YOU CAN USE THE VERSO OF YOUR EXAM AS A DRAFT OR AS ADDITIONAL SPACE TO ANSWER QUESTIONS. STATE CLEARLY HOW YOU ARE USING IT.

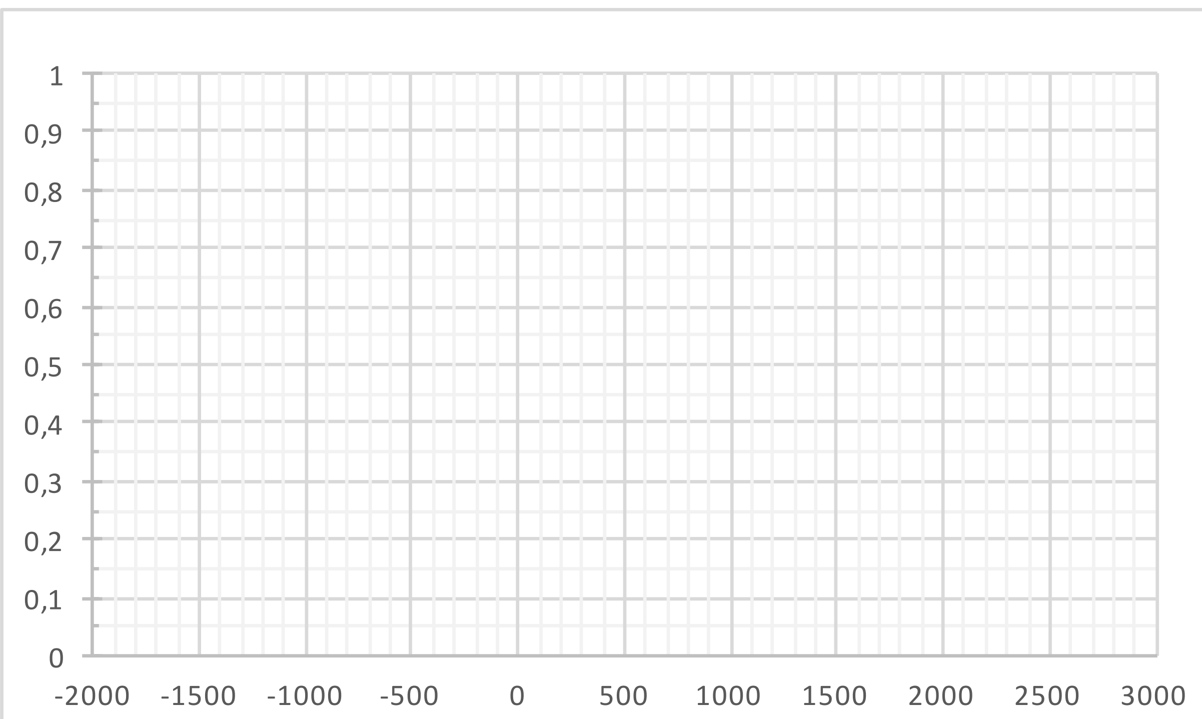
PROBLEM 1 (32 points)

Let's consider a problem encountered by a company that must choose amongst 4 alternatives for the production of a product for which profits are uncertain. The 4 scenarios under consideration form the set of situations the company contemplates and form the set of all elementary events of the problem.

Table of profits obtained depending on chosen alternative and realized scenario

	Scenarios (probabilities)			
Alternatives	**S1 (5 %)**	**S2 (15 %)**	**S3 (20 %)**	**S4 (60 %)**
x_A	-600 $	600 $	500 $	1 200 $
x_B	-200 $	-500 $	-100 $	400 $
x_C	-2 000 $	-100 $	500 $	1 000 $
x_D	3 000 $	1 500 $	-200 $	400 $

a) Trace the cumulative distribution functions associated to the profit generated by each of the 4 alternatives. Identify, if possible, an alternative for which the profit stochastically, but not deterministically, dominates the profit generated by another alternative. **(5 points)**



b) Identify, if possible, an alternative for which the profit is deterministically dominated by the profit generated by another alternative. **(2 points)**

c) Calculate the following statistics for the profit generated by alternatives xA & xB. **(10 points)**

Statistic	Values for xA	Values for xB
Expected value		
Standard deviation		
Probability of financial loss		
Value at risk for a confidence level of 85 %		
Conditional value at risk for a confidence level of 85 %		

d) Based on the following table, compare the risks associated to the profits generated by alternatives xC & xD. **(6 points)**

Statistic	Values for xC	Values for xD
Expected value	585 $	575 $
Standard deviation	772 $	808 $
Probability of financial loss	20 %	20 %
Value at risk for a confidence level of 85 %	100 $	200 $
Conditional value at risk for a confidence level of 85 %	733 $	200 $

e) Describe how to randomly generate one of the 4 scenarios using an unbiased coin so that each scenario has approximately (i.e. ± 1%) the probability described in the table of being selected. **(5 points)**

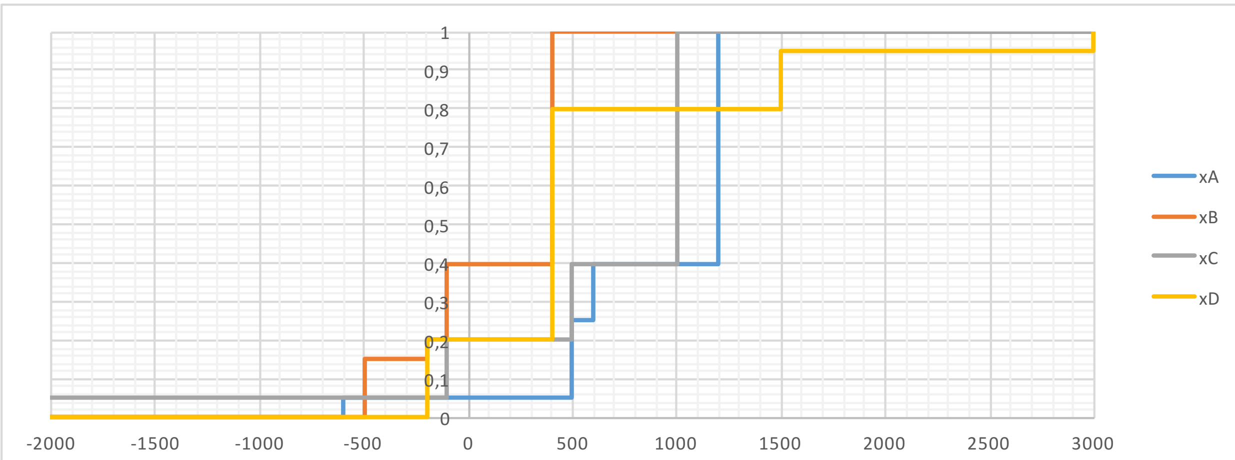
f) Describe how to generate, using a random variable U uniformly distributed over [0, 1], a scenario V on the interval [0, 1] so that the scenario follows this distribution function :

$$F_V(y) = P(V \leq y) = \begin{cases} 0 & si\ y < 0 \\ 1 - (y-1)^2 & si\ 0 \leq y \leq 1 \\ 1 & si\ y > 1 \end{cases}$$

(4 points)

SOLUTION

a) This is a description of the cumulative distribution functions of the profit achieved by the 4 alternatives.



We observe that the cumulative distribution function of xD is always below (and to the right) of the distribution function of xB. It is therefore the case that for all level of profit xD has always less chances to be below this level than xB. xD therefore dominates stochastically xB. But, xD does not dominate deterministically xB since for the scenario S3 it leads to a greater financial loss than xB (-100 for xB, -200 for xD).

xA stochastically dominates xC is not adequate since it also deterministically dominates xC.

b) The alternative xA deterministically dominates the alternative xC since it always leads to a greater profit regardless of which scenarios happens.

c) We can do the following calculations :
For xA :
Expected value = 5%*(-600)+15%*600+20%*500+60%*1200 = 880
Standard deviation =
$\sqrt{5\% * (-600-880)^2 + 15\% * (600-880)^2 + 20\% * (500-880)^2 + 60\% * (1200-880)^2} = 460$
Loss probability = 5%
Value at risk = -500 since we have more than 85% chances of making more than 500$ of profit, so a loss of -500$.
CVaR = -133$ since in the worst 15% scenario we achieve on average a profit of (5%/15%)*(-600)+(10%/15%)*500= 133$, so a loss of less than -133$.

For xB :
Expected value = 5%*(-200)+15%*(-500)+20%*(-100)+60%*400 = 135

Standard deviation =
$\sqrt{5\%(-200-135)^2+15\%*(-500-135)^2+20\%*(-100-135)^2+60\%*(400-135)^2} = 345$
Loss probability = 5%+15%+20%=40%
Value at risk = 200$ since we have exactly a 85% chance of making more than -200$ of profit, so a loss of less than 200$.
CVaR = 500$ since in the worst 15% scenario we achieve on average a profit of -500$, so a loss of 500$.

Statistic	Values for xA	Values for xB
Expected value	880$	135$
Standard deviation	460$	345$
Probability of financial loss	5%	40%
Value at risk for a confidence level of 85 %	-500$	200$
Conditional value at risk for a confidence level of 85 %	-133$	500$

d) The expected value of profit xC is greater than for xD and the standard deviation of xC is lower. This seems to indicate that xC has less risk and a greater potential of returns. We observe however that the difference in expected profit is rather small while xC has a much greater conditional value at risk than xD, which indicates a risk of financial loss greater with xC than with xD. Indeed, xC has a 5% chance of generating a loss of 2000$! We would therefore encourage the company to opt for xD rather than xC.

e) Lots of options to choose from.

Option #1 : Toss the coin 6 times to obtain a uniform value amongst $(\frac{1}{2^6}, \frac{2}{2^6}, \dots, \frac{2^6}{2^6})$. Depending of the result obtained, that we'll call X, generate the scenario amongst S1, S2, S3 & S4 based on the following rule :
$\frac{1}{2^6} \leq X \leq \frac{3}{2^6}$: S1
$\frac{4}{2^6} \leq X \leq \frac{12}{2^6}$: S2
$\frac{13}{2^6} \leq X \leq \frac{25}{2^6}$: S3
$\frac{26}{2^6} \leq X \leq 1$: S4

With this strategy, we obtain the following probabilities :
$$P(S1) = \frac{3}{2^6} \approx 4.7\% \quad P(S2) = \frac{9}{2^6} \approx 14.1\% \quad P(S3) = \frac{13}{2^6} \approx 20.3\% \quad P(S4) = \frac{39}{2^6} = 60.9\%$$

Option #2 (rejection method): Toss the coin 5 times, considering that the five results are P1, P2, P3, P4 & P5, calculate X =P1+2*P2+4*P3+8*P4+16*P5, if the value of X is smaller or equal to 20, generate:
$0 < X \leq 1$: S1
$1 < X \leq 4$: S2

$4 < X \leq 8$: S3
$8 < X \leq 20$: S4
Else, restart the exercise until the obtained X is ≤ 20.

f) We must first establish the inverse distribution function.

$$x = F_V(y) = 1 - (y-1)^2 \Leftrightarrow (y-1)^2 = 1 - x \Leftrightarrow y - 1 = \pm\sqrt{1-x}$$
$$\Leftrightarrow y = 1 \pm \sqrt{1-x} \Leftrightarrow y = 1 - \sqrt{1-x}$$

Where the negative root must be selected, else y would take a strictly higher value than 1 for $x \in [0, 1]$. We obtain the following method :

$$V = 1 - \sqrt{1 - U}$$

<u>PROBLEM 2</u> (23 points)

The company Vulcan Inc. is a business specialized in the production of rubber pieces for trucks. For some time now, it has been trying to convince the Tronka company to choose it as a supplier of pieces for its trucks. The director of Tronka is looking to obtain pieces of a better quality for 3 components of its trucks : motor supports (MS), bumpers (PC) and shock absorbers (A). It is therefore ready to offer a chance to Vulcan Inc. but it is not willing to be responsible for the risks related to the production of prototypes for these new pieces. Indeed, to develop prototypes for each pieces, the engineers estimates that it will cost 8 000$ for (MS), 16 000$ for (PC) and 10 000$ for (A). Except, it is possible that when testing the quality of each prototype, some of them do not meet the standards established by the Tronka company. In the case that any prototype would not meet the standard, Tronka would stop doing business with Vulcan Inc. and would reimburse nothing of the development costs of the prototypes.

Tronka's director additionally suggests to start with the development of pieces (MS). In the eventuality that the prototype (MS) passes the quality test, she would make an order for 100 000 pieces and would accept to make the same exercise for another prototype (PC) or (A). In the eventuality of a second success, Tronka would accept to perform the evaluation of a prototype of the remaining component. As with the pieces (MS), an order of 100 000 pieces would be made for (PC) and (A) in the case where one or the other (or both) passes the test. For each component, the prototype production cost, the unit profit (excluding the prototype production cost), the success probability and the expected order in the event of a success are described below :

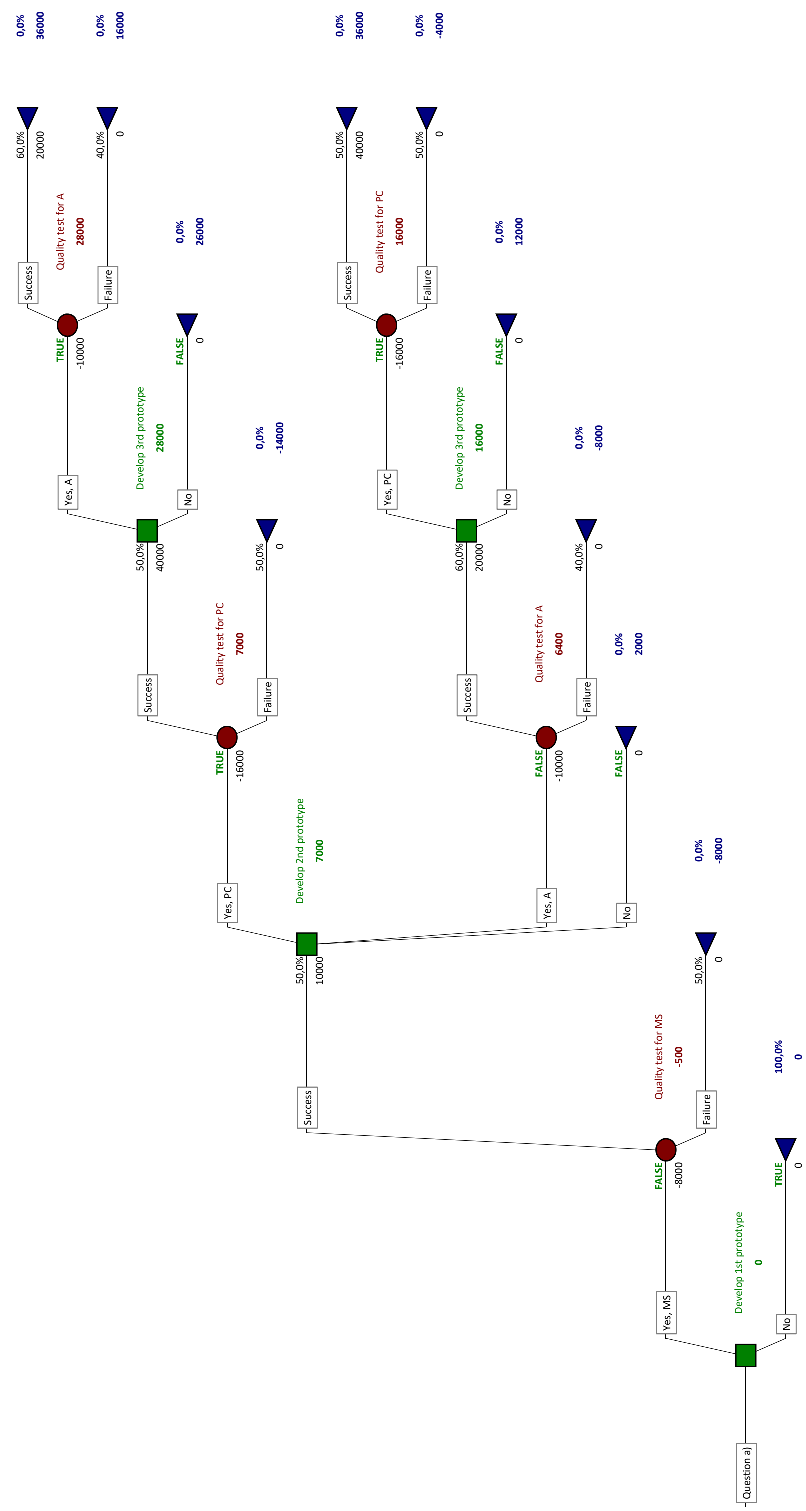
Component	Prototype cost	Unit profit	Success probability	Order if success
Motor supports (SM)	8 000$	0,10 $	50%	100 000 units
Bumpers (PC)	16 000$	0,40 $	50%	100 000 units
Shock absorbers (A)	10 000$	0,20 $	60%	100 000 units

a) Considering that the engineers of Vulcan Inc. agrees that the odds of success for each prototypes are respectively of 50 %, 50 % and 60 % for (MS), (PC), and (A). The success of each prototype is independent from another. Draw and solve a decision tree that allows to identify the optimal strategy for Vulcan Inc. regarding the prototype development for the pieces (MS), (PC) and (A). You can assume that Vulcan Inc. considers the expected profit as the only measure of success of this project. Should Vulcan Inc. engage in this relationship? **(18 points)**

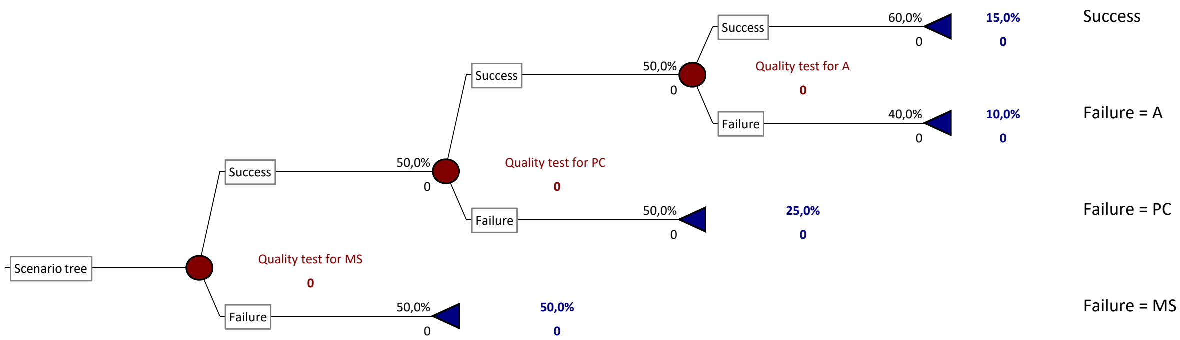
B) Let's now consider that the Vulcan Inc. company decides to go forward with the project by following this development order : MS, PC, A. In the event that one of these prototypes doesn't pass the test, what is the probability for this prototype to be (MS) ? **(5 points)**

SOLUTION

a) Here is the tree and its resolution.



b) Consider the decision tree below in which we identified the elementary events using the names « Success », « Failure=MS », « Failure=PC » and « Failure=A ».

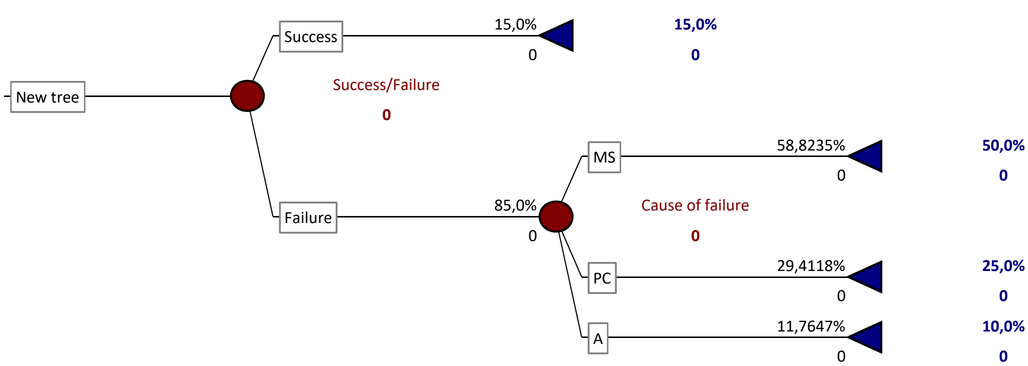


We can apply Baye's theorem as follows :

$$P(Failure = MS|\overline{Success}) = \frac{P(Failure = SM \,\&\, \overline{Success})}{P(\overline{Success})}$$

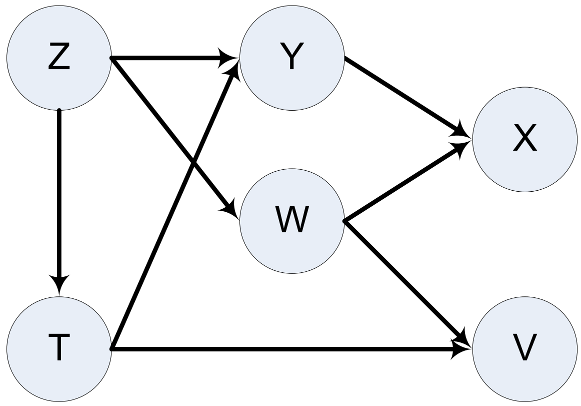
$$= \frac{P(Failure = SM)}{P(Failure = SM) + P(Failure = PC) + P(Failure = A)}$$

$$= \frac{0{,}5}{0{,}5 + 0{,}5 * 0{,}5 + 0{,}5 * 0{,}5 * 0.4} = 58{,}8\%$$



PROBLEM 3 **(14 points)**

We consider the following influence diagram involving the random variables T, V, W, X, Y & Z and satisfying the global Markov property:



a) Identify the four (4) independences that can be deduced from the above network's structure amongst the following 10 proposals : **(5 points)**

Independence	Can be deducted from network ?	Independence	Can be deducted from network ?
$W \perp\!\!\!\perp T \mid Z$		$X \perp\!\!\!\perp V \mid W$	
$Y \perp\!\!\!\perp W$		$Z \perp\!\!\!\perp V$	
$Y \perp\!\!\!\perp V \mid T, X, Z$		$W \perp\!\!\!\perp T$	
$X \perp\!\!\!\perp V \mid W, Y$		$Y \perp\!\!\!\perp V$	
$X \perp\!\!\!\perp V \mid W, T$		$X \perp\!\!\!\perp Z \mid W, Y$	

For example, « $X \perp\!\!\!\perp V \mid W, Y$» describes the fact that X is independent of V if W & Y are known.

b) Enumerate the distribution functions that must be defined in order to obtain a complete and compact model (i.e., that exploits independences) of this problem's uncertainty. **(3 points)**

c) If all variables are of the Bernoulli type (i.e., realization 0 or 1) except for the variables V and W that possess five possible realizations (0, 1, 2, 3 or 4), how many total parameters must be estimated to define these distributions? **(3 points)**

d) If all variables are of the Bernoulli type (i.e., realization 0 or 1) except for the variables V and W that possess five possible realizations (0, 1, 2, 3 or 4), how many total parameters must be estimated to define these distributions in the event that no independence assumption could be made in this problem? **(3 points)**

SOLUTION

a) The 4 independences are : $W \perp\!\!\!\perp T \mid Z$, $X \perp\!\!\!\perp V \mid W, Y$, $X \perp\!\!\!\perp V \mid W,T$, and $X \perp\!\!\!\perp Z \mid W,Y$.

b) The following distributions must be defined :
P(Z), P(T|Z), P(Y|Z,T), P(W|Z), P(X|Y,W), P(V|W,T)

c) The following probabilities must be estimated :

P(Z=1)

P(T=1|Z=0), P(T=1|Z=1)

P(Y=1|Z=0,T=0), P(Y=1|Z=0,T=1), P(Y=1|Z=1,T=0), P(Y=1|Z=1,T=1)

P(W=1|Z=0), P(W=2|Z=0), P(W=3|Z=0), P(W=4|Z=0)
P(W=1|Z=1), P(W=2|Z=1), P(W=3|Z=1), P(W=4|Z=1)

P(X=1|Y=0,W=0), P(X=1|Y=0,W=1), P(X=1|Y=0,W=2), P(X=1|Y=0,W=3), P(X=1|Y=0,W=4), P(X=1|Y=1,W=0), P(X=1|Y=1,W=1), P(X=1|Y=1,W=2), P(X=1|Y=1,W=3), P(X=1|Y=1,W=4)

P(V=1|W=0,T=0), P(V=2|W=0,T=0), P(V=3|W=0,T=0), P(V=4|W=0,T=0)
P(V=1|W=1,T=0), P(V=2|W=1,T=0), P(V=3|W=1,T=0), P(V=4|W=2,T=0)
…
P(V=1|W=4,T=0), P(V=2|W=4,T=0), P(V=3|W=4,T=0), P(V=4|W=4,T=0)

P(V=1|W=0,T=1), P(V=2|W=0,T=1), P(V=3|W=0,T=1), P(V=4|W=0,T=1)
P(V=1|W=1,T=1), P(V=2|W=1,T=1), P(V=3|W=1,T=1), P(V=4|W=2,T=1)
…
P(V=1|W=4,T=1), P(V=2|W=4,T=1), P(V=3|W=4,T=1), P(V=4|W=4,T=1)

In total, 1+2+4+8+10+2*5*4=65 probabilities

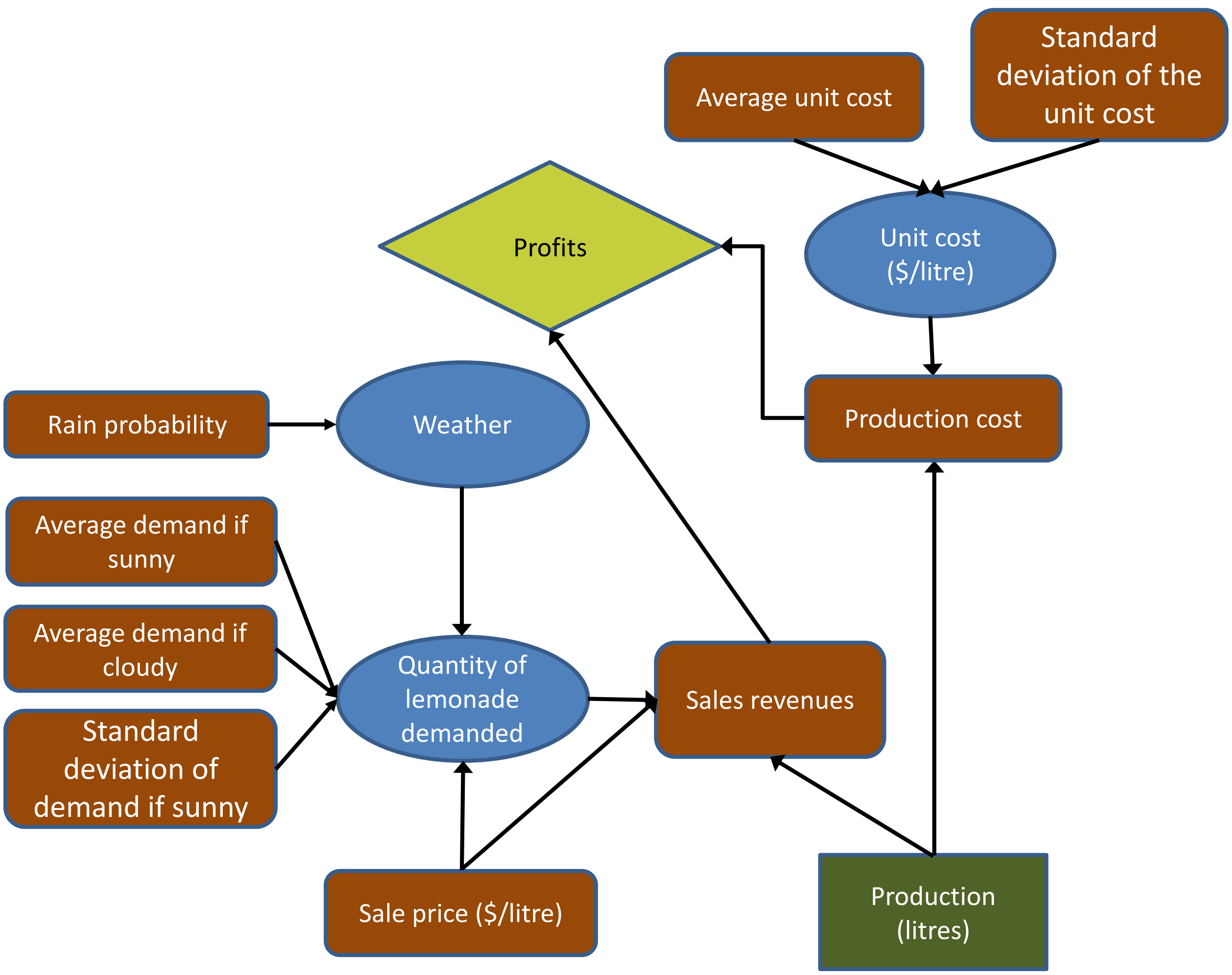
d) The probabilities of every combinations of events for the variables T, V, W, X, Y & Z must be estimated.
P(T=0,V=0,W=0,X=0,Y=0,Z=0)
P(T=0,V=0,W=0,X=0,Y=0,Z=1)
P(T=0,V=0,W=0,X=0,Y=1,Z=0)
P(T=0,V=0,W=0,X=0,Y=1,Z=1)
P(T=0,V=0,W=0,X=1,Y=0,Z=0)
P(T=0,V=0,W=0,X=1,Y=0,Z=1)
…
P(T=1,V=5,W=5,X=1,Y=1,Z=1)

The last probability of P(T=1,V=1,W=3,X=1,Y=1,Z=1) can be deducted from the other by determining what is missing to make 100%. There is therefore $2 * 2 * 2 * 2 * 5 * 5 - 1 = 399$ probabilities to estimate.

<u>PROBLEM 4</u> (31 points)

As mentioned in assignment 3, to earn a bit of pocket money, your nephew sells lemonades glasses on Saturdays in a stand installed along a busy avenue of his neighborhood. Following the recommendations formulated by our group of experts analysts, he decided to fix the price of this drink to 1,50 $ per liter and just printed the posters announcing the price for the day of the sale. Before starting the lemonade production, he wishes to analyse the risks related to a production of 60 liters one last time.

For this new analysis, he made some modifications to the influence diagram in order to highlight clearly what are analyse the assumptions that were made and later perform a sensitivity analysis for some of these.



The document attached to this diagram contains amongst other things the table describing the models he chose to characterize the 3 sources of uncertainty.

Variable		Distribution	Parameters
Weather		Discrete variable	P(rainy)=30% P(cloudy)=60%-P(rainy) P(sunny)=40%
Unit cost		Normal distribution	Mean = 0,5 \$/liter Standard deviation = 0,1 \$/liter
Quantity of lemonade demanded[1]	If « rainy »	None	Quantity = 0
	If « cloudy »	Normal distribution	Mean = 30 liters Standard deviation = 5 liters
	If « sunny »	Normal distribution	Mean = 100 liters Standard deviation = 35 liters

Using the @Risk software, your nephew successfully generated 40 samples of the realized profit if production was of 60 liters. Each sample can be considered independent and identically distributed according to the profit distribution obtained under this decision. These samples are presented in ascending order in the following table :

Ordered samples of the Monte-Carlo simulation					
Rank	Profit	Rank	Profit	Rank	Profit
1	-42,36	15	6,91	29	54,45
2	-41,81	16	8,57	30	55,70
3	-34,63	17	8,61	31	57,09
4	-33,43	18	9,07	32	57,65
5	-32,71	19	12,63	33	60,24
6	-31,26	20	14,99	34	63,37
7	-30,47	21	16,54	35	63,60
8	-28,82	22	25,21	36	64,63
9	-27,34	23	27,70	37	66,13
10	-27,31	24	28,17	38	67,35
11	-27,21	25	29,07	39	69,47
12	-26,26	26	30,15	40	70,78
13	-1,63	27	43,61		
14	-1,53	28	54,09		

[1] Note that for the quantity of lemonade demanded, the conditional uncertainty is modeled separately for each condition : for example, if it is sunny, the conditional density function takes the shape of a normal distribution (mean = 100, standard deviation = 35) while if it rainy the quantity demanded is zero with certainty.

According to this set of samples, the average profit achieved is of 16,98 $ and the standard deviation of this value is of 38,58 $.

a) Calculate a confidence interval of 70 % on the profit's expected value. (You can use the table provided in annex 1.) **(3 points)**

b) How many samples would be needed in order to have a 70% confidence interval (on expected profits) that is smaller than 1,00 $ (You can use the table provided in annex 1.) **(2 points)**

c) Estimate the 35th percentile of the profit as well as a 70% confidence interval for this percentile. (You can use the table provided in annex 1.) **(4 points)**

d) You now initiate a second Monte-Carlo simulation to establish if it is preferable to produce 60 or 30 liters.

Unordered samples of the Monte-Carlo simulation

#	Profit if 60 liters	Profit if 30 liters	#	Profit if 60 liters	Profit if 30 liters
1	-30,16	-15,08	16	59,31	29,65
2	50,48	25,24	17	54,42	27,21
3	17,93	23,61	18	-38,53	-19,26
4	-34,71	-17,36	19	52,85	26,43
5	56,04	28,02	20	4,43	20,96
6	17,10	29,54	21	16,28	30,11
7	49,84	24,92	22	-26,32	-13,16
8	34,17	33,17	23	29,58	32,60
9	-33,23	-16,61	24	-23,00	-11,50
10	58,56	29,28	25	-37,89	-18,95
11	31,67	30,23	26	4,72	21,53
12	-22,45	-11,23	27	59,96	29,98
13	-32,87	-16,43	28	29,22	29,91
14	13,92	27,56	29	2,11	21,58
15	6,86	25,33	30	59,61	29,80

i) Based on this set of samples, what is the probability of making a greater profit by producing 60 liters rather than 30 liters?

ii) What is the level of confidence that this probability is lower than 50% following an approach based on the central limit theorem (You can use the table provided in annex 1.)

iii) What is the level of confidence that this probability is lower than 50% based on a Bayesian approach that would consider that all probabilities are equiprobable (i.e. following the Beta distribution (1,1)) before making the observations presented in the above table? (You can use the table provided in annex 2.)

(9 points)

Considering that all conclusions drawn from any decision model can be influenced by the hypothesis made during its construction, your nephew now wishes to evaluate the sensitivity of the expected profit with respect to certain assumptions in his model. To accomplish this, he generated the Spider diagram and a graph presenting the sensitivity of the expected profit of 3 levels of production (30, 60, and 100 liters) with respect to the expected demand if « sunny ». Answer the following questions using the information presented in those figures.

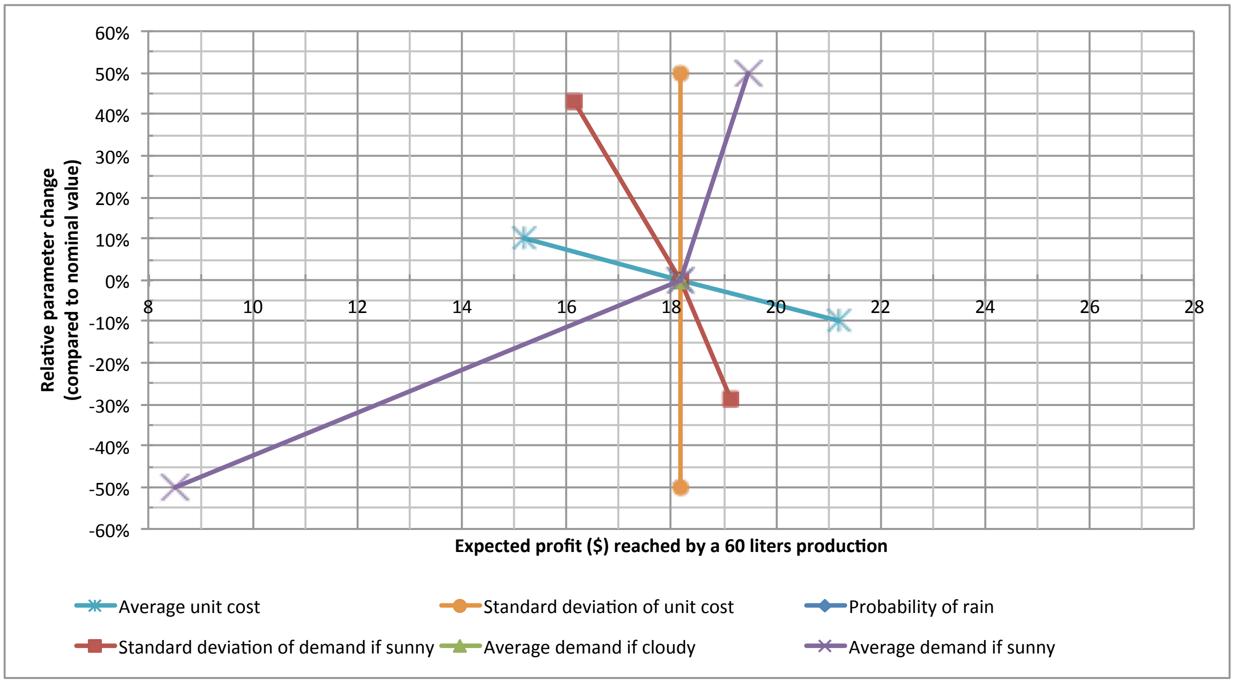


Figure 1: Spider plot

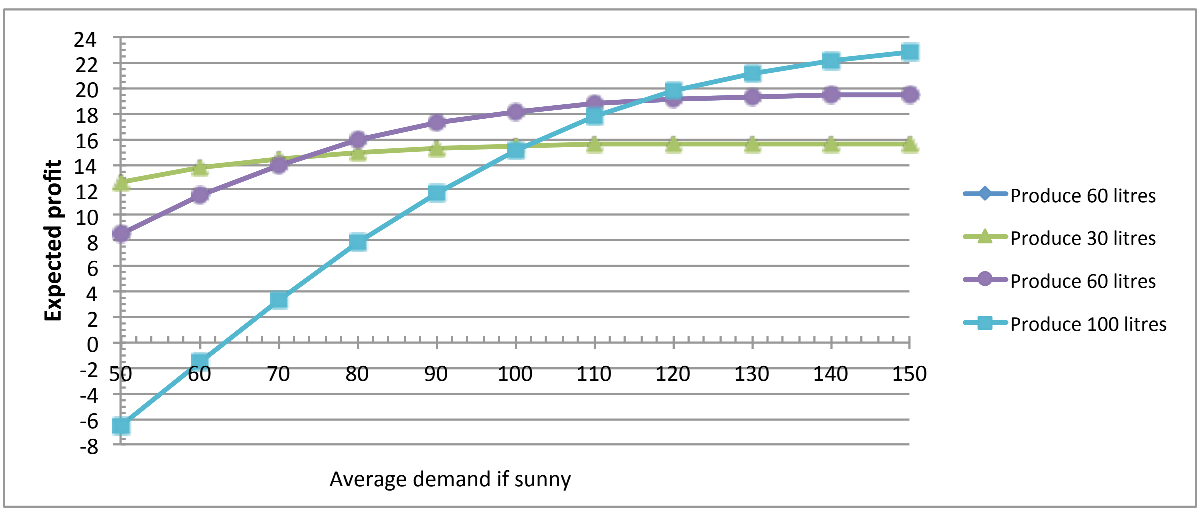


Figure 2: Sensitivity to average demand if sunny

e) Draw the Tornado diagram that should be obtained for this problem. **(3 points)**

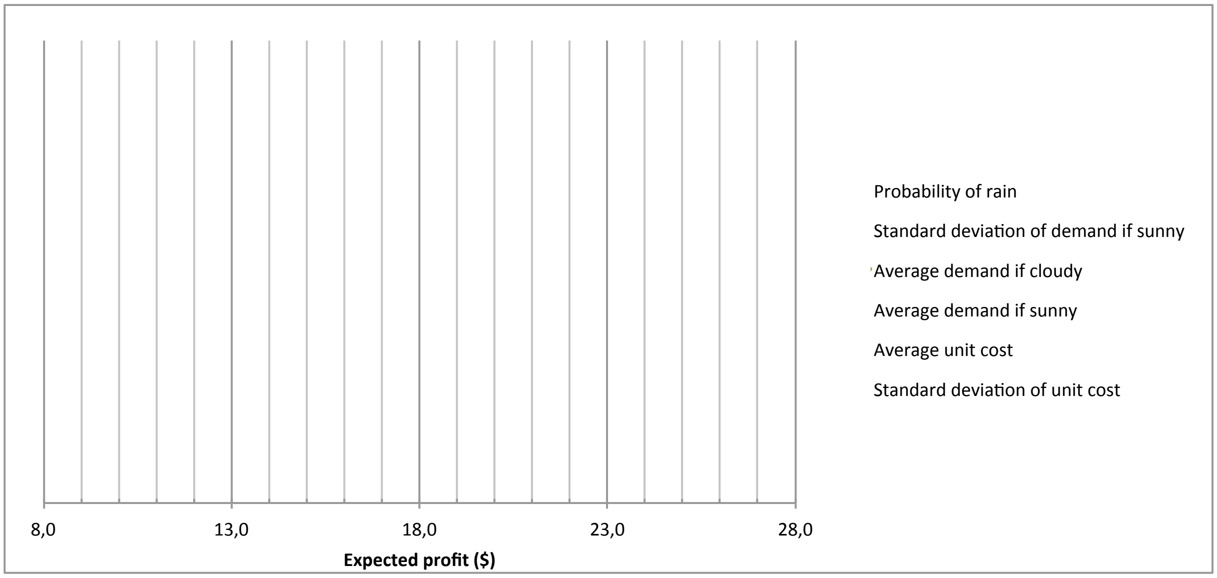


Figure 3: Tornado diagram

f) Using figure 1, determine the chosen confidence interval for the parameters « Standard deviation of demand if sunny » and « Standard deviation of the unit cost ». In the context of a production of 60 liters, explain why a perturbation of the « Standard deviation of demand if sunny » negatively affects the expected profit although it is not the case for the « Standard deviation of the unit cost ». **(4 points)**

Standard deviation of demand if sunny		Standard deviation of the unit cost	
Minimal value	Maximal value	Minimal value	Maximal value

Explanation :

g) Assuming that all other parameters takes their estimated value, according to the sensitivity analysis results, for which values of the « Average demand if sunny » can we say that the expected profit would be greater than 14 $? **(2 points)**

h) Assuming that all other parameters takes their estimated value, according to the sensitivity analysis results, what should be the « average unit cost » for the expected profit to exceed 20 $? **(2 points)**

i) Assuming that all other parameters takes their estimated value, for which values of the « Average demand if sunny » can we say that a production of 60 liters is justified? **(2 points)**

SOLUTION

a) Inverse normal distribution evaluated at 0.85=1-((1-0.7)/2) is equal 1,03643339. The confidence interval is therefore equal to : $\left[16{,}98 - 1{,}0364 \cdot \frac{38{,}58}{\sqrt{40}}, 16{,}98 + 1{,}0364 \cdot \frac{38{,}58}{\sqrt{40}}\right] = [10{,}65, 23{,}30]$.

b) The interval width can be estimated as $2 \cdot 1{,}0364 \cdot \frac{38{,}58}{\sqrt{M}} = \frac{79{,}97}{\sqrt{M}}$, where M is the number of samples. To obtain an interval of width 1, $M = \left(\frac{79{,}97}{1}\right)^2 = 6395{,}2$ samples are required, so 6396 samples.

c) The estimate of the 35th percentile is equal to the value of the $0.35 \cdot 40 = 14^{th}$ sample in the ordered list, so -1,53. To calculate the confidence interval, it suffice to first calculate the Δ : Δ= $1{,}0364 \cdot \sqrt{0.35 \cdot 0.65}/\sqrt{40} = 0{,}0782$. We will find the $40(0{,}35 - 0{,}0782) = 10{,}87 \rightarrow$ 10th index and the $40(0{,}35 + 0{,}0782) + 1 = 18{,}12 \rightarrow$ 19th index. The interval is [-27,31 , 12,63].

d)

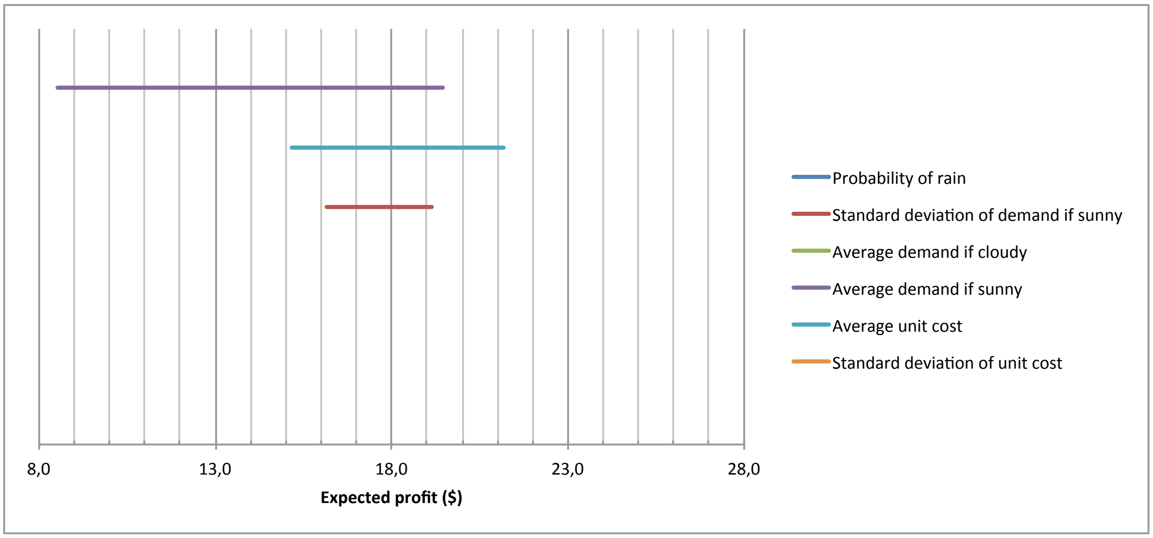
i) We observe 11 cases for which « 60 liters » leads to a greater profit. It is therefore a probability estimated to 11/30=36,67%.

ii) To make an estimate of the confidence level that we have that this frequency is smaller than 50%, we have to return to the model that enables us to establish our confidence intervals. Amongst other things this one states that : $\hat{\theta}_M \geq E[X] - \frac{\Phi^{-1}(1-\alpha)\sigma(X)}{\sqrt{M}}$ with a confidence level of $1-\alpha$. Considering X as the Bernoulli variable that describes the event « The profit is greater with 60 liters than with 30 liters » we obtain that P(« The profit is greater with 60 liters than with 30 liters »)=$E[X] \leq \hat{\theta}_M + \frac{\Phi^{-1}(1-\alpha)\sigma(X)}{\sqrt{M}}$, where $\hat{\theta}_M = \frac{11}{30} = 0{,}3667$ and $\sigma(X) = \sqrt{0{,}3667(1-0{,}3667)^2 + (1-0{,}3667)(0-0{,}3667)^2} = 0{,}49$. We therefore wish that $\hat{\theta}_M + \frac{\Phi^{-1}(1-\alpha)\sigma(X)}{\sqrt{M}} = 0{,}5$. All that remains to be done is isolate $1-\alpha$ in the equation to obtain :

$$1-\alpha = \Phi\left(\left(0{,}5 - \hat{\theta}_M\right)\frac{\sqrt{M}}{\sigma(X)}\right) = \Phi\left(\frac{(0{,}5 - 0{,}3667)\sqrt{30}}{0{,}49}\right) = \Phi(1{,}49) \approx 0{,}935$$

iii) We use a Beta distribution to capture our knowledge that the probability of « 60 liters » leads to a greater profit, i.e. a Bernoulli variable. Following these observations, the Beta distribution will have the parameters $\alpha = 1 + 11 = 12$ and $\beta = 1 + 19 = 20$. The level of confidence that the probability is below 50% is obtained from the distribution function of the Beta distribution.$F_{12,20}(y \leq 50\%) = 92{,}519\%$.

e) Here is the Tornado diagram



Note that the bar for standard deviation of unit cost does not appear because it is of length zero.

f) Standard deviation of demand if sunny :
Minimum = (100%-28%)*35 ≈ 25,2
Maximum = (100%+44%)*35 ≈50,4
Standard deviation of the unit cost :
Minimum = (100%-50%)*0,1 ≈ 0,05
Maximum = (100%+50%)*0,1 ≈ 0,15
Explanation :
For the cost of production, an increase in the standard deviation adds as many scenarios for which the profits increase (realization under the average) as scenarios where the profit diminish (realization above the average). The average effect is therefore null.

Regarding the demand, we first observed that the production of 60 liters is significantly below the average demand. It is therefore the case that for the nominal standard deviation of 35, in most scenarios we are able to sell the whole production. An increase in the standard deviation creates both scenarios of higher demand and scenarios of lower demand compared to an average of 100 liters. The increase in lower demand scenario, leads to more unsold lemonade and therefore to a reduction in profits. This reduction is not compensated by scenarios of higher demand since for those we can only sell the 60 liters.

Additionally, it is interesting to observe that the standard deviation of the production cost doesn't affect the expected profit. We can explain this mathematical phenomena as follows. In the equation that calculates this profit we have :

$$\begin{aligned} E[profit] &= E[revenue - cost] = E[price * \max(demand, production) - unit\ cost * production] \\ &= price * E[\max(demand, production)] - E[unit\ cost] * production \end{aligned}$$

We see that the cost of production affects the profit linearly, so the expected value only depends on the average cost. Regarding the demand, it affects non-linearly the profit, so the expected value will therefore be affected by the standard deviation.

g) In the Spider diagram, we observe that the parameter must be above (100%-20%)*100=80 liters. This is slightly different from what is indicated in figure 2, i.e. 70 liters, because of the fact that the influence of this parameter is non-linear and that a spider diagram presents the linear interpolation of the measured values. The value of 70 liters is therefore the good one here.

h) According to the Spider diagram, the parameter should be smaller than (100%-5%)*0,5=0,475.

i) Based on figure 2, it would be the interval [74, 116].

ANNEX 1

Inverse distribution function for a normal distribution (mean = 0, standard deviation = 1)							
Probability	$F^{-1}(p)$	Probability	$F^{-1}(p)$	Probability	$F^{-1}(p)$	Probability	$F^{-1}(p)$
0,01	-2,32634787	0,26	-0,64334541	0,51	0,02506891	0,76	0,70630256
0,02	-2,05374891	0,27	-0,61281299	0,52	0,05015358	0,77	0,73884685
0,03	-1,88079361	0,28	-0,58284151	0,53	0,07526986	0,78	0,77219321
0,04	-1,75068607	0,29	-0,55338472	0,54	0,10043372	0,79	0,80642125
0,05	-1,64485363	0,30	-0,52440051	0,55	0,12566135	0,80	0,84162123
0,06	-1,55477359	0,31	-0,49585035	0,56	0,15096922	0,81	0,8778963
0,07	-1,47579103	0,32	-0,4676988	0,57	0,17637416	0,82	0,91536509
0,08	-1,40507156	0,33	-0,43991317	0,58	0,20189348	0,83	0,95416525
0,09	-1,34075503	0,34	-0,41246313	0,59	0,22754498	0,84	0,99445788
0,10	-1,28155157	0,35	-0,38532047	0,60	0,2533471	0,85	1,03643339
0,11	-1,22652812	0,36	-0,35845879	0,61	0,27931903	0,86	1,08031934
0,12	-1,17498679	0,37	-0,33185335	0,62	0,30548079	0,87	1,12639113
0,13	-1,12639113	0,38	-0,30548079	0,63	0,33185335	0,88	1,17498679
0,14	-1,08031934	0,39	-0,27931903	0,64	0,35845879	0,89	1,22652812
0,15	-1,03643339	0,40	-0,2533471	0,65	0,38532047	0,90	1,28155157
0,16	-0,99445788	0,41	-0,22754498	0,66	0,41246313	0,91	1,34075503
0,17	-0,95416525	0,42	-0,20189348	0,67	0,43991317	0,92	1,40507156
0,18	-0,91536509	0,43	-0,17637416	0,68	0,4676988	0,93	1,47579103
0,19	-0,8778963	0,44	-0,15096922	0,69	0,49585035	0,94	1,55477359
0,20	-0,84162123	0,45	-0,12566135	0,70	0,52440051	0,95	1,64485363
0,21	-0,80642125	0,46	-0,10043372	0,71	0,55338472	0,96	1,75068607
0,22	-0,77219321	0,47	-0,07526986	0,72	0,58284151	0,97	1,88079361
0,23	-0,73884685	0,48	-0,05015358	0,73	0,61281299	0,98	2,05374891
0,24	-0,70630256	0,49	-0,02506891	0,74	0,64334541	0,99	2,32634787
0,25	-0,67448975	0,50	0	0,75	0,67448975	1,00	∞

ANNEX 2

Cumulative distribution function of the Beta distribution and its inverse for different values of α & β					
α	β	$F_{\alpha,\beta}(0{,}5)$	α	β	$F^{-1}_{\alpha,\beta}(0{,}5)$
31	1	0,00000000	31	1	0,97788854
30	2	0,00000001	30	2	0,94644795
29	3	0,00000023	29	3	0,91467133
28	4	0,00000232	28	4	0,88282171
27	5	0,00001698	27	5	0,85094507
26	6	0,00009610	26	6	0,81905561
25	7	0,00043896	25	7	0,78715908
24	8	0,00166345	24	8	0,75525827
23	9	0,00533692	23	9	0,72335472
22	10	0,01472469	22	10	0,69144930
21	11	0,03537777	21	11	0,65954259
20	12	0,07480639	20	12	0,62763497
19	13	0,14052076	19	13	0,59572672
18	14	0,23656483	18	14	0,56381803
17	15	0,36005007	17	15	0,53190908
16	16	0,50000000	16	16	0,50000000
15	17	0,63994993	15	17	0,46809092
14	18	0,76343517	14	18	0,43618197
13	19	0,85947924	13	19	0,40427328
12	20	0,92519361	12	20	0,37236503
11	21	0,96462223	11	21	0,34045741
10	22	0,98527531	10	22	0,30855070
9	23	0,99466308	9	23	0,27664528
8	24	0,99833655	8	24	0,24474173
7	25	0,99956104	7	25	0,21284092