

Employment during Retirement?

Simulating a German Pension Reform in a Life Cycle Model

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Abstract

In many countries, the ratio of pension entitlement to earnings is declining due to demographic developments. In 2017, the German government introduced a pension reform which allows workers to accumulate more pension capital through continued employment during retirement. I develop an overlapping generations model in which agents choose how much to consume, save, and whether or not to retire. Agents are heterogeneous in their education and dis-utility of labor, which may serve as a proxy for deteriorating health. I find that the reform is welfare enhancing for both types and it creates large incentives for workers to remain employed longer. The model predicts an increase in the effective average retirement age of 2.7 years, from 64.0 to 66.7.

Keywords: Life cycle, Social Security, Retirement, Pension Reform

JEL classification: E24, E61, H55, J22, J26

1 Introduction

According to calculations from the German Ministry of Labor and Social Affairs, the pension level – the ratio of net retirement benefits of an average retiree to the yearly average earnings of a worker – will decrease from 48.0% today to 44.5% in 2030. If no further measures are taken and demographic trends remain the same, it will continue to decrease to 41.7% by 2045. Also, estimates for the contribution rate – the share of gross labor income employers and employees pay to the pension insurance scheme – in 2045 are with 23.6% considerably above the goals originally outlined by the government (DRV 2016). This development is not unique to Germany. As a response, a common goal for governments around the world today is to encourage older workers to continue to participate in the labor force. This thesis will quantitatively evaluate the implications of the so-called *flexible pension* reform, introduced by the German federal government in 2017, on the retirement decision of older workers. Using a computational model allows us to simulate the behavioral response of older workers without having access to data.

Many developed countries face an aging population, a trend that puts pressure on the solvency and sustainability of systems for social security – especially the pension insurance. Projections for a majority of industrialized countries indicate continued low birth rates and further increased life expectancy. This will result in an older, but in aggregate numbers unchanged, population. In the European Union, for instance, the ratio of individuals in working-age to above 65 is expected to decrease from 4:1 to only roughly 2:1 by 2060 (Carone et al. 2016). The financial burden on pension insurances is intensified since, despite an upward trend in longevity, the effective age of retirement has not increased to the same extent. In fact, in the last decades of the 20th century, the effective retirement age has actually decreased since governments implemented measures, such as early retirement, to reduce unemployment (Cremer & Pestieau 2003).

The economic costs of a low labor market participation of older individuals are substantial. In terms of lost output, significantly prolonged benefit payments, and a lower tax base, Herbertsson & Orszag (2003) estimate the associated cost with early retirement to amount up to 10% of GDP for some OECD countries. An additional

concern is the effect this development has on inter-generational redistribution since future generations of workers will eventually have to bear the costs.

Possible answers to the challenges governments face include raised tax rates, social security debt, retirement ages, and decreases in pension benefits for retirees. However, international tax competition limits the government's ability to reform the tax system dramatically (Cremer & Pestieau 2003). A slowly growing but aging population on the other hand increases the share of older voters in the society and hence barriers for a government to introduce significant reductions of retirement benefits as a cost-cutting measure. Maestas & Zissimopoulos (2010) argue that finding cost effective ways to extend working lives is the most effective way to ease these current demographic pressures.

Thus not surprisingly, governments in the U.S., continental Europe, and Scandinavia have in recent years set out the policy objective to incentivize more labor force participation among older individuals. A variety of reforms have been introduced, from increases in both the early and statutory retirement ages to reductions in early retirement incentives, i.e. changes in pension formulae to reduce the implicit tax on continued activity.

Germany, which faces similar challenges as most OECD countries, increased the statutory retirement age by two years through a 2007 pension reform. Building on this reform, the German government reformed the pension system again in 2017. This new system is intended to provide more flexibility for old-age workers. It aims to incentivize them to continue working after they reach their full-retirement age by allowing them to work and collect benefits at the same time while continuing to accumulate pension capital. They are therefore expected to continue working also after they start to collect pension benefits since now, under the new law, they are able to increase their pension capital and thus the net present value of lifetime income considerably when they simultaneously work and collect. The impact on lifetime income results from the fact that a few more months of work increase retirement payments for the rest of the individual's life. Since individuals typically earn the highest wages during later parts of their career, the impact on the pension contribution, and thus on the subsequent retirement payment, is relatively large.

I am interested in the effect of the implemented reform. To this end, I develop a partial equilibrium, discrete time, overlapping generations model in which finitely lived individuals choose how much to consume, save, and whether or not to retire, to study implications on retirement decision of old workers. Individuals in the model are heterogeneous in education and, thus, income profiles as well as in their dis-utility of labor – which is increasing in age and serves as a proxy for e.g. deteriorating health.

Simulating such a model allows me to evaluate quantitative impacts of the *flexible pension* law on the timing of retirement, without having access to data from the post-reform economy in Germany. Since many policy measures in general need a very long time to materialize in the data, long time series are necessary to study policy measures empirically. Employing theoretical models instead has the advantage that these models are able to predict results much sooner.

My results indicate that the reform is welfare enhancing for individuals. It creates large incentives for workers to remain employed longer. The model predicts an increase in the effective average retirement age of 2.7 years, from 64.0 to 66.7. Following the reform, individuals choose to stay employed for longer because this yields a higher net present value of lifetime income than before the reform.

Beginning with a brief background on pension systems and reforms, the German retirement law and the 2017 reform are introduced. Later, after describing and calibrating the model, I proceed with the policy analysis. The remainder of the thesis is then characterized by an attempt to critically evaluate the impact of the reform and to discuss the robustness of the model.

1.1 Related Literature

The literature has long focused on the optimal design of a pension system, i.e. the benefits and costs of having a fully funded, an unfunded/pay-as-you-go (PAYG), or a mixed system. Therefore, differences in pension systems across countries have been studied extensively. Recently, due to demographic pressures and their effect on public finances, the literature focused on policy measures which aim to ensure the

financial soundness of public PAYG pension schemes. In particular, pension reforms which raise the retirement age of older workers have been studied extensively.

[Cremer & Pestieau \(2003\)](#) argue that policies, such as increases in both the early and statutory retirement ages and reductions in early retirement incentives do not only lighten the financial burden for pension insurances but also promote redistribution among retired individuals, thus yielding a “double dividend”. [Vogel et al. \(2014\)](#) find that an upward adjustment in the retirement age has large effects. In their model with endogenous human capital formation, an adjustment in the retirement age leads to a reduction of welfare losses for households by about 3 percentage points.

However, adjustments of the retirement age do not suffice to counteract the rise in longevity. [Carone et al. \(2016\)](#) find that on average in the EU, the duration of retirement will rise by approximately 3 years for men and by 1.5 for women in the long run, despite the recent increases in both the statutory and effective retirement age. [Haan & Prowse \(2014\)](#) develop a dynamic life-cycle model for Germany where individuals optimal behavior is a function of life expectancy and the design of the pension insurance. In line with previous findings, their results indicate that older workers postpone their retirement in response to a rise in longevity in equilibrium. However, this individual adjustment cannot offset the aggregate adverse effect on the government budget. Furthermore, they find that an increase of more than 4 years in the statutory retirement age or an almost 40% cut in the per-year value of public pension benefits would be necessary to offset the fiscal consequences of improved life expectancy. These results highlight the further need for policy reforms which address the fiscal challenges associated with an aging population.

One idea is the introduction of more flexibility for individuals willing to continue to work while at the same time collecting pension benefits. [Carone et al. \(2016\)](#) argue that these reforms, which increase incomes during retirement and effectively extend working lives, are less likely to face political and social resistance than a reduction of generosity in the pension benefits. From an individual’s perspective, working longer positively affects the net present value of lifetime income and can be beneficial for two reasons: Firstly, when postponing retirement, the additional contributions to the pension insurance increase the pension payment and thus lifetime income.

Secondly, the additional labor income, i.e. wages earned while an individual already collects pension benefits, can be used to accumulate more private savings.

The gain in lifetime income has large effects on the retirement decision. [Laun & Wallenius \(2015\)](#) evaluate the recent Swedish pension reform, which is designed to create incentives for continued employment.¹ Their model predicts an average change in the retirement age of 2.5 years as a result of the reform. Interestingly, they find that approximately 60% of the change is due to the individual's gain in net present value of pension benefits which comes from continued employment. Introducing reforms that target an individual's net present value of lifetime income therefore seem promising.

There is a large amount of work on the optimal design of social insurance, retirement, and the claiming of old-age pension benefits. However, most of the literature focuses on the United States (see e.g. [Stock & Wise \(1990\)](#), [French \(2005\)](#), [Coile & Levine \(2007\)](#), and [Laun \(2012\)](#)) or the differences between the U.S. and Europe (see e.g. [Rogerson \(2007\)](#) and [Laun & Wallenius \(2016\)](#)). The literature on the German pension system is focused on previous reforms. [Fehr & Habermann \(2006\)](#) and [Böters & Feil \(2009\)](#) study the distributional and risk-sharing consequences of the 2007 reform.

[Fehr et al. \(2012\)](#) evaluate the increase in the retirement age in a general equilibrium model with overlapping generations à la [Auerbach & Kotlikoff \(1987\)](#). In their model, individuals choose their effective age when to exit from the labor market and the results suggest that raising the retirement age by two years increases effective exit age by about one year. However, the reform does not stabilize the contribution rates in the long run and thus the results indicate a high risk for increased old-age poverty rates. Empirical studies suggest that as a result of the reforms, expected retirement decisions changed. [Coppola & Wilke \(2014\)](#) investigate how the raised retirement age from 65 to 67 influences people's retirement expectations and find, on average, that they expect to retire about two years later. Also actual retirement decisions observable in the data were affected. In 2016, German men, for instance, on average retired at age 63.9 and hence still well below the statutory retirement

¹Sweden introduced a defined-contribution insurance financed on a pay-as-you-go basis with a built-in reduction in generosity and incentives for prolonged employment.

age, which was at 65 and five months (DRV 2017).

I aim to contribute to this literature by adding to the work on Germany. In particular, I provide, to the best of my knowledge, the first account of the most recent pension reform. As mentioned earlier, this particular kind of reform, the introduction of more flexibility, seems promising from a policy perspective and I thus hope to enhance the understanding of the effectiveness and impact of such an approach.

2 The Public Pension System

In this section I conceptually highlight the challenges demographic developments pose on pension systems and describe the German retirement law as well as the 2017 *flexible pension* reform in more detail.

Pension systems differ across countries in their set-up, but most pension systems in today's economies can be categorized into either fully funded or PAYG systems. In a fully funded state pension system, the government taxes the young generation and invests their contribution in order to repay the capital as pension when this generation is old (Wickens 2012). In a PAYG system, pension benefits of the retired generation are paid for out of taxation of the current young generation and not the savings of the old generation. To compare the two, Wickens (2012) demonstrates with simple calculations in an overlapping generations (OLG) Model that the consumption possibilities and welfare of both current and future old generations depend on the relationship between population growth rate and the real interest rate at which individuals save.² In a fully funded system, obviously everyone benefits from an higher market interest rate. In a PAYG system on the other hand, both generations benefit, *ceteris paribus*, from a higher population growth rate.

The intuition behind this result is simple. If the population growth rate is large enough, the return on the “investment” into the system, i.e. paying taxes today and receiving benefits later, is larger than the investment on the capital markets. With a growing population, a new, larger generation of workers will finance the pension

²For simplicity, this calculation implicitly assumes stagnant real wages per worker. In fact, the implied return of a PAYG system is the growth rate of aggregate wages, which are a function of the growth rates of both the population and the real wages per worker (Samuelson 1958)

benefits once one is old. However, low population growth on the other hand causes concern because it implies a growing tax burden for the young generation, given a specific pension level. [Wickens \(2012\)](#) concludes that a PAYG pension scheme is likely to reduce welfare in an economy with slow population growth and, thus, unfunded systems are only feasible for fast-growing populations.

In light of this fundamental challenge, many structural reforms for social security systems are discussed, not only in academia but also in the general public. [Feldstein \(1997\)](#), for instance, proposes a shift from PAYG to a mixed system where a personal investment-based account would complement the pension plan. There, each worker would contribute to both the PAYG and an investment-based personal retirement account which would be invested in diversified equity and bond mutual funds.³ With this approach, he argues, a government could reduce the distortionary effect that a purely tax-financed system has on the labor market while at the same time increase the expected welfare from consumption for households.⁴ In essence, relatively higher returns to capital investments would compensate for a less generous social security.

Critics of such a reform put forward administrative costs, which would absorb much of the excess return, and the increase in risk inherent in an investment-based approach. [Feldstein \(2005\)](#) argues that a government managed fund, which collects the contribution directly from employers, could achieve substantial cost savings. He refers to Sweden, which introduced a mixed system with only small administrative costs, as a prime example. In Germany, for instance, the cost efficiency of this second, private, pillar was in the center of the debate after its introduction (see [Section 2.1](#)). Concerning the increase in risk, Feldstein notes that not only is the PAYG system (due to demographics and political decision-making) inherently uncertain, but also could the investment-based system – with its fundamentally different, but manageable, market risks – act as a diversification in the individual’s pension portfolio, thereby improving protection relative to a PAYG system. Indeed, [Matsen & Thøgersen \(2004\)](#) find that such a mixed system allows individuals to hedge risks and is thus beneficial if individuals face imperfect access to the capital market.

³For a discussion on the feasibility of a transition from a pure PAYG to a mixed system, see [Feldstein & Samwick \(1998\)](#).

⁴For an in-depth welfare analysis, see [Feldstein \(1987\)](#).

Another alternative is the so-called notional defined contribution (NDC) system which also involve individual pension accounts that depend on the contributions of the households. However, individuals receive a notional rate of return on their contributions, which is set by the government (Carone et al. 2016). This interest rate often corresponds to the growth rate of aggregate wages, thus responding to both changes in employment and population growth (Berkel & Börsch-Supan 2004). As such, the rate of return would therefore equal the implied return from a PAYG system (Samuelson 1958), and indeed, Carone et al. (2016) note that the system would remain PAYG financed as there are no real assets accumulated. However, the pension benefit paid during retirement would depend not only on the notional rate of return, but also on the remaining life expectancy (Berkel & Börsch-Supan 2004). Hence, the system would adjust to changes in demographic factors.

2.1 Description of the German Retirement Law

The German pension system is a pre-paid, pay-as-you-go system where the current working population pays with their contribution for the current old who receive pension benefits. Before 1957, Germany had a fully funded pension system, but after about half of the capital stock was lost due to a hyperinflation and the Second World War, it was transformed (Börsch-Supan et al. 2015). The German public pension is intended to extend the standard of living individuals achieved during their work lives. Thus, benefits are proportional to the average of the labor income they received during their entire work life and relative to the average earnings in the economy (Fehr & Uhde 2012). Following a relatively stable phase without additional changes after the transformation, the system underwent a major reform in 1972 which introduced a more flexible rule for workers which contributed at least 35 years to the system, allowing them to receive old-age benefits at age 63 (Börsch-Supan et al. 2015).

Acknowledging the challenges resulting from the demographic changes within the society and several budget crises, in the early 1990's the government introduced substantial cuts to the, up until that point, very generous system. Major reforms took place in the following two decades and Germans have been urged to postpone

their retirement and adjust their saving behaviour (Coppola & Wilke 2014). The first adjustment in 1992 introduced incentives for older individuals to work longer by reducing pension benefits by 0.3% for every month they are collected prior to reaching the full-retirement age. According to Börsch-Supan et al. (2015), this has indeed increased labor force participation, almost symmetrically to the decline after the 1972 reform. Using data from the German Socio-Economic Panel, Börsch-Supan (1992) predicts an increase of the average retirement age by half a year from 58.5 to 59 resulting from the introduction of the 3.6% p.a. adjustments factor.

Almost ten years later, in 2001, a second substantial reform took place. The so-called *Riester reform* (named after Walter Riester, the Federal Minister of Labor and Social Affairs at the time) transformed the pay-as-you-go retirement insurance into a multi-pillar system by introducing a second pillar of subsidized, voluntary, and supplementary funded private pensions, commonly referred to as the *Riester pension*. After its introduction, however, the spread of *Riester pensions* initially progressed slowly. The most common criticisms were (and still are) the lack of widespread distribution, high selling and administrative costs, the calculation of the annuity payments on the basis of (too) high assumptions of life expectancy and lack of market transparency. However, since 2001 the share of households without any supplementary pension has fallen from over 73% at the beginning of the millennium to around 39% today. Currently 16.5 million *Riester contracts* are completed and 44% of eligible households have at least one (Börsch-Supan et al. 2016).

In order to contain the financial burden on households, the *Riester law* also states that contribution from workers and employers to the public scheme must stay below 22% of the labor income until the year 2030 and the ratio of net pension entitlement to pre-retirement earnings above 67%. To ensure that the set targets would be met, the Commission for Sustainability in Financing the German Social Insurance Systems proposed a new formula for the pension value (Kommission 2003). The pension value is an important factor (set by the government) in the calculation of the pension payment of an individual. This formula now contains a sustainability factor⁵ which links benefits to the ratio of individuals not in the labor force and

⁵The sustainability factor takes into account the evolution of the ratio of pensioners to contributors in the adjustment of pensions. As the number of retirees increases, the sustainability factor will decrease and the pension adjustment will be lower. If the number of contributors increases,

those who are currently working.

The commission furthermore proposed a gradual increase of both the normal and early retirement age by two years, proportional to the expected changes in life expectancy, between 2011 and 2035 ([Kommission 2003](#)). The proposed reform was later implemented in 2007. [Carone et al. \(2016\)](#) argue that through the reforms in 1992 and the early 2000's (introduction of contribution and sustainability factors), the public pension system has come close to a Notional Defined Contributions (NDC) system.

2.2 The German Pension System Today

Currently, the monthly pension payments a retiree receives are calculated in the following way:⁶

$$\text{pension payment} = \text{pen. points} \times \text{entry factor} \times \text{pen. value} \times \text{pen. type}. \quad (1)$$

The *pension points* are determined by the relation between the individual's earnings and the average earnings of all insured people during a particular year. If the individual earnings equal the average earnings, the individual receives 1.0 pension point. An individual cannot receive more than 2.14 pension points a year, as there is a contribution ceiling, i.e. income over EUR 76,200 per year will not be counted toward the calculation of the pension points. The *entry factor* takes into account the time when an individual retires. If a worker chooses to claim benefits at exactly the statutory retirement age, this factor equals 1.0. If, however, the individual chooses early retirement, the entry factor will adjust by 0.003 downward for every month before the statutory retirement age the individual starts claiming, thereby lowering the monthly benefits by 3.6% p.a. On the contrary, if the individual chooses to retire after the full-retirement age, the entry factor will increase by 0.005 for every

the sustainability factor increases the adjustment. In this way, both the effects of an extended life expectancy, the development of births, and employment on the financing of statutory old-age pension insurance are transferred to pensioners. Source: [Federal Ministry of Labour and Social Affairs](#).

⁶The information in the following paragraph is obtained from the German version of the [website](#) of the German Statutory Pension Insurance Scheme.

month in which the benefits are not claimed which is equivalent to an increase in the retirement payment of 6% p.a. The *pension value* is the monetary value of one pension point. This value is determined by the so-called pension adjustment formula which includes both the sustainability factor and a wage component that ties the pension value to the general development of wages in the economy. Currently, the pension value is EUR 31.03 per month. Lastly, the *pension type factor* is 1.0 for the regular old-age pension.

Table 1 shows the development of the average income, the contribution assessment ceiling, pension value, and the contribution rate in Germany during the last 60 years.

Table 1: Time series assessment bases pension payment

Year	Avg. Inc. ^{a)} (EUR)	Contrib. Ceil. ^{b)} (EUR)	Val. Pension Pts. ^{a)} (EUR)	Contrib. rate ^{a)} (%)
1960	3119	5215	3.24	14.00
1970	6822	11044	6.60	17.00
1980	15075	25769	14.00	18.00
1990	21446	38654	20.24	18.70
2000	27740	52765	24.84	19.30
2010	31144	66000	27.20	19.90
2011	32100	66000	27.47	19.90
2012	33002	67200	28.07	19.60
2013	33659	69600	28.14	18.90
2014	34514	71400	28.61	18.90
2015	35363	72600	29.21	18.70
2016	36267	74400	30.45	18.70
2017	37103	76200	31.03	18.70

a) Source: German Statutory Pension Insurance Scheme

b) Source: Federal Ministry of Justice and Consumer Protection

The contributions to the retirement insurance which finance the old-age pension benefits are paid by both employees and employers. They currently add up to 18.7% of the gross wage and are split equally. The contributions are paid by workers in regular employment, i.e. self employed and so called mini-jobs (with a monthly wage

below EUR 450) are exempted.

Pensions are taxed at the progressive personal income tax. In pension taxation, the tax-exempt amount plays an important role. How much of the pension is taxed depends on the year of retirement. The calculation of the tax-exempt amount is based on the annual gross pension. For individuals who already retired prior to 2005, only 50% of the gross annual pension will be taxed. The tax exemption is a fixed amount and will remain unchanged in subsequent years. This also applies if the pension continues to increase due to pension adjustments. Due to the basic amount of tax exemption (which amounts to EUR 8,820 in 2017), many retirees still do not have to pay any taxes when they have no further taxable income apart from their pension. [DRV \(2016\)](#) estimates that nearly three-quarters of pensioner households are currently completely tax-exempt. Pensioners who have paid average premiums throughout their working lives and have no noteworthy extra income are expected to be required to pay taxes on their pensions for the first time in the coming years.

Every year, the percentage of the taxable portion of the pension increases by two percentage points for the respective retirees. With a pension start in the year 2017 it thus amounts to 74% of the yearly gross pension. After 2020, the taxable portion of the pension only increases by one percentage point for newcomers. All pensions starting in 2040 or later will be fully taxed.

2.3 The Reform

In the beginning of 2017, the German federal government introduced the so-called *flexible pension law* (Gesetz zur Flexibilisierung des Übergangs vom Erwerbsleben in den Ruhestand und zur Stärkung von Prävention und Rehabilitation im Erwerbsleben, 'Flexirentengesetz'). As the name suggests, the new law is intended to provide more flexibility. It is targeted towards old-age workers and it aims to incentivize them to continue working after they reach their full-retirement age, by allowing them to work and collect benefits at the same time while continuing to accumulate pension capital ([DRV 2016](#)).

In the old system, pre-reform, employers paid a pension contribution to the retire-

ment insurance if an employee decided to continue working while collecting pension benefits. Employees, on the other hand, did not pay a contribution (DRV 2016). Therefore, employees would stop accumulating additional pension capital as soon as they started to claim benefits. The employers payments would benefit the overall pension insurance system and thus contribute to a financially sound pension system. However, it would not directly affect an individuals' pension claim.

Now, employees that work and collect are allowed to pay a contribution too. If individuals choose to continue paying a contribution to the pension system, they continue to accumulate more private pension capital. As such, the pension points which determine the pension payment will now be accumulated until the individual stops working. Hence, before the reform pension payments were a function of the sum of pension points up until the statutory retirement.⁷ Post reform, however, they are a function of the sum of pension points up until the period when the individual retires – regardless of whether this is before or after the statutory retirement age.

Furthermore, the new flexible pension law allows workers who choose early retirement to work while collecting benefits. Prior to the reform their pension payments were reduced in case they earned more than EUR 450 per month. From July 2017 on, retirees can earn up to EUR 6,300 per year without any reduction. Earnings above the threshold will lead to a reduction of the monthly pension payment, i.e. 40% of the labor income will be deducted from the pension payment (DRV 2016).⁸

3 The Model

I consider a partial equilibrium, discrete time, overlapping generations model in which finitely lived individuals choose how much to consume, save, and whether or not to retire. I model one period as one year. Individuals enter the model at age 25 and live for 58 periods with certainty, since life expectancy of a 65 year old in 2016 was 17.57 years (DRV 2017). The individuals are heterogeneous in their education, resulting in heterogeneous exogenous wage schedules and heterogeneous in their dis-utility of labor.

⁷Or until the individual stops working – in case of early retirement.

⁸Continued employment after the statutory retirement age does not reduce pension payments.

3.1 Preferences

In the model, individuals want to maximize the discounted utility of consumption and leisure

$$\max_{c_a, l_a} \sum_{a=1}^I \beta^a \left[\ln(c_{a,s}) - b_a(s) l_{a,s} \right], \quad (2)$$

where a is age, $s \in \{\text{high school, university}\}$ is type, β is the discount factor, $c_{a,s}$ is consumption, and $l_{a,s}$ the age-dependent labor supply. The vector of parameters $b_a(s)$ governs the heterogeneity of the dis-utility of labor which is dependent on age and type. I assume that $b_a(\text{high school}) < b_a(\text{university})$ to ensure that high school graduates work even though they receive a lower wage than university graduates.⁹ Finally, as the maximum age is 83, $I = 58$.

Individuals maximize their utility subject to the budget constraint

$$c_{a,s} + k_{a+1,s} = (1 + r(1 - \tau_k))k_{a,s} + (1 - \tau_w(y_{a,s}))y_{a,s} + (1 - \tau_w(R_{a,s}))R_{a,s}, \quad (3)$$

where $y_{a,s} = w_{a,s}l_{a,s}$ denotes labor income as $w_{a,s}$ is the age dependent wage. $\tau_w(y_{a,s})$ is a progressive tax on labor income, $(1 - \tau_w(R_{a,s}))R_{a,s}$ net retirement benefits which depend on the age and previous labor income, and τ_k the withholding tax on capital income. The retirement benefits, $R_{a,s}$, are calculated according to [Equation \(1\)](#).

The government uses the tax revenue to finance the retirement benefits. I abstract from bequests. Following [Laun & Wallenius \(2015\)](#), (1) I impose a no-borrowing constraint, $k_{a,s} \geq 0$. The constrain ensures that people work when they are young, even though they receive a relatively low wage. (2) I assume a discrete labor choice; the individual either works full time or not at all, $l_{a,s} \in \{0, \bar{l}\}$. This assumption is motivated by the well-known fact that hours worked are clustered around a certain, country specific level of hours and zero hours in the data – a feature that standard utility functions cannot replicate ([French 2005](#)). Fixed costs of work (e.g. the commute and the set-up time) generate a reservation wage, below which individuals

⁹Instead of modeling e.g. deteriorating health, I assume an increasing dis-utility from labor in later periods. For a detailed discussion of this assumption, see [Section 4.1](#).

do not work. They are a common way to explain the data (Cogan 1981).

For simplicity, work is supplied inelastically in the model and there is no stochastic employment opportunity. As it is common in the literature, I assume preferences are separable and consistent with balanced growth, which dictates the $\ln(\cdot)$ choice.

3.2 Recursive Formulation

Since the nature of the problem does not change, I can write individuals' decision problem in recursive form. In every period, an individual knows the value for age a , assets k_a , average income and amount of pension points which are relevant for the calculation of the pension benefits, R_a , pension status, and work status. They then decide how much to consume, how much to save, and whether or not to work. As Laun & Wallenius (2015), I assume individuals claiming pension benefits can work but once individuals stop working, they cannot return to work.

In order to limit the computational complexity, I abstract from a number of features of the German pension law. In the model, I do allow individuals to retire early, i.e. before they reached the statutory retirement age of 65. According to the German pension law, they are only eligible for early retirement once they are 63. If they decide to retire early, i.e. to stop working at e.g. age 63, they receive reduced pension benefits. Obviously they forego the possibility to accumulate more pension points, which entails a lower pension payment than if they would have retired at a later point. Also, their pension payment is reduced by the early retirement adjustment factor of 3.6% p.a. The German retirement law allows individuals that retired early to work, but 40% of the labor income above the threshold of EUR 5,400 (pre-reform) and EUR 6,300 (post-reform) will be deducted from the pension payment.

I abstract from the possibility to combine early retirement and work in the model. Once an individual stops working, there is no possibility to continue working part time. This feature limits the ability to mimic all features of the reform, i.e. the EUR 900 increase in the threshold. However, this is arguably of minor importance for the calculations of the net present value of lifetime income and, thus, I am confident that my results will not have a large bias due to this assumption.

Furthermore, I assume that once individuals reach the statutory retirement age, they start claiming benefits – even if they decide to continue working. Hence, in the model, individuals will not make use of the possibility to delay their pension claim in order to receive the increase in the retirement payment of 6% p.a. These assumptions are necessary because they greatly reduce the choice set and thus computational complexity. To support this simplification, I notice that [Laun & Wallenius \(2015\)](#) find that agents will apply for benefits once eligible, and since this choice is not important in their model, [Laun & Wallenius \(2016\)](#) abstract from modeling the optimal timing when individuals should start collecting benefits. I too highlight the importance of the decision of whether or not to work at an old age.

To solve the model, I express the maximization problem an individual faces in a recursive fashion. The value of a state in period a is defined as

$$V(k_a) = \max_{\{c_s, l_s\}_{s=a}^I} \sum_{a=0}^I \beta^a \left[\ln(c_a) - b_a l_a \right] \quad (4)$$

$$\text{s.t. } c_{a,s} + k_{a+1,s} = (1 + r(1 - \tau_k))k_{a,s} + (1 - \tau_w(y_{a,s}))y_{a,s} + (1 - \tau_w(R_{a,s}))R_{a,s},$$

which can be re-written in a recursive formulation, the so-called Bellman Equation ([Bellman 1957](#)):

$$V(k_a) = \max_{\{k_{a+1}, l_a\}} \left\{ \ln(c_a) - b_a l_a + \beta V(k_{a+1}) \right\}. \quad (5)$$

The consumption Euler equation which determines the life-cycle consumption profile reads

$$u'(c_a) = \beta(1 + r(1 - \tau_k))u'(c_{a+1}), \quad (6)$$

where $u'(c_a)$ is the derivative of the utility function with respect to consumption in a particular period. See [Appendix A.1](#) for a derivation of [Equations \(5\) and \(6\)](#).

Using a recursion working backwards from the last period of life, I can find the value functions and decision rules for an individual. Since death is certain beyond age I , the value function at $I + 1$ is identical to zero. Using the budget constraint in [Equation \(3\)](#) to substitute for consumption in the Bellman Equation ([Equation \(5\)](#)), the problem in a given period a reduces to the choice of how much to save and whether or not to work. Consumption is then solved for as the residual and is used in order to calculate the utility for each level of beginning-of-period assets and each employment state. The value of this utility level is passed on to the previous period where the decision rule and the value function for period $I - 1$ are calculated.

That is, for a given capital endowment, employment status and pension status, I search for the optimal savings choice that solves the above problem. Repeating this procedure, I am able to solve for the decision rule in every period. I solve the model 21 times, i.e. once for every year in which an individual could stop working. Then I compare the value of all different retirement ages to find the optimal retirement decision. Using the data, I calibrate the model such that the optimal retirement decision in the baseline model corresponds to what we observe, approximately 64 years.

The computational [Appendix A](#) provides a more detailed description of the recursive formulation of the problem ([Appendix A.1](#)) and on how the model is solved ([Appendix A.2](#)).

4 Calibration

In this section, I discuss the values I have assigned to the parameters in the model. I calibrate the model to mimic the German economy. As is standard in the literature, I restrict the sample to employed males.

4.1 Preference Parameters

The discount factor β and the dis-utility of labor $b_a(s)$ are the preference parameters that need to be assigned values. Similar to [Laun & Wallenius \(2015\)](#), I choose the

value of the discount factor to avoid any life cycle effects on consumption. From Equation (6), the consumption Euler Equation, follows that consumption is stable over time if the left hand side equals the right hand side of the equation. Thus $\beta = \frac{1}{1+r(1-\tau_k)}$, where $r = 0.03$ is the annual interest rate and $\tau_k = 0.26$ is the withholding tax. Hence, β is set to 0.98 in the model. The resulting effect is referred to as consumption smoothing, i.e. individuals' desire for a stable path of consumption, which implies that they shift their consumption from periods with higher income to periods with lower income. Because the retirement, a low income period, follows the earning period, consumption smoothing leads to a humped-shaped age path of asset holding (Modigliani 1986).

The parameters governing the dis-utility of labor are important for matching the retirement age decision in the model and in the data. They are a way to ensure that individuals stop working when they are old. I assume that the dis-utility of labor is increasing in age and takes on an exponential form, so that the marginal impact of the dis-utility of labor increases with every year. Hence, working for one more year is less painful when individuals are young compared to when they are old.

One argument for why workers receive less utility from working when they are old is the deteriorating health conditions of older individuals. French (2005), Laun & Wallenius (2015), and Laun & Wallenius (2016) stress the importance of the negative impact of health on the labor supply of old workers. In Figure 1, I plot the estimated probability of being in bad health for four five-year age bins. I estimate the probability using data on Germany from the Survey on Health, Aging and Retirement in Europe (SHARE).¹⁰ In the survey, individuals indicate their self assessed health on a scale from 1 to 5, where 1 means poor, 2 fair, 3 good, 4 very good, and 5 excellent health. I define answers below 3 as *bad health* and fit a quadratic function to the data. I exclude individuals younger than 55 and older than 75 due to small

¹⁰This thesis uses data from SHARE Wave 6 (DOI: 10.6103/SHARE.w6.610), see Börsch-Supan et al. (2013) for methodological details.

The SHARE data collection has been primarily funded by the European Commission through FP5 (QLK6-CT-2001-00360), FP6 (SHARE-I3: RII-CT-2006-062193, COMPARE: CIT5-CT-2005-028857, SHARELIFE: CIT4-CT-2006-028812) and FP7 (SHARE-PREP: No 211909, SHARE-LEAP: No 227822, SHARE M4: No 261982). Additional funding from the German Ministry of Education and Research, the Max Planck Society for the Advancement of Science, the U.S. National Institute on Aging (U01_AG09740-13S2, P01_AG005842, P01_AG08291, P30_AG12815, R21_AG025169, Y1-AG-4553-01, IAG_BSR06-11, OGHA_04-064, HHSN271201300071C) and from various national funding sources is gratefully acknowledged (see www.share-project.org).

sample sizes in those age groups. The estimated probability is in line with findings by (French 2005, p.407), who plots a similar graph, although for a larger sample.

Including a health sector, and possibly a disability insurance, in the model would lead to a great increase in computational complexity and thus, for simplicity, I assume an exponentially increasing dis-utility of labor instead of modeling the underlying causal channels. As already mentioned, I choose these parameters so the model matches the employment rates of older workers in the data. Figure 1 also visualizes the dis-utility of labor for both types. The vector $b_a(s)$ is defined on an interval $B(s) = [b_{min}(s), b_{max}(s)]$ where $b_{min}(s), b_{max}(s) \geq 0$. Assuming an exponential form and an arbitrary – positive but small – value for $b_{min}(s)$, I am able to calibrate $b_{max}(s)$ to match the employment rates of older worker in the data. Utilizing a bisection method in the algorithm, I am able to approximate the value for $b_{max}(s)$ for which the model predicts the observed retirement behavior. In the baseline case $B(\text{high school}) = [0.1, 2.84]$ and $B(\text{university}) = [0.1, 3.02]$.

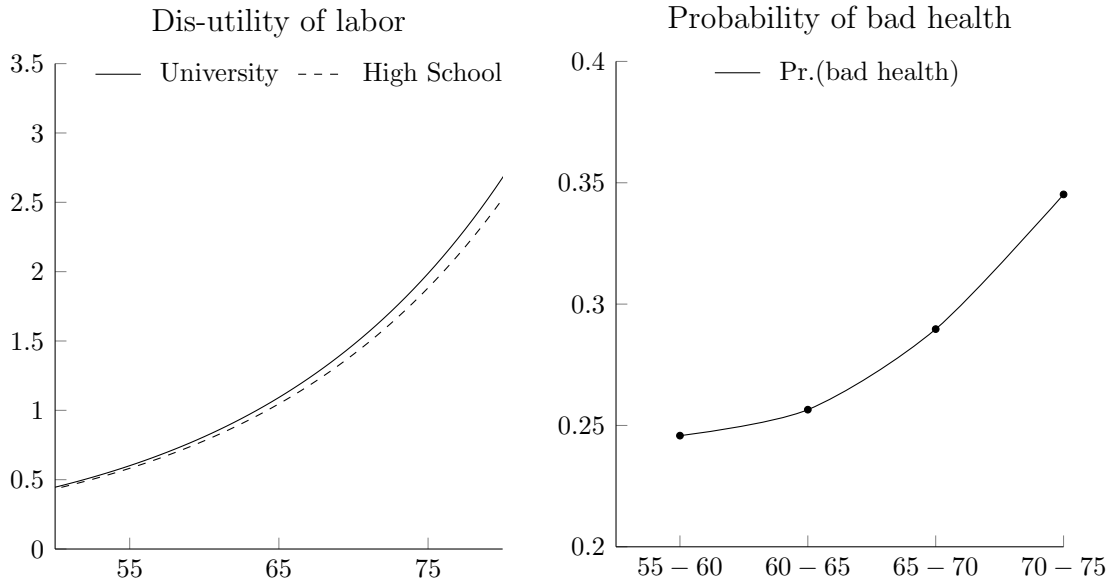


Figure 1: Dis-utility of labor and probability of being in bad health
Source: Probability of bad health estimated using SHARE data

4.2 Earnings Profile

I construct the exogenous age-varying wage profile using data from the [Luxembourg Income Study](#) for Germany. I use data for university and high school educated males

between ages 25 and 63 and, following [Laun & Wallenius \(2015\)](#), fit a quadratic function separately for either education levels to the data. In my sample, 31.6% of men have a university degree. In order to minimize the impact of outliers, I drop the largest 1% and the smallest 1% of observations. The exogenous income profile is estimated as:

$$y_{a,s} = \begin{cases} -59,716.91 + 4,467.88 \times \min(63, a) - 45.23 \times (\min(63, a))^2 & \text{if } s = h, \\ -105,073.20 + 7,479.68 \times \min(63, a) - 75.24 \times (\min(63, a))^2 & \text{if } s = u, \end{cases} \quad (7)$$

where h = high school and u = university.

The plotted earnings functions are shown in [Figure 2](#). It displays a hump-shaped profile, where wages increase in earlier years, level out in the later periods and decrease slightly shortly before retirement. Clearly, university graduates earn higher wages than high school graduates throughout their entire work-life. At peak, their labor income is approximately 1.6 times higher than for their high school graduate counterparts. Interestingly, the general hump-shape profile, however, is very similar for both cases.

Note that the data is cross-sectional and we thus have to consider the possibility of selection issues. One key problem is that we cannot observe an income profile for individuals who do not work. Therefore, I choose to exclude data from workers older than 63 years and to hold the estimated income at this age constant for the remaining periods. I choose this restriction as it corresponds to the age where workers are eligible for early retirement for the first time.

Employees who are compulsorily insured pay their own contribution but employers and employees each pay half of it. The contribution of the employee is deducted from the salary and the amount of the contribution depends on the salary. Currently, the total contribution is 18.6%, hence employees pay 9.3% of their income. However, contributions are only to be paid up to a certain amount of earnings, the income threshold. This threshold is currently EUR 6,500 per month.

The pension points are calculated according to [Equation \(1\)](#). Recall that if the individual earnings equal the average earnings, i.e. EUR 37,103, the individual



Figure 2: Labor income and contribution-income profiles
Source: Wage profiles estimated using LIS data

receives one pension point. An individual cannot receive more than 2.14 pension points a year, as there is a contribution ceiling, i.e. income over EUR 76,200 will not be counted toward the calculation of the pension points.

Figure 2 also plots the contribution ceiling which is used to calculate the pension points in case an individual earns more. In the data, this contribution ceiling is only binding for the university type. Their labor is below the contribution ceiling in the first 12 years. At age 37, their labor income exceeds the contribution ceiling and their additional income is no longer considered for the accumulation of pension capital. Hence, university graduates receive the maximum amount of pension points, i.e. 2.14 points a year, for more than 25 years. For high school graduates, this contribution ceiling is never binding as their labor income is always below EUR 76,200 a year. Hence, they receive more pension capital relative to their wage than university graduates since for them, every Euro of income is counted toward the calculation of pension points.

4.3 Taxes

The government levies taxes to finance government spending and to pay the retirement benefits. It collects two different taxes, a progressive personal income tax,

$\tau_w(y_{a,s})$, and a withholding tax on capital income, τ_k as well as the 9.3% pension contribution from workers. The withholding tax on capital income is 25% and, additionally, a solidarity surcharge of 5.5% is levied. Hence, the effective capital income tax equals 26.38% and, thus, $\tau_k = 0.26$. This abstracts from the fact that individuals with an income tax rate below the withholding tax only pay their personal income tax rate on capital income.¹¹

The German personal income tax schedule is formula based and progressive in income. I model the tax code according to the OECD Revenue Statistics ([OECD 2017](#)), with data retrieved from the [OECD Tax Database](#). There are five brackets with different marginal taxes. Income below EUR 8,820 per year is tax exempt, while a taxable income above EUR 256,304 is subject to the highest marginal tax rate of 45%. The tax calculations are based on the amount of annual taxable income X , rounded to the next full Euro, such that T , the income tax liability (excluding the flat contribution to the pension system), is given by

$$T = \begin{cases} 0 & \text{if } X \leq 8,820, \\ (1,007.27 Y + 1,400) Y & \text{if } 8,821 \leq X \leq 13,769, \\ (223.76 Z + 2,397) Z + 929.57 & \text{if } 13,770 \leq X \leq 54,057, \\ 0.42 X - 8,475.44 & \text{if } 54,058 \leq X \leq 256,303, \\ 0.45 X - 16,164.53 & \text{if } 256,304 \leq X, \end{cases} \quad (8)$$

where $Y = \frac{X-8,820}{10,000}$, $Z = \frac{X-13,769}{10,000}$ are parameters set by the government. [Figure 3](#) gives an overview of marginal tax rates for taxable incomes below EUR 100,000.

Pensions are taxed at the progressive personal income tax, but currently 26% of the pension payment are still tax exempt ([DRV 2016](#)). The remaining part is further subject to the basic amount of tax exemption.

For analytic convenience, I assume the government throws away spending not used to finance the retirement benefits. This is equivalent to assume it spends the revenue on something individuals do not value, i.e. that does not affect the marginal utility

¹¹The data is retrieved from the [German personal income tax code](#).

from consumption in the individuals' utility function (Ljungqvist & Sargent 2004, p. 413). A prime example of such spending is the provision of public goods such as military spending or law enforcement.

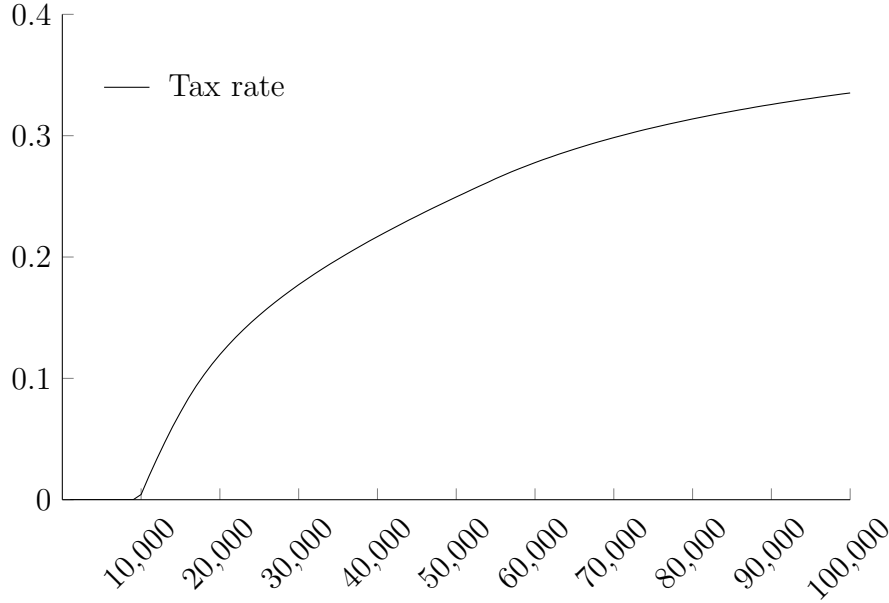


Figure 3: Marginal income tax rate
Source: Author's rendering of [OECD data \(2018\)](#)

5 The Calibrated Economy – Baseline Model

In the following section, I outline the calibrated model and discuss how well it predicts the behavior of individuals in the data before the reform. [Table 2](#) summarizes the parameter calibration.

In order to calibrate the model, I require it to match important features of the data. The main indicator I use for the model calibration is the effective retirement by older workers. To make predictions about the change in behavior induced by the reform, I adjust the model in such a way that it predicts this behavior correctly. Only then can I proceed and implement the reform to study causal links between reform and changes in behavior. The baseline model predicts an optimal retirement age of 64 years for individuals of both types. This main target is set by the data. As calculated by [DRV \(2017\)](#), in 2016, German men on average retired at age 63.9 and hence still well below the statutory retirement age, which was at 65 and five

Table 2: Calibrated parameter values

Parameter	Value	Target	Source
I	58	Final model period	Link
r	0.03	Annual interest rate	
τ_k	0.26	Tax on capital income	Link
τ_w	See Figure 3	Marginal income tax rates	Link
β	0.98	Discount factor	
y	See Figure 2	Income profile	Link
High school			
$b_{min}(h)$	0.1	min. dis-utility of labor	
$b_{max}(h)$	2.84	max. dis-utility of labor	
University			
$b_{min}(u)$	0.1	min. dis-utility of labor	
$b_{max}(u)$	3.02	max. dis-utility of labor	

months. Therefore, the model replicates this central feature of the pre-pension reform German economy.

[Figure 4](#) plots the profiles for the two remaining decision variables, consumption and savings. Note that consumption is flat for the latter part of the life cycle as I have calibrated the discount factor in a way to avoid life cycle effects in consumption, i.e. in the model, individuals smooth their consumption from periods with higher income to periods with lower income due to their desire for stability. However, due to the credit constraint in younger ages, we observe an increase in consumption in earlier periods. Individuals would like to further smooth their consumption by borrowing against future income. The credit constraint prevents them from doing so and thus, they consume all their income which is increasing in time.

Because the retirement, a low income period, follows the earning period, consumption smoothing leads to a humped-shaped age path of asset holding. Notice here again the effect of the borrowing constraint, as individuals start saving only when they reached an income level where this constraint is not binding anymore. This

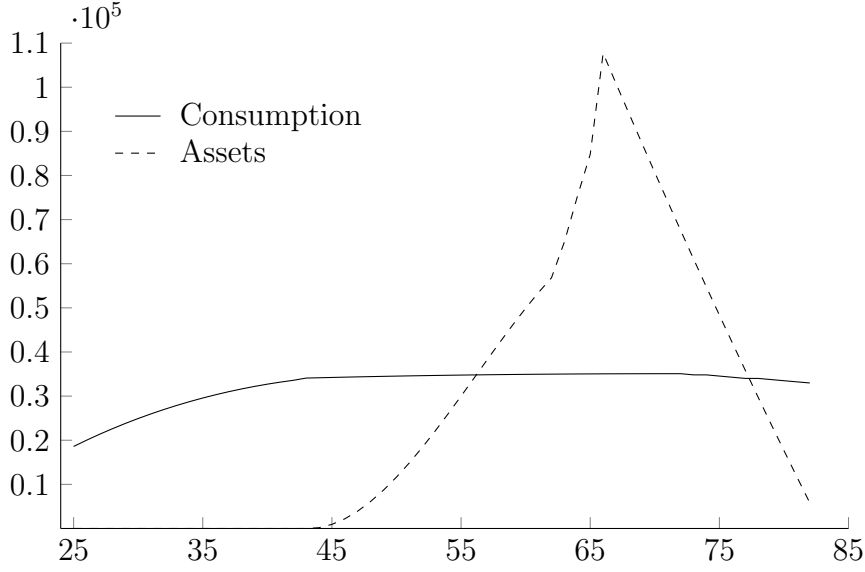


Figure 4: Consumption/Asset profiles (university type)

coincides with a flattening of the consumption profile. Finally, in the beginning of the retirement period, individuals start to dis-save until they exhaust their assets.

6 Results

This section applies the model to analyze the *flexible pension* reform. I outline the implications of the pension reform as predicted by the model and evaluate quantitatively the behavioral response (the timing of the retirement decision) of older workers. I am particularly interested in whether the reform creates incentives for continued employment of older individuals. As illustrated in [Section 2.3](#), the German retirement system has undergone a major change. The restriction on older workers, which prevented them from further contributing to the retirement insurance, was lifted and replaced with a more flexible solution. Older workers now have the choice to continue to accumulate pension capital through employment beyond the statutory retirement age.

In fact, the reform removed the implicit tax on continued employment in the old system. Payments by individuals now directly benefit their pension claim and consequently their lifetime income. In particular, the pension points that determine the pension payment, $R_{a,s}$ (see [Equation \(1\)](#)), will now be accumulated until the time when the individual stops working.

I find the German *flexible pension* reform does indeed create large incentives for workers to remain employed longer. In fact, the model predicts an increase in the effective average retirement age of 2.7 years, from 64.0 to 66.7. Following the reform, individuals choose to stay employed for longer because this yields a higher net present value of lifetime consumption than before the reform. To understand the findings, consider the decision of a 63-year-old individual. Working for one more year comes at the costs of suffering from more dis-utility of labor, but these increased costs are outweighed by the increase in utility the individual receives from an increase in consumption possibilities.

As Figure 5 visualizes, the reform affects both asset holdings and life-cycle consumption of the individuals for the university type.¹² After the reform, individuals accumulate more capital over the life cycle and consume more in every given year. Given the increased consumption possibility, the value from optimal behavior, the value of discounted lifetime utility, increases by 0.05% for the university type and by 0.10% for the high school type after the implementation (not shown in the figure). Hence, the results imply the reform is welfare enhancing for both types.

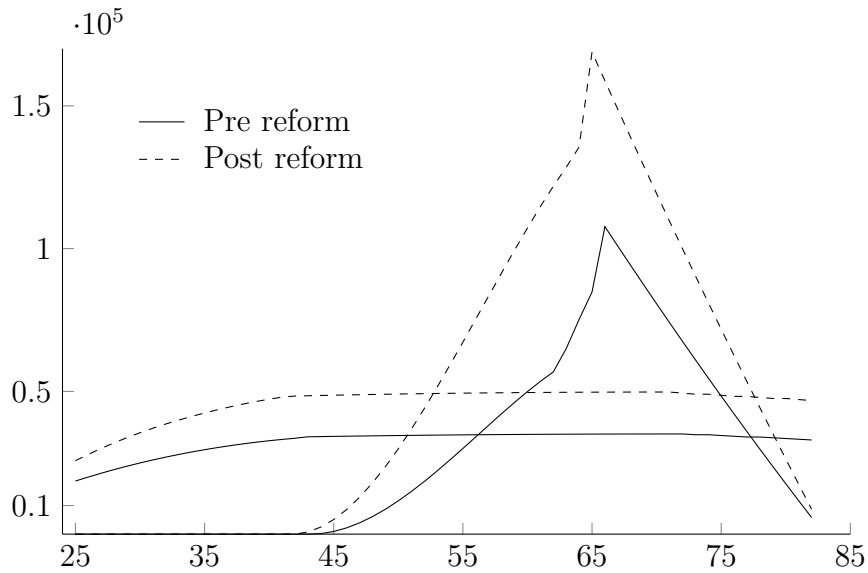


Figure 5: Consumption/Asset profiles (university type)

Note, however, the reform has an heterogeneous effect on the labor supply decision of either type. As a result, high school graduates increase their effective retirement age by three years, university graduates work only for two more years.

¹²To conserve space I only show the plots for one type; the plots are qualitatively similar across types.

As such, the labor supply effect is larger for high school individuals. This is intuitive, since their contribution constraint is not binding. Recall [Figure 2](#), which plots the relationship between gross income and contribution-income – the calculation basis which determines the individual contribution to the pension system. Both gross labor income and contribution-income are identical for high school individuals since their income is below the contribution ceiling. However, for university graduates, the contribution ceiling is binding during the years in which they receive higher wages, and hence, they accumulate less pension points relative to their income than the high school types. This helps to explain why the effect of continued employment is less pronounced for the university type, since for them, accumulating more pension capital is relatively less effective.

Furthermore, since working during retirement results in an increased income, as individuals receive the pension benefits in addition to their labor income, they face a higher marginal tax rate. If university graduates continue to work while they collect retirement benefits, they must pay a relatively higher marginal tax rate on their pension benefits than high school graduates. This higher marginal tax rate discourages them to accumulate more pension capital as they advance in age and thus, for them, continued employment is relatively less attractive than for the high school type.

7 Sensitivity Analysis

To test the robustness of the results of the model and to increase the understanding of the relations between the model parameters and the results, I conduct sensitivity analyses where I vary several features of the model. The parameters governing the dis-utility of labor are important for matching the retirement age decision in the model and in the data. [Figure 6](#) plots the dis-utility of labor for the university type for a variety of specifications for the upper limit of dis-utility of labor, $b_{max}(u)$.¹³ Since there are no comparable model approaches in the literature, there is no direct answer as to whether the magnitude of the change in $b_{max}(s)$ I use as a robustness

¹³To conserve space I only show the plots for one type; the plots are qualitatively similar across types.

check aligns with previous literature. However, the model predictions should not completely change when the key parameter is somewhat altered, by say 3% or 5% in either direction.

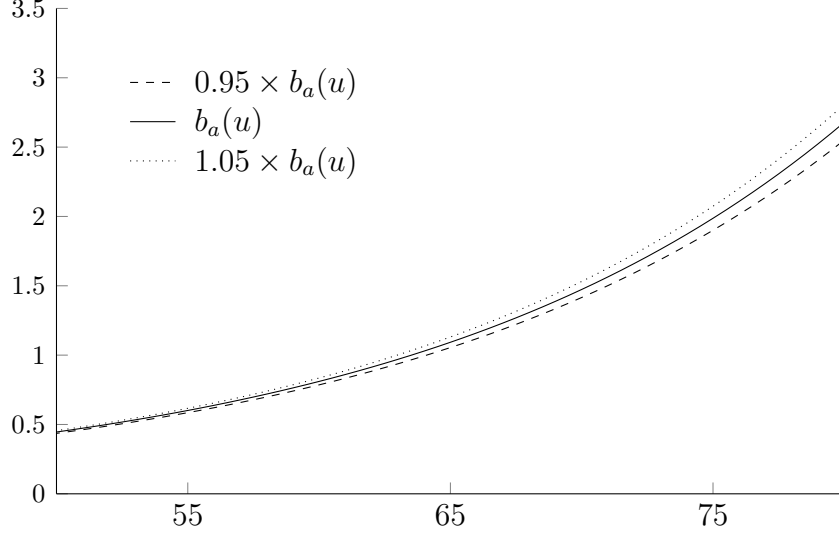


Figure 6: Sensitivity analysis $b_{max}(u)$

The sensitivity analysis suggests that my result, that the reform is effective in encouraging older workers to continue working, is generally robust to changes in the model parameters. For instance, no specification indicates a zero or even negative effect on the retirement behavior in response to the reform. Table 3 shows the effective retirement ages after the implementation of the reform for a variety of specifications for the upper limit of dis-utility of labor, $b_{max}(s)$. Lowering the parameter by e.g. 3% does not affect the average retirement effect, an even lower value actually increases the average retirement age by three years.

Increasing $b_{max}(s)$ obviously decreases the size of the predicted effect, i.e. for a value e.g. 5% larger than the original specification, the effective average retirement age is only 1.7 years larger than before the reform. Changing the dis-utility by even more, e.g. by 10%, leads to pre-reform retirement different from 64, which is inconsistent with the data. In essence, only when I introduce large deviations from the baseline calibration, the magnitude of the predicted effect changes.

Moreover, I keep the distribution of university and high school graduates fixed throughout the analysis. In my sample, the share of men with a university degree is 31.6%. We know, however, that the share of university graduates in the overall

Table 3: Sensitivity analysis $b_{max}(s)$

	Parameter	Value	Eff. retirement	Avg. eff. retirement
High school				
	$0.95 \times b_{max}(h)$	2.69	67.0	67.0
	$0.97 \times b_{max}(h)$	2.75	67.0	66.7
	$0.99 \times b_{max}(h)$	2.81	67.0	66.7
<i>Baseline</i>	$b_{max}(h)$	2.84	67.0	66.7
	$1.01 \times b_{max}(h)$	2.87	67.0	66.7
	$1.03 \times b_{max}(h)$	2.93	66.0	66.0
	$1.05 \times b_{max}(h)$	2.95	66.0	65.7
University				
	$0.95 \times b_{max}(u)$	2.87	67.0	67.0
	$0.97 \times b_{max}(u)$	2.93	66.0	66.7
	$0.99 \times b_{max}(u)$	2.99	66.0	66.7
<i>Baseline</i>	$b_{max}(u)$	3.02	66.0	66.7
	$1.01 \times b_{max}(u)$	3.05	66.0	66.7
	$1.03 \times b_{max}(u)$	3.11	66.0	66.0
	$1.05 \times b_{max}(u)$	3.17	65.0	65.7

population rose during the last 15 years, see [Figure 7](#). Changes in the population structure have implications for the effectiveness of the reform. As the university types react with a relatively smaller increase in employment in response to the reform, the aggregate effects of the reform would be smaller if the share of university graduates would rise further. Consider a scenario where the share of individuals with an university degree is 50%.¹⁴ The predicted average retirement age for such a scenario would be 2.5 years higher than for the pre-reform economy.

¹⁴Note, I only consider males with at least a high school degree in my sample. Obviously, the share of individuals with tertiary education in the population is not policy invariant. However, for the purpose of this example, I consider an exogenous increase.

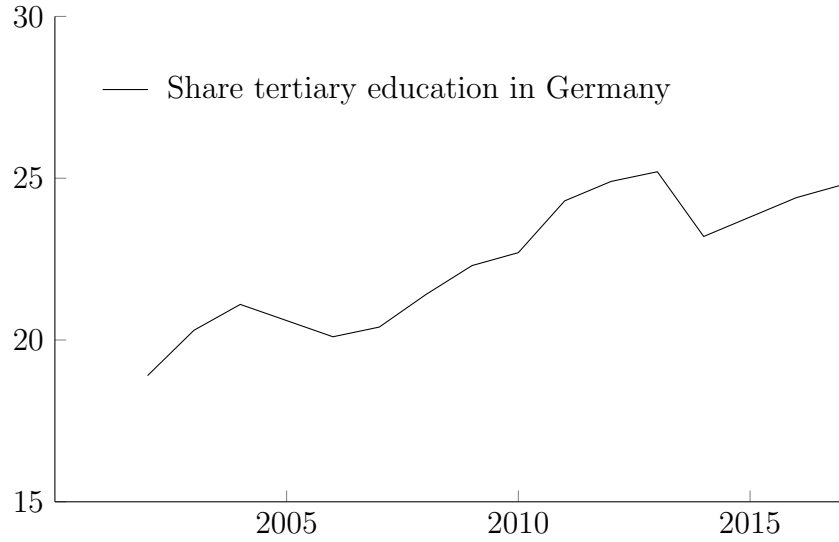


Figure 7: Share of population with tertiary education
Source: Authors rendering of [Eurostat data \(2018\)](#)

8 Discussion

The results of the simulation suggest that the *flexible pension* reform is effective in encouraging older workers to continue to participate in the labor force. They indicate that introducing more flexibility for older workers to their decision making can be beneficial in two ways. Firstly, it may be beneficial for individuals since the reform, given the model assumptions, seems to enlarge consumption possibilities and to enhance welfare. Secondly, since individuals typically earn the highest wages during the latter part of their working life, their contributions, which benefits the overall solvency and sustainability of the pension system, are largest during the time prior to retirement. Hence, incentivizing workers to continue working after they reach their full-retirement age has a relatively large impact on the total amount of pension contributions.

The magnitude of the results is in line with findings of [Laun & Wallenius \(2015\)](#) for the most recent Swedish pension reform. Their model predicts an increase in the average retirement age of two and a half years from 62.1 to 64.6. However, this reform entailed both a reduction in generosity with impact on the present value of lifetime benefits from regular old-age pensions and a change in the occupational pension scheme. Both parts of the reform are quantitatively important in their model, which raises the question of whether my model overestimates the labor supply effect. For

comparison, [Fehr et al. \(2012\)](#) find that raising the retirement age in Germany by two years increases effective retirement age by about one year.

Therefore, I conduct careful sensitivity analyses where I vary the dis-utility of labor parameter, $b_{max}(s)$. I find that the results are generally robust to changes in this parameter. While changing the dis-utility leaves the direction of the effect on the retirement decision unchanged, introducing large deviations from the baseline calibration changes the magnitude of the predicted effect of the reform.

A potential source of an upward bias is the specification of the government spending. Recall that in my model, I assume that the government throws away revenues not used to finance the retirement benefits. This is equivalent to assuming that it spends the revenue on something individuals do not value. Another option would be to assume that the government balances its budget by rebating unused revenue through a lump-sum transfer to the individuals. Given that the effective labor tax rate has not declined following the reform, the lump-sum transfer would adjust in such a way that the budget balance is restored post-reform. Hence, I am unable to evaluate if, for instance, the government must run a budget deficit (which has to be financed by e.g. future tax increases) following the reform.

In their sensitivity analysis, [Laun & Wallenius \(2015\)](#) find that including this general equilibrium effect leads to a reduction in the effect size by 0.2 years, i.e. in the partial equilibrium version, they find an increase in the average retirement age of 2.7 years, compared to the 2.5 years predicted by the general equilibrium version of the model. This suggests that a general equilibrium model may predict a slightly lower labor market response for the flexible pension reform than my model.

Furthermore, given that we observe heterogeneous responses to the reform from both types of individuals, it is interesting to notice what consequences a shift to more university graduates in a society could have on the pension system. The results also highlight the dampening effect which the contribution ceiling has on individuals with income above the ceiling. Together with a progressive tax code, the contribution ceiling discourages workers with higher wages from continued employment. Hence, the quantitatively larger share of the positive effect of the reform comes from high school graduates, and with more university graduates, it would likely be smaller

than currently predicted.

Since I abstract from potentially important features in the decision making process of individuals, I must acknowledge the limitations of the model. I do highlight the importance of the decision of whether or not to work at an old age, which is of first-order importance for the evaluation of the effectiveness of the reform. However, since I only capture the impact of health by introducing a preference parameter as a proxy for deteriorating health in older ages, I am, for instance, not able to introduce health investments or the decision of whether or not to claim disability insurance, as potentially interesting decision variables into the model. Also, since I abstract from general equilibrium effects of the reform, the model is silent on the effects of government spending. From [Rogerson \(2007\)](#) and [Ragan \(2013\)](#), we know that the use of government revenue can have implications for labor supply choices of individuals, thus potentially affecting the effect size of the reform. Larger and more computationally intense models which include uncertainty in longevity, shocks to health, a disability insurance, and the potential for individuals to invest in future health may thus be useful to study the German pension reform in future research.

Lastly, the model is silent on the portfolio choice of individuals when they provide for their pension. Given [Matsen & Thøgersen \(2004\)](#) emphasis on the importance of the interplay between public pension system and private savings, one potentially interesting addition to the model could be to allow individuals to invest in various asset types additionally to their pension savings. This could shed light on the impact of the reform on the portfolio composition of individuals.

9 Conclusion

In many countries, the ratio of net pension entitlement to pre-retirement earnings is declining due to demographic developments. Like many other countries, Germany has introduced several reforms to improve the sustainability of the pension system. In this thesis, I evaluate the most recent reform, the so-called flexible pension reform, which was implemented in 2017. It allows workers to accumulate pension capital through continued employment during retirement.

To this end, I develop a partial equilibrium, discrete time, overlapping generations model in which finitely lived individuals choose how much to consume, save, and whether or not to retire, to study the implications on the retirement decision of old workers. Model households are heterogeneous in education and, thus, in income profiles, as well as in their dis-utility of labor. This dis-utility is increasing in age and serves as a proxy for deteriorating health as workers grow older.

I use the model to study what potential lessons there are to be learned from this reform and find the reform is welfare enhancing for both types. It creates large incentives for workers to remain employed longer. The model predicts an increase in the effective average retirement age of 2.7 years, from 64 to 66.7. Following the reform, individuals choose to stay employed for longer because this yields a higher net present value of lifetime income than before the reform.

The focus in this thesis is on the recent German pension reform. Potential avenues for future research include extensions of the model to a general equilibrium framework where individuals' health, labor market response to changes in taxation/-government spending, and portfolio choices can be evaluated.

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A Computational Appendix

A.1 Recursive Formulation

Following [McCandless \(2008\)](#), I formulate the individual's problem recursively and derive the consumption Euler equation which gives the optimal intertemporal consumption decision. To simplify notation, I suppress the dependence on type s . In the model, the amount of assets available at age a , k_a , is predetermined and known. The individuals choose the amount of assets that will be available in period $a + 1$, k_{a+1} , and thus re-writing the budget constraint in [Equation \(3\)](#) as

$$c_a = \frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a + R_a - k_{a+1} \right) \quad (\text{A.1})$$

allows us to express the value of the objective function as

$$V(k_a) = \max_{\{k_s, l_{s-1}\}_{s=a+1}^I} \sum_{a=0}^I \beta^a \left[\ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a + R_a - k_{a+1} \right) \right) - b_a l_a \right], \quad (\text{A.2})$$

where the value of the value function, $V(k_a)$, is the discounted value of utility when the maximization problem has been solved and when k_a was the initial amount of assets ([McCandless 2008](#), 53). Since the problem is recursive in nature, the value function in the next period can be written as

$$V(k_{a+1}) = \max_{\{k_s, l_{s-1}\}_{s=a+2}^I} \sum_{a=0}^I \beta^a \left[\ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_{a+1} + (1 - \tau_w(y_{a+1}))w_{a+1} l_{a+1} + R_{a+1} - k_{a+2} \right) \right) - b_{a+1} l_{a+1} \right]. \quad (\text{A.3})$$

By separating the period a problem from that of future periods, I can write the value function in [Equation \(A.2\)](#) as

$$\begin{aligned}
V(k_a) = & \max_{\{k_{a+1}, l_a\}} \left\{ \ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a + R_a - k_{a+1} \right) \right) - b_a l_a \right. \\
& + \max_{\{k_s, l_{s-1}\}_{s=a+2}^I} \sum_{a=1}^I \beta^a \left[\ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a \right. \right. \right. \\
& \left. \left. \left. + R_a - k_{a+1} \right) \right) - b_a l_a \right] \Big\}. \tag{A.4}
\end{aligned}$$

Adjusting the indices in the second part of the sum yields the Bellman equation ([Bellman 1957](#)):

$$\begin{aligned}
V(k_a) = & \max_{\{k_{a+1}, l_a\}} \left\{ \ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a + R_a - k_{a+1} \right) \right) - b_a l_a \right. \\
& + \beta \max_{\{k_s, l_{s-1}\}_{s=a+2}^I} \sum_{a=0}^I \beta^a \left[\ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_{a+1} + (1 - \tau_w(y_{a+1}))w_{a+1} l_{a+1} \right. \right. \right. \\
& \left. \left. \left. + R_{a+1} - k_{a+2} \right) \right) - b_{a+1} l_{a+1} \right] \Big\} \\
= & \max_{\{k_{a+1}, l_a\}} \left\{ \ln \left(\frac{1}{1 + \tau_c} \left((1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a + R_a - k_{a+1} \right) \right) \right. \\
& \left. - b_a l_a + \beta V(k_{a+1}) \right\}. \tag{A.5}
\end{aligned}$$

To proceed, I assume that the value function exists and has a first order derivative.

The resulting first-order condition w.r.t k_{a+1} is

$$\frac{-1}{(1 + r(1 - \tau_k))k_a + (1 - \tau_w(y_a))w_a l_a + R_a - k_{a+1}} + \beta \frac{\partial V(k_{a+1})}{\partial k_{a+1}} = 0. \tag{A.6}$$

Assuming the conditions¹⁵ for the Envelope theorem (Benveniste & Scheinkman (1979)) hold, I can find $\frac{\partial V(k_{a+1})}{\partial k_{a+1}}$ by taking the partial derivative of the Bellman equation in Equation (A.5) w.r.t k_t :

$$\frac{1+r}{(1+r(1-\tau_k))k_a + (1-\tau_w(y_a))w_al_a + R_a - k_{a+1}}. \quad (\text{A.7})$$

Using the result in Equation (A.6) and evaluating Equation (A.7) in period $a+1$, I obtain the consumption Euler equation:

$$\begin{aligned} & \frac{1}{(1+r(1-\tau_k))k_a + (1-\tau_w(y_a))w_al_a + R_a - k_{a+1}} = \\ & \beta \frac{1+r}{(1+r(1-\tau_k))k_{a+1} + (1-\tau_w(y_{a+1}))w_{a+1}l_{a+1} + R_{a+1} - k_{a+2}} \\ & \Leftrightarrow \\ & u'(c_a) = \beta(1+r(1-\tau_k))u'(c_{a+1}). \end{aligned} \quad (\text{A.8})$$

¹⁵Using the following notation

$$V(x_t) = \max[F(k_t, y_t)\beta V(x_{t+1})]$$

s.t.

$$x_{t+1} = G(x_t, y_t)$$

that gives the Benveniste-Scheinkman derivative as

$$V'(x_t) = F_x(x_t, y_t) + \beta V'(G(x_t, y_t))G_x(x_t, y_t)$$

holds under the following assumptions

1. $x_t \in X$, where X is a convex set with a nonempty interior
2. $F(\cdot, \cdot)$ is concave and differentiable
3. $G(\cdot, \cdot)$ is concave and differentiable and invertible in y_t
4. $y_t \in Y$, where Y is a convex set with a nonempty interior.

A.2 Solution Algorithm

This section draws on [Imrohoroglu et al. \(1998\)](#) which describe in detail the algorithm used to solve for a stationary equilibrium in overlapping generations models. The MATLAB code for the computation of the baseline, as well as of the reform scenario, is available upon request.

The state x of an individual is given by

- age;
- assets;
- pension points;
- work status (age at which stopped working, if no longer working); and
- pension status (age at which started claiming pension benefits, if claiming).

An individual knows x in the beginning of period and decides

- how much to consume;
- how much to save;
- whether or not to work; and
- whether or not to claim pension benefits. ¹⁶

I require the asset holdings k_a to fall on a discrete grid of points $G = \{g_1, \dots, g_m\}$, where $g_i \geq 0$ due to the non-borrowing constraint. Let [Equation \(4\)](#) be the maximized value of the objective function of an age- a individual. The optimization problem is one of finite-state, finite-horizon dynamic programming. Using a recursion working backwards from the last period of life, I can find the value functions and decision rules for the individual. Using the budget constraint in [Equation \(3\)](#) to substitute for consumption in [Equation \(5\)](#), the problem in period a reduces to choosing how much to save and whether or not to work, i.e. choosing the asset holdings tomorrow k_{a+1} and the labor supply l_a . For individuals that are eligible and claim benefits as well as for individuals that are at age 63 and older, the control space

¹⁶For simplicity, I assume latest when individuals reach the statutory retirement age, they start to claim benefits. For a discussion of this assumption, see [Section 3.2](#).

in a given period is an $m \times 2$ matrix $X_a = \{x_a = (k_{a+1}, l_a) : k_{a+1} \in G; l_a \in \{0, 1\}$, i.e. they choose how much to save and whether or not to work. I assume once individuals stop working, they cannot return to work. Hence, for individuals that stopped working, the control space does not contain l_a anymore and is thus a $m \times 1$ vector $\tilde{X}_a = \{\tilde{x}_a = k_{a+1} : k_{a+1} \in G\}$. Consumption, which is a continuous variable, is solved for as the residual and is used in order to calculate the utility for each level of beginning-of-period assets, employment state, and pension state realization.

Since death is certain beyond age I , the value function at $I + 1$ is identical to zero. Hence the solution to

$$V(k_I) = \max_{\{l_I\}} \ln(c_I) - b_I l_I, \quad (\text{B.1})$$

subject to

$$c_I = (1 + r(1 - \tau_k))k_I + (1 - \tau_w(y_I))y_I + R_I, \quad (\text{B.2})$$

is an $m \times 1$ vector decision rule $D_I = \{d_I = l_I : l_I \in \{0, \bar{l}\}\}$ for age- I individuals. Notice that $k_{I+1} = 0$ since there is no bequest motive and death is certain after period I . The value function $V(k_I)$, which is an $m \times 2$ matrix with entries that correspond to the value of utility at the different possible asset levels and work status, is passed on to the next step where the decision rule and the value function for period $I - 1$ (and similarly for all other periods) are calculated in the following way

$$V(k_{I-1}) = \max_{\{k_I, l_{I-1}\}} \left\{ \ln(c_I) - b_I l_I + \beta V(k_I) \right\}, \quad (\text{B.3})$$

subject to

$$c_{I-1} + k_I = (1 + r(1 - \tau_k))k_{I-1} + (1 - \tau_w(y_{I-1}))y_{I-1} + (1 - \tau_w(R_{I-1}))R_{I-1}. \quad (\text{B.4})$$

For every given $x_{I-2} \in X_{I-2}$, the value of $x_{I-1} \in X_{I-1}$ that solves the above problem is obtained by evaluating the objective function at each point in the choice set X .

This value is reported in the first row of the $m \times 2$ decision rule $D_{I-1} = \{d_{I-1} = (k_I, l_{I-1}) : k_I \in G; l_{I-1} \in \{0, \bar{l}\}\}$. Repeating this procedure for all possible initial states, the entire matrix D_{I-1} is filled, i.e. for a given k_{I-2} and employment status as well as pension status, which depends on the type of the individual, I search over $k_{I-1} \in G$ that solves the above problem and report the value as an element of the decision rule.

Simultaneously, the value function V_{I-1} is found as an $m \times 2$ matrix with entries corresponding to the right-hand side of the above objective function evaluated at the decision rule D_{I-1} . Then, given a state in the initial period and a retirement age, I am able to pin down the optimal consumption and savings profile of an individual and the associated value with these profiles.

I solve the model 21 times. Once I have solved the model for all possible retirement ages, i.e. every year in which the individual could stop working, I compare the value of all different retirement ages to find the optimal retirement decision. Using the data, I then calibrate the model such that the optimal retirement decision in the baseline model corresponds to what we observe, approximately 64 years.

A.3 Capital Grid

The value function in [Equation \(4\)](#) is defined over a continuous choice set, i.e. I assume savings is a continuous choice variable, with an infinite amount of possible values for k_a . When I solve the model numerically, I am not able to consider all possible values for savings. I overcome this problem by creating a discrete state space for all possible savings. The non-borrowing constraint defines a minimum of zero for the state space and I create an arbitrary maximum that will never be binding. I pick 1,000 evenly spaced grid points between the minimum and the maximum. Thus, savings fall onto the discrete space $G = \{0, \dots, k^{max}\}$, where $k^{max} = 500,000$. If an estimate for savings \hat{k}_a does not appear on the grid, it must lay between two known points. Spline interpolation is utilized to approximate the value of [Equation \(4\)](#), $V(\hat{k}_a)$, at this point. I choose k^{max} to be almost ten times larger than the maximum of the exogenous age-varying net wage profile. Conducting robustness checks with a larger k^{max} and a finer grid leave my results unaffected.