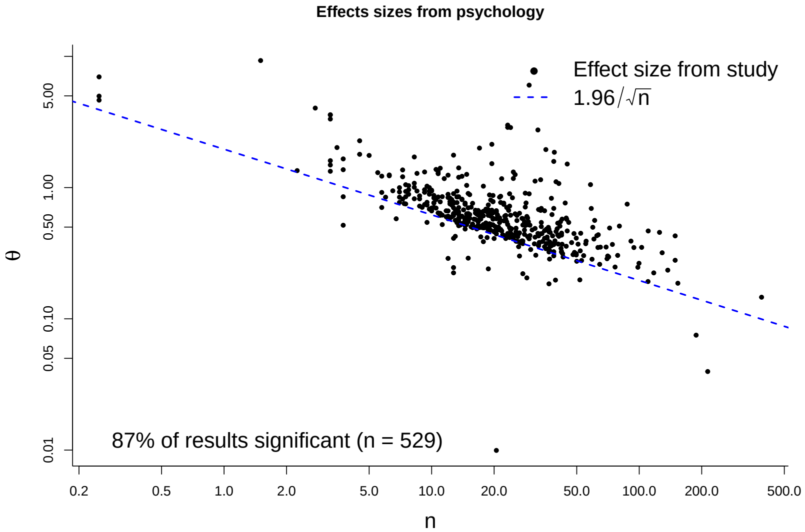
A Bayesian Meta-analysis Method that Corrects for Publication Bias

Jonas Moss

University of Oslo

??, 2018
?? 2018

This is What *p*-hacking Looks Like!



Motyl et al. (2017): The state of social and personality science

Motivation and Setup

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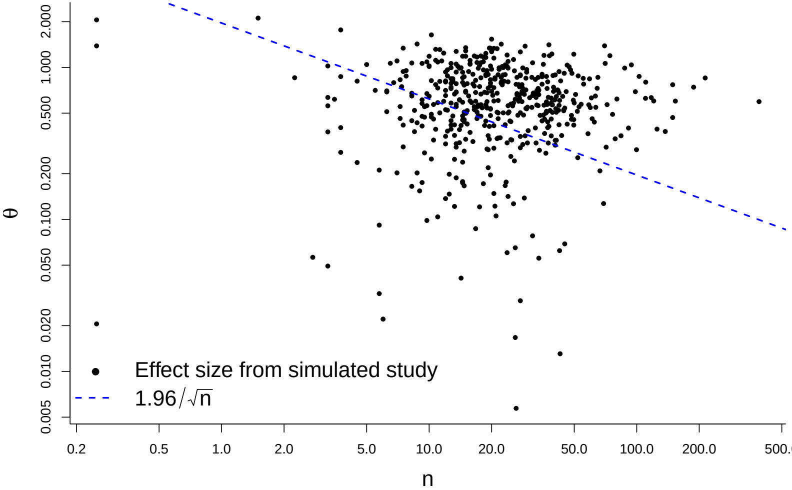
- **Question:** Is the classical model realistic in presence of p-hacking?

What the Previous Plot *Should Have* Looked Like!



Simulated data, random effects (mean: 0.6, sd: 0.25)

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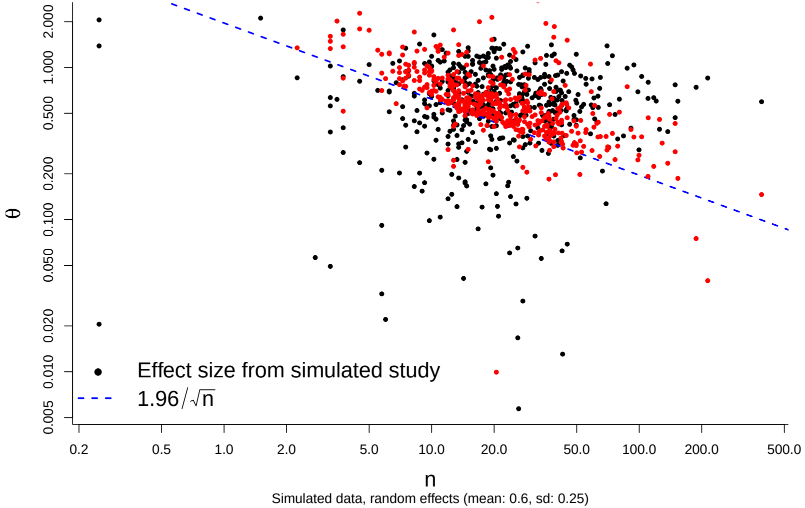

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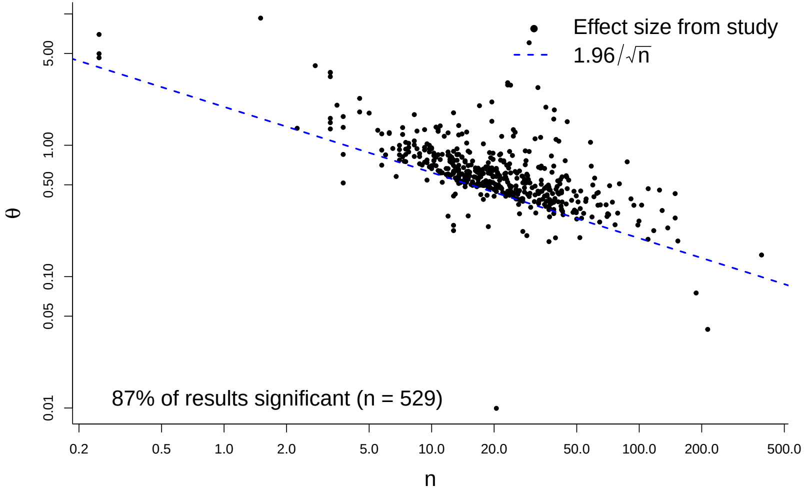
- **Problem**: The first plot also contains studies that weren't affected by selection for significance!

Revisiting the First Plot



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- Computationally feasible due to STAN.

Example I: Meta-analysis of a Field

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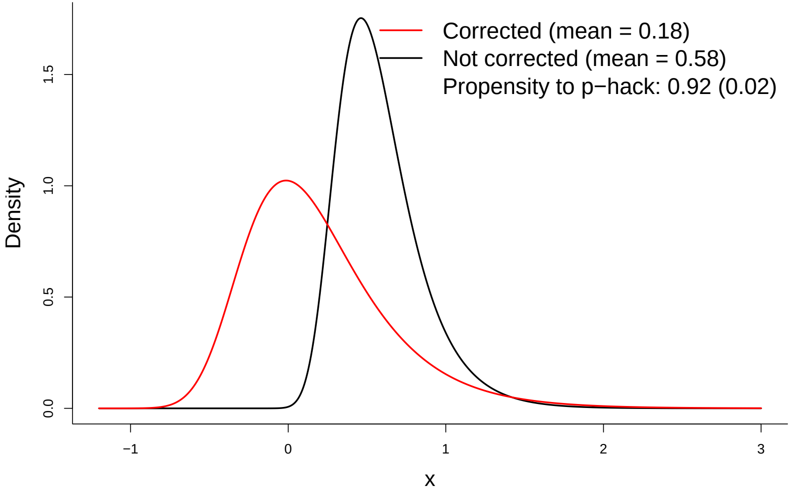
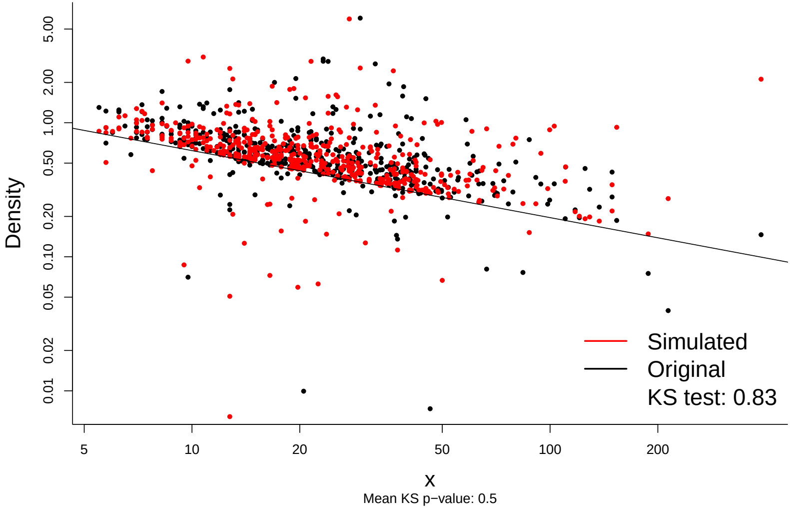
- **Priors:** $\theta_0 \sim N\left(0,1\right)$, $\sigma \sim \mathrm{Exp}\left(1\right)$, $p \sim \mathrm{Uniform}$

Posterior Predictive Distributions

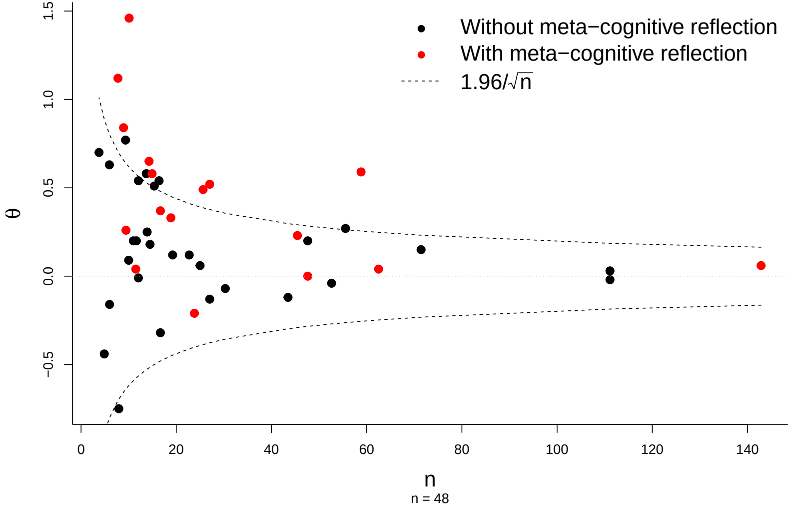


A Simulation from the Posterior



Example II: Ordinary Meta-analysis

Bangert-Drowns, Hurley & Wilkinson (2004)



Is there a difference between red and black?

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Model	θ_0	θ_M	σ $	p$
Corrected $	0.02 (0.06)$ $	0.14 (0.16)$	0.1(0.1) $0	.3 (0.07
Not corrected $	0.14 (0.05)$ $	0.21 (0.10)$	0.2(0.05) $N	A$

-

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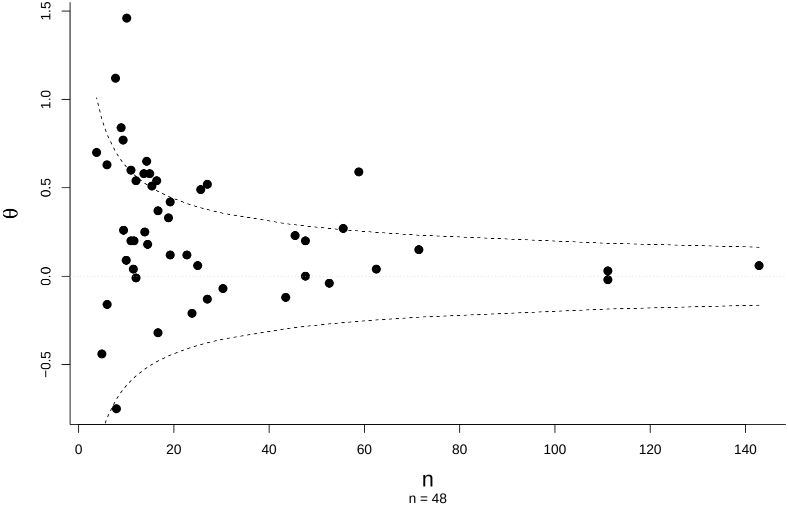
- Publication is easier with larger *n*.

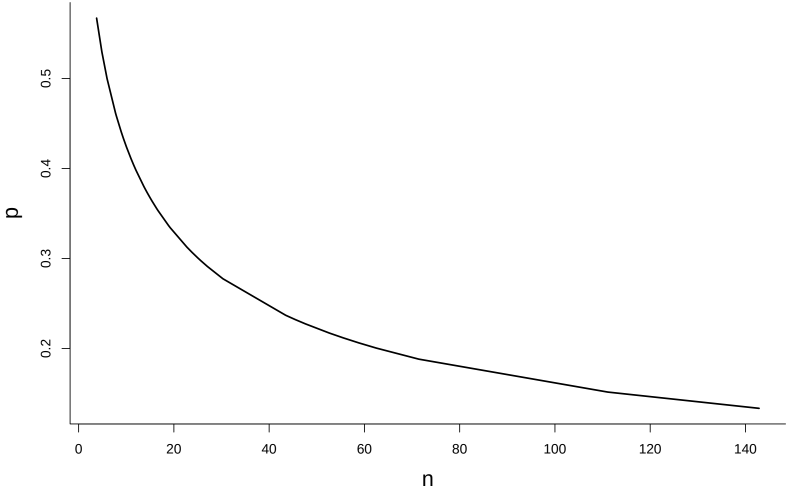
Visually Evidence for Logistic Regression on p



Results without Meta-cognition

Model	θ_0	σ	p_0	p_1
Random effects	0.07(0.07)	0.1(0.05)	$-0.7(0.3)$	$-0.5(0.3)$
Fixed effects	0.04(0.07)	NA	$-0.8(0.3)$	$-0.6(0.4)$

The Shape of the *p*-hacking Propensity



Conclusion

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- An `R`-package at GitHub:

`straussR`

Statistical Reanalysis under Selection for Significance

`https://github.com/JonasMoss/straussR`

References

Bangert-Drowns, Robert L, Marlene M Hurley, and Barbara Wilkinson. 2004. “The Effects of School-Based Writing-to-Learn Interventions on Academic Achievement: A Meta-Analysis.” *Review of Educational Research*.

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