Modelling complex systems problem sheet 1

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Fashion and Fads, solutions

This section will describe two models concerning phone usage. If x is the number of students owning iphones, and $y = n - x$ are owners of phones with other brands, n is the total number of phones with corresponding students.

An independent decision model

If one random phone breaks at each time step. The student will then with probability p buy an iphone, or with probability $1 - p$ buy another brand. These probabilities are unaffected by previous ownerships. This process may be modeled by investigating the number of students owning either an iphone or the other brand and how that number changes through time.

Picking the number of students owning iphones, we begin by investigating the probability that it is an iphone that breaks. The total number of phones is n of which x are iphones, given that it is totally random which phone breaks, the probability that it is an iphone is then given by $\frac{x}{n}$. Thus if we pick a random real number, $r \in (0, 1)$, and this number is smaller than the probability that an iphone breaks, we say that it is an iphone that breaks, else we do nothing. If an iphone breaks, the number of students owning an iphone in the next time step is reduced by one. Regardless of if an iphone breaks or not we then check if a new r is smaller than the probability of buying an iphone, p; if it is, then an iphone owner is added to the next time step. The number of students owing other brands is completely ignored during the whole process, because it is the complementary value to the number of iphone owners.

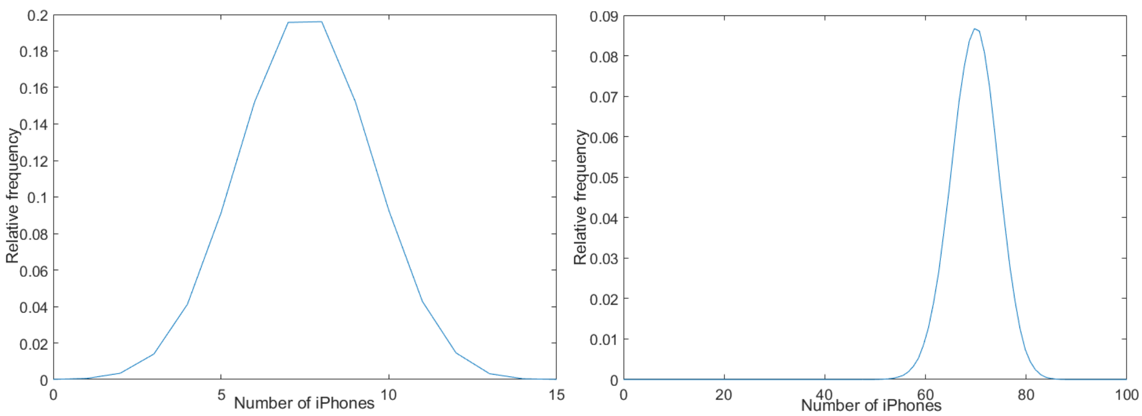


Figure 1: The relative frequency of students with iPhones with 15 possible students and probability 0.5 to the left, and 100 students with probability 0.7 to the right

A master equation for an independent decision model

The master equation $\pi(i, t+1)$ for an independent decision model with four cases is given by

$$\pi(i,t+1) = \pi(i-1,t)\frac{N-(x-1)}{N}p+\pi(i+1,t)\frac{x+1}{N}(1-p)+\pi(x,t)\left(\frac{x}{N}p+\frac{N-x}{N(1-p)}\right), \quad (1)$$

which when $t \to \infty$ looses its dependence on t. If we make the ansats that

$$\pi(i) = \binom{N}{i} p^i (1-p)^{N-i}, \quad (2)$$

which when plugged into equation 1 we get

$$\begin{aligned}\pi(i) = &\binom{N}{i-1} p^{i-1}(1-p)^{N-(i-1)}\frac{N-(i-1)}{N}p \\ &+\binom{N}{i+1} p^{i+1}(1-p)^{N-(i+1)}\frac{i+1}{N}(1-p) \\ &+\binom{N}{i} p^{i}(1-p)^{N-i}\Big(\frac{i}{N}p+\frac{N-i}{N}(1-p)\Big).\end{aligned} \quad (3)$$

Expanding equation 3 we get

$$\left(\frac{(N-1)!}{(i-1)!(N-i)!}+\frac{(N-1)!}{i!(N-i-1)!}\right)\Big(p^i(1-p)^{N-i+1}+p^{i+1}(1-p)^{N-i}\Big), \quad (4)$$

which is in turn simplified to the right hand side of equation 2.

A decision model

Implementing the same decision model as before, but now with probability, q, that the student who's phone broke will ask two other random students and if both of them have the same brand that brand will be chosen as the brand of the students new phone.

Simulating with the total number of students $N = 15$ for several different q it is seen that for $q \approx 0.15$ we have a large variance and both the maximum and the minimum of iPhones is taken several times.

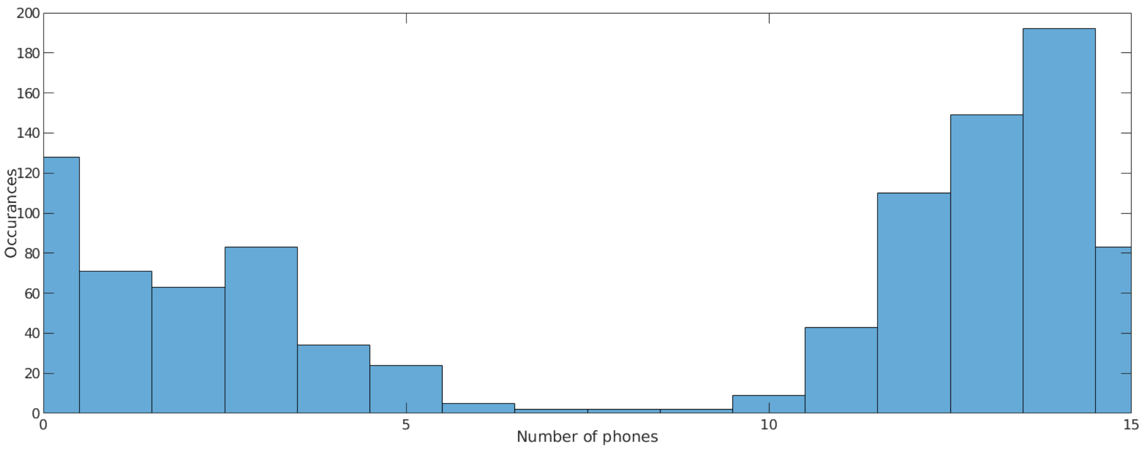
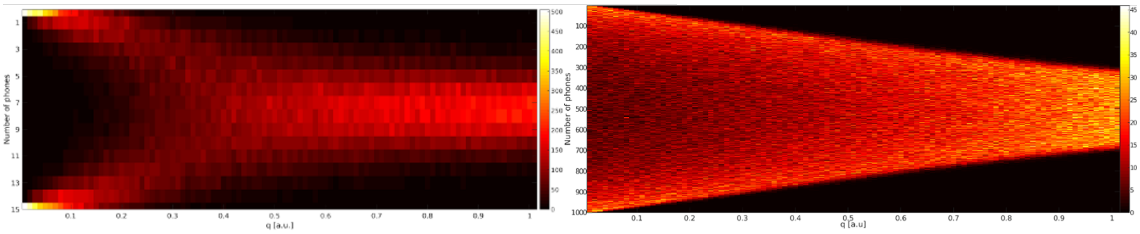


Figure 2: Histogram showing the number of times (vertical axis) a specific amount (horizontal axis) of phones were owned.

This can be seen in the histogram in figure 2. Although because of probabilities the histogram will look slightly different every time one simulates.



Running the simulation for large number of times steps and looking at the bifurcation diagram when the number of phones are $N = 15$ and $N = 1000$. We can see that although they are similar, there are a few differences. while the bifurcation diagram on the left shows a clear split for lower q's the bifurcation diagram on the right does not. This is because of the sheer volume of possible states for $N = 1000$; we can expect that many if not all of those states will be occupied at least once, thus it is expected that for longer simulation runs we may be able see that the diagram does indeed bifurcate.

Cellular Automata

1 dimensional elementary cellular automata simulator

Implementing a one dimensional elementary cellular automata (CA) simulator may be done quite straight forwardly by using an input of a numbered rule, this rule will then be converted to binary code and act as a look-up table for a $3x1$ search vector that travels along an array looking for patterns of ones and zeros. Using rule 110 one obtains an image similar to figure 3

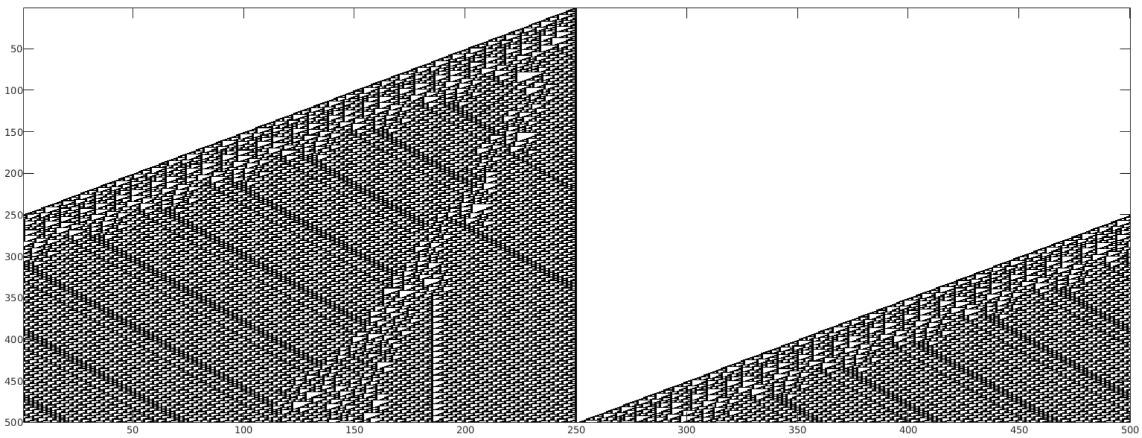


Figure 3: Result of applying rule 110 to a one dimensional CA simulator and propagating it through time

if propagated through time.

2 dimensional cellular automata model

If we consider a painter on a canvas that must conform to rules, such as, if on a white pixel, paint that pixel black and turn 90 degrees to the right then move forward one pixel. Or if on a black pixel, paint that pixel white, turn 90 degrees left and move forward one pixel.

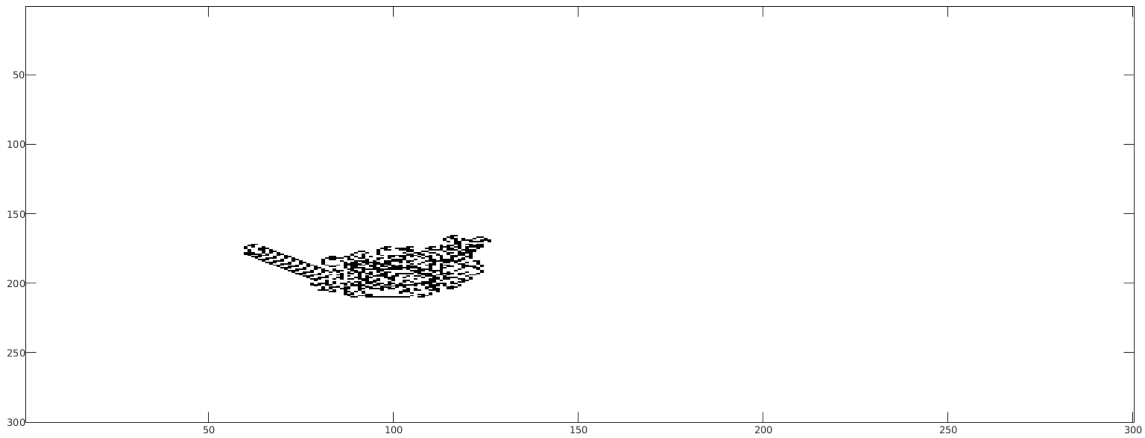


Figure 4: Results of simulating Langton's ant through time.

This model is also known as Langton's ant, which looks like figure 4 when propagated through time.

Pupulation dynamics

Implementing a simple stochastic model for population dynamics

Implementing a model with the following rules:

- If exactly two individuals land on the same site they produce b offspring. These new individuals pass to the next generation.
- If three or more individuals occupy the same site then they all die and produce no offspring.
- Empty sites or sites containing just one individual also produce no offspring.

gives a system with a bifurcation diagram similar to figure 5

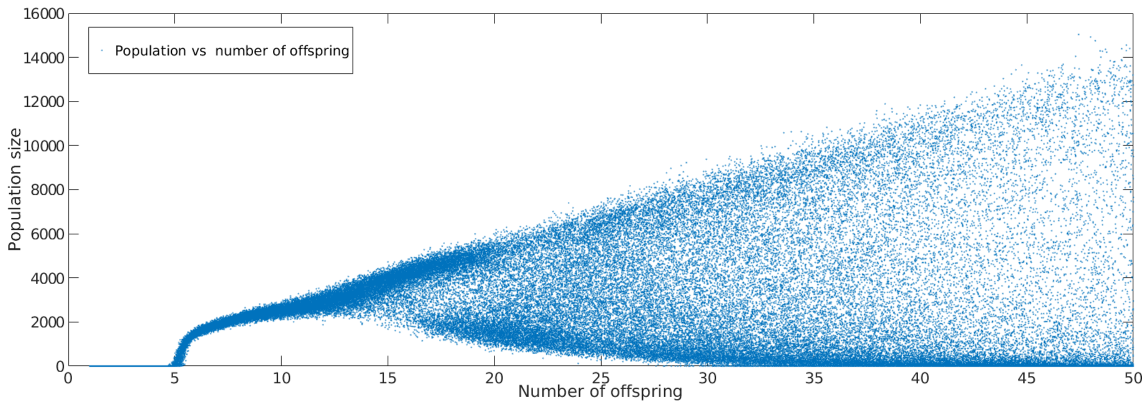


Figure 5: Population size as a function of number of offspring, with $N = 1000$ food sites.

when simulated with 1000 food sites.

Looking at population sizes with different number of offpring, b, for 20 generations we get figure 6

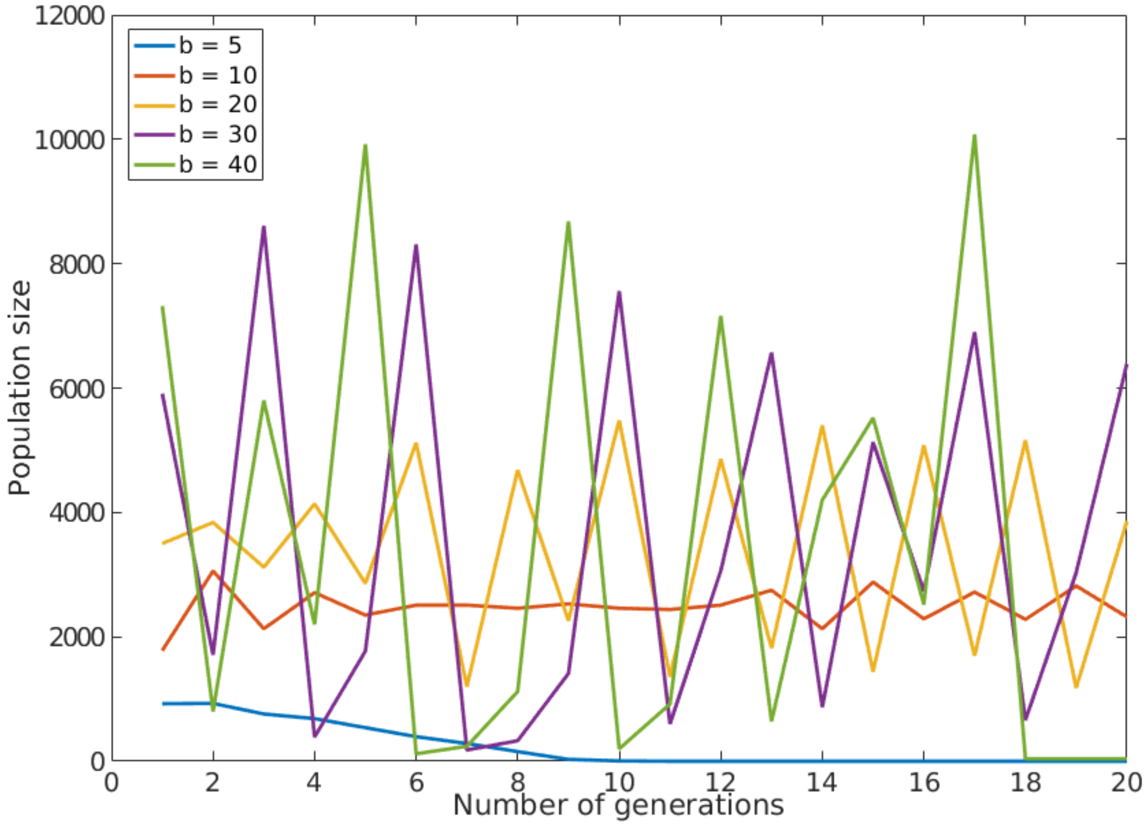


Figure 6: population size with different number of offspring, b.

which shows how the population size changes through generations with different number of offspring.