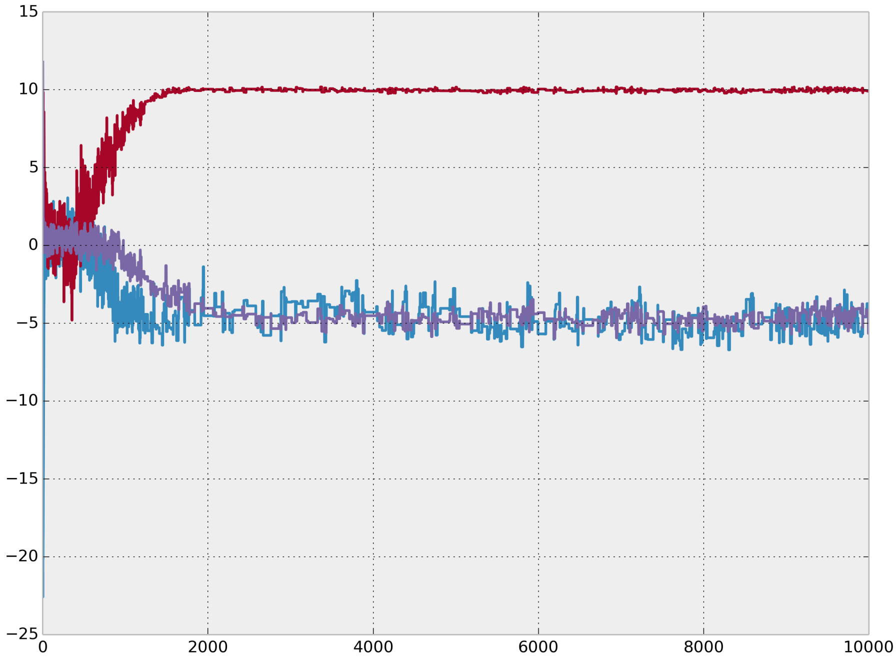

Figure 1:

22nd November 2016

Diagnostics

1. In Figure 1, we see a traceplot from three different runs of the same MCMC algorithms:

   (a) Has the MCMC chain converged?

2. Suppose that we have two MCMC chains,

   $$X_1, X_2, \ldots, X_n,$$

   and

   $$Y_1, Y_2, \ldots, Y_n,$$

   where both the chains have the same stationary distribution. The estimated variances are $\bar{V}[X_t] = 0.1, \bar{V}[Y] = 1$, and the estimated autocorrelations are $A\hat{C}F_X(1) = 0.999$ and $A\hat{C}F_Y(1) = 0.5$. Which is a better estimate of the expectation of the stationary distribution $\bar{X}$ or $\bar{Y}$? Better in what sense?

3. Figure 2 shows the traceplot of $X_{1:6000}$. Since $X_{1:6000}$ are samples from a Markov chain with posterior distribution $\pi(x)$ an estimator of the $E[X]$

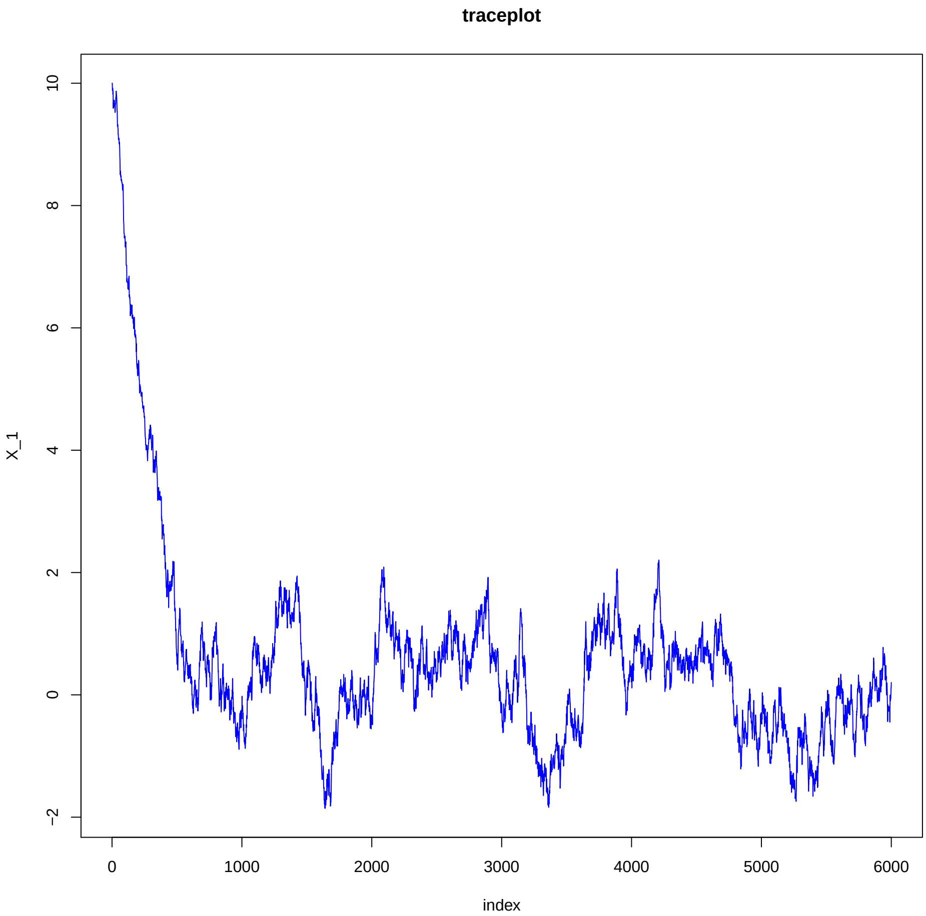

Figure 2:

is to take $\frac{1}{6000}\sum_{i=1}^{6000} X_i \approx 0.2$. Is this a good estimator can you suggest improvement?

4. Below is output results from a run with STAN.

```
## Inference for Stan model: model_string.
## 3 chains, each with iter=30000; warmup=10000; thin=1;
## post-warmup draws per chain=20000, total post-warmup draws=60000.
##
##          mean se_mean  sd  2.5%   25%   50%  97.5% n_eff Rhat
## mu       12.7       0 1.4  10.0  12.7  15.4 25754
1
## sigma     6.0       1 1.0   4.4   5.8   8.3 100
1
## lp__    -45.7       0 1.0 -48.4 -45.3 -44.6 17964
1
```

(a) Which is most reliable the estimate of μ or σ?

(b) To compute the quantiles one just take the quantiles of the Markov chain. Which quantile is there a difference in the uncertainty for the 50% quantile compared to the 95% quantile ?

5. (Harder) Given an AR(1) processes:

$$X_t = aX_{t-1} + \epsilon_t,$$

where $\epsilon_t \sim N(0, \sigma^2)$, for $t = 1, 2 \ldots$, compute:

(a) $E[X_t X_{t-1}]$. Hint: Use that ϵ_t and X_{t-1} are independent and $E[\epsilon_t] = 0$.

(b) Compute the $ACF(1) = \frac{E[X_t X_{t-1}]}{\sqrt{V[X_t]V[X_{t-1}]}}$ for the processes. Hint use that $V[X_t] = E[X_t^2] = V[X_{t-1}] = E[X_{t-1}^2]$.

(c) Compute the $ACF(2) = \frac{E[X_t X_{t-2}]}{\sqrt{V[X_t]V[X_{t-2}]}}$.