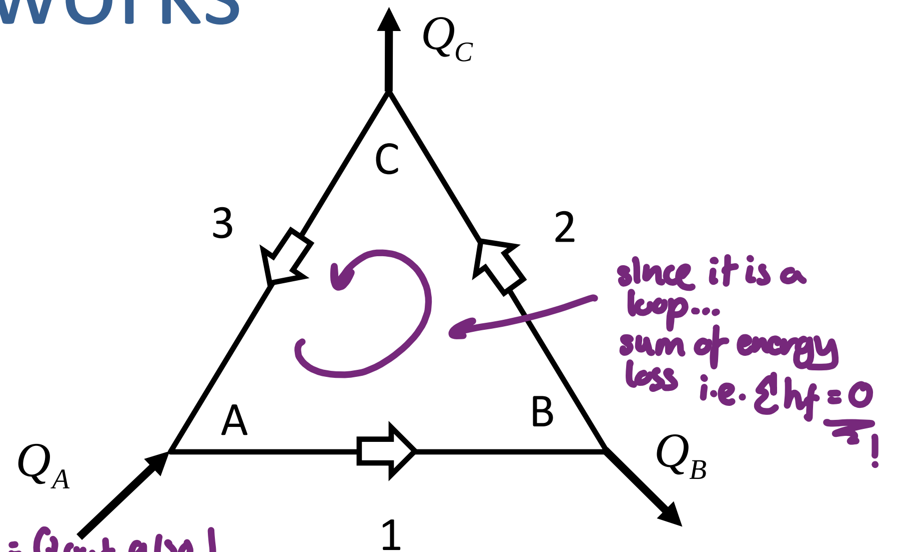
Single Loop Pipe Networks

Given Q_A, Q_B, Q_C; K_1, K_2, K_3. Find Q_1, Q_2, Q_3?

(i) Assign positive flow direction (arbitrary)

(ii) Mass conservation

$$\left.\begin{aligned} Q_A + Q_3 &= Q_1 \\ Q_1 &= Q_B + Q_2 \\ Q_2 &= Q_C + Q_3 \end{aligned}\right\} \text{Qin = Qout}$$



- Only 2 independent equations! $\underbrace{Q_A = Q_B + Q_C}_{}$ is given.

Qin = Qout also !

can be subbed in to get rid 1 eqn.

(iii) Steady so Bernoulli balance around loop

$$\sum h_f = 0 = h_{f1} + h_{f2} + h_{f3} = K_1|Q_1|Q_1 + K_2|Q_2|Q_2 + K_3|Q_3|Q_3$$

- Mathematically closed, 3 unknowns and 3 simultaneous (coupled) equations
- Single loop gives single nonlinear equation (from Bernoulli balance)
- Can be solved analytically or with an iterative solution
- Numerical solution (Newton-Raphson...), existing computer codes

Single Loop Pipe Networks - solution

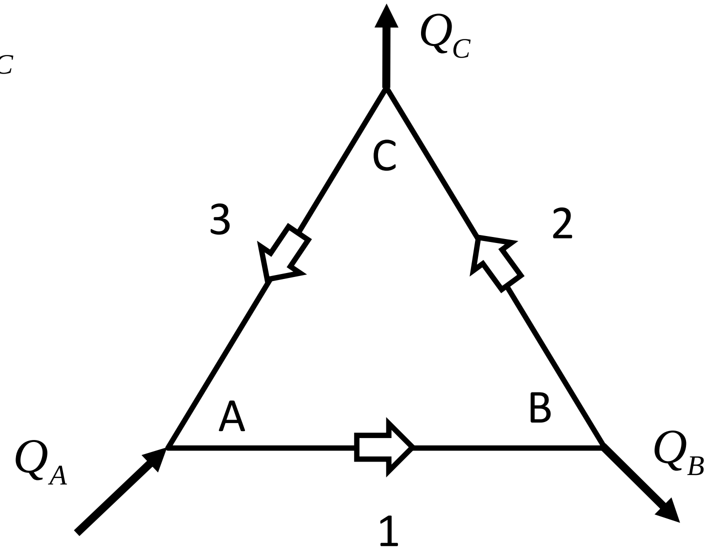
Q_A, Q_B, Q_C; K_1, K_2, K_3 are all known and $Q_A = Q_B + Q_C$

(ii) Mass conservation

$$Q_A + Q_3 = Q_1 \Rightarrow Q_3 = Q_1 - Q_A$$

$$Q_1 = Q_B + Q_2 \Rightarrow Q_2 = Q_1 - Q_B$$

$$Q_2 = Q_C + Q_3$$



Sub the above into the Bernoulli balance equation

$$K_1|Q_1|Q_1 + K_2|Q_2|Q_2 + K_3|Q_3|Q_3 = 0$$

$$\Rightarrow K_1|Q_1|Q_1 + K_2|Q_1 - Q_B|(Q_1 - Q_B) + K_3|Q_1 - Q_A|(Q_1 - Q_A) = 0$$

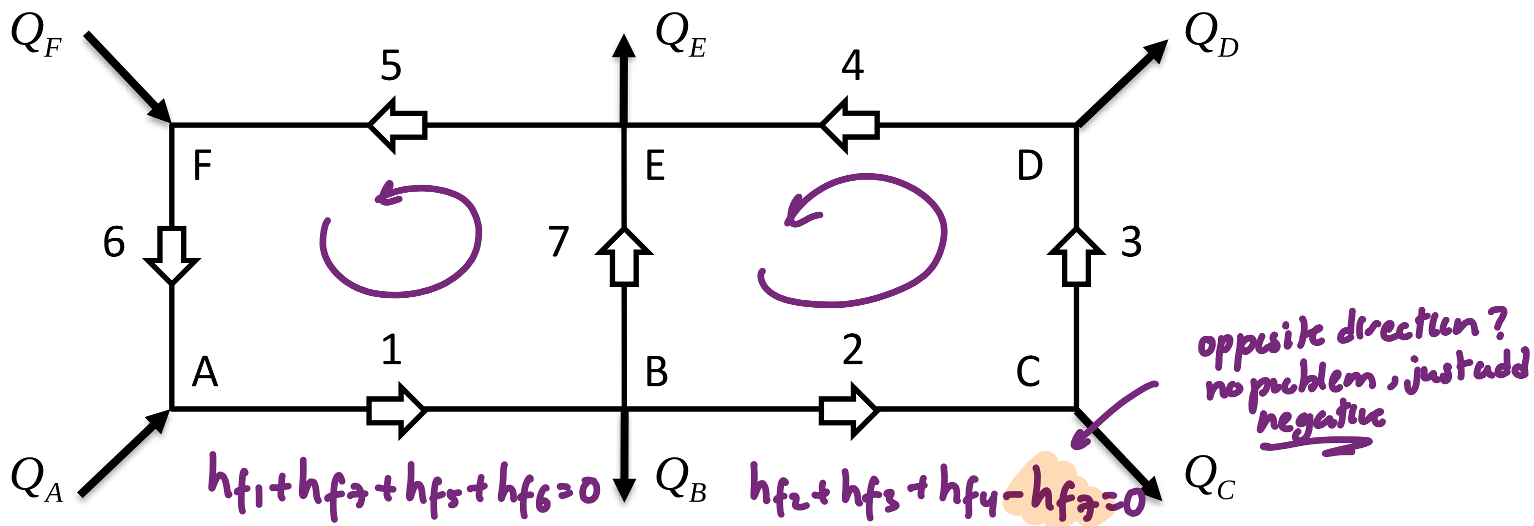
Modulus signs just help with directions, solve and then use physics (e.g. diagram) to determine directions

$$K_1 Q_1^2 + K_2 (Q_1 - Q_B)^2 + K_3 (Q_1 - Q_A)^2 = 0$$

$$\Rightarrow (K_1 + K_2 + K_3) Q_1^2 - 2(K_2 Q_B + K_3 Q_A) Q_1 + \ (K_2 Q_B^2 + K_3 Q_A^2) = 0$$

Quadratic in Q_1, just use the quadratic formula to solve!

Double Loop Pipe Network



$h_{f1} + h_{f7} + h_{f5} + h_{f6} = 0$ $h_{f2} + h_{f3} + h_{f4} - h_{f7} = 0$

Given Q_A, Q_B, Q_C, Q_D, Q_E, Q_F; K_1, K_2, K_3, K_4, K_5, K_6, K_7. Find Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7?

(i) Assign positive flow direction (arbitrary)

(ii) Mass conservation

$$Q_A + Q_6 = Q_1 \qquad Q_1 = Q_B + Q_2 + Q_7 \qquad Q_2 = Q_C + Q_3$$

$$Q_3 = Q_D + Q_4 \qquad Q_4 + Q_7 = Q_E + Q_5 \qquad Q_F + Q_5 = Q_6$$

- Only 5 **independent** equations! $Q_A + Q_F = Q_B + Q_C + Q_D + Q_E$ is given.

Double Loop Pipe Network (cont.)

(iii) Steady so Bernoulli balance around each loop

$$K_1|Q_1|Q_1 + K_7|Q_7|Q_7 + K_5|Q_5|Q_5 + K_6|Q_6|Q_6 = 0$$

$$K_2|Q_2|Q_2 + K_3|Q_3|Q_3 + K_4|Q_4|Q_4 - K_7|Q_7|Q_7 = 0$$

- Mathematically closed, 7 unknowns and 7 simultaneous (coupled) equations
- Double loop gives two nonlinear equation (from Bernoulli balance)
- Cannot be solved analytically (see tutorial 6, Q4) an iterative solution is required
- Numerical solution (Newton-Raphson...), existing computer codes
- Hardy-Cross Method is easily implemented in practice (see e.g. Potter & Wiggert (2002))

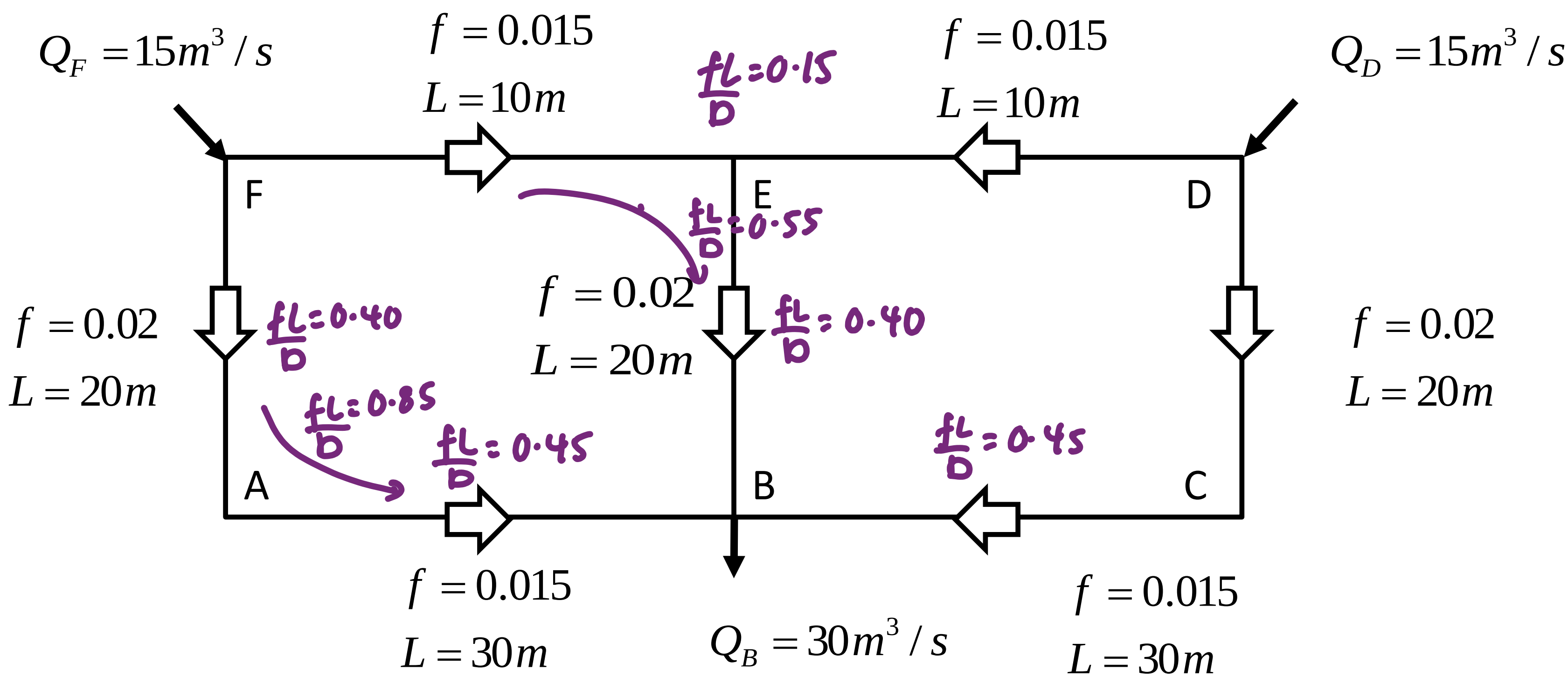
during exams:

- don't need to fully solve (impossible)
- just need to know the steps of:
  - assign initial flow direction
  - write down the eqns
  - if given some info, eg. K values, iterate some times to get good enough solution.

Application Example

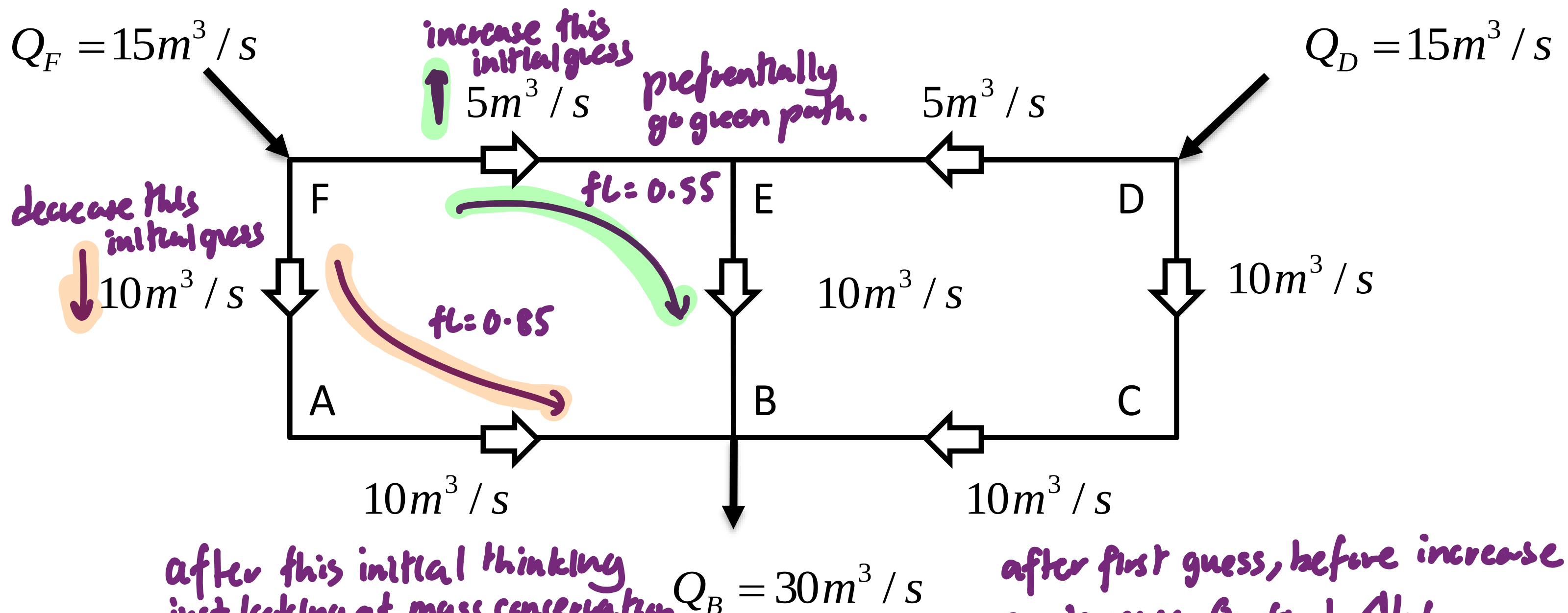
1. Observe Q_{in} and Q_{out}.
   Predict what direction Q will flow.
2. Find $\frac{fL}{D}$ of each pipe

Consider the following pipe system:



- Inflow (Q_F and Q_D) and outflow (Q_B) are given
- All pipe diameters are D=1m
- Volume flow rates Q in each pipe are unknown

Step 1: Guess Flow in Each Pipe

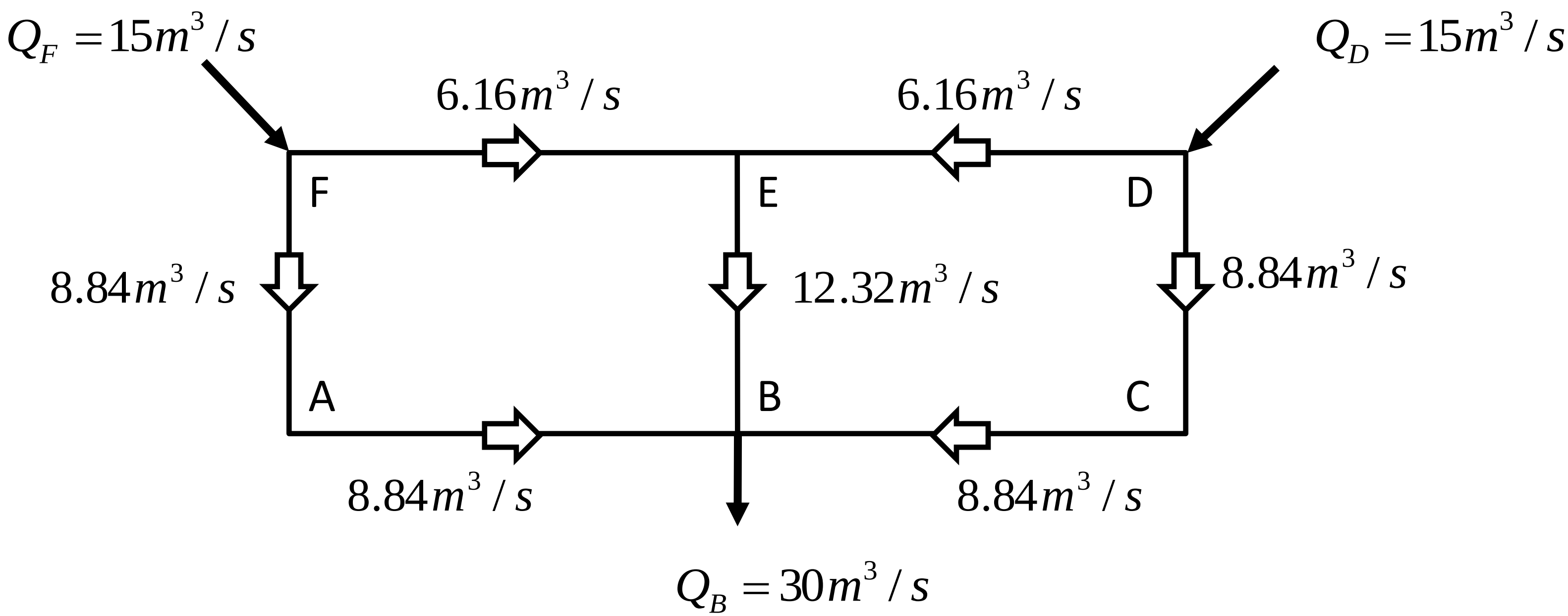


after this initial thinking just looking at mass conservation and splitting, we should look at fL/D! (D constant in this question)

after first guess, before increase or decrease Q, find Σh_f (bernoulli balance) and see whether how close/far it is from $\underline{\underline{0}}$

- Make an intelligent first guess how the flow is distributed
- Mass continuity must be observed here!!!
- This initial step becomes more difficult for larger networks

Step 2: Apply Hardy-Cross Procedure



Discussion of next level thinking:

- The paths FEB (30m) and DEB (30m) are shorter than FAB (50m) and DCB (50m)

  => As a result, pipe EB carries more flow than pipes FA and DC

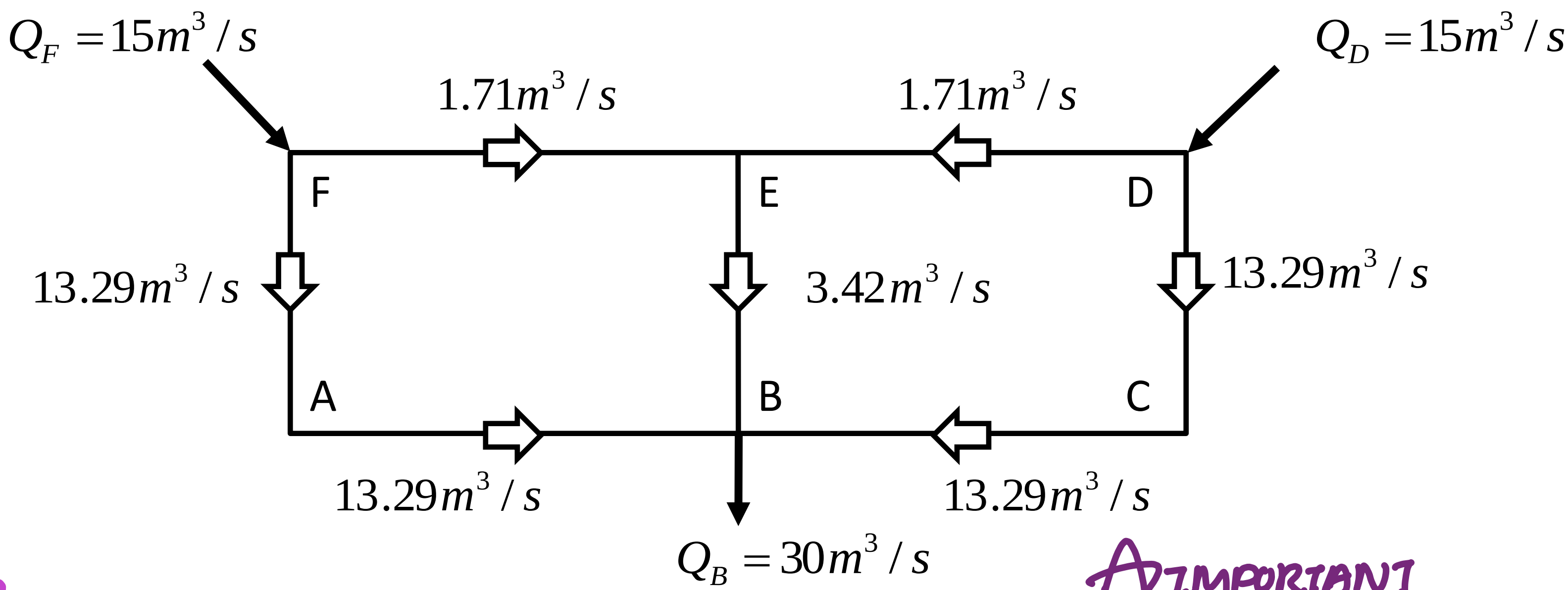
- The flow symmetry results from the symmetry in the pipe specification (***L***, ***f*** and ***D***)

symmetry can be one of the good observation? (if there's symmetry)

You can now complete: Tutorial 5

Understanding Pipe Systems

Let us assume that the diameter of the pipe segment EB is reduced to D = 0.5 m; all other pipe diameters remain D = 1.0 m. Adopting the Hardy-Cross procedure for this case gives:



IMPORTANT

- The **resistance** of pipe segment EB **increases** significantly, $K \sim D^{-5} \Rightarrow K$ inc 32 times
- As a result, the **flow** through this segment **reduces** significantly
- The **flow** always follows the path of the **lowest resistance**!!!