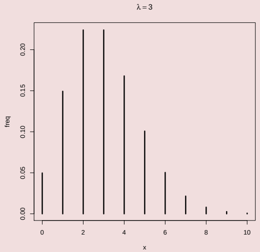
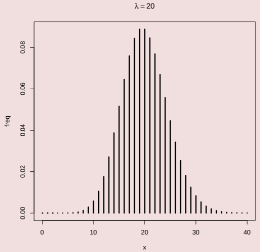
Chapter 11

Poisson regression

$$
\begin{aligned}
y &\sim Po(\lambda), \\
\mathbb{E}[y] &= \lambda, \\
\mathbb{V}[y] &= \lambda.
\end{aligned}
$$

- Counts (non negative integers) without upper limit
- One parameter, λ.
- Variance equal to mean.
- For large λ close to normal.



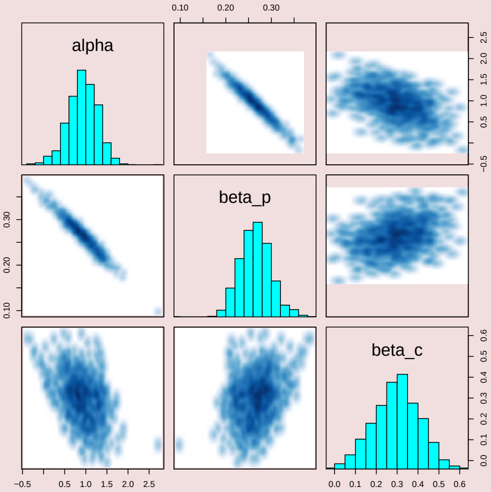


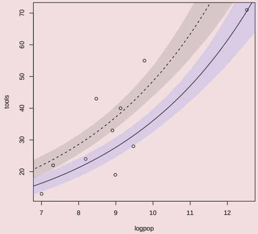
	culture	population	contact	total_tools
1	Malekula	1100	low	13
2	Tikopia	1500	low	22
3	Santa Cruz	3600	low	24
4	Yap	4791	high	43
5	Lau Fiji	7400	high	33
6	Trobriand	8000	high	19
7	Chuuk	9200	high	40
8	Manus	13000	low	28
9	Tonga	17500	high	55
10	Hawaii	275000	low	71

Poisson model

$$
\begin{aligned}
tools_i &\sim Po(\lambda_i) \\
g(\lambda_i) &= \alpha + \log(population_i)\beta_p + contact_i\beta_c \\
\alpha &\sim N(0, 10) \\
\beta_p &\sim N(0, 10) \\
\beta_c &\sim N(0, 10)
\end{aligned}
$$

Back to: Poisson model, posterior fit





Fitted simple_fit through stan then:

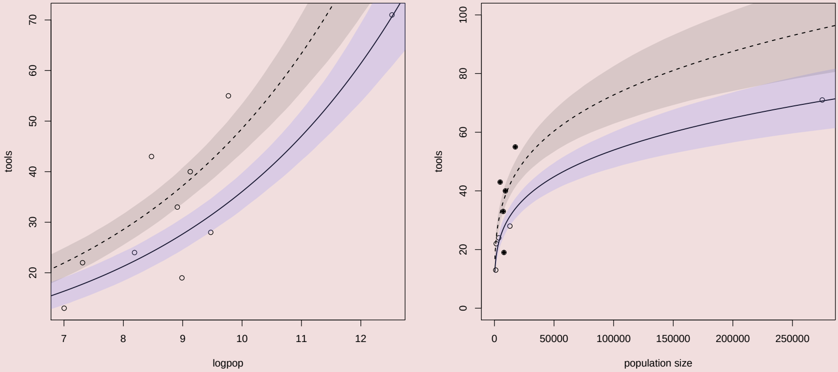
```
log_pop.seq <- seq(from = 6, to = 13, length.out=100)
samples <- extract(simple_fit)
lambda_mat <- sapply(1:length(samples$alpha), function(i){

return(exp(samples$alpha[i] + log_pop.seq * samples$beta_p[i]))
})
lambda_mat_c <- sapply(1:length(samples$alpha), function(i){

return(exp(samples$alpha[i] + log_pop.seq * samples$beta_p[i] +
samples$beta_c[i]))
})
```

Complete code in Rmarkwdown on homepage later this week.

Use the right scale



Height data

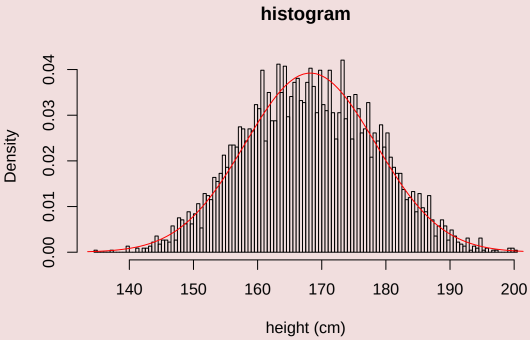


Figure : Height, and fitted normal distribution

Height data

- If we had the geneder of the person we could easily fit a model

$$
\begin{aligned}
y_i &\sim N(\mu_i, \sigma_i) \\
\mu_i &= \alpha_m m_i + \alpha_f(1 - m_i) \\
\sigma_i &= \sigma_m m_i + \sigma_f(1 - m_i) \\
\alpha_m &\sim N(170, 100) \\
\alpha_f &\sim N(170, 100) \\
\sigma_f &\sim HC(0, 5) \\
\sigma_m &\sim HC(0, 5)
\end{aligned}
$$

- The density of the total population:

$$p(y) = p(y|m=1)p(m=1) + p(y|m=0)p(m=0)$$

Density

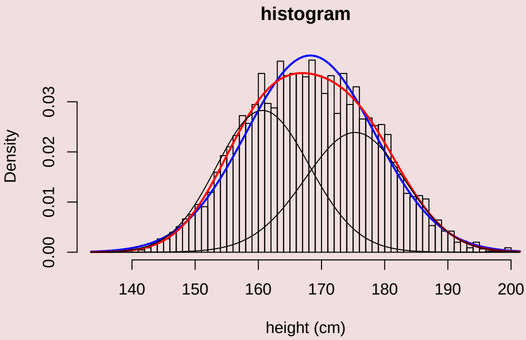


Figure : Height, and fitted normal distribution using gender

Missing

- But if we don't have gender?

Missing

- But if we don't have gender?
- Lets pretend we have.

Missing

$$
\begin{aligned}
y_i &\sim pN(\alpha_1, \sigma_1) + (1-p)N(\alpha_2, \sigma_2) \\
\alpha_1 &\sim N(170, 100) \\
\alpha_2 &\sim N(170, 100) \\
\sigma_1 &\sim HC(0, 5) \\
\sigma_2 &\sim HC(0, 5) \\
p &\sim U[0, 1].
\end{aligned}
$$

Density for mixture

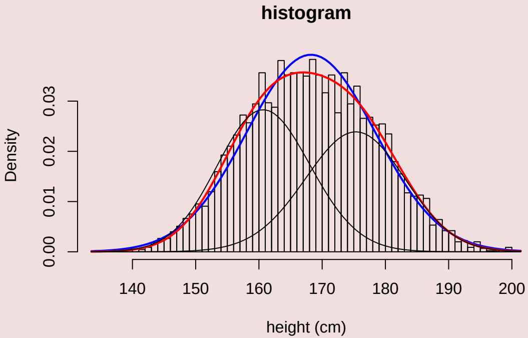


Figure : Height, and fitted normal distribution for the mixture distribution

Density for mixture

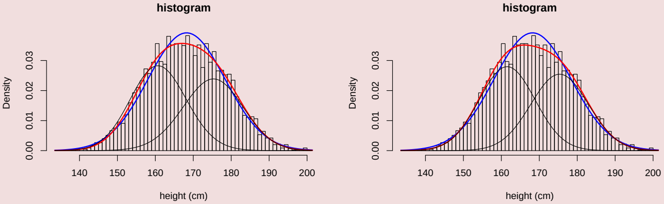


Figure : Height, and fitted normal distribution for the mixture distribution

Mixture for Classification

An alternative formulation is to use:

$$
\begin{aligned}
y_i &\sim N(\alpha_{x_i}, \sigma_{x_i}) \\
x_i &\sim Bin(1, p) \\
\alpha_0 &\sim N(170, 100) \\
\alpha_1 &\sim N(170, 100) \\
\sigma_0 &\sim HC(0, 5) \\
\sigma_1 &\sim HC(0, 5) \\
p &\sim U[0, 1].
\end{aligned}
$$

Mixture for Classification

- Better for interpretation.
- Not possible to fit in Stan. Can't handle discrete unobserved variables.

Rstan mixture

```
parameters{
            ordered[2] mu;
            real<lower=0> sigma1;
            real<lower=0> sigma2;
            real<lower=0,upper=1> p;
}
model{
            mu      ~ normal(170, 100);
            sigma1 ~ cauchy(0,5);
            sigma2 ~ cauchy(0,5);
            for(i in 1:N)
                 target += log_sum_exp(bernoulli_lpmf(0|p) + normal_lpdf(y[i]| mu[1], sigma1 ),
                          bernoulli_lpmf(1|p) + normal_lpdf(y[i]| mu[2], sigma2));
}
```

Posterior

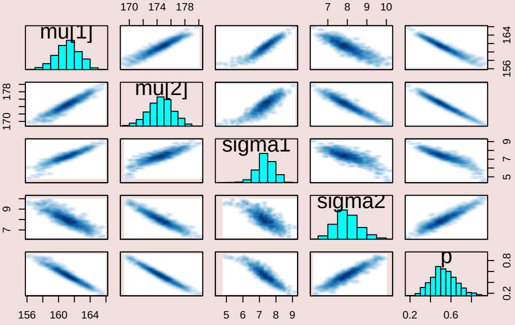


Figure : Posterior distribution of the parameters

Mixture for Classification

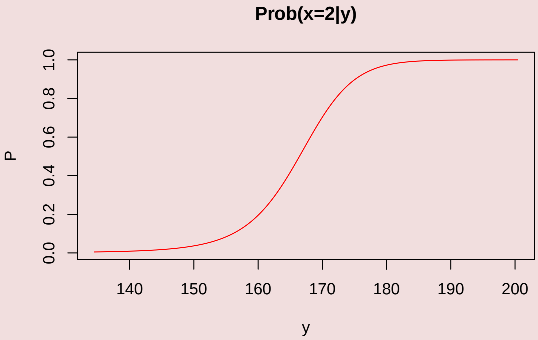


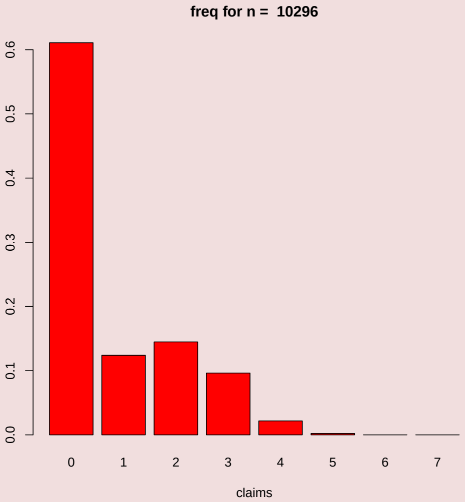
Figure : Using Height to determine class

Definition Mixtures

- Distribution of a mixing distribution.
- General form of two dimensional mixture distribution

$$g(x) = p_1 g_1(x) + (1 - p_1) g_2(x).$$

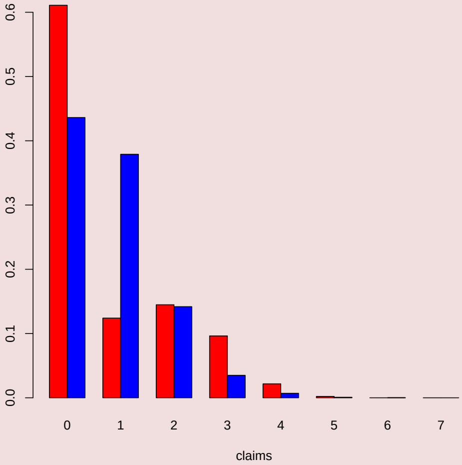
Car insurance, number of claims of past 5 years



first Poisson model

$$
\begin{aligned}
y_i &\sim Po(\lambda_i) \\
g(\lambda_i) &= \alpha \\
\alpha &\sim N(0, 10)
\end{aligned}
$$

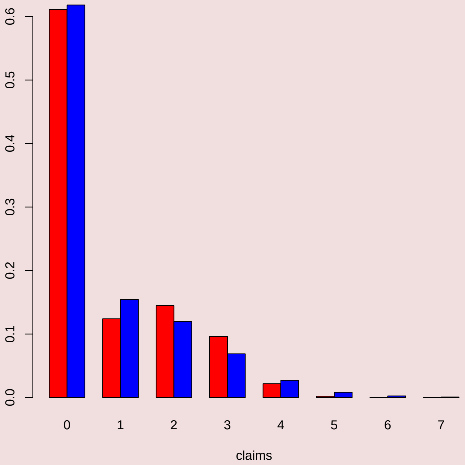
fit



first Poisson model

$$
\begin{aligned}
y_i &\sim p\delta_0 + (1-p)Po(\lambda_i) \\
g(\lambda_i) &= \alpha \\
\alpha &\sim N(0, 10) \\
p &\sim U[0, 1]
\end{aligned}
$$

fit



Including covariates

$$
\begin{aligned}
y_i &\sim p\delta_0 + (1-p)Po(\lambda_i) \\
g_1(\lambda_i) &= \alpha + x\beta_x \\
g_2(p) &= \alpha_0 + x\beta_x^0 \\
&\dots
\end{aligned}
$$

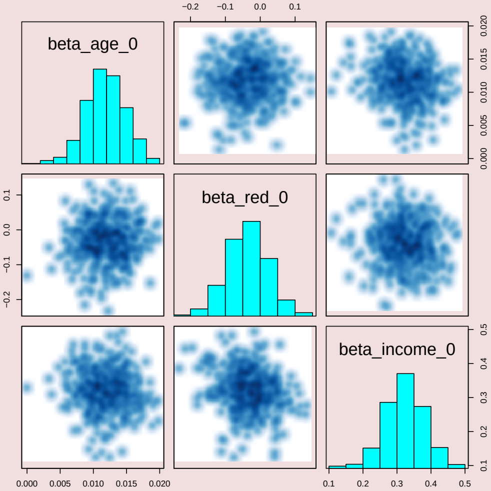


Figure : Covariates for p

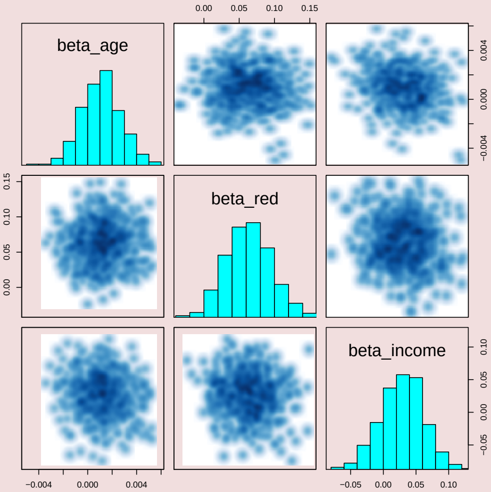


Figure : Covariates for λ

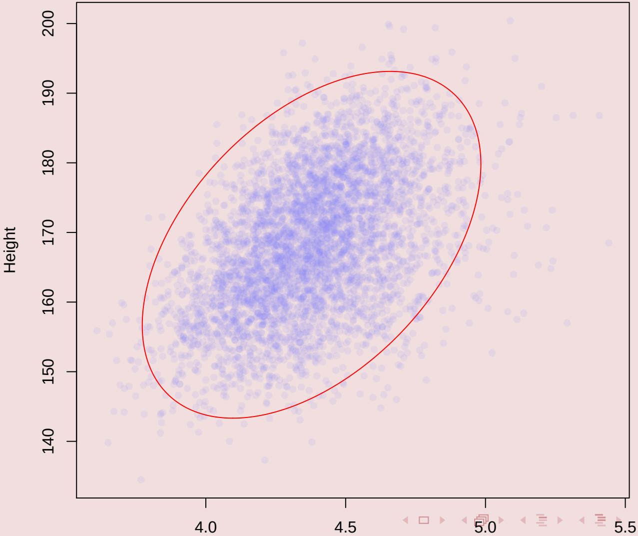
Beyond one dimension

- Multivariate Normal, X $d-$dimensional

$$p(X|\mu,\Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{0.5}} e^{-\frac{1}{2}(X-\mu)^T\Sigma^{-1}(X-\mu)}$$

- Mean, $\mathbb{E}[X] = \mu$,
- Covariance, $\mathbb{C}[X] = \Sigma$.
- Conditional distribution are also normal.

Height and Weight

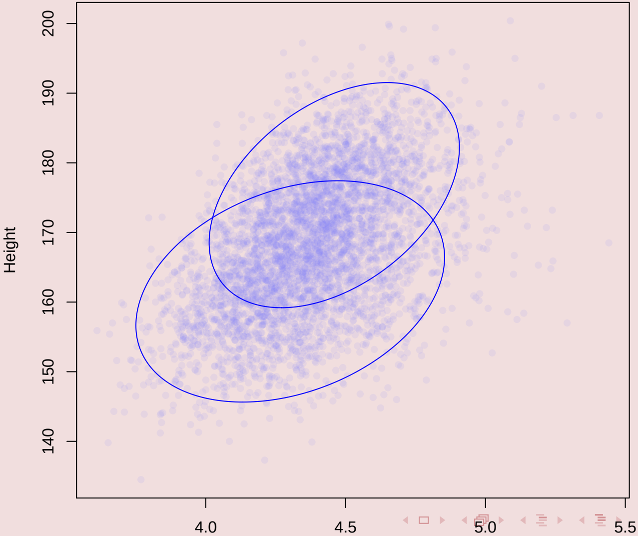


Advanced mixture

$$[height_i, log(weight_i)] \sim N(\mu_{m_i}, \Sigma_{m_i})$$
$$m_i \sim Bin(1, p)$$

- Here μ_i is 2-d vector.
- Here Σ_i is 2x2 matrix.
- Complete model for $m_i, height_i, log(weight_i)$.
- To predict height given gender and weight, regression model.
- If we lost gender?

Height and Weight



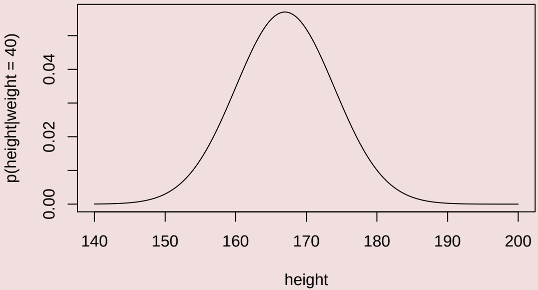


Figure : Height given log(weight)

Non linear regression

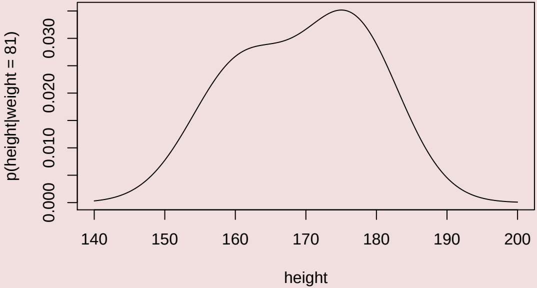


Figure : Height given log(weight)

Non linear regression

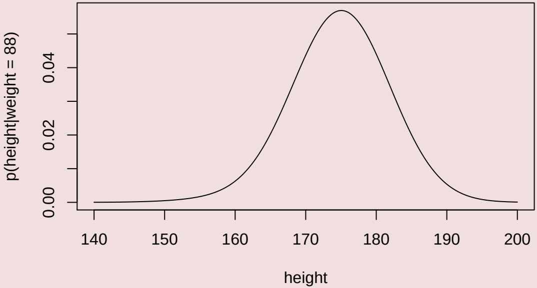


Figure : Height given log(weight)