Chapter 2

Bayes Formula

- $\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$.

Figure: Thomas Bayes

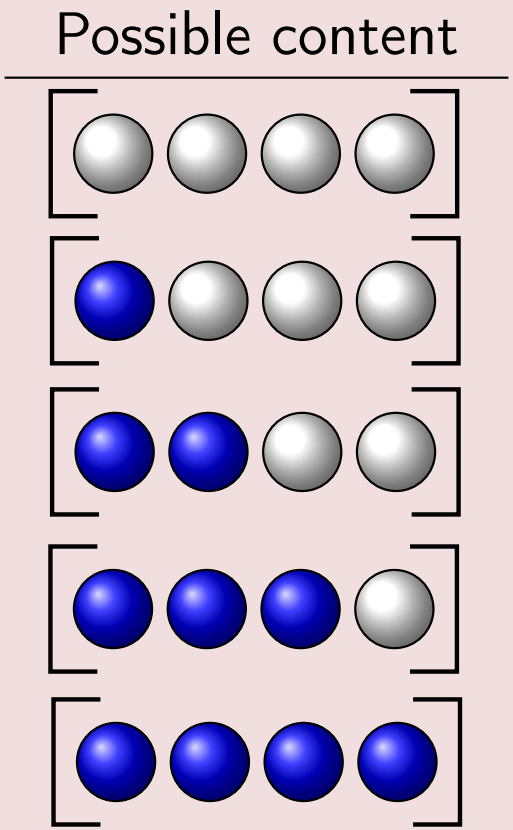
Bayes Formula

- $\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$.
- $\mathbb{P}(A|B) \propto \mathbb{P}(B|A)\mathbb{P}(A)$, (this is often hard to grasp)

Figure: Thomas Bayes

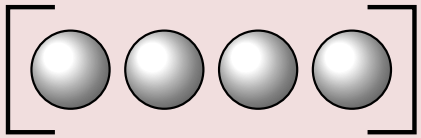
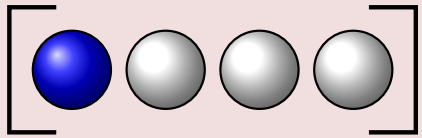
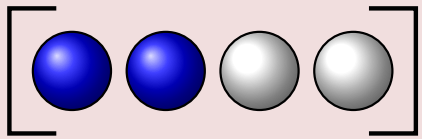
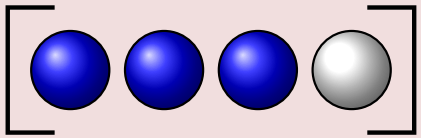
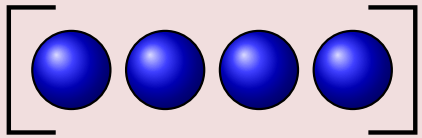
The setup

A priori five possibilities, all equally likely:



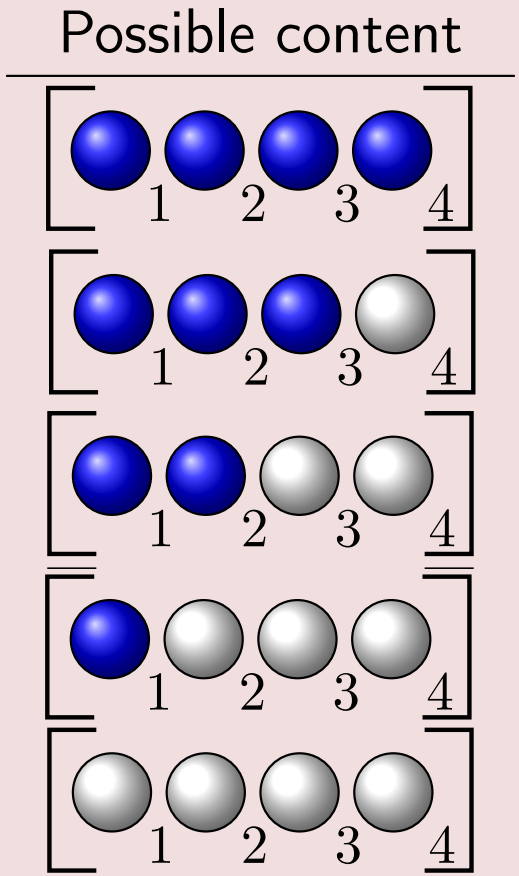
The prior

A priori five possibilities, all equally likely:

Possible content	prior count
	1
	1
	1
	1
	1

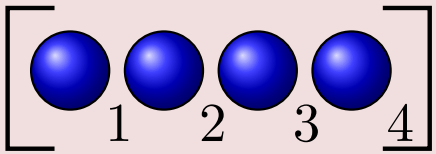
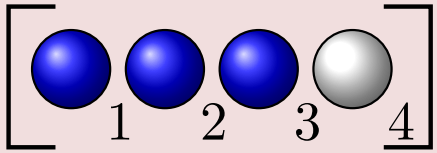
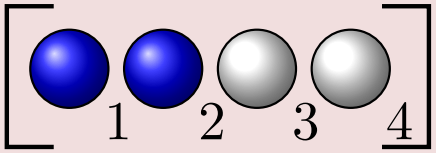
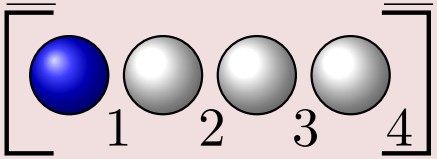
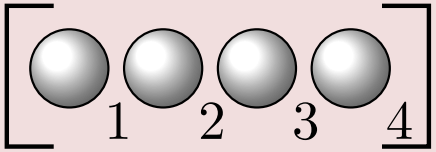
The likelihood

Which unique marble correspond to a white marble?

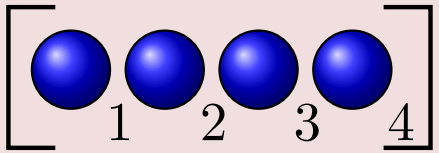
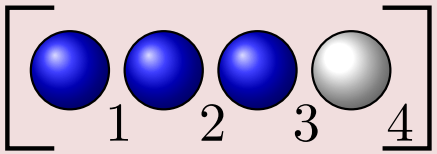
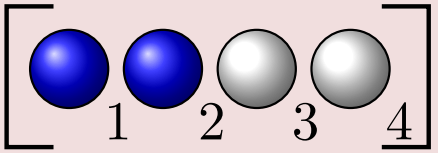
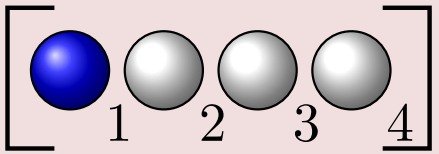
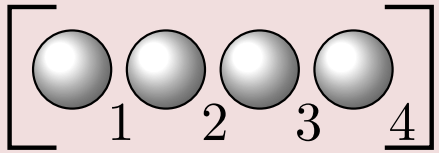


The likelihood

Which unique marble correspond to a white marble?

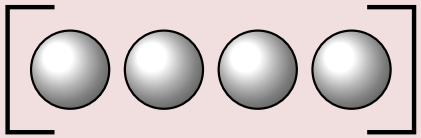
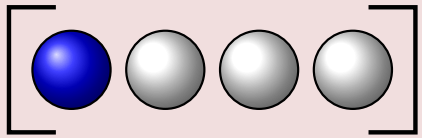
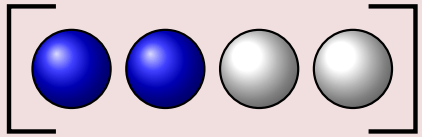
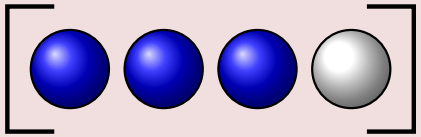
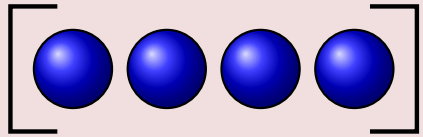
Possible content	ways to produce a white marble
	0
	1
	2
	3
	4

The likelihood

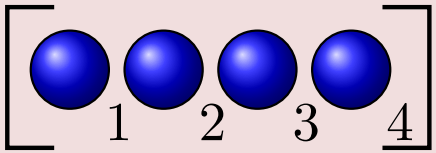
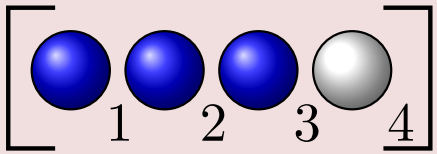
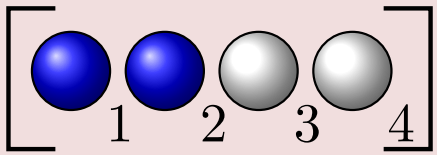
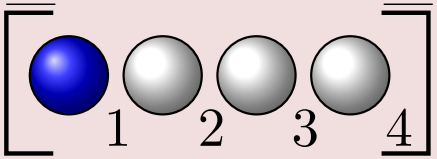
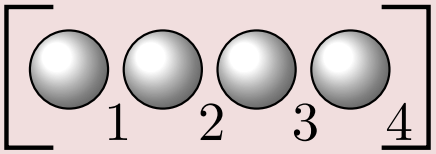
Possible content	white	blue
	0	4
	1	3
	2	2
	3	1
	4	0

Prior again

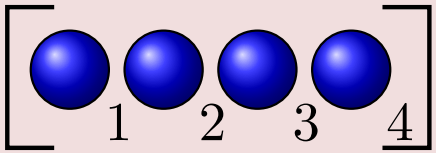
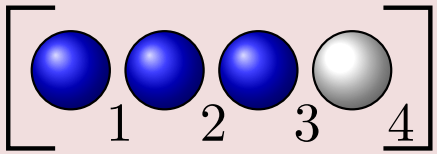
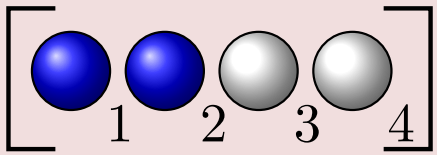
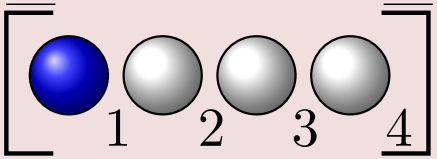
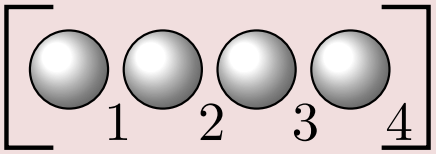
A priori five possibilities, all equally likely:

Possible content	$\mathbb{P}(\cdot)$
	$\frac{1}{5}$
	$\frac{1}{5}$
	$\frac{1}{5}$
	$\frac{1}{5}$
	$\frac{1}{5}$

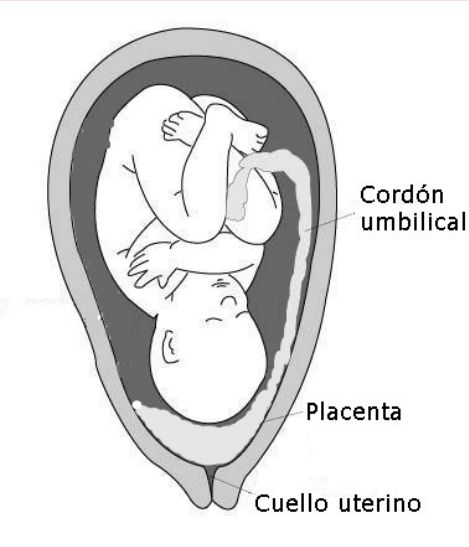
Likelihood again

Possible content	$\mathbb{P}(white \mid content)$	$\mathbb{P}(blue \mid content)$
	$\frac{0}{4}$	$\frac{4}{4}$
	$\frac{1}{4}$	$\frac{3}{4}$
	$\frac{2}{4}$	$\frac{2}{4}$
	$\frac{3}{4}$	$\frac{1}{4}$
	$\frac{4}{4}$	$\frac{0}{4}$

Likelihood again

Possible content	$\mathbb{P}(white \mid content)$	$\mathbb{P}(blue \mid content)$
	$\frac{0}{4}$	$\frac{4}{4}$
	$\frac{1}{4}$	$\frac{3}{4}$
	$\frac{2}{4}$	$\frac{2}{4}$
	$\frac{3}{4}$	$\frac{1}{4}$
	$\frac{4}{4}$	$\frac{0}{4}$

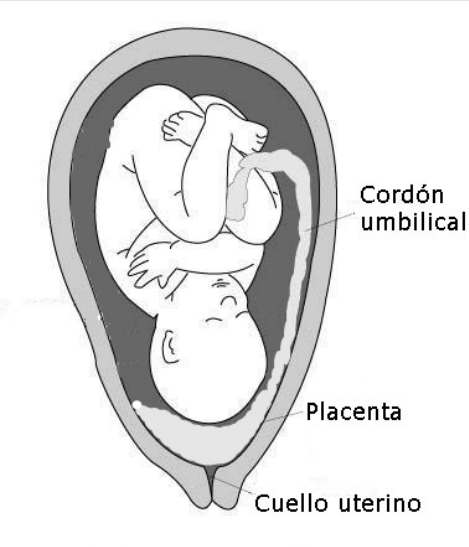
Placenta praevia



Placenta praevia
Grade IV

- Placenta praevia, is a condition occurring in $0.4 - 0.5\%$ of all labors. It is due to that the placenta is in the lower part uterus.

Placenta praevia



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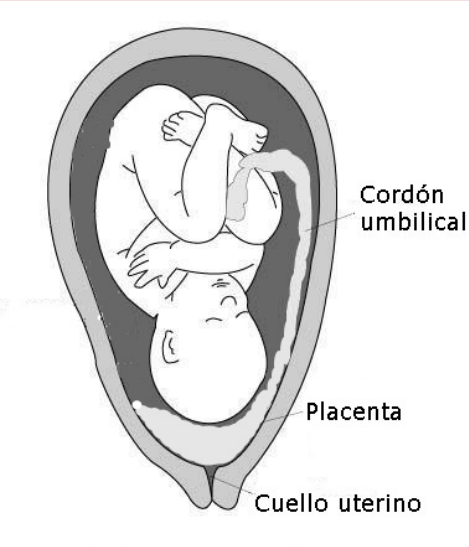
Placenta praevia



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- We study the probability, p, of a female baby given the mother had Placenta praevia.
- What is a reasonable prior for p?

Posterior of p

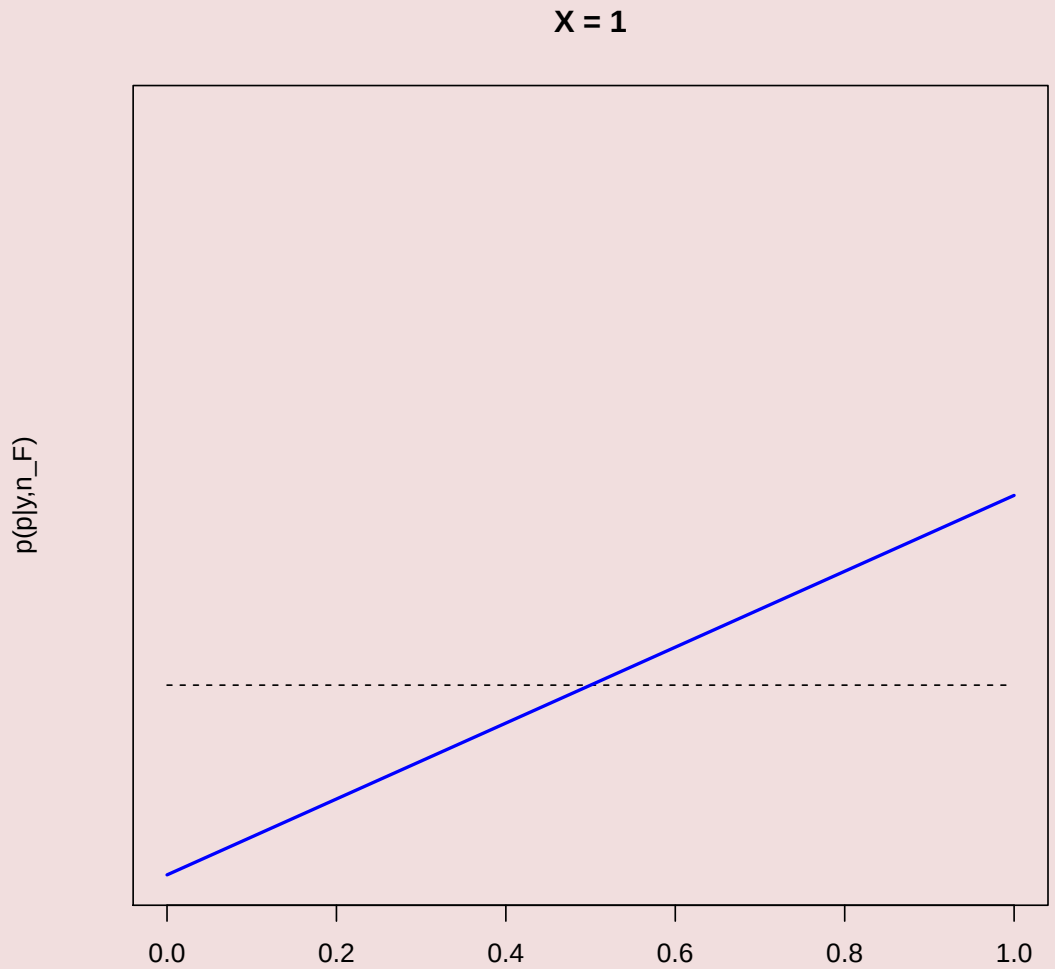
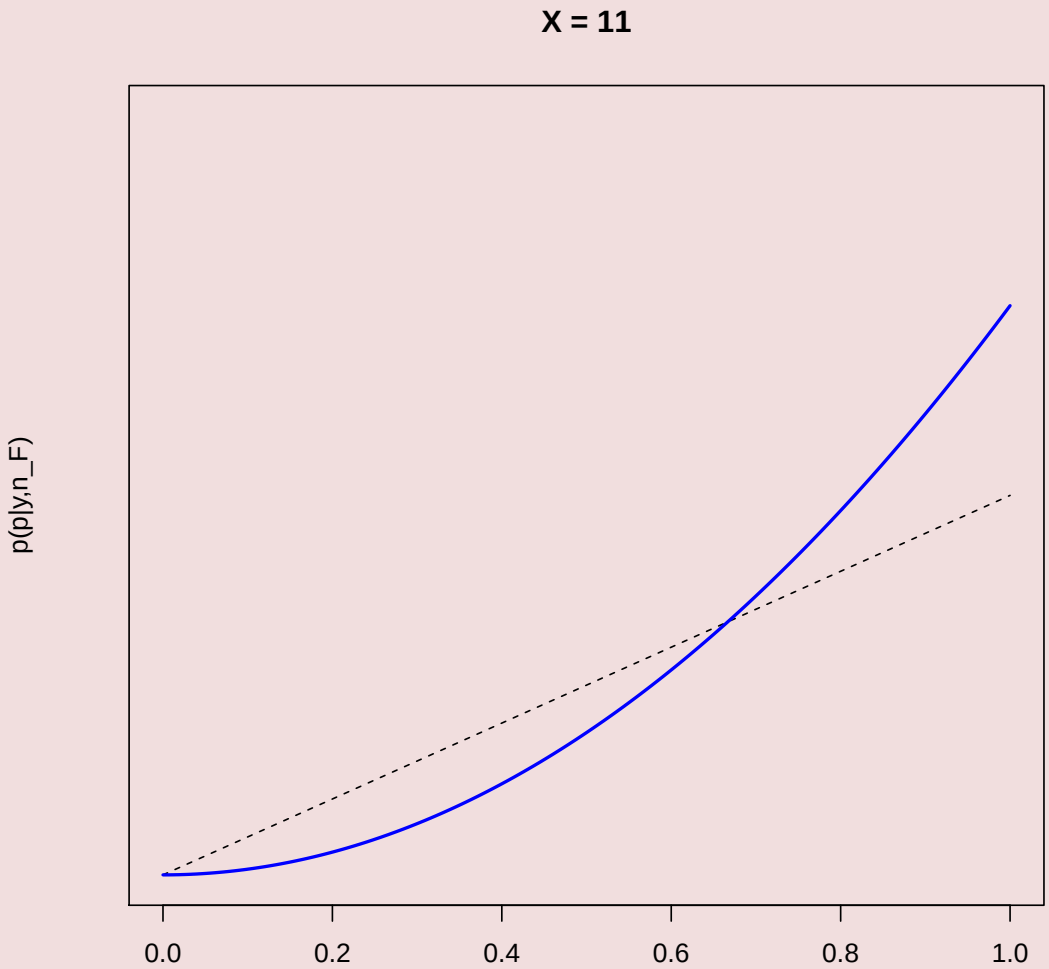


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p



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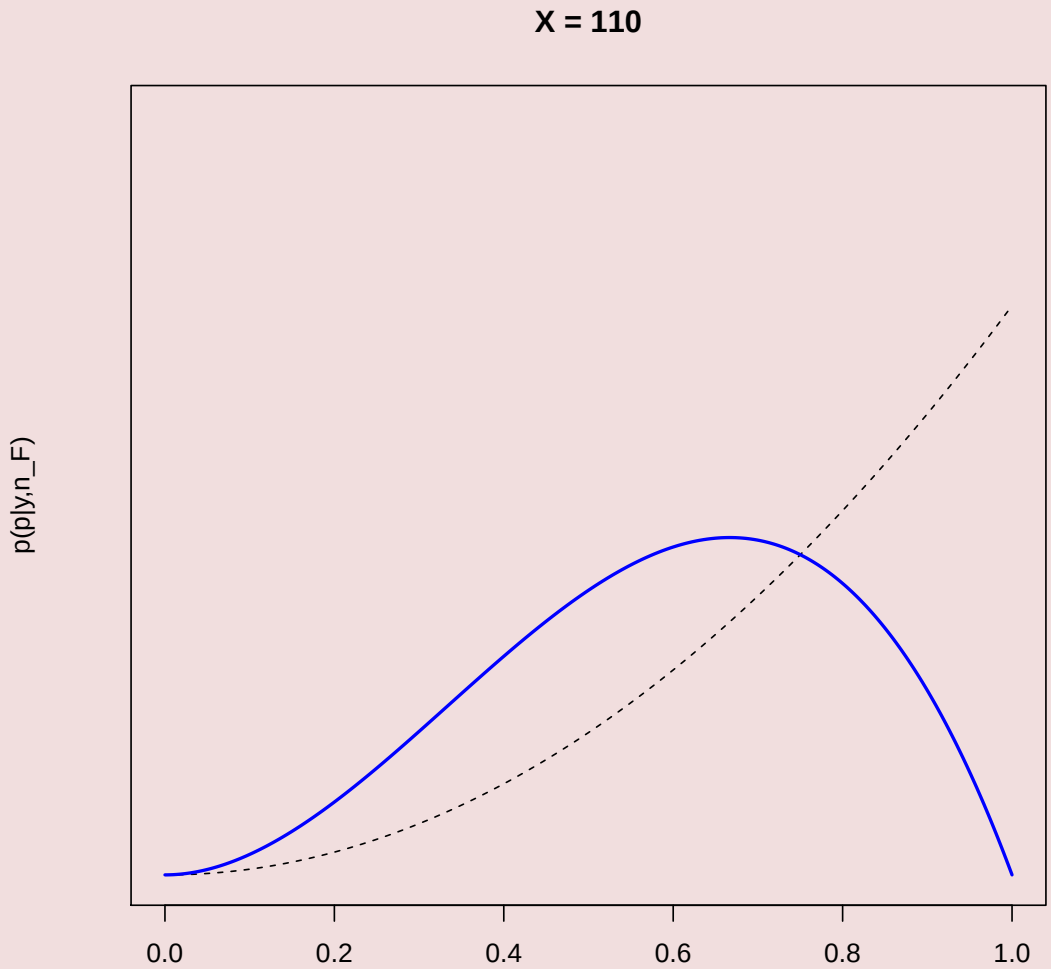

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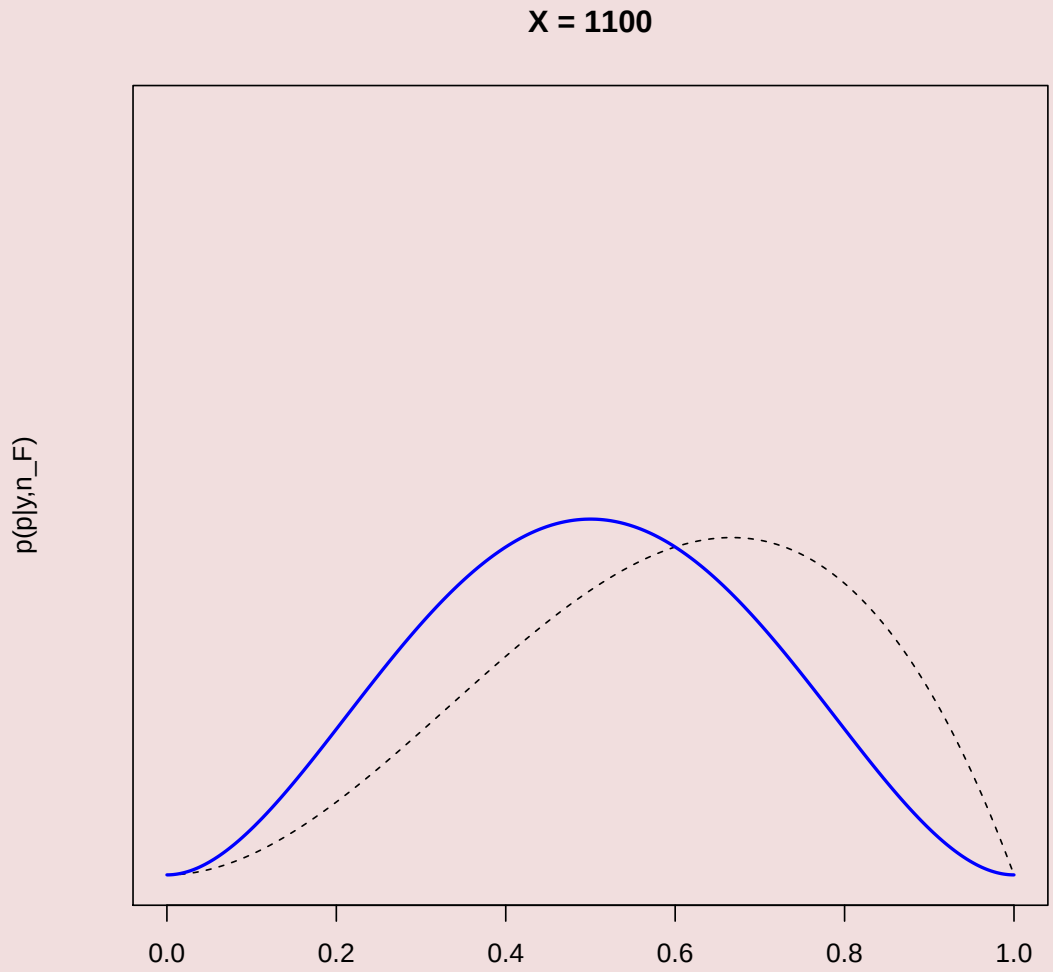

Figure: $p(\overset{\text{p}}{p}|x_1, \ldots, x_n)$

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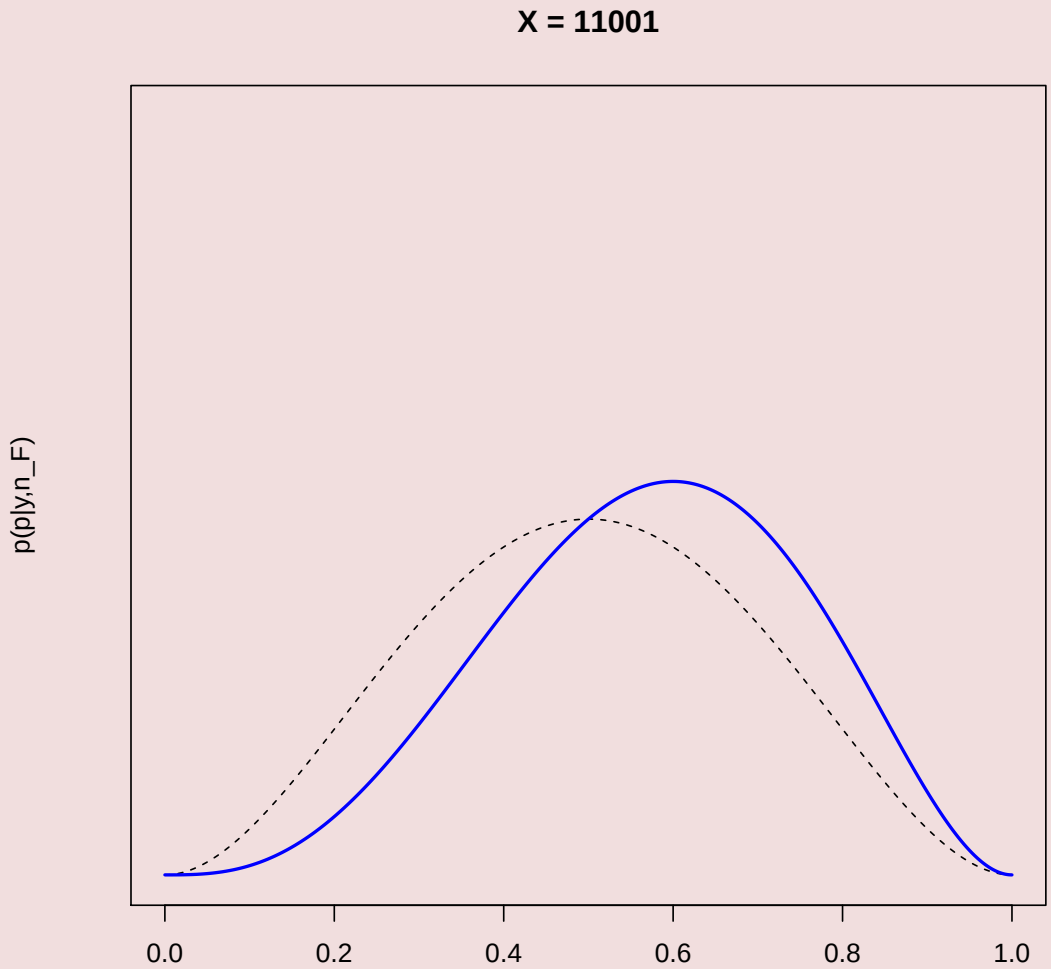

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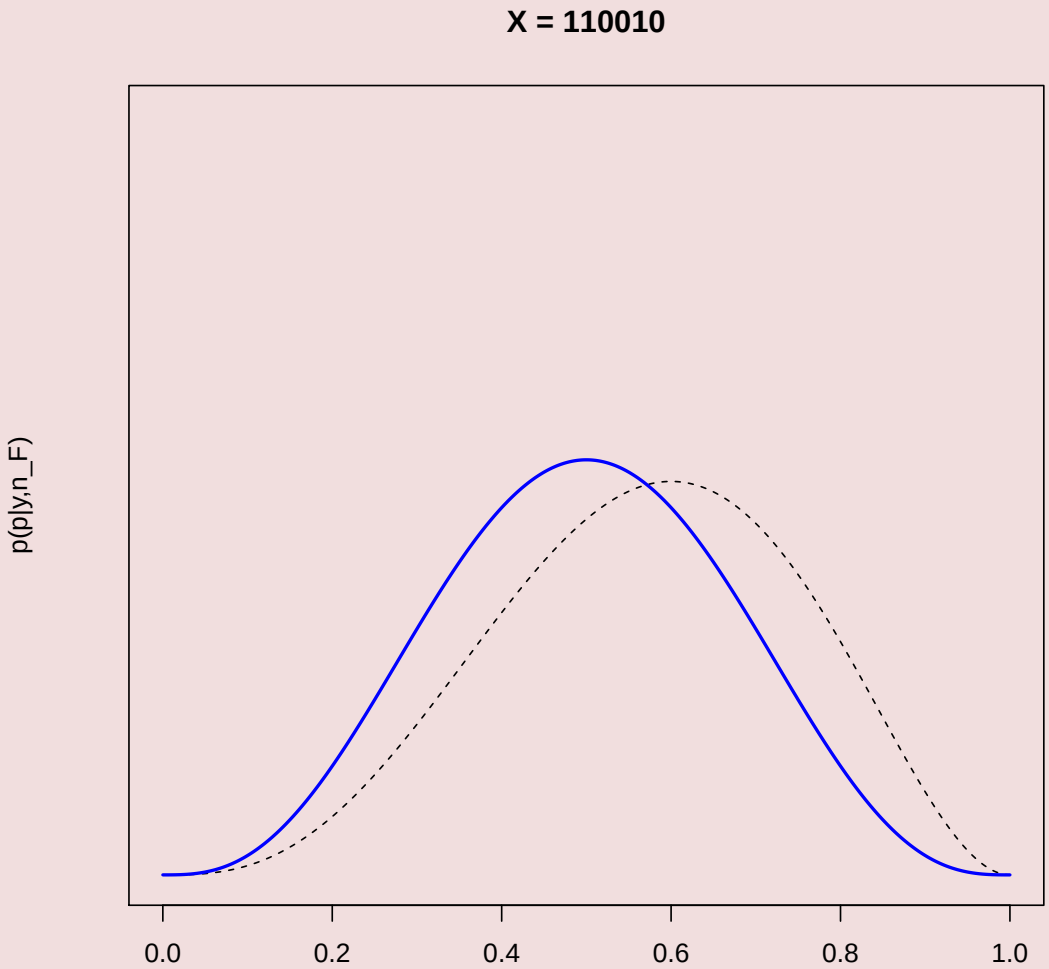
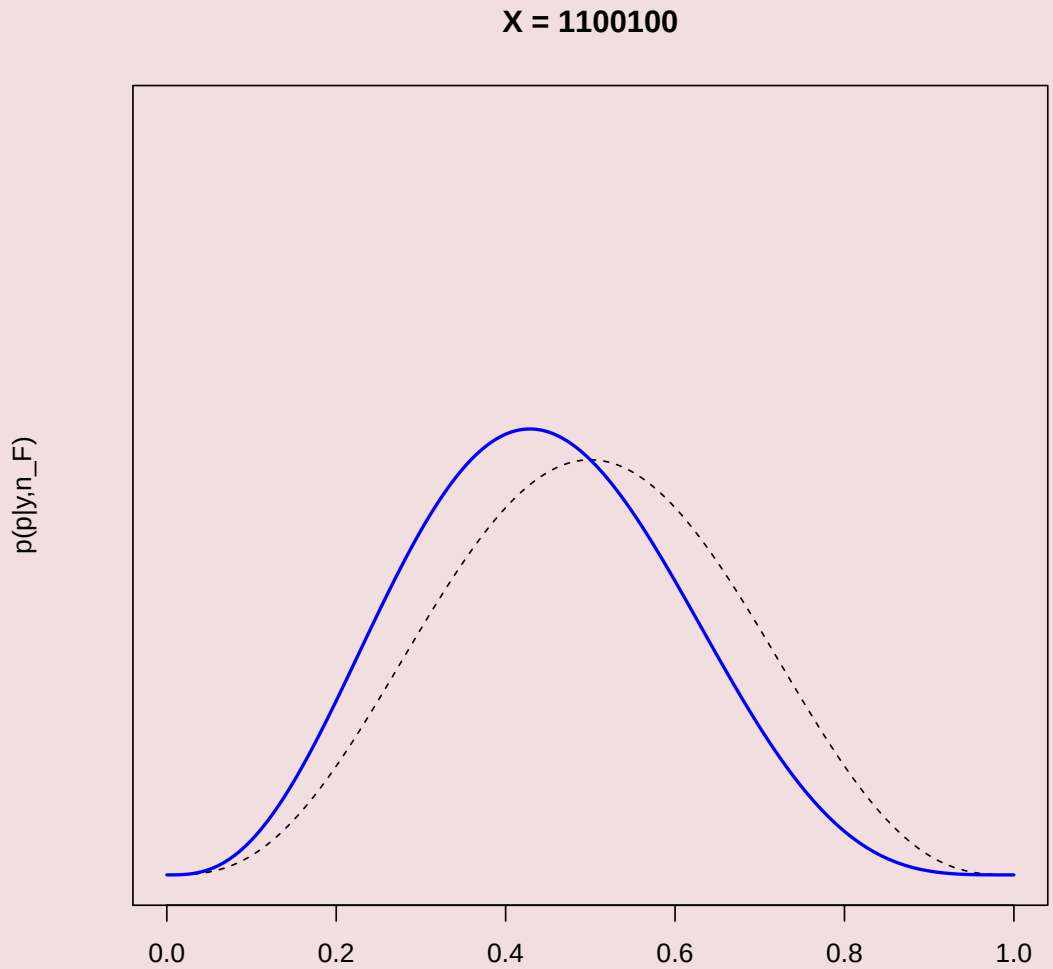


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p



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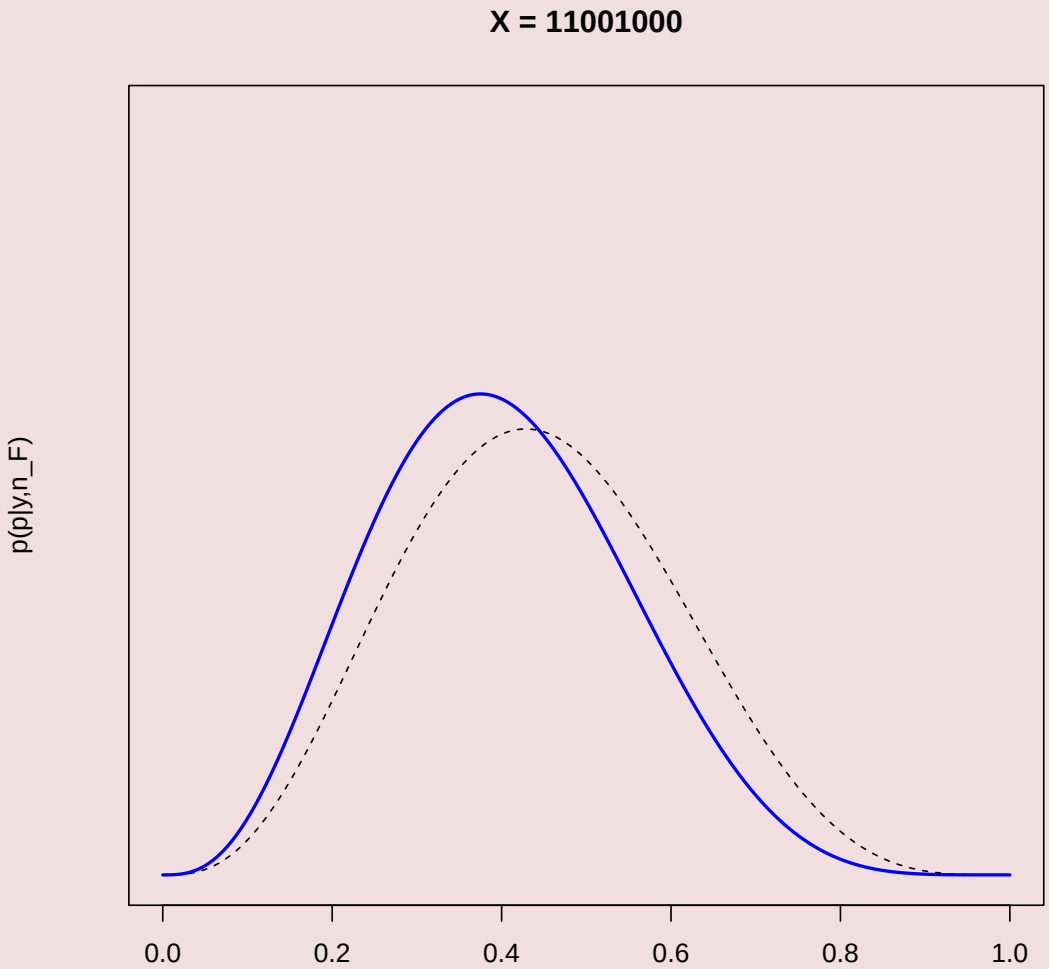

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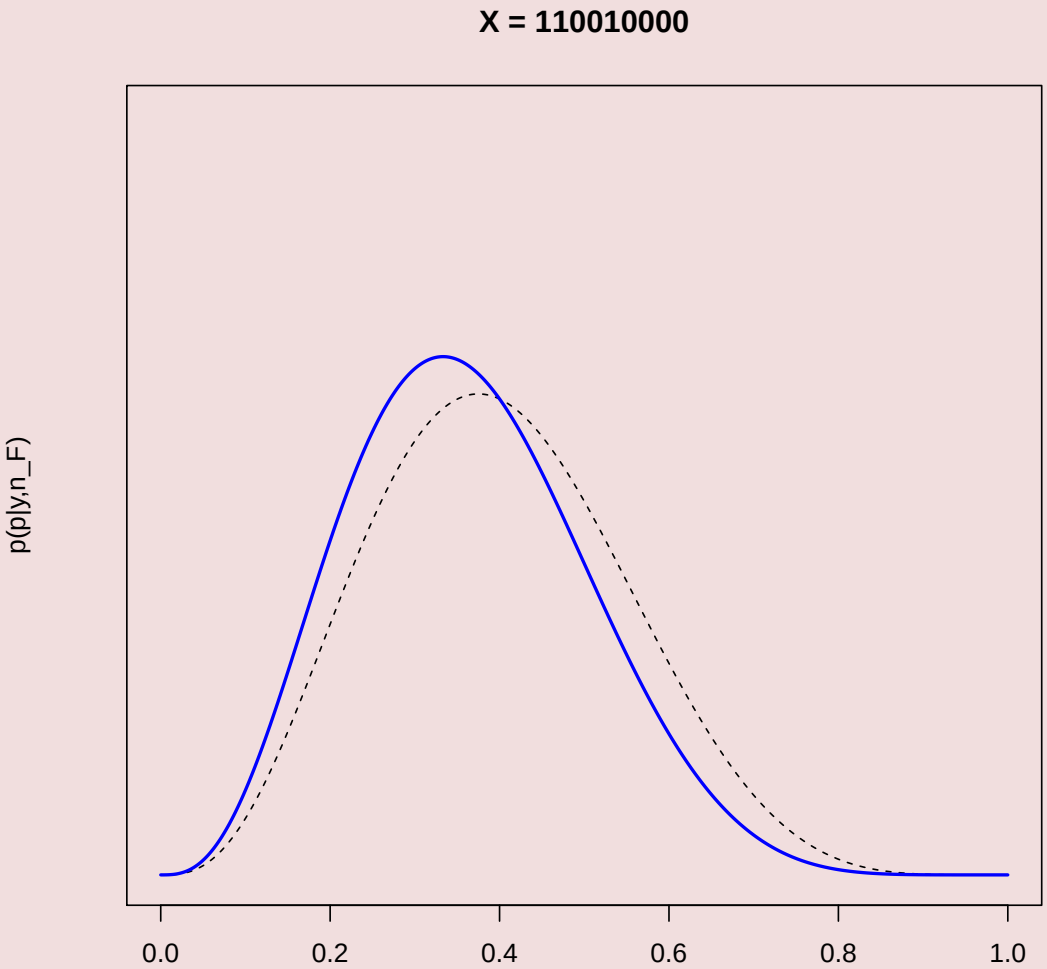

Figure: $p(p|\overset{\text{p}}{x}_1, \ldots, x_n)$

Proportional to, $\propto$

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$$f(x) \propto 2x,$$
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$$f(x) \propto cx, \text{ if } c > 0.$$

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$$f(x) \propto 4x,$$
$$f(x) \propto cx, \text{ if } c > 0.$$

- For $f(x) = x$:

$$f(x) \not\propto x^2,$$
$$f(x) \not\propto x(x+1),$$
$$f(x) \not\propto -x.$$

- In Bayesian statistics important since it preserve ratios (odds).

Posterior of p

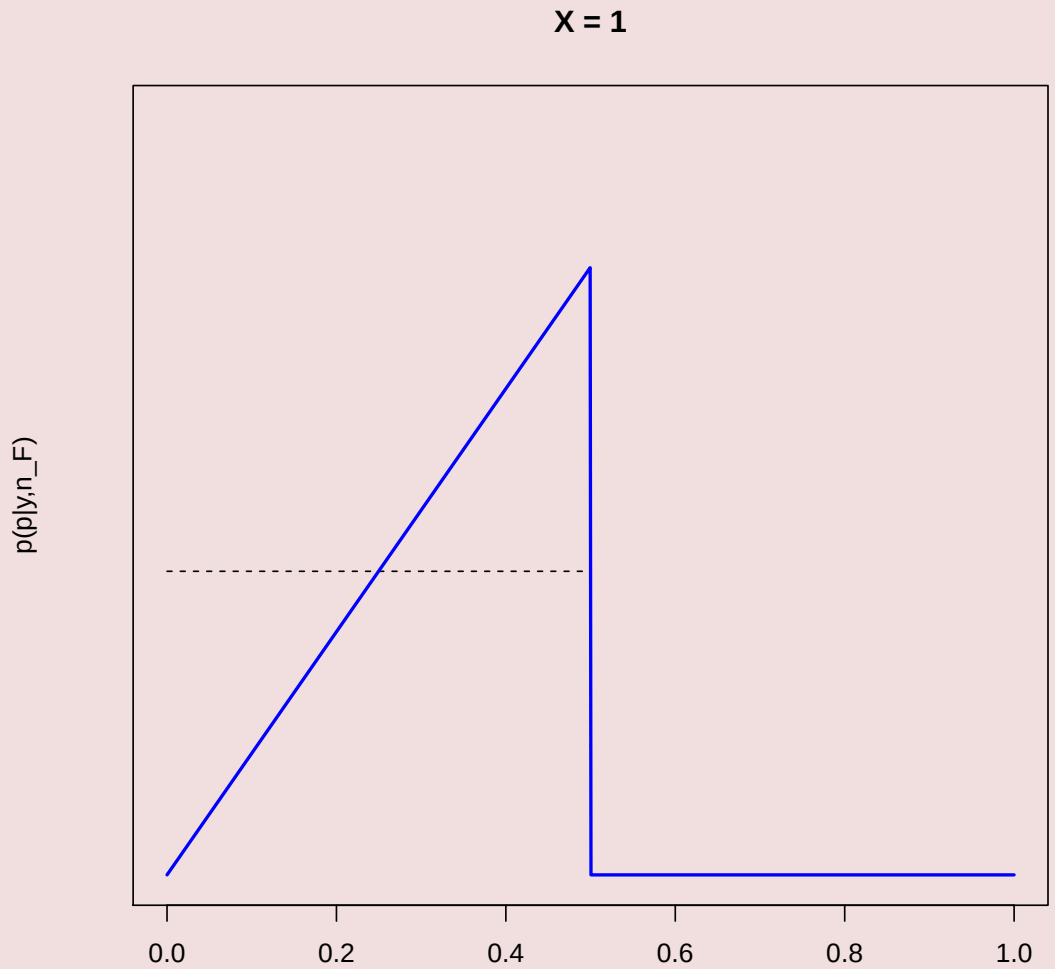


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p

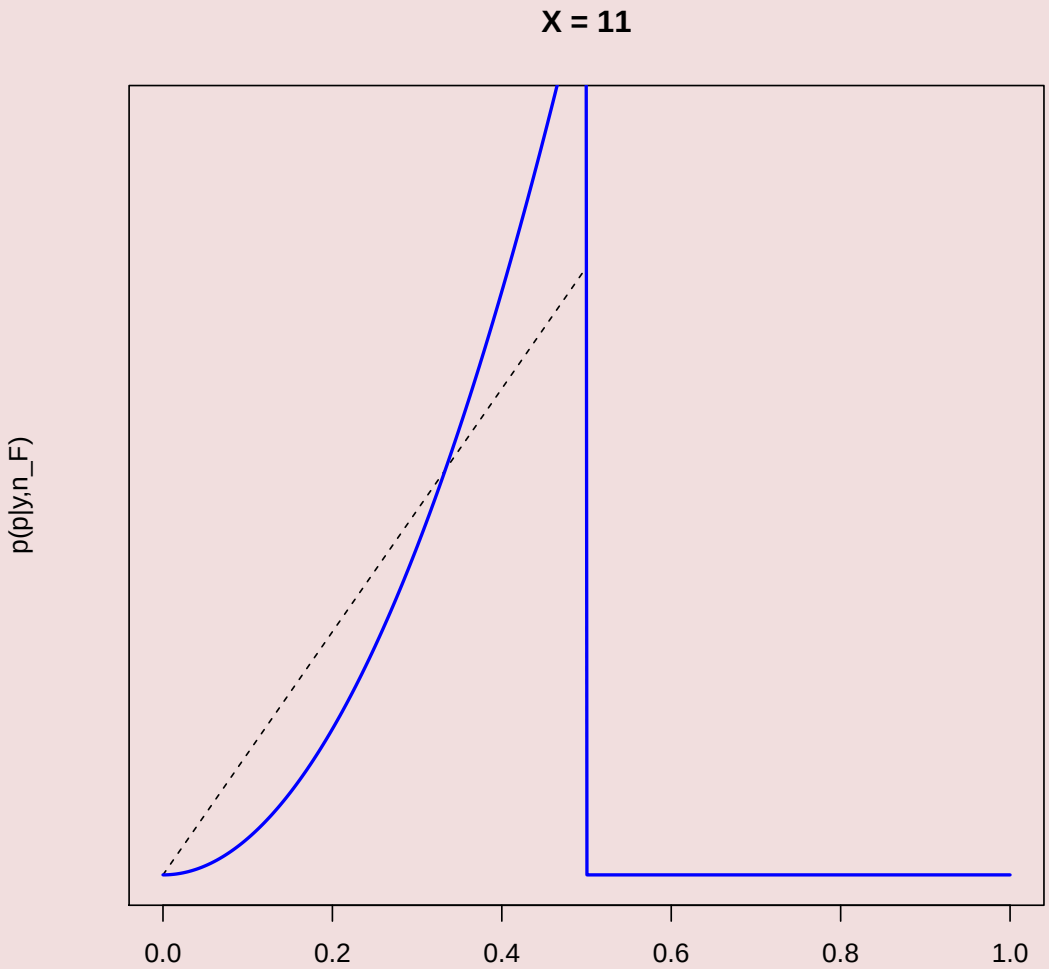


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p

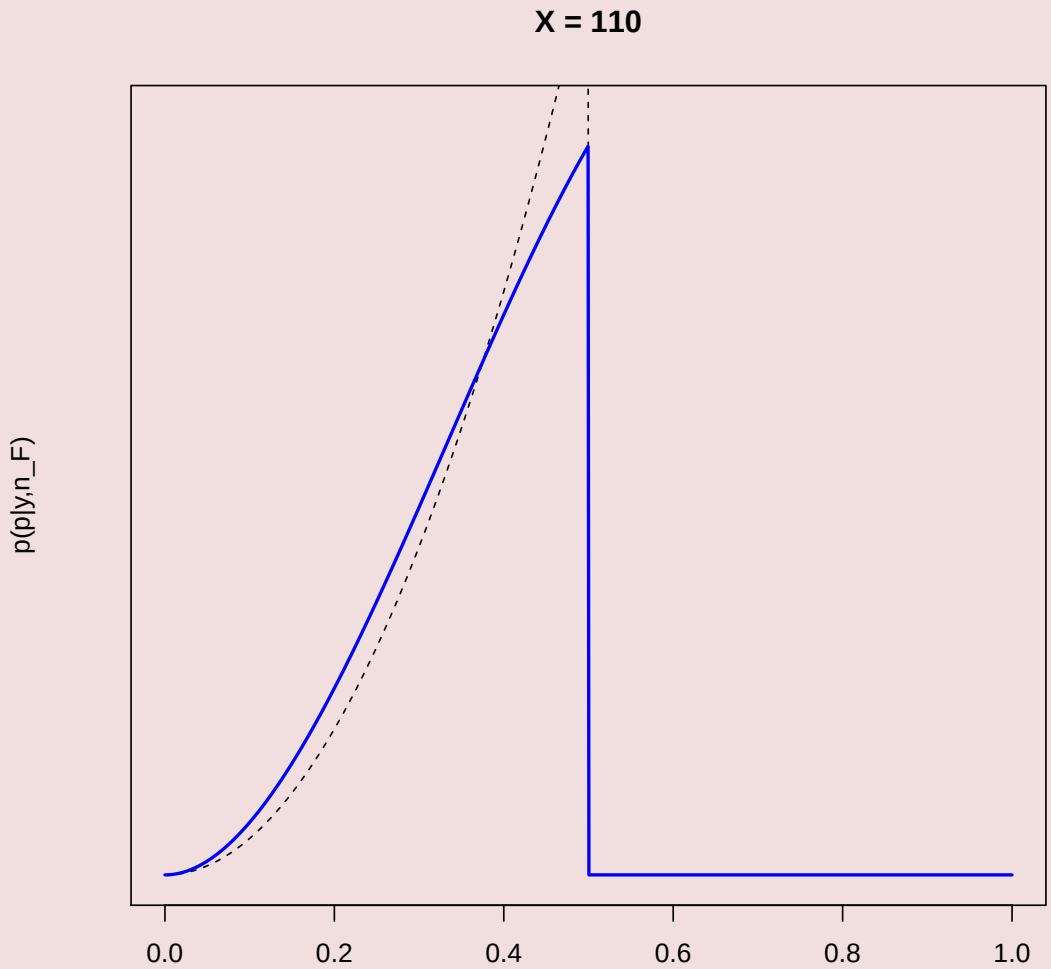


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p

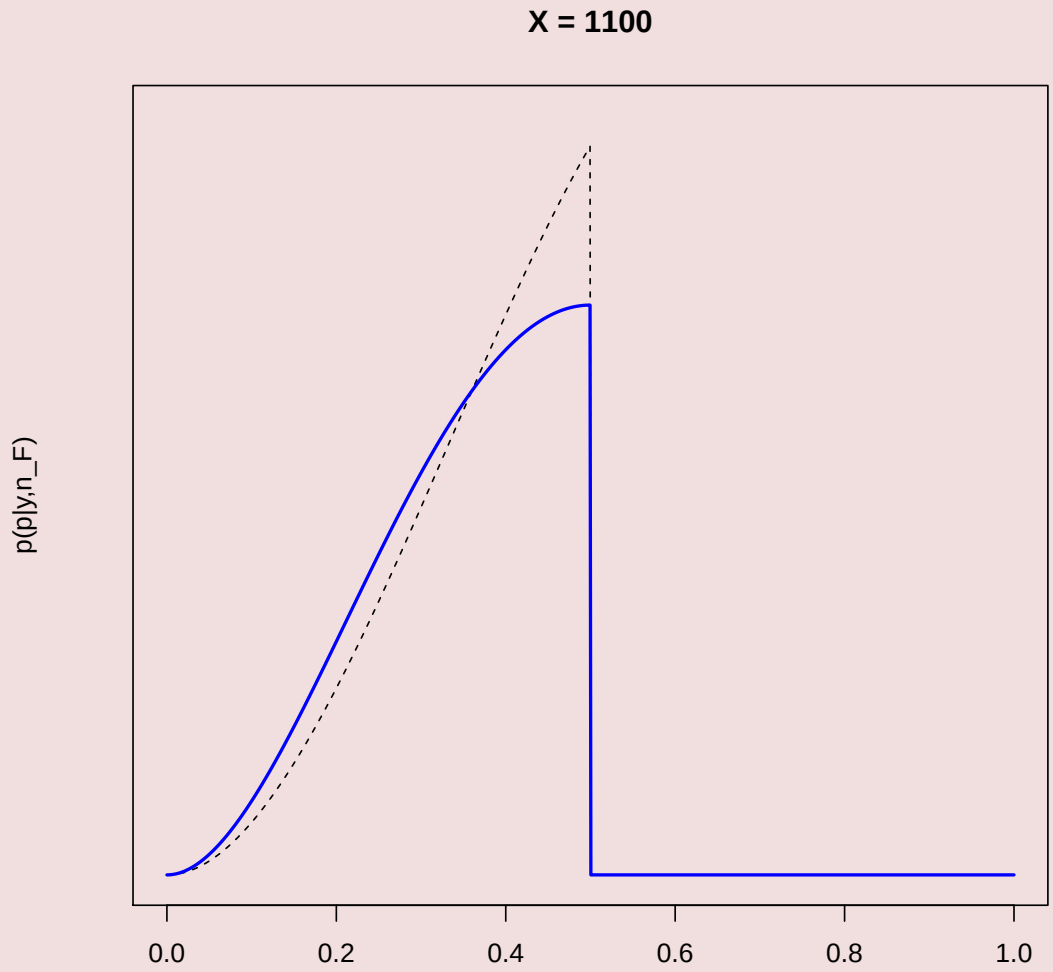


Figure: $p(\overset{\text{p}}{p}|x_1, \ldots, x_n)$

Posterior of p

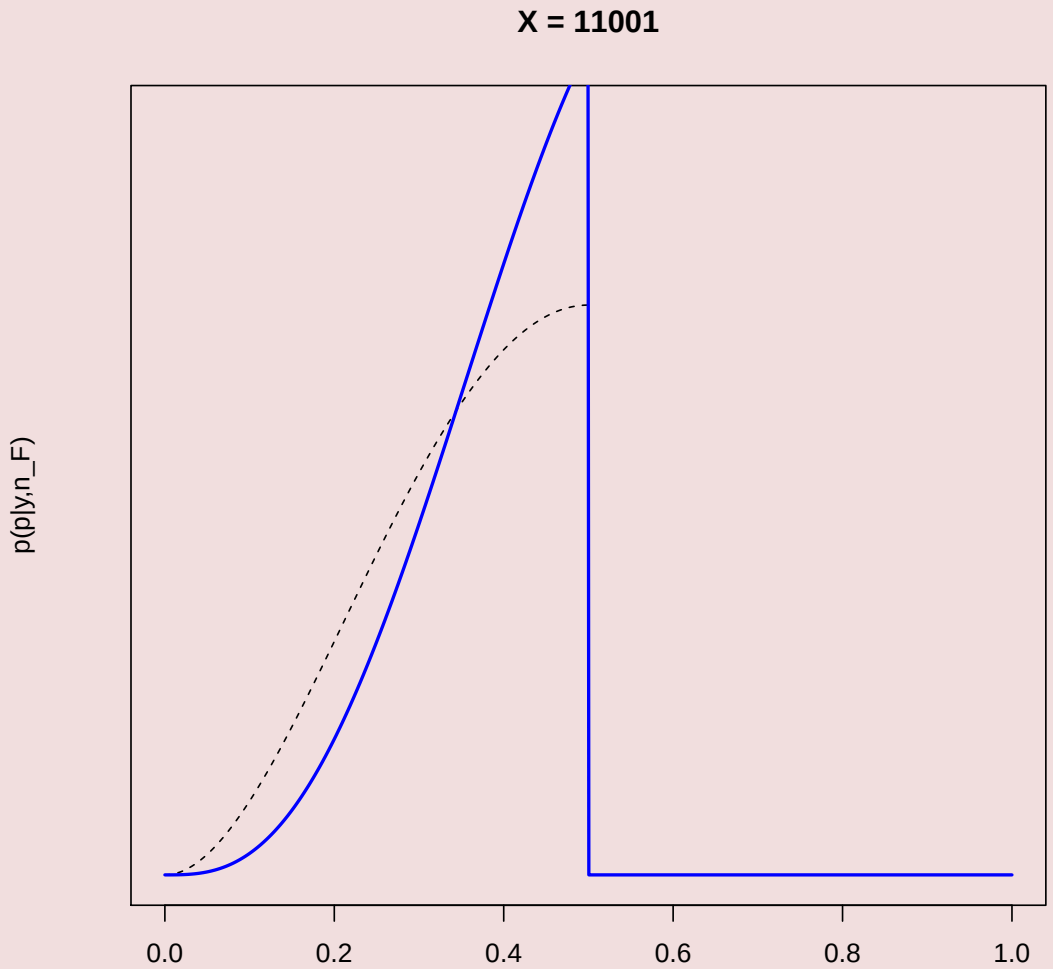


Figure: $p(\overset{\text{p}}{p}|x_1, \ldots, x_n)$

Posterior of p

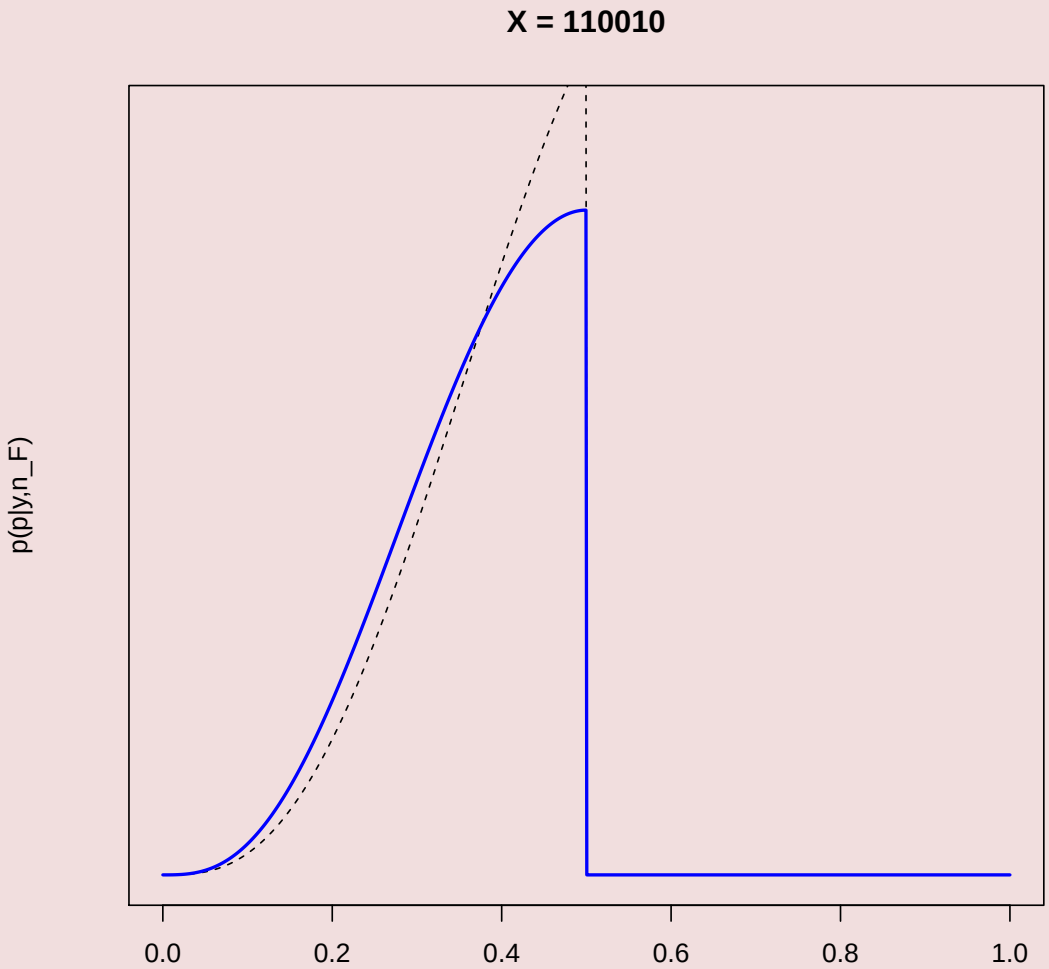


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p

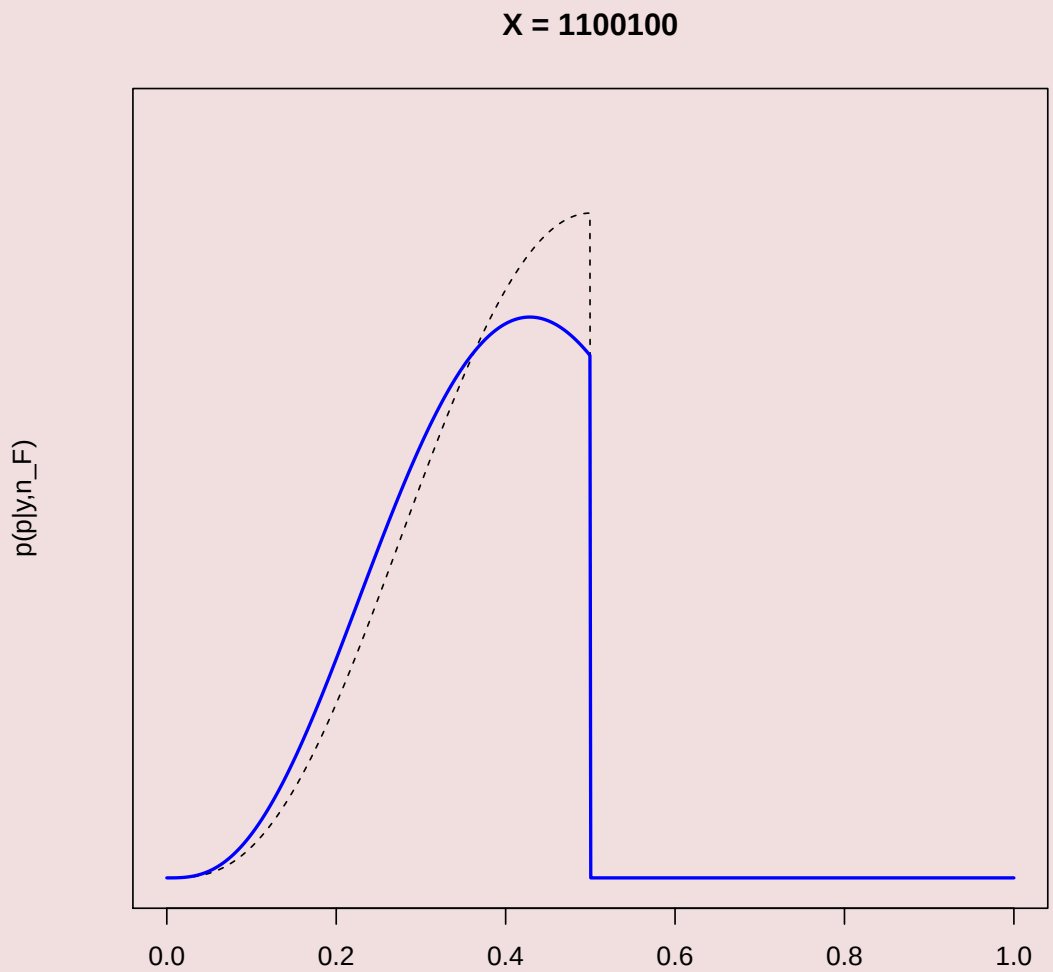


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p

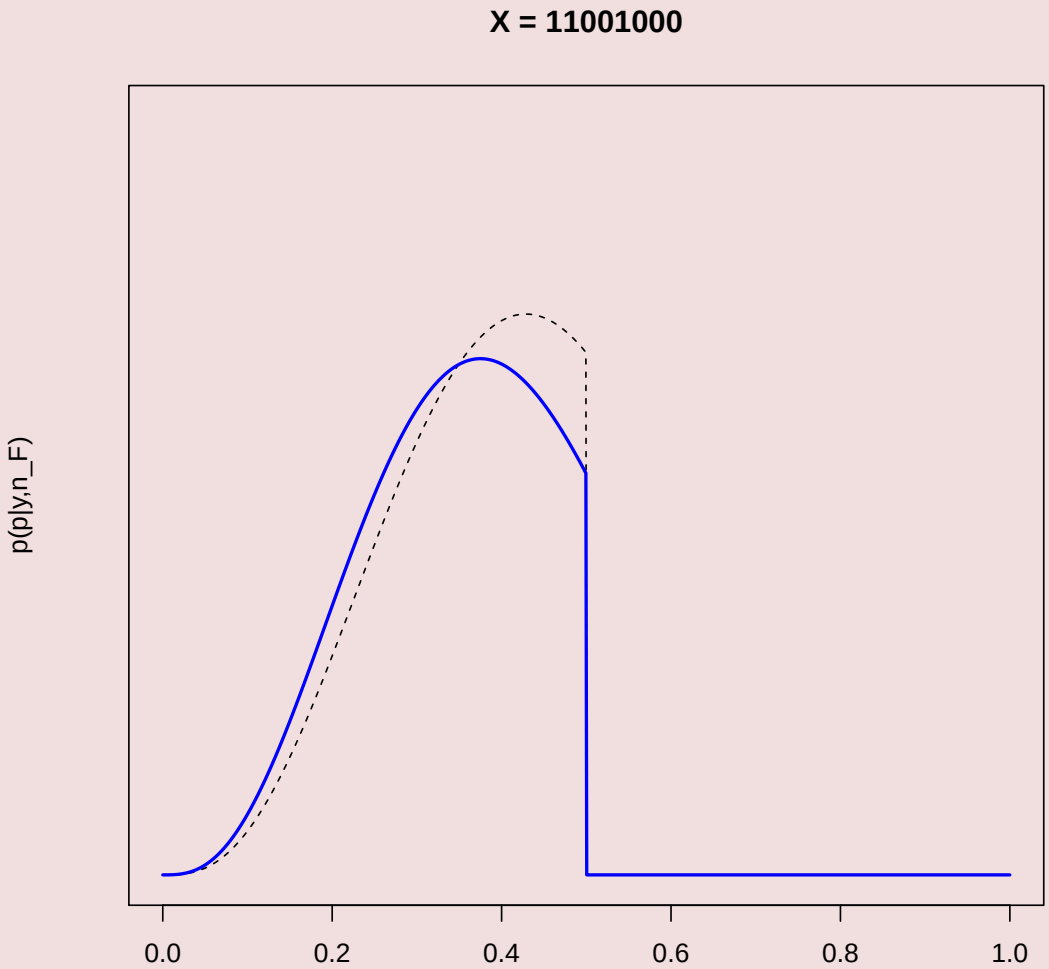


Figure: $p(p|x_1, \ldots, x_n)$

Posterior of p

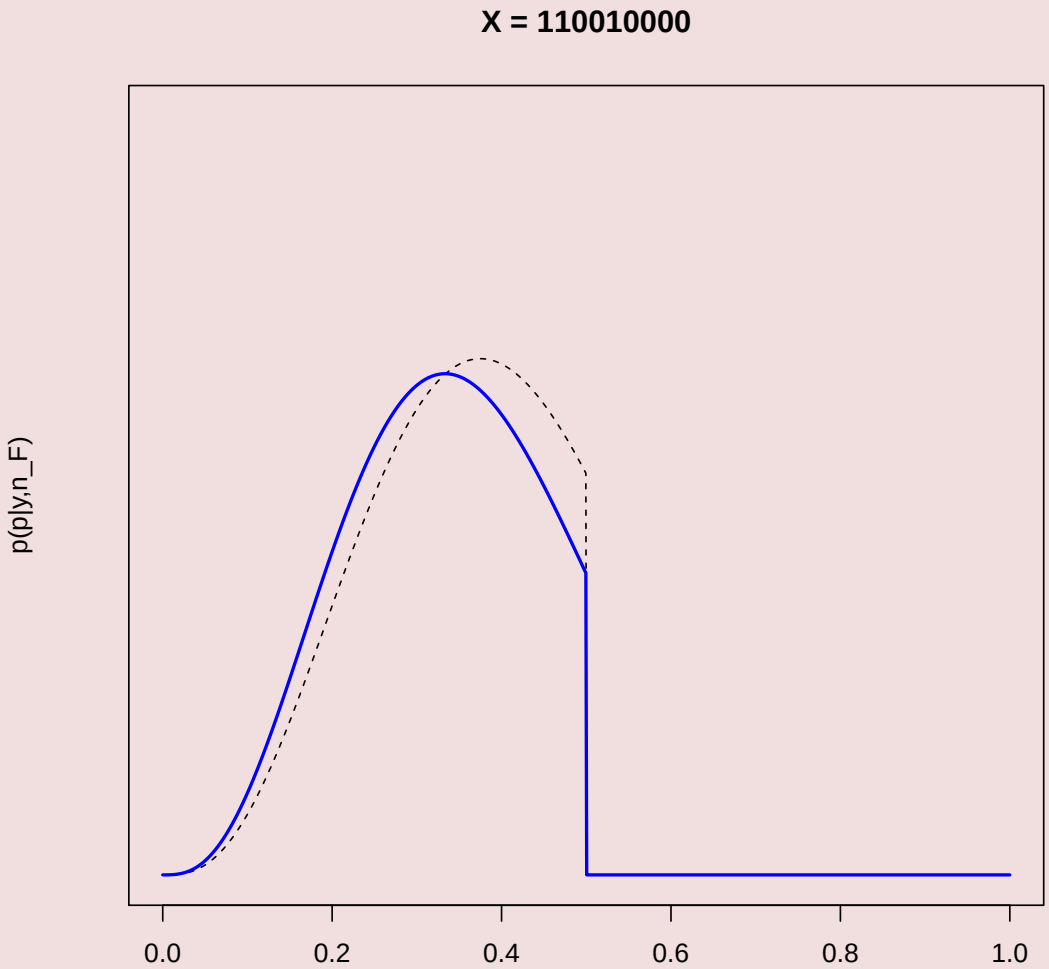
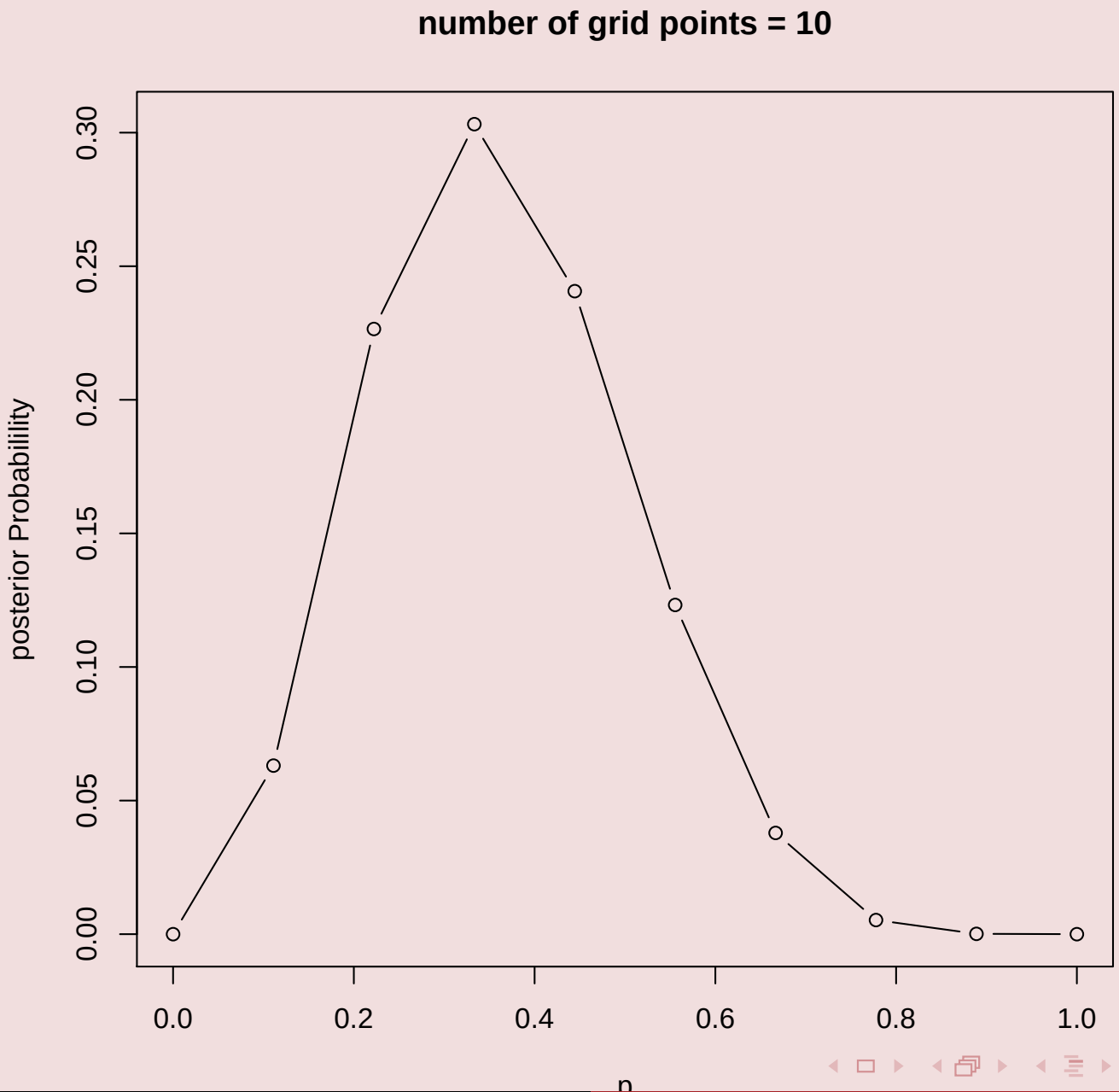


Figure: $p(p|x_1, \ldots, x_n)$

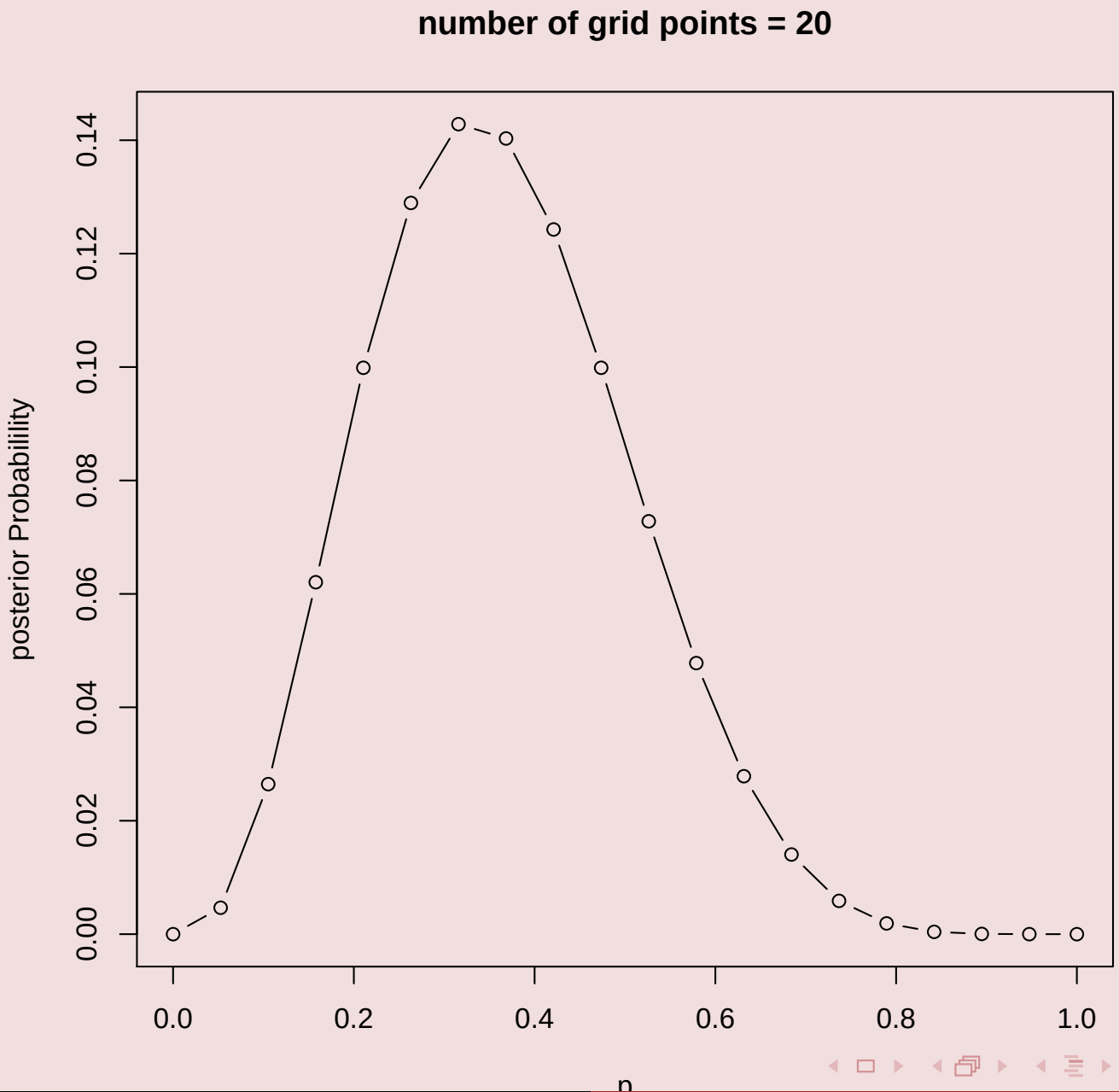
Producing the Figure in R, grid approximation

```
n_grid <- 10 # how fine grid
p_grid <- seq( from  =0, to =1 , length.out = n_grid)

#prior
prior <- rep(1, n_grid)

#posterior
likelihood <- dbinom(x =3, size= 9, prob = p_grid)

# unnormalized posterior
unstd.posterior <- likelhiood * prior

# normalized posterior
porsterior <- unstd.posterior  / sum(unstd.posterior )

plot(p_grid, posterior, type='b', ylab='posterior Probabilility', xlab='p',
     main = paste("number of grid points =", n_grid, sep = ""))
```


number of grid points = 10
posterior Probability
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.2
0.4
0.6
0.8
1.0
p

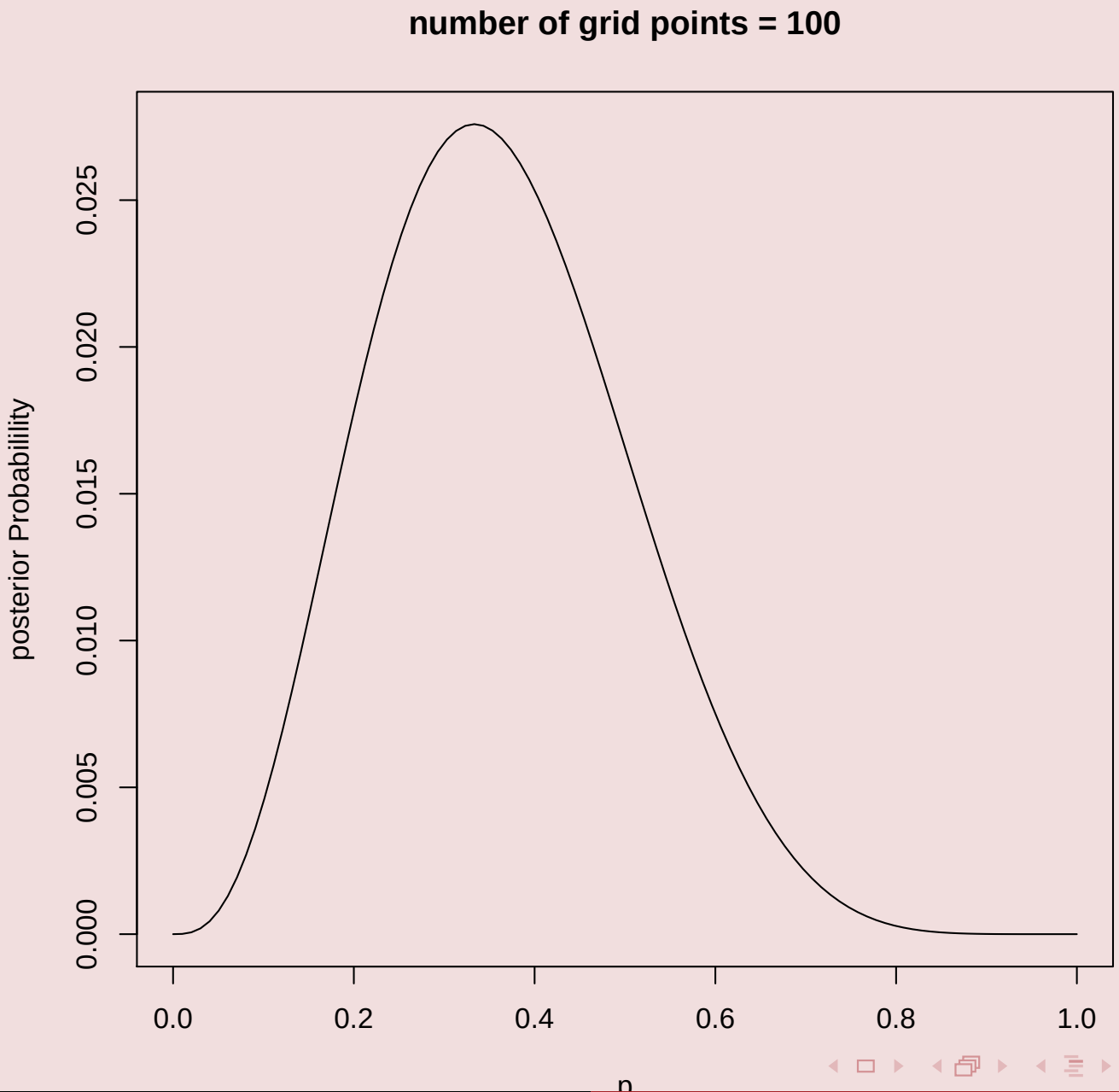
Producing the Figure in R, grid approximation

```
n_grid <- 20 # how fine grid
p_grid <- seq( from  =0, to =1 , length.out = n_grid)

#prior
prior <- rep(1, n_grid)

#posterior
likelihood <- dbinom(x =3, size= 9, prob = p_grid)

# unnormalized posterior
unstd.posterior <- likelhiood * prior

# normalized posterior
porsterior <- unstd.posterior  / sum(unstd.posterior )

plot(p_grid, posterior, type='b', ylab='posterior Probabilility', xlab
main = paste("number of grid points =", n_grid, sep = ""))
```


number of grid points = 20
posterior Probability
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.0
0.2
0.4
0.6
0.8
1.0
p

Producing the Figure in R, grid approximation

```
n_grid <- 100 # how fine grid
p_grid <- seq( from  =0, to =1 , length.out = n_grid)

#prior
prior <- rep(1, n_grid)

#posterior
likelihood <- dbinom(x =3, size= 9, prob = p_grid)

# unnormalized posterior
unstd.posterior <- likelhiood * prior

# normalized posterior
porsterior <- unstd.posterior  / sum(unstd.posterior )

plot(p_grid, posterior, type='l', ylab='posterior Probabilility', xlab
main = paste("number of grid points =", n_grid, sep = ""))
```


number of grid points = 100
posterior Probability
0.000
0.005
0.010
0.015
0.020
0.025
0.0
0.2
0.4
0.6
0.8
1.0
p