A Bayesian Meta-analysis Method that Corrects for Publication Bias

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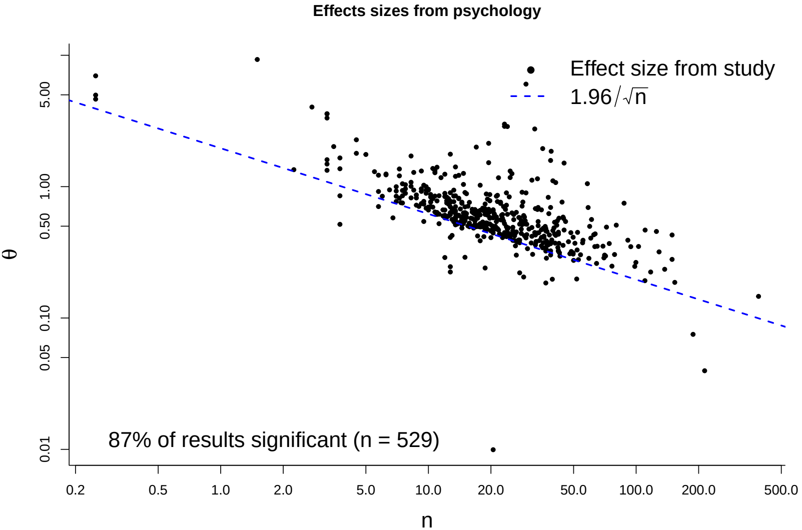
Nordstat 2018

This is What p-hacking Looks Like!



Motyl et al. (2017): The state of social and personality science

Motivation and Setup

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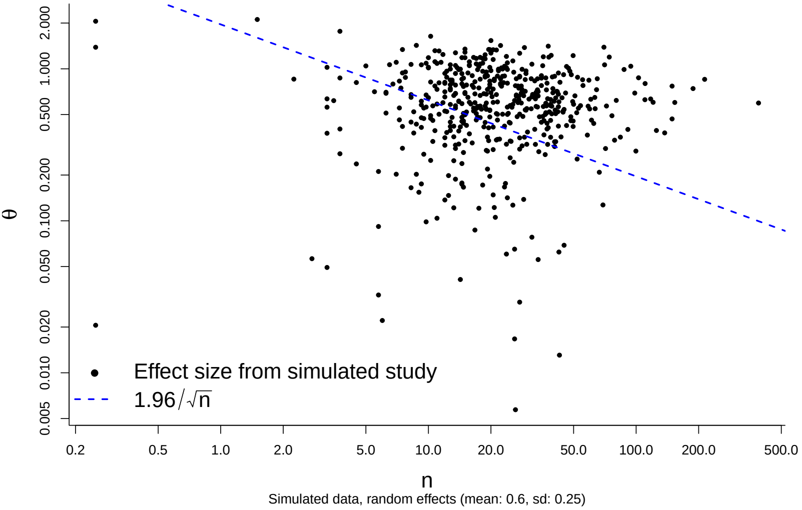
- **Question:** Is the classical model realistic in presence of p-hacking?

What the Previous Plot *Should Have* Looked Like!



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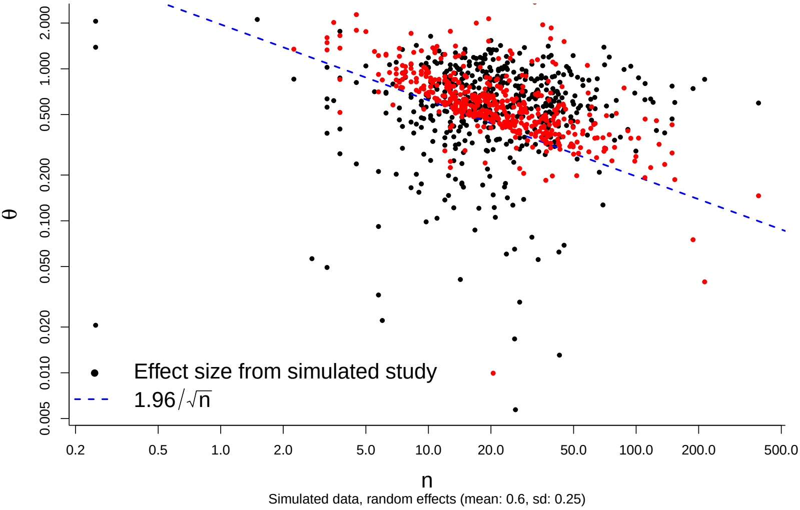

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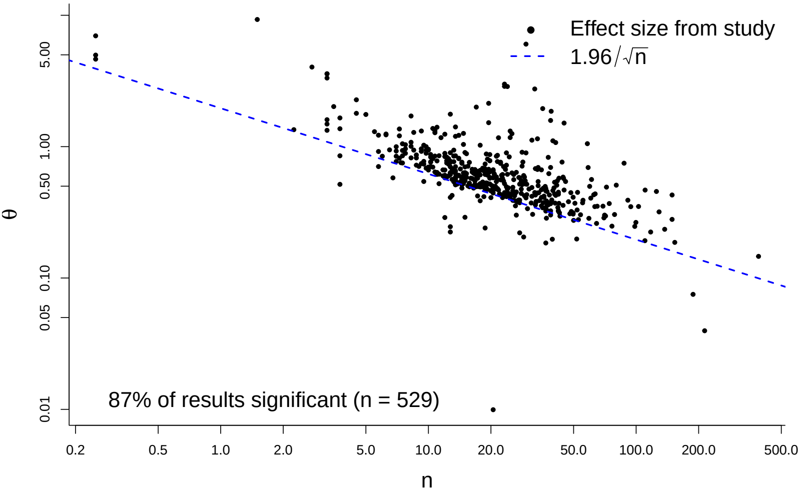
- **Problem**: The first plot also contains studies that weren't affected by selection for significance!

Revisiting the First Plot



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- On to examples!

Example I: Meta-analysis of a Field

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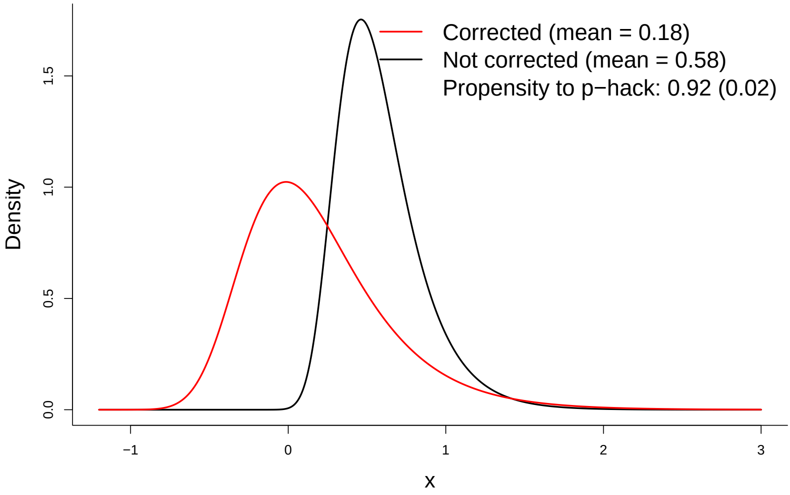
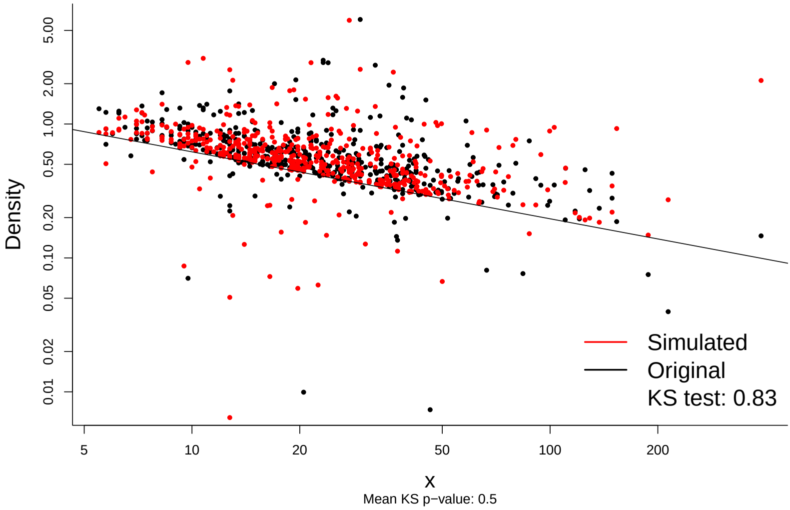
Posterior Predictive Distributions

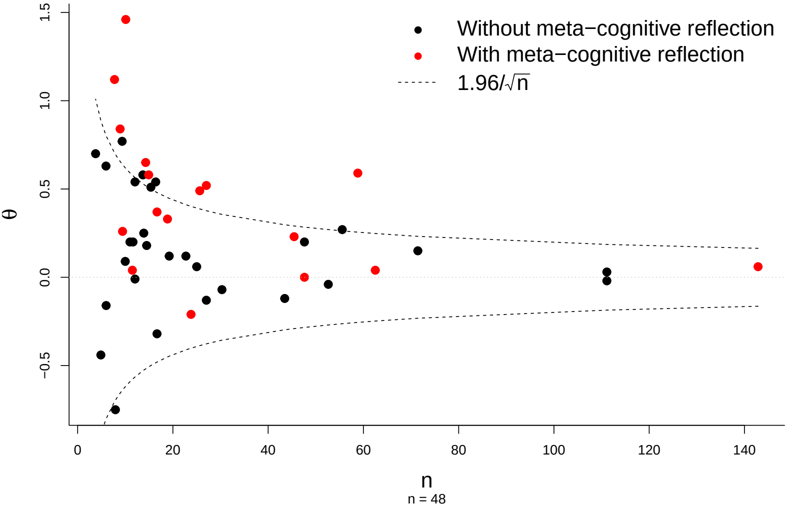


A Simulation from the Posterior



Example II: Ordinary Meta-analysis

Bangert-Drowns, Hurley & Wilkinson (2004)



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Model	θ_0	θ_M	σ	p
Corrected	0.02(0.06)	0.14(0.16)	0.1(0.1)	0.3(0.07)
Not corrected	0.14(0.05)	0.21(0.10)	0.2(0.05)	*NA*

-

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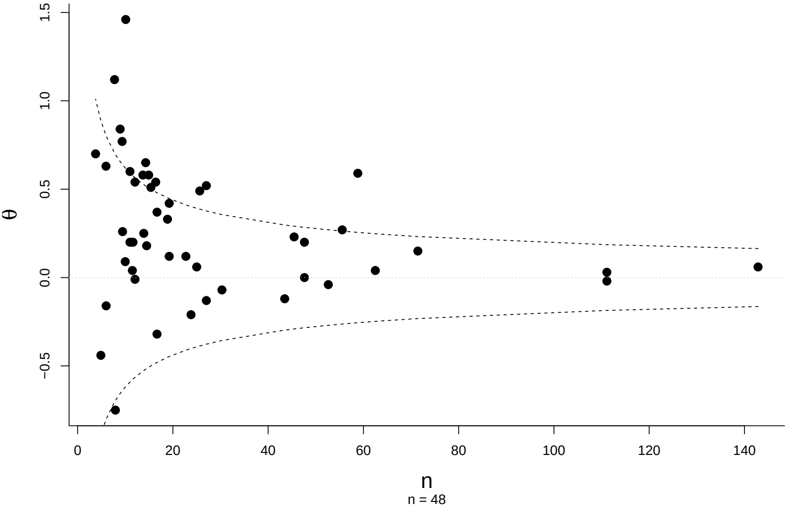
- Publication is easier with larger n.

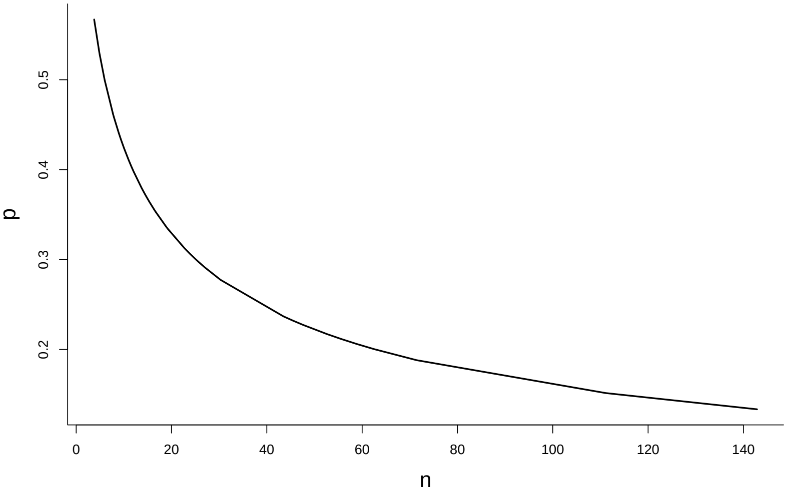
Visually Evidence for Logistic Regression on p



Results without Meta-cognition

Model	θ_0	σ	p_0	p_1
Random effects	0.07(0.07)	0.1(0.05)	$-0.7(0.3)$	$-0.5(0.3)$
Fixed effects	0.04(0.07)	*NA*	$-0.8(0.3)$	$-0.6(0.4)$

The Shape of the p-hacking Propensity



Conclusion

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- An R-package at GitHub:

`straussR`

Statistical Reanalysis under Selection for Significance

`https://github.com/JonasMoss/straussR`

References

Bangert-Drowns, Robert L, Marlene M Hurley, and Barbara Wilkinson. 2004. "The Effects of School-Based Writing-to-Learn Interventions on Academic Achievement: A Meta-Analysis." *Review of Educational Research*.

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