Chapter 3

Inference from posterior distribution

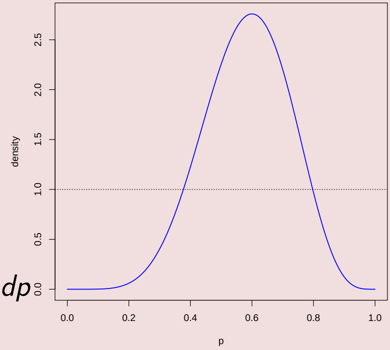
- The posterior distribution

$$p(p|n,y) \propto p^y(1-p)^{n-y}\mathbb{I}_{[0,1]}(p)$$

.

- What is the probability that p is less then 0.5:

$$\mathbb{P}(p<0.5|n,y) = \int_0^{0.5} p(p|n,y)dp$$



Sampling from the posterior

- Grid approximation of the posterior:

```
y = 6
n = 10
p_grid <- seq(from = 0, to  = 1, length.out = 1000)
prior  <- rep(1, 1000)
likelihood <- dbinom(x = y, size = n, prob = p_grid)
posterior  <- likelihood * prior
posterior  <- posterior / sum(posterior)
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- Sampling:

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samples <- sample(p_grid,
                  prob = posterior,
                  size = 10000,
                  replace = T)
```

Posterior distribution

What is the probabilility that p is less then 0.5?

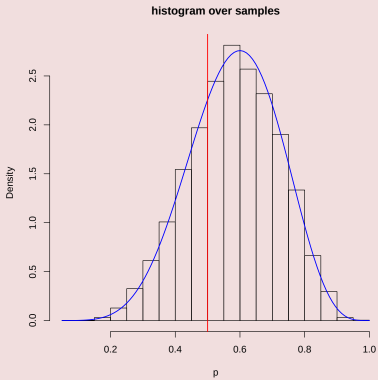

Figure : Histogram of draws from the posterior. Blue line is the posterior distribution

Calculating $\mathbb{P}(p < 0.5)$

- $\mathbb{P}(p|n = 10, y = 6) = \int_0^{0.5} p(p|n = 10, y = 6)dp = 0.274$

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- Approximation by sampling $(p_1, p_2, \ldots, p_s)$:

$$P_{est} = \frac{1}{10^4} \sum_{i=1}^{10^4} \mathbb{I}_{[0,0.5)}(p_i) = 0.281$$

In code:

```
samples <- sample(p_grid ,
                  prob = posterior ,
                  size = 10000,
                  replace = T)
P_est <- mean(samples < 0.5)
```

Approximation

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samples <- sample(p_grid,
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- $s = 2$ three runs of the code

$$P^1_{est} = 0.5, P^2_{est} = 0, P^3_{est} = 0.$$

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- $s = 10^5$ three runs of the code

$$P^1_{est} = 0.272, P^2_{est} = 0.274, P^3_{est} = 0.274.$$

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$$p(y^*|n^*) = \int p(y^*|n^*, p)p(p)dp.$$

- The predictive distribution after observing the data (y, n)

$$p(y^*|n^*, n, y) = \int p(y^*|n^*, p)p(p|n, y)dp.$$

Sampling from the posterior

- Prior predictive distribution

```
samples <- runif(n = 10000, min = 0, max =1)
ystar   <- rbinom(n = 10000,
                  size = 16,
                  prob = samples)
```

Sampling from the posterior

- Prior predictive distribution

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- Posterior predictive distribution

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n = 10
p_grid <- seq(from = 0, to  = 1, length.out = 1000)
prior  <- rep(1, 1000)
likelihood <- dbinom(x = y, size = n, prob = p_grid)
posterior  <- likelihood * prior
posterior  <- posterior / sum(posterior)
samples <- sample(p_grid,
                  prob = posterior,
                  size = 10000,
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ystar   <- rbinom(n = 10000,
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```

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- An approximation of the percentile interval (PI) is obtained by taking quantiles from samples.
- Highest posterior interval (HPDI) is the smallest possible interval.

Bayesian intervals

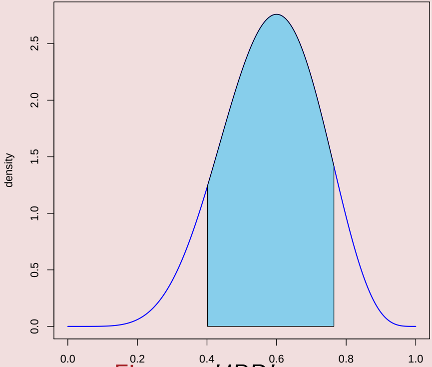

Figure : $HPDI_{80}$

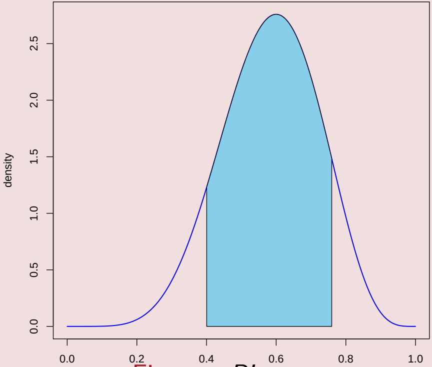

Figure : PI_{80}