Capacitance Calculation

NP-NP Interaction

For two nanoparticles i and j are assumed to be spherical conductors with radius r_i and r_j sit in an insulating environment of permittivity ϵ_{env}. The spheres are connected to each other by an insulating molecule of permittivity ϵ_{mol}. The distance between the sphere's centers is d.

For an isolated sphere in an environment with permittivity ϵ_{env}, the self-capacitance is straightforward:

$$C_{ii} = 4\pi\epsilon_{env}r_i$$

The mutual capacitance between two spheres depends on both their sizes and the distance between them. When the spheres are not in contact, the mutual capacitance can be expressed as a series expansion arising from solving the Laplace's equation for the potential in between, where each term in the series represents the interaction between higher-order multipoles on the spheres:

$$C_{ij} = 2\pi\epsilon_{mol}(\frac{r_i r_j}{d})\sum_{n=0}^{\infty}(\frac{r_i^n + r_j^n}{d^n})$$

NP-Electrode Interaction

When estimating the nanoparticle-electrode interaction one could assume that the electrode dentoted j is much larger than nanoparticle i, i.e. $r_j \to \infty$ resembling a spherical conductor near a conducting plane. This reduces the mutual capacitance to

$$C_{ij} = 2\pi\epsilon_{mol}r_i$$

Eventually, if now one wants to estimate the potential of an floating electrode U which is attached to a single nanoparticle i of potential ϕ_i we get

$$U = \frac{C_{ii}}{C_{ii} + C_{ij}}\phi_i$$