Case on "Bond portfolio management"

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1 Questions

Exercise 1

Suppose an insurance company sold two "Guaranteed Investment Contracts" (GICs) today that will require the company to pay back to investors the following amounts:

- GIC1: the insurance company will have to pay \$100,000 in 2 years.
- GIC2: the insurance company will have to pay \$110,000 in 3 years.

The term structure of interest rates is flat at $r_\infty(0, T) = 2.00\%, \forall T$ (continuous compounding).

The company wants to immunize these liabilities by investing in the following assets:

- Floating-Rate Bond (FRB): 4-year maturity, paying $c(T_i) = r_1(T_{i-1})$ annually. Assume that the index $r_1(T_{i-1})$ is equivalent to the continuous rate at time T_{i-1}. For example, the first coupon will be $c(1) = r_1(0) = e^{0.02} - 1$.

  (Alternatively, we can think of this FRB as a sequence of 1-year term deposits, or revolving investments in 1-year ZCBs.)

- Coupon-Bearing Bond (CBB): 5-year maturity, paying $c = 5\%$ annually.

Questions:

1. Determine the amount that the insurance company has to invest in each asset today to immunize the liabilities.
   Note: Ignore the convexity condition.
2. Assume that interest rates change to $r_\infty(0, T) = 2.40\%, \forall T$, immediately after the portfolio allocation determined in the previous question. Assume that interest rates remain the same over the whole investment period and therefore that all future cash flows can be reinvested at that same rate. Determine the final wealth of the insurance company, i.e., the terminal value at time $T = 3$ after paying the last liability.

Remark: You can assume that the weights in each asset determined in question 1 remain constant over all future time periods. In fact, given how we are assuming that interest rates vary in this example, the final result does not change with whatever weights you choose after the initial allocation. Under more general conditions, we should rebalance the assets frequently to ensure that the Duration of the Assets tracks the Duration of the Liabilities closely.

Exercise 2

Repeating the previous exercise, assume that:

1. The liabilities are the same.
2. At the initial moment (time $t = 0$), everything is the same. The insurance company decides to buy the same notional values: $N_{FRB} = 114\,929.71$ and $N_{CBB} = 74\,311.96$.
3. One year after that, at time $t = 1$, interest rates are still at $r_\infty(0, T) = 2.40\%, \forall T$. The current value of the portfolio of assets is still the same, $V_A(1) = 202\,478.70$.

However, now the company wants to rebalance the portfolio at time $t = 1$ to immunize against possible further changes in interest rates.

Furthermore, now there are more assets available. In addition to the two bonds already in the portfolio, the following can also be traded:

- 6-month Zero Coupon Bond (ZCB)
- 2-year Coupon Bearing Bond, paying 2% annually.

The rebalancing procedure should satisfy the following:

- Minimize transaction costs
- The total amount of assets will be fully reinvested, without adding or subtracting any funds at this moment.
- Satisfy both the duration and convexity conditions.
- No asset should represent more than 50% of the portfolio

Determine the optimal new immunizing portfolio at time $t = 1$ (indicate the new Notional of each asset).

2 What you need to submit

1. Your answers should be submitted through Moodle.
2. The main report should be a pdf file with all the answers. You will get points for being both concise and complete. That is, write everything that is relevant, and nothing more. Additionally, you will get points for writing a clean, well organized report, that is easy to follow.
3. You may submit an auxiliary excel file (or similar) showing your detailed calculations. This should be regarded only as an "appendix" to the previous pdf file.