procedure *push*(f, h, v, w)
  assert $e_f(v) > 0$ and $h(v) > h(w)$ and $(v, w) \in G_f$
  $e \leftarrow (v, w)$
  if e is a forward edge **then**
    $\delta \leftarrow \min(e_f(v), c(e) - f(e))$
    increase $f(e)$ by δ
  else
    $e \leftarrow (w, v)$
    $\delta \leftarrow \min(e_f(v), f(e))$
    decrease $f(e)$ by δ
  decrease $e_f(v)$ by δ
  increase $e_f(w)$ by δ

procedure *relabel*(f, h, v)
  assert $e_f(v) > 0$ and for all edges $(v, w) \in E_f$ we have $h(w) \geq h(v)$
  $h(v) \leftarrow h(v) + 1$

function *preflow_push*(G, s, t)
  for each node u **do** $e_f(u) \leftarrow 0$
  $h(s) \leftarrow n$
  for each node $u \neq s$ **do** $h(u) \leftarrow 0$
  for each edge (s, v) **do**
    $f(s, v) \leftarrow c(s, v)$
    $e_f(v) \leftarrow c(s, v)$
    $e_f(s) \leftarrow e_f(s) - c(s, v)$
  for each edge (u, v) such that $u \neq s$ **do** $f(u, v) \leftarrow 0$
  while there is a node $v \neq t$ with $e_f(v) > 0$ **do**
    if there is a node w such that $h(v) > h(w)$ and $(v, w) \in G_f$ **then**
      $push(h, f, v, w)$
    else
      $relabel(h, f, v)$
  return f

We will next prove the correctness of the preflow-push algorithm. Initially the preflow f and height function h are compatible. Each push satisfies the capacity constraints due to how the δ is calculated, and each relabel increases the height of a node v by one. This could violate the compatibility of f and h. The relevant condition for compatibility is:

For all edges $(v, w) \in E_f$ *we have* $h(v) \leq h(w) + 1$

If it is the case $h(v) > h(w)$ then a push and not a relabel is performed, and in the other case, $h(v) = h(w)$ the height of v is incremented by one, and this still