Write in matrix standard form: Maximize

$$z = 10x_1 + 11x_2$$

subject to

$$\begin{cases} x_1 + 2x_2 \leq 150 \\ 3x_1 + 4x_2 \leq 200 \\ 6x_1 + x_2 \leq 175 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}.$$

SOLUTION

$$X_1 + 2X_2 + X_3 \leq 150$$
$$3X_1 + 4X_2 + X_4 \leq 200$$
$$6X_1 + X_2 + X_5 \leq 175$$

(i) ensure $X_i \geq 0$

(ii) ensure non-negativity of $B = \begin{bmatrix} 150 \\ 200 \\ 175 \end{bmatrix}$

(iii) convert $\leq$ to $=$ and add slack variable

$$X_1 + 2X_2 + X_3 = 150$$
$$3X_1 + 4X_2 + X_4 = 200$$
$$6X_1 + X_2 + X_5 = 175$$

(iv) convert $\geq$ to $=$ and add surplus

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 3 & 4 & 0 & 1 & 0 \\ 6 & 1 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 150 \\ 200 \\ 175 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} X_3 \\ X_4 \\ X_5 \end{bmatrix} \quad C = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Write in matrix standard form: Maximize

$$z = 10x_1 + 11x_2$$

subject to

$$\begin{cases} x_1 + 2x_2 \geq 150 \\ 3x_1 + 4x_2 \geq 200 \\ 6x_1 + x_2 \geq 175 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}.$$

① ✓
② ✓
③ ✓
④ $X_1 + 2X_2 - X_3 = 150$

$3X_1 + 4X_2 - X_4 = 200$

$6X_1 + 1X_2 - X_5 = 175$ ✓

$Z = 10X_1 + 11X_2 - MX_6 - MX_7 - MX_8$

⑤ $X_1 + 2X_2 - X_3 + X_6 = 150$

$3X_1 + 4X_2 - X_4 + X_7 = 200$

$6X_1 + X_2 - X_5 + X_8 = 175$

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & -1 & 0 & 0 & 1 & 0 \\ 6 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 150 \\ 200 \\ 175 \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{bmatrix} \quad X_0 = \begin{bmatrix} X_6 \\ X_7 \\ X_8 \end{bmatrix} \quad C = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 0 \\ 0 \\ -M \\ -M \\ -M \end{bmatrix} \quad C_0 = \begin{bmatrix} -M \\ -M \\ -M \end{bmatrix}$$

, where M is a large constant.

Write in matrix standard form: Minimize

$$z = 2x_1 - 3x_2$$

subject to

$$\begin{cases} 2x_1 + 5x_2 \geq -100 \\ 5x_1 - 2x_2 \leq -80 \end{cases}.$$

① X_1 & X_2 are no considered this

$X_1 = X_3 - X_4$
$X_2 = X_5 - X_6$, where X_3, X_4, X_5, and $X_6 \geq 0$

② $-2(X_5 - X_4) - 5(X_5 - X_6) \geq 100$

$-5(X_3 - X_4) - 2(X_5 - X_6) \leq 80$

③+④ $-2X_3 + 2X_4 - 5X_5 + 5X_6 + X_7 = 100$

$-5X_3 + 5X_4 - 2X_5 + 2X_6 - X_8 = 80$

⑤ $Z = 2X_1 - 3X_2 + MX_9$

$-2X_3 + 2X_4 - 5X_5 + 5X_6 + X_7 = 100$

$-5X_3 + 5X_4 - 2X_5 + 2X_6 - X_8 + X_9 = 80$

$A = \begin{bmatrix} -2 & 2 & -5 & 5 & 1 & 0 & 0 \\ -5 & 5 & -2 & 2 & 0 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 100 \\ 80 \end{bmatrix}$

$X = \begin{bmatrix} X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ -2 \\ -3 \\ 3 \\ 0 \\ 0 \\ M \end{bmatrix}$

$X_0 = \begin{bmatrix} \\ \\ X_9 \end{bmatrix}$

Write in matrix standard form: Maximize

$$z = 10x_1 + 11x_2$$

subject to

$$\begin{cases} x_1 + 2x_2 \geq 150 \\ 3x_1 + 4x_2 \geq 200 \\ 6x_1 + x_2 \geq 175 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}.$$

(i) (v)

(ii) (v)

(iii) $\leq \rightarrow =$ add slack

(iv) $\geq \rightarrow =$ add surplus

$$X_1 + 2X_2 - X_3 = 150$$

$$3X_1 + 4X_2 - X_4 = 200$$

$$6X_1 + X_2 - X_5 = 175$$

(v)

$$X_1 + 2X_2 - X_3 + X_6 = 150$$

$$3X_1 + 4X_2 - X_4 + X_7 = 200 \qquad Z = 10X_1 + 11X_2 - MX_6 - MX_7 - MX_8$$

$$6X_1 + X_2 - X_5 + X_8 = 175$$

(vi)

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & -1 & 0 & 0 & 1 & 0 \\ 6 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 150 \\ 200 \\ 175 \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{bmatrix} \qquad X_a = \begin{bmatrix} X_6 \\ X_7 \\ X_8 \end{bmatrix} \qquad C = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

X_4

$-M$

$-M$

$-M$

Day 1 01/22/2020

- We have a minor quit today

Day 2 01/24/2020

Simplex method

EX: Maximize

$$Z = 3X_1 + 4X_2$$

Subject to

$$2X_1 + X_2 \leq 6$$

$$2X_1 + 3X_2 \leq 9$$

$$X_1, X_2 \geq 0$$

	X_1	X_2	X_3	X_4		
X_3	2	1	1	0	6	$\frac{6}{1}$
X_4	2	3	0	1	9	$\frac{9}{3}$
	-3	-4	0	0		

Initial simplex tableau

$X_1 = 0, X_2 = 0 \rightarrow$ Point A

- get 1 in the spot of the pivot elmt

X_3	$\frac{4}{3}$	0	1	$-\frac{1}{3}$	3
X_2	$\frac{2}{3}$	1	0	$\frac{1}{3}$	3
	$-\frac{1}{3}$	0	0	$\frac{4}{3}$	12

$\frac{3}{\frac{4}{3}} = \frac{9}{4}$ ← pivot element

$\frac{3}{\frac{2}{3}} = \frac{9}{2}$

$X_1 = 0, X_4 = 0$

$X_3 = 3, X_2 = 3$

$(0, 3) \rightarrow$ Point D

he talked about why row variables changed.

	X_1	X_2	X_3	X_4	
X_1	1	0	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{9}{4}$
X_2	$\frac{2}{3}$	1	0	$\frac{1}{3}$	3
	$\frac{1}{3}$	0	0	$\frac{4}{3}$	12

$\frac{3}{4} \cdot r_1 \rightarrow r_1$

$r_2 \rightarrow r_2 - r_1(\frac{2}{3}) = 0 \quad 1 \quad -\frac{1}{2} \quad \frac{1}{2}$

$r_3 \rightarrow r_3 - \frac{1}{3}(r_1) = 0 \quad 0 \quad -\frac{3}{12} \quad (\frac{4}{3}) + \frac{1}{12}$

	X_1	X_2	X_3	X_4	
X_1	1	0	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{9}{4}$
X_2	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
	0	0	$\frac{1}{4}$	$\frac{15}{12}$	$\frac{51}{4}$

- We are finished when there are no negative numbers in the bottom row.

$C_0^T \cdot B$ where $B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ $C_0 = \begin{bmatrix} 0 \\ -M \\ -M \end{bmatrix}$

$\Rightarrow$

$$\begin{bmatrix} 0 & -M & -M \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = (0)(1) - (M)(2) - M(4)$$

$$= -6M$$

$C_0^T \cdot A - C^T$, where $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & -1 & 0 & 1 \end{bmatrix}$

$$C_0^T \cdot A - C^T = \begin{bmatrix} -4M & -3M & -3M & 0 & M & -M & -M \end{bmatrix} - \begin{bmatrix} 2 & 3 & 4 & 0 & 0 & -M & -M \end{bmatrix}$$

$$C_0^T A - C^T = \begin{bmatrix} -2-4M & -3-3M & -4-3M & 0 & M & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ we split up M and non M terms into two rows

- Second quiz next Friday (01/31)