Hammerstein Model Identification Workflow

This document outlines the mathematical stages and workflow for building a Hammerstein model from experimental input–output data.

1. Static Nonlinearity Extraction

At sufficiently low excitation frequency, the system behaves as memoryless, so the output $y(t)$ satisfies:

$$y(t) \approx f_{\text{static}}(u(t))$$

where:

- $u(t)$ is the input signal.
- $y(t)$ is the measured output.
- $f_{\text{static}}(\cdot)$ is a nonlinear mapping (e.g., piecewise-linear, polynomial, sigmoidal) that is estimated by fitting y versus u under quasi-steady-state conditions.

Once identified, f_{static} captures all *memoryless* distortion (the low-frequency harmonic plateau).

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2. Complex Frequency Response Function

For a set of higher test frequencies $\{f_0\}$, record steady-state time series $u_k = u(t_k)$ and $y_k = y(t_k)$ over an integer number of periods. Compute the complex fundamental amplitudes via synchronous detection:

$$Y_1(f_0) = \frac{2}{N} \sum_{k=0}^{N-1} (y_k - \bar{y})\, e^{-j2\pi f_0 t_k}, \quad U_1(f_0) = \frac{2}{N} \sum_{k=0}^{N-1} (u_k - \bar{u})\, e^{-j2\pi f_0 t_k}.$$

The empirical frequency response is then:

$$\widehat{G}(j\omega_0) = \frac{Y_1(f_0)}{U_1(f_0)}, \quad \omega_0 = 2\pi f_0.$$

This complex-valued data captures both magnitude and phase of the dynamic block in a best linear approximation. Accordingly we throw away higher harmonics, nonlinearity, etc. which means nonlinearity is only produced by the static nonlinearity while $G(s)$ just filters and delays.

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3. Dynamic Block Identification (All-Pole Model)

Assume the dynamic block is an all-pole system of order n:

$$G(s) = \frac{K}{\prod_{i=1}^{n}(1 + \frac{s}{p_i})}, \quad p_i > 0,$$

which in the frequency domain becomes:

$$G(j\omega) = \frac{K}{\prod_{i=1}^{n}(1 + j\omega/p_i)}.$$

Fit the parameters $\{K, p_i\}$ by minimizing the least-squares error over real and imaginary parts:

$$\min_{K,\{p_i\}} > 0 \sum_k \left|\Re\{G(j\omega_k)\} - \Re\{\widehat{G}(j\omega_k)\}\right|^2 + \left|\Im\{G(j\omega_k)\} - \Im\{\widehat{G}(j\omega_k)\}\right|^2.$$

Enforcing $p_i > 0$ guarantees all poles are in the left half-plane (stable).

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4. Hammerstein Model Assembly

1. **Static block**: $w(t) = f_{\text{static}}(u(t))$.
2. **Linear block**: $y_{\text{model}}(t) = G(s)\,w(t)$, where $G(s)$ has numerator

   $$N(s) = K \prod_{i=1}^{n} p_i,$$

   and denominator

   $$D(s) = \prod_{i=1}^{n}(s + p_i).$$

The complete Hammerstein model is therefore the cascade $u \to f_{\text{static}} \to G(s) \to y$.