IMPERIAL COLLEGE LONDON

MEng Examination 2022

PART II

This paper is also taken for the relevant examination for the Associateship

CIVE50004: ENVIRONMENTAL ENGINEERING

20 May 2022: 12:00 – 15:00 BST

An extra 30 minutes will be added on to the times shown above in order for you to scan and upload your answers.

*This paper contains **THREE** questions.*

*Answer **ALL THREE** questions.*

All questions carry equal marks.

Formulae sheets are provided at the end of the examination paper.

Q1. (Answer ALL parts of this question; total of 40 marks)

(a) Explain in detail why the effect of groundwater velocity is generally assumed not to be significant when applying Darcy's law. Provide two examples of where this assumption can breakdown. What can be done to overcome this limitation?

should mention: $\phi = gz + \frac{u^2}{2} + \int_{P_0}^{P} \frac{dP}{\rho}$

so that total head = hydraulic head. $= z + \frac{P}{\rho g}$ $\left(\frac{u^2}{2g} = 0\right)$ ✓ It's ok to make this assumption as groundwater flows very slowly in soils.

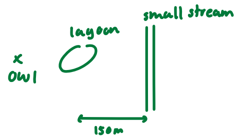
assumption of $u=0$ breaks down when:
1. in the vicinity of wells being pumped at high rate.
2. in highly permeable formations / region.

can be solved by using Forchheimer eqn: $q + \frac{kb^2}{g} q^2 = -k\frac{dh}{dx}$

[8 marks]

(b) A waste lagoon overlies a shallow unconfined homogeneous and isotropic sand aquifer, with a hydraulic conductivity of 2 m day^{-1} and an effective porosity of 0.2. The company managing the site needs to know the potential risks from chemicals leaking into the underlying groundwater. The lagoon is 150 metres due west of a small stream, which runs north-south and is hydraulically connected to the aquifer. Three observation wells (OWs) have been installed around the lagoon, with OW1 due west of the centre of the lagoon. The easting and northing coordinates, along with casing elevations and dips, for the observation wells are given in Table 1.1.

info given: $K = 2$ m/day, $n_e = 0.2$



Provide a sketch of the problem and, using the information provided, determine the Darcy velocity (find q) in order to demonstrate that the lagoon poses a risk to the river. Calculate the time a chemical travelling with the groundwater would take to reach the stream.

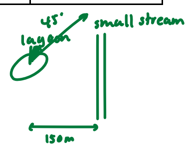
[Show all your working and do not solve using Matlab.]

I can find i from the table below — K is given already! then $q = -Ki$!

h = casing elev. − dip

Well ID	Easting [m]	Northing [m]	Casing elevation [m]	Dip [m]	
OW1 (set this as origin)	4568 (0)	1425 (0)	139.70	4.10	135.60
OW2	4620 (52)	1455 (30)	138.52	4.97	133.55
OW3	4620 (52)	1395 (−30)	137.83	2.78	135.05

Table 1.1 Observation well data



final answer: $t = \frac{150\sqrt{2}}{0.03536} = 600$ days

[12 marks]

$h = Ax + By + C$

$135.60 = C$

$133.55 = 52A + 30B + 135.60$ minus $135.05 = 52A - 30B + 135.60$

$-1.5 = 60B$

$B = -0.025$, $A = -0.025$

$i = \sqrt{0.025^2 + 0.025^2}$
$= 0.03536$

$q = 2 \times 0.03536$
$= 0.07071$ m day^{-1}

$v = \frac{q}{n_e}$
$= \frac{0.07071 \text{ m day}^{-1}}{0.2}$
$= 0.3536$ m day^{-1}

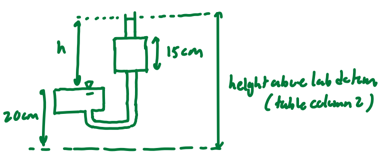
$v = \frac{d}{t} \rightarrow t = \frac{d}{v}$ but d is not 150 as flow is not to the right!

to find flow direction:
$\theta = \tan^{-1}\left(\frac{A}{B}\right) = \tan^{-1}\left(\frac{1}{1}\right)$
$= 225°$ (why not write 45°? cause by looking at h we can guess how flow, flows)
$\theta_{flow} = 45°$

big x → small x, med x; flow
hence i direction is opposite 225°

(c) A water company is investigating a 110 m thick (H for $T = KH$), confined fractured sandstone aquifer for use as a water resource. As part of this, they are undertaking measurements to ascertain key aquifer properties.

(i) A 15 cm diameter cylindrical sample ($D = 15 \times 10^{-2}$) of sandstone, 15 cm long, was placed in a falling head permeameter. The following timed water levels above a temporary lab datum in a 10 mm diameter cylindrical tubing ($d = 10 \times 10^{-3}$) are given in Table 1.2. The height of the outfall of the lower reservoir is 20 cm above the temporary lab datum. Using the data provided calculate the hydraulic conductivity of the rock sample.



$Q = AK\frac{h}{L}$ $Q = -a\frac{dh}{dt}$

$AK\frac{h}{L} = -a\frac{dh}{dt}$

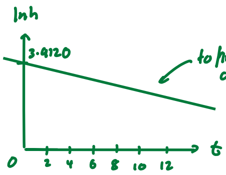
$\int \frac{1}{h} dh = \int -\frac{AK}{aL} dt$

$\ln h = -\frac{AK}{aL} t + c$

plot $\ln h$ against t and gradient will be $-\frac{AK}{aL}$

Time [min]	Height above lab datum [cm]
0	50.0
2	44.2
4	40.6
6	37.5
8	36.1
10	33.8
12	31.0

h	ln h
30	3.4012
24.2	3.1863
20.6	3.0252
17.5	2.8622
16.1	2.7788
13.8	2.6246
11.0	2.3978



to find this gradient of line of best fit
I use linear regression function in calculator!

$m\,[\text{cm/min}] = -0.07821$

$m = -\frac{AK}{aL}$

$K = -\frac{maL}{A} = -\frac{-0.07821\ \text{min}^{-1} \times \cancel{\pi/4}(1^2)\,\cancel{\text{cm}^2} \times 15\text{cm}}{\cancel{\pi/4}(15^2)\,\cancel{\text{cm}^2}} = 5.214\times10^{-3}\ \text{cm min}^{-1}$

$= 8.69\times10^{-7}\ \text{m s}^{-1}$

Table 1.2 Falling head permeameter water levels against time.

[7 marks]

(ii) Contractors for the water company subsequently did a pumping test on the aquifer. The abstraction well was pumped at a constant rate of 1 megalitre per day over a period of 4 hours. Timed drawdowns at an observation well sited 30 m from the abstraction well are provided in Table 1.3. From these data calculate the hydraulic conductivity of the aquifer. How does this value compare with that obtained in part (i)?

constant rate pumping test.
to find S and T,
if we have T we can find k by $T=KH$, H is given $H=110\text{m}$

use Jacob's Large Time Approx.

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

$$s \approx \underbrace{\frac{Q_w}{4\pi T}\left[\ln\left(\frac{4T}{r^2 S}\right) - 0.5772\right]}_{\text{y-intercept}} + \underbrace{\frac{Q_w}{4\pi T}}_{\text{gradient.}} \ln t$$

we just want gradient of s-$\ln t$ graph.

gradient $= 0.28237$ m (linear regression calculator)

$\frac{Q_w}{4\pi T} = 0.28237\text{m}$

$T = \frac{1000\text{m}^3\,\text{day}^{-1}}{4\pi \times 0.28237\text{m}} = 281.82\ \text{m}^2\text{day}^{-1}$

$K = \frac{T}{H} = \frac{281.82\text{m}^2\text{day}^{-1}}{110\text{m}}$

$= 2.562\ \text{m day}^{-1}$

$\approx 3\times10^{-5}\ \text{m s}^{-1}$

$1\text{M}\ell = 1000\text{m}^3$

Time [min]	Drawdown [m]	ln t
20	0.67	
50	0.92	
80	1.06	
110	1.15	
140	1.22	
170	1.27	
200	1.32	
230	1.36	

Table 1.3 Timed drawdowns from constant rate pumping well.

[8 marks]

(iii) Provide an explanation for any discrepancy between the two and comment on their significance for modelling the aquifer. Would you recommend the aquifer as a water resource?

[5 marks]

remember: K from falling head permeameter (ex-situ) is a lot smaller than K from pumping test (in-situ)
in-situ condition has crack which increases permeability.

A very unique question, where not all month we need irrigation I!

Q2. (Answer ALL parts of this question; total of 40 marks)

A green roof is installed on a building. The rooftop area is flat and has a surface area of 120 m^2. The green roof consists of a layer of soil planted with grass ($K_c = 1.0$). A nearby weather station gives the following average meteorological conditions for the area (Table 2.1):

Table 2.1. Average meteorological conditions

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$E_{t,0}$ [mm month^{-1}]	28	47	72	125	150	188	168	140	95	52	33	22
P [mm month^{-1}]	240	362	218	117	64	42	28	34	66	146	166	198

The soil layer is 10 cm deep, and has the following characteristics:

Saturation point [m^3 m^{-3}]	0.42
Field capacity [m^3 m^{-3}]	0.18
Wilting point [m^3 m^{-3}]	0.05

$TAW = (0.18 - 0.05) \times 10cm = 1.3cm$

(a) Calculate the $\underbrace{\text{total water requirement}}_{E_t}$ of the green roof vegetation (in m^3 year^{-1}).

$I = E_p - P + R$
don't know I and R
X

$\Delta S = P - E - Q - R$
long term $\Delta S = 0$
but still don't know Q and R
X

$E_t = K_s K_c E_{t,0}$
$K_s = 0$ (irrigation)
usually we don't know K_c since it varies throughout season.
But this question is an exception! Grass $K_c = 1.0$!
$\therefore E_t = 1.0 \times 1.0 \times E_{t,0}$.

To get E_t in m^{3}year^{-1}
sum all the E_t
mm month^{-1} → mm year^{-1}
then times Area!
$E_t = 1120$ mm year^{-1} $= 134.4$ m^3 year^{-1}

[4 marks]

(b) The engineer decides to capture the excess roof runoff (Q) into a tank and use it to irrigate (I) the grass during summer. What is the minimum size of the tank to ensure sufficient water storage? List the main assumptions you have made in your calculation.

This question actually want to find what is the I required.
Usually $I = E_p - P + R$ easy, but we need to only ΣI that is +ve!

$\cancel{\Delta S}^{0} = P - E - Q - \cancel{R}^{0}$
assume ΔS over long-period $= 0$
assume there is no recharge, $R = 0$

$Q = P - E$
to able to store EXCESS Q
$Q > 0$ (if $Q < 0$ there is no excess Q)
$P - E > 0$
$\therefore P > E$ (Oct to Mar)

$Q = \Sigma P - \Sigma E$ (Oct - Mar)
$= (146-52) + (166-33) + (198-22) + \ldots$
$= 1076$ mm
$= 129.12$ m^3 (volume we can store up to)
(actually no need calc this, because what we want to find is actually I required)

$I = E_p - P$ (we need to irrigate during month where $E_p > P$) → Apr - Sep
$\Sigma E_p - \Sigma P = (125-117) + (150-64) + \ldots = 515$ mm
$= 61.8$ m^3
(volume we required for irrigation)
choose this volume!

[6 marks]

(c) How much runoff generation would be avoided by the combination of green roof and storage tank, compared to a normal roof (in m^3 year^{-1})?

$Q - I = 129.12 - 61.8 = 67.32$ m^{3}year^{-1}

[4 marks]

(d) Calculate the minimum irrigation frequency, if you know that the grass has a depletion factor of 0.6.

$TAW = (0.18 - 0.05) \times 10cm = 1.3cm$
$RAW = pTAW = 0.6 \times 1.3cm = 0.78cm$
(maximum dose)

minimum freq. $= \frac{I}{RAW} = \frac{515mm}{7.8mm} = 66.02 = 63$ /year X

minimum freq. $= \frac{\max(I)}{RAW} = \frac{146mm}{7.8mm} = 18.7 = 19$ times a month.
(maximum I → June $= 188 - 42 = 146$)

[4 marks]

(e) An automatic irrigation system is installed. Explain what measurement method you would recommend to measure soil moisture.

[2 marks]

(f) List and briefly explain the different ways in which climate change may affect the green roof design and how this can be anticipated.

[8 marks]

(g) Urbanisation can have a major impact on the terrestrial water cycle. Explain and discuss which hydrological processes may be affected, and how.

- unit hydrograph have shorter duration
- higher peak discharge, shorter time to peak.

flood chapter.

- impervious surface → increase runoff, reduce soil infiltration. → reduce baseflow in river.

increase surface runoff. flood risk reduce groundwater recharge.

increase transport of surface pollutant.

[12 marks]

- evaporation, E ↓, RR ↑

- higher water demand, more extraction points → depletion of groundwater aquifers.

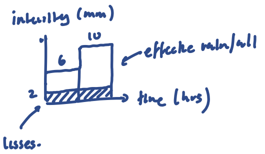
→ changes (lower) water table.

groundwater chapter.

Q3. (Answer ALL parts of this question; total of 40 marks)

(a) A catchment has a 1-hour 5mm unit hydrograph given in Table 3.1. A rainfall event has the intensity of 6 and 10 mm per two hours. If the uniform loss is 2 mm, calculate the flood hydrograph and the peak flow produced by the storm.

we need effective (net) rainfall not total.



so 0-2 hours 4 mm
2-4 hours 8 mm.

since $\Delta T_1 = 1h$ and $\Delta T_2 = 2h$
$\Delta T_2 = 2\Delta T_1$,

we can obtain a 2hrs UH:

	5mm 1h		10mm 2h	1mm 2h
0	0	0	0	0
1	0.8	0	0.8	0.08
2	2.3	0.8	3.1	0.31
3	1.8	2.3	4.1	0.41
4	1.0	1.8	2.8	0.28
5	0.7	1.0	1.7	0.17
6	0	0.7	0.7	0.07
7	0	0	0	0

Table 3.1 1-hour 5mm unit hydrograph

Time (h)	Flow ($m^3 s^{-1}$)
0	0
1	0.8
2	2.3
3	1.8
4	1.0
5	0.7
6	0

after finding 5mm 2hrs UH,

hr	4mm 2h	8mm 2h	total
0	0	0	0
1	0.32	0	0.32
2	1.24	0	1.24
3	1.64	0.64	2.28
4	1.12	1.48	2.60
5	0.68	3.28	3.96
6	0.28	2.24	2.52
7	0	1.36	1.36
8	0	0.56	0.56
9	0	0	0

peak flow = $3.96 m^3 s^{-1}$

* this is correct, marking scheme are wrong!

[14 marks]

(b) Peaks over threshold (POT) is one of the methods to select samples for flood frequency analysis from observed flows.

(i) From the flow dataset given in Table 3.2, select the flood POTs that satisfy the criteria for being independent so that they can be used for the frequency analysis and justify your solution. Assume the average time to rise of 15 hours.

ATR = 15h
3×ATR = 45h

Table 3.2 Table of flow selected for flood frequency analysis

5 days ✓ 1 day ✗ 4 days ✓ 2days = 48h > 45h ✓

Days	2 ✓		6		7 ✓		9		11
Peak flows (m3 day^{-1})	153		131		136		165		175 highest
Min flow between peaks (m^3 day^{-1})		63		81		74		127 ~~>116.67~~ ~~>110~~	$\frac{2}{3} \times$ ~~175~~ 165 = ~~116.67~~ 110

$63 < \frac{2}{3} \times 153$

$< \frac{2}{3} \times 136 = 91$

(compare with earlier in time)

[6 marks]

(ii) Table 3.3 lists 11 peaks over threshold (POT) daily river flows that were selected for flood frequency analysis during the 6-year period from 2000-2005.

Table 3.3 Peaks over threshold observations for daily flows

$\underbrace{F(q_d) = P(Q \le q_m) = \frac{m-0.44}{N+0.12}}_{\text{Gringorten from data sheet.}}$ $\quad \frac{m-0.44}{11+0.12}$

$N = 11$ (this N is no. POTs not n, number of years!)

Year	Max. daily flow ($m^3 s^{-1}$)		$F(q_d)$	T [year]
2000	52	40	0.0503	0.5743
2000	66	51	0.1402	0.6343
2001	109	52	0.2302	0.7085
2002	40	61	0.3201	0.8022
2002	86	66	0.4100.	0.9244
2002	87	78	0.5000	1.0909
2003	122	86	0.5899	1.3300
2003	97	87	0.6798	1.7034
2004	78	97	0.7697	2.3684
2005	51	109	0.8597	3.8877
2005	61	122	0.9496	10.8220

Define and explain terms in the equation for the calculation of the return period for POT method and calculate the return period for selected flood peaks.

$$T = \underbrace{\frac{n}{t}}_{\text{correction factor:}} \times \frac{1}{\underbrace{P(q > q_d)}_{\text{probability of exceedance.}}} = \frac{n}{t} \times \frac{1}{1-F(q_d)}$$

n (no. years), t (no. selected POTs)

$F(q_d)$ can be found with multiple method. Easiest one being Gringorten Formula (can use Weibull too)

[8 marks]

(c) You are asked to design a rainfall forecasting system for flood warning. Explain which system you would propose for the lead times between 2-6 hours and why. Briefly describe how the selected system operates.

[6 marks]

(d) Explain the three differences between the conceptual models of soil zone in urban areas for pervious and impervious areas. Include a sketch of the system to support your answers.

[6 marks]

Formulae Sheet

A. Introduction to Hydrology (Prof Wouter Buytaert)

Catchment water balance	$\Delta S = P - E - Q - R$
Density of solids	$\rho_S = \dfrac{M_S}{V_S}$
Dry bulk density	$\rho_B = \dfrac{M_S}{V_T}$
Total bulk density	$\rho_T = \dfrac{M_S + M_L}{V_T}$
Porosity	$\varepsilon = \dfrac{V_L + V_G}{V_T}$
Void ratio	$e = \dfrac{V_L + V_G}{V_S}$
Gravimetric moisture content	$\theta_G = \dfrac{M_L}{M_S}$
Volumetric moisture content	$\theta = \dfrac{V_L}{V_T}$
Interrelating formula	$\theta = \theta_G \dfrac{\theta_B}{\theta_W}$
Penman Monteith	$E_a = K_c K_s E_{t,0}$
River flow	$Q = \int v(A)\,dA = \bar{v}A$
Rectangular weir	$Q = KbH^{1.5}$
Hydropower equation	$P = \varepsilon_t\, \varepsilon_h\, h\, \rho\, Q\, g$

Irrigation $I = E_p - P + R$

Total available water $TAW = 1000\,(\theta_{FC} - \theta_{WP})\,Z_r$

Readily available water $RAW = p\,TAW$

B. Groundwater Systems (Prof Adrian Butler)

Darcy's law

$q_i = -K\dfrac{dh}{di}$, where i represents a coordinate direction (e.g. x, y, z)

Water table elevation in an unconfined aquifer

$$h^2 - h_0^2 = \frac{W}{K}\left(Lx - x^2\right) + \left(h_1^2 - h_0^2\right)\frac{x}{L}$$

Total flow per unit width in an unconfined aquifer

$$Q' = W\left(x - \frac{L}{2}\right) + \frac{K}{2L}\left(h_0^2 - h_1^2\right)$$

Well pumping confined aquifer without recharge (Thiem equation)

$$s = h_e - h = \frac{Q_w}{2\pi T}\ln\left(\frac{r_e}{r}\right)$$

Well pumping unconfined aquifer with recharge

$$h_e^2 - h^2 = \frac{Q_w}{\pi K}\ln\left(\frac{r_e}{r}\right) + \frac{W}{2K}\left(r^2 - r_e^2\right)$$

Theis solution

$s = \dfrac{Q_w}{4\pi T}W(u)$, where $u = \dfrac{r^2 S}{4Tt}$

Jacob's large time approximation

$$s \approx \frac{Q_w}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right]$$

C. Flood Risk Estimation and Management (Dr Ana Mijic)

Weibull formula $P_m(Q \leq q_m) = \frac{\mathrm{m}}{N+1}$

Gringorten formula $P_m(Q \leq q_m) = \frac{\mathrm{m}-0.44}{N+0.12}$

Gumbel distribution $F(q_d) = \exp\left[\, -\exp\left(\frac{\alpha - q_d}{\beta}\right)\right]$

Gumbel variate $z = -\ln\{-\ln[F(q_d)]\}$

Probability for sequence of years $P(x, n) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

Moment matching

$$\alpha = \mu - 0.5772\,\beta$$
$$\beta = (6^{\frac{1}{2}}/\pi)\sigma$$

L-moment matching

$$L_1 = \frac{1}{n}\sum_{j=1}^{n} q_j$$
$$L_2 = \frac{2}{n}\sum_{j=2}^{n}\left[\frac{(j-1)q_j}{n-1}\right] - L_1$$
$$\alpha = L_1 - 0.5772\,\beta$$
$$\beta = \frac{L_2}{\ln(2)}$$

Normal equation for flood forecasting $(R^T R)\theta = R^T X$