

**BEWEGING EN TRILLINGEN**

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**Linkages: The Colibri**

PAPER

Titularis

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# Introduction

In this paper we will discuss a kinematic and inverse dynamical analysis of a Colibri bird. The Colibri is represented by a nine-bar linkage system with eleven joints and a slider. A drawing of the system can be found in figure 1. The names on the figure are used through the entire paper. Joints A, D, G, H and K are connected to the ground. Bar two is the driver of the system and rotates around joint A with a constant speed. Point C is a slider and will always stay directly under joint A. The length of bars four and eight are variable to make the desired motion of the system possible. The initial values of the parameters can be found in figure 2.

This paper will start with the motion analysis and a discussion about any possible dead configurations. After this a kinematic analysis of the system is done to determine the positions, velocities and accelerations of the bars and joints. After checking these results, a dynamic analysis is used to calculate the internal and external forces of the system.

# Motion analysis and dead configurations

## Motion analysis

A motion analysis is used to determine the degree of freedom of the system.

n = 9 (the number of links) The angle between bars eight and nine and bars four and five are fixed so they both count as one bar. They are named separately to make the calculations easier to understand.

f1 = 11 joints that remove two degrees of freedom (A,B,C,D,E,F,G,H,I,J,K)

f2 = 1 joint that removes one degree of freedom (C)

Joint C is used twice in the calculation of the mobility, one as a joint with one degree of freedom and once as a slider with two degrees of freedom.

This gives us a mobility of one which means one variable can be chosen to determine the entire system. In this case phi2, the angle which drives bar two is chosen.

## Dead configurations

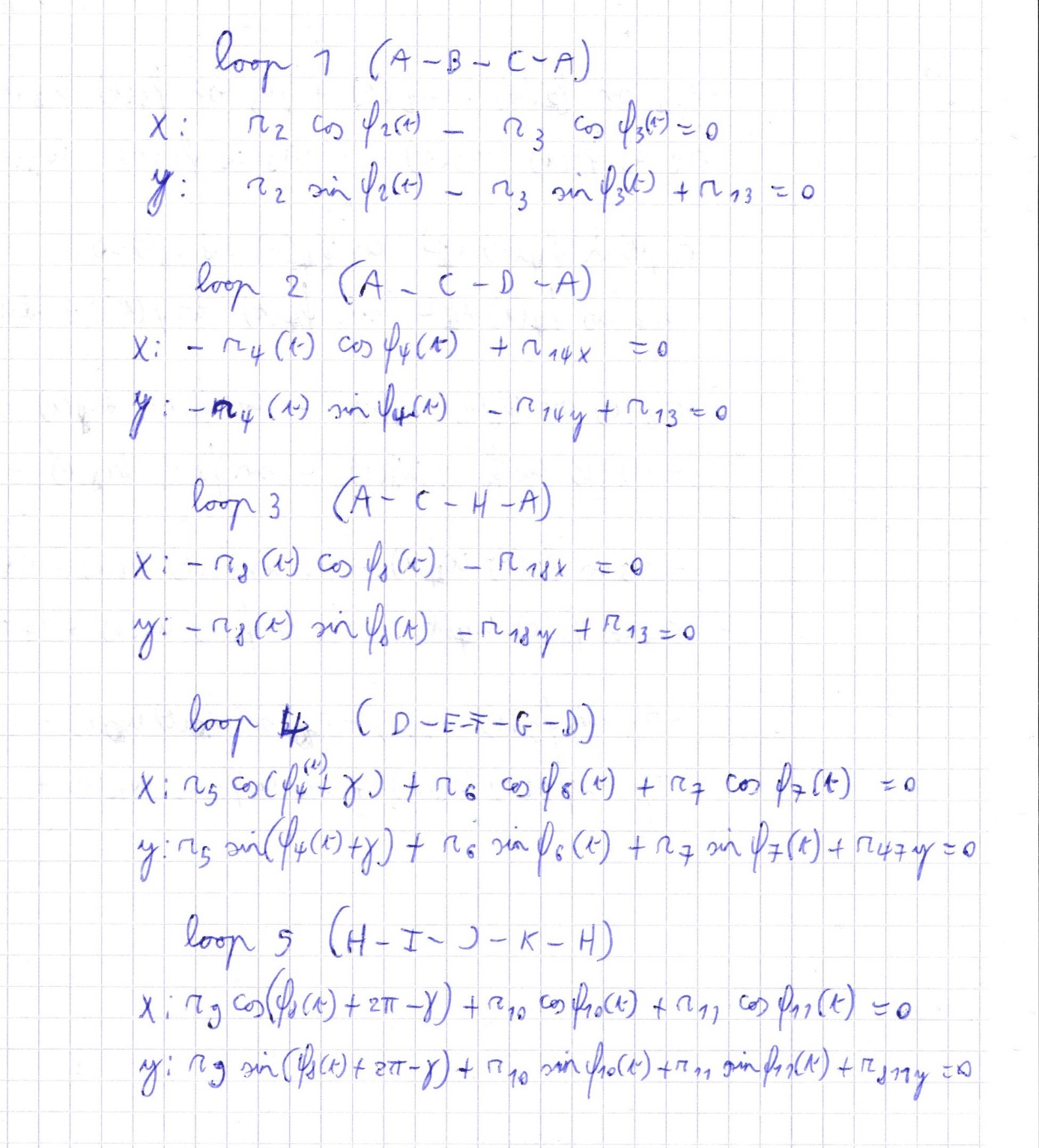
The system has no dead configurations. Phi2 doesn’t have any limitations.

# Kinematic analysis

## Loop equations

There are ten unknown parameters of the system left. Angles three till eleven, the length of bar 4, the length of bar 8 and the distance between joints A and C.   
The entire system can be determined by ten independent equations or five loops in which each loop has an x and a y equation. The loop equations can be found in figure 3.

Phi5 and phi9 can be written in function of phi4 and phi8 as shown in loops four and five.



## Position analysis

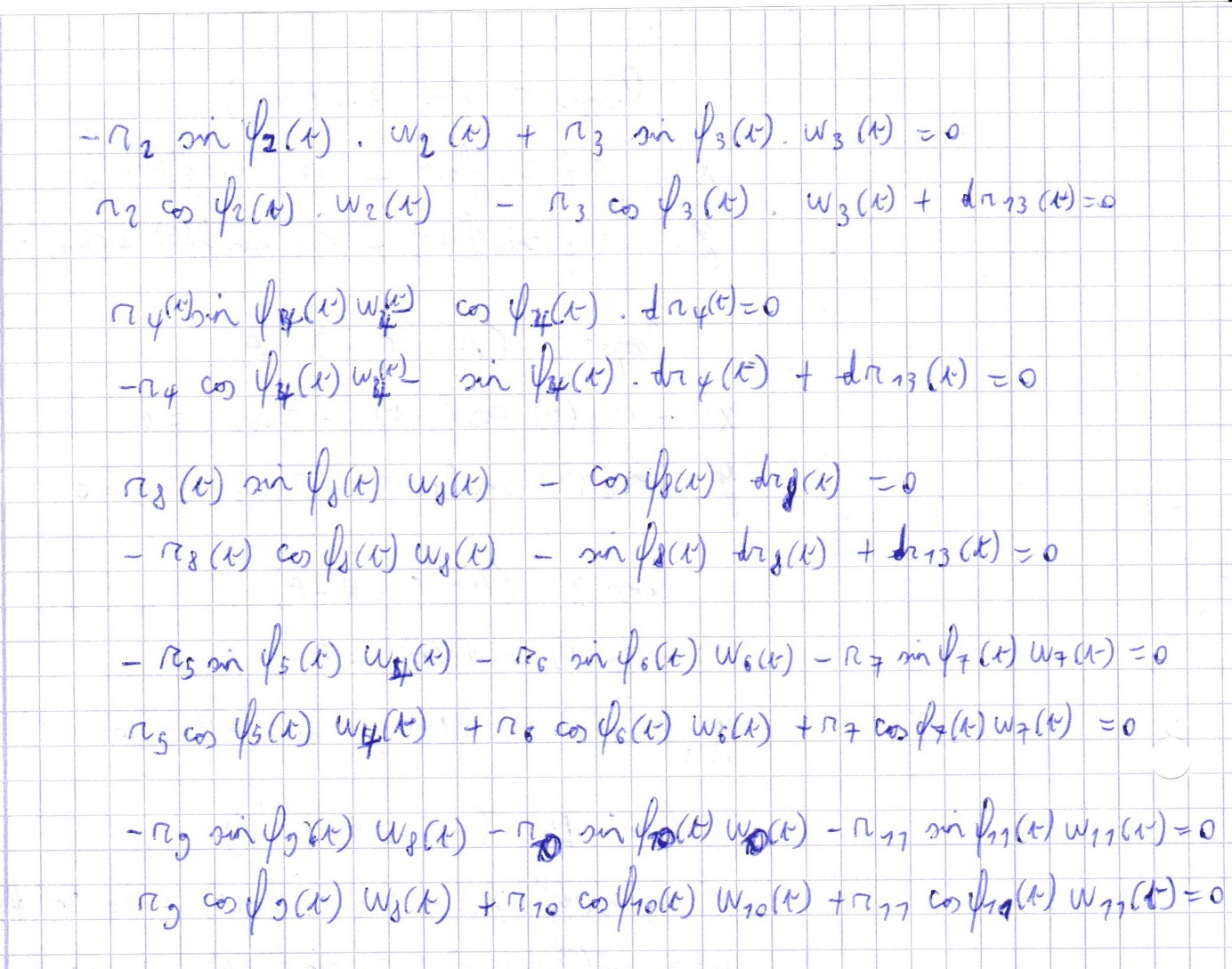
Matlab solves the system of ten equations and ten variables using fsolve. Fsolve calculates the angles in time based on the initial values given and phi2.

The results can be found in figures 4 and 5.

The symmetry of the system can be found in these figures, phi6 and phi10 both start at an angle of 90 degrees and will change in opposite direction. R4 and r8 are always the same length. This is already a good indicator that the position analysis is correct.

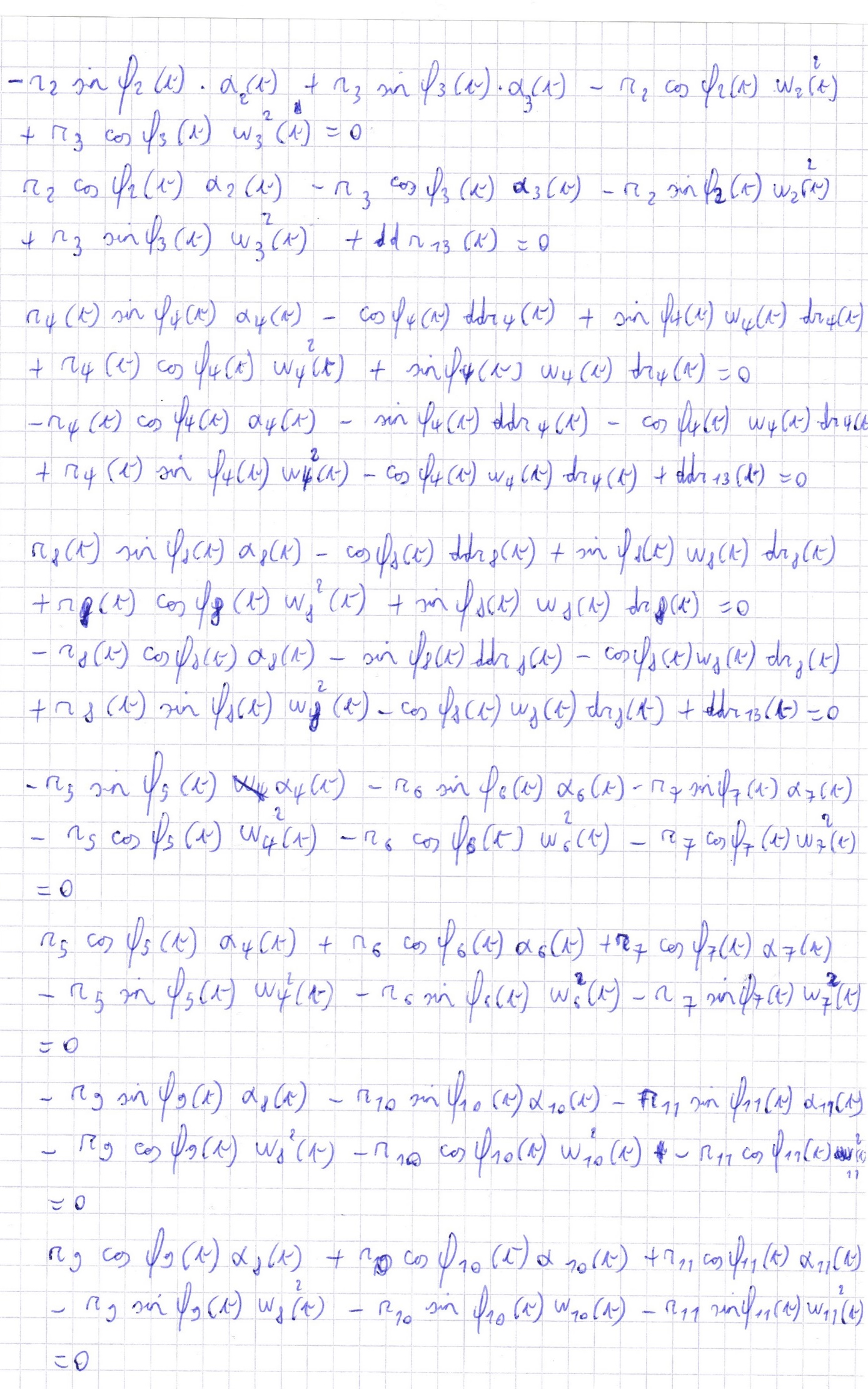
## Velocity analysis

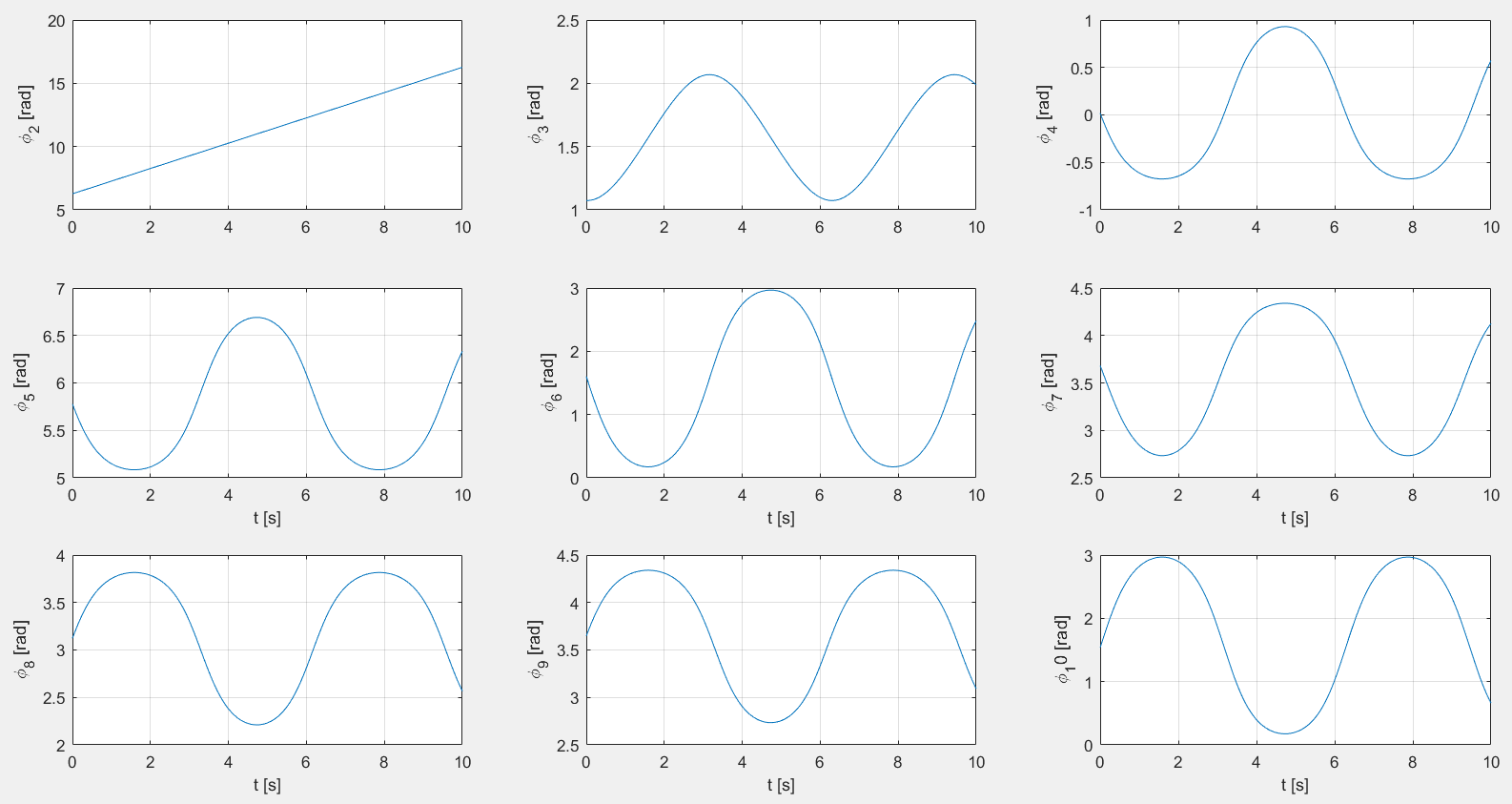
The velocity analysis can be done based on the closure equations by taking the time derivative of these equations. This yields a set of ten equations in ten unknowns. These unknowns can be calculated using matlab. The derived equations can be found in figure 6. The results can be found in figures 7 and 8. In these figures the symmetry can again be found.

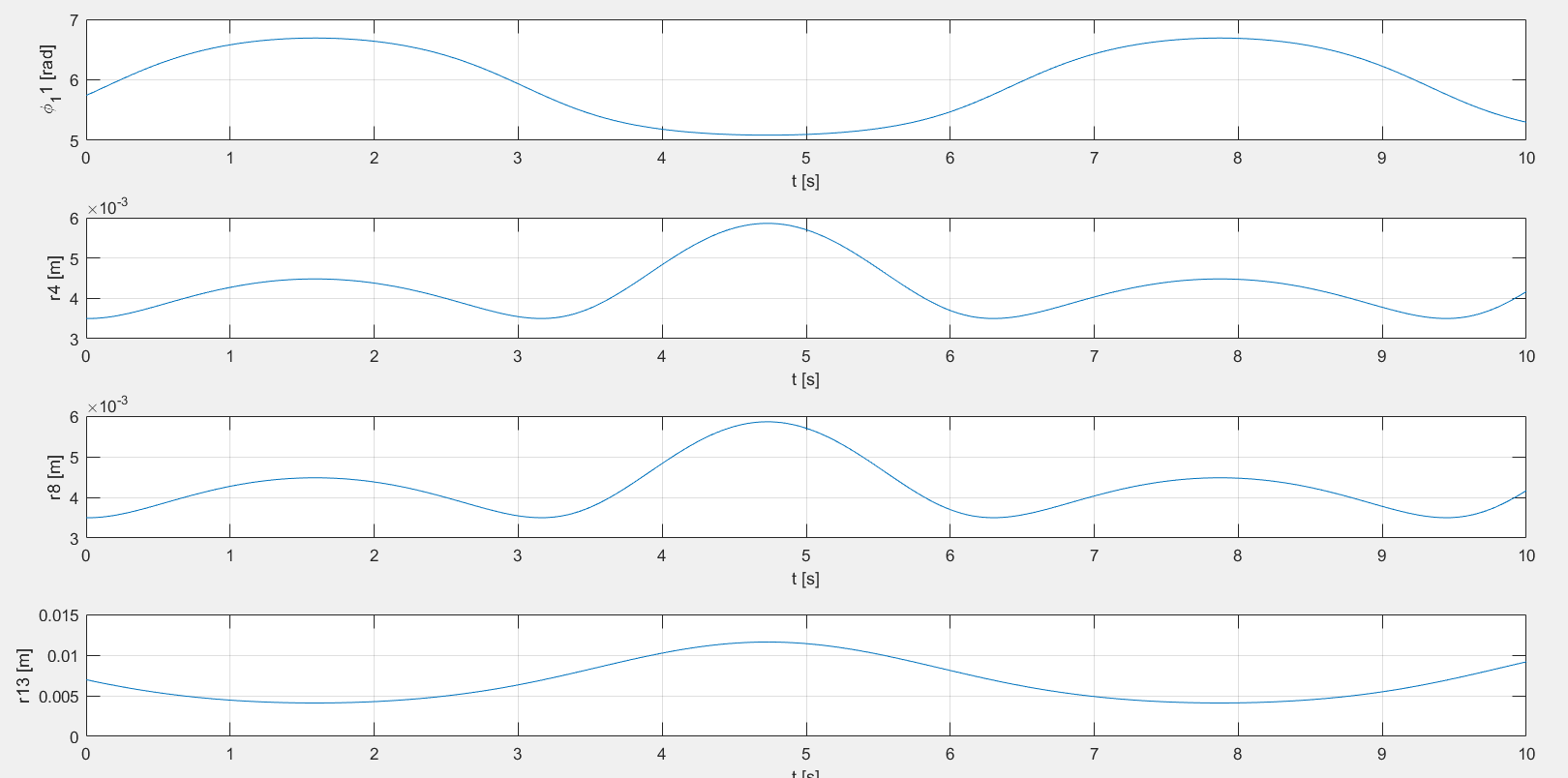


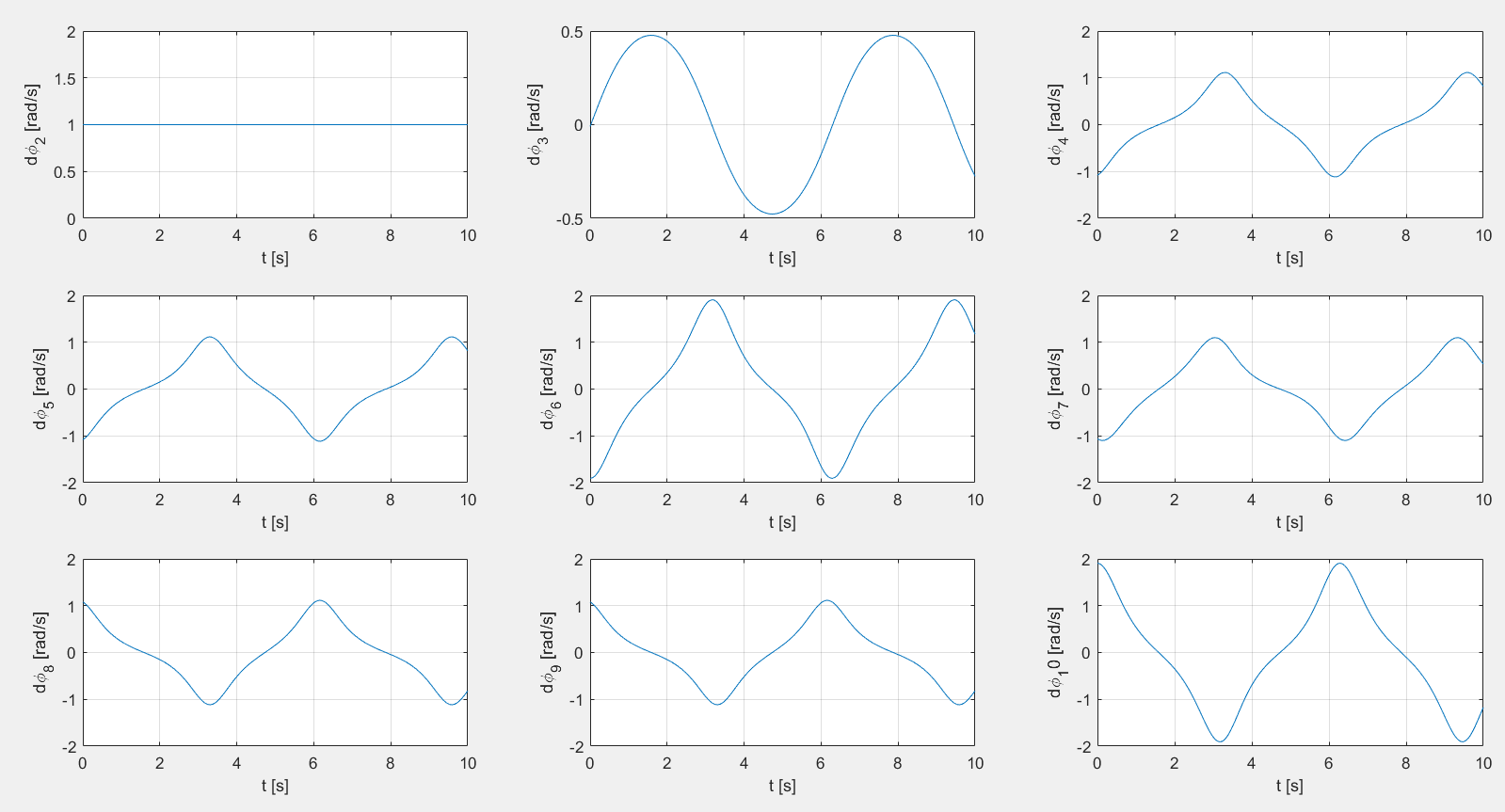
## Acceleration analysis

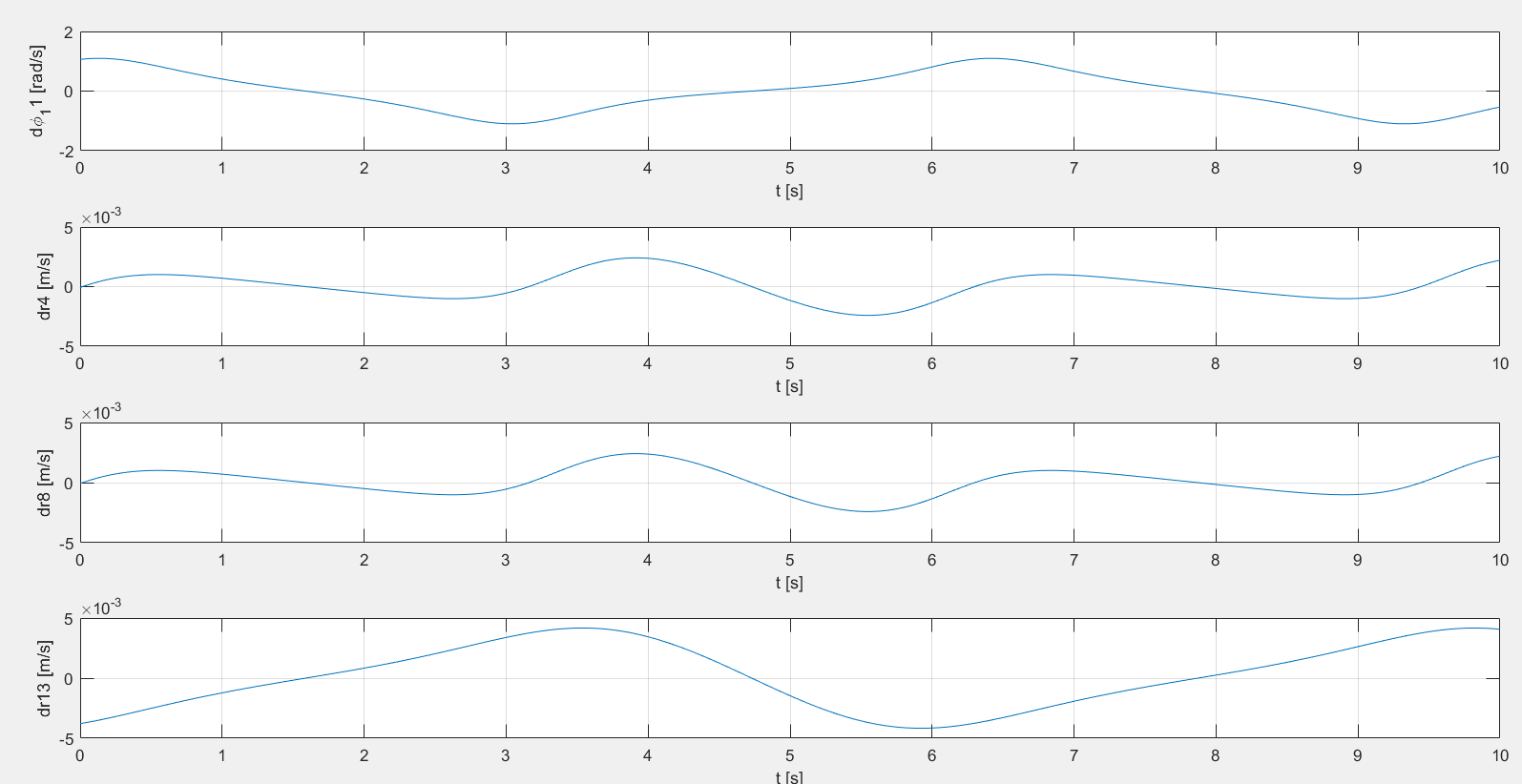
The acceleration analysis is done by taking the derivative of the velocity analysis equations. This yields another ten equations in ten unknowns which can be calculated with matlab. The equations can be found in figure 9 and the results can be found in figures 10 and 11. And the same clear symmetry of the system can be seen.

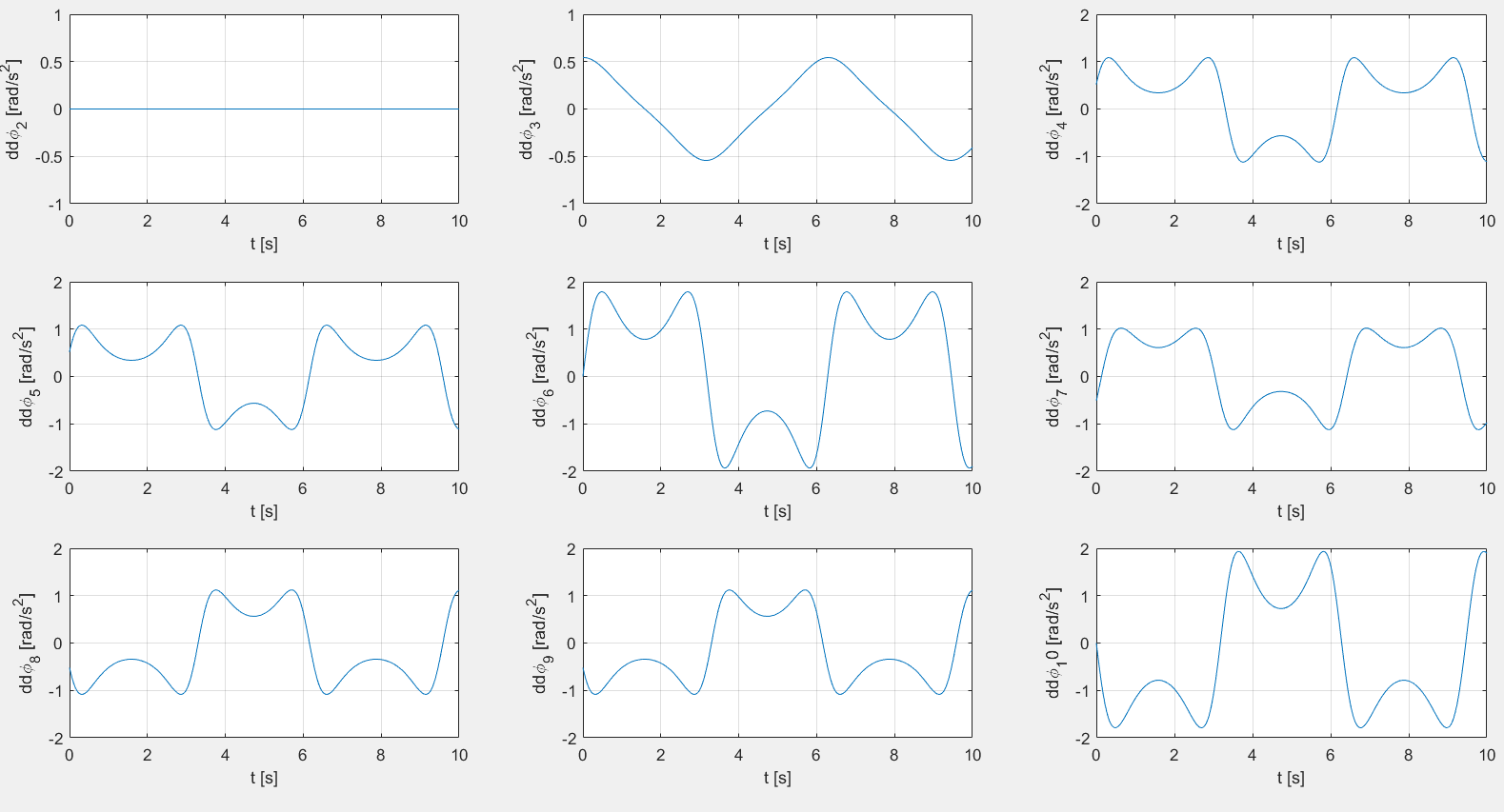


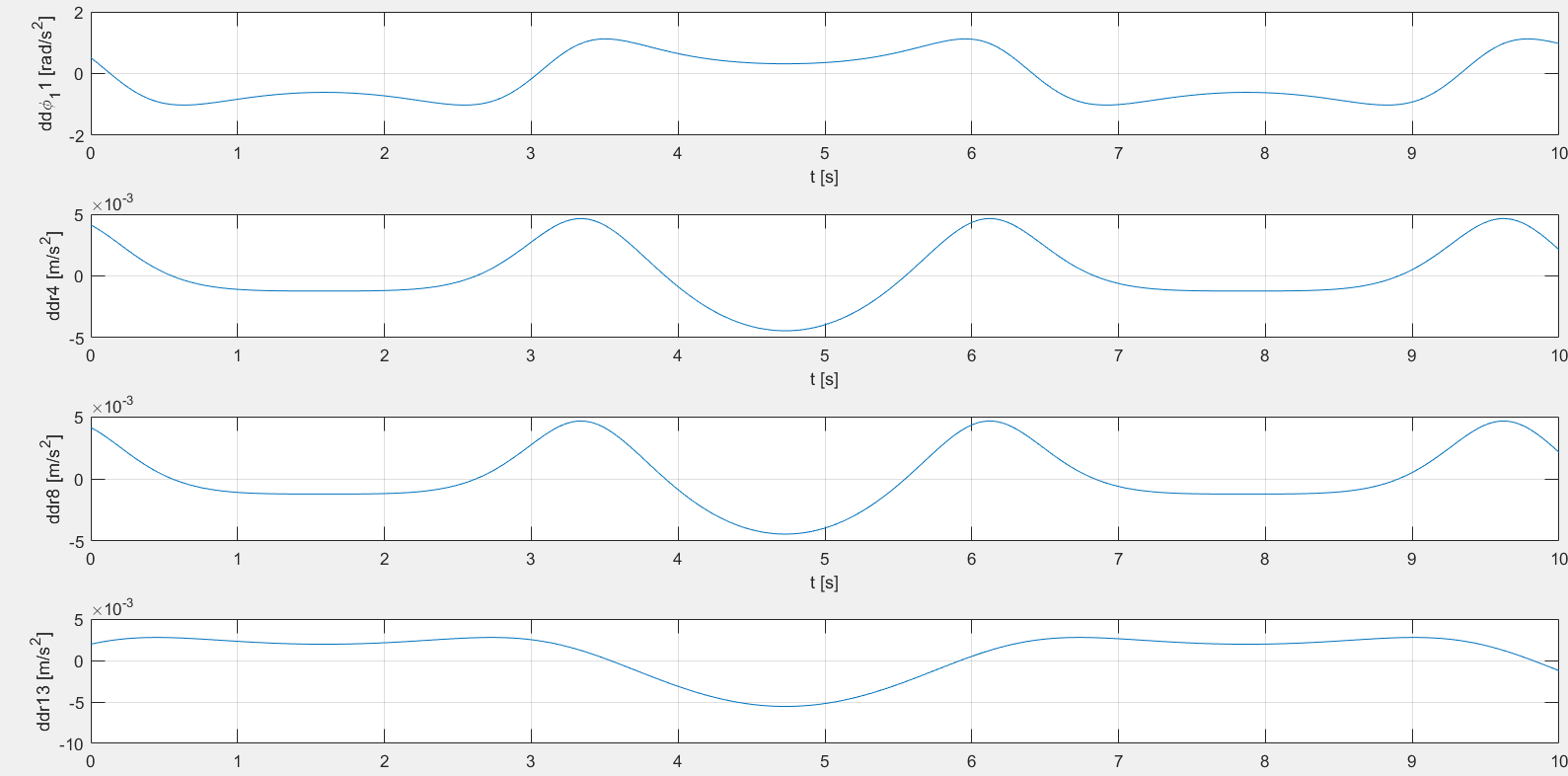












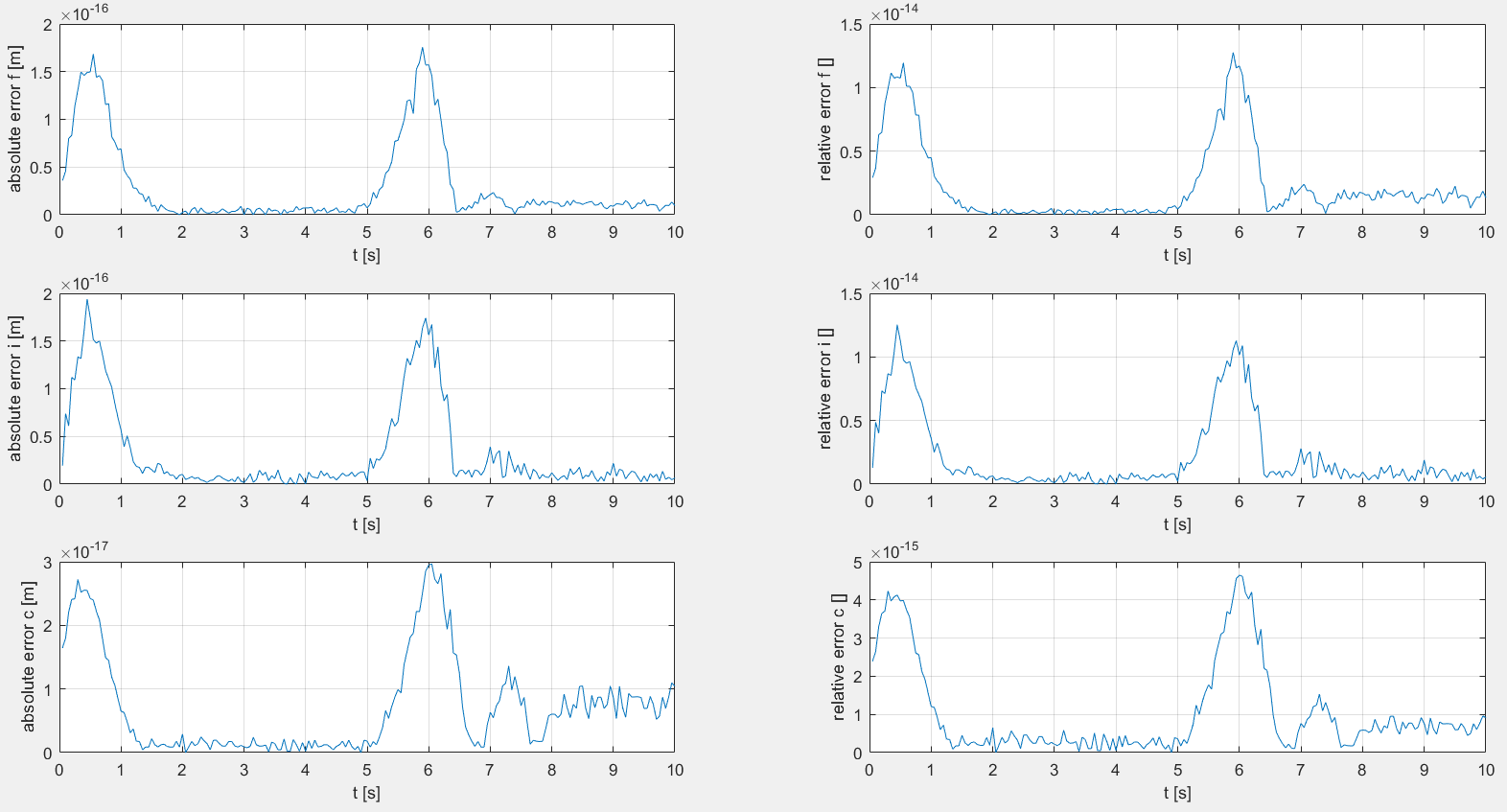
## Control

All angles, velocities and accelerations have been analytically calculated, using the loop equations, in the sections above. These results must be checked by using other methods to calculate the positions, velocities and accelerations. First the position will be checked by a simple animation made in matlab to see if the system moves correctly, after this the position of a point will be calculated starting from two different points. Next the velocity and the acceleration will be checked using the numerical derivatives and by using two different points. Three points will be tested each time, point C and a point in each wing to make sure every subpart moves correctly.

### Position

By running the added matlab files, it is clear that the system makes correct movements.

The position of F is calculated using D-E-F and using G-F. The position of I is calculated using K-J-I and using H-I. The position of C is calculated using A-B-C and using D-C. The errors can be found in figure 12.



### Velocity

Figure 13 -14

### Acceleration

Figure 15-16

# Dynamic analysis

Once the kinematic model has been completed and tested, the linkage is analysed analytically in order to design the motor and the linkages.

## Centre of gravity

As the most important force in the linkage is the inertial force, a good estimation of each centre of gravity (cog) is essential. The nomenclature is as such: is the coordinate over the i-axis of the centre of gravity of link x seen from the relative frame of point A.

Link 4-5 and 8-9 are linked and can thus be seen as one bar. Their mass density and cross area are assumed to be identical. As the link lengths and varied in the kinematic analysis, the maximal length is taken as the true length of the link, necessary for the position of the total centre of gravity.

Eg. if the cog of bar 1 is on 5 meters along the positive x-axis with respect of A, .

At first the position of the centre of gravity in the frame of reference of each link is calculated.

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| Centre of gravity in the relative frame of reference | X | Y |
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| 3 |  |  |
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| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| wing |  |  |

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| --- | --- |
| Centres of gravity |  |
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## Masses and moments of inertia

In a moving system, the mass and inertial moment can have a big influence on the behaviour of the linkage. As an estimation every link is assumed to have an identical thickness of 2mm. In the report on which this linkage is based, the usage of epoxy glass laminate is recommended. This material has a density . Or in other words the links weigh 5.65 g/m. By multiplying it with the length of a link, that link’s mass can be obtained: . Note that the wings are also simplified as links.

Apart from link 4, 5, 8 and 9 each moment of inertia is calculated in its centre of gravity. As later on the moment equation of link 4-5 and 8-9 will be calculated in their fixed points, respectively D and H, their moment of inertia also has to be calculated on that spot using the Steiner’s formula

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As the moments of inertia are calculated on the same spot, the total moment of inertia of link 4-5 and 8-9 is simply the sum of the inertia of each link. and are the moment of inertia in the centre of gravity

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| Moments of inertia |  |
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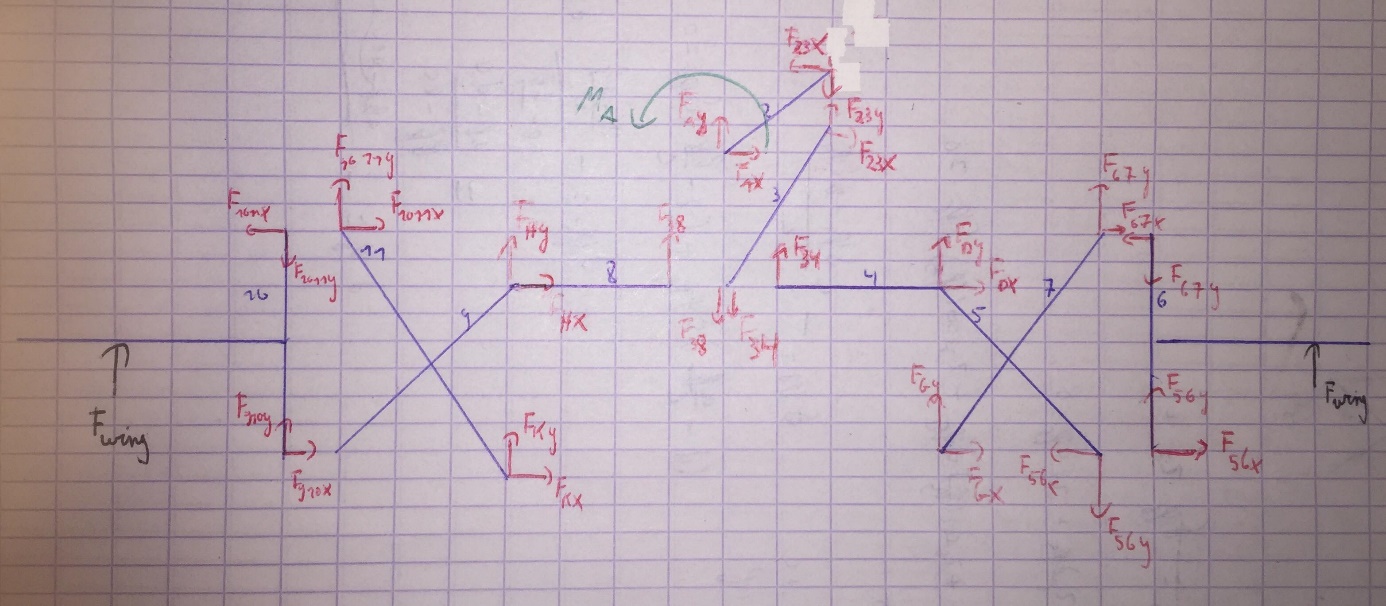
## Acceleration and velocities

The last parameters that should be calculated before are the accelerations and velocities of the centres of gravity. These will be used for the controls of our calculations.

Each link can be seen as a bar rotating around the origin in their relative frame of reference. As these frames don’t rotate relative to the absolute frame of reference, the speed and acceleration can always be derived from the following formula for a point A in a frame of reference with as origin O:

By meticulously applying these exact equations to each link, starting from its closest fixed point, the accelerations and velocities can be found, as can also be seen in the matlab code. If the link is connected to a fixed point, the absolute motion of the origin is naturally set to zero.

## Force equations

The force on each bar can be calculated with the second law of Newton. This gives three equations for every bar: the force equation in x- and y-direction and the torque equation. As there are 8 individual links, this leads to 24 equations. As you can see on image 13, this matches exactly the number of unknown forces and moments. Each bar will be discussed separately. 

Bar 2 has four unknown forces and one unknown moment. The equations are

Bar 3 is a special case: even though the top part is as usual, the bottom part has three different forces: the reaction force of bar 4 and 8 and the external force of C. Note that the slider limits the force in C to be only horizontal.

Bar 45 has 5 reaction forces and a fixed point. In order to facilitate the torque equation, the moment around point D has been taken.

Bar 6 has apart from his 4 usual reaction forces also a known external force . This adds an extra term in the equations.

Bar 7 is a normal bar with 4 reaction forces and a fixed point in G

Bar 89, 10 and 11 are mirrors of 45, 6 and 7 respectively:

Bar89 has a fixed point on H:

Bar10 also has a force pushing on its wing:

Bar11 is fixed to point K:

In order to solve these equations with a matrix method, the 24 equations are arranged in an Ax=b format. x contains the 24 unknowns.

## Results