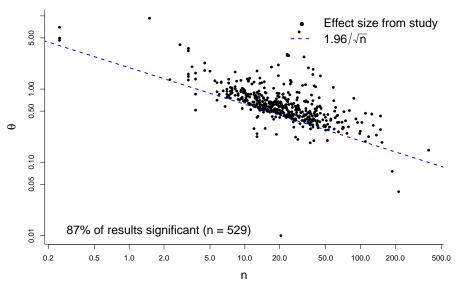
A Bayesian Meta-analysis Method that Corrects for Publication Bias

Jonas Moss

University of Oslo

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This is What *p*-hacking Looks Like!



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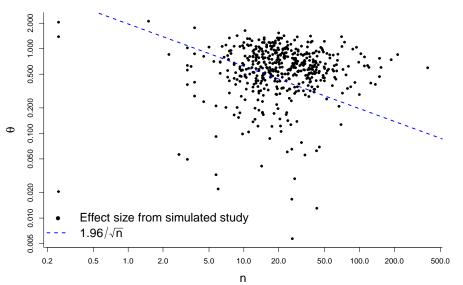
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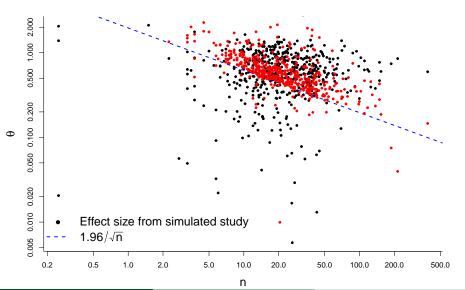
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- Question: Is the classical model realistic in presence of *p*-hacking?

What the Previous Plot Should Have Looked Like!



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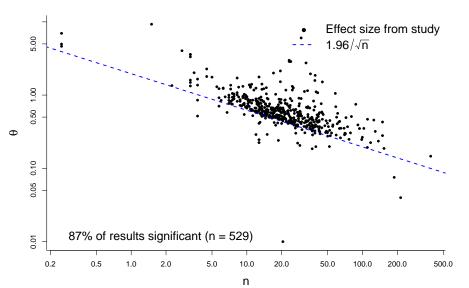
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- **Problem**: The first plot also contains studies that weren't affected by selection for significance!

Revisiting the First Plot



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- On to examples!

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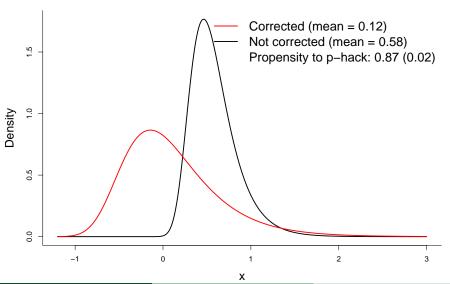
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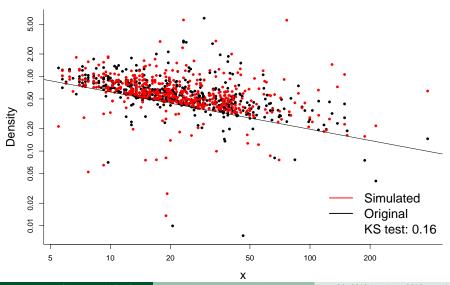
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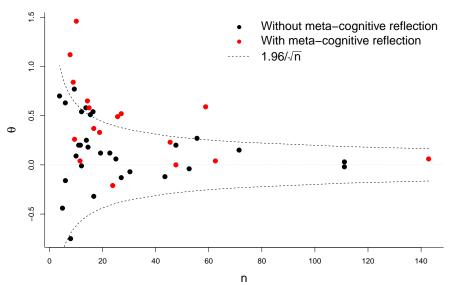
Posterior Predictive Distributions



A Simulation from the Posterior



Bangert-Drowns, Hurley & Wilkinson (2004)



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Model	$ heta_{0}$	θ_{M}	σ	р
Corrected	0.02(0.06)	0.14(0.16)	0.1(0.1)	0.3(0.07)
Not corrected	0.14(0.05)	0.21(0.10)	0.2(0.05)	NA

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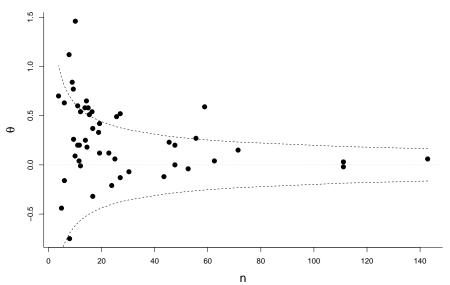
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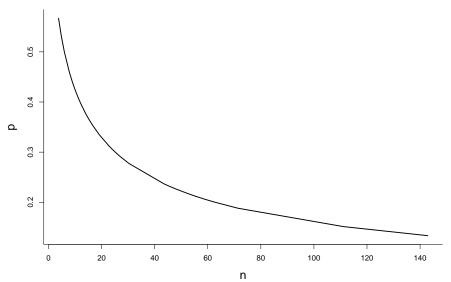
Visually Evidence for Logistic Regression on *p*



Results without Meta-cognition

Model	θ_0	σ	p_0	p_1
Random effects Fixed effects	0.07(0.07) 0.04(0.07)	0.1(0.05) <i>NA</i>	` ,	-0.5(0.3) $-0.6(0.4)$

The Shape of the *p*-hacking Propensity





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- An R-package at GitHub:

straussR

Statistical Reanalysis under Selection for Significance

https://github.com/JonasMoss/straussR