Tutorial 3: Steady Pipe Flow (3·1 Bernouilli's and Energy Lost)

Question 1

steady state : $U_1 = U_2$

(Lecture 4) Steady water flow is established in a very long length of 300mm internal diameter pipeline. At section one, the elevation is +90m and the pressure is 275kPa. At section two, 300m distant, the elevation is +75m and the pressure is 345kPa. Determine

i. the head loss from 1 to 2

ii. the direction of flow

iii. the shear stress at the pipe wall

iv. the friction factor if the steady discharge is $0.14\text{m}^3/\text{s}$

[Ans: i) 7.86 m, iii) 19.28 Pa, iv) 0.039]

(assume flow from 1 to 2)

i. energy at 1 = energy at 2 + energy lost

$$z_1 + \frac{p_1}{\rho g} + \cancel{\frac{U_1^2}{2g}} = z_2 + \frac{p_2}{\rho g} + \cancel{\frac{U_2^2}{2g}} + \Delta h$$

steady state, $U_1 = U_2 \rightarrow \frac{U_1^2}{2g} - \frac{U_2^2}{2g} = 0$

$$z_1 - z_2 + \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \Delta h$$

$$90 - 75 + \frac{275\times10^3}{1000\times9.81} - \frac{345\times10^3}{1000\times9.81} = \Delta h$$

$\Delta h = 7.864$ m

ii. Since Δh from 1 to 2 is +ive...
flow is from 1 to 2 (head lost if negative means opposite)

iii. energy lost, Δh → mostly due to friction (major lost)

$\Delta h = \frac{\tau_0 P L}{\rho g A}$ or you can do $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right) = \frac{4L\tau_0}{\rho D g}$ $\tau_0 = \frac{f}{8}\rho U^2$, $f = \frac{8\tau_0}{\rho U^2}$

$$\tau_0 = \frac{h_f \rho g A}{P L}; \quad P = \pi D,\ A = \frac{\pi}{4}D^2$$

$= 19.28$ Pa

iv. $\tau_0 = \frac{f}{8}\rho \underline{U^2}$ $Q = UA$, $A = \frac{\pi}{4}D^2$

$$U = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.14}{\frac{\pi}{4}(300\times10^{-3})} = 1.98$$

$$f = \frac{\tau_0 \times 8}{\rho U^2}$$

$$= \frac{19.28\times8}{1000\times1.98^2}$$

$= 0.039$ //

Question 2

(Lecture 4) A steady flow Q of water discharges from 100m length of horizontal pipe to free atmosphere. All relevant dimensions are illustrated in Figure 1. The pipe wall is smooth.

i. What is the discharge Q if water is assumed to be an ideal fluid?

ii. What is the discharge Q if water is a real fluid if you just consider the major losses (h_f)?

iii. For the discharge of a real fluid calculated above, estimate the minor head loss h_L due to the contraction at the entrance. Do you think this is significant?

iv. Repeat the calculation under (ii) for $k_s/D = 0.002$.

v. Check both results using the Colebrook-White equation (the use of Matlab and *fsolve* is recommended).

[Ans: i) 43.5×10^{-3} m^3/s, ii) 7.46×10^{-3} m^3/s, iv) 6.15×10^{-3} m^3/s]

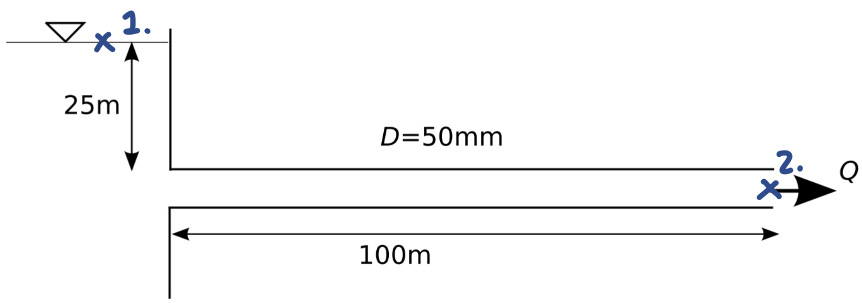


Figure 1: Reservoir with horizontal pipe discharging to free atmosphere.

i. energy at 1 = energy at 2 + energy lost

$$z_1 + \cancel{\frac{P}{\rho g}} + \cancel{\frac{U_1^2}{2g}}^{\,0} = z_2 + \cancel{\frac{P}{\rho g}} + \frac{U_2^2}{2g} + \cancel{h_f}$$

Patm = 0　　　Patm = 0

$h_f = 0$ cause ideal fluid has no shear stress → no friction.

$$z_1 - z_2 = 25 + \frac{50\times10^{-3}}{2}$$
$$= 25.025\ m$$

$$u_2^2 = 2g(25.025)$$

$$u_2 = 22.15\ m/s$$

ii. $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right)$

missing f, to find f we need both k_s/D and Re
k_s/D 0 cause smooth pipe

quite a neat solution, we have $z_2 - z_1 = \frac{U_2^2}{2g} + h_f$, where $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right)$

- we want to find U_2, we are missing f...
- to find f we use moody's diagram,
- which requires k_s/D and Re
- $k_s/D = 0$ (smooth) but $Re = \frac{UD}{\nu}$ where U is unknown
- so we are REQUIRED to iterate for U!

$$Re = \frac{\rho U L}{\mu}$$
$$= \frac{U\times50\times10^{-3}}{10^{-6}}$$

let's iterate for U:
let U = 10 m/s
$Re = 500\,000 \xrightarrow{\text{moody diagram}} f = 0.013$

$$h_f = 0.013 \times \frac{100}{50\times10^{-3}} \times \frac{10^2}{2\times9.81} = 132.52m$$

$$z_1 - z_2 = \frac{U_2^2}{2g} + h_f$$

$25.025 \neq 132.52$

U is too big, try U=5...

let $U = 5 m/s$ $\quad$ $hf = 0.01 \times \ldots = 38.23$

$Re = 250000 \xrightarrow{\text{moody}} f = 0.015$

$25.025 \neq 1.274 + 38.23$

let $U = 3.8 m/s$ $\quad$ $hf = 0.01 \times .. = 24.287$

$Re = 190000 \xrightarrow{\text{moody}} f = 0.0165$

$25.025 \approx 0.736 + 24.287$ ✓

iii Entrance lost from a reservoir (big) to a pipe with diameter 50 mm (small):

A_2/A_1	0 (entrance)	0.2	0.4	0.6	0.8	1.0
ξ	0.5	0.45	0.38	0.28	0.14	0

$A_2 << A_1$ $\quad$ $A_2/A_1 \approx 0$

$h_l = 0.5 \frac{U^2}{2g}$ $\quad$ but $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right) = 33\frac{U^2}{2g}$

66 times differences! yes h_e is negligible

- left side of moody diagram 64/Re
- can ignore minor lost most of the time.

Question 3

(Lecture 4) Engine oil (mass density 890kg/m^3, viscosity 0.08kg/ms) flows from a large tank A to a large tank B through the small smooth rectangular slot shown in Figure 2. The flow rate is 30mm^3/s. Adopt reasonable values for any additional information you may need. What is the pressure of the air trapped in tank A ?
[Ans: 147 Pa]

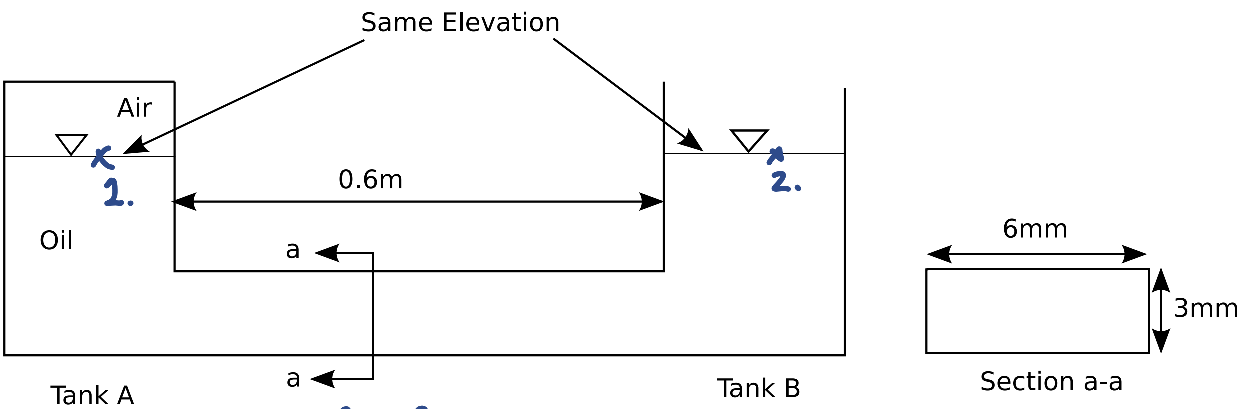


Figure 2: Tank system A-B.

$$\cancel{z_1} + \frac{p_1}{\rho g} + \cancel{\frac{u_1^2}{\rho g}}^{\,0} = \cancel{z_2} + \cancel{\frac{p_2}{\rho g}}^{\,P_{atm}=0} + \cancel{\frac{u_2^2}{2g}}^{\,0} + h_f + h_l$$

$z_1 = z_2$

assume it's a steady state : $u_1 = u_2 = 0$

$$\frac{p_1}{\rho g} = h_f + h_l$$

$$= 861 \times \frac{0.6}{4\times10^{-3}} \frac{(1.67\times10^{-3})^2}{2g}$$

100,000× larger!

$$+ 1.5\frac{(1.67\times10^{-3})^2}{2g}$$

$$p_1 = 0.01836 \times 890 \times 9.81$$

$$= 160 \text{ Pa}$$

A_2/A_1	0 (entrance)	0.2	0.4	0.6	0.8	1.0
ξ	0.5	0.45	0.38	0.28	0.14	0

A_1/A_2	0 (exit)	0.1	0.25	0.5	0.75	1
ξ	1	0.81	0.5625	0.25	0.0625	0

$$h_l = 0.5\frac{U^2}{2g} + 1\frac{U^2}{2g} = 1.5\frac{U^2}{2g}$$

to find $h_f = f\frac{L}{D}\left(\frac{U^2}{2g}\right)$

need to f, requires moody diagram, which require Re, and maybe k_s/D if ks is large.

$$Q = UA$$

$$\frac{30\times10^{-3}\times3}{6\times10^{-3}\times3\times10^{-3}} = U$$

$U = 1.67\times10^{-3}$ m/s

$$Re = \frac{UD}{\nu} = \frac{1.67\times10^{-3}\times 4R_H}{0.08/890} = 0.074$$

$$R_H = \frac{ab}{2(a+b)} = \frac{6\times3}{2(6+3)} = 1$$

small Re, left of moody diagram where:

$$f = 64/Re = 861.2$$

Question 4

(Lecture 4) Water flows steadily from a large reservoir through a pipe system A-G as illustrated in Figure 3. There are elbows at B and C and a gate valve at D. The pipe diameter is 50mm, except for section EF where the diameter is 25mm. The pipes are smooth. Adopt reasonable values for any additional information you may need.

moody: $k_s/D = 0$!

i. What reservoir water surface elevation Z is necessary to sustain a steady water flow velocity of 2m/s at G when the gate valve is fully open?

$U = 2$ m/s

ii. What is the elevation of the total energy line just before E?

iii. What is the elevation of the hydraulic line just after F?

[Ans: i) 50.8 m, ii) 43.8 m, iii) 1.47 m]

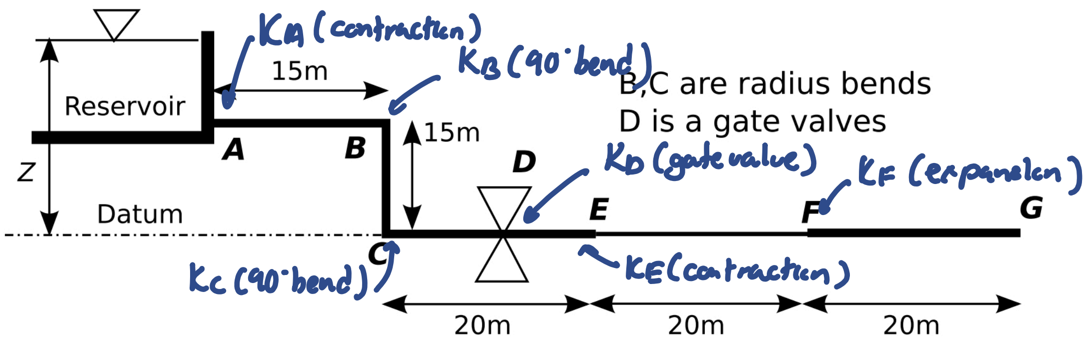


Figure 3: Pipe system with segments A-G. **Note:** this question formed the basis for one exam question in 2009/2010. This should give you an indication of the level to be expected.

(i) Bernoulli Balance:

Energy at reservoir = Energy at G

$$\underset{z}{\cancel{z_A}} + \underset{0}{\cancel{\frac{P_A}{\rho g}}} + \underset{0}{\cancel{\frac{U_A^2}{2g}}} = \underset{0}{\cancel{z_G}} + \underset{0}{\cancel{\frac{P_G}{\rho g}}} + \frac{U_G^2}{2g} + \text{energy lost}$$

$$Z = \Big(\overbrace{f_{50}\frac{L_{50}}{D_{50}}}^{\text{friction loss in } d=50 \text{ pipes}} + K_A + K_B + K_C + K_D\Big)\frac{U_{50}^2}{2g}$$

$$+ \Big(\underbrace{f_{25}\frac{L_{25}}{D_{25}}}_{\text{friction loss in } d=25\text{mm pipes}} + \overbrace{K_E + K_F}^{K_E,\, K_F \text{ is using } U_{25}}\Big)\frac{U_{25}^2}{2g}.$$

as expansion / contraction takes the higher velocity $U_{25} > U_{50}$

note that EF ($d = 25$ mm) will have different U than the other pipes ($d = 50$ mm)
→ different f too as $f(Re, k_s/D)$

$Q_{25} = Q_{50}$

$U_{25}A_{25} = U_{50}A_{50}$

$U_{25}\cancel{\frac{\pi}{4}}D_{25}^2 = U_{50}\cancel{\frac{\pi}{4}}D_{50}^2$

U at G ($d = 50$): 2 m/s

$U_{25} = \frac{2 \times 50^2}{25^2} = 8$ m/s

$Re = \frac{UL}{\nu}$ (where $L = D$ in term of pipes)

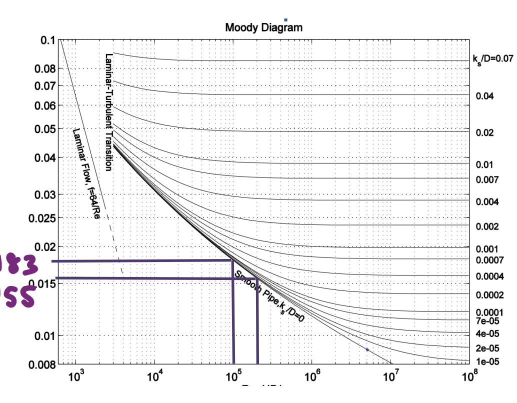
$Re_{25} = \frac{U_{25}D_{25}}{\nu} = \frac{8 \times 25 \times 10^{-3}}{10^{-6}} = 200\,000$

$Re_{50} = \frac{U_{50}D_{50}}{\nu} = \frac{2 \times 50 \times 10^{-3}}{10^{-6}} = 100\,000$

Subbing all the K-values:
($K_A = 0.5$, $K_B = K_C = 0.3$, $K_D = 0.15$, $K_E \approx 0.45$, $K_F = 0.5625$)

$Z = 50.8$ m

$f_{50} \sim 0.0183$
$f_{25} \sim 0.0155$



ii. Energy at reservoir = Energy before E + Energy lost

$$50.8 = H_E + \Big(f_{50}\frac{L_{AE}}{D_{50}} + K_A + K_B + K_C + K_D\Big)\frac{U_{50}^2}{2g}$$

$H_E = 46.8$ m (this include: $\underset{0}{\cancel{z_E}} + \frac{P_E}{\rho g} + \frac{U_E^2}{2g}$)

iii. Energy at E = Energy just after F + Energy Lost between E, F

$$46.8 = h_F + \frac{U_F^2}{2g} + \left(f_{25}\frac{L_{EF}}{D_{25}} + K_E + K_F\right)\frac{U_{25}^2}{2g}$$

U_F (just after H) : $U_{50} = 2\,m/s$

$h_F = 2.58\,m$

Question 5

(Lecture 4) Oil (use $\rho = 900\text{kg/m}^3$, $\nu = 4 \times 10^{-5}\text{m}^2/\text{s}$) flows from reservoir A through gate B to reservoir C at $0.03\text{m}^3/\text{s}$ in a 150mm diameter smooth pipe (Figure 4). The gate at B is partially closed such that $\xi = 2.5$. Determine the elevation of reservoir A (relative to the same datum as the elevation at C) and sketch the Energy Grade Line.

[Ans: i) 2.26 m]

$k_s/D = 0$

given: $Q = 0.03\,m^3/s$

$d = 150 \times 10^{-3}\,m$

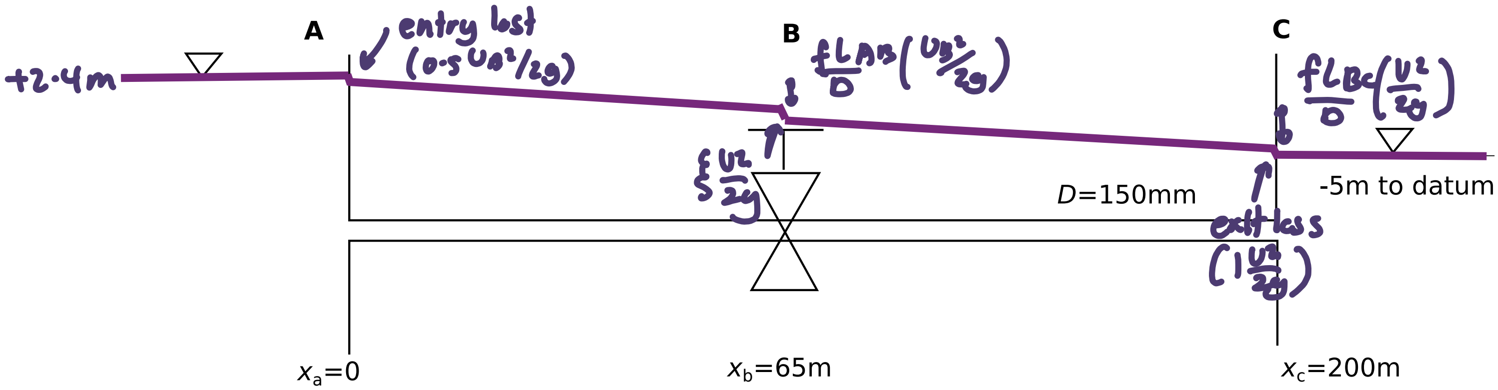


Figure 4: Reservoirs A and C connected by a pipe with a gate at B.

K_A (entry lost) $= 0.5$

K_C (exit lost) $= 1$

$Q_B = U_B A_B$

$U_B = \frac{0.03}{\frac{\pi}{4}(150\times10^{-3})^2} = 1.698$

energy at A = energy at C + energy lost.

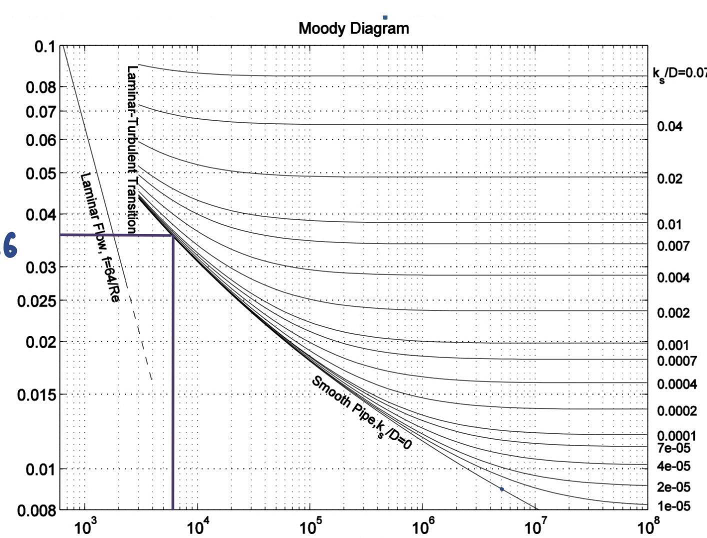
$$z_A + \cancel{\frac{P_A}{\rho g}} + \cancel{\frac{U_A^2}{2g}} = z_C + \cancel{\frac{P_C}{\rho g}} + \cancel{\frac{U_C^2}{2g}} + \frac{U_B^2}{2g}\left(K_A + K_B + K_C + \frac{fL}{D}\right)$$

0 0 0 0

$$z_A = -5 + \frac{1.698^2}{2\times9.81}\left(0.5 + 2.5 + 1 + \frac{0.036\times200}{150\times10^{-3}}\right) = 2.64\,m$$

$$Re = \frac{UL}{\nu} = \frac{1.698\times150\times10^{-3}}{4\times10^{-5}} = 6367.5$$

~ 0.036



Question 6

(Lecture 4) Determine the discharge of the water system (20°C) shown in Figure 5. The friction factor and the pipe diameter apply throughout. Note that the pump discharge is a function of the head as illustrated in Figure 6.
[Ans: $0.265 \times 10^{-3}\ m^3/s$]

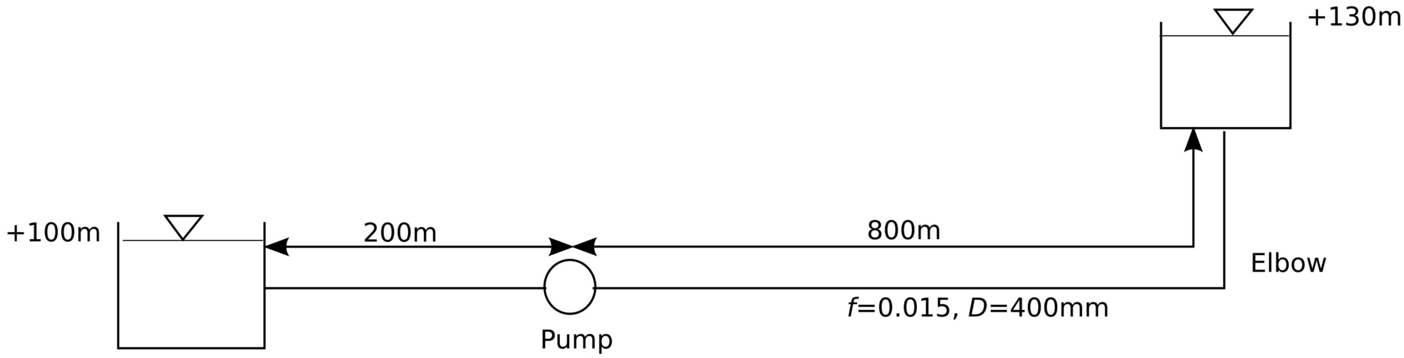


Figure 5: Water system with pump and elbow.

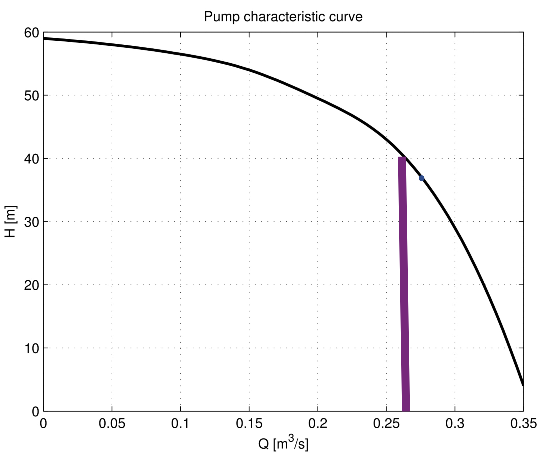


Figure 6: Pump discharge characteristic.

energy input (pump) + energy before = energy after + energy lost.

$$H(Q) + z + \cancelto{0}{\frac{p}{\rho g}} + \cancelto{0}{\frac{U^2}{2g}} = z + \cancelto{0}{\frac{p}{\rho g}} + \cancelto{0}{\frac{U^2}{2g}} + \frac{U^2}{2g}\left(\underbrace{0.5}_{\text{entry}} + \underbrace{0.5}_{\text{elbow}} + \underbrace{1}_{\text{exit}} + \overbrace{\frac{fL}{D}}^{\text{friction.}}\right)$$

$$H(Q) + 100 = 130 + \frac{625Q^2}{2g\pi^2}(2 + 37.5)$$

$$H(Q) = 30 + 127.49Q^2$$

iterate: Q_0 (initial guess) $= 0.15\ m^3/s$

$$Q = UA$$
$$Q = U\left(\frac{\pi}{4}D^2\right)$$
$$Q = U\left(\frac{\pi}{4}(400 \times 10^{-3})^2\right)$$
$$U = \frac{25Q}{\pi}$$
$$U^2 = \frac{625Q^2}{\pi^2}$$

$\underbrace{H(0.15)}_{\approx 54m} - 30 - 127.49(0.15)^2 \approx 21.13$ (too big)
try bigger Q

$Q_1 = 0.25$

$\underbrace{H(0.25)}_{\approx 43m} - 30 - 127.49(0.25)^2 \approx 5.03$ (too big)

$Q_2 = 0.3$

$\underbrace{H(0.3)}_{\approx 29m} - 30 - 127.49(0.3)^2 \approx -12.47$ (too small)

$Q_3 = 0.275$

≈ -2.14

$Q_4 = 0.263$

≈ 1.182

$$\boxed{Q \approx 2.265}$$