TECHNISCHE UNIVERSITÄT BERLIN
Electrical Engineering and Computer Science
Internet of Things for Smart Buildings
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Model Predictive Control **SS 20**

P5: NMPC for inverted pendulum

In this project you will be working an a very popular example for unstable an chaotic systems: The inverted double pendulum on a cart. A representation of the system is given in figure 1. The system consists of two joints and two sticks and is mounted on a cart that can move left to right in one dimension. Your first task is to create a state-space representation of this system, similarly to [1], in CasADi. You may choose

$$x = \left[x_{\text{cart}}, \theta_1, \theta_2, \dot{x}_{\text{cart}}, \dot{\theta}_1, \dot{\theta}_2\right] \tag{1}$$

as states. Your input to the system should be a force ($-4\,\text{N} \leq F \leq -4\,\text{N}$) that is acting on the cart, as depicted in 1. By moving the cart, it is possible to invert the pendulum and hold it in its upright equilibrium, starting from a hanging stable position. It is your task to implement an MPC controller with CasADi to find an optimal trajectory and invert the pendulum. For this, it will be necessary to choose an appropriate discretization scheme for the derived non-linear ODE system. The parameters describing the system are the mass of the cart $m_{\text{cart}} = 0.6\,\text{kg}$, the mass of the two rods $m_{\text{rod},1} = m_{\text{rod},2} = 0.2\,\text{kg}$ and their length $l_{\text{rod},1} = l_{\text{rod},2} = 0.5\,\text{m}$. The inertia of the rods then results in

$$J_{\text{rod},i} = \frac{m_{\text{rod},i} \cdot l_{\text{rod},i}^2}{3}$$

for $i = 1, 2$.

Furthermore, you should have two different positions on the track where the pendulum is errected. Your MPC controller should find optimal trajectories to transition from on to the other set-point. To make things more interesting, we assume that obstacles exist (simple geometric shapes). These obstacles have to be avoided while transitioning. It is your choice, where and how to place these obstacles. Investigate trajectories for different placements of obstacles. Finally, you are asked to create a 2D animation of the inverted pendulum using, for example, Matplotlib, to present your results.

References

[1] W. Zhong and H. Rock, "Energy and passivity based control of the double inverted pendulum on a cart," in *Proceedings of the 2001 IEEE International Conference on Control Applications (CCA'01)(Cat. No. 01CH37204)*. IEEE, 2001, pp. 896–901.

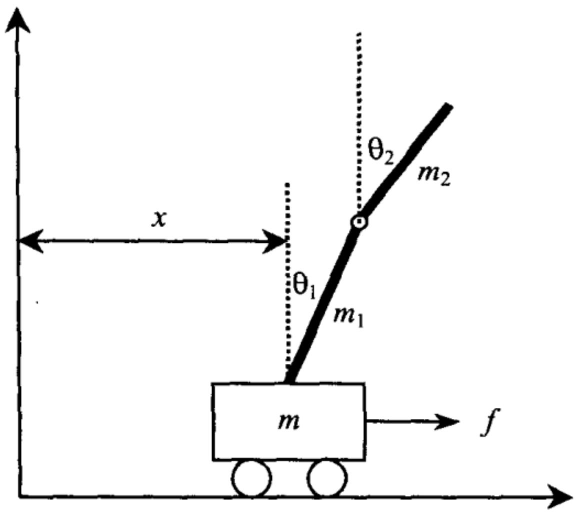

Figure 1: Double inverted pendulum on a cart [1].