Submission 3.1

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Cheshvan 2014

1. Lxy : x loves y
   No

2. Mxy :x is the mother of y
   No

3. Cxy : x is the child of y
   Yes

4. Hxy : x is the height of y
   No

5. Hxy : y is the height of x
   No

6. Fxy : y is the first (legal) wife of x
   No (not everybody is married)

7.

1.	$\forall_x(f(x,0)=x)$	P_1
2.	$\forall_x(f(x,g(x))=0)$	P_2
3.	$\forall_x\forall_y f(x,y)=f(y,x)$	P_3
4.	$\exists_x(f(x,0)=x)$	EG(1)
5.	$\exists_x(f(x,g(x))=0$	EG(2)
6.	$\exists_x\exists_y(f(x,y)=f(y,x))$	EG(3)
7.	$f(0,g(0))=0$	EI(5)
8.	$f(0,g(0))=f(g(0),0)$	EI(6)
9.	$f(g(0),0)=0$	Subst(7,8)
10.	$\exists_a(f(g(a),0)=a)$	EG(9)
11.	$\exists_a(a=g(a))$	Subst(4,10)

1.	$\forall_x(f(x,0)=x)$	P_1
2.	$f(0,0)=0$	$UI(1)$
3.	$\exists_y(f(y,y)=y)$	$EG(2)$

8. $f:\mathbb{R}^2 \implies \mathbb{R}$
   $g:\mathbb{R} \implies \mathbb{R}$
   $f(x,y)=x+y$
   $g(x)=-x$

9. Neither

10. Injective but not surjective

11. Both

12. Surjective but not injective

13. Both
    Every natural number has a unique representation in roman numerals, therefore f is surjective. No two natural numbers have the same representation in Roman Numberals. Therefore f is injective.

14. Surjective but not injective
Consider that $f(1) = a$ and $f(27) = a$ and $1 \neq 27$, so f is not injective. Additionally, and that the domain is $\mathbb{R}$, while the codomain is restricted to letters a through z, and every element in B is pointed to by at least one element in A. Therefore f is surjective.

15. Injective but not surjective
$\neg\exists x(f(x) = 1 \wedge x \in Domain(f))$. Therefore f is not surjective. If $f(x_1) = y = f(x_2)$, then $2x_1 = 2x_2$ so $x_1 = x_2$. Therfore f is injective.

16. Both
Given any $y \in Codomain(f)$, $f(y-1) = y$, so $\forall y \exists x(y \in Codomain(f) \implies (f(x) = y \wedge x \in Domain(f)))$, so f is surjective. If $f(x_1) = y = f(x_2)$, then $x_1 - 1 = x_2 - 1$ so $x_1 = x_2$. Therefore, f is injective.

17. Neither
$\neg\exists x(f(x) = -1 \wedge x \in Domain(f))$, so f is not surjective. $f(-2) = 4 = f(2)$, but $2 \neq -2$, so f is not injective.

18. Both
Given any $y \in Codomain(f)$, $f(\sqrt[3]{y}) = y$. Therefore $\forall y \exists x(y \in Codomain(f) \implies (f(x) = y \wedge x \in Domain(f)))$. Therefore f is surjective. If $f(x_1) = y = f(x_2)$, then $x_1^3 = x_2^3$. $f(\sqrt[3]{x_1}) = f(\sqrt[3]{x_2})$, where $\sqrt[3]{x}$ is a function, so $x_1 = x_2$. Therfore f is injective.

19. Injective but not surjective
$\neg\exists x(f(x) = -1 \wedge x \in Domain(f))$. Therefore f is not surjective. $f(x_1) = f(x_2) \implies f(ln(x_1)) = f(ln(x_2))$, where $ln(x)$ is a function, $\implies e^{ln(x_1)} = e^{ln(x_2)} \implies x_1 = x_2$. Therefore f is injective.

20. Neither
The number 2 exists in the Codomain of f, but $\sin^{-1} 2$ does not exist. Therefore, not all things in the Codomain of f exist in the image, so if is not surjective. $f(0) = 0 = f(2\pi)$. Therefore f is not injective.

21. Surjective but not injective
Given any $y \in Codomain(f)$, $f(\sin^{-1} y) = y$. Therefore $\forall y \exists x(y \in Codomain(f) \implies (f(x) = y \wedge x \in Domain(f)))$. Therefore f is surjective.

$f(0) = 0 = f(2\pi)$. Therefore f is not injective.

22. Surjective but not injective
Given any $y \in Codomain(f)$, $f(y) = y$. Therefore $\forall y \exists x(y \in Codomain(f) \implies (f(x) = y \wedge x \in Domain(f)))$. Therefore f is surjective.

$f(.2) = 0 = f(.1)$, but $\frac{1}{2} \neq \frac{1}{3}$.