Motivation

➢

Overview

1. Introduction
2. Background: PDR on Hardware
3. PDR on Software
4. Implementation in Ultimate
5. Evaluation
6. Related Work
7. Future Work
8. Conclusion

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1. Introduction

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2.1 Preliminaries: Boolean Transition System

- A Boolean Transition System S = (X, I, T) consists of
  - Set of boolean variables X
  - A conjunction representing the initial state I
  - A propositional formula T over variables in X and $X' = \{x \in X \mid x' \in X'\}$, called Transition Relation

- States in S are cubes containing each variable from X with a boolean valuation of it

  ➔Finite number of states: $2^{|X|}$

- Transitions @Todo

2.1 Preliminaries: Formulas

- Given a formula φ over X, we get a primed formula φ' by replacing each variable with its corresponding variable in X'

- A literal is a variable or ist negation
- A cube is a conjunction of literals
- A clause is a disjunction of literals
  - ➔ Negation of a cube is a clause and vice versa

- A Safety Property P is a formula over X that should be satisfiable by every state reachable from I
  - ➔ $\bar{P}$ being a set of bad states

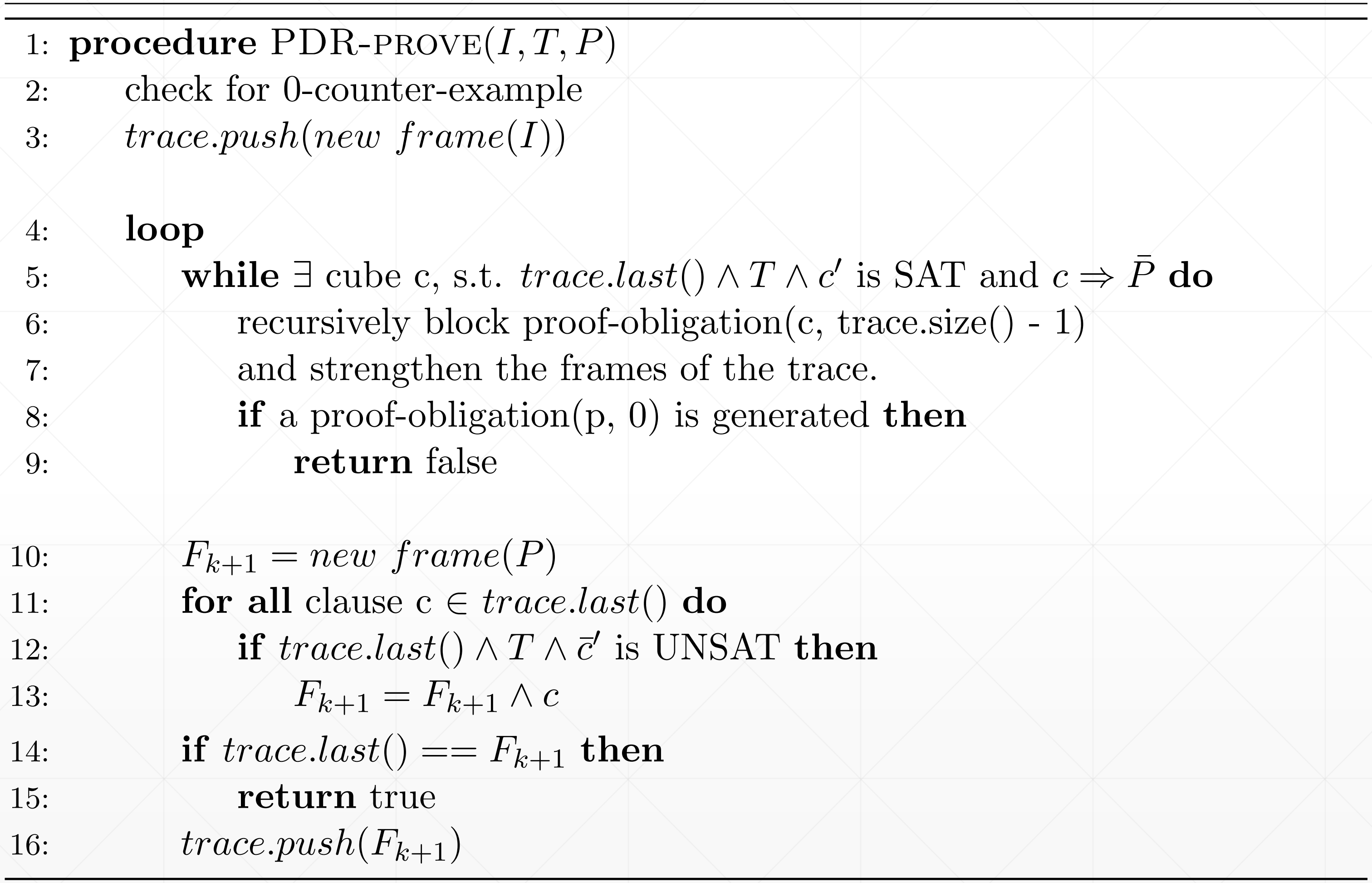
2.2 Algorithm: Overview

- PDR on hardware checks if states in $\bar{P}$ are reachable from I

- For that it uses cubes of clauses, called Frames
  - Frame F_i represents an over-approximation of reachable states in at most i transitions from I

- PDR maintains sequence of frames $[F_0, F_1, \dots, F_k]$, called trace

2.2 Algorithm: Pseudo-Code



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procedure PDR-PROVE($I, T, P$)
    check for 0-counter-example
    $trace.push(new\ frame(I))$

    loop
        while $\exists$ cube c, s.t. $trace.last() \wedge T \wedge c'$ is SAT and $c \Rightarrow \bar{P}$ do
            recursively block proof-obligation(c, trace.size() - 1)
            and strengthen the frames of the trace.
            if a proof-obligation(p, 0) is generated then
                return false

        $F_{k+1} = new\ frame(P)$
        for all clause c $\in$ $trace.last()$ do
            if $trace.last() \wedge T \wedge \bar{c}'$ is UNSAT then
                $F_{k+1} = F_{k+1} \wedge c$
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        $trace.push(F_{k+1})$
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2.2 Algorithm: Pseudo-Code

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procedure PDR-PROVE($I, T, P$)
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            return true
        $trace.push(F_{k+1})$
```

2.2 Algorithm: Checking for 0-Counter-Example

- Is $I \wedge \bar{P}$ satisfiable?

→ If satisfiable:

- Algorithm terminates and returns that a bad state is reachable

→ If unsatisfiable:

- Algorithm initializes the first frame in the trace: $F_0 = I$ and continues

2.2 Algorithm: Pseudo-Code

```
procedure PDR-PROVE($I, T, P$)
    check for 0-counter-example
    $trace.push(new\ frame(I))$
    loop
```

Next Transition Phase

```
        while $\exists$ cube c, s.t. $trace.last() \wedge T \wedge c'$ is SAT and $c \Rightarrow \bar{P}$ do
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```

2.2 Algorithm: Next Transition Phase

- Checking if the next state is a good state

  ➢ Let $[F_0, F_1, \dots, F_k]$ be the current trace

  ➢ Is $F_k \land T \land \overline{P'}$ satisfiable?

➔ If satisfiable:

- Take satisfying assignment $\vec{x} = \left\{x_1, x_2, \dots, x_{|X|}, x'_1, x'_2, \dots, x'_{|X'|}\right\}$
- The algorithm gets new bad state: $b = x_1 \land x_2 \land \dots \land x_{|X|}$
- Construct the tuple $t = (b, k)$, called proof-obligation

2.2 Algorithm: Next Transition Phase

- Checking if the next state is a good state:
  - ➢Let $[F_0, F_1, \dots, F_k]$ be the current trace
  - ➢Is $F_k \land T \land \overline{P'}$ satisfiable?

➔ If unsatisfiable:

- Continue with the next phase

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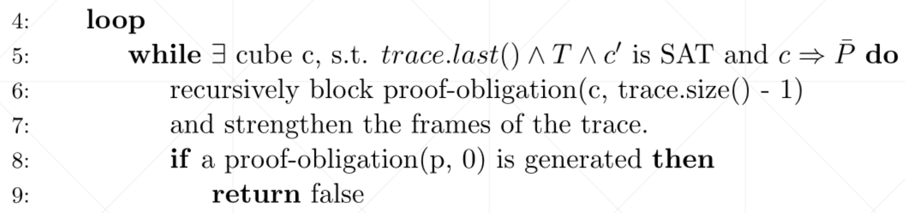
Blocking-Phase

2.2 Algorithm: Blocking-Phase



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```

Note: useful to have
Piece of pseudo-code?

- Proving that new bad states are not reachable

  - ➢ If there are proof-obligations:
    - Algorithm takes proof-obligation (b, i)
    - Tries to block bad state b by checking $F_{i-1} \wedge T \wedge b'$ for satisfiability

    ➔ If satisfiable:
    - Frame F_{i-1} is not strong enough to block b
    - Take satisfying assignment $\vec{x} = \left\{x_1, x_2, \ldots, x_{|X|}, x'_1, x'_2, \ldots, x'_{|X'|}\right\}$
    - The algorithm gets another new bad state: $c = x_1 \wedge x_2 \wedge \ldots \wedge x_{|X|}$
    - Construct new proof-obligation $\mathrm{u} = (c, i - 1)$

2.2 Algorithm: Blocking-Phase

- Proving that new bad states are not reachable

  - ➢ If there are proof-obligations:
    - Algorithm takes proof-obligation (b, i)
    - Tries to block bad state b by checking $F_{i-1} \land T \land b'$ for satisfiability

      ➔ If unsatisfiable:
      - Algorithm strenghthens F_i with $\bar{b}$

        ➔ $F_i = F_i \land \bar{b}$
      - Blocking bad state b at F_i

2.2 Algorithm: Blocking-Phase

- This continues recursively until:
  - There are no proof-obligations left

    ➔ Algorithm continues with the next phase

  - A proof-obligation $(d, 0)$ is created

    ➔ Proving that a bad state can be reached, terminating the algorithm

2.2 Algorithm: Pseudo-Code

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Propagation-Phase

2.2 Algorithm: Propagation-Phase

- Propagating learned information:
  - ➢After no proof-obligations are left, the algorithm initzializes new frame $F_{k+1} = \mathrm{P}$
  - ➢Algorithm passes on learned informations, e.g, which states are blocked:
    - For each clause c in F_k check: $F_k \wedge T \wedge \bar{c}'$ for satisfiability

    ➔ If satisfiable:
      - Do nothing, continue with next clause

2.2 Algorithm: Propagation-Phase

- Propagating learned information:
  - After no proof-obligations are left, the algorithm initzializes new frame $F_{k+1} = \mathrm{P}$
  - Algorithm passes on learned informations, e.g, which states are blocked:
    - For each clause c in F_k check: $F_k \wedge T \wedge \bar{c}'$ for satisfiability

      → If unsatisfiable:
      - Algorithm strengthens F_{k+1} with c

        → $F_{k+1} = F_{k+1} \wedge c$

2.2 Algorithm: Propagation-Phase

- Check for termination:

  ➢ After all clauses have been tested, algorithm checks if $F_k \equiv F_{k+1}$

  - If so, the algorithm has found a fixpoint and terminates

    ➔ No states of $\bar{P}$ are reachable

  - If not, the algorithm continues with a new Next Transition Phase

2.2 **Algorithm:** Propagation-Phase

- Check for termination

➢ After all clauses have been tested, algorithm checks if $F_k \equiv F_{k+1}$

- If so, the algorithm has found a fixpoint and terminates

  ➔ No states of $\bar{P}$ are reachable

- If not, the algorithm continues with a new Next Transition Phase

➢ Algorithm repeats the three phases until a fixpoint is found, or a proof-obligation $(d, 0)$ is created

2.2 Algorithm: Pseudo-Code TEMPLATE

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```

2.3 Example

2.4 Possible Improvements

- Blocking one state at a time is ineffective:
  - Generalize blocked states

    ➔ Eliminate insignificant cubes from states, that are not used by UNSAT-cores

- Ternary Simulation to reduce proof-obligations:
  - Extend binary variables with a new value: unknown
  - Check state variables of proof-obligations for importance

    ➔ Eliminate unimportant state variables

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3.1 Preliminaries

- To use PDR on software, we need to lift the algorithm from propositional-logic based systems to first-order logic based systems

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- To use PDR on software, we need to lift the algorithm from propositional-logic based systems to first-order logic based systems

- For that we first need new definitions

3.1 Preliminaries: Control Flow Graph

- A control flow graph (CFG) $A = (X, L, G, \ell_0, \ell_E)$ is a graph consisting of
  - A finite set of variables X
  - A finite set of locations L
  - A finite set of transitions $G \subseteq L \times FO \times L$

    ➔ FO being a quantifier free first-order logic formula over variables in X and $X' = \{x \in X \,| x' \in X'\}$
  - An initial location $\ell_0 \in L$
  - An error location $\ell_E \in L$

3.1 Preliminaries: Control Flow Graph

- The transition formula $T_{\ell_1 \to \ell_2}$ from location ℓ_1 to location ℓ_2 is defined as:

  - $T_{\ell_1 \to \ell_2} = \begin{cases} (\ell_1, t, \ell_2), & (\ell_1, t, \ell_2) \in G \\ false, & otherwise \end{cases}$

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➔ Global Transition Formula $T = \bigvee_{(\ell_1, t, \ell_2) \in G} T_{\ell_1 \rightarrow \ell_2}$

3.2 Lifted Algorithm: Pseudo-Code

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procedure LIFTED-PDR-PROVE($L, G$)
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    $\ell_0.trace.push(new\ frame(true))$
    for all $\ell \in L \backslash \{\ell_0, \ell_E\}$ do
        $\ell.trace.push(new\ frame(false))$
    $level := 0$

    loop
        for all $\ell \in L \backslash \{\ell_E\}$ do
            $\ell.trace.push(new\ frame(true))$
        $level := level + 1$
        get initial proof-obligations

        while $\exists$ proof-obligation $(t, \ell, i)$, do
            Recursively block proof-obligation
            if a proof-obligation$(p, \ell, 0)$ is generated then
                return false

        for $i = 0;\ i \leq level;\ i := i + 1$ do
            for $\ell \in L \backslash \{l_E\}$ do
                if $\ell.trace[i] \neq \ell.trace[i-1]$ then
                    break
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```

3.2 Lifted Algorithm: Checking for 0-Counter-Example

- Is $\ell_0 = \ell_E$?

→ Yes:

- Algorithm terminates, returning that ℓ_E is reachable

→ No:

- Algorithm continues

3.2 Lifted Algorithm: Pseudo-Code

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        for $i = 0;\ i \leq level;\ i := i + 1$ do
            for $\ell \in L \backslash \{l_E\}$ do
                if $\ell.trace[i] \neq \ell.trace[i-1]$ then
                    break
            return true
```

3.2 Lifted Algorithm: Local Traces

- There is no global trace $[F_0, F_1, \dots, F_k]$

  ➔ Every location $\ell \in L \setminus \{\ell_E\}$ has its own local trace $[F_{0,\ell}, F_{1,\ell}, \dots, F_{k,\ell}]$

  ➔ Lifted frames are cubes of first-order formulas

  ➔ @ToDo, explain changes to proofobligations

3.2 Lifted Algorithm: Pseudo-Code

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            if a proof-obligation$(p, \ell, 0)$ is generated then
                return false

        for $i = 0;\ i \leq level;\ i := i + 1$ do
            for $\ell \in L \setminus \{l_E\}$ do
                if $\ell.trace[i] \neq \ell.trace[i-1]$ then
                    break
            return true
```

3.2 Lifted Algorithm: Initialization

- Initialize each local frames:

  - $F_{0,\ell} = \begin{cases} true, & \ell = \ell_0 \\ false, & otherwise \end{cases}$

3.2 Lifted Algorithm: Pseudo-Code

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                if $\ell.trace[i] \neq \ell.trace[i-1]$ then
                    break
            return true
```

3.2 Lifted Algorithm: Next Level Phase

- Initializing the next level:

  - ➢ Let k be the current level
    - ➔ Every location $\ell \in L \setminus \{\ell_E\}$ has trace $[F_{0,\ell}, \dots, F_{k,\ell}]$

  - ➢ Algorithm initzializes new level k+1 for all locations $\ell \in L \setminus \{\ell_E\}$
    - ➔ Adding new frame $F_{k+1,\ell} = true$

3.2 Lifted Algorithm: Next Level Phase

- Initializing the next level:
  - ➢ Let k be the current level
    - ➔ Every location $\ell \in L \setminus \{\ell_E\}$ has trace $[F_{0,\ell}, \dots, F_{k,\ell}]$
  - ➢ Additionally, the algorithm computes initial proof-obligations:
    - Because of the structure of CFGs, we always know the transitions to ℓ_E
      - ➔ Check G for transitions of the form (ℓ, t, ℓ_E)
      - ➔ For each transition, get proof-obligation (t, ℓ, k)
      - ➔ @ToDo explain lifted proof-obligations

3.2 Lifted Algorithm: Pseudo-Code

```
procedure PDR-PROVE$(I, T, P)$
    check for 0-counter-example
    $trace.push(new\ frame(I))$

    loop
        while $\exists$ cube c, s.t. $trace.last() \wedge T \wedge c'$ is SAT and $c \Rightarrow \bar{P}$ do
            recursively block proof-obligation(c, trace.size() - 1)
            and strengthen the frames of the trace.
            if a proof-obligation(p, 0) is generated then
                return false

        $F_{k+1} = new\ frame(P)$
        for all clause c $\in trace.last()$ do
            if $trace.last() \wedge T \wedge \vec{c}'$ is UNSAT then
                $F_{k+1} = F_{k+1} \wedge c$
        if $trace.last() == F_{k+1}$ then
            return true
        $trace.push(F_{k+1})$
```

```
procedure LIFTED-PDR-PROVE$(L, G)$
    check for 0-counter-example
    $\ell_0.trace.push(new\ frame(true))$
    for all $\ell \in L\backslash\{\ell_0, \ell_E\}$ do
        $\ell.trace.push(new\ frame(false))$
    $level := 0$

    loop
        for all $\ell \in L\backslash\{\ell_E\}$ do
            $\ell.trace.push(new\ frame(true))$
        $level := level + 1$
        get initial proof-obligations

        while $\exists$ proof-obligation $(t, \ell, i)$, do
            Recursively block proof-obligation
            if a proof-obligation$(p, \ell, 0)$ is generated then
                return false

        for $i = 0;\ i \leq level;\ i := i + 1$ do
            for $\ell \in L\backslash\{l_E\}$ do
                if $\ell.trace[i] \neq \ell.trace[i-1]$ then
                    break
            return true
```

3.2 Lifted Algorithm: Blocking-Phase

- Blocking-Phase no longer nested in preceding-phase
  - ➔ No longer optional
    - ➔ In each iteration we have at least the initial proof-obligations

3.2 Lifted Algorithm: Blocking-Phase

- Checking if bad transitions are reachable:
  - ➢ Algorithm takes proof-obligation (t, ℓ, i) with the lowest i
    - For each predecessor location ℓ_{pre} of ℓ check if $F_{i-1,\ell_{pre}} \wedge T_{\ell_{pre} \to \ell} \wedge t'$ is satisfiable

      ➔ If satisfiable:
      - t could not be blocked at ℓ on level i
      - Get new proof-obligation $(p, \ell_{pre}, i-1)$

        ➔ p being the weakest precondition of t and $T_{\ell_{pre} \to \ell}$

3.2 Lifted Algorithm: Blocking-Phase

- Checking if bad transitions are reachable:

  - ➢ Algorithm takes proof-obligation (t, ℓ, i) with the lowest i
    - For each predecessor location ℓ_{pre} of ℓ check if $F_{i-1,\ell_{pre}} \wedge T_{\ell_{pre}\to\ell} \wedge t'$ is satisfiable

      ➔ If unsatisfiable:
      - t is blocked at ℓ on level i
      - Strengthen each frame $F_{j,\ell}$, $j \leq i$ with $\bar{t}$

      ➔ $F_{j,\ell} = F_{j,\ell} \wedge \bar{t}$

3.2 Lifted Algorithm: Blocking-Phase

➢ This continues recursively until:

- There are no proof-obligations left

  ➔ Algorithm continues with the next phase

- A proof-obligation $(d, \ell, 0)$ is created

  ➔ Proving that there exists a feasible path to ℓ_E

3.2 Lifted Algorithm: Pseudo-Code

```
procedure PDR-PROVE($I, T, P$)
    check for 0-counter-example
    $trace.push(new\ frame(I))$

    loop
        while $\exists$ cube c, s.t. $trace.last() \wedge T \wedge c'$ is SAT and $c \Rightarrow \bar{P}$ do
            recursively block proof-obligation(c, trace.size() - 1)
            and strengthen the frames of the trace.
            if a proof-obligation(p, 0) is generated then
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        $F_{k+1} = new\ frame(P)$
        for all clause c $\in trace.last()$ do
            if $trace.last() \wedge T \wedge \bar{c}'$ is UNSAT then
                $F_{k+1} = F_{k+1} \wedge c$
        if $trace.last() == F_{k+1}$ then
            return true
        $trace.push(F_{k+1})$
```

```
procedure LIFTED-PDR-PROVE($L, G$)
    check for 0-counter-example
    $\ell_0.trace.push(new\ frame(true))$
    for all $\ell \in L \backslash \{\ell_0, \ell_E\}$ do
        $\ell.trace.push(new\ frame(false))$
    $level := 0$

    loop
        for all $\ell \in L \backslash \{\ell_E\}$ do
            $\ell.trace.push(new\ frame(true))$
        $level := level + 1$
        get initial proof-obligations

        while $\exists$ proof-obligation $(t, \ell, i)$, do
            Recursively block proof-obligation
            if a proof-obligation$(p, \ell, 0)$ is generated then
                return false

        for $i = 0;\ i \leq level;\ i := i + 1$ do
            for $\ell \in L \backslash \{l_E\}$ do
                if $\ell.trace[i] \neq \ell.trace[i-1]$ then
                    break
            return true
```

3.2 Lifted Algorithm: Propagation-Phase

- No more propagation of learned information

- Only checking for termination
  - ➔ Trying to find a global fixpoint:
    - Is there an i where $F_{i-1,\ell} = F_{i,\ell}$ for every $\ell \in L \setminus \{\ell_E\}$?

3.2 Lifted Algorithm: Propagation-Phase

- No more propagation of learned information

- Only checking for termination
  - ➔ Trying to find a global fixpoint:
    - Is there an i where $F_{i-1,\ell} = F_{i,\ell}$ for every $\ell \in L \setminus \{\ell_E\}$?

  - ➔ Yes:
    - Algorithm terminates returning that there is no feasible path to ℓ_E

3.2 Lifted Algorithm: Propagation-Phase

- No more propagation of learned information

- Only checking for termination
  - ➔ Trying to find a global fixpoint:
    - Is there an i where $F_{i-1,\ell} = F_{i,\ell}$ for every $\ell \in L \setminus \{\ell_E\}$?

    ➔ No:
    - Algorithm continues with the next level

3.3 Example

3.4 Possible Improvements

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3. PDR on Software
4. **Implementation in Ultimate**
5. Evaluation
6. Related Work
7. Future Work
8. Conclusion

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4.2 Implemented Improvements

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6. Related Work

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7.1 Further Improvements

Overview

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8. Conclusion