The Generalised Bohm Criterion For a Flowing Plasma

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Abstract

A paper published by Baalrud and Hegna in 2011, 'Kinetic theory of the presheath and the Bohm Criterion', is reviewed and compared to literature to investigate if it indicates a value of γ for ions. This is important as the value of γ is required to know the velocity at which ions have to be travelling at in order to enter the sheath if the ions are 'warm': i.e. when the ion temperature is not assumed to be zero. The conventional 'Generalised Bohm Criterion', as derived by Riemann, fails for certain ion distributions, for example a shifted Maxwellian which represents a flowing plasma. Therefore a shifted Maxwellian is put into the criterion as derived by Baalrud and Hegna. Their integral definitions for fluid flow (shift) velocity and temperature are checked with numerical methods from Python. They are shown to hold true. Their result implies ions have a γ of one. Other approaches in literature imply values of 5/3 or 3. It could be that Baalrud and Hegna's result is true for large dust grains but more investigation needs to be done to confirm this.

1 Introduction

A plasma is a substance which is ionised to a point where electromagnetic forces dominate all others. When an isolated charge is introduced into a plasma, the plasma counteracts the charge to reduce the potential and move towards an equilibrium. For example if a negative charge is placed inside a plasma, positive charges, namely ions, move to surround the charge and shield it from the rest of the plasma; creating a 'sheath'. This size of the sheath is on the order of a Debye length, a characteristic length that is dependent on the parameters of the plasma: the temperature of the electrons and ions and their respective densities. Thus in the reference frame of an ion several Debye lengths away from the sheath (and not moving too close) it would not experience any forces from the charge and its surrounding sheath.

At the edge of a plasma, when it comes into contact with a surface, a similar sheath is formed. Firstly the ions are assumed to be cold. As electrons are typically moving faster than the ions (due to their lighter mass and higher temperature), they will leave the plasma and charge the surface negatively and so the plasma becomes positive; i.e. the electron flux on the surface is greater than the ion flux. As this charge builds up, more electrons will be reflected and ions attracted. The ions form a sheath, like before, to counteract the negative build up of charge. A steady state is formed once the fluxes are equal, and the particle losses in the plasma are balanced by ionization of neutral atoms[1].

Bohm was the first to derive the condition at the sheath boundary and his derivation made several assumptions [2]. Firstly the electrons have a Maxwellian velocity distribution at a temperature T_e (Kelvin) and the ions are cold. The electrostatic potential is defined as 0 at the sheath edge, where the ion and electron density (which are functions of the potential) are equal. The electron density is also assumed to follow the Boltzmann relation. Finally there are no collisions so ion energy is conserved. Using the ion energy conservation and the continuity of ion flux (as there is no ionization in the sheath) the ion density can be found. This combined with the electron density is substituted into Poisson's equation. It is then multiplied by E and integrated over z(the sheath edge is perpendicular to z). The right hand side of the resulting expression must be positive so a Taylor expansion is performed and an inequality is found:

$$u_s^2 \geq \frac{k_b T_e}{M_i}, \tag{1}$$

where u_s is the ion speed at the sheath edge,M_i is the mass of the ions and k_b is Boltzmann's constant[3]. The mathematical treatment of a sheath at a plasma boundary therefore has revealed that the ions have to be travelling at or above a certain speed to enter the sheath: the Bohm velocity $u_B = \sqrt{k_b T_e / M_i}$. If an ion enters at a speed below this, the sheath will grow until this condition is satisfied. To fulfill this criterion the ions are accelerated by a small but finite electric field in a transitional region from the bulk of the plasma to the edge of the sheath: the quasi-neutral presheath. The width of the presheath, denoted by l, is much greater than the sheath, so if a cross section of the plasma is drawn on the presheath scale, the sheath cannot be resolved. Therefore the sheath and presheath have to be treated separately. In the presheath treatment it is discovered that the ions must be travelling slower than u_B in the presheath, else there is a singularity and the quasineutrality breaks down [1]:

$$u_s^2 \leq \frac{k_b T_e}{M_i} \tag{2}$$

Solutions at the sheath edge are found by matching the sheath and presheath solutions; Eq1 and Eq2 become equalities, so the ions have to be travelling at u_B at the sheath edge.

When the ions are assumed to be cold the Bohm velocity is only a function of electron temperature. What happens when the ions are warm? For example, ions are warm in a tokomak. A useful application for knowing what happens when the ions are warm is the interaction of a dust grain in a tokomak reactor. When there is a plasma in a tokomak reactor, dust grains are scraped off the walls of the reactor and their interaction with the plasma could seriously affect the performance of the reactor. Therefore understanding what happens with these dust grains is an important area of research. In particular for large dust grains (large when comparing the dust grain radius to the Debye length) as then the sheath around such a dust grain could be assumed to be planar.

One would assume that the Bohm velocity is equal to the sound speed at the sheath edge:

$$u_s \geq \left[\frac{k_b T_e + \gamma k_b T_i}{M_i}\right]^{\frac{1}{2}} \tag{3}$$

This includes a $\gamma k_b T_i$ term, where γ is the power from the adiabatic ideal gas law: $PV^{\gamma} = constant$, the ratio of specific heats. The value for γ however is unclear. In this report various approaches and their attempts to find an appropriate value of γ will be presented and discussed.

2 The Generalised Bohm Criterion

First of all, Bohm made several assumptions and Harrison and Thompson were one of the first to remove the assumption on the ion distribution function, which is a function in six-dimensional space (x, y, z, v_x, v_y, v_z) [4]:

$$M_i \langle v^{-2} \rangle^{-1} \geq k_b T_e \tag{4}$$

However, like Bohm, the electron density was assumed to follow the Boltzmann relation. Riemann expanded on Harrison and Thompson's work to generalise it for the electron distribution functions as well[5][6]:

$$\frac{1}{M_i} \int \frac{f_i(v)}{v_z^2} d^3v \leq -\frac{1}{m_e} \int \frac{\partial f_e(v)}{\partial v_z} d^3v, \tag{5}$$

where f_i and f_e are the ion and electron distribution functions respectively. The sheath boundary is perpendicular to the z direction. This is well known as the 'Generalised Bohm Criterion' and is widely accepted in literature. However the distribution functions for the ions and electrons in industrial cases, such as in tokomaks, are not normally known so this does little to help. Furthermore this condition doesn't work for all distribution functions. If the ion distribution function is finite at $v = 0$ the integrand on the left hand side becomes singular and the condition breaks down as the integral diverges to infinity. It is also pointed out by Baalrud and Hegna that the derivation that leads to Riemann's condition has two issues[7]. First of all, in the derivation of the 'conventional' Bohm Criterion, the collision operator is neglected in Vlasov's equation. This is only valid if the plasma is in equilibrium, which implies that the electrons and ions have identical Maxwellian distributions. This would cause the left hand side to diverge and leads to a contradiction. Moreover to get the left hand side of the inequality, integration by parts is used, however the function on which integration by parts is used, $(1/v_z)\partial f_s/\partial v_z$ is not continuously differentiable[8].

3 Baalrud and Hegna

3.1 Derivation

In 2011 Baalrud and Hegna published a paper titled "Kinetic theory of the presheath and the Bohm criterion". In this they claim to have derived a general Bohm criterion without making any assumptions of the distribution functions of the species in the plasma nor assuming the ions were cold. Baalrud and Hegna used the same sheath criterion used in the derivation for the conventional 'Generalised Bohm Criterion'. The derivation of which is shown below[7]:

Firstly, Poisson's equation is expanded about the sheath edge where $\Phi(Potential) = 0$:

$$\nabla^2\Phi = -4\pi[\rho(\Phi = 0) + \frac{d\rho}{d\Phi}\Big|_{\Phi=0}\Phi + ...], \tag{6}$$

where ρ is the charge density. The problem is one dimensional and the first non-vanishing term of the expansion is:

$$\frac{d^2\Phi}{dz^2} = -4\pi\frac{d\rho}{d\Phi}\Big|_{\Phi=0}\Phi \tag{7}$$

This is multiplied by $\frac{d\Phi}{dz}$ and integrated with respect to z:

$$\int\frac{d\Phi}{dz}\frac{d^2\Phi}{dz^2}dz = \int -4\pi\frac{d\rho}{d\Phi}\Big|_{\Phi=0}\Phi\frac{d\Phi}{dz}dz \tag{8}$$

This becomes:

$$(\frac{d\Phi}{dz})^2 + 4\pi\frac{d\rho}{d\Phi}\Big|_{\Phi=0}\Phi^2 = constant \tag{9}$$

Since $E = -\nabla\Phi = -\frac{d\Phi}{dz}$, where E is the electric field:

$$\frac{E^2}{4\pi} + \frac{d\rho}{d\Phi}\Big|_{\Phi=0}\Phi^2 = constant \tag{10}$$

In the limit that $\Phi \to 0$ and $z/\lambda_D \to -\infty$ the constant is zero[9](λ_D is the Debye length) . This is done in the sheath length scale. This can also be realised as when $\Phi = 0$, $E = 0$. It is then rearranged:

$$\frac{d\rho}{d\Phi}\Big|_{\Phi=0} = -\frac{E^2}{4\pi\Phi^2} \tag{11}$$

This implies that:

$$\frac{d\rho}{d\Phi}\Big|_{\Phi=0} \leq 0 \tag{12}$$

Since $dn_s/d\Phi = -(1/E)dn_s/dz$ and $\rho = \sum_s n_s q_s$ (where subscript s denotes a species: ions or electrons):

$$\sum_s \frac{q_s}{E}\frac{dn_s}{dz}\Big|_{z=0} \leq 0 \tag{13}$$

This is the sheath criterion.

Now Baalrud and Hegna start their derivation from the kinetic equation for a species s

$$\frac{\partial f_s}{\partial t} + \underline{v}\cdot\frac{\partial f_s}{\partial\underline{x}} + \frac{q_s}{m_s}\underline{E}\cdot\frac{f_s}{\partial\underline{v}} = C(f_s), \tag{14}$$

where $\underline{v}$ is the velocity vector and $C(f_s)$ is a collision operator. The density moment of the equation is

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial\underline{x}}\cdot(n_s\underline{V_s}) = 0, \tag{15}$$

where $\underline{V_s}$ is the fluid flow velocity of a species s. The momentum moment is

$$m_S n_s(\frac{\partial\underline{V_s}}{\partial t} + \underline{V_s}\cdot\frac{\underline{V_s}}{\partial\underline{x}}) = n_s q_s\underline{E} - \frac{\partial p_s}{\partial\underline{x}} - \frac{\partial}{\partial\underline{x}}\cdot\Pi_s + \underline{R_s}, \tag{16}$$

where m_s is the mass of the species, p_s is the scalar pressure defined as n_sT_s(where the temperature is in electron volts), Π_s is the stress tensor and R_s is the frictional force density. Now the following assumptions are made: the plasma is in steady state, only spatial variation in f_s due to the E field in the presheath and sheath and the E field is in the $\hat{z}$ direction: the problem has become one dimensional as the sheath is planar. Therefore the density moment, Eq.23 becomes a continuity equation

$$n_s\frac{dV_{z,s}}{dz}+V_{z,s}\frac{dn_s}{dz}=0, \tag{17}$$

where $V_{z,s}$ is the fluid flow velocity of a species s in the $\hat{z}$ direction. The momentum moment, Eq.16, becomes:

$$m_sn_sV_{z,s}=n_sq_sE-\frac{dp_s}{dz}-\frac{\Pi_{zz,s}}{dz}+R_{z,s} \tag{18}$$

Substituting the continuity equation, Eq.17, into the sheath criterion, Eq.13, yields:

$$\sum_s\frac{q_sn_s}{EV_{z,s}}\frac{dV_{z,s}}{dz}\bigg|_{z=0}\leq 0 \tag{19}$$

The momentum equation, Eq.18, is then substituted into Eq.19:

$$\sum_s q_s\left[\frac{q_sn_s-(n_sdT_s/dz+d\Pi_{zz,s}/dz-R_{z,s})/E}{m_sV_{z,s}^2-T_s}\right]_{z=0}\leq 0 \tag{20}$$

The temperature derivative, stress tensor and frictional force density are higher order moments. So the equation is not closed. However to achieve closure Baalrud and Hegna make several, somewhat questionable, assumptions. Firstly they state: "In many plasmas...the temperature and stress moments vary on spatial scales much longer than the Debye length." They go on to add that these scales are "typically on the order of the presheath length scale." On top of that that frictional force density term is controlled by collisions so is on the collision length scale[10]. They add that: "the gradient length scale of the electrostatic potential approaches the Debye length at the sheath edge." Therefore the terms in the parentheses are $\mathcal{O}(\lambda_D/l)\ll 1$ smaller than the q_sn_s term. Finally they state that many presheath models are based on a limit of $\lambda_D/l\to 0$ so the E field at the sheath edge becomes infinite[7]. Therefore Eq.20 reduces to:

$$\sum_s\frac{q_s^2n_s}{m_sV_{z,s}^2-T_s}\bigg|_{z=0}\leq 0 \tag{21}$$

For one species of singly charged ions and neglecting the electron fluid flow velocity the condition, in fluid variables, becomes:

$$V_{z,i}\geq\left[\frac{T_e+T_i}{M_i}\right]^{\frac{1}{2}} \tag{22}$$

3.2 Results and Discussion

Baalrud and Hegna have defined the fluid variables from their final criterion in terms of velocity-space integrals of the distribution function[7]. Firstly the distribution functions are normalised to the density

$$n_s=\int d^3vf_s, \tag{23}$$

the fluid flow velocity

$$V_s=\frac{1}{n_s}\int d^3v\underline{v}f_s, \tag{24}$$

and temperature

$$T_s=\frac{1}{n_s}\int d^3v\frac{1}{3}m_s\underline{v_r}^2f_s=\frac{1}{2}m_sv_{T_s}^2, \tag{25}$$

where v_{T_s} is the most probable velocity or 'thermal velocity' and v_r is a relative velocity defined as

$$\underline{v_r}=\underline{v}-\underline{V_s} \tag{26}$$

This implies that Eq.22 in integral form is:

$$\frac{1}{n_i}\int d^3v\underline{v_z}f_i\geq\left[\frac{1}{3n_i}\int d^3v\underline{v_r}^2f_i+\frac{1}{3n_e}\frac{m_e}{M_i}\int d^3v\underline{v_r}^2f_e\right]^{\frac{1}{2}} \tag{27}$$

Now considering a shifted Maxwellian for the ions and a stationary Maxwellian for the electrons, it is not evident that the fluid flow velocity and temperature integrals for these distribution functions output the real shift and temperature of the function. The integrals for the ion fluid flow velocity at the sheath edge and the temperature for both species were solved to check their definitions. Firstly, for a stationary Maxwellian, the electron temperature integral is relatively straightforward to solve analytically. A stationary Maxwellian distribution in velocity space is:

$$f_e(v_x, v_y, v_z) = \frac{n_e}{\pi^{\frac{3}{2}}}\Big(\frac{m_s}{2T_e}\Big)^{\frac{3}{2}} \exp\Big(-(v_x^2+v_y^2+v_z^2)\frac{m_s}{2T_s}\Big) \tag{28}$$

Using the fact the thermal velocity definition from Eq.25, this is rewritten as:

$$f_e(v_x, v_y, v_z) = \frac{n_e}{\pi^{\frac{3}{2}} v_{T_e}^3} \exp\Big(-\frac{v_x^2+v_y^2+v_z^2}{v_{T_e}^2}\Big) \tag{29}$$

Then a coordinate transformation to spherical polars is made including the Jacobian. This leaves:

$$f_e(v, \phi, \theta) = \frac{n_e}{\pi^{\frac{3}{2}} v_{T_e}^3} \exp(-\frac{v^2}{v_{T_e}^2}) v^2 \sin\theta \tag{30}$$

The triple integral for the electron temperature then becomes:

$$\int_0^\infty \int_0^{2\pi} \int_0^\pi \frac{m_e}{3\pi^{\frac{3}{2}} v_{T_e}^3} \exp\Big(-\frac{v^2}{v_{T_e}^2}\Big) v^4 sin\theta d\theta d\phi dv \tag{31}$$

The integrals over θ and ϕ are straightforward and the integral over v is solved using a standard result to give

$$\frac{1}{2} m_e v_{T_e}^2 \equiv T_e \tag{32}$$

A flowing Maxwellian distribution for the ions is shown below:

$$f_i(v_x, v_y, v_z) = \frac{n_e}{\pi^{\frac{3}{2}} v_{T_e}^3} \exp\Big(-\frac{v_x^2+v_y^2+(v_z-u)^2}{v_{T_e}^2}\Big), \tag{33}$$

where u is the drift velocity in the z direction. This was also changed to spherical coordinates but solved numerically using the python scipy.integrate.dblquad method. The temperature of the function was set to 5eV and the calculated temperature is shown below in Figure 2

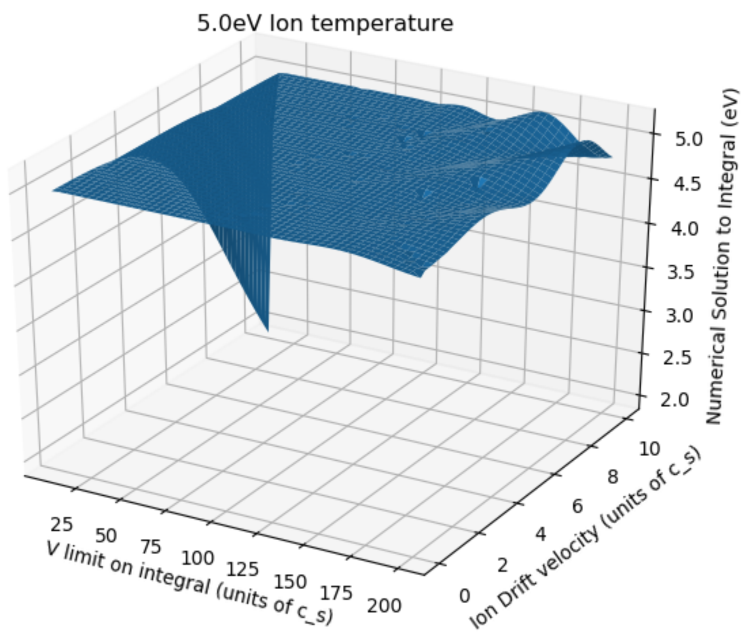

Figure 1: 3D plot of Ion Temperature at 5eV, electrons at 1eV, varying the drift velocity and upper limit on the v integral, both normalised with respect to the cold ion Bohm speed.

The cold ion Bohm speed is:

$$c_s = \sqrt{\frac{T_e}{M_i}}, \tag{34}$$

where T_e is in electron volts. The ion drift velocity and upper limit on the v integral are varied and the calculated temperature is along the z axis. One expects a flat plane in $x - y$ and this is displayed. The reason why there is a drop at high drift velocities and low v integral limits is because a significant portion of the function is outside this limit. Furthermore it is noticed that at higher limits there is a larger deviation from $5eV$. This could originate from the dblquad method used, it uses the trapezium rule to solve the integral numerically. The greater the width of the integral limits the wider the segments the function is split into, leading to a larger error in the result.

The fluid flow velocity (shift/drift), in the z direction for the ion distribution is also calculated using the python dblquad method:

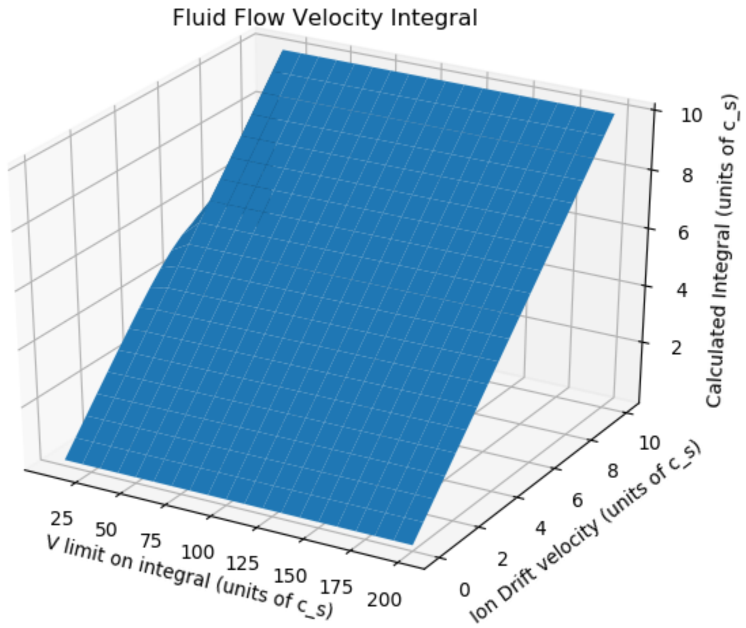

Figure 2: 3D plot of Ion fluid flow varying the drift velocity and upper limit on the v integral, both normalised with respect to the cold ion Bohm speed.

As shown the integral gives the expected result. Like the ion temperature, a similar dip is noticed at high drift and low v integral limit. However the same deviation from the true result at higher integral limits is not found. This could be because the integrand is different and therefore the same inaccuracy from the dblquad method doesn't have an effect. The right hand side of the integral form, Eq27, is then computed and plotted for a varying ion temperature at a fixed fluid flow velocity of $10c_s$, with electrons at 1eV throughout.

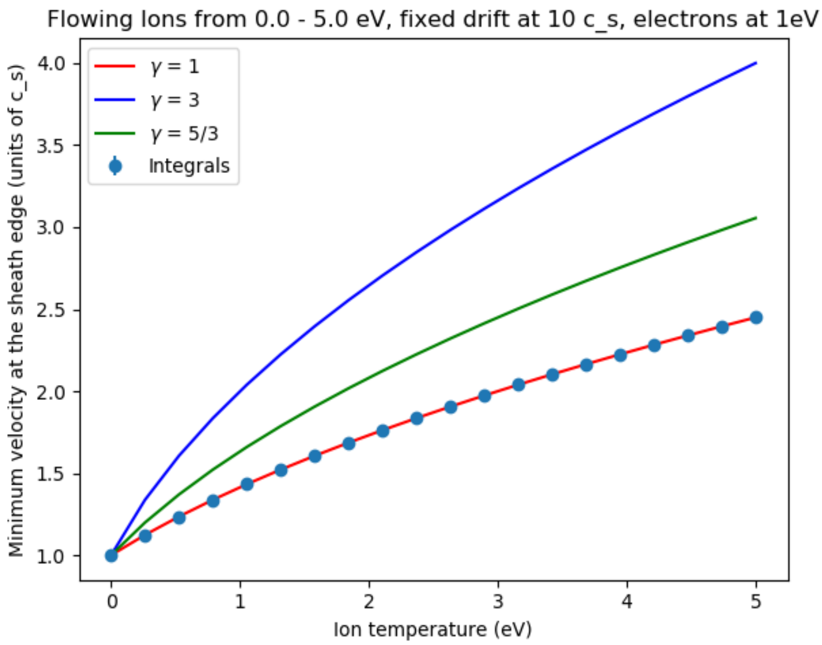

Figure 3: Plot of RHS of integral inequality compared with γ of 1,5/3 and 3

The integrals are shown to match the fluid variable equation, Eq22, which has a γ of one, so the velocity-space integral definitions hold true. Therefore Eq27 works for functions that aren't Maxwellian, as typically temperature is only defined for Maxwellian distribution functions. The temperature from their integral definition leads to a physically sensible result that can be used in the formula: Eq22. The result from Baalrud and Hegna is quite surprising. This result, eq.22, implies that ions have a γ of one. This factor, γ is also equal to $1 + \frac{2}{n_d}$, where n_d is the number of degrees of freedom. It is also the ratio of specific heats: c_p/c_v. Electrons, due to their very light mass, move around with high velocities so they are considered to be isothermal. Note that if a system is isothermal, it is equivalent to the adiabatic law with a γ of one: $PV = constant$. The electrons therefore have infinite degrees of freedom. The result from Baalrud and Hegna's work therefore implies that the ions are also isothermal and have infinite degrees of freedom. This is surprising as one would have expected that their heavier mass would inhibit their ability to have infinite degrees of freedom. There has been little other indication that the value of γ for ions is one.

Nevertheless the assumptions made after Eq20 are somewhat dubious. For example using the fact that presheath models often have the electric field to be infinite at the sheath edge highlights an important point. In the derivation of the sheath criterion, Eq13, the electric field is instead assumed to be 0 at the sheath edge. This highlights the nature of the sheath and the presheath as they are on such different length scales that at the same point in space in one length scale E is 0 and in the other it is infinite. To attempt to investigate what happens at their mutual boundary the two length scales are matched. On top of that by ignoring the spatial gradient in temperature they are stating that the temperature varies so little at that point that it is isothermal, which could be why isothermal ions are found as the result. They also state that the assumption that these terms vary on the presheath scale is only in "many plasmas" there are plasmas in which these assumptions are invalid, rendering their condition untrue.

The work by Baalrud and Hegna could be true in the case of very large dust grains. The reason for this is that if the dust grain is not large compared to the Debye length a third length scale must be considered in the mathematical treatment: the radius of the dust grain. A third length scale could invalidate all the assumptions Baalrud and Hegna made after Eq20. This is speculation however and more work needs to be done to investigate this conjecture.

4 Elsewhere in Literature

4.1 S Kuhn, K-U Riemann

Comparing the work by Baalrud and Hegna to similar work done by S Kuhn et al (2006) and K-U Riemann (1995), who found the 'Generalised Bohm Criterion' from section2, it is noticed that they use a different definition for γ[11][9].

Their definition of γ arises from this gradient of the pressure, the 'local polytropic law':

$$\nabla p \equiv \nabla(nkT) \equiv \gamma kT \nabla n \tag{35}$$

where this temperature is in Kelvin, leads to $\gamma(\underline{r}, t)$, the 'local polytropic coefficient' being defined as:

$$\gamma(\underline{r}, t) \equiv 1 + \frac{n}{T}\frac{dT}{dn} \tag{36}$$

Note that this gamma is a function of $\underline{r}, t$ so it varies over space and time unlike the γ as used in the ion sound speed. In the appendix of the paper by Kuhn et al they claim that this coefficient is analogous to the other γ:

$$\gamma(\underline{r}, t) \equiv \frac{c_p - c}{c_v - c}, \tag{37}$$

where c_p is the specific heat at constant pressure and c_v is the specific heat at constant volume. These two γ are therefore equal when c is zero. If the 'local polytropic law' is used in Baalrud and Hegna's derivation in Eq.18, then Eq.20, with temperature in electron volts, becomes:

$$\sum_s q_s \left[\frac{q_s n_s - (d\Pi_{zz,s}/dz - R_{z,s})/E}{m_s V_{z,s}^2 - \gamma(\underline{r})T_s}\right]_{z=0} \leq 0 \tag{38}$$

Note that there is no longer a spatial derivative in the temperature, it has been absorbed into this alternative $\gamma(\underline{r})$. If the same assumptions are made, i.e. that E is infinite, the stress moment and frictional force density vary on the presheath length scale and one species of singly charged ions, the ion sound speed result is found:

$$V_{z,i} \geq \left[\frac{T_e + \gamma(\underline{r})T_i}{M_i}\right]^{\frac{1}{2}}, \tag{39}$$

with temperature in electron volts. There is no temporal dependence as earlier on in the derivation the assumption is made that the plasma is in steady state. This then gives no indication to the value of γ for the ions.

K-U Riemann also made the statement that Harrison-Thompson's result, Eq.4, is only compatible with fluid analysis with a γ of 3[9]. However it also stated that if in the fluid model it is known that $\gamma \neq 3$ it is incorrect to use $\gamma = 3$ in the kinetic analysis[9].

4.2 Stangeby

A fluid analysis approach was carried out using two slightly different ion source terms by Stangeby[12]. The fluid equations are particle, energy and momentum conservation. The electrons are assumed to follow the Boltzmann relation. The two ion source solutions used were formulated by Emmert et al[13] and Bissell and Johnson[14][15]. The sound speed is found by finding the point at which the fluid equations breakdown due to a singularity, when the derivatives are equal to zero, as that is where the sheath edge is defined. First in the adiabatic, collisionless case a value of 3 was found for γ. In the adiabatic strongly collisional case $\gamma = 5/3$ was found. For intermediate collisionality the fluid model became far more complex and only for the case where $T_e = T_i$ was a combined ion and electron γ indicated: 8/3. If $\gamma_{electron} = 1$, then it could be said that this indicates the γ for ions is 5/3.

4.3 C Willis

In stationary plasmas, Willis compared SCEPTIC[16] (a Particle-in-Cell, code) with a modified OML (orbital-motion-limited) model for large dust grains (radius of 100 Debye lengths). A Particle-in-Cell code simplifies simulating all the particles in a plasma, as there are typically 10^8 particles per cubic metre, by representing many 'real' particles as one macro-particle. Moreover the electromagnetic fields are calculated on a grid. OML is a charging model that calculates the potential of a dust grain in a plasma. The potential is calculated by SCEPTIc and compared to OML models with varying values of γ. Willis states that $\gamma = 5/3$ is the closest value however the figure showing this comparison is not convincing[17].

SCEPTIC was also compared to a modified SOML (Shifted Orbital Motion Limited) for a flowing plasma. Again the floating potential is calculated with SCEPTIC and compared to modified SOML models with varying values of γ. Willis states that $\gamma = 5/3$ is the most appropriate but again the fit is not convincing[17].

Finally, Willis calculated the Mach cones around a large dust grain due to the flowing plasma with warm ions. Similar to how air creates a cone when an object travels supersonically, a double cone is made around a dust grain in a supersonic plasma flow. This cone denotes the presheath around the dust grain. If the flow of the ions is greater than Bohm criterion then there is no presheath required to accelerate them. Therefore the warm ion Bohm speed can be found at the point where there is no presheath upstream of the dust grain. This situation is simulated by Willis and found persuasive evidence that implied that the value of $\gamma = 3$[17].

5 Conclusion

In conclusion a definitive value for γ for ions was not found. The integral definitions by Baalrud and Hegna were checked and shown to be true. Their work implies that the value of γ for warm ions is one, which is quite a surprising result as this means the ions have infinite degrees of freedom like the electrons even with their heavier mass. There is a possibility that the work by Baalrud and Hegna is valid for very large dust grains, so that the length scale of the dust grain radius can be ignored. Nevertheless this is speculation and must be further investigated. It was discovered that the γ as used by Kuhn and Riemann was defined differently, it was only analogous to the regular definition as a ratio of specific heats in a certain case. If this definition was used in Baalrud and Hegna's derivation the ion sound speed would be re-derived leading to no indication about its value for warm ions. Riemann also stated that only $\gamma = 3$ in a kinetic approach was compatible with fluid analysis.

There are also indications that γ takes several other values. Willis found compelling evidence to suggest that $\gamma = 3$ by investigating the Mach cones, from a supersonic flowing plasma, around a large dust grain. He also found $\gamma = 5/3$ by comparing the potential calculated by SCEPTIC to what OML and SOML models predicted for stationary and flowing plasmas. However the graphs that show this are not convincing. Stangeby on the other hand carried out a fluid analysis approach and found that $\gamma = 3$ in the collisionless case and $\gamma = 5/3$ in the strong and intermediate collsionality regime. It must be noted that the result for the intermediate collisionality was only found in the case where the electron and ion temperatures were equal.

6 Bibliography

References

[1] M Coppins, *Low Temperature Plasmas and Industrial Applications*

[2] D Bohm, *Minimum Ionic Kinetic Energy for a Stable Sheath*, in *The Characteristics of Electrical Discharges in Magnetic Field*, edited by A. Guthrie and R.K. Wakerling, Ch. 3, 77 (McGraw-Hill, New York, 1949)

[3] M Lieberman, A Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, Ch. 6, 156-158 (1994)

[4] E R Harrison and W B Thompson Proc. Phys. Soc. 74 145 (1959)

[5] K-U Riemann Phys. Fluids 24 2163 (1981)

[6] K-U Riemann J. Phys. D: Appl. Phys. 24 493 (1991)

[7] S D Baalrud and C C Hegna Plasma Sources Sci. Technol. 20 025013 (2011)

[8] Fernsler R F, Slinker S P and Joyce G Phys. Rev. E 71 026401 (2005)

[9] K-U Riemann IEEE Trans. Plasma Sci. 23 709 (1995)

[10] R N Franklin J. Phys. D: Appl. Phys. 36 2821 (2003)

[11] S Kuhn et al Phyiscs of Plasmas 13 013503 (2006)

[12] P C Stangeby, *The Plasma Boundary of Magnetic Fusion Devices*, Ch.10 p416-420 (2000)

[13] A G Emmert et al Phys. Fluids 23 803 (1980)

[14] Bissell et al Phs. Fluids 30 779 (1987)

[15] Bissell et al Phys. Fluids B 1 1133 (1989)

[16] Open Source SCEPTIC code: https://github.com/plasmasim/sceptic

[17] C Willis, *Dust in Stationary and Flowing Plasmas* (2012)