PROJECT 3
PACKING CANS IN BOXES

SYNOPSIS

You've been hired as a consultant to the Lagrange Shipping Company. Your task is to help the company design rectangular shipping cartons that will maximize the amount of product that can be shipped in a given size truck.

BACKGROUND INFORMATION

1. All merchandise shipped by the Lagrange Shipping Company comes in cylindrical steel cans. The cans come in various diameters and heights, as shown in the table.

Type of can	diameter	height	number of cans
A	12 in	16 in	$4x$
B	16 in	24 in	$2x$
C	24 in	24 in	x

   The values shown as the number of cans of each type represent the relative numbers of each type of can shipped. The company ships a continuing supply of these types of cans.

2. The cans are packed in rectangular shipping cartons.

3. The cartons are shipped in semi trailers with inside dimensions of 99 inches wide, 110 inches high, 52 feet long.

4. The steel used for the cans is approximately 1/4 inch thick and has a density of 500 pounds per cubic foot.

5. The material packed in the cans (which are packed full) has a density of 100 pounds per cubic foot.

6. A carton cannot be safely handled if its contents weigh more than 10,000 pounds.

7. Although a number factors affect the costs incurred by the shipping company, by far the largest is the number of trailer loads required to ship the product. The company pays a fixed amount per trailer load, regardless of the weight or volume of the load, so the company wishes to maximize the amount of product it carries in each load.

8. The company does not want to have a backlog of cans to ship, so it has had a rule that each trailer's shipment must carry the three types of cans in the proportions shown above. The company is willing to consider the possibility of slightly relaxing this requirement by allowing the proportions to be different in one truck if they are balanced out by one other truck.

9. The company does not wish to have more than four sizes of cartons.

YOUR TASK

Determine the numbers and sizes of cartons that the company should use; the numbers, sizes, and arrangements of cans with each type of carton; and the arrangement of cartons in the truck. Note that a near-optimal solution is sufficient.

HOW TO PROCEED

Convert the information in the problem into an optimization problem with constraints. You may even find it helpful to consider smaller optimization problems within the main optimization problem since for any given carton size, you will want to pack in as many cans as possible. Then, using a combination of methods from this class, any technology you wish, common sense, and your ingenuity, devise a solution.

WHAT YOU SHOULD SUBMIT

1. A typed **title page** following the format from the previous projects.
2. A typed one–page or two-page **letter** addressed to the company's director of operations presenting your solution. This letter should not include the mathematics that led to your solution. Do, however, include a precise summary of the information requested in "your task" as described above.
3. A complete **report** of your solution to the problem. This should contain a description of your approach to the problem, the precise mathematical statement of the problem you solved (including objective functions and constraints), the details of the mathematics you did, and the same summary of your results included in the letter. If you used technology in the solution of the problem, indicate how you did so and the results. Be sure to include complete bibliographic details for any references you consult.