YOU CAN USE THE VERSO OF YOUR EXAM AS A DRAFT OR AS ADDITIONAL SPACE TO ANSWER QUESTIONS. STATE CLEARLY HOW YOU ARE USING IT.

PROBLEM 1 (28 points)

Let's consider a problem encountered by a company that must choose amongst 4 alternatives for the production of a product which costs are uncertain. The 3 scenarios under consideration form the set of situations the company contemplates and form the set of all elementary events of the problem.

Table of costs obtained depending on the chosen alternative and realized scenario

	Scenarios (probabilities)		
Alternatives	**S1 (75%)**	**S2 (10%)**	**S3 (15%)**
$\mathbf{x_A}$	1500	0	1600
$\mathbf{x_B}$	750	2000	1100
$\mathbf{x_C}$	1100	1500	0
$\mathbf{x_D}$	1100	1500	100

a) Identify, if possible, an alternative for which the cost is stochastically dominated, but not deterministically dominated, by the cost generated by another alternative. **(4 points)**

b) Identify, if possible, an alternative for which the cost is deterministically dominated by the cost generated by another alternative. **(2 points)**

c) Calculate the following statistics for the cost generated by alternatives xA. **(14 points)**

Statistic	**Values for xA**
Expected value	
Standard deviation	
Probability that no cost is incurred	
Value at risk for a confidence level of 85%	
Conditional value at risk for a confidence level of 85%	
Value at risk for a confidence level of 70%	
Conditional value at risk for a confidence level of 70%	

d) Describe how one could generate a scenario among S1, S2, and S3 randomly based on a random variable that is uniformly distributed on the interval [0, 1]. The probability of obtaining S1, S2, or S3 should respectively be 75%, 10%, and 15%. **(4 points)**

e) Based on the following table, compare the risks associated to the costs generated by alternatives xB and xC. **(4 points)**

Statistic	**Values for xB**	**Values for xC**
Expected value	\$927,50	\$975
Standard deviation	\$378,31	\$426,47
Probability that no cost is incurred	0%	15%
Value at risk for a confidence level of 90%	\$1100	\$1100
Conditional value at risk for a confidence level of 90%	\$2000	\$1500

PROBLEM 2 (22 points)

A company must decide on the construction of a production facility. The company may decide to build a large or a small facility; in the latter case, the company may also decide to increase the capacity of the facility after two years of operation. Three possible scenarios are considered for the demand of the products it will produce in the facility:

- Scenario #1: Low demand for 10 years
- Scenario #2: High demand for the first 2 years, low demand for the next 8 years
- Scenario #3: High demand for 10 years

The table below summarizes what is the net present value of the cash flow generated by the construction of each type of facility under each type of demand scenario and the probabilities of each of these scenarios.

Net present value (in millions of dollars)

	Scenario #1 (50%)	**Scenario #2 (25%)**	**Scenario #3 (25%)**
Big facility	-5	6	10
Small facility	1	4	4
Small facility with increased capacity after 2nd year	-3	5	8

a) Draw and solve a decision tree that allows to identify whether this company should build a large or small facility and whether (and under which conditions) it should increase the capacity of the small facility. What is the optimal strategy for this company? You can assume that the company considers the expected net present value as the only measure of success for this project. **(10 points)**

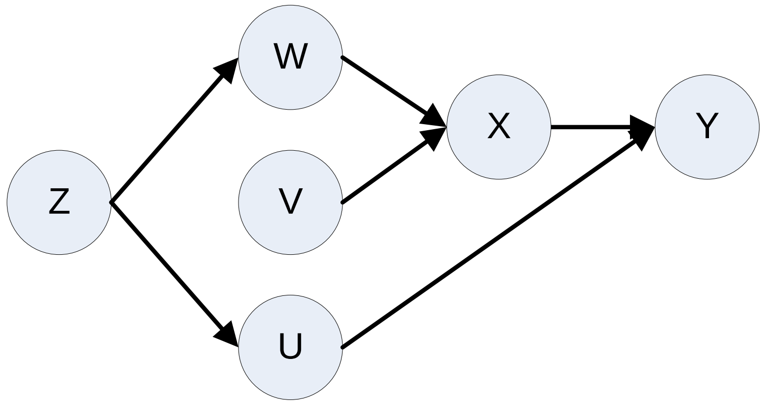
b) Consider now that the company has the opportunity to learn exactly what the demand for the products will be in the next 10 years before deciding which facility to construct. What would be the largest amount of money that this company would be willing to pay to obtain this information? **(6 points)**

c) Consider now that the company is instead able to hire the services of a consulting firm that will predict only what the demand will be in the next 2 years. What would be the largest amount of money that this company would be willing to pay to obtain this information? **(6 points)**

<u>PROBLEM 3</u> (26 points)

We consider the following influence diagram involving the random variables U, V, W, X, Y and Z and satisfying the Markov property presented in class:

Figure 1: Influence Diagram



a) Identify the 5 independences that can be deduced from the above network's structure amongst the following 10 proposals: **(5 points)**

Independence	Can be deduced from the diagram? (Yes / No)	Independence	Can be deduced from the diagram? (Yes / No)
$V \perp\!\!\!\perp W$		$Z \perp\!\!\!\perp V$	
$V \perp\!\!\!\perp W \mid Y$		$Z \perp\!\!\!\perp Y$	
$X \perp\!\!\!\perp U \mid Y$		$V \perp\!\!\!\perp W \mid Z$	
$X \perp\!\!\!\perp U \mid Z$		$Z \perp\!\!\!\perp Y \mid U$	
$Z \perp\!\!\!\perp W$		$Y \perp\!\!\!\perp Z \mid U,X$	

For example, “$Y \perp\!\!\!\perp Z \mid U,X$” describes the fact that Y is independent of Z if U and X are known.

b) Enumerate the distribution functions that must be defined in order to obtain a complete and compact model (i.e., that exploits independences) of this problem's uncertainty. **(3 points)**

c) If all variables are Bernoulli random variables (with realizations 0 or 1) except for the variables V and W that possess four possible realizations (0, 1, 2, or 3), how many parameters must be estimated in total to define these distributions? **(3 points)**

d) If all variables are Bernoulli random variables (with realizations 0 or 1) except for the variables V and W that possess four possible realizations (0, 1, 2, or 3), how many parameters must be estimated in total to define these distributions in the event that no independence assumption could be made in this problem? **(3 points)**

e) Considering that you have identified the following distribution models for realizations of the pair (Z, W), use Bayes' theorem to reformulate this model in a way that can be used in the **Influence Diagram** presented in **Figure 1**. **(6 points)**

Value for W (w)	**Probability P(W=w)**
0	18%
1	22%
2	40%
3	20%

Value for W (w)	**Value for Z (z)**	**Probability P(Z=z | W=w)**
0	0	20%
0	1	80%
1	0	50%
1	1	50%
2	0	50%
2	1	50%
3	0	80%
3	1	20%

f) You are now considering characterising a conditional probability model for variable U when Z is known. You would actually like the distribution of U when Z is known to follow a Poisson distribution. Describe the model that would be obtained using the following data (each entry being obtained independently from each other) when employing a Bayesian approach.
(6 points)

Index of the observation	**Observed value for Z**	**Observed value for U**	**Index of the observation**	**Observed value for Z**	**Observed value for U**
1	0	6	9	1	1
2	0	17	10	1	2
3	0	14	11	1	1
4	0	14	12	1	3
5	0	11	13	1	5
6	0	14	14	1	4
7	0	11	15	1	3
8	0	6	16	1	1

<u>PROBLEM 4</u> (24 points)

Let's consider a similar company as discussed in problem 2. This time the company can only choose between building a large or a small facility. The company has run a Monte-Carlo simulation in order to compare the risks of the two alternatives. Doing so it obtained 40 samples for each alternative, each of which should be considered an independent and identically distributed realization from the distribution of net present value (NPV) achieved by each facility type. These samples are presented in the following two tables in the order from the smallest to the largest simulated scenario. (Note that the units of NPV are in thousands of dollars, i.e. K$)

Ordered samples obtained from Monte-Carlo simulation of <u>Large Facility</u>							
Rank	**Value (in K$)**	**Rank**	**Value (in K$)**	**Rank**	**Value (in K$)**	**Rank**	**Value (in K$)**
1	-153	11	54	21	119	31	149
2	-63	12	59	22	123	32	151
3	-52	13	70	23	126	33	156
4	-50	14	87	24	131	34	157
5	-47	15	91	25	137	35	162
6	-5	16	93	26	140	36	163
7	3	17	108	27	141	37	171
8	35	18	110	28	145	38	172
9	40	19	113	29	146	39	176
10	51	20	113	30	147	40	181

Ordered samples obtained from Monte-Carlo simulation of <u>Small Facility</u>							
Rank	**Value (in K$)**	**Rank**	**Value (in K$)**	**Rank**	**Value (in K$)**	**Rank**	**Value (in K$)**
1	-42	11	17	21	47	31	84
2	-31	12	24	22	48	32	85
3	-23	13	28	23	56	33	94
4	-23	14	33	24	58	34	95
5	-10	15	34	25	63	35	106
6	-4	16	35	26	66	36	109
7	2	17	35	27	70	37	122
8	3	18	40	28	72	38	123
9	11	19	41	29	76	39	130
10	14	20	47	30	79	40	145

According to this sample set, the expected net present value of the large facility is $91,25K and its standard deviation is $79,44K, while for the small facility the mean is $48,98K and the standard deviation is $46,62K.

a) Calculate a 86% confidence interval for the expected value of the net present value of the large facility. (You can use the table provided in the **ANNEX**.) **(4 points)**

b) How many samples would be needed in order for the 86% confidence interval calculated in **a)** to have a size that is smaller than $0,1K? (You can use the table provided in the **ANNEX**.) **(2 points)**

c) Estimate the probability that the net present value for the large facility is strictly lower than \$0K. Considering that this probability is in fact the expected value of the random variable defined as

$$Z = \begin{cases} 1 & if\ the\ NPV\ is < 0\$ \\ 0 & if\ the\ NPV\ is \geq 0\$ \end{cases}$$

determine a 86% confidence interval for this probability. (You can use the table provided in the **ANNEX**.) **(6 points)**

d) Estimate the 20^{th} percentile of the net present value of the large facility and an 86% confidence interval for this percentile. (You can use the table provided in the **ANNEX**.) **(6 points)**

e) Based on the results of this Monte-Carlo simulation, what would be the best type of facility to build? Present your conclusion by discussing the compromise between the two alternatives in terms of risk and returns. **(6 points)**

ANNEX

Inverse distribution function for a normal distribution (mean = 0, standard deviation = 1)							
Probability	$F^{-1}(p)$	Probability	$F^{-1}(p)$	Probability	$F^{-1}(p)$	Probability	$F^{-1}(p)$
0,01	-2,32634787	0,26	-0,64334541	0,51	0,02506891	0,76	0,70630256
0,02	-2,05374891	0,27	-0,61281299	0,52	0,05015358	0,77	0,73884685
0,03	-1,88079361	0,28	-0,58284151	0,53	0,07526986	0,78	0,77219321
0,04	-1,75068607	0,29	-0,55338472	0,54	0,10043372	0,79	0,80642125
0,05	-1,64485363	0,30	-0,52440051	0,55	0,12566135	0,80	0,84162123
0,06	-1,55477359	0,31	-0,49585035	0,56	0,15096922	0,81	0,8778963
0,07	-1,47579103	0,32	-0,4676988	0,57	0,17637416	0,82	0,91536509
0,08	-1,40507156	0,33	-0,43991317	0,58	0,20189348	0,83	0,95416525
0,09	-1,34075503	0,34	-0,41246313	0,59	0,22754498	0,84	0,99445788
0,10	-1,28155157	0,35	-0,38532047	0,60	0,2533471	0,85	1,03643339
0,11	-1,22652812	0,36	-0,35845879	0,61	0,27931903	0,86	1,08031934
0,12	-1,17498679	0,37	-0,33185335	0,62	0,30548079	0,87	1,12639113
0,13	-1,12639113	0,38	-0,30548079	0,63	0,33185335	0,88	1,17498679
0,14	-1,08031934	0,39	-0,27931903	0,64	0,35845879	0,89	1,22652812
0,15	-1,03643339	0,40	-0,2533471	0,65	0,38532047	0,90	1,28155157
0,16	-0,99445788	0,41	-0,22754498	0,66	0,41246313	0,91	1,34075503
0,17	-0,95416525	0,42	-0,20189348	0,67	0,43991317	0,92	1,40507156
0,18	-0,91536509	0,43	-0,17637416	0,68	0,4676988	0,93	1,47579103
0,19	-0,8778963	0,44	-0,15096922	0,69	0,49585035	0,94	1,55477359
0,20	-0,84162123	0,45	-0,12566135	0,70	0,52440051	0,95	1,64485363
0,21	-0,80642125	0,46	-0,10043372	0,71	0,55338472	0,96	1,75068607
0,22	-0,77219321	0,47	-0,07526986	0,72	0,58284151	0,97	1,88079361
0,23	-0,73884685	0,48	-0,05015358	0,73	0,61281299	0,98	2,05374891
0,24	-0,70630256	0,49	-0,02506891	0,74	0,64334541	0,99	2,32634787
0,25	-0,67448975	0,50	0	0,75	0,67448975	1,00	∞