MATH 371

OPERATIONS RESEARCH

Chapter 9 - Scheduling Models

1. Production problems

   - Scenario: A single product is to manufactured over a number of successive time periods to meet pre-specified demands. Units of the product can be shipped or stored. Production costs and storage costs are known.
   - Objective: Determine a production schedule which will meet all future demands at minimum total cost (total production cost plus total storage cost, since total shipping cost is assumed fixed).
   - Method: Convert to a transportation problem by considering time periods during which production can take place as sources and time periods in which units will be shipped as destinations.
   - Variables: x_{ij} is the number of units to be produced during time period i for shipment during time period j. c_{ij} is the unit production cost during time period i plus the cost of storing a unit of product from time period i until time period j. For $i > j$, c_{ij} is made very large to force x_{ij} to be zero.

2. Transshipment problems

   - Scenario: There are sources, having supplies, and destinations, having demands, but there are also junctions through which goods can be shipped. A junction can be a source, a destination, or neither.
   - Objective: Develop a transportation schedule that will meet all demands at minimum total cost.
   - Method: Convert to a transportation problem by making every junction both a source and a destination. Each junction is assigned a supply equal to its original supply (or zero if it was not originally a source) plus the total number of units in the system. Each junction is assigned a demand equal to its original demand (or zero if it was not originally a destination) plus the total number of units in the system. The cost to transport a unit from a junction (as a source) to itself (as a destination) is zero.

3. Assignment problems

   - Scenario: Schedule workers to jobs on a one-to-one basis. The number of workers is assumed equal to the number of jobs, which can be accomplished by creating either fictitious workers or jobs, as needed. The time c_{ij} required for worker i to complete job j is known.
   - Objective: Schedule every worker to a job so that all jobs are completed in the minimum total time.
   - Method (Hungarian method): Construct the $n \times n$ cost matrix, with one worker per row and one job per column.
     - In each row, locate the smallest element and subtract it from every element in that row. Then repeat for each column. The revised cost matrix will have at least one zero in each row and column.
     - Determine whether there exists a feasible assignment involving only zero costs in the revised cost matrix. That is, determine whether the revised cost matrix has n zero entries no two of which are in the same row or column. If such an assignment exists, it is optimal. Otherwise, go to step 3.
     - Cover all zeros in the revised cost matrix with as few horizontal and vertical lines as possible. Locate the smallest value in the cost matrix not covered by a line. Subtract this number from every element not covered by a line and add it to every element covered by two lines.
     - Return to step 2.

4. The traveling salesperson problem

- Scenario: An individual must leave a base location, visit $n - 1$ other locations (each once and only once), and then return to the base. The cost of traveling between each pair of locations, c_{ij} is given, with c_{ij} not necessarily equal to c_{ji}.
- Objective: Schedule a minimum-cost itinerary.
- Method: Convert to an assignment problem as follows. Label the locations 1, 2, ..., n. Consider a set of n workers and n jobs. The cost of an assignment c_{ij} is the cost of traveling directly from location i to location j. Every feasible solution to the traveling salesperson problem corresponds to a feasible solution to the associated assignment problem. However, the assignment problem will possess feasible solutions (corresponding to non-cyclic permutations) which do not represent a feasible solution of the traveling salesperson problem. Apply the Hungarian method to the cost matrix. If the result corresponds to a feasible itinerary, that itinerary must be optimal. If not, use a variation of branch and bound. Branching is on c_{pq}, where $p \to q$ is any one of the assignments in the current first approximation. One new cost matrix is obtained by replacing c_{pq} by a prohibitively large number; the other new matrix is obtained by replacing c_{qp}, as well as all elements in the p^{th} row or q^{th} column except c_{pq} itself, by a prohibitively large number.
- Nearest-neighbor method:
  - Locate the smallest element in the cost matrix, circle it, and include the corresponding link in the itinerary.
  - If the newly circled element is c_{pq}, replace all other elements in the p^{th} row and all other elements in the q^{th} column, as well as c_{qp} by a prohibitively large number.
  - Locate the smallest uncircled element in the latest cost matrix. Tentatively adjoin its corresponding link to the (incomplete) itinerary. If the resulting itinerary is feasible, circle the designated cost and go to step 5.
  - If the resulting itinerary is infeasible, remove the latest link from the itinerary and replace its corresponding cost by a prohibitively large number. Go to step 3.
  - Determine whether the itinerary is complete. If so, accept it as the near-optimal one. If not, go to step 2.