MATH 371

LECTURE

CHAPTER 1

EXAMPLES

1. Example: In a factory, machine 1 produces 8-inch pliers at the rate of 60 units per hour and 6-inch pliers at the rate of 70 units per hour. Machine 2 produces 8-inch pliers at the rate of 40 units per hour and 6-inch pliers at the rate of 20 units per hour. It costs \$50 per hour to operate machine 1, while machine 2 costs \$30 per hour to operate. The production schedule requires that at least 240 units of 8-inch pliers and at least 140 units of 6-inch pliers must be produced during each 10-hour day. Which combination of machines will cost the least money to operate?

Project Summary

Let z denote the cost, x the number of hours Machine 1 operates, and y the number of hours Machine 2 operates.

	Per Hour for Machine 1	Per Hour for Machine 2	Totals
8-inch pliers	60	40	240
6-inch pliers	70	20	140
Cost	\$50	\$30	

Formulation

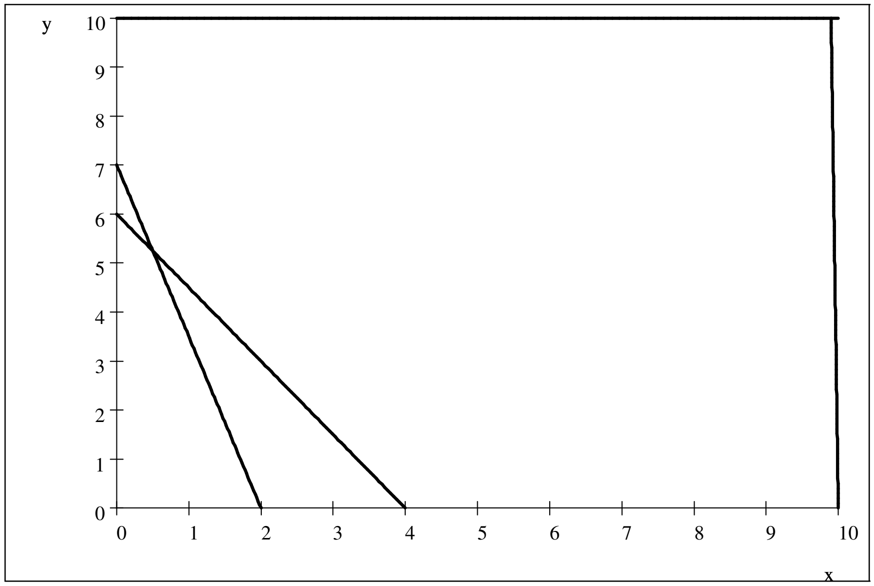
Minimize

$$z = 50x + 30y,$$

subject to

$$\begin{cases} 60x + 40y \geq 240 \\ 70x + 20y \geq 140 \\ 0 \leq x \leq 10 \\ 0 \leq y \leq 10 \end{cases}.$$

Solution:



Corner Points

Point	Coordinates	Value of $z = 50x + 30y$	
A	$(0, 7)$	210.00	
B	$\left(\frac{1}{2}, \frac{21}{4}\right)$	182.50	minimum cost
C	$(4, 0)$	200.00	
D	$(10, 0)$	500.00	
E	$(10, 10)$	800.00	
F	$(0, 10)$	300.00	

2. A chain saw requires 4 hours of assembly and a wood chipper 6 hours. A maximum of 48 hours of assembly time is available. The profit is \$150 on a chain saw and \$220 on a chipper. How many of each should be assembled for maximum profit?

$$\begin{cases} x = \text{number of chain saws} \\ y = \text{number of wood chippers} \end{cases}$$

	x	y	TOTAL
assembly time	4	6	48
profit	150	220	

Objective function: Maximize

$$z = 150x + 220y$$

Constraints:

$$\begin{aligned} 4x + 6y &\leq 48 \\ x &\geq 0, \ y \geq 0 \end{aligned}$$

Corner Points

	(x, y)	$z = 150x + 220y$
A	$(0, 0)$	0
B	$(12, 0)$	\$1800
C	$(0, 8)$	\$1760

For a maximum profit of \$1800, assemble 12 chain saws and 0 wood chippers.

3. Deluxe coffee is to be mixed with regular coffee to make at least 50 pounds of a blended coffee. The mixture must contain at least 10 pounds of deluxe coffee. Deluxe coffee costs \$6 per pounds and regular coffee \$5 per pound. How many pounds of each kind of coffee should be used to minimize costs?

$$\begin{cases} x = \text{number of pounds of deluxe coffee} \\ y = \text{number of pounds of regular coffee} \end{cases}$$

	x	y	TOTAL
amount of coffee	1	1	50
amount of deluxe	1	0	10
cost	6	5	

Objective function: Minimize

$$z = 6x + 5y$$

Constraints:

$$\begin{aligned} x + y &\geq 50 \\ x &\geq 10 \\ x &\geq 0, \;\; y \geq 0 \end{aligned}$$

Corner Points

	(x, y)	$z = 6x + 5y$
A	$(50, 0)$	\$300
B	$(10, 40)$	\$260

For lowest cost, make 10 pounds of deluxe coffee and 40 pounds of regular coffee.

4. A company is considering two insurance plans with the types of coverage and premiums shown in the table.

	Policy A	Policy B
Fire/Theft	\$10,000	\$15,000
Liability	\$180,000	\$120,000
Premium per unit	\$50	\$40

(a) The company wants at least \$300,000 of fire/theft insurance and at least \$3,000,000 of liability insurance from these plans. How many units should be purchased from each plan to minimize the cost of the premiums What is the minimum premium?

$$\begin{cases} x = \text{number of units of policy A} \\ y = \text{number of units of policy B} \end{cases}$$

	x	y	TOTAL
Fire/Theft	\$10,000	\$15,000	\$300,000
Liability	\$180,000	\$120,000	\$3,000,000
Premium per unit	\$50	\$40	

Objective function: Minimize

$$z = 50x + 40y$$

Constraints:

$$\begin{aligned} 10,000x + 15,000y &\geq 300,000 \\ 180,000x + 120,000y &\geq 3,000,000 \\ x &\geq 0,\ y \geq 0 \end{aligned}$$

Corner Points

	(x, y)	$z = 50x + 40y$
A	$(30, 0)$	\$1500
B	$(6, 16)$	\$940
C	$(0, 25)$	\$1000

For a minimum premium, buy 6 units of policy A and 16 units of policy B.

(b) Suppose the premium for policy A is reduced to \$25. Now how many units should be purchased from each plan to minimize the cost of the premiums? What is the minimum premium?

Corner Points

	(x, y)	$z = 25x + 40y$
A	$(30, 0)$	\$750
B	$(6, 16)$	\$790
C	$(0, 25)$	\$1000

For a minimum premium, but 30 units of policy A and 0 units of policy B.

5. Kim Walrath has a nutritional deficiency and is told to take at least 2400 mg of iron, 2100 mg of vitamin B-1, and 1500 mg of vitamin B-2. One Maxivite pill contains 40 mg of iron, 10 mg of B-1, and 5 mg of B-2 and costs 6¢. One Healthovite pill provides 10 mg of iron, 15 mg of B-1, and 15 mg of B-2 and costs 8¢. What combination of Maxivite and Healthovite pills will meet the requirements at lowest cost? What is the minimum cost?

$$\begin{cases} x = \text{number of Maxivite Vitamins} \\ y = \text{number of Healthovite Vitamins} \end{cases}$$

	x	y	TOTAL
iron	40	10	2400
B-1	10	15	2100
B-2	5	15	1500
cost	0.06	0.08	

Objective function: Minimize

$$z = 0.06x + 0.08y$$

Constraints:

$$\begin{aligned} 40x + 10y &\geq 2400 \\ 10x + 15y &\geq 2100 \\ 5x + 15y &\geq 1500 \\ x &\geq 0,\ y \geq 0 \end{aligned}$$

Corner Points

	(x, y)	$z = 0.06x + 0.08y$
A	$(300, 0)$	\$18.00
B	$(120, 60)$	\$12.00
C	$(30, 120)$	\$11.40
D	$(0, 240)$	\$19.20

For a minimum cost, she should order 30 Maxivite vitamins and 120 Healthovite vitamins.

6. A machine shop manufactures two types of bolts. The bolts require time on each of three groups of machines, but the time required on each group differs, as shown in the table.

	I	II	III
Type 1	0.1 min	0.1 min	0.1 min
Type 2	0.1 min	0.4 min	0.02 min

Production schedules are made up one day at a time. In a day, there are 240, 720, and 160 minutes available, respectively, on these machines. Type 1 bolts sell for 10¢ and type 2 bolts for 12¢. How many of each type of bolt should be manufactured per day to maximize revenue? what is the maximum revenue?

$$\begin{cases} x = \text{number of Type 1 bolts} \\ y = \text{number of Type 2 bolts} \end{cases}$$

	x	y	TOTAL
Machine I	0.1	0.1	240
Machine II	0.1	0.4	720
Machine III	0.1	0.02	160
Price	0.10	0.12	

Objective function: Maximize

$$z = 0.10x + 0.12y$$

Constraints:

$$\begin{aligned} 0.10x + 0.10y &\leq 240 \\ 0.10x + 0.40y &\leq 720 \\ 0.10x + 0.02y &\leq 160 \\ x &\geq 0,\ y \geq 0 \end{aligned}$$

Corner Points

	(x, y)	$z = 0.10x + 0.12y$
A	$(0, 0)$	0
B	$(1600, 0)$	\$160.00
C	$(1400, 1000)$	\$260.00
D	$(800, 1600)$	\$272.00
E	$(0, 1800)$	\$216.00

For a maximum revenue of \$272, manufacture 800 Type 1 bolts and 1600 Type 2 bolts.

7. A town has budgeted \$250,000 for the development of new rubbish disposal areas. Seven sites are available, whose projected capacities and development costs are given below. Which sites should the town develop?

Site	A	B	C	D	E	F	G
Capacity, tons/wk	20	17	15	15	10	8	5
Cost, \$1000	145	92	70	70	84	14	47

Solution

Let x_1 denote the number of rubbish disposal areas to be developed at site A. Thus, x_1 is either 0 or 1. Likewise, let x_2, x_3, ..., x_7 denote the number of rubbish disposal areas to be developed at sites B, C, ..., G. Then we have the following mathematical program:

$$\text{Maximize } z = 20x_1 + 17x_2 + 15x_3 + 15x_4 + 10x_5 + 8x_6 + 5x_7$$

subject to

$$145x_1 + 92x_2 + 70x_3 + 70x_4 + 84x_5 + 14x_6 + 47x_7 \leq 250$$

$$x_1 \leq 1,\ x_2 \leq 1,\ ...,\ x_7 \leq 1$$

$$\text{all variables nonnegative and integral}$$

8. A semiconductor corporation produces a particular solid-state module that it supplies to four different television manufacturers. The module can be produced at each of the corporation's three plants, although the costs vary because of differing production efficiencies at the plants. Specifically, it costs \$1.10 to produce a module at plant A, \$0.95 at plant B, and \$1.03 at plant C. Monthly production capacities of the plants are 7500, 10000, and 8100 modules, respectively. Sales forecasts project monthly demand at 4200, 8300, 6300, and 2700 modules for television manufactureres I, II, III, and IV, respectively. If the cost (in dollars) for shippping a module from a factory to a manufacturer is as shown below, find a production schedule that will meet all needs at minimum total cost.

	I	II	III	IV
A	0.11	0.13	0.09	0.19
B	0.12	0.16	0.10	0.14
C	0.14	0.13	0.12	0.15

Solution

Let x_{11} denote the number of televisions produced at plant A and shipped to manufacturer I, and similarly for x_{ij}, $1 \leq i \leq 3$, $1 \leq j \leq 4$. Then we have the following mathematical program:

$$\begin{aligned}\text{Minimize } z &= 1.10\,(x_{11}+x_{12}+x_{13}+x_{14}) + 0.95\,(x_{21}+x_{22}+x_{23}+x_{24}) \\ &\quad +1.03\,(x_{31}+x_{32}+x_{33}+x_{34}) \\ &\quad +(0.11x_{11}+0.13x_{12}+\cdots+0.15x_{34})\end{aligned}$$

subject to

$$\begin{aligned} x_{11}+x_{12}+x_{13}+x_{14} &\leq 7500 \\ x_{21}+x_{22}+x_{23}+x_{24} &\leq 10000 \\ x_{31}+x_{32}+x_{33}+x_{34} &\leq 8100 \\ x_{11}+x_{21}+x_{31} &\geq 4200 \\ x_{12}+x_{22}+x_{32} &\geq 8300 \\ x_{13}+x_{23}+x_{33} &\geq 6300 \\ x_{14}+x_{24}+x_{34} &\geq 2700 \end{aligned}$$

all variables nonnegative and integral