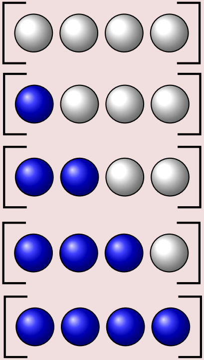
Chapter 2

The setup

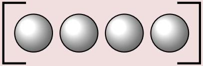
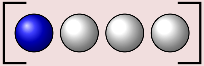
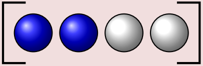
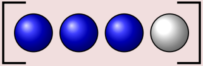
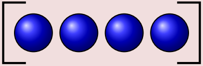
A priori five possibilities, all equally likely:

Possible content



The prior

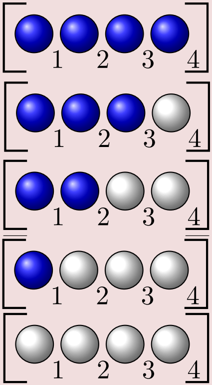
A priori five possibilities, all equally likely:

Possible content	prior count
	1
	1
	1
	1
	1

The likelihood

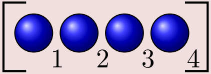
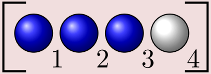
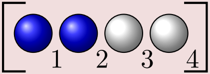
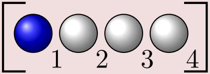
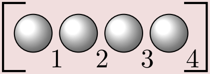
Which unique marble correspond to a white marble?

Possible content

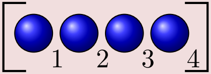
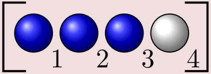
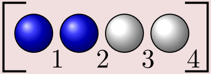
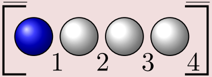
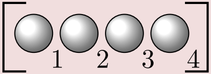


The likelihood

Which unique marble correspond to a white marble?

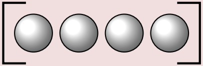
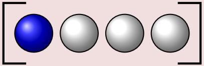
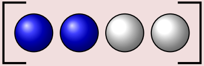
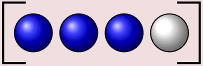
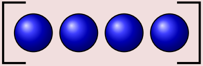
Possible content	ways to produce a white marble
	0
	1
	2
	3
	4

The likelihood

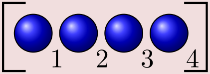
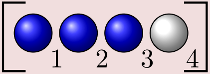
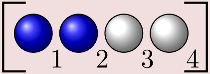
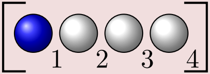
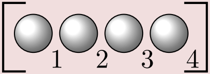
Possible content	white	blue
	0	4
	1	3
	2	2
	3	1
	4	0

Prior again

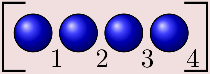
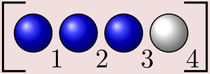
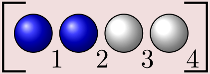
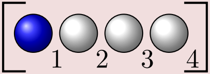
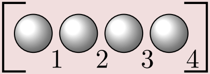
A priori five possibilities, all equally likely:

Possible content	$\mathbb{P}(\cdot)$
	$\frac{1}{5}$
	$\frac{1}{5}$
	$\frac{1}{5}$
	$\frac{1}{5}$
	$\frac{1}{5}$

Likelihood again

Possible content	$\mathbb{P}(white \mid content)$	$\mathbb{P}(blue \mid content)$
	$\frac{0}{4}$	$\frac{4}{4}$
	$\frac{1}{4}$	$\frac{3}{4}$
	$\frac{2}{4}$	$\frac{2}{4}$
	$\frac{3}{4}$	$\frac{1}{4}$
	$\frac{4}{4}$	$\frac{0}{4}$

Likelihood again

Possible content	$\mathbb{P}(white \mid content)$	$\mathbb{P}(blue \mid content)$
	$\frac{0}{4}$	$\frac{4}{4}$
	$\frac{1}{4}$	$\frac{3}{4}$
	$\frac{2}{4}$	$\frac{2}{4}$
	$\frac{3}{4}$	$\frac{1}{4}$
	$\frac{4}{4}$	$\frac{0}{4}$

Bayes Formula

- $\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$.

Figure : Thomas Bayes

Bayes Formula

- $\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$.
- $\mathbb{P}(A|B) \propto \mathbb{P}(B|A)\mathbb{P}(A)$, (this is often hard to grasp)

Figure : Thomas Bayes

Proportional to, $\propto$

- A function is proportional to an other function if it is equal up to a constant depend on the argument of the function.

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- For $f(x) = x$:

$$
\begin{aligned}
f(x) &\propto 2x, \\
f(x) &\propto 4x, \\
f(x) &\propto cx, \text{ if } c > 0.
\end{aligned}
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\end{aligned}
$$

- For $f(x) = x$:

$$
\begin{aligned}
f(x) &\not\propto x^2, \\
f(x) &\not\propto x(x+1), \\
f(x) &\not\propto -x.
\end{aligned}
$$

- In Bayesian statistics important since it preserve ratios (odds).

Prior through math

- Let p be the probability of drawing a white marble.

Prior through math

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- The, marble bag, prior is equivalent to

$$\mathbb{P}(p) = \begin{cases} \frac{1}{5} & \text{for } p = \frac{0}{4}, \\ \frac{1}{5} & \text{for } p = \frac{1}{4}, \\ \frac{1}{5} & \text{for } p = \frac{2}{4}, \\ \frac{1}{5} & \text{for } p = \frac{3}{4}, \\ \frac{1}{5} & \text{for } p = \frac{4}{4}. \end{cases}$$

This is a probability distribution of the random probability, p of drawing a white marble.

The likelihood in math

- We denote, X, the random variable

$$X = \begin{cases} 1 & \text{if a white marble is drawn,} \\ 0 & \text{if a blue marble is drawn.} \end{cases}$$

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$$X = \begin{cases} 1 & \text{if a white marble is drawn,} \\ 0 & \text{if a blue marble is drawn.} \end{cases}$$

- The probability distribution of X given a p is

$$\mathbb{P}(X|p) = p^X(1-p)^{1-X}.$$

This is the likelihood.

The posterior in math

- After observing n independent draws, $x_1, x_2, \ldots, x_n$ one get

$$\mathbb{P}(p|x_1, \ldots, x_n) \propto \mathbb{P}(p) \prod_{i=1}^{n} \mathbb{P}(x_i|p)$$

The posterior in math

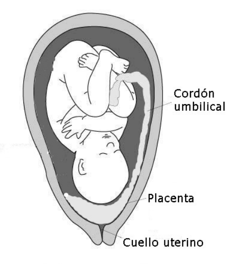
- After observing n independent draws, $x_1, x_2, \ldots, x_n$ one get

$$\mathbb{P}(p|x_1,\ldots,x_n) \propto \mathbb{P}(p)\prod_{i=1}^{n}\mathbb{P}(x_i|p)$$

- Putting in the prior we get

$$\mathbb{P}(p|x_1,\ldots,x_n) \propto \begin{cases} \frac{1}{5}\left(\frac{0}{4}\right)^{\sum_{i=1}^n x_i}\left(1-\frac{0}{4}\right)^{n-\sum_{i=1}^n x_i} & \text{for } p=\frac{0}{4}, \\ \frac{1}{5}\left(\frac{1}{4}\right)^{\sum_{i=1}^n x_i}\left(1-\frac{1}{4}\right)^{n-\sum_{i=1}^n x_i} & \text{for } p=\frac{1}{4}, \\ \frac{1}{5}\left(\frac{2}{4}\right)^{\sum_{i=1}^n x_i}\left(1-\frac{2}{4}\right)^{n-\sum_{i=1}^n x_i} & \text{for } p=\frac{2}{4}, \\ \frac{1}{5}\left(\frac{3}{4}\right)^{\sum_{i=1}^n x_i}\left(1-\frac{3}{4}\right)^{n-\sum_{i=1}^n x_i} & \text{for } p=\frac{3}{4}, \\ \frac{1}{5}\left(\frac{4}{4}\right)^{\sum_{i=1}^n x_i}\left(1-\frac{4}{4}\right)^{n-\sum_{i=1}^n x_i} & \text{for } p=\frac{4}{4}. \end{cases}$$

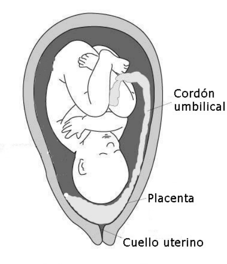
Placenta praevia



Placenta praevia
Grade IV

- Placenta praevia, is a condition occurring in $0.4 - 0.5\%$ of all labors. It is due to that the placenta is in the lower part uterus.

Placenta praevia



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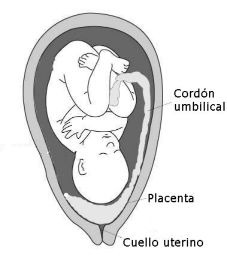

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- We study the probability, p, of a female baby given the mother had Placenta praevia.
- What is a reasonable prior for p?

Posterior of p

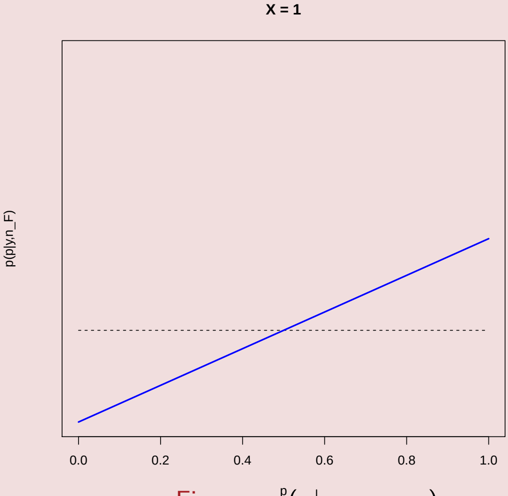
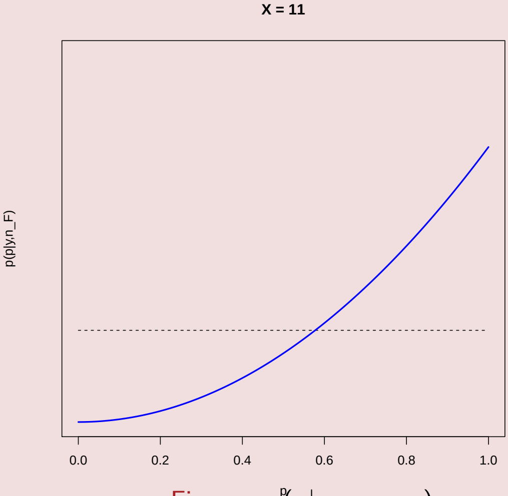


Figure : $p(p|x_1, \ldots, x_n)$

Posterior of p



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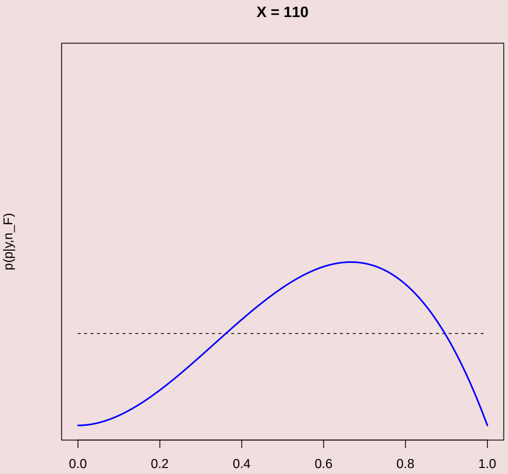

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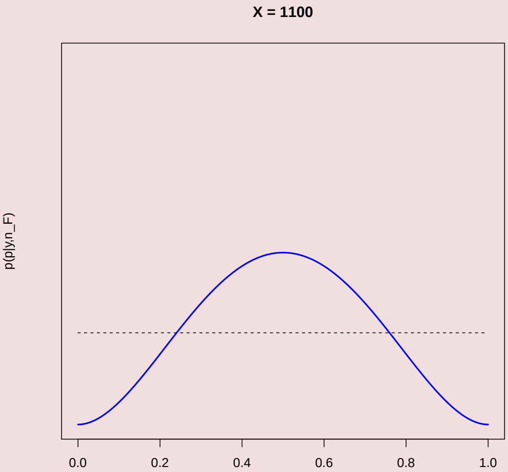

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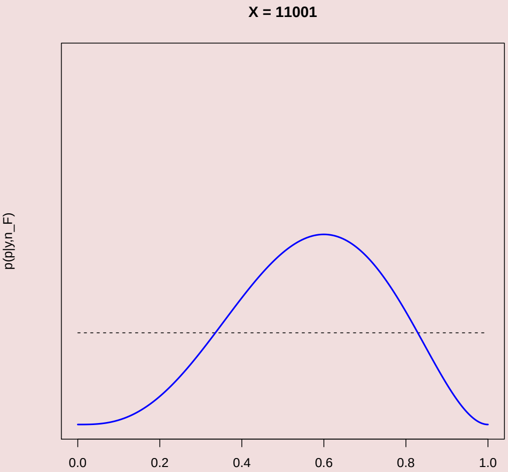

Figure : $p(\overset{p}{p}|x_1, \ldots, x_n)$

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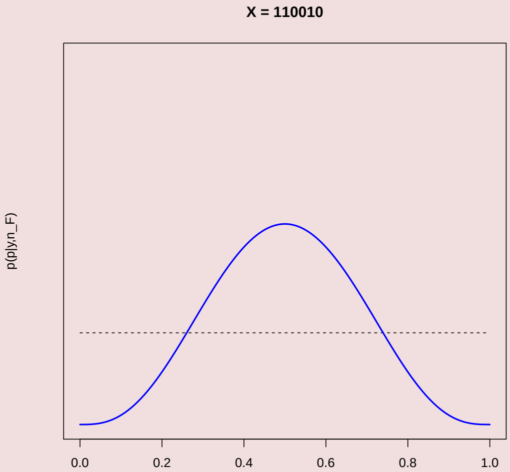

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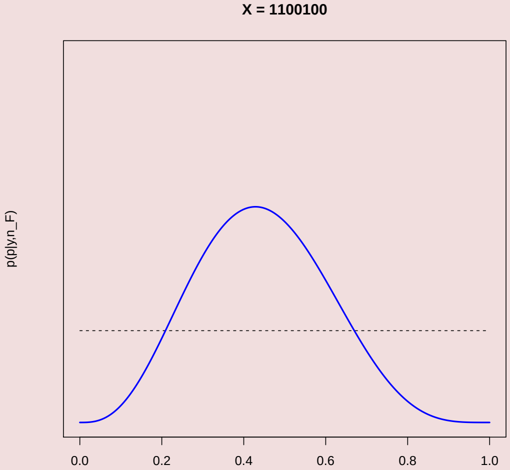

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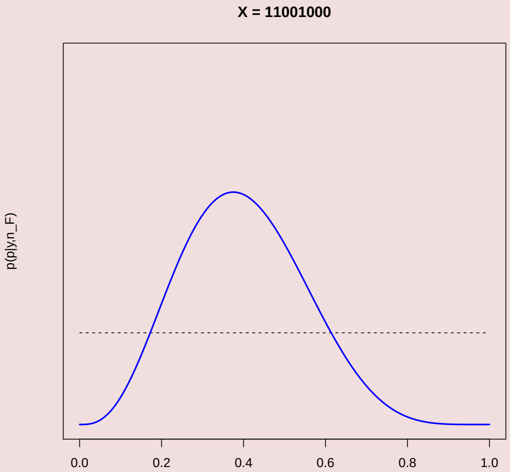

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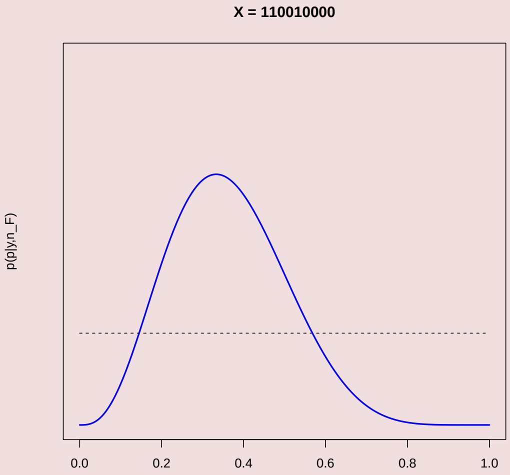

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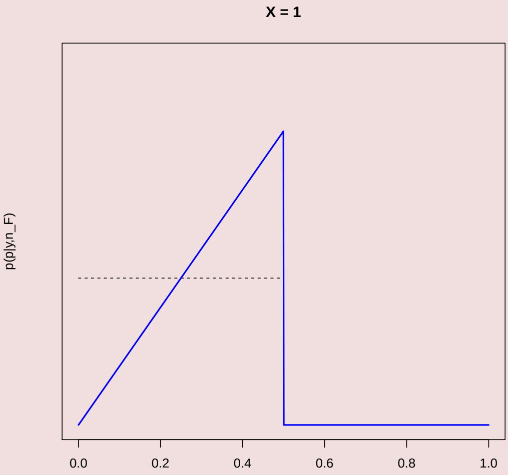

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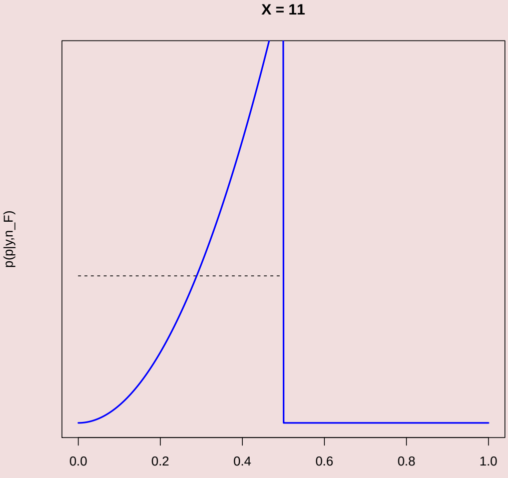

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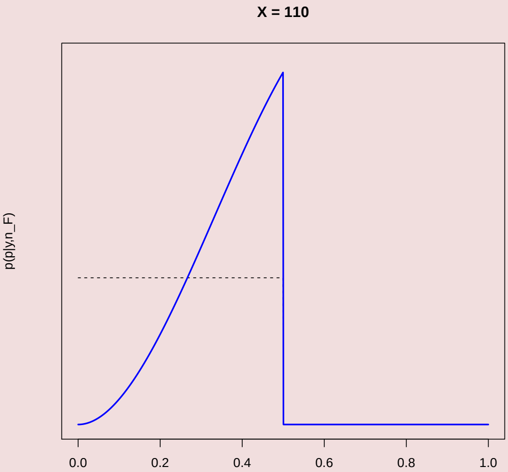

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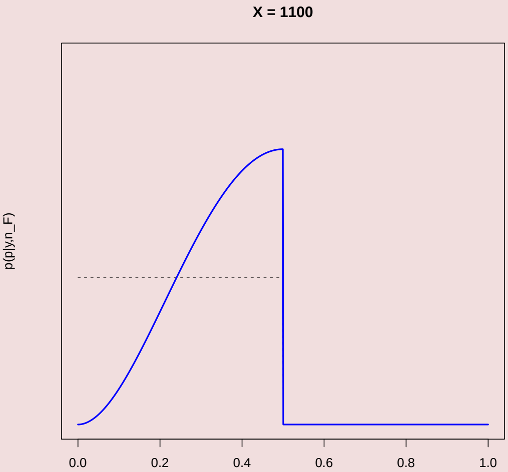

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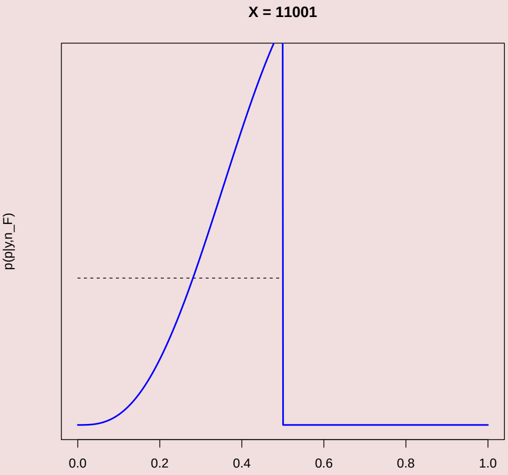
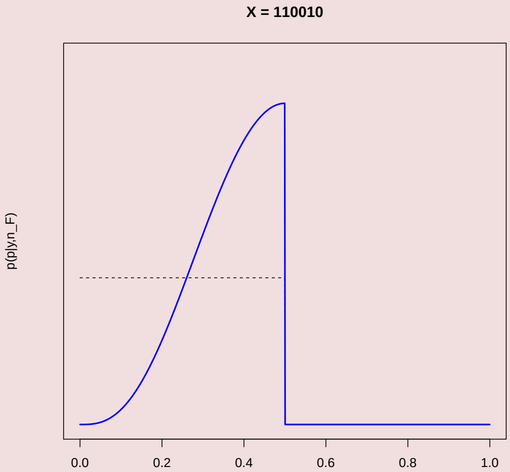


Figure : $p(\overset{\text{p}}{p}|x_1, \ldots, x_n)$

Posterior of p



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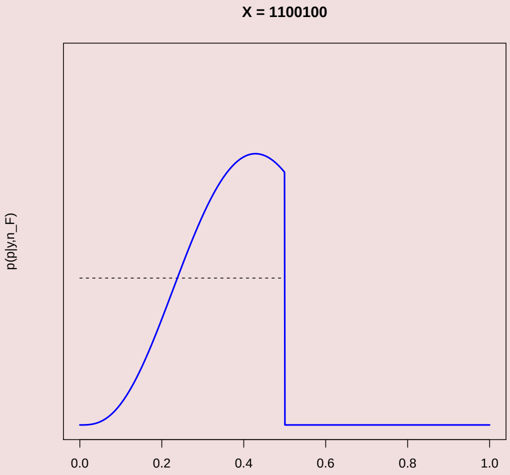

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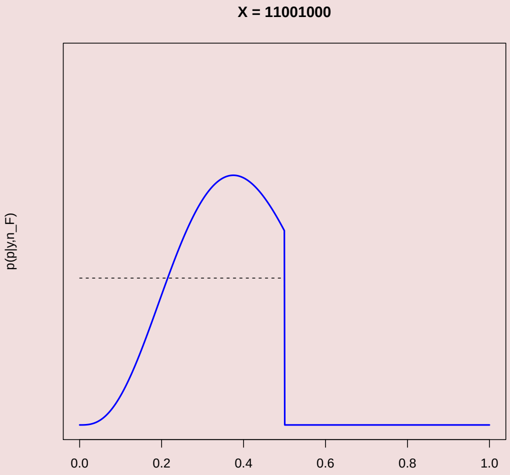

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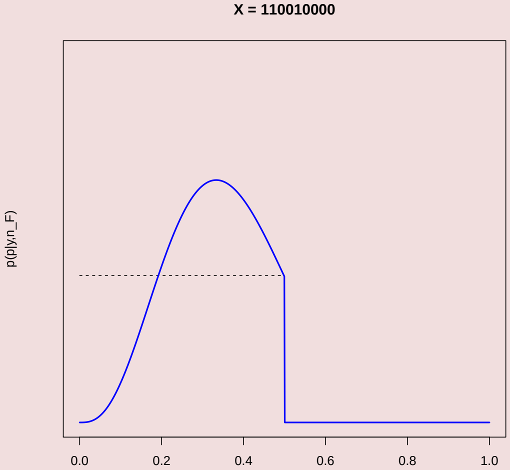
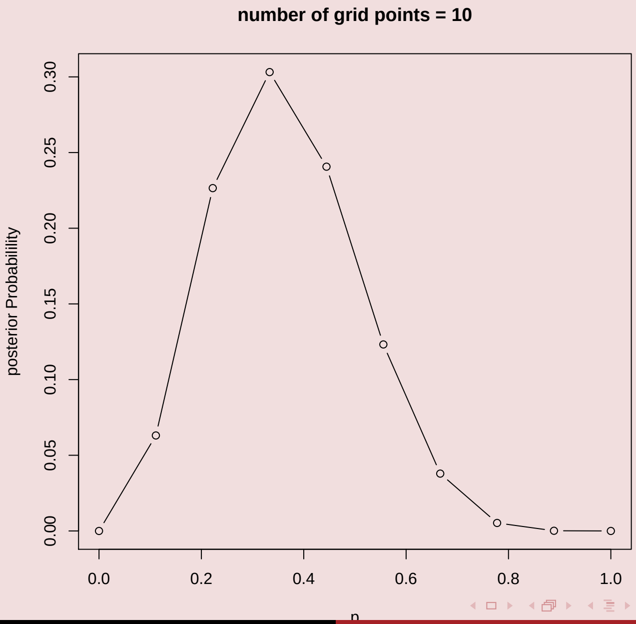


Figure : $p(p|x_1, \ldots, x_n)$

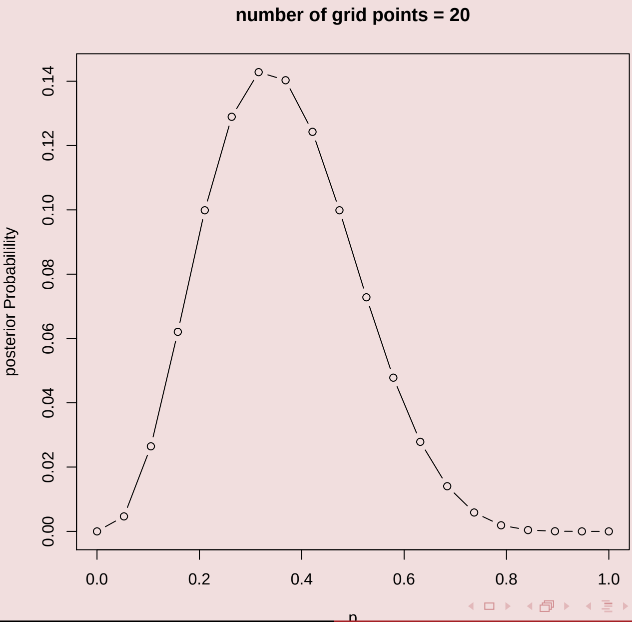
Producing the Figure in R, grid approximation

```
n_grid <- 10 # how fine grid
p_grid <- seq( from  =0, to =1 , length.out = n_grid)

#prior
prior <- rep(1, n_grid)

#posterior
likelihood <- dbinom(x =3, size= 9, prob = p_grid)

# unnormalized posterior
unstd.posterior <- likelhiood * prior

# normalized posterior
porsterior <- unstd.posterior  / sum(unstd.posterior )

plot(p_grid, posterior, type='b', ylab='posterior Probabilility', xlab='p',
     main = paste("number of grid points = ", n_grid, sep = ""))
```


number of grid points = 10
posterior Probabililty
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.2
0.4
0.6
0.8
1.0
p

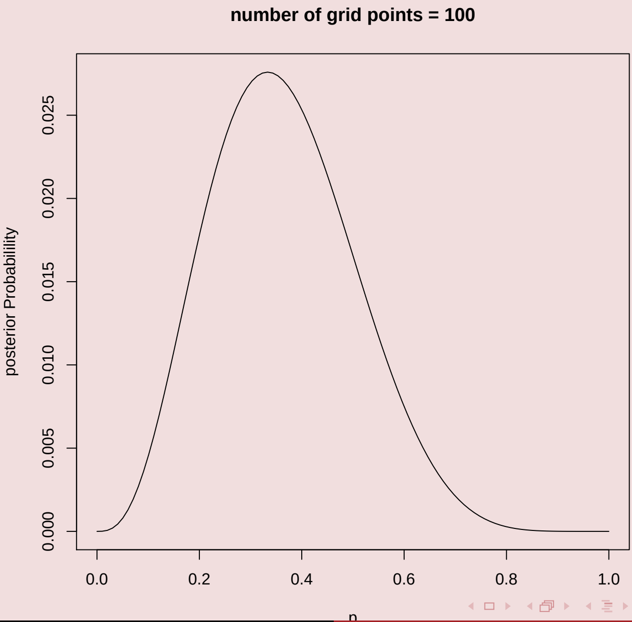
Producing the Figure in R, grid approximation

```
n_grid <- 20 # how fine grid
p_grid <- seq( from  =0, to =1 , length.out = n_grid)

#prior
prior <- rep(1, n_grid)

#posterior
likelihood <- dbinom(x =3, size= 9, prob = p_grid)

# unnormalized posterior
unstd.posterior <- likelhiood * prior

# normalized posterior
porsterior <- unstd.posterior  / sum(unstd.posterior )

plot(p_grid, posterior, type='b', ylab='posterior Probabilility', xlab='p',
main = paste("number of grid points = ", n_grid, sep = ""))
```


number of grid points = 20
posterior Probabilility
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.0
0.2
0.4
0.6
0.8
1.0
p

Producing the Figure in R, grid approximation

```
n_grid <- 100 # how fine grid
p_grid <- seq( from  =0, to =1 , length.out = n_grid)

#prior
prior <- rep(1, n_grid)

#posterior
likelihood <- dbinom(x =3, size= 9, prob = p_grid)

# unnormalized posterior
unstd.posterior <- likelhiood * prior

# normalized posterior
porsterior <- unstd.posterior  / sum(unstd.posterior )

plot(p_grid, posterior, type='l', ylab='posterior Probabilility', xlab='p',
main = paste("number of grid points = ", n_grid, sep = ""))
```


number of grid points = 100
posterior Probabilility
0.000
0.005
0.010
0.015
0.020
0.025
0.0
0.2
0.4
0.6
0.8
1.0
p