main

February 20, 2018

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        from scipy.optimize import minimize
        %matplotlib inline
```

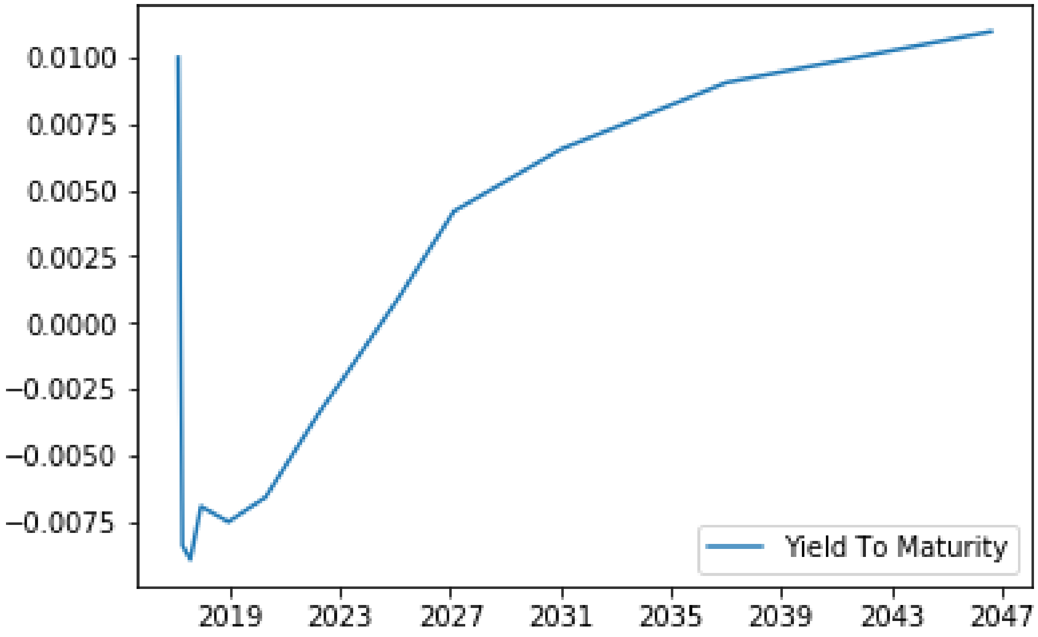
0.0.1 Loading data

```
In [2]: data = pd.read_excel('data.xlsx')
        data.columns
        data['nr_days'] = data['End_date'] - data['Start_date']
        data['price'] = data['price'] / 100
        data['coupon'] = data['coupon']/100
        data['nr_years'] = data['nr_days'].apply(lambda x: int(x / np.timedelta64(1,'D'))) / 360
        data
```

```
Out[2]:       price  month  coupon  Coupons pr year Start_date   End_date    nr_days  \
        0   1.00021      1  0.0000                1 2017-02-02 2017-02-15    13 days
        1   1.00626     12  0.0000                1 2017-02-03 2017-12-15   315 days
        2   1.01397     24  0.0000                1 2017-02-04 2018-12-14   678 days
        3   1.00153      3  0.0000                1 2017-02-05 2017-04-12    66 days
        4   1.02142     36  0.0000                1 2017-02-06 2020-04-17  1166 days
        5   1.01776     60  0.0000                1 2017-02-07 2022-04-08  1886 days
        6   1.00411      6  0.0000                1 2017-02-08 2017-07-26   168 days
        7   1.10358     72  0.0150                1 2017-02-09 2023-02-15  2197 days
        8   1.03233     96  0.0050                1 2017-02-10 2025-02-15  2927 days
        9   0.98297    120  0.0025                1 2017-02-11 2027-02-15  3656 days
        10  1.64471    180  0.0550                1 2017-02-12 2031-01-04  5074 days
        11  1.56051    240  0.0400                1 2017-02-13 2037-01-04  7265 days
        12  1.33147    360  0.0250                1 2017-02-14 2046-08-15 10774 days

             nr_years
        0    0.036111
        1    0.875000
        2    1.883333
        3    0.183333
        4    3.238889
        5    5.238889
```

```
6     0.466667
7     6.102778
8     8.130556
9    10.155556
10   14.094444
11   20.180556
12   29.927778
```

```
In [3]: def ListOfPeriods(n, T, t=0):

            last_coupon = int(np.floor(n*(T-t)))
            period_adder = n*(T-t) - last_coupon

            return [(i + period_adder)/n  for i in range(1, last_coupon + 1)]


        def BondPrice(c, n, T, ys, t=0):

            """Returns price, for given YTM"""

            LOP = ListOfPeriods(n, T, t=t)

            discounted_coupons = [(c/n) * np.exp(-(i-t)*(ys)) for i in LOP]
            FV = np.exp(-(T-t)*(ys))

            B = np.sum(discounted_coupons) + FV
            return B


        def YTM(B, c, n, T, t=0, ys_guess=0.01):

            def ObjectiveFunc(ys):

                return (B - BondPrice(c, n, T, ys, t=t))**2

            bound_ys = [(-1 , 1)]

            solution = minimize(fun= ObjectiveFunc, x0=ys_guess, method='SLSQP', bounds=bound_ys

            return solution.x[0]

In [4]: data['YTM'] = data.apply(lambda x: YTM(B = x['price'], c=x['coupon'], n= x['Coupons pr y

In [5]: f, ax = plt.subplots(1,1)
        data.sort_values('End_date', inplace=True)
        ax.plot(data['End_date'], data['YTM'], label='Yield To Maturity')
        ax.legend()
        plt.savefig('YTM.png')
```



1 Question 2

```
In [6]: class NielsonSiegelModel(object):

            def __init__(self):
                pass

            @staticmethod
            def NielsonSiegel(b1, b2, b3, lam1, T):

                return b1 + b2*((1-np.exp(-T/lam1))/(T/lam1)) + b3*(((1-np.exp(-T/lam1))/(T/lam1


            def VectorRates(self, b1, b2, b3, lam1, vector_T):

                """ Returns vector of rates"""
                return [self.NielsonSiegel(b1, b2, b3, lam1, Ti) for Ti in vector_T]


            def VectorPrices(self, vector_c, vector_n, vector_T, vector_r):

                list_of_prices = []
                for i in range(len(vector_c)):
                    c, n, T, ys = vector_c[i], vector_n[i], vector_T[i], vector_r[i]
                    list_of_prices.append(BondPrice(c, n, T, ys, t=0))

                return list_of_prices

            def NSM(self, data, x_guess):

                """
                x_guess: (list)
                =======
                b1 = x[0]
                b2 = x[1]
                b3 = x[2]
                lam1 = x[3]

                =======
                returns : vector of x
                """

                vector_c, vector_n, vector_T, vector_p_market = data['coupon'], data['Coupons pr

                def ObjectiveFunc(x):
                    b1 = x[0]
                    b2 = x[1]
                    b3 = x[2]
                    lam1 = x[3]

                    vector_r = self.VectorRates(b1, b2, b3, lam1, vector_T) # vector_r is endoge
                    vector_p_model = self.VectorPrices(vector_c, vector_n, vector_T, vector_r)

                    res = np.sum(np.array(np.array(vector_p_market) - np.array(vector_p_model))*
                    return res

                bound=[(-2,2),(-2,2),(-30,30),(-30,30)]

                solution = minimize(fun= ObjectiveFunc, x0=x_guess, method='SLSQP', bounds=bound

                return solution
```

```
In [7]: x_guess = [0.01, -0.01, 0 , 2]
        NSM = NielsonSiegelModel()
        sol = NSM.NSM(data, x_guess)
        x_list = list(sol.x)
        b1, b2, b3, lam1 = x_list[0], x_list[1], x_list[2], x_list[3]
        print('b1: ',b1,'b2: ',b2,'b3: ',b3, 'lam1:', lam1)
        b1_NS, b2_NS, b3_NS, lam1_NS = b1, b2, b3, lam1
```

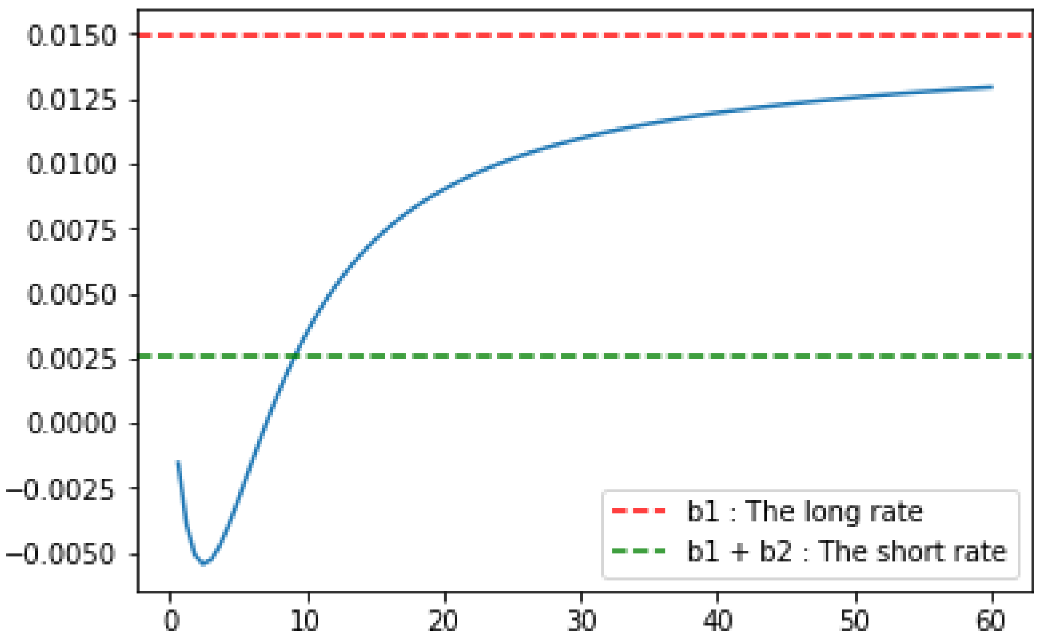
```
b1:  0.014907040259 b2:  -0.0123318899496 b3:  -0.0468055972558 lam1: 2.00034667642
```

```
In [8]: MeanSquaredErrorNelsonSiegel = sol.fun
        print(MeanSquaredErrorNelsonSiegel)
```

```
0.00043023479318226665
```

```
In [9]: x_= np.linspace(0,60,num=100)
        y_NSM = [NSM.NielsonSiegel(b1, b2, b3, lam1, i) for i in x_]

        f, ax = plt.subplots(1,1)
        ax.plot(x_, y_NSM)
        ax.axhline(b1, label= 'b1 : The long rate', color='red', ls='--')
        ax.axhline(b1+b2, label= 'b1 + b2 : The short rate', color='green', ls='--')
        ax.legend()
        plt.savefig('Nelson-Siegel.png')
```

```
/Users/Jeppe/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:9: RuntimeWarning: inva
```



```
In [10]: class SvenssonModel(object):

             def __init__(self):
                 pass

             @staticmethod
             def Svensson(b1, b2, b3, b4, lam1, lam2, T):
```

```
    return b1 + b2*((1-np.exp(-T/lam1))/(T/lam1)) + b3*(((1-np.exp(-T/lam1))/(T/lam


def VectorRates(self, b1, b2, b3, b4, lam1, lam2, vector_T):

    """ Returns vector of rates"""
    return [self.Svensson(b1, b2, b3, b4, lam1, lam2, Ti) for Ti in vector_T]


def VectorPrices(self, vector_c, vector_n, vector_T, vector_r):

    list_of_prices = []
    for i in range(len(vector_c)):
        c, n, T, ys = vector_c[i], vector_n[i], vector_T[i], vector_r[i]

        list_of_prices.append(BondPrice(c, n, T, ys, t=0))

    return list_of_prices

def SM(self, data, x_guess):

    """
    x_guess: (list)
    =======
    b1 = x[0]
    b2 = x[1]
    b3 = x[2]
    b4 = x[3]
    lam1 = x[4]
    lam2 = x[5]

    =======
    returns : vector of x
    """

    vector_c, vector_n, vector_T, vector_p_market = data['coupon'], data['Coupons p

    def ObjectiveFunc(x):
        b1 = x[0]
        b2 = x[1]
        b3 = x[2]
        b4 = x[3]
        lam1 = x[4]
        lam2 = x[5]

        vector_r = self.VectorRates(b1, b2, b3, b4, lam1, lam2, vector_T) # vector_
        vector_p_model = self.VectorPrices(vector_c, vector_n, vector_T, vector_r)
```

```
                     res = np.sum(np.array(np.array(vector_p_market) - np.array(vector_p_model))
                     return res

                 bound=[(-2,2),(-2,2),(-2,2),(-2,2), (-10,10),(-10,10)]

                 solution = minimize(fun= ObjectiveFunc, x0=x_guess, method='SLSQP', bounds=boun

                 return solution
```

```
In [11]: x_guess = [0.014, -0.012, -0.468, 0, 2, 2]
         SM = SvenssonModel()
         sol = SM.SM(data, x_guess)
         x_list = list(sol.x)
         b1, b2, b3, b4, lam1, lam2 = x_list[0], x_list[1], x_list[2], x_list[3], x_list[4], x_l
         b1_SM, b2_SM, b3_SM, b4_SM, lam1_SM, lam2_SM = b1, b2, b3, b4, lam1, lam2
```

```
/Users/Jeppe/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:57: RuntimeWarning: ove
/Users/Jeppe/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:15: RuntimeWarning: ove
/Users/Jeppe/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:16: RuntimeWarning: ove
```

```
In [12]: print('b1 :', b1, 'b2 :', b2, 'b3 :', b3,'b4 :', b4, 'lam1 :', lam1, 'lam2', lam2)
```

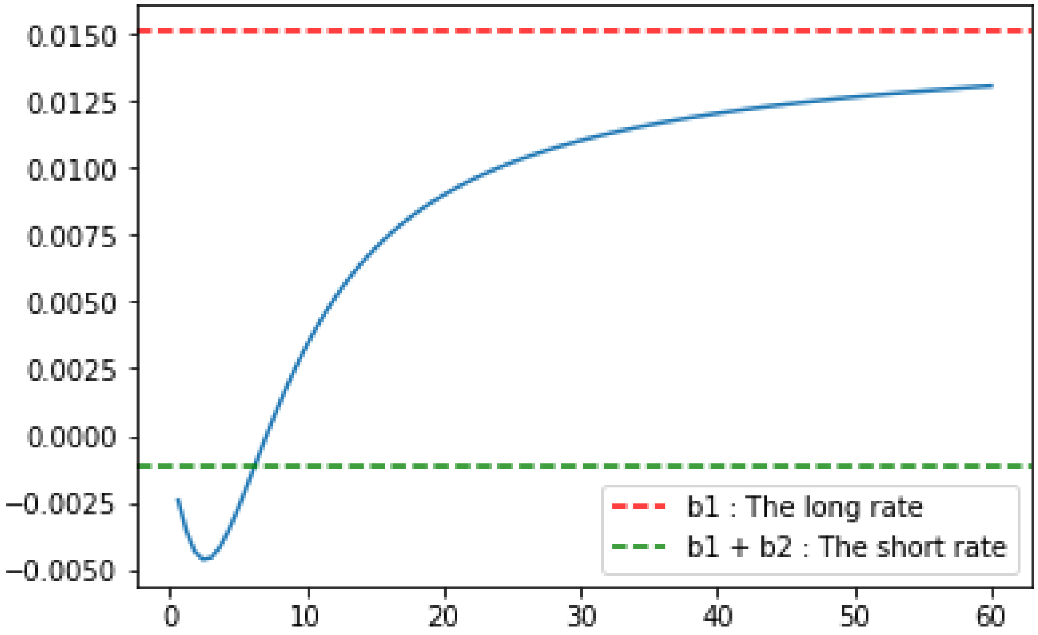
```
b1 : 0.0150950624961 b2 : -0.0162137997828 b3 : 0.00360089378392 b4 : -0.0544871532228 lam1 : 0.
```

```
In [13]: MeanSquaredErrorSvensson = sol.fun
         print(MeanSquaredErrorSvensson)
```

```
0.0003997039553169198
```

```
In [14]: x_= np.linspace(0,60,num=100)
         y_SM = [SvenssonModel.Svensson(b1, b2, b3, b4, lam1, lam2, i) for i in x_]

         f, ax = plt.subplots(1,1)
         ax.plot(x_, y_SM)
         ax.axhline(b1, label= 'b1 : The long rate', color='red', ls='--')
         ax.axhline(b1+b2, label= 'b1 + b2 : The short rate', color='green', ls='--')
         ax.legend()
         plt.savefig('Svensson.png')
```

```
/Users/Jeppe/anaconda3/lib/python3.6/site-packages/ipykernel/__main__.py:9: RuntimeWarning: inva
```

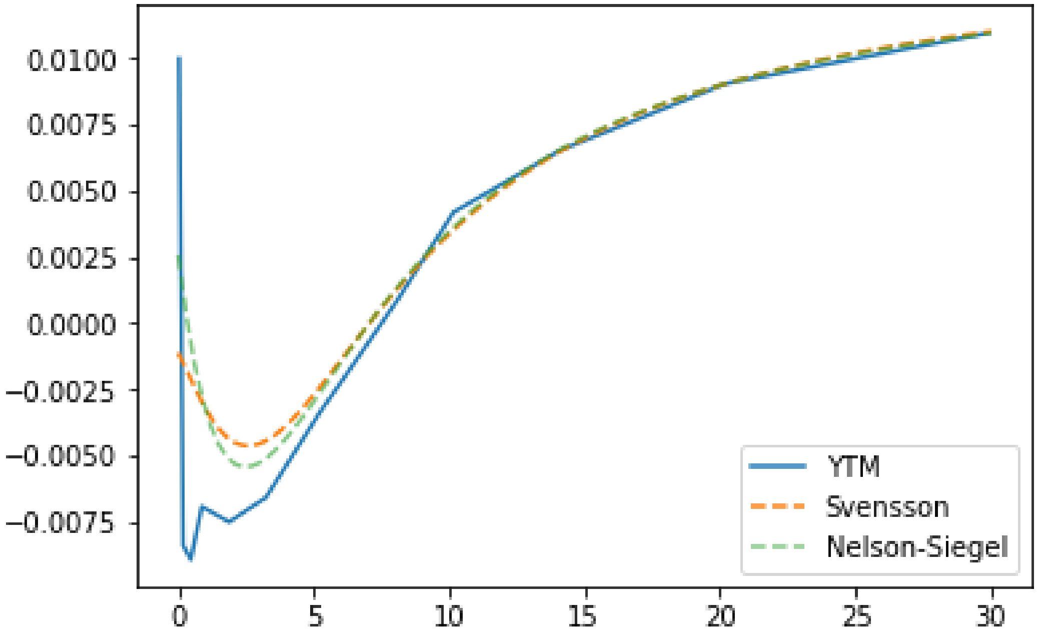


```
In [15]: x_compare = [1/52, 0.5, 1, 5, 10, 30]
         SM_rates = [SvenssonModel.Svensson(b1_SM, b2_SM, b3_SM, b4_SM, lam1_SM, lam2_SM, i) for
         NS_rates = [NSM.NielsonSiegel(b1_NS, b2_NS, b3_NS, lam1_NS, i) for i in x_compare]
```

```
In [16]: print(NS_rates)
```

```
[0.0024106873470304082, -0.00096489435833784724, -0.0032408950357681043, -0.0029654541346972024,
```

```
In [22]: x_= np.linspace(0.00001,30,num=100)
         SM_rates = [SvenssonModel.Svensson(b1_SM, b2_SM, b3_SM, b4_SM, lam1_SM, lam2_SM, i) for
         NS_rates = [NSM.NielsonSiegel(b1_NS, b2_NS, b3_NS, lam1_NS, i) for i in x_]

         f, ax = plt.subplots(1,1)
         ax.plot(data['nr_years'], data['YTM'])
         ax.plot(x_, SM_rates, label = 'Svensson', ls='--')
         ax.plot(x_, NS_rates, label = 'Nelson-Siegel', ls ='--', alpha=0.6)

         ax.legend()
         plt.savefig('ThreeFigures.png')
```


0.0100
0.0075
0.0050
0.0025
0.0000
−0.0025
−0.0050
−0.0075
0
5
10
15
20
25
30
YTM
Svensson
Nelson-Siegel