Compressive Sensing with Applications to Near-Field to Far-Field Antenna Pattern Extrapolation

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Outline

Introduction Project Overview

- **Compressive Sensing (CS):** A signal processing technique for recovering sparse signals from undersampled measurements.
- **Problem Statement:** Near-field to Far-field Antenna Pattern Extrapolation.
- **Project Goal:** Apply compressive sensing methods to improve antenna measurement techniques through efficient data acquisition and accurate field reconstruction.
- **Supervisors:** Prof. Dirk Slock, Dr. Zilu Zhao, and Dr. Fangqing Xiao.

Sparse Bayesian Learning (SBL)

- **Core Idea:** Inducing sparsity in weight vectors through an evidence maximization over a parameterized Gaussian prior.
- **Observation Model:** $\mathbf{t} = \Phi \mathbf{w} + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$.
- **Prior:** $p(\mathbf{w}; \gamma) = \prod_{i=1}^{M} (2\pi\gamma_i)^{-\frac{1}{2}} \exp(-\frac{w_i^2}{2\gamma_i})$.
- Hyperparameters γ inferred via Type-II maximum likelihood.

EM-Based SBL and its Scalability

- **EM Algorithm:** Alternates between E-step (posterior computation) and M-step (hyperparameter update).
  - E-step: $\mathbf{\Sigma}_w = \left(\beta\Phi^\top\Phi + \mathrm{diag}(\gamma^{-1})\right)^{-1}$, $\boldsymbol{\mu}_w = \beta\mathbf{\Sigma}_w\Phi^\top\mathbf{t}$.
  - M-step: $\gamma_i = \mu_{w,i}^2 + (\Sigma_w)_{ii}$.
- **Limitation:** High computational cost due to $D \times D$ matrix inversion in each iteration.
- **Covariance-Free EM (CoFEM):**
  - Avoids explicit posterior covariance computation.
  - Estimates statistics using linear systems and Rademacher probe vectors.

Stein's Unbiased Risk Estimate (SURE)

- **Stein's Lemma:** Relates expectation of a function of a Gaussian variable to its derivative.
- **SURE Principle:** Provides an unbiased estimator for the risk ($\mathbb{E}\|\boldsymbol{\mu} - \hat{\boldsymbol{\mu}}\|_2^2$) without knowing the true mean $\boldsymbol{\mu}$.
- **Application:** Enables hyperparameter optimization by minimizing the SURE.

$$\hat{\lambda} = \arg\min_{\lambda \in \Lambda} \left(\|\mathbf{y} - \hat{\boldsymbol{\mu}}_\lambda\|_2^2 + 2\sigma^2 \sum_{i=1}^{n} \frac{\partial \hat{\mu}_{\lambda,i}}{\partial y_i}(\mathbf{y}) \right)$$

Synthetic Data Generation

- Generated data using a standard compressive sensing model: $\mathbf{t} = \Phi\mathbf{w} + \mathbf{e}$.
- Components:
  - Measurement vector $\mathbf{t} \in \mathbb{R}^N$.
  - Sensing matrix $\Phi \in \mathbb{R}^{N \times D}$ (e.g., DFT matrix or Gaussian random matrix).
  - Sparse signal $\mathbf{w} \in \mathbb{R}^D$ with user-defined sparsity ρ.
  - Additive Gaussian noise $\mathbf{e} \sim \mathcal{N}(0, \sigma^2)$.
- Setup allows control over N, D, ρ, and σ for reproducible evaluation.

Choice of Algorithm: SBL over AMP

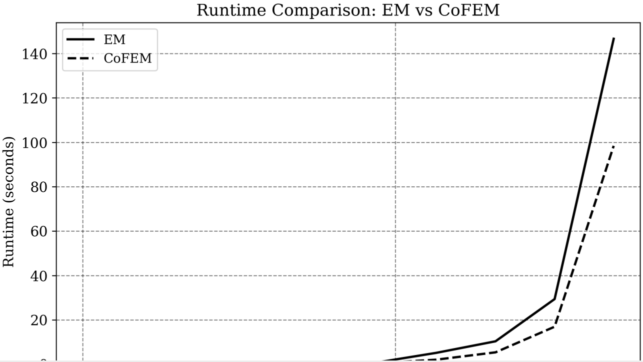
- **Approximate Message Passing (AMP):** Powerful but relies heavily on i.i.d. sub-Gaussian measurement matrices.
- **Project Context:** Application to antenna pattern extrapolation involves structured, non-random measurement matrices (e.g., FFT bases).
- **Decision:** Chose EM-based SBL for its robustness and generality, making fewer assumptions about the measurement matrix.

Fast Marginal Likelihood Algorithm (Tipping's Algorithm)

- Efficient basis selection and pruning strategy for SBL.
- Optimizes hyperparameters via Type II Maximum Likelihood.
- **Iterative Process:**
  1. Initialize hyperparameters and noise variance.
  2. Compute posterior statistics.
  3. Update hyperparameters by adding, deleting, or re-estimating basis functions to maximize marginal likelihood.
  4. Iterate until convergence, guided by SURE for MSE minimization.
- Handles both real and complex-valued inputs.

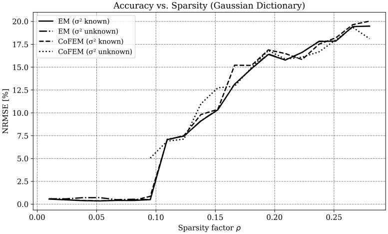
EM vs. CoFEM Runtime Comparison

- CoFEM significantly reduces runtime compared to standard EM, especially with increasing dimensionality.
- For N=500, CoFEM achieved 98.4 seconds vs. EM's 146.6 seconds.

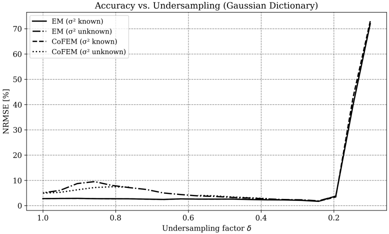


Accuracy Comparison: Gaussian Matrices

- EM and CoFEM exhibited nearly identical reconstruction accuracy in Gaussian measurement systems.
- CoFEM with unknown noise variance performed on par with variants assuming known noise level.



(A) NRMSE vs. sparsity level ρ.



(B) NRMSE vs. undersampling factor δ.

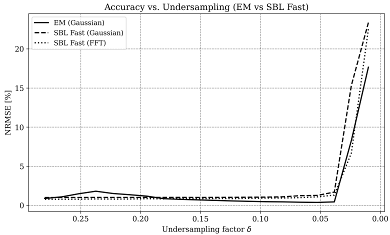
Figure: Reconstruction accuracy for EM and CoFEM algorithms with

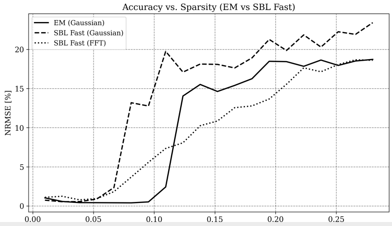
Challenges with Fourier Matrices (EM/CoFEM)

- EM algorithm showed unstable behavior and sensitivity to initialization for complex-valued (FFT-based) dictionaries.
- Efforts to adapt EM for this case were discontinued due to unreliability.

SBL-SURE (Tipping's FML) Performance

- Demonstrated robust performance across both Gaussian and complex-valued systems.
- Accuracy in Gaussian systems closely matched EM's, with EM showing marginal improvements.
- **Key Finding:** In complex-valued systems (Fourier matrices), Tipping's method outperformed its own Gaussian results, suggesting better adaptation to structured dictionaries.





Conclusion

- Successfully investigated and applied compressive sensing techniques for antenna pattern extrapolation.
- Implemented and evaluated various sparse learning algorithms (EM, CoFEM, SBL-SURE/FML).
- SBL with SURE (Tipping's FML algorithm) proved to be the most robust and efficient method, particularly for structured (Fourier) measurement matrices.
- This work demonstrates the potential of CS methods in improving antenna measurement techniques.

Future Work

- Reformulate EM algorithm to reduce computational complexity for high-dimensional problems (e.g., using Woodbury identity).
- Explore further optimization of algorithms for specific antenna models.
- Validate findings with real-world antenna measurement data.
- Extend the framework to other types of electromagnetic problems.

Questions? Thank you!