Submission 2.2

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October 2014

1 Problems 2.8.3

- 1. Since $f(x) = ax^2 + bx + c$ is a quadratic function, then $\frac{-b \pm \sqrt{b^2 4ac}}{2a} = x_0$ where $f(x_0) = 0$. $\frac{-b \pm \sqrt{b^2 4ac}}{2a} = x_0 = \pm 1$, so $f(\pm 1) = 0$.
- 2. The conclusion is stated as a premise.
- 3. S_{xy} : x is continuous at y f: our fcn c: the point in question (in this case, 0) C_{xy} : x converges to y

$$\begin{array}{lll} S_{fc} & \iff \forall_x (C_{xc} \Longrightarrow C_{f(x)f(c)}) & \text{Our definition of continuous.} \\ \neg S_{fc} & \iff \neg \forall_x (C_{xc} \Longrightarrow C_{f(x)f(c)}) & \text{Our definition of not continuous.} \\ & \iff \exists_x \neg (\neg C_{xc} \lor C_{f(x)f(c)}) \\ & \iff \exists_x (C_{xc} \land \neg C_{f(x)f(c)}) \end{array}$$

You have to find a sequence x, such that it converges to c, but f(x) is not continuous at f(c).

4. S_{xy} : x converges to y. x_n : x is a series. $\forall_{x_n}(S_{x_n 3} \Longrightarrow S_{x_n^2-1,3^2-1})$

2 Arguments in English 2.8.4

- 1. b Invalid Unsound
 - d C_{xy} : x is caused by y 1. $\forall_y \exists_x (C_{xy})$ 2. $\exists_y \forall_x (C_{xy})$
- 2. b Valid Invalid
 - d H_{xy} : x hates y f: Fred a: Al 1. $\forall_x(H_{xa} \Longrightarrow H_{fx})$ P_1 2. $\forall_x H_{ax}$ P_1 3. H_{aa} UI(2) 4. $H_{aa} \Longrightarrow H_{fa}$ UI(1) 6. H_{fa} MP(3,4) 7. H_{af} UI(2) 8. $H_{af} \land H_{fa}$ Conj(6,7)

3. b Valid

Soundness is difficult to determine

- d L_x : x is large and hostile
 - I_x : x is impervious to pesticides
 - i: An insect in this house
 - 1. $\forall_i(L_i)$
 - $2. \exists_i (I_i)$ P_2
 - 3. L_x UI(1)
 - EI(2)4. I_x
 - 5. $L_x \wedge I_x$ Conj(3,4)
 - 6. $\exists_i (L_i \wedge I_i)$ EG(5)

4. b Valid

Sound

- d S_x : x can succeed at the university
 - B_x : x is bright

 M_x : x is mature

s: student

- 1. $\exists_s \neg S_s$
- P_1 $2. (B_s \wedge M_x) \implies (S_s)$ P_2
- 3. $\neg(S_s) \implies \neg(B_s \land M_x)$ CP(2)
- 4. $\neg(S_s) \implies \neg B_s \vee \neg M_x$ DM(3)
- 5. $\neg S_x$ EI(1)
- 6. $\neg B_x \lor \neg M_x$ MP(4,5)
- 7. $\exists_x (\neg B_x \vee \neg M_x)$ EG(6)
- b Valid 5.

Sound

- d P_x : x is a pig
 - 1. $\exists_x \exists_y \exists_z ((x \neq y) \land (x \neq z) \land (z \neq y) \land P_x \land P_y \land P_z)$
 - 2. $\exists_x \exists_y ((x \neq y) \land (x \neq a) \land (a \neq y) \land P_x \land P_y \land P_a)$ EI(1)
 - 3. $\exists_x \exists_y ((x \neq y) \land P_x \land P_y)$ Simp(2)

b Valid Sound 6.

- d L_{xy} : x likes y
 - p: Popeye
 - o: Olive Oyl
 - 1. $\forall x(L_{xo} \implies L_{px}) \quad P_1$
 - 2. $\forall x(L_{ox})$ P_2
 - UI(2)3. L_{oo}
 - 4. $L_{oo} \implies L_{po}$ UI(1)
 - 5. L_{po} MP(3,4)
 - 6. L_{op} UI(2)
 - 6. $L_{po} \wedge L_{op}$ Conj(5,6)
- a S_x : x is sound

 C_x : x has a true conclusion

a: this argument

$$\neg C_a \\ \neg \exists_x (S_x \land \neg C_x) \\ \neg S_a$$

b Valid

Sound

- c Assume there is an interpretation in which $\neg C_x$ and $\neg \exists_x (S_x \land \neg C_x)$ are both true, but $\neg S_x$ is false. So there must be some element of UD, call it 1, such that $(1 \ni S)$ and $\neg (1 \ni C)$. However, the second premise would be false in this interpretation. $\rightarrow \leftarrow$ No such interpretation.
- d S_x : x is sound

 C_x : x has a true conclusion

a: this argument

- 1. $\neg C_a$ P_1 2. $\neg \exists_x (S_x \land \neg C_x)$ P_2 3. $\forall_x \neg (S_x \land \neg C_x)$ QEx(2)4. $\forall_x (\neg S_x \lor \neg \neg C_x)$ DM(3)5. $\forall_x (\neg S_x \lor C_x)$ DN(4)6. $\neg S_a \lor C_a$ UI(5)7. $S_a \Longrightarrow C_a$ CSis(6)
- 7. $S_a \Longrightarrow C_a$ CSis(6 8. $\neg C_a \Longrightarrow \neg S_a$ CP(7)
- 4. $\neg S_a$ MP(1,8)
- 8. a O_x : x weighs over 200 pounds
 - j: Jones' killer
 - s: Smith

$$\begin{array}{c}
O_j \\
\neg O_s \\
\hline
O_s \neq O_j
\end{array}$$

- c Assume there is an interpretation in which O_x and $\neg O_y$, but not $O_y \neq O_x$. So there must be two elements of UD, call them 1 and 2, such that $1 \ni O$ and $\neg (2 \ni O)$, and 2 = 1.
- d O_x : x weighs over 200 pounds
 - j: Jones' killer
 - s: Smith

Simith
$$O_{j} \qquad P_{1}$$

$$\neg O_{s} \qquad P_{2}$$

$$\neg (O_{s} \iff O_{j})$$

$$\forall_{x} \forall_{y} (x = y \implies (P_{x} \iff P_{y})) \qquad \text{Leibniz}$$

$$\forall_{x} \forall_{y} (\neg (P_{x} \iff P_{y}) \implies x \neq y) \qquad \text{CP}$$

$$O_{s} \neq O_{j} \qquad \text{MP}$$

9. b Valid

Not sound

- d L_{xy} : x likes y
 - m: mandy
 - a: andy

1.
$$\forall_x(L_{xm})$$
 P_1
2. $\forall_x(L_{mx} \iff (m=a))$ P_2
3. L_{mm} $UI(1)$
4. $L_{mm} \iff (m=a)$ $UI(2)$
5. $(L_{mm} \implies (m=a)) \land ((m=a) \implies L_{mm})$ Equiv(4)
6. $L_{mm} \implies (m=a)$ $Simp(5)$
7. $m=a$ $MP(3,6)$

10. b Valid

Sound

- d A_{xy} : x if afraid of y
 - h: Mr. Hyde
 - j: Dr. Jekyll

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1. \forall_x(A_{xh})

2. \forall_x(A_{hx} \iff (x=j))

3. A_{hh} UI(1)

4. A_{hh} \iff (h=j) UI(2)

5. (L_{hh} \implies (h=j)) \land ((h=j) \implies L_{hh}) Equiv(4)

6. L_{hh} \implies (h=j) Simp(5)

7. j=h
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3 Arguments in quantificational logic 2.8.5

- 2. b Assume the premise is true. In order for it to be true in all cases that an element of F must be within G, all elements of F must be contained within G. Therefore, if an element is not in G, it cannot be within F.
 - $\begin{array}{ccc} & 1. \ \forall_x (F_x \Longrightarrow G_x) & P_1 \\ \text{c} & 2. \ F_x \Longrightarrow G_x & \text{UI}(1) \\ & 3. \ \forall_x (\neg G_x \Longrightarrow \neg F_x) & \text{UG}(2) \end{array}$