Generating Synthetic Turbulence in Astrophysical Plasmas

Final Presentation

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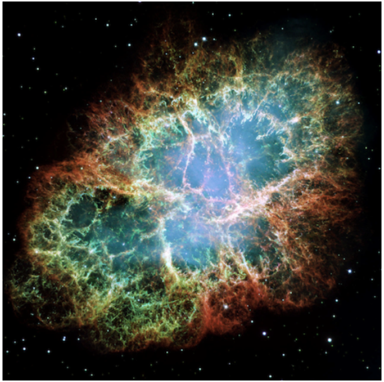
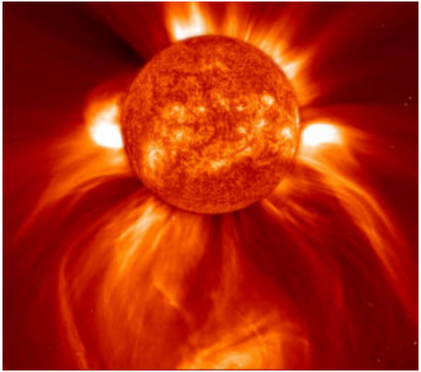
Background & Motivation

Turbulence in Plasmas?

> Supernovae, solar wind, turbulent accretion flows etc.
> Cosmic Ray Scattering and Influences Solar Particles (Yan, Lazarian 2004)





Standard Method

- Studies of turbulence, e.g: Cosmic Ray Diffusion by turbulence or Turbulent Star Formation, are typically done using MHD simulations (Eyink et. al, 2013)
- Takes 0.1-1 million CPU hours
- Want to generate turbulence data cheaply

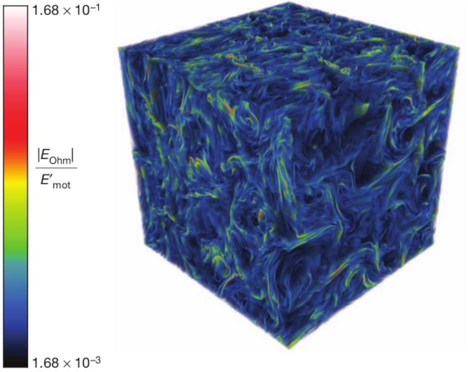


Figure: A 1024^3 point cube of the Electric Field. Electric field magnitude shown by colours: indicates turbulence.

GS95 Theory

Most widely accepted theory of astrophysical turbulence is the Goldreich-Sridhar theory (1995). It predicts a spectrum for the turbulence in 3D:

$$E(k_{\|}, k_{\perp}) = k_{\perp}^{-10/3} \exp\left(-\frac{k_{\|}}{k_{\perp}^{2/3}}\right), \tag{1}$$

where $E^{1/2}$ is the amplitude of the waves, $k_{\perp}$ and $k_{\|}$ are the perpendicular and parallel components of the wavevector respectively w.r.t the local mean magnetic field. The scalar field, Φ, can be initialised in real or k space:

$$\Phi(\underline{r}) = E^{1/2} cos(\underline{k} \cdot \underline{r} + \psi) \iff \widetilde{\Phi}(\underline{k}) = E^{1/2} \exp(i\psi), \tag{2}$$

where ψ is a random phase. Here we are initialising the above spectrum using $k_{\|}$ & $k_{\perp}$ w.r.t. the global mean field. The challenge is to produce a GS95 spectrum w.r.t. the local mean field. For this we describe two methods.

Squares Method

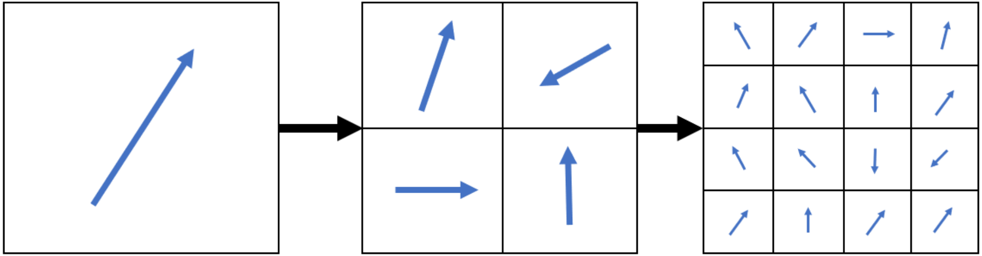


Figure: Domain split up into smaller areas, scalar field set according to local mean magnetic field. Keep splitting until Nyquist Frequency reached.

- This requires some trial and error as the range of wavenumbers used in each domain is rather arbitrary

Displacement Method

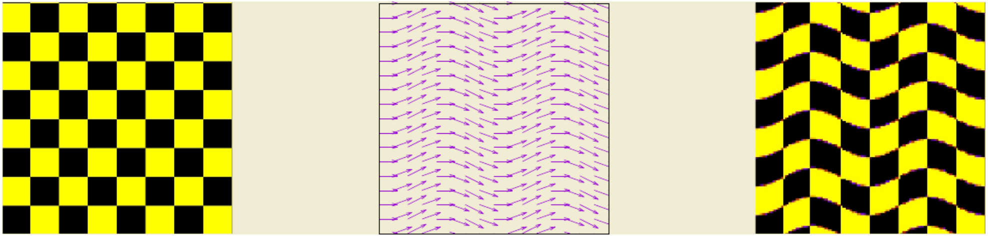


Figure: Given an initial scalar field (left) and a magnetic field (centre), the scalar field should be deformed to follow the magnetic field (right).

MHD induction equation for a perfect conductor: $\frac{\delta \mathbf{B}}{\delta t} = \nabla \times (\mathbf{v} \times \mathbf{B})$, where **B** is the magnetic field and **v** is the velocity. Making an ansatz this becomes:

$$\mathbf{B}^{(i+1)} - \mathbf{B}^{(i)} = \nabla \times (\delta\xi^{(i)} \times \mathbf{B}^{(i)}), \tag{3}$$

where $\delta\xi^{(i)}$ is the spatial displacement. A second ansatz is made: $\delta\xi_*^{(i)} = \delta\mathbf{R}^{(i)} \times \mathbf{B}^{(i)}$, where $\delta\mathbf{R}^{(i)}$ is the displacement field that we desire.

GS95 Spectrum Verification

In both squares & displacement method a magnetic field was setup:

$$B_x(x, y) = 1 \tag{4}$$

$$B_y(x, y) = 0.5sin(2x) \tag{5}$$

When the energy spectra, E, is integrated separately over $k_\perp$ & $k_{||}$ the following relations are found:

$$E(k_\perp) \propto k_\perp^{-5/3}, \; E(k_{||}) \propto k_\perp^{-2} \tag{6}$$

Another measure of performance is the structure function, which depends on the scale of the turbulence in real space: $l_\perp$ and $l_{||}$ w.r.t. the local mean magnetic field. A key result from GS95 is the relation between $k_\perp$ and $k_{||}$:

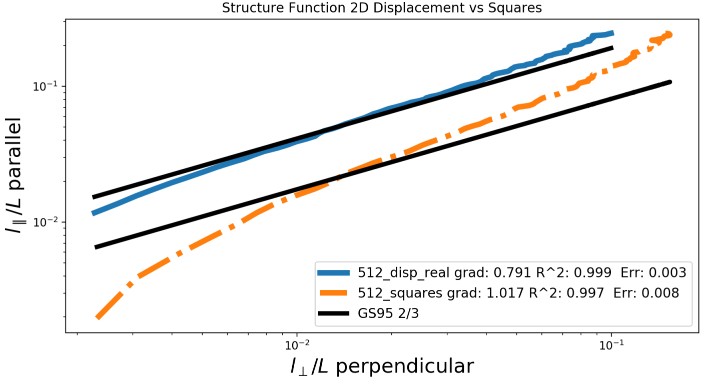
$$k_{||} \propto k_\perp^{2/3}. \tag{7}$$

As $k \propto 1/l$, the same relation applies for $l_\perp$ and $l_{||}$ as used in the structure function, implying a slope of $2/3$ on a logarithmic plot.

2D Real Displacement vs Squares

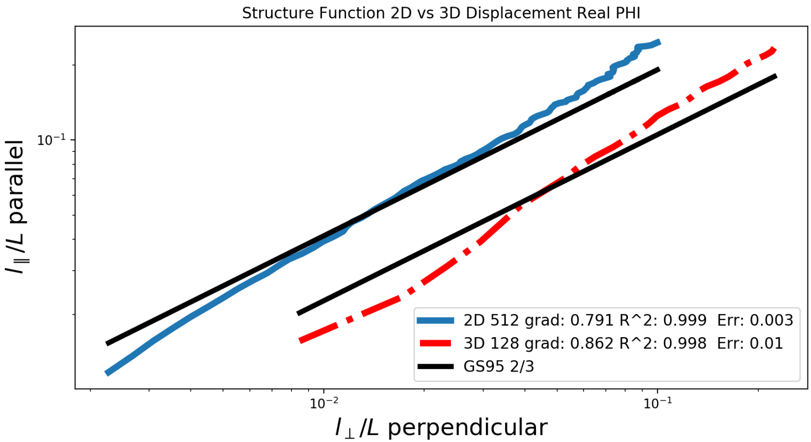
- The structure function w.r.t the local mean magnetic field:

$$SF(l_{\|}, l_{\perp}) = \langle(\mathbf{B}(r + l_{\|}\hat{b}_{\|} + l_{\perp}\hat{b}_{\perp}) - \mathbf{B}(r))^2\rangle \quad (8)$$



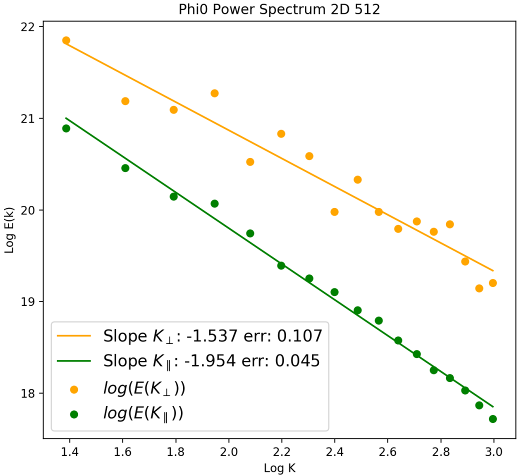
- Squares method less accurate. Displacement method extended to 3D.

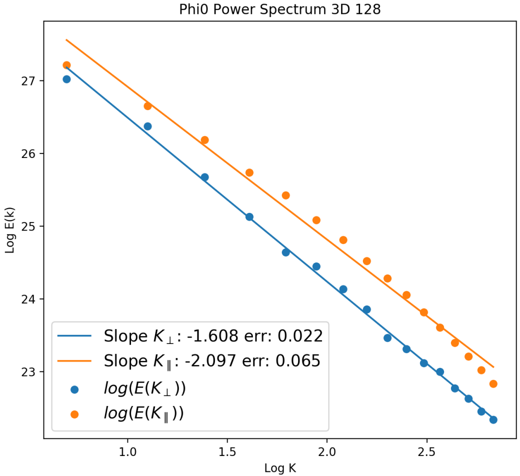
2d vs 3d Real Displacement



> This shows that the 3D method works less accurately but this could be due to the low resolution.

2D & 3D Real Disp. Verification



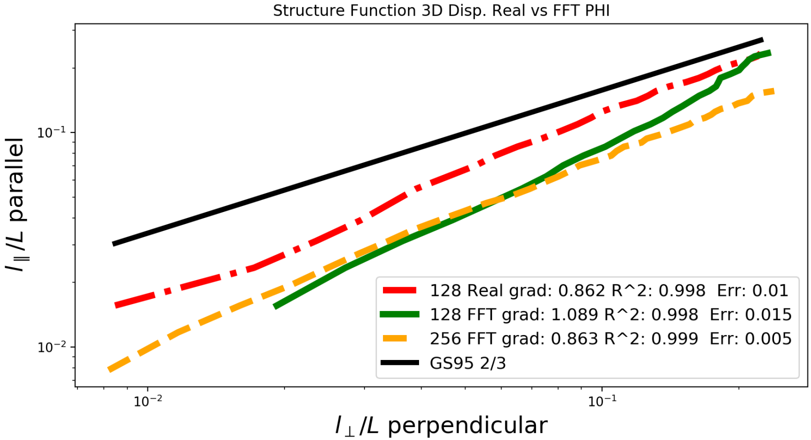


- These show that the phi0 initialisation in real space worked as intended in 2D & 3D.
- As both seem to agree with: $E(k_\perp) \propto k_\perp^{-5/3}$ and $E(k_\parallel) \propto k_\perp^{-2}$

Issue with Time - FFT

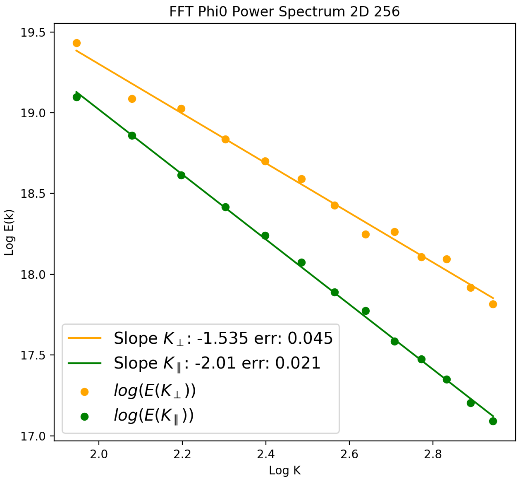
- Displacement method run times:
  - 2d 512 resolution $\sim$ 40 minutes
  - 3d 64 res. $\sim$ 40 minutes
  - 3d 128 res. $\sim$ 43 hours
- Need to speed up the process to create data at a meaningful resolution
- Attempted OpenMP in Fortran - but phi init. not thread safe
- Instead initialise phi in k-space and use inverse fourier transform back to real space
- This reduced the time from 43 hours to 4 minutes for 3d 128

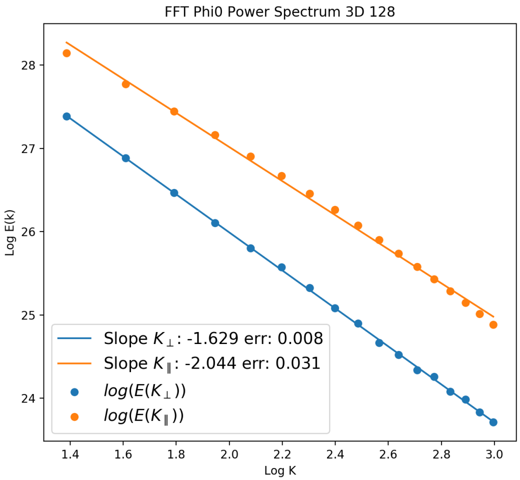
3d Disp. Real vs FFT



- Here the FFT method performs as accurately as the one in real space
- However this plot displays why higher resolutions are required as 128 FFT less accurate

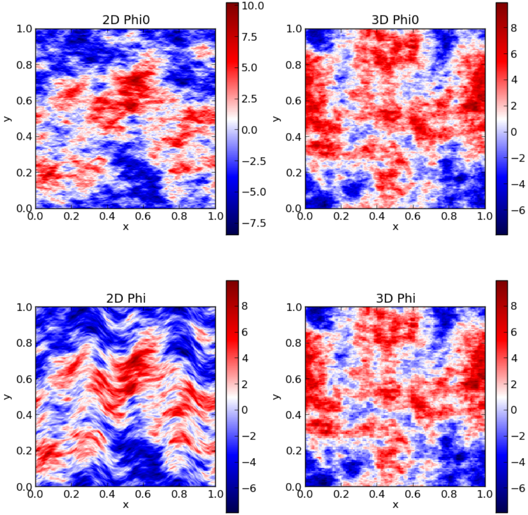
2D & 3D Disp. Verification FFT





- Both show that the phi0 initialisation in k space worked as intended.
- As both seem to agree with: $E(k_\perp) \propto k_\perp^{-5/3}$ and $E(k_\parallel) \propto k_\perp^{-2}$

FFT Disp. Verification



- > An X,Y slice of the Phi0 and Phi arrays
- > In both 2D & 3D added sinusoidal mag. field in Y varying with X
- > In 2D Phi (bottom right) can see displacement method working
- > 3D Phi no sinusoidal displacement seen
- > More investigation required

Conclusion & Further Work

- Displacement performs better than Squares
- Power Spectrum implies displacement method works as intended
- Nevertheless still not perfect: structure function doesn't match theory
- Successfully extended to 3D, but very slow
- Attempted speed up with FFT method - power spectrum implies successful
  - But FFT method still uses too much RAM (45GB for 3D 512 single precision arrays) - need more memory efficiency
  - FFT method also gives less accurate results in 3D phi plots
  - Need closer look at FFT method
- Port methods to python to use MPI4py to safely parallelize the process and access higher resolutions
- Compare results in 3D for varying M_A, (i.e. changing the magnetic field variation)

Thank you!

Contact

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