Day 1 1/13/2020

- attendance and stuff right now.

- what is ops research
  - constranted optimization
- A few quizzes and tests
- Projects are the biggest portion of the course
  - some will contain programming (hopefully more)
- Course home/syllabus
- I will need to give a presentation on April 15th
- No final exam!!

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$$ax + by = r$$
$$cx + dy = s$$

$$x = \frac{r - by}{a}$$
$$x = \frac{s - dy}{c}$$

$$ax + by - (cx + dy) = r - s$$
$$ax - cx + by - dy = r - s$$
$$x(a-c) + y(b-d) = r - s$$

$$(a-c)x = r - s - y(b-d)$$

$$x = \frac{r - s - y(b-d)}{(a-c)}$$

- his solution

$$\begin{cases} ax + by = r \xrightarrow{d} adx + byd = rd \\ cx + dy = s \xrightarrow{-b} -bcx - bdy = -sb \end{cases}$$

$$adx - bcy = rd - sb$$

$$x(ad - bc) = rd - sb$$

$$x = \frac{rd - sb}{ad - bc}$$

- program needs to determine
  - no solution
  - infinitely many
  - special cases $x=0$, $y=0$, $z=0$

- Notes

$60x + 70y = 50$ minimum 240 x

minimum 140 y

$40x + 20y = 30$

Objective function $\Rightarrow z = 50x + 30y$

Subject to $\begin{cases} 60x + 40y \geq 240 \\ 70x + 20y \geq 140 \\ \end{cases}$

- talking about shading graphs
- feasible region is the shaded area with the constraints

Key fact: the solution to a linear programming problem occurs at a vertex of the feasible region

- we only need to check the coordinates of the points of the shaded region.

- Integral $\Rightarrow$ coefficients are integers.

Day 2 01/15/2020

- Just got done doing an example

Ex Maximize $Z = 3x_1 + 4x_1$

Subject to
$$\begin{cases} 2x_1 + x_2 \leq 6 \\ 2x_1 + 3x_2 \leq 9 \\ x_1 \geq 0, \; x_2 \geq 0 \end{cases}$$

Simplex method (chapter 2)

$$A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$x \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- Review matrix multiplican

$$\underset{(2\times 2)}{\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}} \underset{(2\times 1)}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\underset{2 \times 1}{\begin{bmatrix} 2x_1 + x_2 \\ 2x_1 + 3x_2 \end{bmatrix}} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

Ex: Maximize

$$Z = 10x_1 + 11x_2$$

Subject to

$$\left.\begin{aligned} x_1 + 2x_2 &\le 15 \\ 3x_1 + 4x_2 &\le 200 \\ 6x_1 + x_2 &\le 175 \\ x_1 \ge 0,\ x_2 &\ge 0 \end{aligned}\right\}$$

Step 1: $x \ge 0 \rightarrow$ done

Step 2: $B \ge 0 \rightarrow$ done

Step 3: Convert $\le \rightarrow =$

Slack variables

$$\begin{aligned} x_1 + 2x_2 \le 15 &\longrightarrow x_1 + 2x_2 + x_3 = 150 \\ 3x_1 + 4x_2 \le 200 &\longrightarrow 3x_1 + 4x_2 + x_4 = 200 \\ 6x_1 + x_2 \le 175 & \quad\quad 6x_1 + x_2 + x_5 = 175 \end{aligned}$$

Step 4: Convert $\ge \rightarrow =$

Step 5: Ensure feasable Solution, with artificial variables $\longrightarrow$ done

Step 6 Construct matrices

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 3 & 4 & 0 & 1 & 0 \\ 6 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 150 \\ 200 \\ 175 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

Ex: ~~Minimize~~ Maximize

$$z = 10x_1 + 11x_2$$

Subject to

$$\left.\begin{aligned} x_1 + 2x_2 &\geq 15 \\ 3x_1 + 4x_2 &\geq 200 \\ 6x_1 + x_2 &\geq 175 \\ x_1 \geq 0,\ x_2 &\geq 0 \end{aligned}\right\}$$

Step 1: $x \geq 0 \rightarrow$ done

Step 2: $B \geq 0 \rightarrow$ done

Step 3: Convert $\leq \rightarrow =$

Step 4: Convert $\geq \rightarrow =$

Surplus variables
• Subtracting instead of add.

$$x_1 + 2x_2 \leq 15 \longrightarrow x_1 + 2x_2 - x_3 + x_6 = 150$$

$$3x_1 + 4x_2 \leq 200 \longrightarrow 3x_1 + 4x_2 - x_4 + x_7 = 200$$

$$6x_1 + x_2 \leq 175 \qquad 6x_1 + x_2 - x_5 + x_8 = 175$$

Step 5: Ensure feasable solution, with artificial variables $\longrightarrow$ done

Step 6 Construct matrices

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & -1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & -1 & 0 & 0 & 1 & 0 \\ 6 & 1 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 150 \\ 200 \\ 175 \end{bmatrix} \quad C = \begin{bmatrix} 10 \\ 11 \\ 0 \\ 0 \\ 0 \\ -M \\ -M \\ -M \end{bmatrix}$$

$$X_0 = \begin{bmatrix} x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

$Z = 10x_1 + 11x_2 - Mx_6 - Mx_7 - Mx_8$ , $M$ is a big constant

Day 3 01/17/2020

- talking about project 2!
- Continuation from last time
- Converting a linear programming problem to standard form

Ex minimize

$$z = 2x_1 - 3x_2$$

Subject to

$$2x_1 + 5x_2 \geq -100$$

$$5x_1 - 2x_2 \leq -80$$

Solution

$$x_1 = x_3 - x_4$$

$$x_2 = x_5 - x_6$$

$$x_{\{3,4,5,6\}} \geq 0$$

Step 2: ensure nonnegativity of B

$$-2(x_3 - x_4) - 5(x_5 - x_6) \leq 100$$

$$-5(x_3 - x_4) + 2(x_5 - x_6) \geq 80$$

Step 3 and 4: replace $\geq$ & $\leq$ with $=$

$$-2(x_3 - x_4) - 5(x_5 - x_6) + x_7 = 100$$

$$-5(x_3 - x_4) + 2(x_5 - x_6) - x_8 = 80$$

- Distribute

$$-2x_3 + 2x_4 - 5x_5 + 5x_6 + x_7 = 100$$

$$-5x_3 + 5x_4 + 2x_5 - 2x_6 - x_8 + x_9 = 80$$

adding new slack variable

$$z = 2(x_3 - x_4) - 3(x_5 - x_6) + Mx_9$$

M is a large positive constant

$$A = \begin{array}{c} \begin{matrix} x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{matrix} \\ \begin{bmatrix} -2 & 2 & -5 & 5 & 1 & 0 & 0 \\ -5 & 5 & 2 & -2 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

$$B = \begin{bmatrix} 100 \\ 80 \end{bmatrix}$$

$$X = [x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9]^T$$

$$X_0 = [x_7 \ x_9]^T$$

$$C = [2 \ -2 \ -3 \ 3 \ 0 \ 0 \ 1]^T$$

The Simplex method (chapter 3)

EX: maximize $z = 3x_1 + 4x_2$

$$2x_1 + x_2 \le 6$$

$$2x_1 + 3x_2 \le 9$$

$$x_1, x_2 \ge 0$$

Step 3: Slack

$$2x_1 + x_2 + x_3 = 6$$

$$2x_1 + 3x_2 + x_4 = 9$$

$$\{x_1, x_2, x_3, x_4\} \ge 0$$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad X_0 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

$C_0$ is the part of $c$ that corresponds with $x_0$

$$C_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

| | $X^T$ | |
|---|---|---|
| $X_0$ | $A$ | $B$ |
| | $C_0^T \cdot A - C^T$ | $C_0^T \cdot B$ |

$\Downarrow$

| | $[x_1\ x_2\ x_3\ x_4]$ | |
|---|---|---|
| $\begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$ | $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 6 \\ 9 \end{bmatrix}$ |
| | $[3\ 4\ 0\ 0]$ | $C_0^T B \rightarrow 0$ |

Initial simplex tableau

now rref

Step 1: locate most negative number in bottom row excluding last column

- for our example this is negative four

$[3\ \ 4\ \ 0\ \ 0]$ this is the work column

Step 2: form ratios:

$$\frac{\#\text{ in last column}}{\#\text{ in work column}}$$

Smallest of these is the pivot element

- $\frac{6}{1}$ or $\frac{9}{3}$ ← our pivot element

$$\begin{bmatrix} 2 & 1 & 1 & 0 \end{bmatrix}$$

$[2 \; 3 \; 0 \; 1]$

Step 3: get a 1 in the pivot element spot and zeros in the rest of that element.

divide

$r_2$ by 3

$r_2 \to r_2 \cdot \frac{1}{3} = \frac{2}{3} \quad 1 \quad 0 \quad \frac{1}{3}$

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & 1 & 1 & 0 & 6 \\ \frac{2}{3} & 1 & 0 & \frac{1}{3} & 3 \\ \hline -3 & -4 & 0 & 0 & \end{array}\right]$$

$r_3 \to r_3 + 4r_2 = [-3 \; -4 \; 0 \; 0] + [\frac{8}{3} \; 4 \; 0 \; \frac{4}{3}]$

$r_1 \to r_1 - r_2 = [2 \; 1 \; 1 \; 0] - [\frac{2}{3} \; 1 \; 0 \; \frac{1}{3}]$

$$\Rightarrow \left[\begin{array}{cccc|c} \frac{4}{3} & 0 & 0 & -\frac{1}{3} & 3 \\ \frac{2}{3} & 1 & 0 & \frac{1}{3} & 3 \\ \hline -\frac{1}{3} & 0 & 0 & \frac{4}{3} & 12 \end{array}\right]$$