Get logic'd

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August 2014

1 Day 1

If I am deceived, I exist.
If I am not deceived, I do not exist.
I exist.

2 Day 2: Validity

An argument is **valid** if and only if the truth of its premises guarantees the truth of the conclusion. (i.e. there is no case in which all premises are true and the conclusion is false)
An argument is **vacuously valid** if and only if it is valid and has inconsistent premises.
An argument is **sound** if it is both valid and the premises are true.
A set of sentences is **consistent** if and only if there is a case that all sentences are true.
An argument is **defeasibly valid** if and only if, if the premises are true, it's reasonable to expect the conclusion to be true as well.
A **counterexample** to an argument is a circumstance in which the premises are true and the conclusion is false.
A proof technique is called **formal** if and only if it can be carried out by a computer.
Theorem-argument exchange theorem?

3 Day 3: Operators

If I am deceived, then I exist.
If I am not deceived, then I exist
A: I am deceived.
B: I exist.

Logical Operators
negation $\neg$ not

biconditional $\iff$ if and only if, necessary and sufficient, exactly when
conditional $\implies$ if-then, A only if B
disjunction $\vee$ or
conjunction $\wedge$ and

$((p \implies q) \wedge p) \implies q$ Modus ponens
$((p \implies q) \wedge !q) \implies !p$ Modus tollens

4 Day 4: More definitions and truth tables

Case: An assignment of truth values to atomic sentences.

If Einstein's theory of relativity is correct, light bends in the vicinity of the sun.
Light bends in the vicinity of the sun.
Einstein's theory of relativity is correct.
E:Einstein's theory of relativity is correct.
L:Light bends in the vicinity of the sun.

E	L	$E \implies L$	L	E
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

A	G	L	$A \implies \neg G$	$L \implies A$	$G \implies \neg L$
T	T	T	F	T	F
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \implies B$	$A \iff B$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

5 Day 5: To prove validity

Raymond Smullyan

5.1 Formal Ways

Truth Tables, see Day 4

5.2 Informal Ways

Conditional Proof
Assume all premises are true, apply definitions, past theorems. Pure light of reason!.
Show the conclusion is also true.
Proof by contradition
Assume argument is invalid (i.e., there's a counter example, i.e. there's a case where all prepositions are true and c is false.)

6 Day 6:Logical equivalencies

Don't (drink and drive).
Don't drink or don't drive.
Drink nand drive.
Drink only if not driving.

6.1 Logical Equivalencies

DeMorgan's Law:$\neg(A \wedge B) \iff (\neg A \vee \neg B)$
Conditional Disjunction:$(A \implies B) \iff (\neg A \vee B)$
Contra-positive:$(A \implies B) \iff (\neg B \implies \neg A)$
Definition of Equivalence:$(A \iff B) \iff ((A \implies B) \wedge (B \implies A))$
Distribution of And over Or:$(A \wedge (B \vee C)) \iff ((A \wedge B) \vee (A \wedge C))$
Distribution of Or over And:$A \vee (B \wedge C) \iff ((A \vee B) \wedge (A \vee C))$

6.2 Some definitions and stuff

Main operator: The most "outer" operator in a logical statement.
Tautology: Always true; Logically equivalent.

Logically equivalent : Tautology :: Contradictory sentences : Contradiction
Proofs of iff: Prove $A \implies B$ and $B \implies A$ to prove $A \iff B$.

7 Day 7:Rules of Inference

$$\frac{A}{\begin{array}{l}(A \implies B) \implies B \\ (A \vee B)\end{array}}$$

7.1 Natural Deduction

1.	$A \implies \neg G$	P_1
2.	$L \implies A$	P_2
3.	$G \implies \neg A$	$ContraPos(1)$
4.	$\neg A \implies \neg L$	$ContraPos(2)$
6.	$G \implies \neg L$	$HypotheticalSyllogism$

1.	$M \iff H$	P_1
2.	$M \wedge \neg H$	P_2
3.	M	$Simp(2)$
4.	H	$DefOfEq(3)$
5.	$\neg H$	$Simp(2)$
6.	$H \vee E$	$Add(4)$
7.	E	

1.	$O \iff E$	P_1
2.	$\neg H \implies \neg E$	P_2
3.	$H \implies \neg E$	P_3
4.	$H \vee \neg H$	$Taut$
5.	$\neg E \vee \neg E$	$CD(2,3,4)$
6.	$\neg E$	Rep
7.	$\neg E \vee \neg O$	Add

8 Day 8

A sentence is a **contradiction** iff it can be worked into the form $A \wedge \neg A$.
A sentence is a **tautology** iff it can be worked into the form $A \vee \neg A$.
$\neg(G \wedge M)$
$\neg M \implies S$
$S \implies \neg G$
$\neg M \implies \neg G$
$G \implies M$
$G \vee \neg M$
$\neg G \vee \neg M$
Valid
Assume all premises are true. By DeMorgan's Law, $\neg G \vee \neg M$

9 Day 12

Universal quantification: $\forall$ for all
Existential quantification: $\exists$ there exists

Subjects are lowercase letters, predicates are uppercase letters.

M_s	"Socrates is a man"
$\forall_x(M_x \implies R_x)$	"For all x, if x is a man, x is mortal"
R_s	"Socrates is mortal"

10 Day 13

$c:$	Charlie
$P_x:$	x is a pig
$E_{xy}:$	x = y
P_c	

"I have at least 2 pigs."
$\exists_x \exists_y (x \neq y \wedge P_x \wedge P_y)$
P_x: x is a Pig

"I have only 2 pigs."
$\exists_x \exists_y \forall_z (x \neq y \wedge x \neq z \wedge y \neq z \wedge P_x \wedge P_y \wedge \neg P_z)$ No! Bad!

$\exists_x \exists_y \forall_z [(x \neq y \wedge P_x \wedge P_y) \wedge \neg(x \neq y \neq z \wedge P_z)]$
$\exists_x \exists_y (x \neq y \wedge P_x \wedge P_y \wedge \forall_z ((x \neq z \wedge y \neq z) \implies \neg P_z))$
$\exists_x \exists_y (x \neq y \wedge P_x \wedge P_y \wedge \forall_z (P_z \implies (x = z \vee y = z))$

Leibniz's Law: Indiscernibility of Identicals.
$\forall_a \forall_b (a = b \implies (P_a \iff P_b))$
$\forall P$ would be invalid, because it is $2^{\underline{nd}}$ order logic.

11 Day 14

S_x: x is sound
C_x: x's conclusion is true
$\neg S_x \wedge \neg C_x \wedge \neg \exists_x (S_x \wedge C_x)$

"Consider the case in which.." NOT "Assume..."

Here is an invalid argument:

$\exists_x \exists_y (P_x \wedge P_y \wedge x \neq y)$
$\exists_x \exists_y \exists_z (P_x \wedge P_y \wedge P_z \wedge x \neq y \wedge x \neq z \wedge y \neq z)$

$UD : 1, 2, 3 \leftarrow Universe of discourse$
$P_x = 1, 2 \leftarrow extensions$

case:TFL::interpretation:QL

Universe of Discourse(UD): The universe we're discoursin' about.

Everything has a cause. Therefore, something is the cause of everything.
C_{xy}: x was caused by y.

$$\frac{\forall_x(\exists_y(C_{xy}))}{\exists_y(\forall_x(C_{xy}))}$$

$UD : \{1, 2\}$
$C_{xy} = \{(1, 2), (2, 1)\}$

12 day 15

Universes of Discourse cannot be empty, but predicates can.

$$\begin{array}{l} \neg\exists_x \ni (F_x \wedge G_x) \\ \forall_x(G_x \implies H_x) \\ \hline \forall_x(F_x \implies \neg H_x) \end{array}$$

$UD = \{x\}$
$F = \{x\}$
$G = \{\}$
$H = \{x\}$

To informally prove validity:

$$\frac{\forall_x(F_x \implies G_x)}{\forall_x(\neg G_x \implies \neg F_x)}$$

Consider the case in which the premises are true and the conclusion is false. In this case, it must be true that there exists an x that is in F, but not in G, so that the conclusion is false. However, in this case the premise is also false.

Assume there is an interpretation in which $\forall_x(F_x \implies G_x)$ and not $\forall_x(\neg G_x \implies \neg F_x)$. So there must be some element of the UD, call it 1, such that $\neg(1 \ni G)$ but $1 \ni F$ (b/c conc. false). But then, in this interpretation, $F1 \implies G1$ is false, so $\forall_x(F_x \implies G_x)$ is false. $\rightarrow\leftarrow$ No such interpretation.

13 Day 16

C_{xy}: x created y
"Nothing created everything"
$\neg\exists_x\forall_y(C_{xy})$
$\forall_x\neg\forall_y(C_{xy})$

Quantifier Exchange
$\neg\forall_x(...x...) \iff \exists_x\neg(...x...)$
$\neg\exists_x(...x...) \iff \forall_x\neg(...x...)$

$$
\begin{aligned}
f(x) \text{ is discontinuous } \iff \quad & \neg\forall_\epsilon\exists_\delta\forall_x(|x-c|<\delta \implies |f(x)-f(c)|<\epsilon) \\
& \exists_\epsilon\neg\exists_\delta\forall_x(...) \\
& \exists_\epsilon\forall_\delta\neg\forall_x(...) \\
& \exists_\epsilon\forall_\delta\exists_x\neg(\neg|x-c|<\delta \vee |f(x)-f(c)|<\epsilon) \\
& \exists_\epsilon\forall_\delta\exists_x(|x-c|<\delta \wedge |f(x)-f(c)|\geq\epsilon)
\end{aligned}
$$

14 Day 17

S_{xy}: x is continuous at y
f : our fcn
c : the point in question (in this case, 0)
C_{xy} : x converges to y
you may use fcn notation in your sentence.

$$
\begin{aligned}
\neg S_{fc} \quad & \iff \neg\forall_x(C_{xc} \implies C_{f(x)f(c)}) \\
\neg S_{fc} \quad & \iff \neg\forall_x(C_{xc} \implies C_{f(x)f(c)}) \\
& \iff \exists_x\neg(\neg C_{xc} \vee C_{f(x)f(c)}) \\
& \iff \exists_x(C_{xc} \wedge \neg C_{f(x)f(c)})
\end{aligned}
$$

UD = {a,b,...}
P_x :
a: Alec
b: Basab

1.	$\exists_x$	P
2.	P_c	$EI(1)$
3.		

This argument is unsound, for its conclusion is false, and no sound argument has a false conclusion. S_x : X is sound.
C_x : X's conclusion is true.

$\neg C_x$	1. $\neg C_a$		P_1
$\neg\exists_x(S_x \wedge \neg C_x)$	2. $\forall_x(\neg C_x \implies \neg S_x)$		P_2
$\forall_x(\neg C_x \implies \neg S_x)$	3. $\neg C_a \implies \neg S_a$		UI(2)
$\neg S_x$	4. $\neg S_x$		MP(1,3)