Deep Learning Lab 3 Report

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All the tests here are done using only the first batch of data.

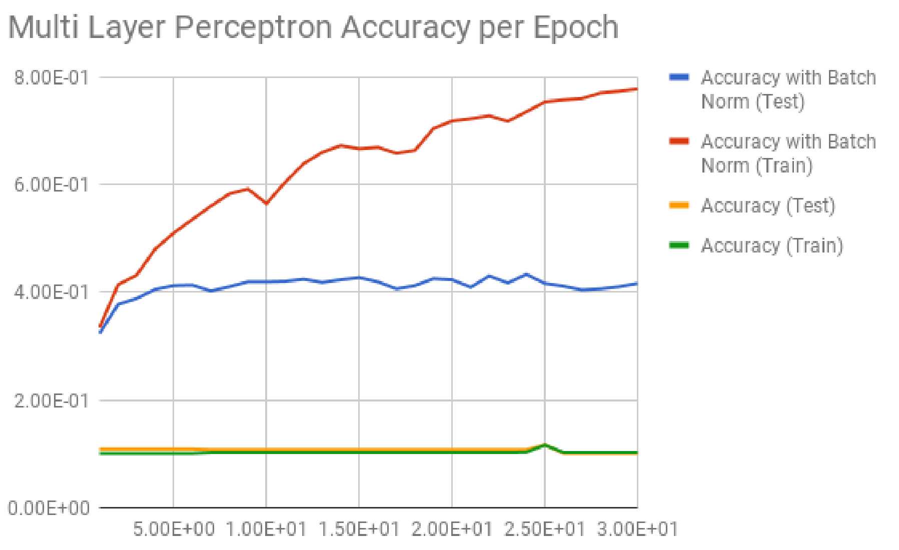
1 Analytic Gradient Checks

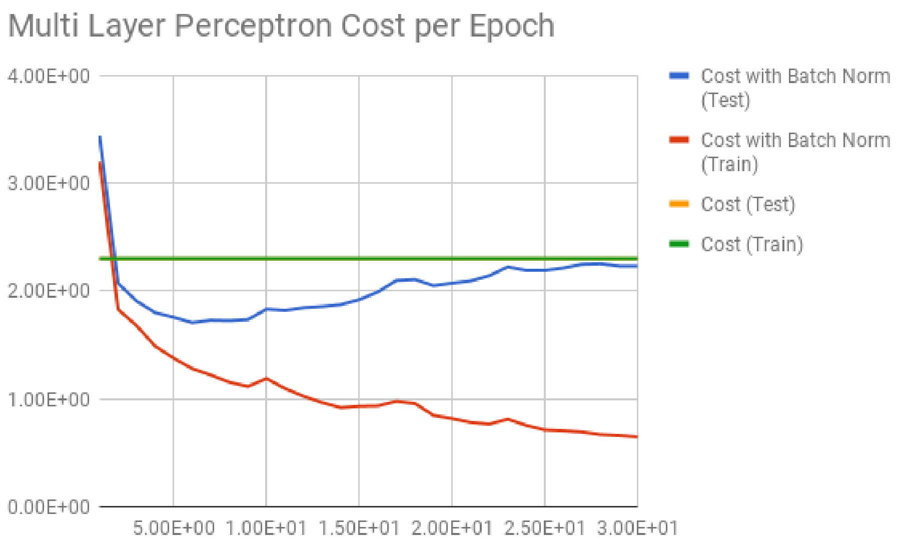
I found the max difference between my gradient computations and the analytic gradient computations for 10 random initialization of the matrix and 10 randomly selected vectors of size 100. The max differences are shown below. There is some discrepancy between the analytic and numerical methods, which may suggest some bug in my gradient code.

$\lvert W_{grad} - W_{slow} \rvert$	$\lvert b_{grad} - b_{slow} \rvert$	$\lvert W_{grad} - W_{fast} \rvert$	$\lvert b_{grad} - b_{fast} \rvert$
5.643239e-05	2.662484e-11	4.620460e-03	4.518226e-07
1.083007e-04	2.316801e-11	1.124352e-02	4.517568e-07
1.888498e-02	1.822172e-11	2.856370e-01	4.518098e-07
2.448560e-02	2.301777e-11	2.637259e-03	4.517750e-07
9.152127e-04	2.615631e-11	2.168040e-02	4.517268e-07
3.832875e-02	2.889897e-11	1.626128e-02	4.517098e-07
4.299322e-05	1.613767e-11	7.286867e-03	4.517778e-07
2.085649e-04	3.041160e-11	1.559779e-02	4.517694e-07
8.994879e-05	2.790417e-11	4.172718e-03	4.518894e-07
1.391745e-05	1.948573e-11	1.794065e-03	4.518356e-07

2 3 Layer Network With and Without Batch Normalization

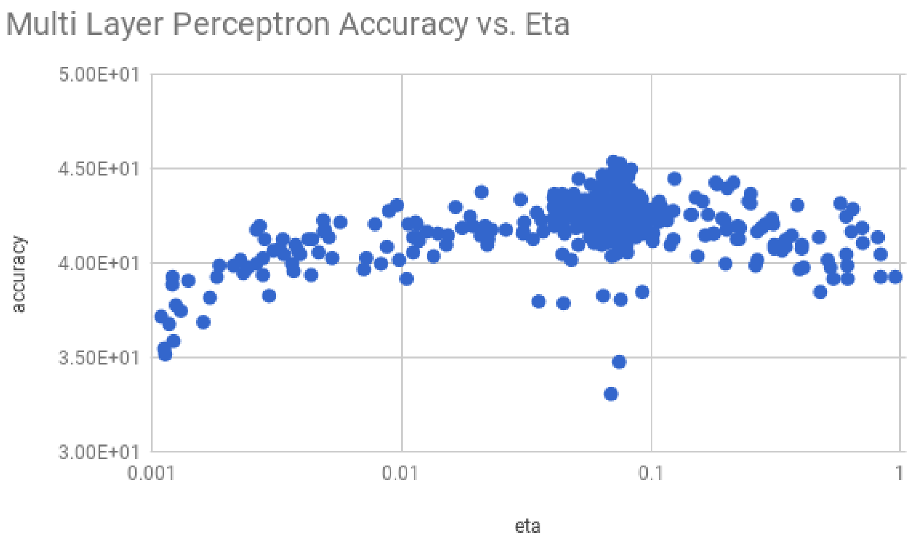
Below are graphs of accuracy and cost with and without batch normalization. Without batch normalization there is virtually no decrease in the cost of the model.

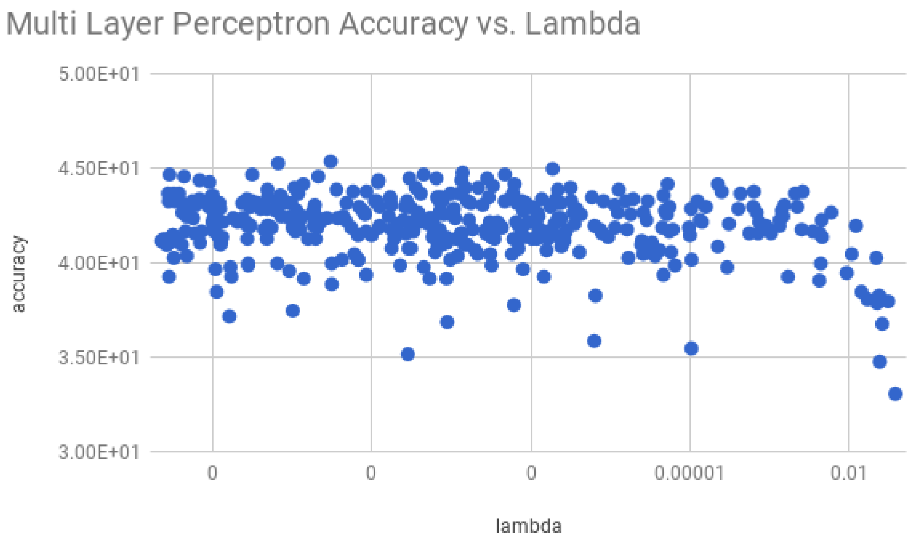




3 Searching For Eta and Lambda

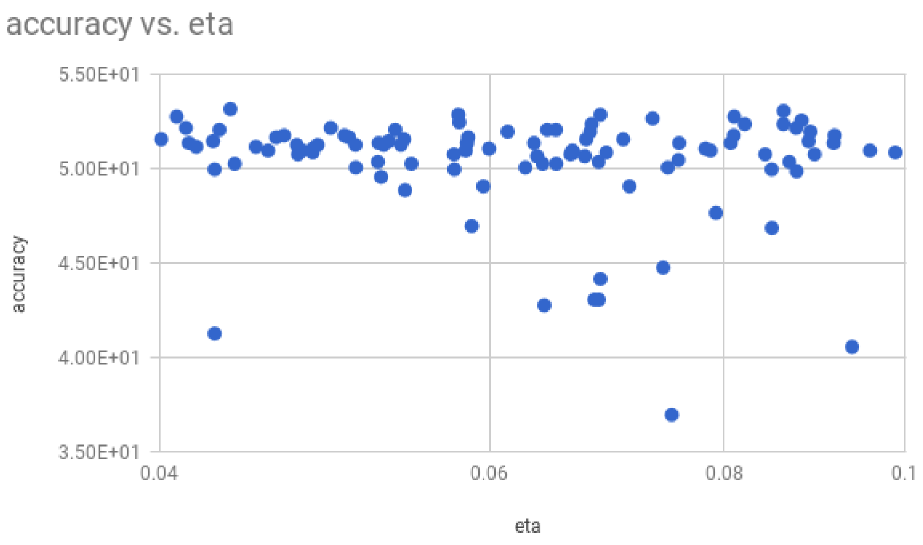
I searched for `eta` from 0.001 to 1, and from 0.05 to 0.1. I searched for `lambda` from $1e - 15$ to 1 and $1e - 15$ to $1e - 8$. The highest test accuracy was 45.7E% and the lowest cost was 1.70%.

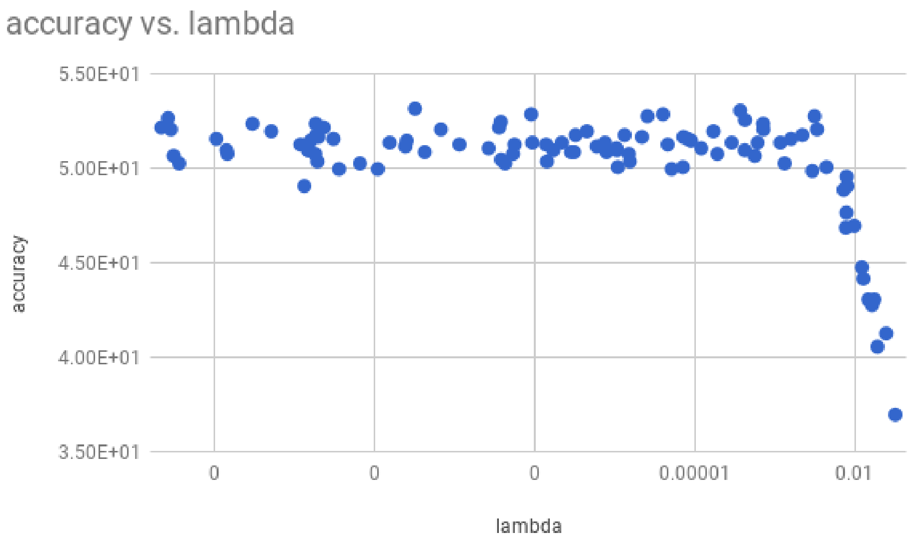




4 2 Layer Network

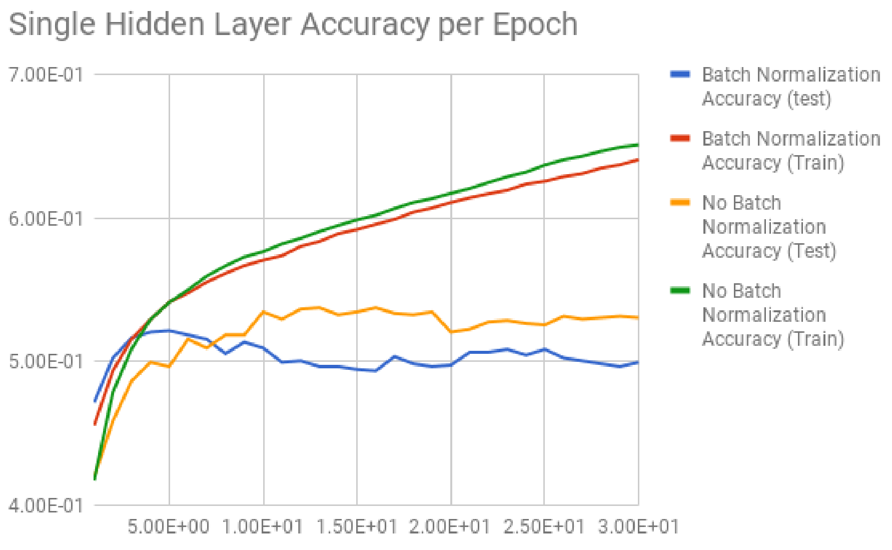
I performed a search for eta and lambda for a 2 layer network with 50 hidden nodes and with normalization. The results are below.

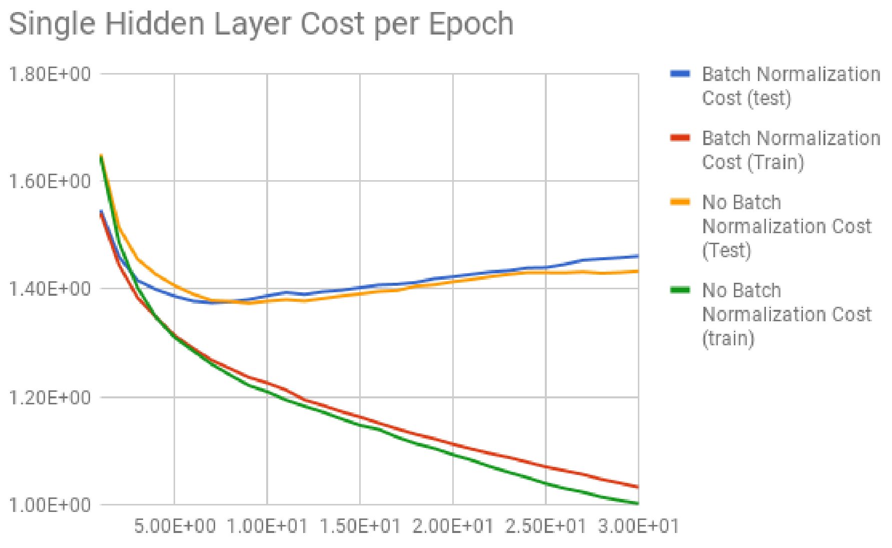




Different values of eta made little difference, and any reasonably small value for lambda made little difference, so I chose `eta = 0.0861` and `lambda = 1e-12`. The hyperparameters for the network with and without normalization are below:

```
batch size:  200 , eta:  0.0861 , epochs:  30 , lambda:  1e-12 , rho:
0.90 , eta_decay:  0.95 , eta_rate:  1 , nodes:  3072 50 10
batch size:  200 , eta:  0.0449 , epochs:  30 , lambda:  1.662508e-12
, rho:  0.90 , eta_decay:  0.95 , eta_rate:  1 , nodes:  3072 50 10
```





With batch normalization the network reaches peak accuracy after < 5 epochs, but without batch normalization it takes around 15.

5 Extra Credit - Optimizing Network

I added early stopping, multiple layers, and an additional fine-tuning training cycle. I also did an exhaustive search for eta. My best accuracy was 55.24%, though the architecture did not always reach this high an accuracy. The architecture and training method could reliably reach an accuracy of 53.95%, which is a 2% improvement over 1 hidden layer of 50 nodes.

I first added early stopping, where the network keeps track of its best weights and returns that. Since this can only improve a network, I added it first. I then implemented the alternate activation function described in the next section. Since it overtrains very quickly, I tried it with hidden layers of size 30 and 40 instead, but this did not change the effectiveness.

I trained with early stopping, ReLu, all data, and the following hyper-parameters:
`batch_size: 200, epochs: 10, hidden: [50 50], rho: 50, decay: 0.9` for several values of eta. I searched for `eta` and `lambda` for the next few architectures, because I wasn't sure how changing the layers would change the optimal hyper parameters. I got the following results:

eta	lambda	accuracy	cost
6.226471e-02	1.543103e-13	5.314685e+01	1.371600e+00
5.455565e-02	7.246277e-12	5.354645e+01	1.320756e+00
8.256417e-02	9.705649e-12	5.374625e+01	1.341339e+00
8.599156e-02	9.526210e-13	5.374625e+01	1.363473e+00
5.155291e-02	5.989056e-12	5.464535e+01	1.317072e+00

The best accuracy was 54.6%. I ran the same architecture, but with 30 nodes in the second layer. This performed worse.

eta	lambda	accuracy	cost
7.902129e-02	1.019798e-12	5.304695e+01	1.360670e+00
5.544915e-02	1.036286e-11	5.304695e+01	1.365088e+00
7.343569e-02	6.439861e-12	5.324675e+01	1.340537e+00
9.268782e-02	8.139354e-13	5.344655e+01	1.333692e+00
5.075492e-02	1.489555e-14	5.374625e+01	1.330342e+00

I then ran the architecture with three layers of 50 nodes:

eta	lambda	accuracy	cost
7.769074e-02	1.679157e-11	5.354645e+01	1.329145e+00
6.617725e-02	1.266021e-14	5.354645e+01	1.366043e+00
5.581979e-02	2.417803e-12	5.384615e+01	1.346099e+00
5.709754e-02	1.012062e-12	5.414585e+01	1.340506e+00
7.319992e-02	2.264835e-13	5.414585e+01	1.354794e+00

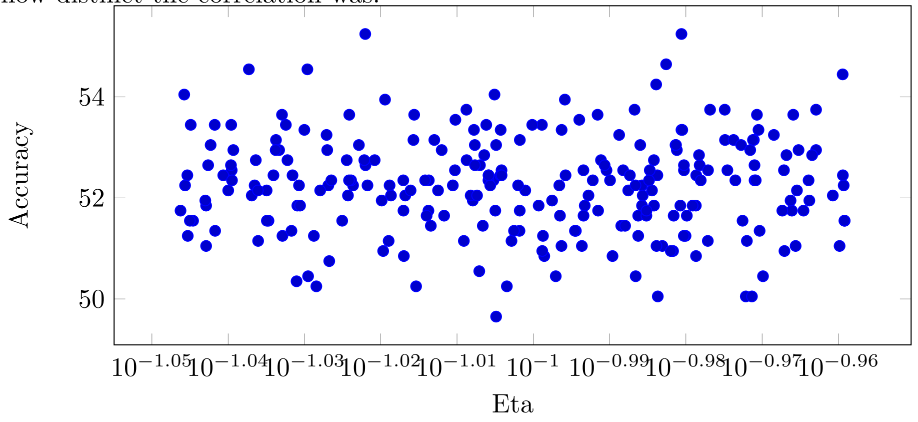
The article mentioned alternating large and small layers, so I tried 50 nodes, 75 nodes, and then 50 nodes again:

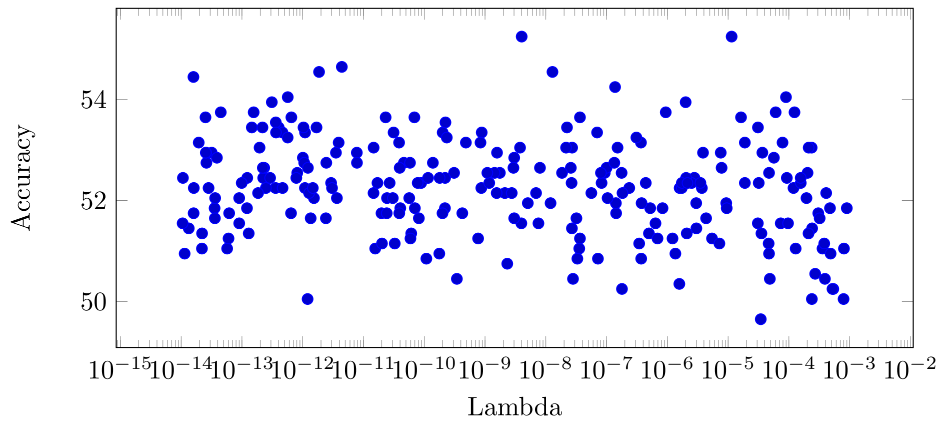
eta	lambda	accuracy	cost
9.473716e-02	1.902223e-14	5.314685e+01	1.339430e+00
8.879906e-02	5.986626e-11	5.324675e+01	1.339534e+00
1.131646e-01	8.952013e-14	5.324675e+01	1.339695e+00
1.034871e-01	1.551209e-14	5.354645e+01	1.348949e+00
1.060341e-01	1.985480e-14	5.484515e+01	1.367991e+00

Then I tried 50 nodes, 100 nodes, and then 50 nodes again. This had the best performance. I conducted another very fine search between 0.09 and 0.11 and got a max accuracy of 55.24%:

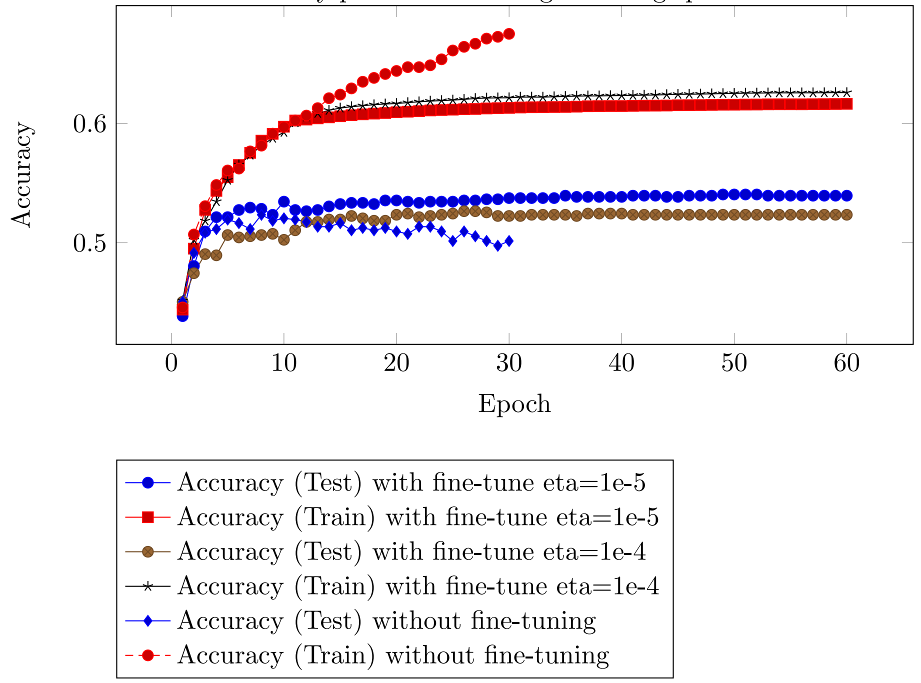
eta	lambda	accuracy	cost
1.099213e-01	1.591922e-14	5.444555e+01	1.348196e+00
9.353371e-02	1.857539e-12	5.454545e+01	1.333234e+00
9.189513e-02	1.287525e-08	5.454545e+01	1.341914e+00
1.042203e-01	4.401982e-12	5.464535e+01	1.387249e+00
9.517717e-02	4.009366e-09	5.524476e+01	1.323682e+00
1.047042e-01	1.142718e-05	5.524476e+01	1.350019e+00

Since this network had the highest performance, and a good bit of variance in values for lambda, I plotted lambda vs accuracy and eta vs accuracy to see how distinct the correlation was.





Each range of values for lambda and eta has a lot of variance, and when I trained with any of these values I found that I could not reliably get a high accuracy. However, the paper given described pre-training with a large eta (like I have done here) and then training again with a very small eta that is several places smaller. So, I pre-trained with `eta=0.1` and `lambda = 1e-12`, and then fine-tuned with `eta=1e-5` and `eta=1e-4` for an additional 50 epochs. I used `eta=0.1` because I could not find any practical advantage to using specific values of eta.



I got the most reliable and best results with an initial eta of 0.1 and a fine-tune eta of $1e - 5$. I achieved a test accuracy of 53.95%, and a test cost of 1.323132 with this method. For comparison, here's the results with just one hidden layer of 50:

eta	lambda	accuracy	cost
9.300533e-02	2.548970e-12	5.144855e+01	1.375621e+00
7.390973e-02	2.192645e-13	5.144855e+01	1.376336e+00
6.772294e-02	2.049641e-12	5.154845e+01	1.386340e+00
5.652835e-02	2.562800e-14	5.154845e+01	1.393548e+00
5.278486e-02	6.282998e-14	5.194805e+01	1.382188e+00

6 Extra Credit - Different Activation function

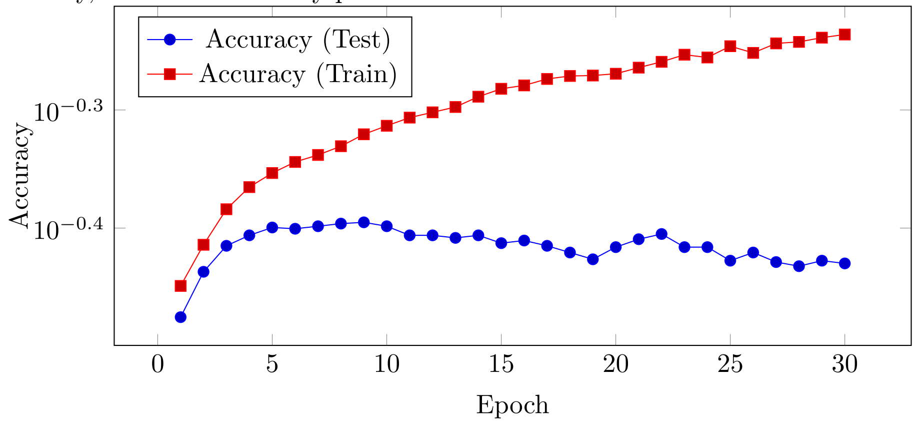
I used the activation function

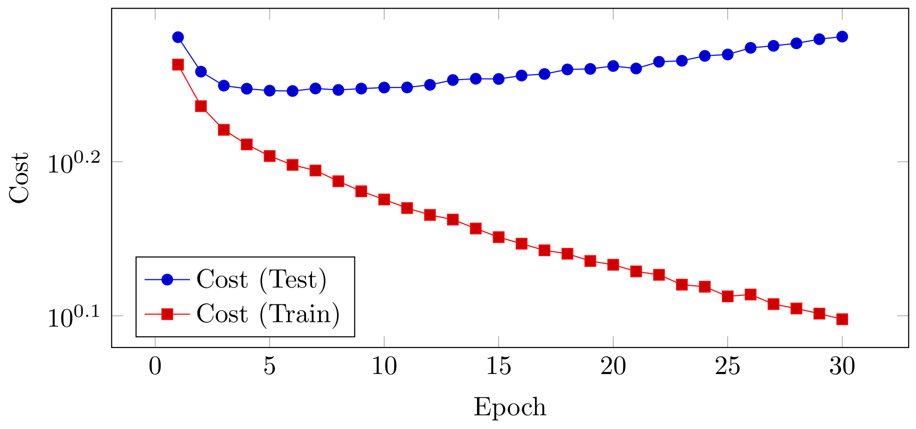
$$\varphi(x) = \frac{2}{1+e^{-x}} - 1$$

with derivative

$$\varphi'(x) = \frac{(1+\varphi(x))(1-\varphi(x))}{2}$$

with the following hyper-parameters:
`batch size: 200, eta: 0.0725, epochs: 30, lambda: 1.662508e-10, rho: 0.90, eta_decay: 0.95, nodes: 3072 50 10`
It trained much faster ReLu, but it also overtrained immediately after reaching its peak. With the above hyperparameters, it peaked in 9 epochs with a test accuracy of 40.3%. As seen below, training accuracy continues to increase steadily, while test accuracy plummits.




Cost
Epoch
$10^{0.2}$
$10^{0.1}$
0
5
10
15
20
25
30
Cost (Test)
Cost (Train)