Submission 4.1

Jonas Wechsler

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1. P_0 : If only one student has a black card, they know that they are the only one with a black card because they do not see anybody else with a black card.
   Assume that it takes exactly k repetitions for k students with black cards.

2. $P_0 : 1^3 + 5 * 1 = 6, \frac{6}{6} = 1 \wedge 1 \epsilon \mathbb{N}$
   P_k : Assume $k^3 + 5k$ is divisible by 6.
   $$\begin{aligned} P_{k+1} : \quad & (k+1)^3 + 5(k+1) \\ & k^3 + 3k^2 + 3k + 1 + 5k + 5 \\ & k^3 + 3k^2 + 8k + 6 \\ & (k^3 + 5k) + 3k(k+1) + 6 \end{aligned}$$
   A sum of numbers that are multiples of 6 is also a multiple a multiple of 6. We have already assumed that $k^3 + 5k$ is a multiple of 6 and we have already shown that 6 is a multiple of 6. If k is even, then $3k$ is a multiple of 6, so $3k(k+1)$ is also a multiple of 6. If k is odd, then $k+1$ is even and $3(k+1)$ is a multiple of 6. So, $3k(k+1)$ is a multiple of 6. Because the expression $(k^3 + 5k) + 3k(k+1) + 6$ is a multiple of 6, it is also divisible by 6.

3. P_0 : 6 can be represented with 3 2 cent coins.
   Assume n cents can be represented with 2 and 7 cent coins.
   There are 2 cases for n+1. If n+1 is even, it can be represented with 2 cent coins. If n+1 is odd, then n+1-7 is even, and can be represented with 2 cent coins and a 7 cent coin.

4. $P_0 : |1| \leq |1|$
   P_k : Assume $|x_1 + x_2 + ... + x_n| \leq |x_1| + |x_2| + ... + |x_n|$
   $P_{k+1} : |x_1 + x_2 + ... + x_n + x_{n+1}| \leq |x_1| + |x_2| + ... + |x_n| + |x_{n+1}|$
   In the case that x_{n+1} is negative, $|x_1 + ... + x_n| \geq |x_1 + ... + x_{n+1}|$, and $|x_1| + ... + |x_n| \leq |x_1| + ... + |x_{n+1}|$, meaning that $|x_1 + ... + x_{n+1}| \leq |x + 1| + ... + |x_{n+1}|$. In the case that x_{n+1} is zero, $P_{k+1} = P_k$. In the case that x_{n+1} is positive, $|x_1 + ... + x_n| + |x_{n+1}| = |x_1 + ... + x_{n+1}|$, so $|x_1 + ... + x_{n+1}| \leq |x_1| + ... + |x_{n+1}|$.

5. $P_0 : (1+x)^1 \geq 1 + x$
   P_k : Assume $(1+x)^n \geq 1 + nx$
   $$\begin{aligned} P_{k+1} : \quad & (1+x)^(n+1) \geq 1 + (n+1)x \\ & (1+x)(1+x)^n \geq 1 + xn + x \\ & (1+x)(1+x)^n \geq (1+x) + nx \\ & (1+x)^n + x(1+x)^n \geq x + 1 + nx \end{aligned}$$
   $x(1+x)^n \geq x$, because $(1+x)^n \geq 1$, so if $(1+x)^n \geq 1 + nx$, then $(1+x)^n + x(1+x)^n \geq x + 1 + nx$.

6. (a) $s(k)$ is a function. $dom(s) = \mathbb{N} \geq 1$
   $codom(s) = \mathbb{R} > 0$

$P_0:$ $\quad e - s(1) < \frac{1}{(1+1)!}(1)$
$e - \frac{1}{1} - \frac{1}{1} < frac12$
$e - 2 < \frac{1}{2}$

(b) P_k $\quad e - s(k) < \frac{1}{(k+1)!}(1 + \frac{1}{k+1} + \frac{1}{(k+1)^2} + ...)$ $\quad \frac{1}{k!} < \frac{1}{(k+1)!}\frac{1}{(k+1)^{k+1}}$ because $k! >$

P_{k+1} $\quad e - s(k+1) < \frac{1}{(k+1)!}(1 + \frac{1}{k+1} + \frac{1}{(k+1)^2} + ... + \frac{1}{(k+1)^{k+1}})$
$e - s(k) - \frac{1}{k!} < \frac{1}{(k+1)!}(1 + ...) + \frac{1}{(k+1)!}\frac{1}{(k+1)^{k+1}}$
$(k+1)!$ and $\frac{1}{(k+1)!} < 1$ and $\frac{1}{(k+1)^{k+1}}$.

(c)