Exploring group theory for use in procedural content generation

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Introduction

- *Procedural Content Generation* (PCG): the algorithmic creation of objects
- Interesting objects have *regularity*: proportion, repetition, symmetry
- PCG often deals with generating regular features

- *Group Theory* (GT): the mathematical study of *symmetry*
- Symmetry: the property of an object to look the same after transformation: reflection, rotation
- Any *regularity* can be described in terms of a symmetric transformation

In this light, GT and PCG seem related. Can we take advantage of this connection?

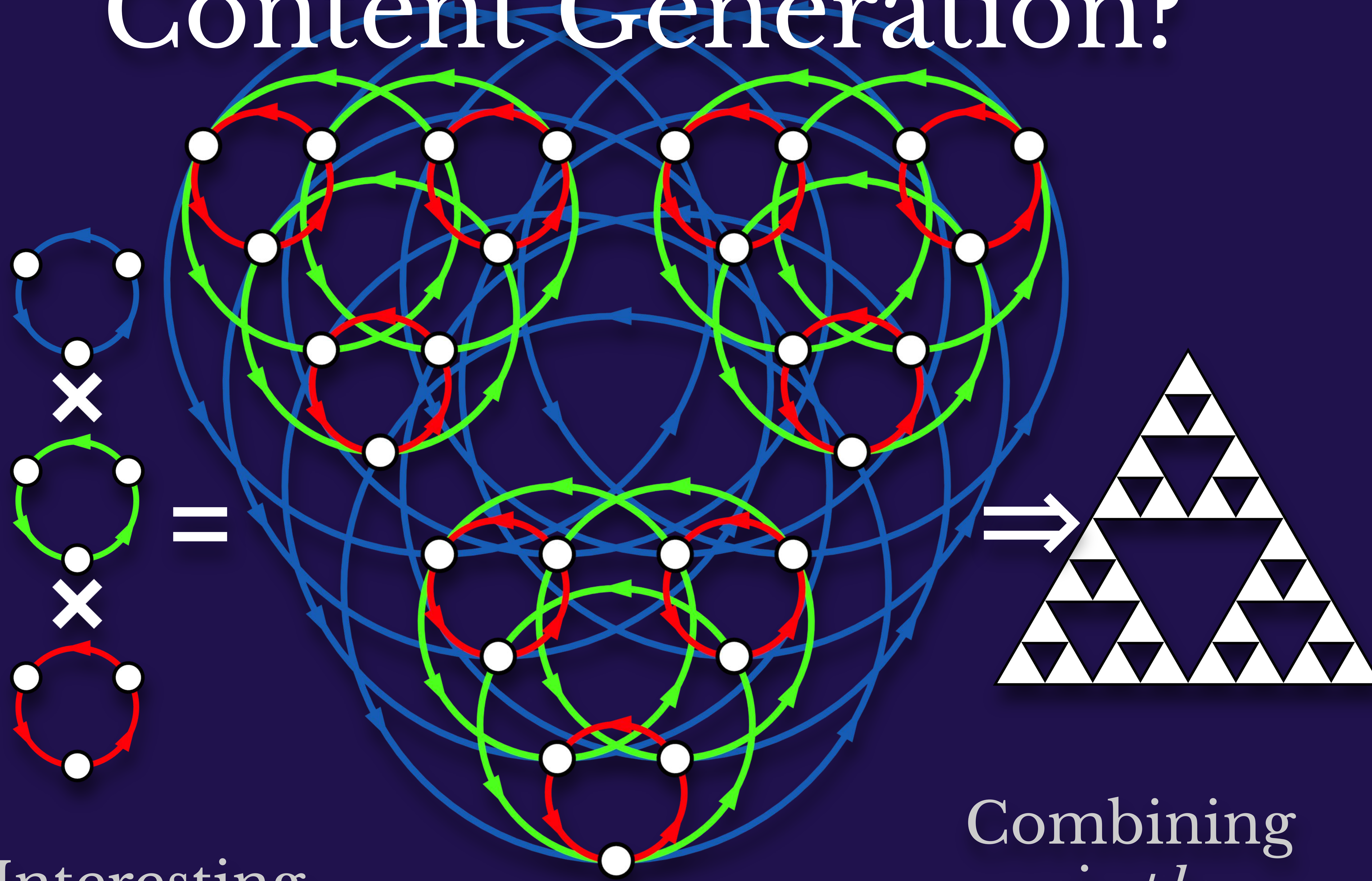
Research

Because interesting objects have symmetry, and we can use the tools of GT (Group products) to *generate* symmetric objects, it is worth asking: Can GT act as a unifying theory for PCG?

This *ongoing* research is exploring the hypothesis with questions like:

- How can we describe PCG artifacts as groups?
- How can we represent groups programmatically?
- What tools of GT can we take advantage of for PCG?

Is Group Theory a Unifying Formalism for Procedural Content Generation?



Interesting objects have *symmetry*

Combining *simple* symmetries yields *intricate* objects

Definitions

Group (Standard): a set S and an operation that combines any two elements of the set to produce a third element of the set, in such a way that the operation is associative, an identity element e exists and every element x has an inverse x^{-1}.

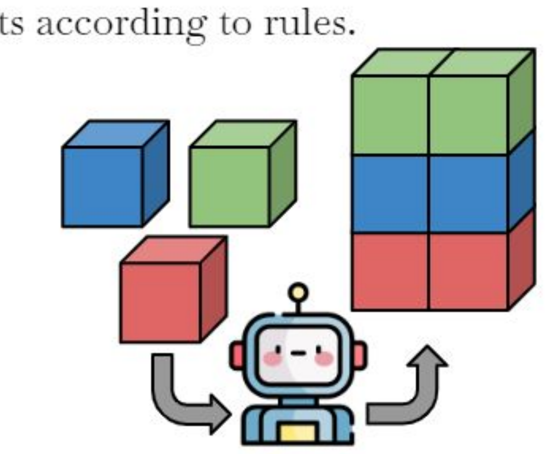
Group (Functional-1): a set S and an explicit function f that takes two elements of the set and outputs a third element of the set, such that the function is associative, there exists an identity element e such that $f(e, x) = x$ for all x and there exists an inverse element such that $f(x, x^{-1}) = e$ for all x.

Group (Functional-2): a set S of elements and a set F of functions of the same order that takes one element of the set and outputs one element of the set, such that $f(x, y) = z$ of the previous definition has an equivalent $\chi(y) = z$ for all x, y, z in S.

Group Multiplication: Given groups G (with operation *) and H (with operation Δ), the direct product $G \times H$ is defined as follows:

- The underlying set is the Cartesian product, $G \times H$. That is, the ordered pairs (g, h), where $g \in G$ and $h \in H$.
- The binary operation on $G \times H$ is defined component-wise: $(g_1, h_1) \cdot (g_2, h_2) = (g_1 * g_2, h_1 \Delta h_2)$

Procedural Content Generation: A set of elements with an AI that combines these elements according to rules.



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