Property Directed Reachability in Ultimate

Bachelor Thesis Presentation

By **Jonas Werner**
Advised by **Dr. Daniel Dietsch**

25.09.2018

Introduction: Motivation

- Property Directed Reachability (**PDR**) was first devised as a hardware verification technique in 2010 by Aaron Bradley[1]

1: Aaron R. Bradley. Sat-based model checking without unrolling. In *VMCAI*, volume 6538 of *Lecture Notes in Computer Science*, pages 70–87. Springer, 2011.

Introduction: Motivation

- Property Directed Reachability (**PDR**) was first devised as a hardware verification technique in 2010 by Aaron Bradley[1]

  ➔ Surprisingly won 3rd place at CAV 2010 hardware checking competition[2]

1: Aaron R. Bradley. Sat-based model checking without unrolling. In *VMCAI*, volume 6538 of *Lecture Notes in Computer Science*, pages 70–87. Springer, 2011.

2: Hwmcc10 results. http://fmv.jku.at/hwmcc10/results.html. Accessed: 2018-07-20

Introduction: Motivation

- Property Directed Reachability (**PDR**) was first devised as a hardware verification technique in 2010 by Aaron Bradley[1]

  ➔ Surprisingly won 3rd place at CAV 2010 hardware checking competition[2]

> **"This new method appears to be the most important contribution to bit-level formal verification in almost a decade"** [3]

1: Aaron R. Bradley. Sat-based model checking without unrolling. In *VMCAI*, volume 6538 of *Lecture Notes in Computer Science*, pages 70–87. Springer, 2011.

2: Hwmcc10 results. http://fmv.jku.at/hwmcc10/results.html. Accessed: 2018-07-20

3: Niklas Een, Alan Mishchenko, and Robert Brayton. 2011. Efficient implementation of property directed reachability. In Proceedings of the International Conference on Formal Methods in Computer-Aided Design (FMCAD '11). FMCAD Inc, Austin, TX, 125-134.

Introduction: Motivation

- Property Directed Reachability (**PDR**) was first devised as a hardware verification technique in 2010 by Aaron Bradley[1]

  ➔ Surprisingly won 3rd place at CAV 2010 hardware checking competition[2]

“This new method appears to be the most important contribution to bit-level formal verification in almost a decade” [3]

- Using PDR on software may have similar performance!

1: Aaron R. Bradley. Sat-based model checking without unrolling. In *VMCAI*, volume 6538 of *Lecture Notes in Computer Science*, pages 70–87. Springer, 2011.

2: Hwmcc10 results. http://fmv.jku.at/hwmcc10/results.html. Accessed: 2018-07-20

3: Niklas Een, Alan Mishchenko, and Robert Brayton. 2011. Efficient implementation of property directed reachability. In Proceedings of the International Conference on Formal Methods in Computer-Aided Design (FMCAD '11). FMCAD Inc, Austin, TX, 125-134.

Introduction: Motivation

➢ Our Goals:

- Use PDR on software in the verification framework Ultimate[1]
  - ➔ Combining Trace Abstraction and PDR
  - ➔ Comparison to existing techniques

1: Ultimate. https://ultimate.informatik.uni-freiburg.de. Accessed: 2018-07-20.

Overview

- **How does our PDR algorithm work?**
  - Preliminaries
  - Running Example
  - Related Work

- **How do we use PDR in Ultimate?**
  - Combination of Trace Abstraction and our PDR algorithm
  - Implemented Improvements

Overview

- **Evaluation**
  - Comparison of Trace Abstraction using PDR and Trace Abstraction using Nested Interpolants

- **What can be done in the future?**
  - Implementing more Improvements

PDR Algorithm: Preliminaries

Control flow graph (CFG) $A = (X, L, E, \ell_0, \ell_E)$ is a graph consisting of

- Finite set of first-order variables X
- Finite set of locations L
- Finite set of transitions E $\subseteq L \times FO \times L$

  ➔ FO is a quantifier free first-order logic formula over variables in X and $X' = \{x \in X \mid x' \in X'\}$
- Initial location $\ell_0 \in L$
- Error location $\ell_E \in L$

PDR Algorithm: Datastructures

Frame $F_{i,\ell}$:

- Represents a first-order formula
- ℓ is the corresponding location
- i is the corresponding iteration

➔ Each location has multiple assigned frames

Proof-Obligation (p, ℓ, i) :

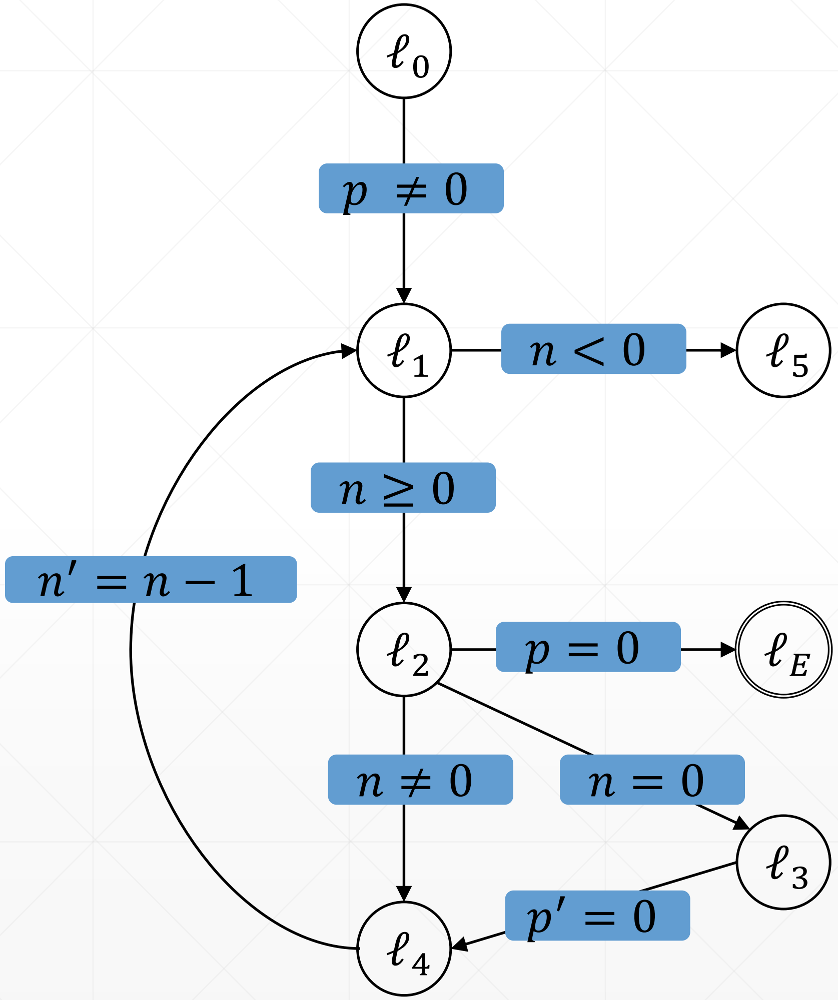
- p is a first-order formula
- ℓ is the corresponding location
- i is the corresponding iteration

➔ Need to be blocked

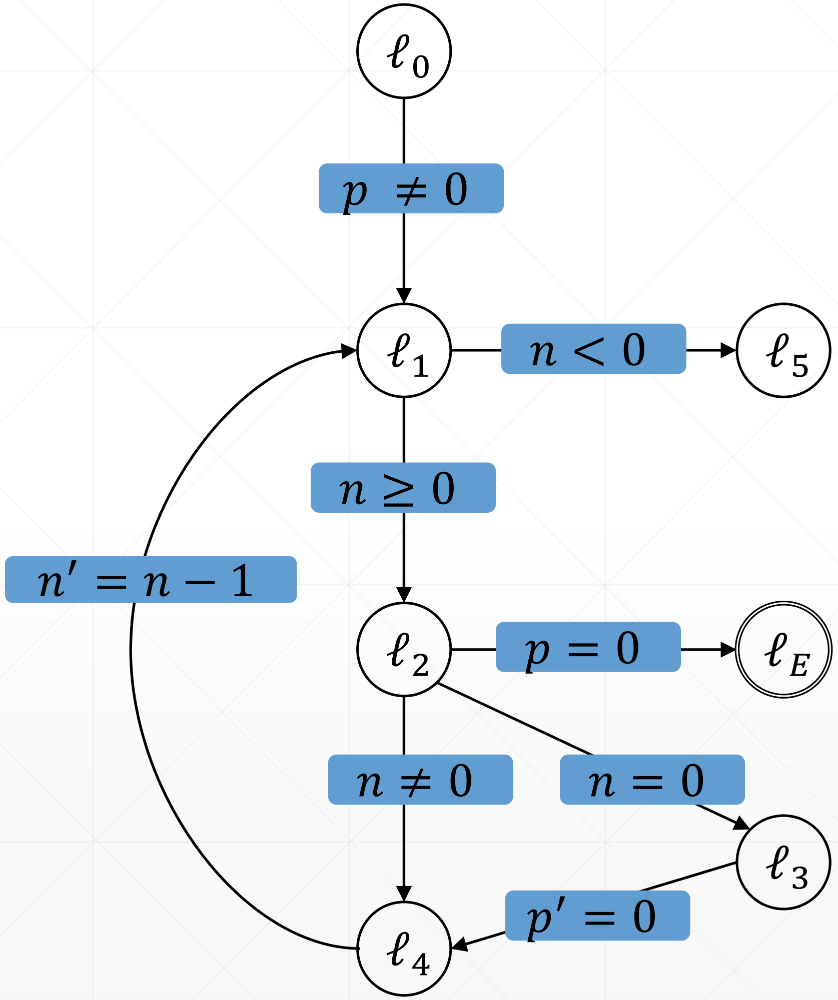
PDR Algorithm: Description

- Starts with checking for a **0-Counter-Example**
- **Global Initialization**
- Repeats three phases until termination:
  1. **Next Iteration Initialization**
  2. **Blocking-Phase**
  3. **Propagation-Phase**

Example: Running Example

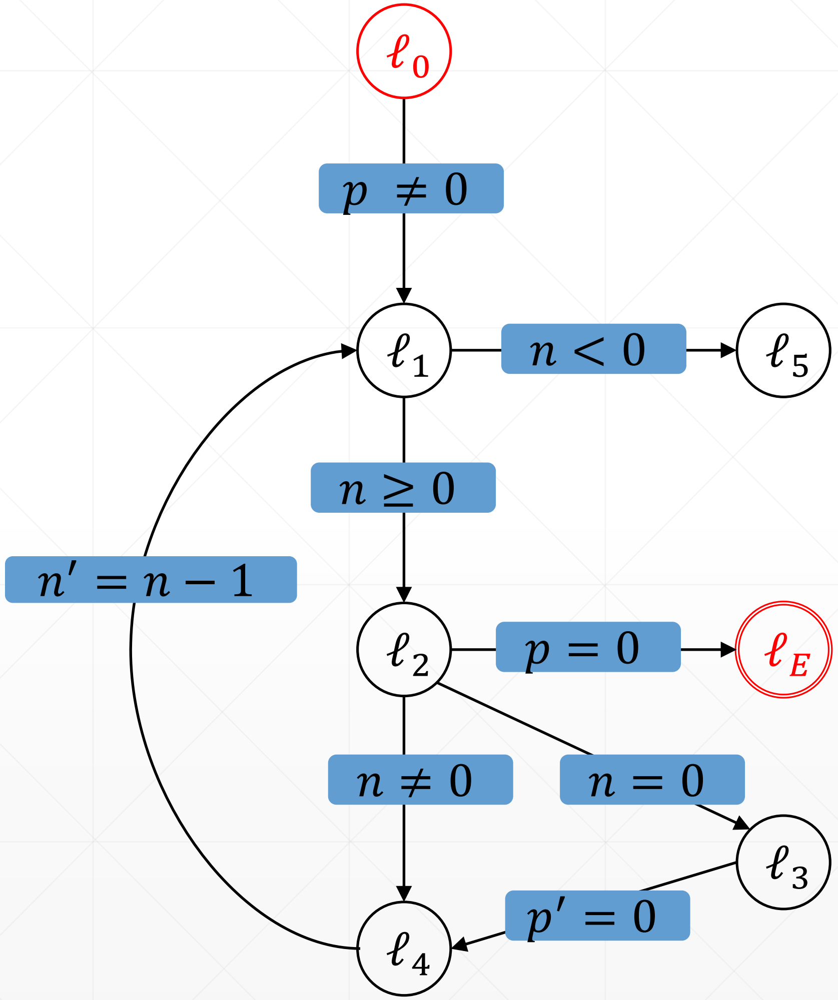


Example:



1. Step: Check for 0-Counter-Example

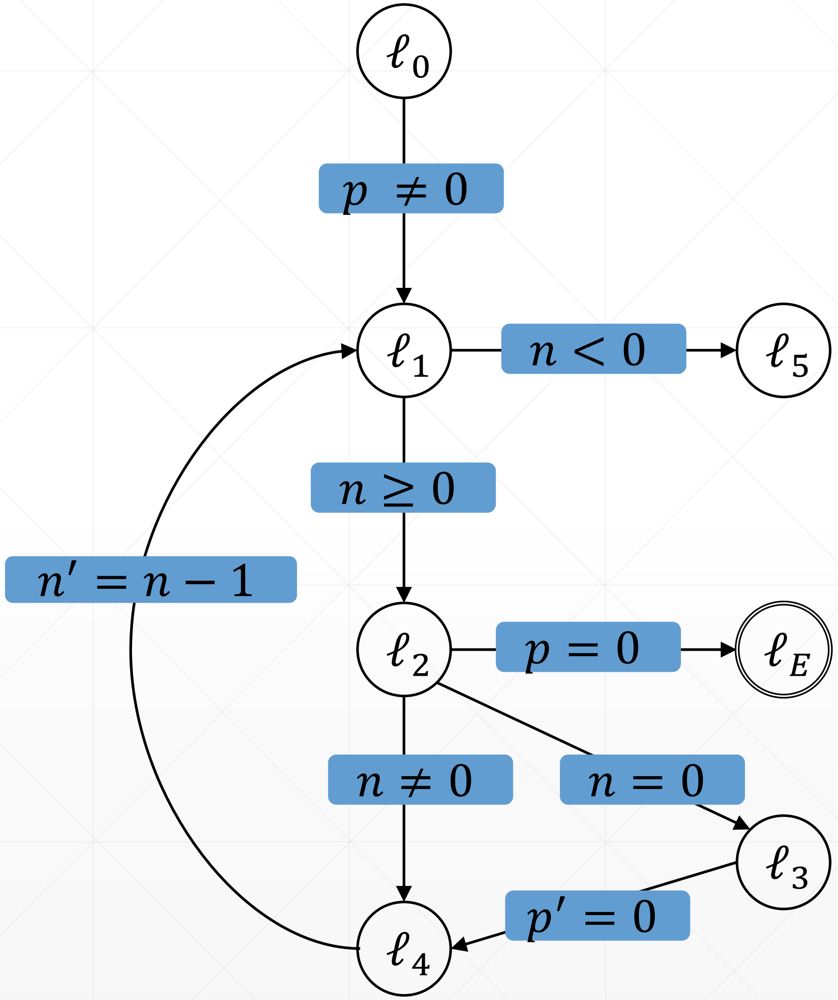
Example:



1. Step: Check for 0-Counter-Example

- Is $\ell_0 = \ell_E$?
  ➔ No. Continue with initialization

Example:

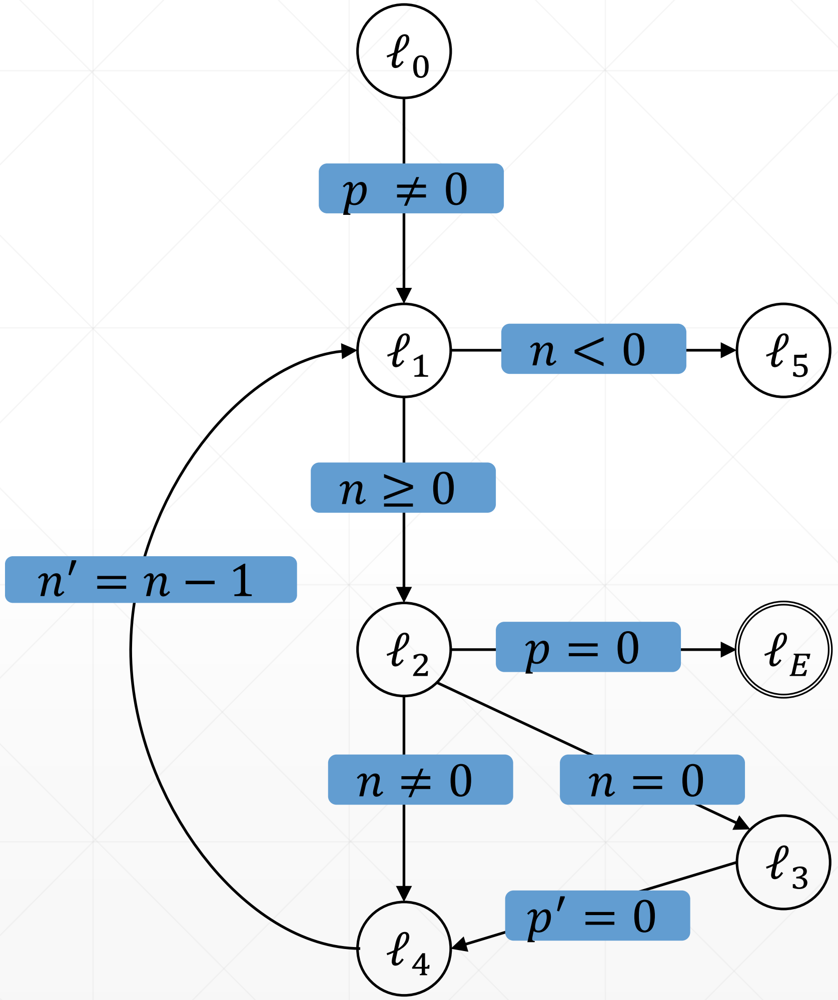


location	0
ℓ_0	
ℓ_1	
ℓ_2	
ℓ_3	
ℓ_4	

2. Step: Global Initialization

- $F_{0,\ell} = \begin{cases} \text{true}, & \ell = \ell_0 \\ \text{false}, & otherwise \end{cases}$

Example:

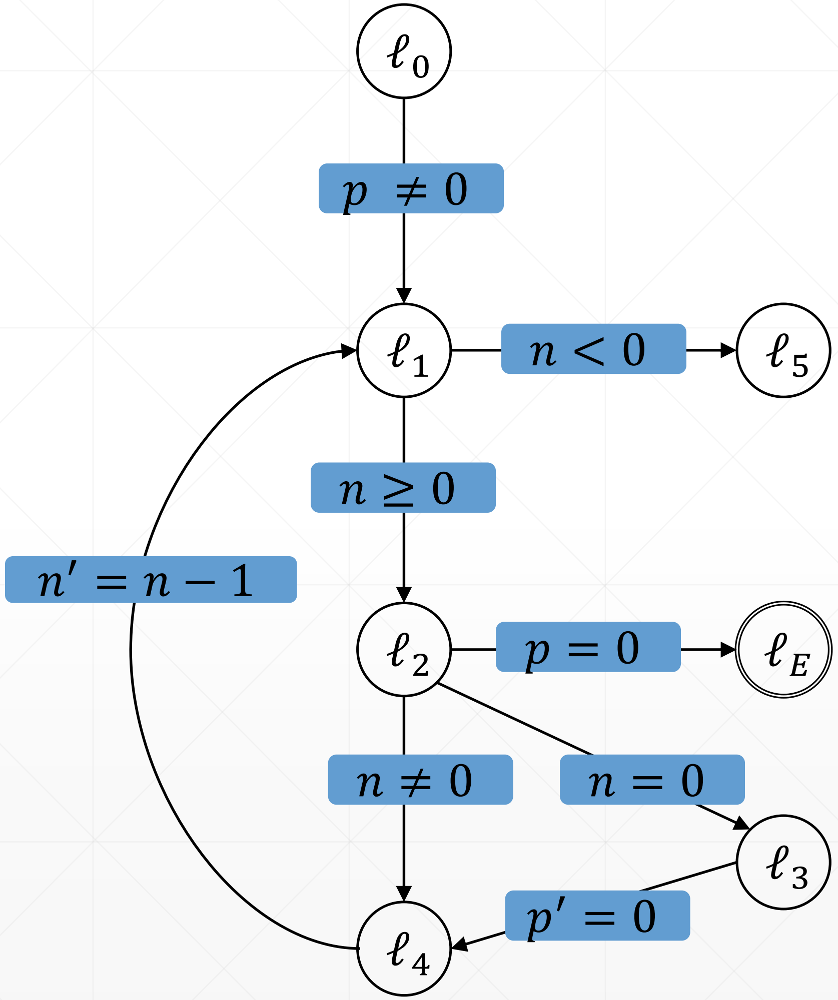


location	0
ℓ_0	t
ℓ_1	f
ℓ_2	f
ℓ_3	f
ℓ_4	f

2. Step: Global Initialization

- $F_{0,\ell} = \begin{cases} \text{true}, & \ell = \ell_0 \\ \text{false}, & otherwise \end{cases}$

Example:

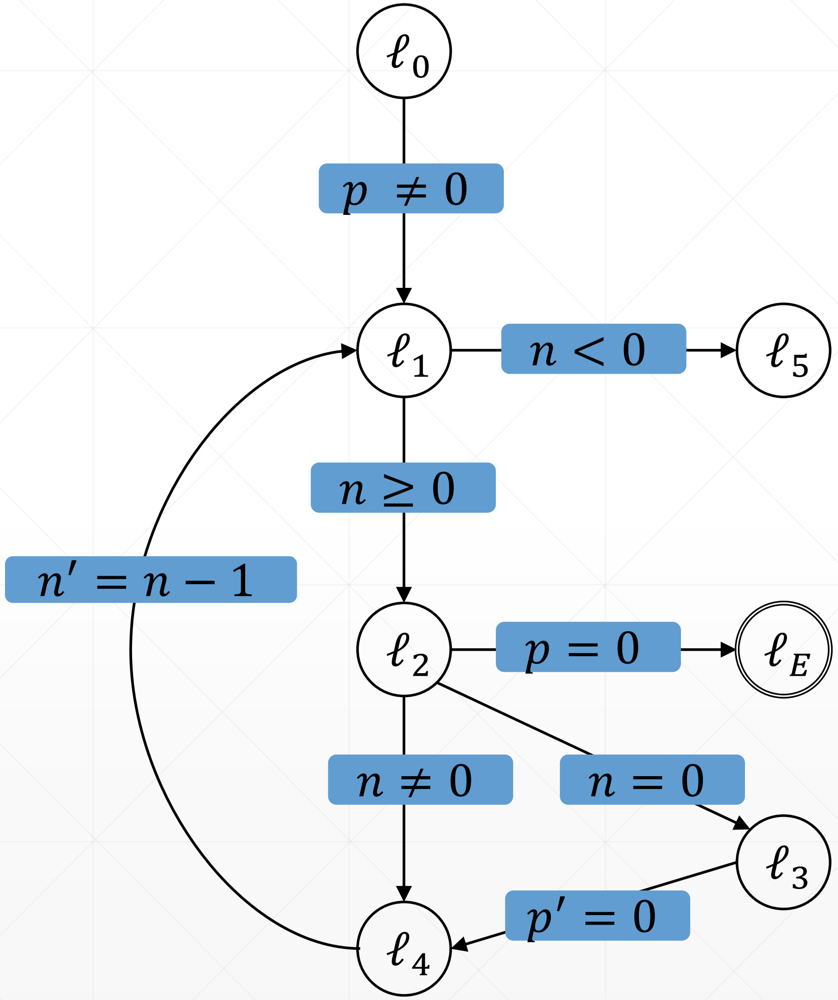


location	0	1
ℓ_0	t	
ℓ_1	f	
ℓ_2	f	
ℓ_3	f	
ℓ_4	f	

3. Step: Iteration 1 Initialization

- Initialize iteration 1 frames as true

Example:

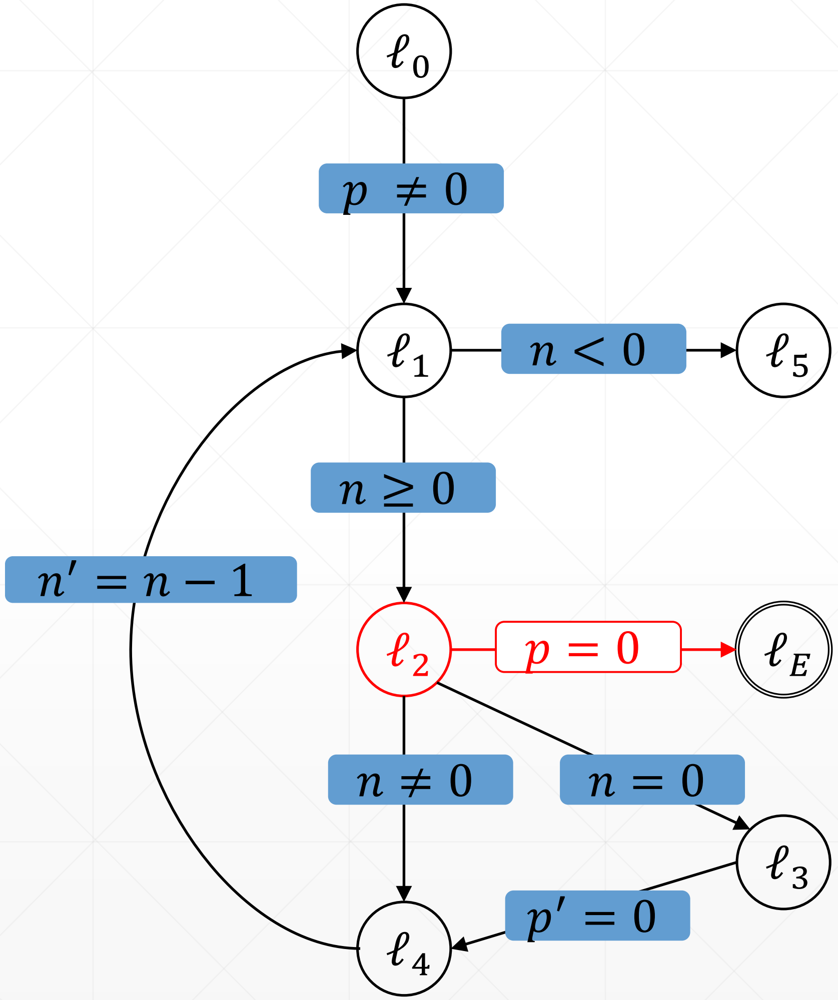


location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	f	t
ℓ_3	f	t
ℓ_4	f	t

3. Step: Iteration 1 Initialization

- Initialize iteration 1 frames as true

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	f	t
ℓ_3	f	t
ℓ_4	f	t

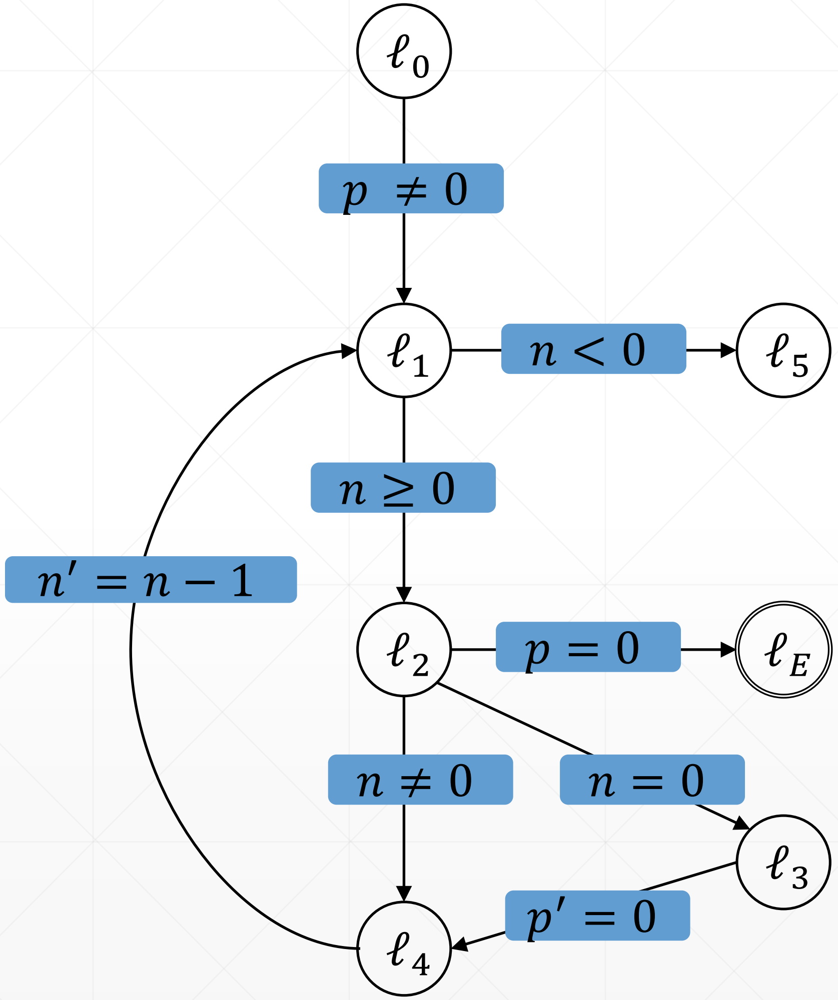
3. Step: Iteration 1 Initialization

- Get initial proof-obligation:
  ➔ $(p = 0, \ell_2, 1)$

Proof-Obligations:

- $(p = 0, \ell_2, 1)$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	f	t
ℓ_3	f	t
ℓ_4	f	t

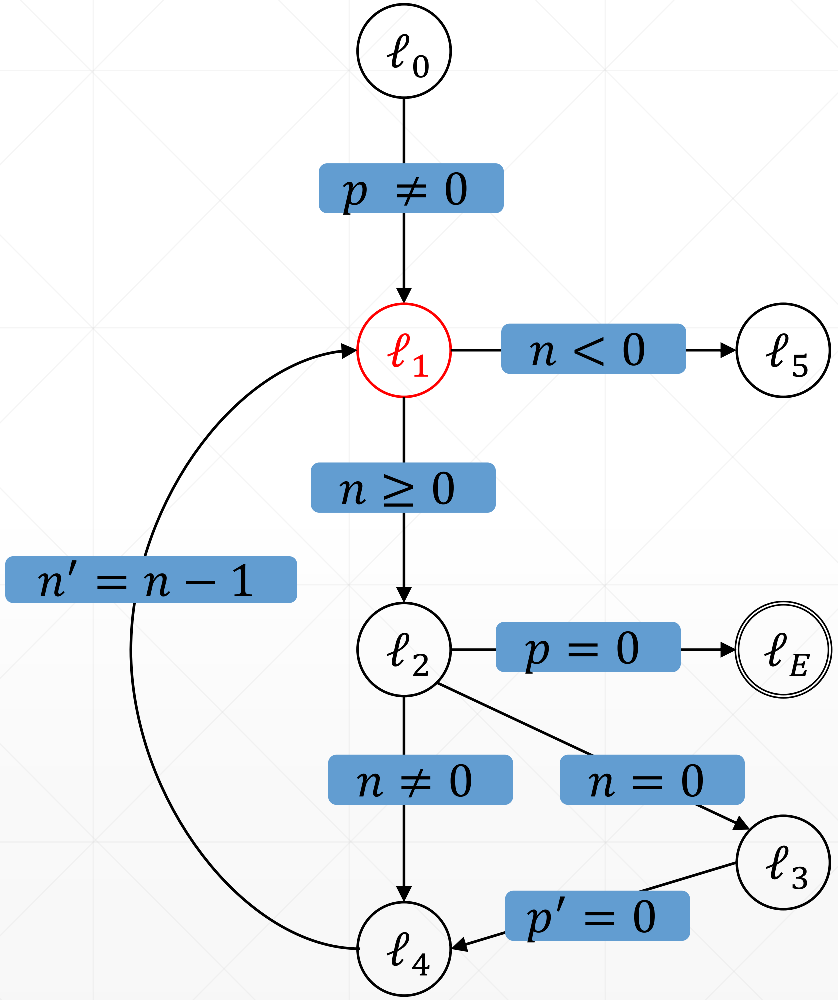
4. Step: Iteration 1 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 1)$

Proof-Obligations:

- $(p = 0, \ell_2, 1)$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	f	t
ℓ_3	f	t
ℓ_4	f	t

4. Step: Iteration 1 Blocking-Phase:

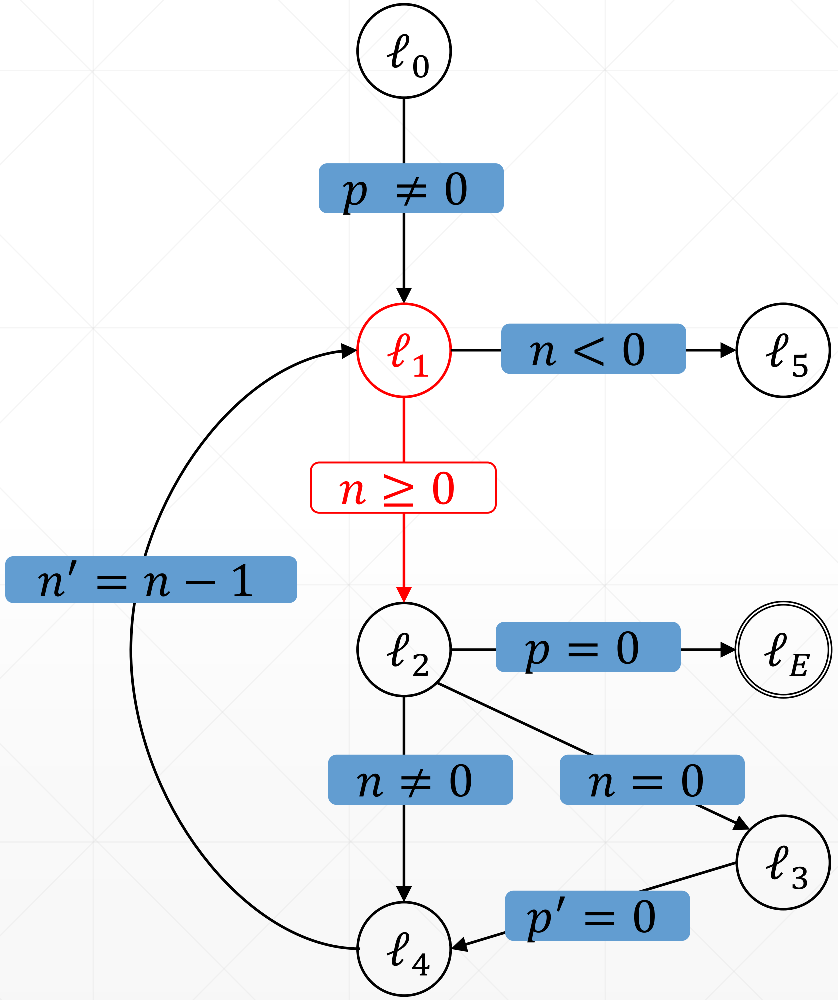
- Try to block $(p = 0, \ell_2, 1)$

- Predecessor ℓ_1:
  $F_{0,\ell_1} \wedge T_{\ell_1 \rightarrow \ell_2} \wedge obligation'$

Proof-Obligations:

- $(p = 0, \ell_2, 1)$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	f	t
ℓ_3	f	t
ℓ_4	f	t

4. Step: Iteration 1 Blocking-Phase:

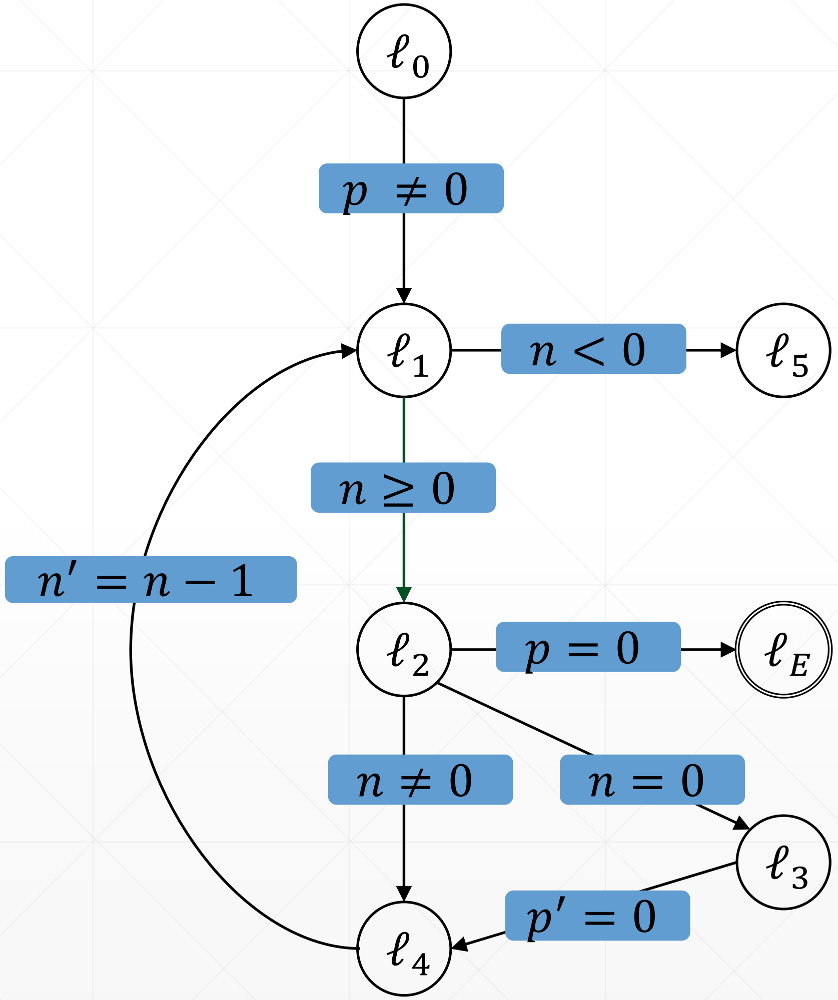
- Try to block $(p = 0, \ell_2, 1)$

- Predecessor ℓ_1:
  $f \wedge n \geq 0 \wedge p' = 0$

Proof-Obligations:

- $(p = 0, \ell_2, 1)$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	f	t
ℓ_3	f	t
ℓ_4	f	t

4. Step: Iteration 1 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 1)$

- Predecessor ℓ_1:

  $f \wedge n \geq 0 \wedge p' = 0$

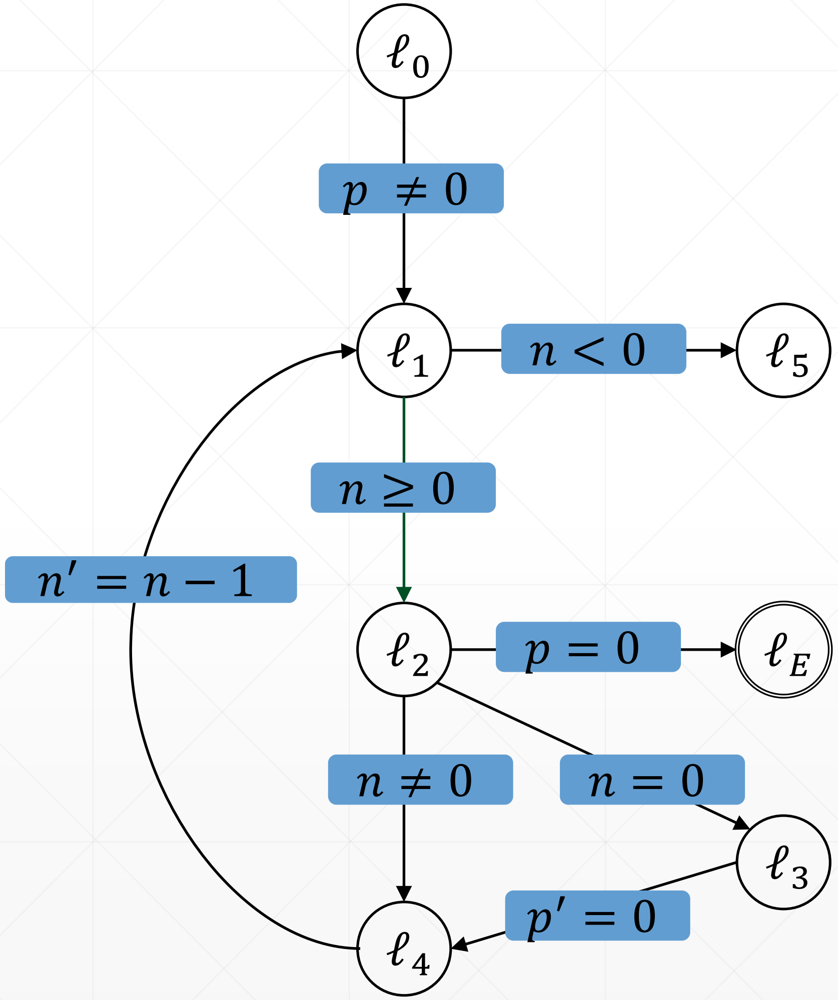
  ➔ Unsatisfiable

  ➔ Strengthen frames $F_{0,\ell_2}, F_{1,\ell_2}$

Proof-Obligations:

- ∅

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t
ℓ_4	f	t

4. Step: Iteration 1 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 1)$

- Predecessor ℓ_1:

  $f \wedge n \geq 0 \wedge p' = 0$

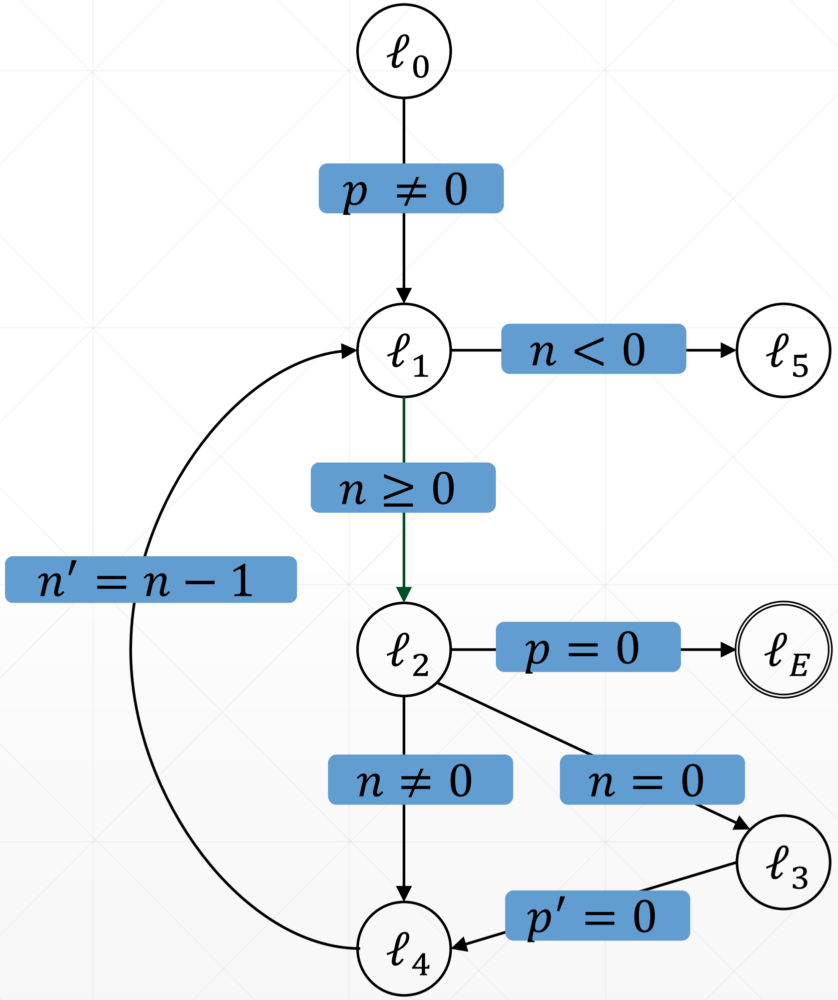
  → Unsatisfiable

  → Strengthen frames $F_{0,\ell_2}, F_{1,\ell_2}$

Proof-Obligations:

- ∅

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t
ℓ_4	f	t

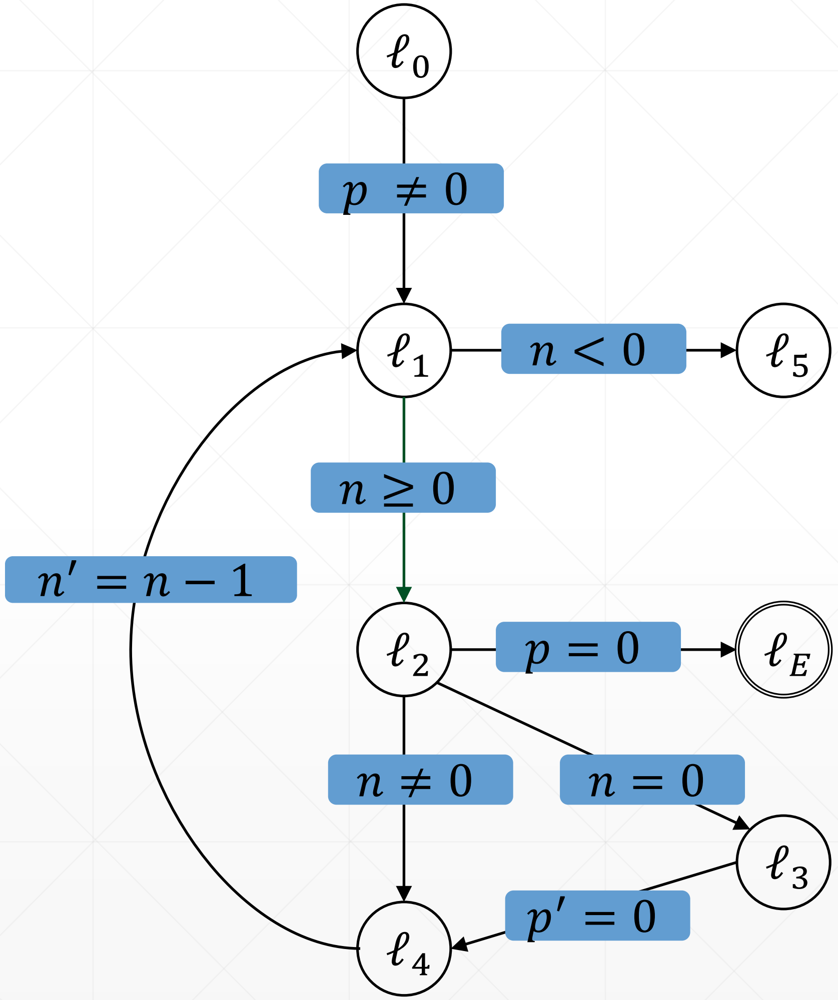
5. Step: Iteration 1 Propagation-Phase

➢ Is there a global fixpoint?

Proof-Obligations:

- $\emptyset$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t
ℓ_4	f	t

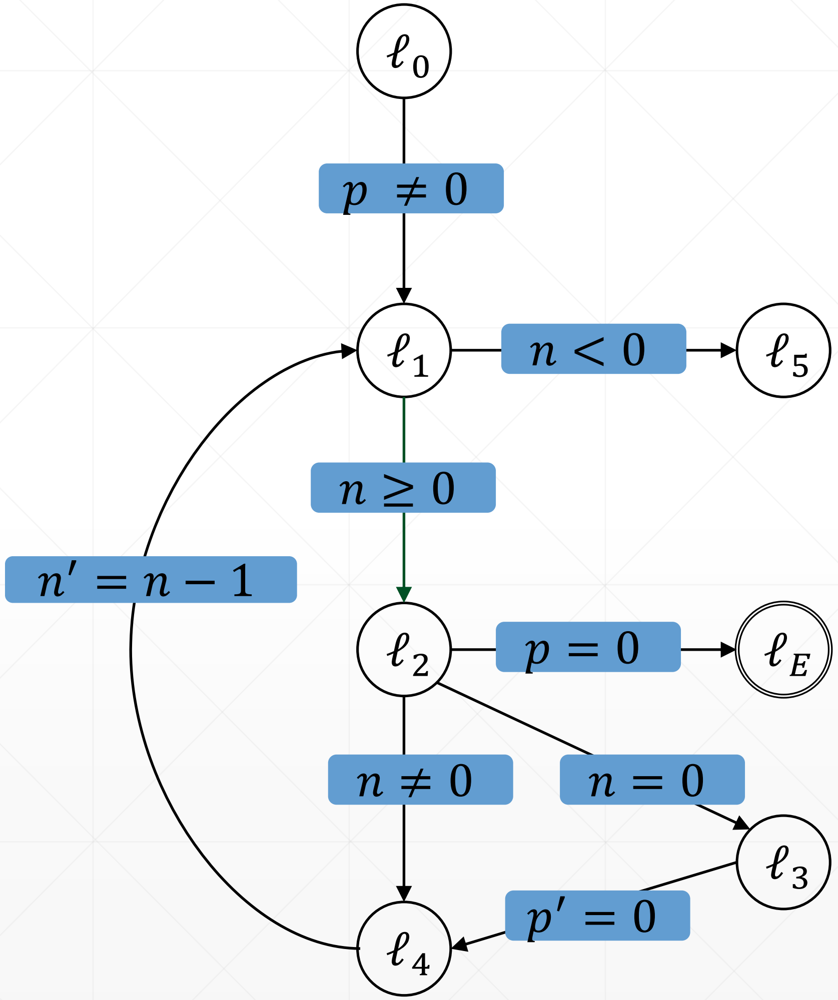
5. Step: Iteration 1 Propagation-Phase

- Is there an i where $F_{i-1,\ell} = F_{i,\ell}$ for $\ell \in L \setminus \{\ell_E\}$?

Proof-Obligations:

- $\emptyset$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t
ℓ_4	f	t

5. Step: Iteration 1 Propagation-Phase

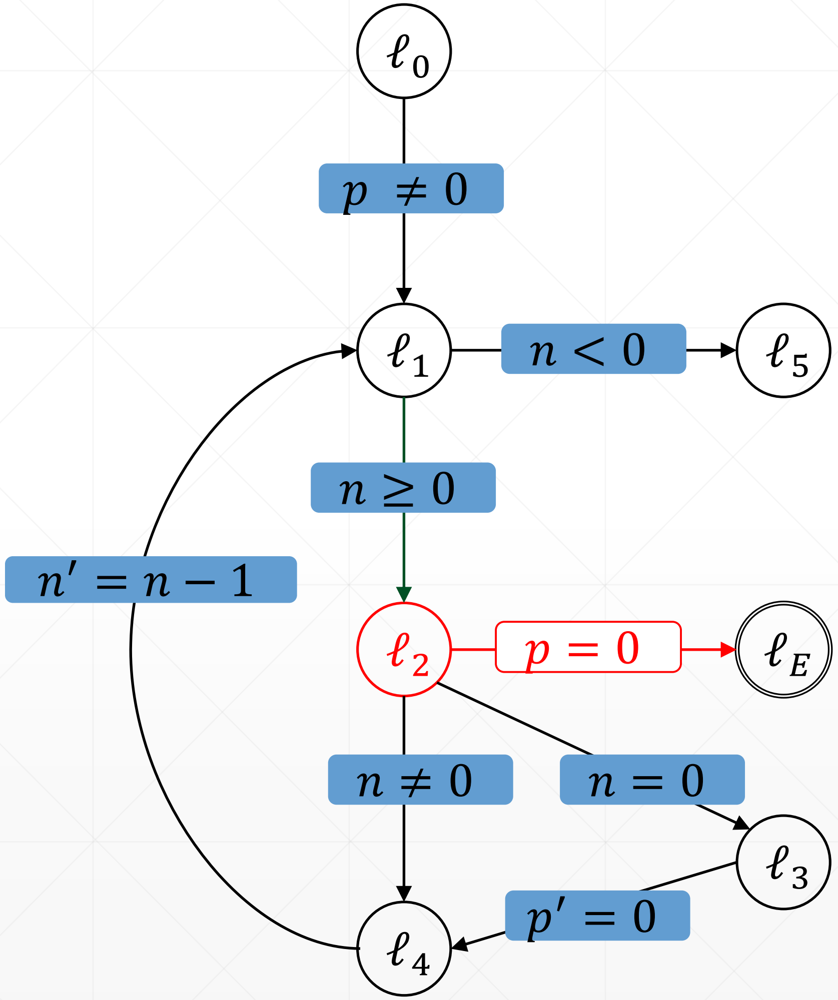
- Is there an i where $F_{i-1,\ell} = F_{i,\ell}$ for $\ell \in L \setminus \{\ell_E\}$?

➔ No. Continue with iteration 2

Proof-Obligations:

- $\emptyset$

Example:



location	0	1
ℓ_0	t	t
ℓ_1	f	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t
ℓ_4	f	t

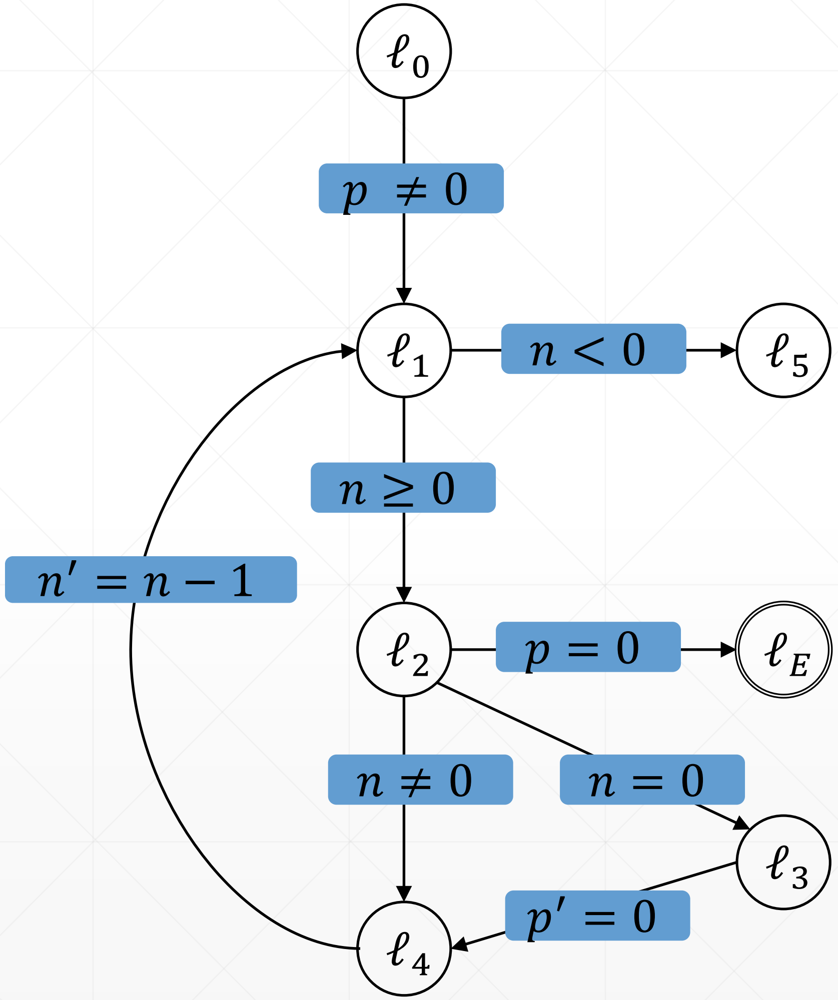
6. Step: Iteration 2 Initialization

- Initialize new frames
- Add initial proof-obligation $(p = 0, \ell_2, 2)$

Proof-Obligations:

- $\emptyset$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	f	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

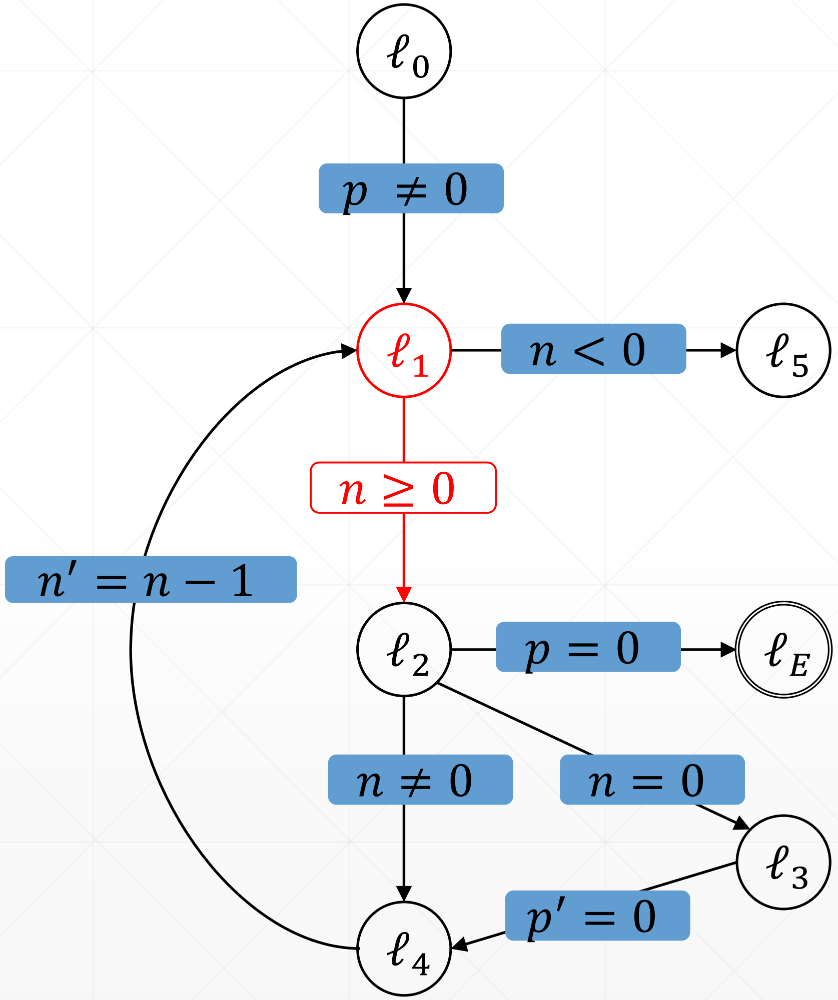
6. Step: Iteration 2 Initialization

- Initialize new frames
- Add initial proof-obligation $(p = 0, \ell_2, 2)$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	f	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

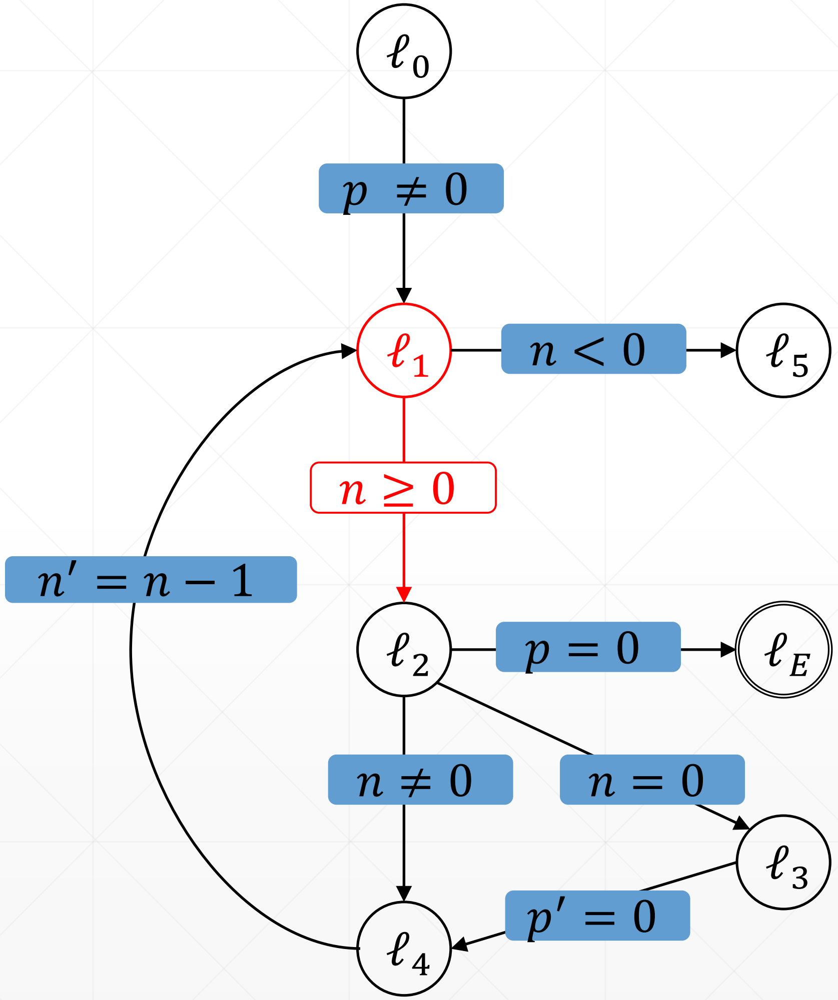
7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 2)$
  - Predecessor ℓ_1:
    $t \ \wedge \ n \geq 0 \ \wedge \ p' = 0$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	f	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 2)$

- Predecessor ℓ_1:

  $t \wedge n \geq 0 \wedge p' = 0$

  ➔ Satisfiable!

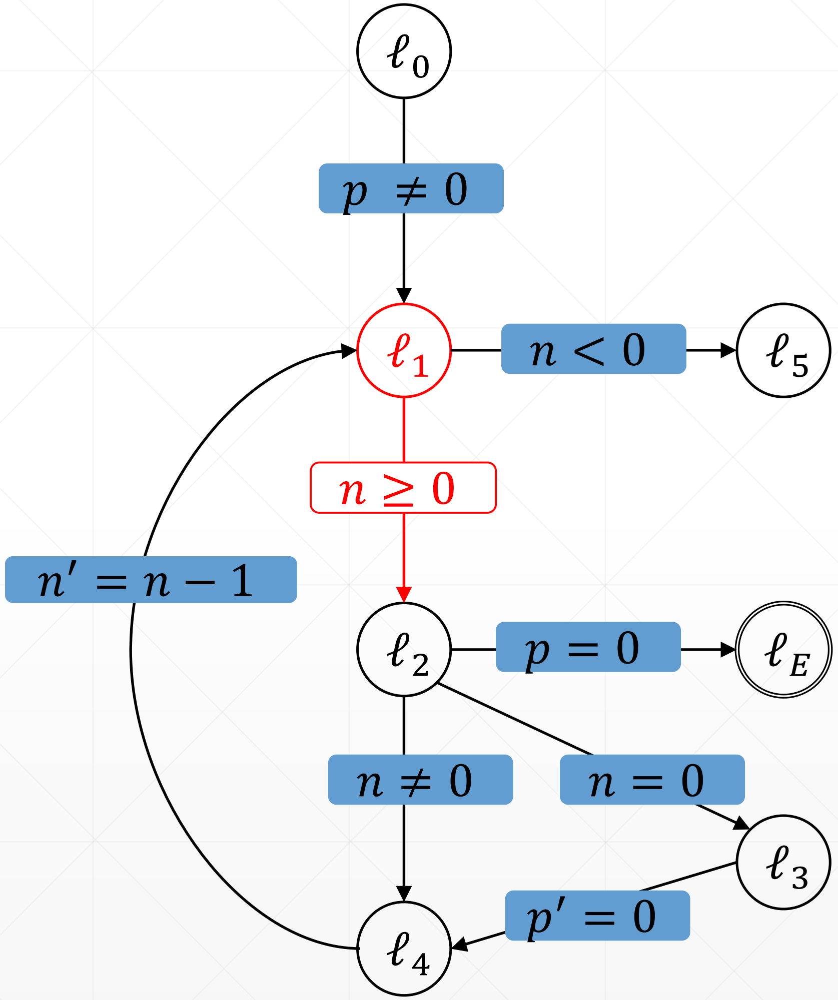
  ➔ $wp(n \geq 0, p' = 0) = (p = 0)$

  ➔ New proof-obligation $(p = 0, \ell_1, 1)$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	f	t	t
ℓ_2	$f \land p \neq 0$	$t \land p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 2)$
- Predecessor ℓ_1:

  $t \land n \geq 0 \land p' = 0$

  ➔ Satisfiable!

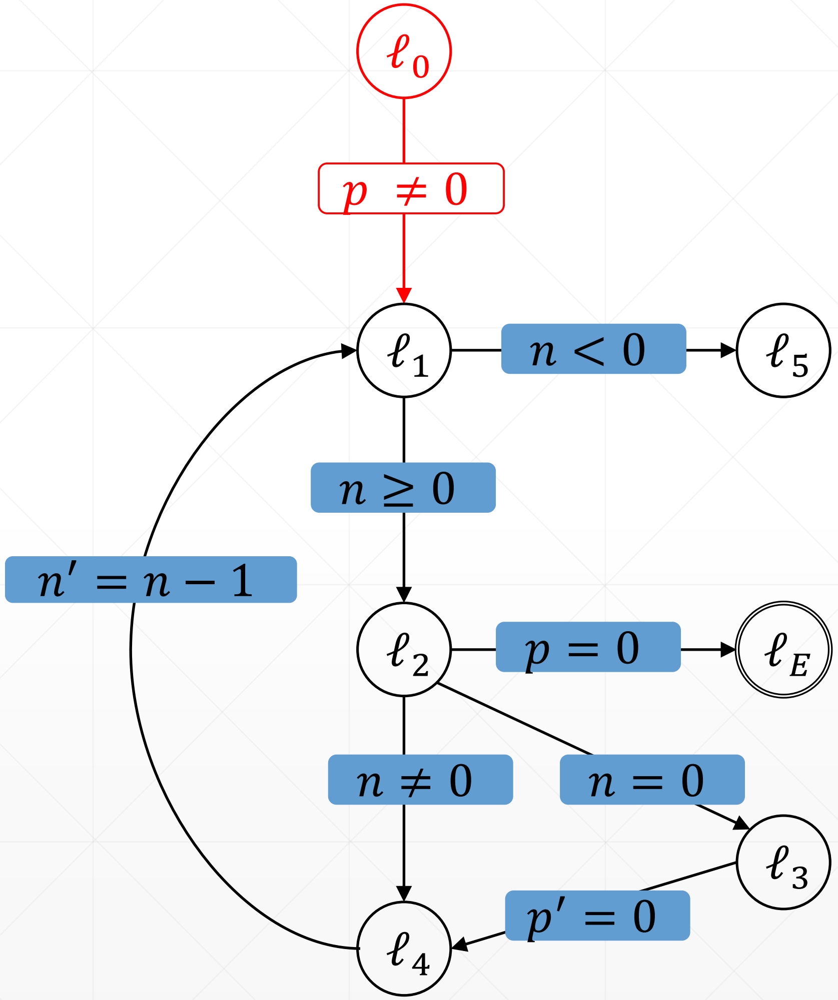
  ➔ $wp(n \geq 0, p' = 0) = (p = 0)$

  ➔ New proof-obligation $(p = 0, \ell_1, 1)$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	f	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_1, 1)$

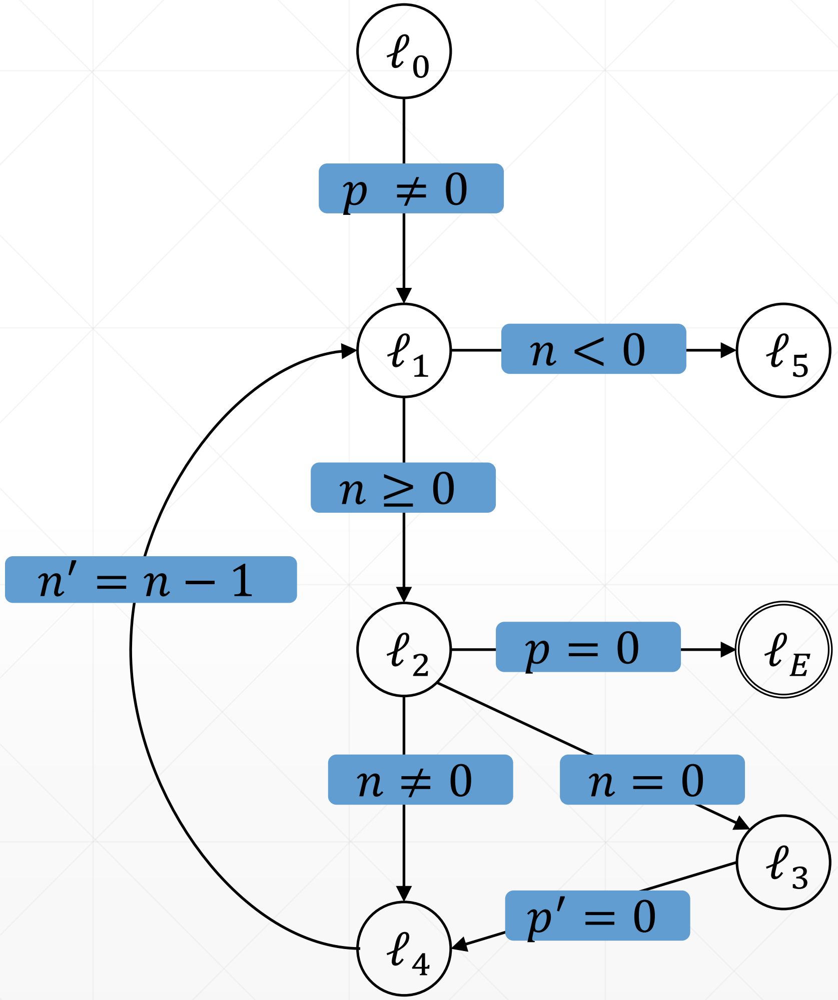
- Predecessor ℓ_0:

  $t \wedge p \neq 0 \wedge p' = 0$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_1, 1)$

- Predecessor ℓ_0:

  $t \wedge p \neq 0 \wedge p' = 0$

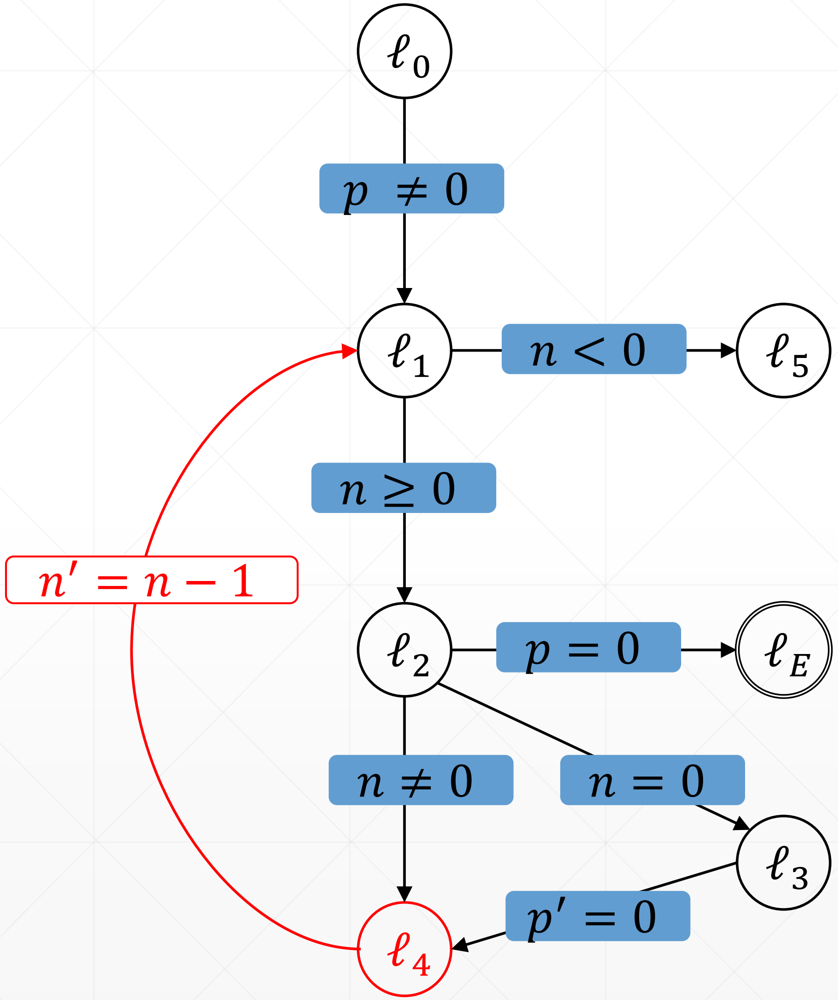
  ➔ Unsatisfiable!

  ➔ Strengthen frames $F_{0,\ell_1}, F_{1,\ell_1}$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

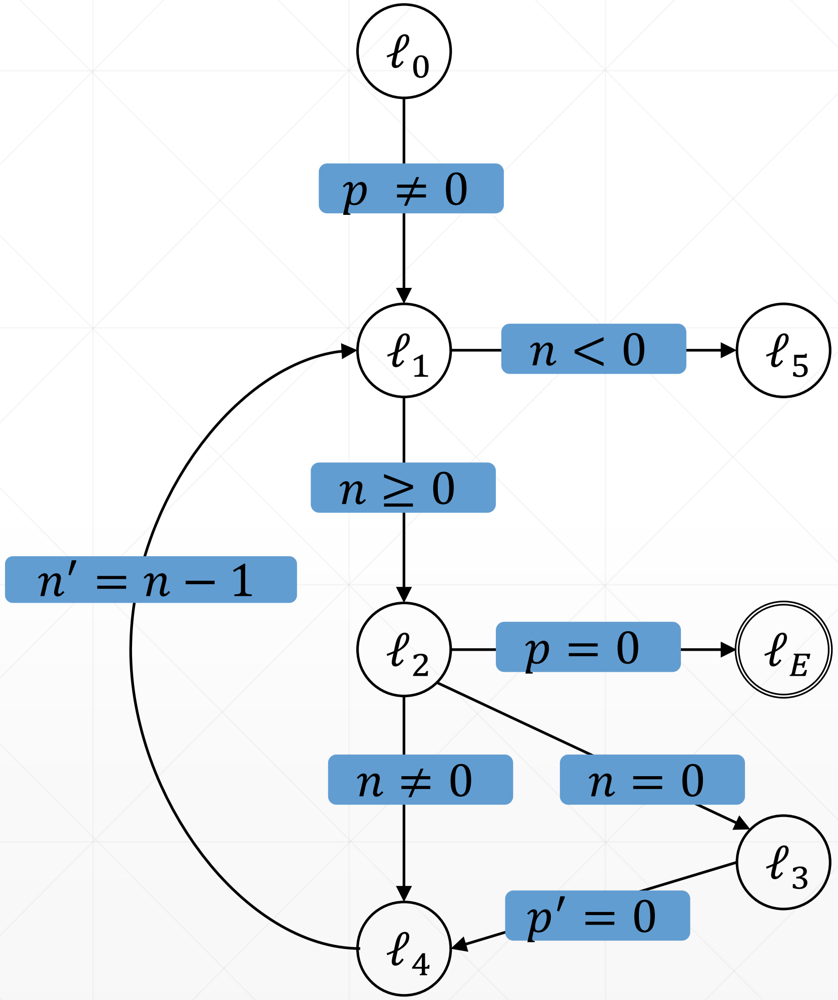
- Try to block $(p = 0, \ell_1, 1)$

- Predecessor ℓ_4:
  $f \wedge n' = n - 1 \wedge p' = 0$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_1, 1)$

- Predecessor ℓ_4:

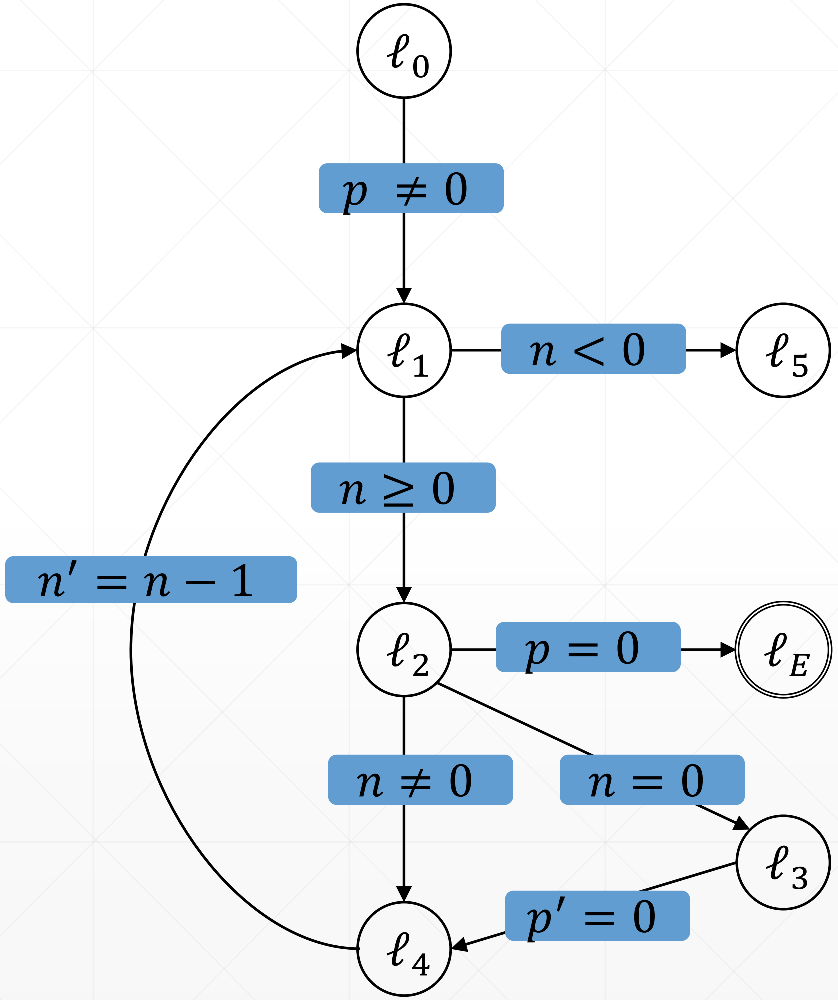
  $f \wedge n' = n - 1 \wedge p' = 0$

  ➔ Unsatisfiable!

Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_1, 1)$

- Predecessor ℓ_4:

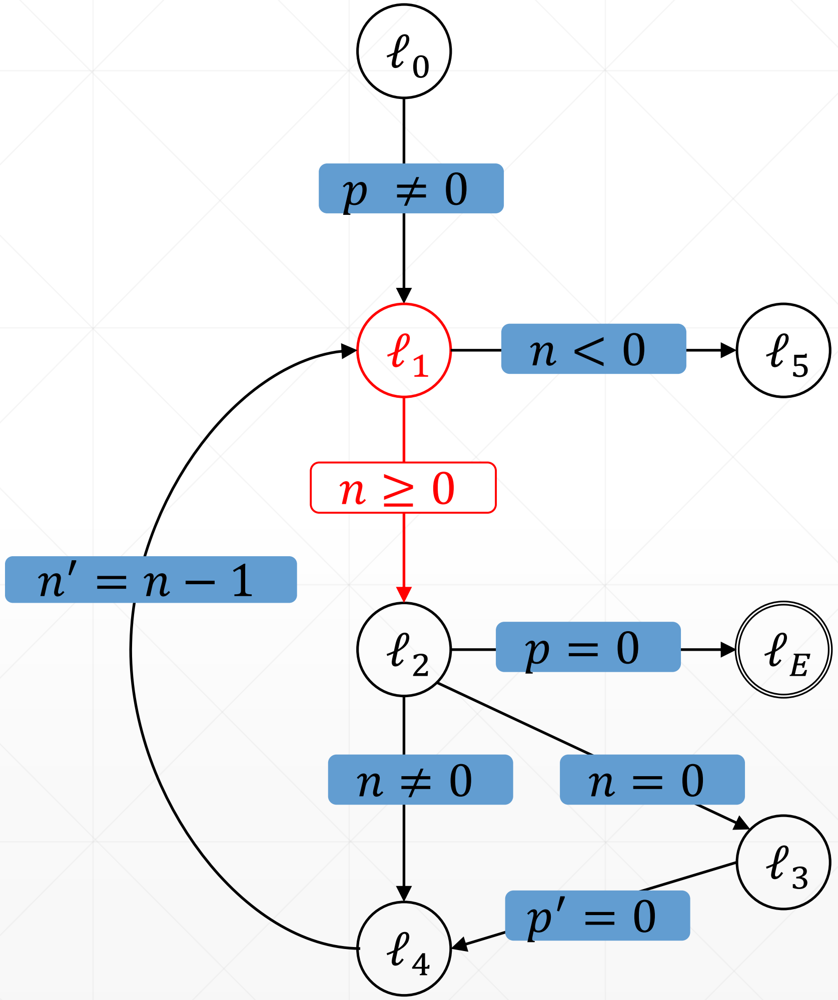
  $f \wedge n' = n - 1 \wedge p' = 0$

  ➔ Unsatisfiable!

Proof-Obligations:

- $(p = 0, \ell_2, 2)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t
ℓ_4	f	t	t

7. Step: Iteration 2 Blocking-Phase:

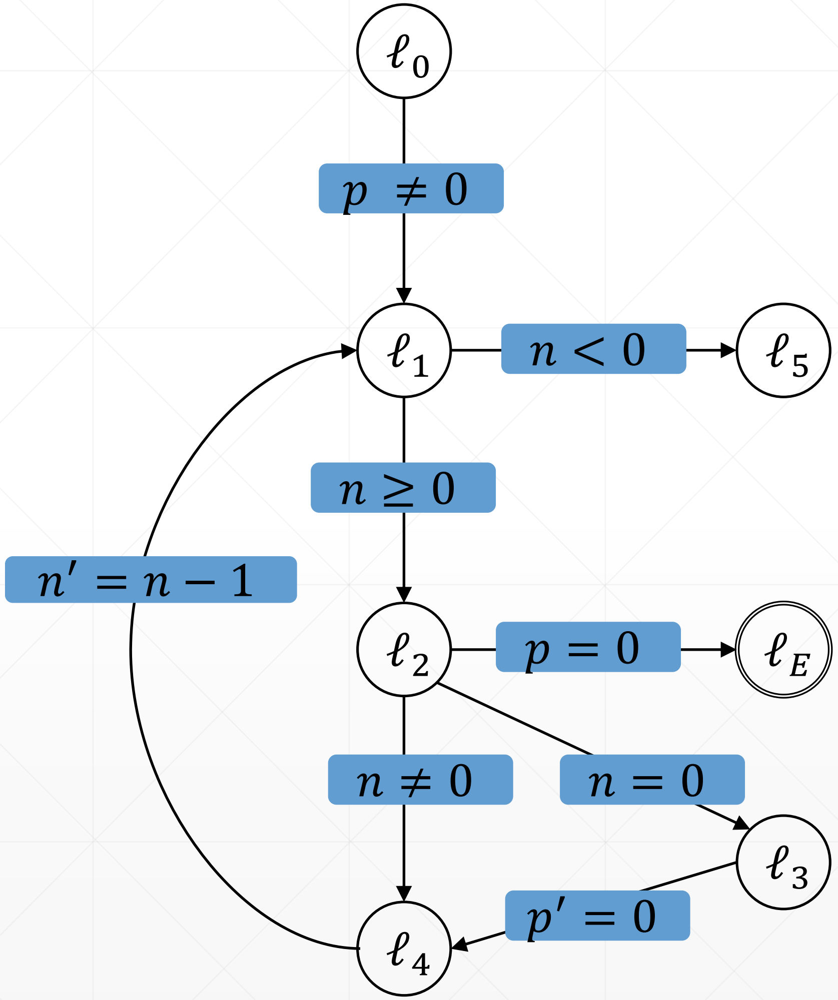
- Try to block $(p = 0, \ell_2, 2)$ again

- Predecessor ℓ_1:
  $t \wedge p \neq 0 \wedge n \geq 0 \wedge p' = 0$

Proof-Obligations:

- $(p = 0, \ell_2, 2)$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t	t
ℓ_4	f	t	t

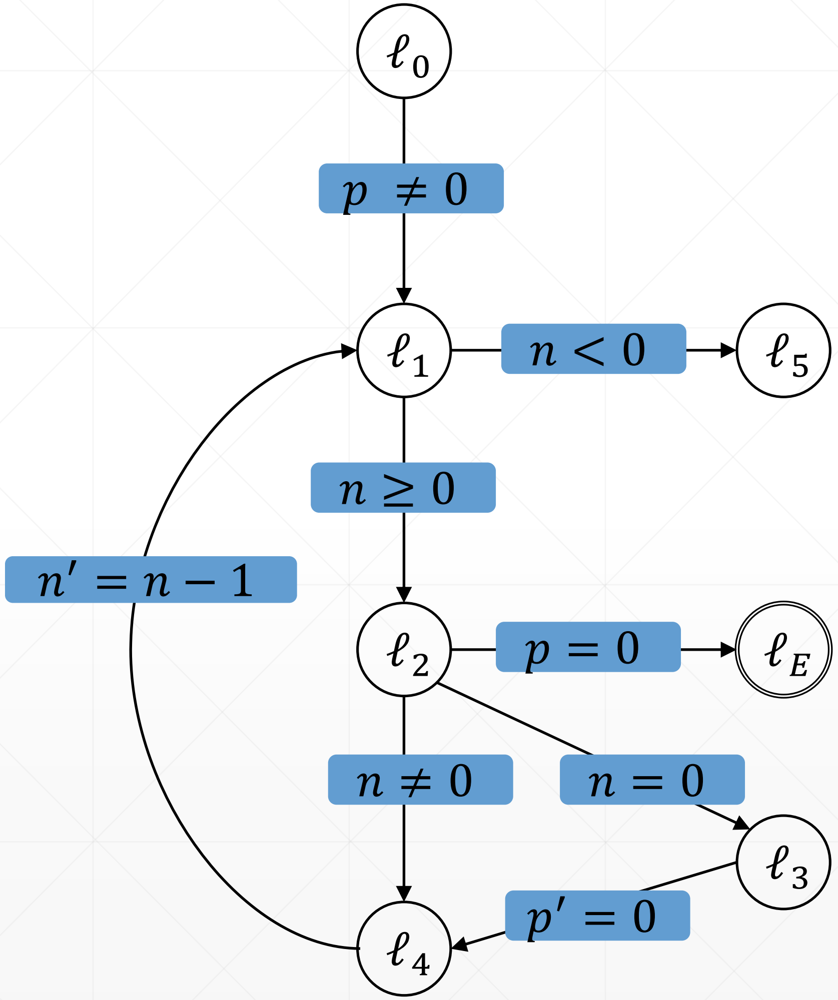
7. Step: Iteration 2 Blocking-Phase:

- Try to block $(p = 0, \ell_2, 2)$ again
- Predecessor ℓ_1:
  $t \wedge p \neq 0 \wedge n \geq 0 \wedge p' = 0$
  ➔ Unsatisfiable!
  ➔ Strengthen frames F_{2,ℓ_2}

Proof-Obligations:

- $\emptyset$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t	t
ℓ_4	f	t	t

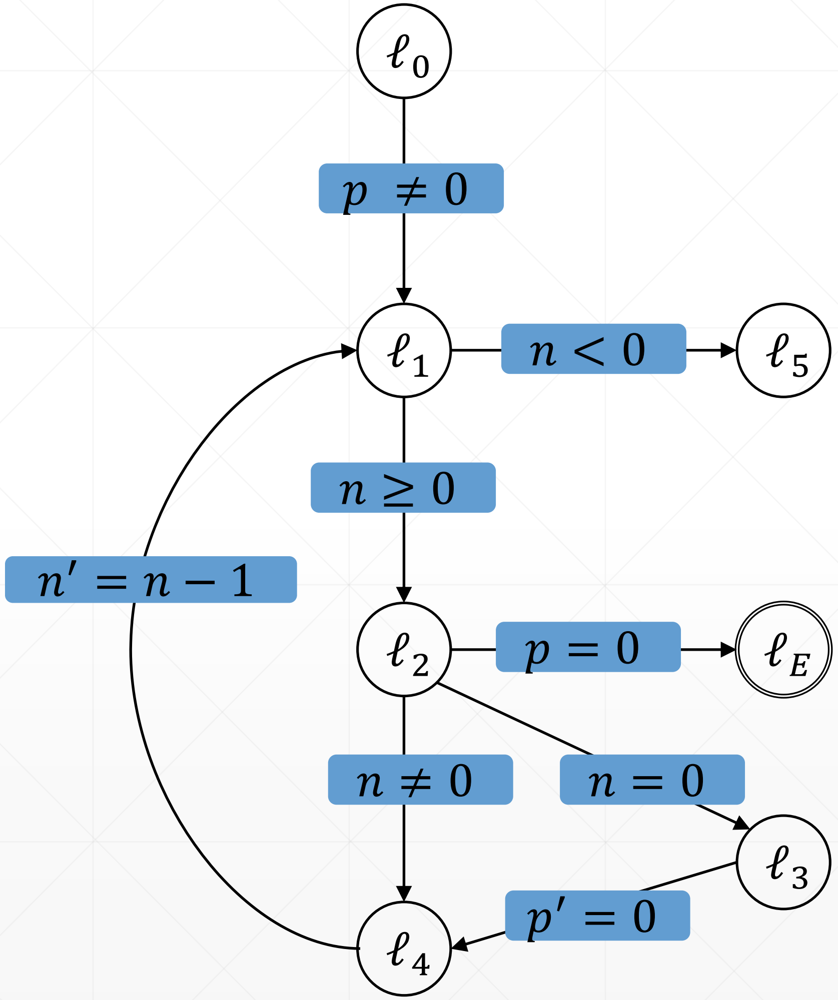
8. Step: Iteration 2 Propagation-Phase:

- Is there a global fixpoint?
  - ➔ No. Continue with Iteration 3

Proof-Obligations:

- $\emptyset$

Example:



location	0	1	2
ℓ_0	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t	t
ℓ_4	f	t	t

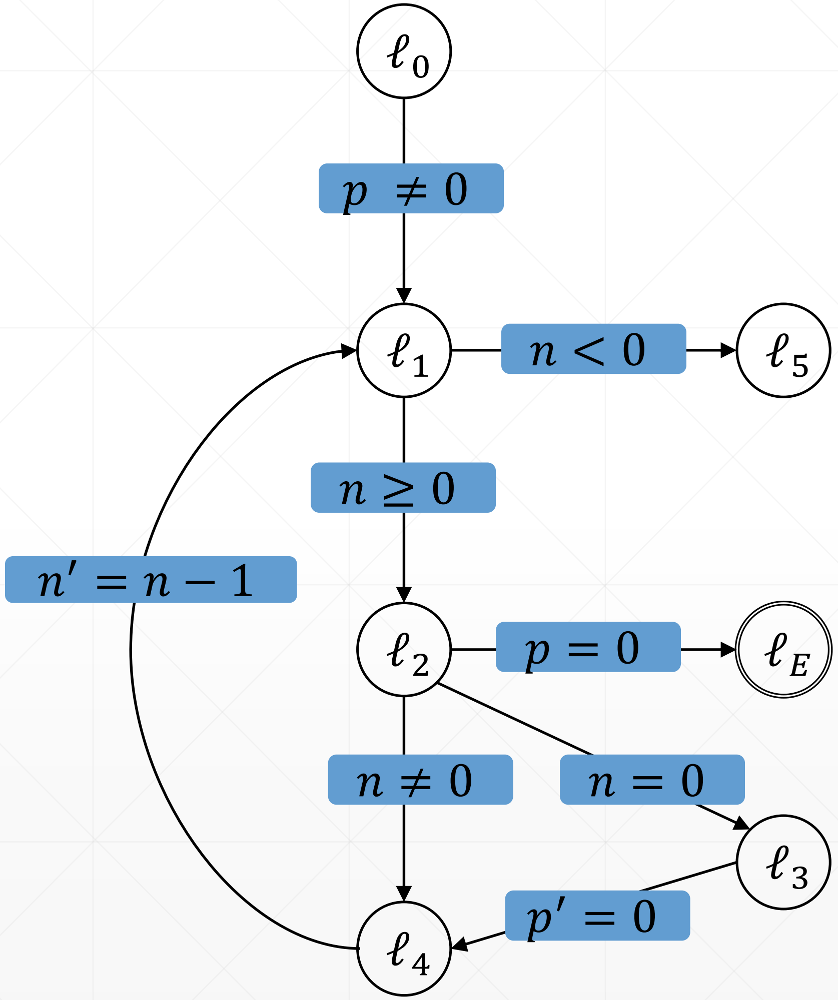
9. Step: Iteration 3 Initialization

- Initialize new frames
- Get initial proof-obligations

Proof-Obligations:

- $\emptyset$

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t	t
ℓ_4	f	t	t	t

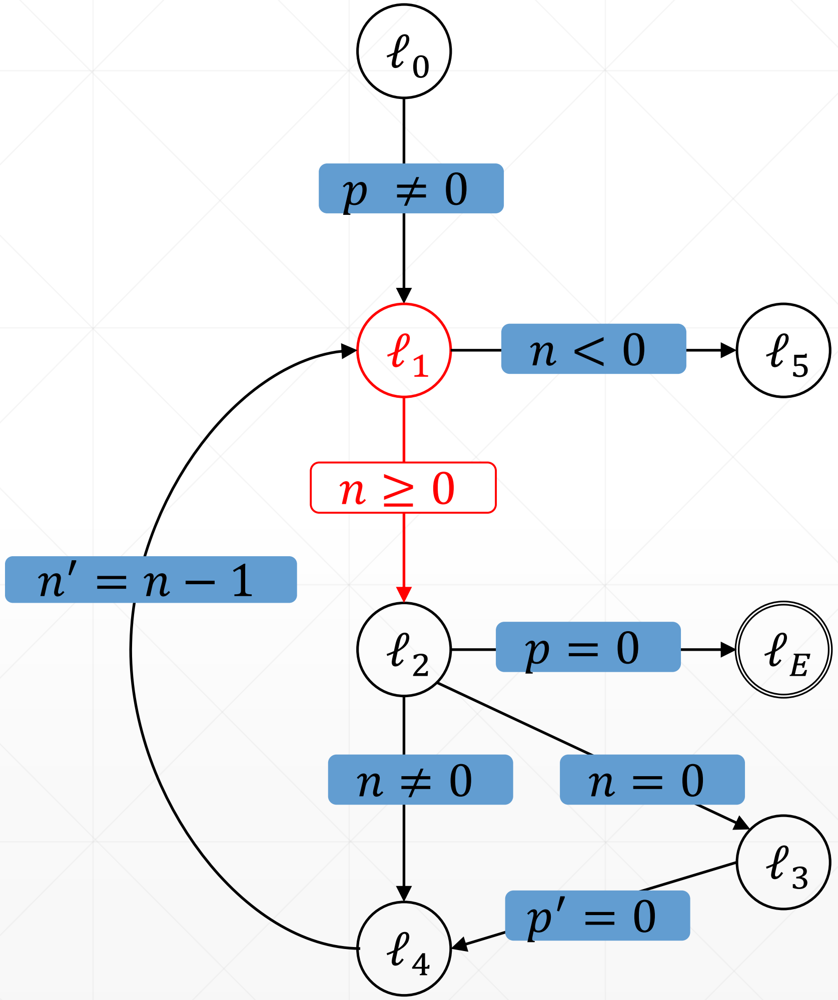
9. Step: Iteration 3 Initialization

- Initialize new frames
- Get initial proof-obligations

Proof-Obligations:

- $(p = 0, \ell_2, 3)$

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t	t
ℓ_4	f	t	t	t

10. Step: Iteration 3 Blocking-Phase

- Try to block $(p = 0, \ell_2, 3)$

- Predecessor ℓ_1:

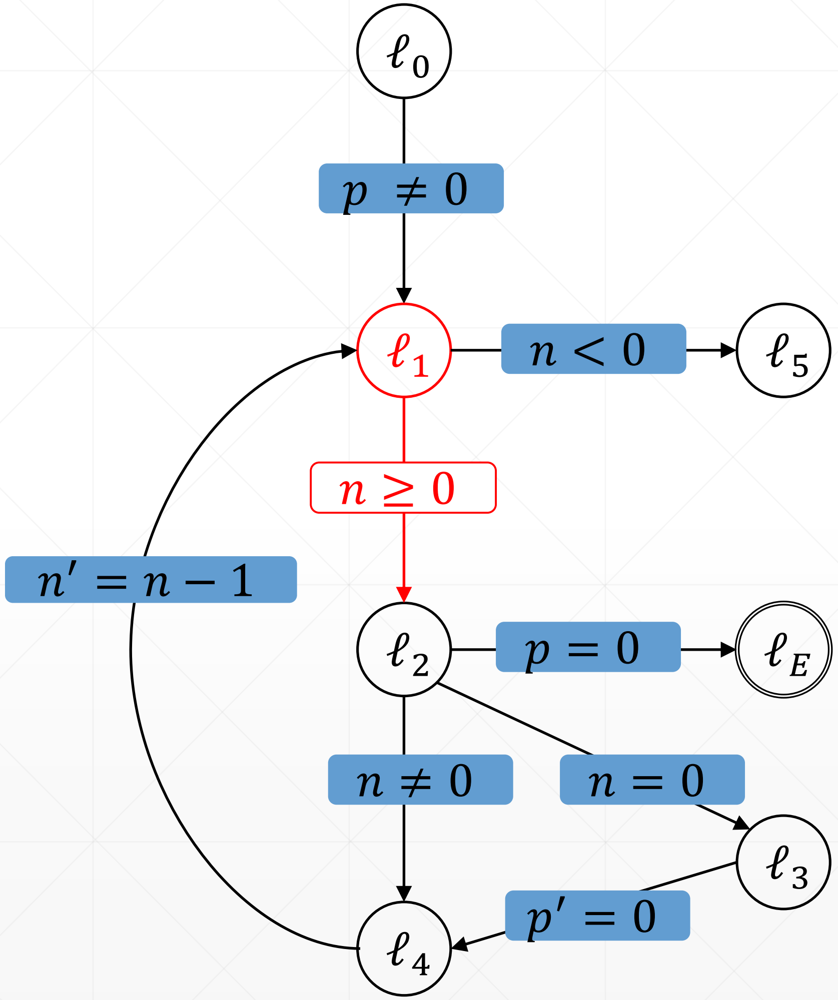
  $t \ \wedge \ n \geq 0 \ \wedge \ p' = 0$

  ➔ Like the Iteration before this is satisfiable

Proof-Obligations:

- $(p = 0, \ell_2, 3)$

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t	t
ℓ_4	f	t	t	t

10. Step: Iteration 3 Blocking-Phase

- Try to block $(p = 0, \ell_2, 3)$

- Predecessor ℓ_1:

  $t \wedge n \geq 0 \wedge p' = 0$

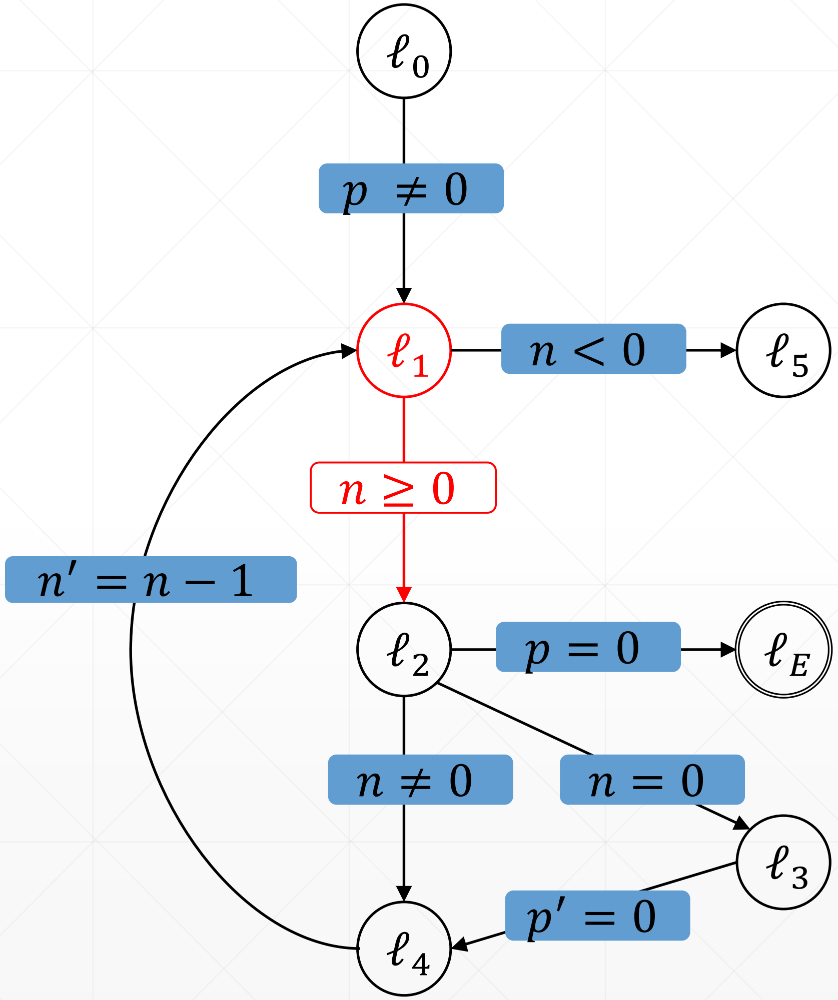
  ➔ Get same proof-obligation as before but on Iteration 2

  ➔ $(p = 0, \ell_1, 2)$

Proof-Obligations:

- $(p = 0, \ell_2, 3)$

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t	t
ℓ_4	f	t	t	t

10. Step: Iteration 3 Blocking-Phase

- Try to block $(p = 0, \ell_2, 3)$
  - Predecessor ℓ_1:

    $t \wedge n \geq 0 \wedge p' = 0$

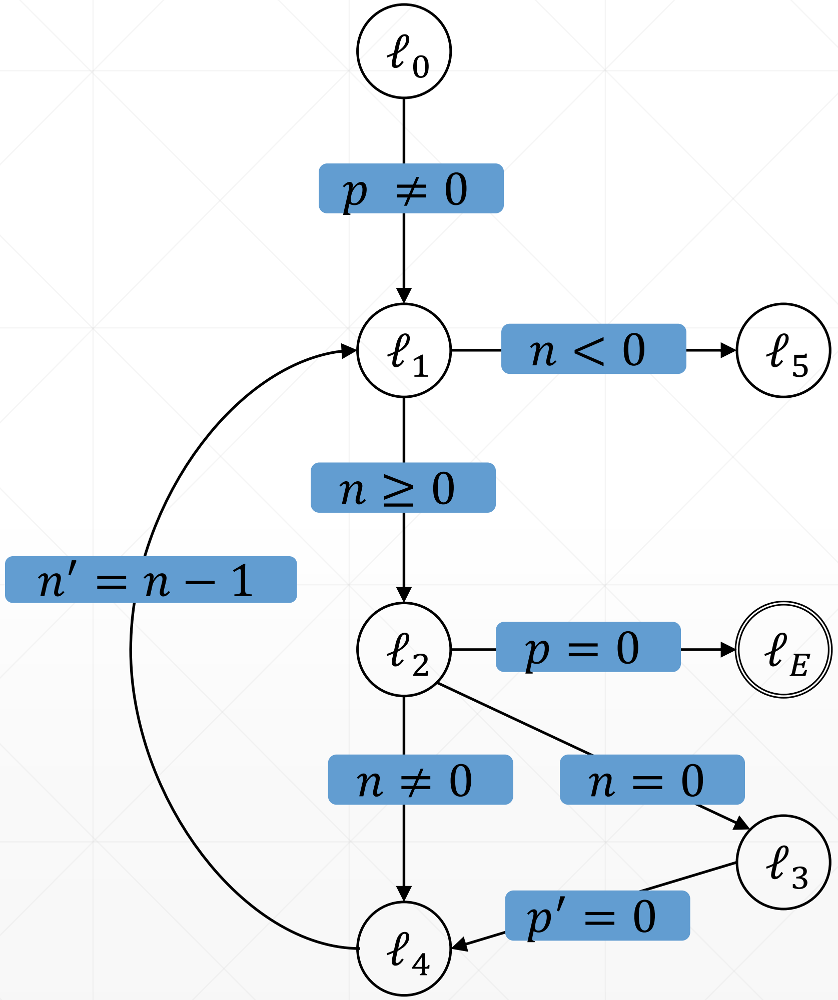
    ➔ Get same proof-obligation as before but on Iteration 2

    ➔ $(p = 0, \ell_1, 2)$

Proof-Obligations:

- $(p = 0, \ell_2, 3)$
- $(p = 0, \ell_1, 2)$

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \land p \neq 0$	$t \land p \neq 0$	t	t
ℓ_2	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	t
ℓ_3	f	t	t	t
ℓ_4	f	t	t	t

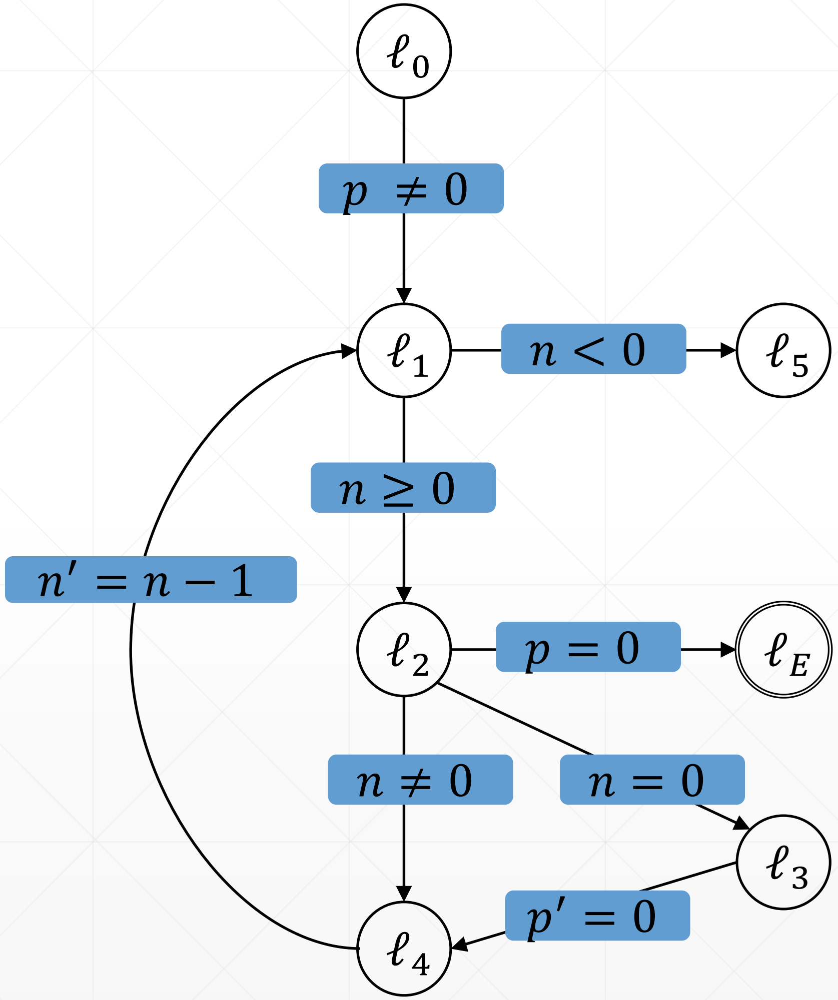
10. Step: Iteration 3 Blocking-Phase

- There are a lot of repetitions
  - ➔ Duplicate proof-obligations

Proof-Obligations:

- $(p = 0, \ell_2, 3)$
- $(p = 0, \ell_1, 2)$

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	f	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t

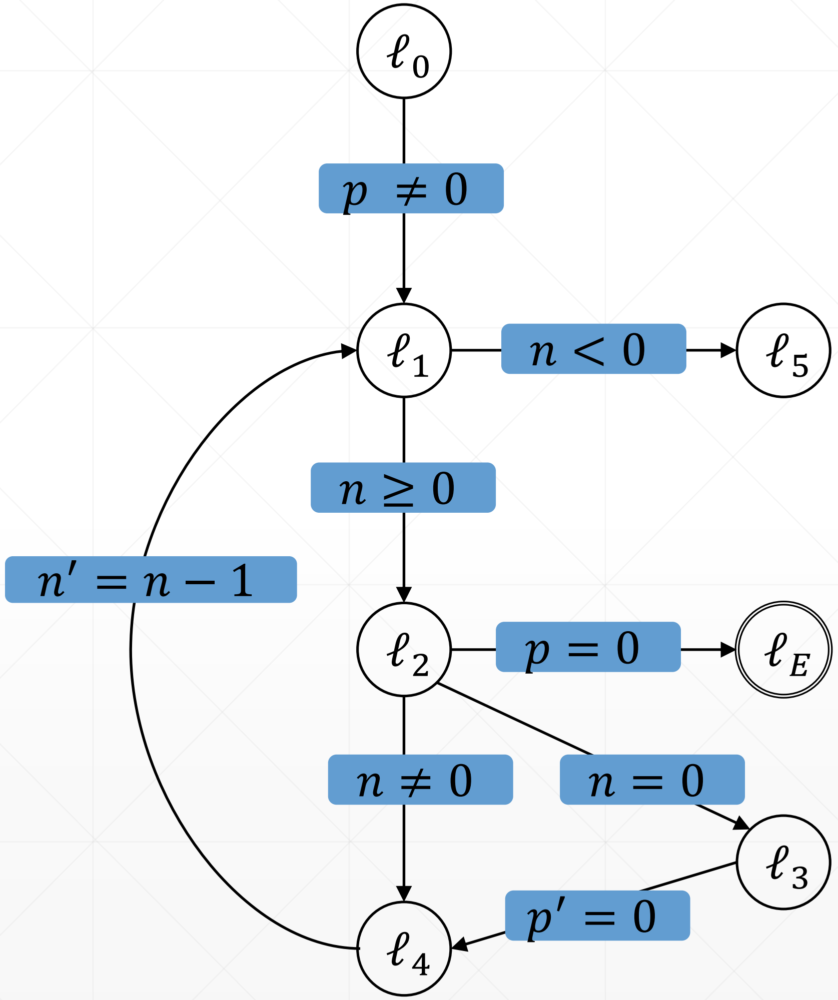
10. Step: Iteration 3 Blocking-Phase

- There are a lot of repetitions
  - ➔ Duplicate proof-obligations

Proof-Obligations:

- ∅

Example:



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	t
ℓ_2	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$
ℓ_3	f	t	t	t
ℓ_4	$f \land p \neq 0$	$t \land p \neq 0$	t	t

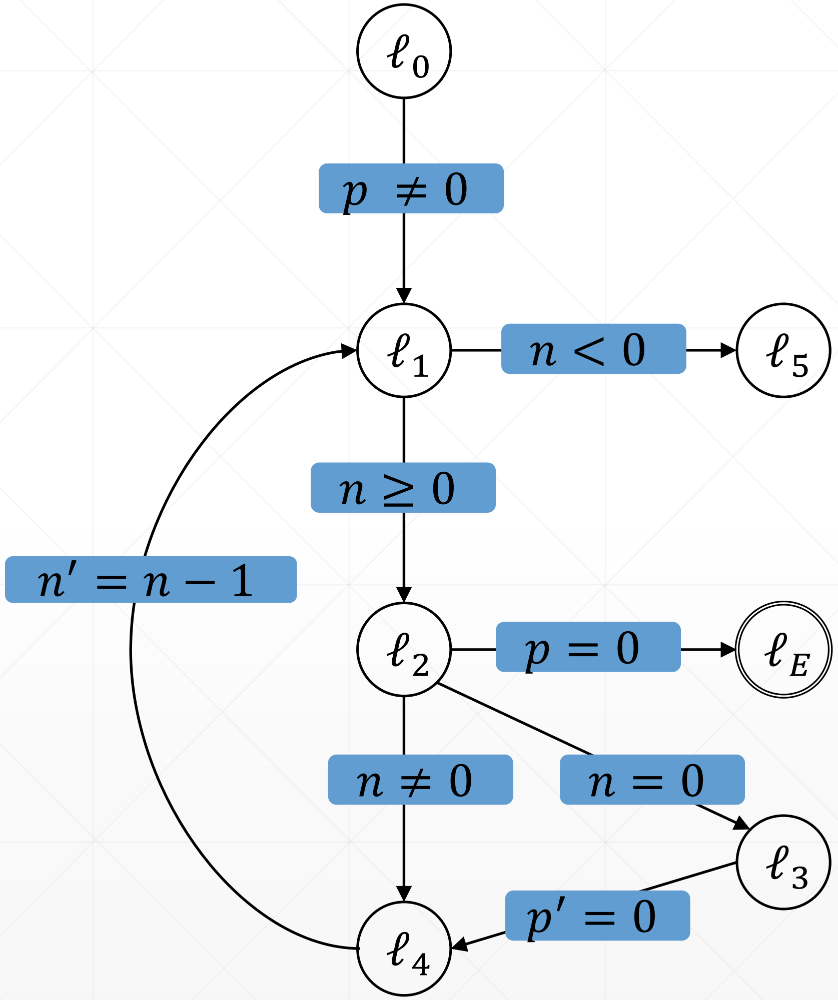
11. Step: Iteration 3 Propagation-Phase

- Is there a global fixpoint?
  - ➔ No. Continue with Iteration 4

Proof-Obligations:

- $\emptyset$

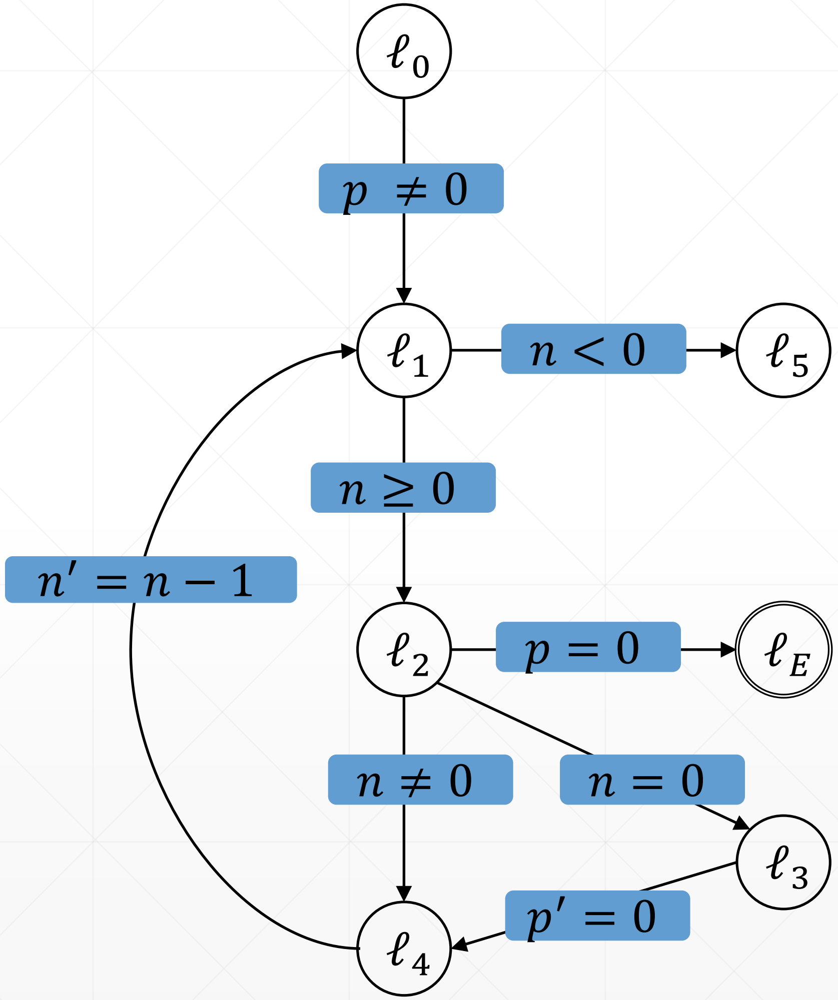
Example:



location	0	1	2	3	4
ℓ_0	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t	t

11. Step: Iteration 4 Initialization

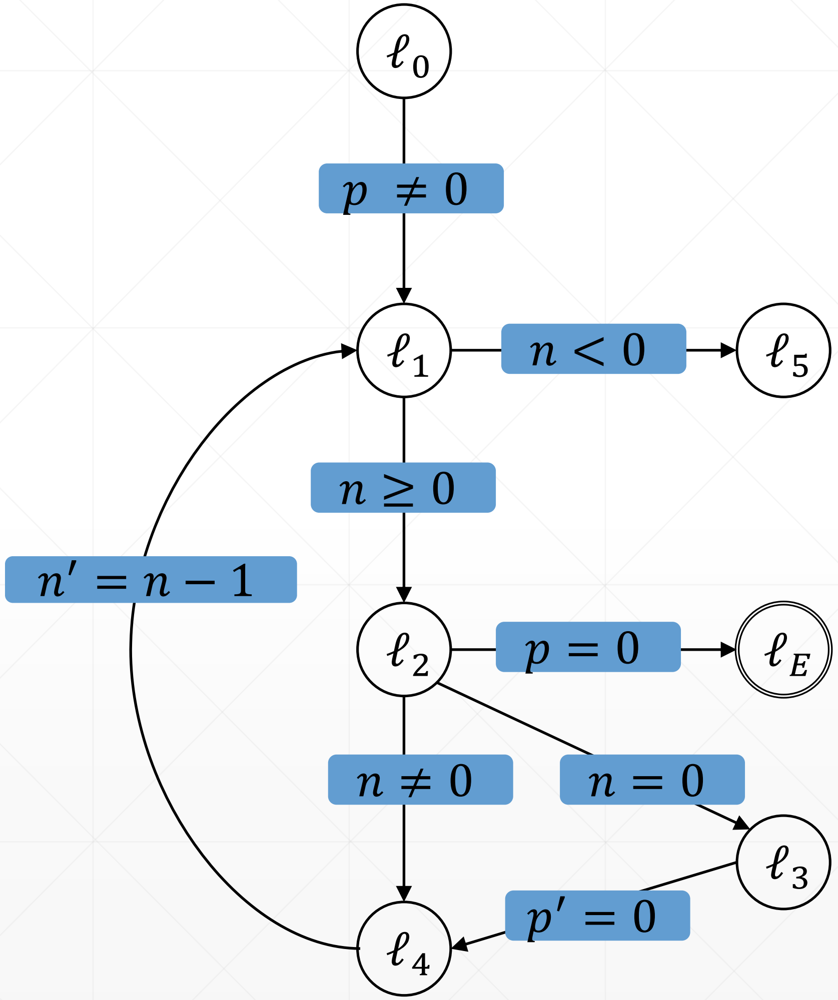
Example:



location	0	1	2	3	4
ℓ_0	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	f	t	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	t	t	t

12. Step: Iteration 4 Blocking-Phase

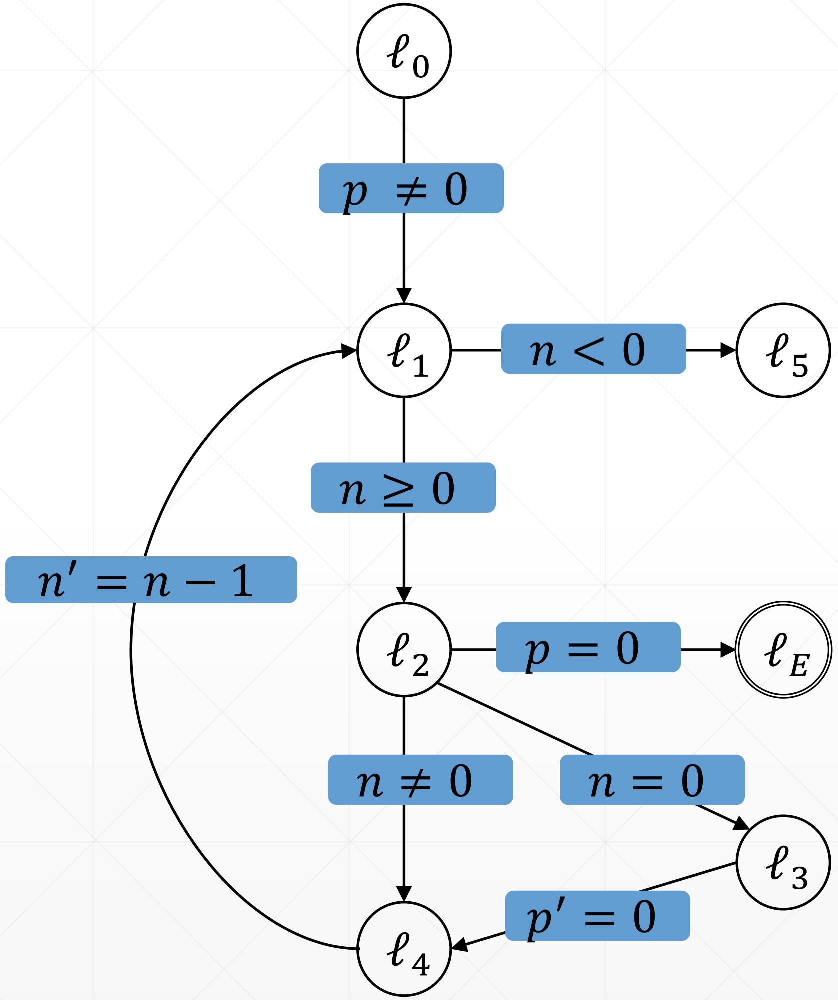
Example:



location	0	1	2	3	4
ℓ_0	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	$f \wedge f$	$t \wedge f$	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t

12. Step: Iteration 4 Blocking-Phase

Example:

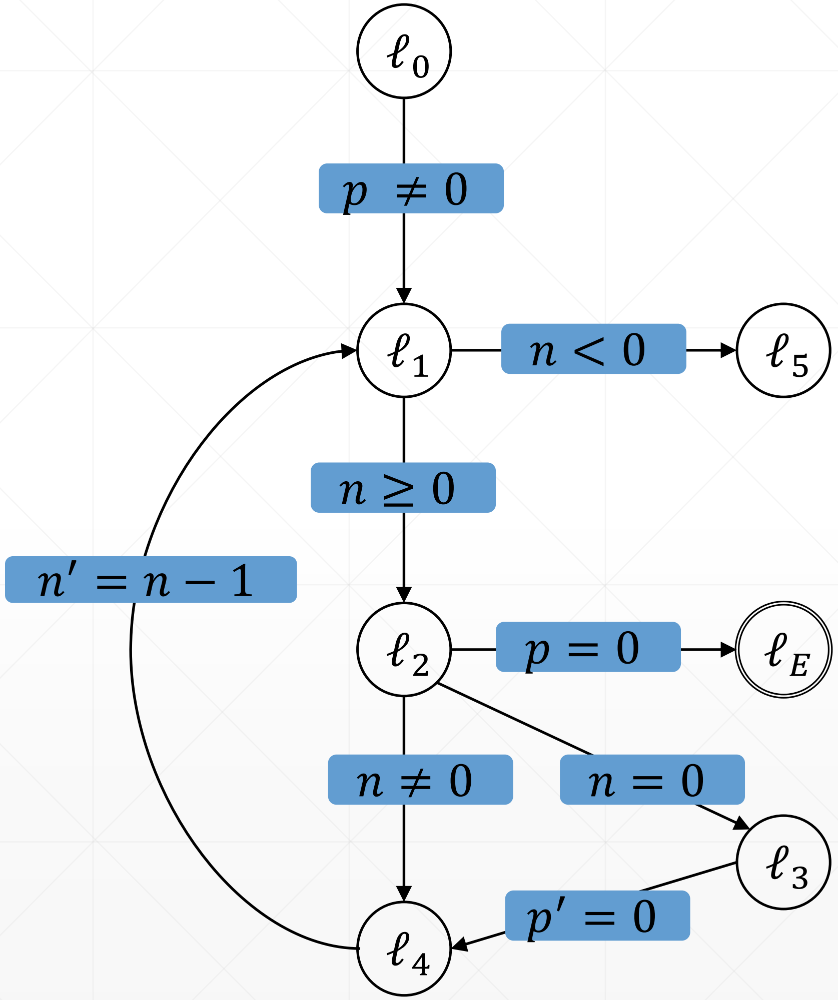


location	0	1	2	3	4
ℓ_0	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	$f \wedge f$	$t \wedge f$	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t

13. Step: Iteration 4 Propagation-Phase

- Is there a global fixpoint?
  - ➔ No. Continue with Iteration 5

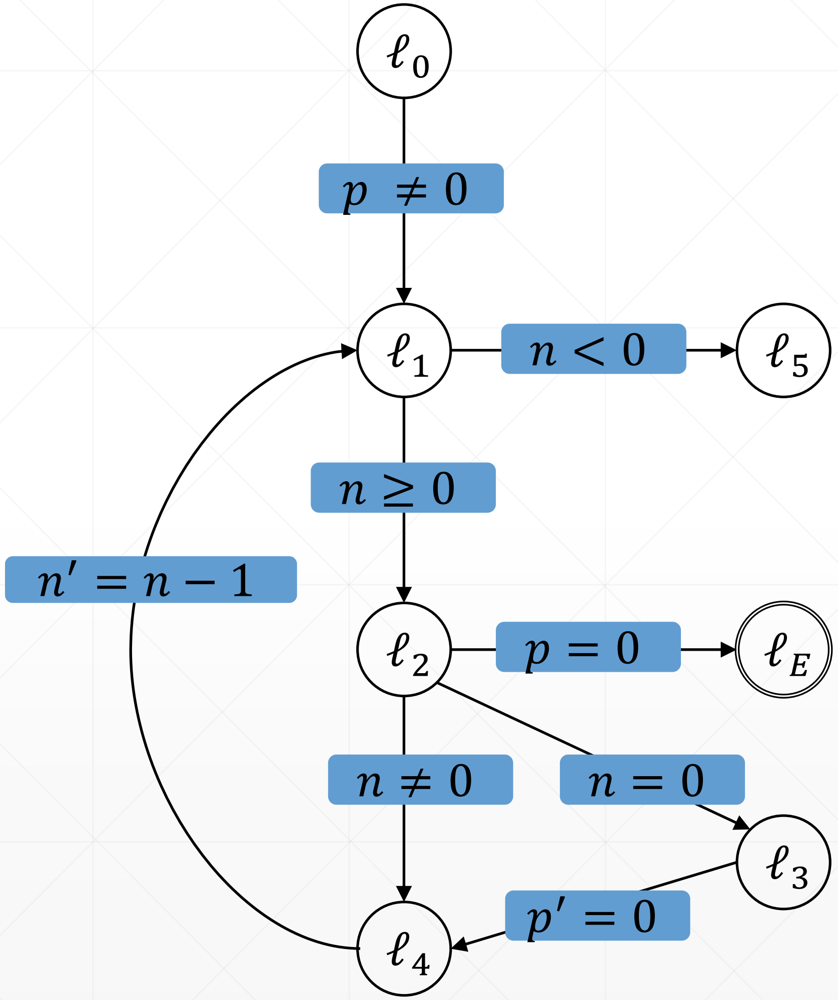
Example:



location	0	1	2	3	4	5
ℓ_0	t	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_3	$f \wedge f$	$t \wedge f$	t	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t	t

14. Step: Iteration 5 Initialization

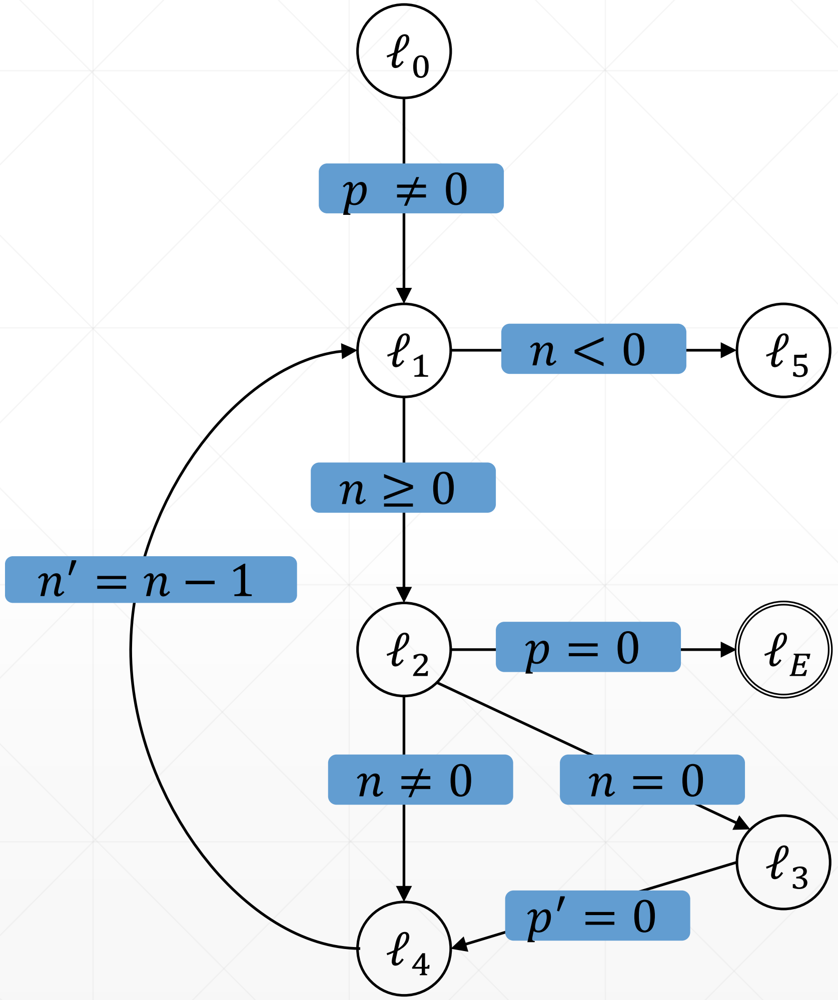
Example:



location	0	1	2	3	4	5
ℓ_0	t	t	t	t	t	t
ℓ_1	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	t	t
ℓ_2	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	t
ℓ_3	$f \land f$	$t \land f$	t	t	t	t
ℓ_4	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	t	t	t

15. Step: Iteration 5 Blocking-Phase

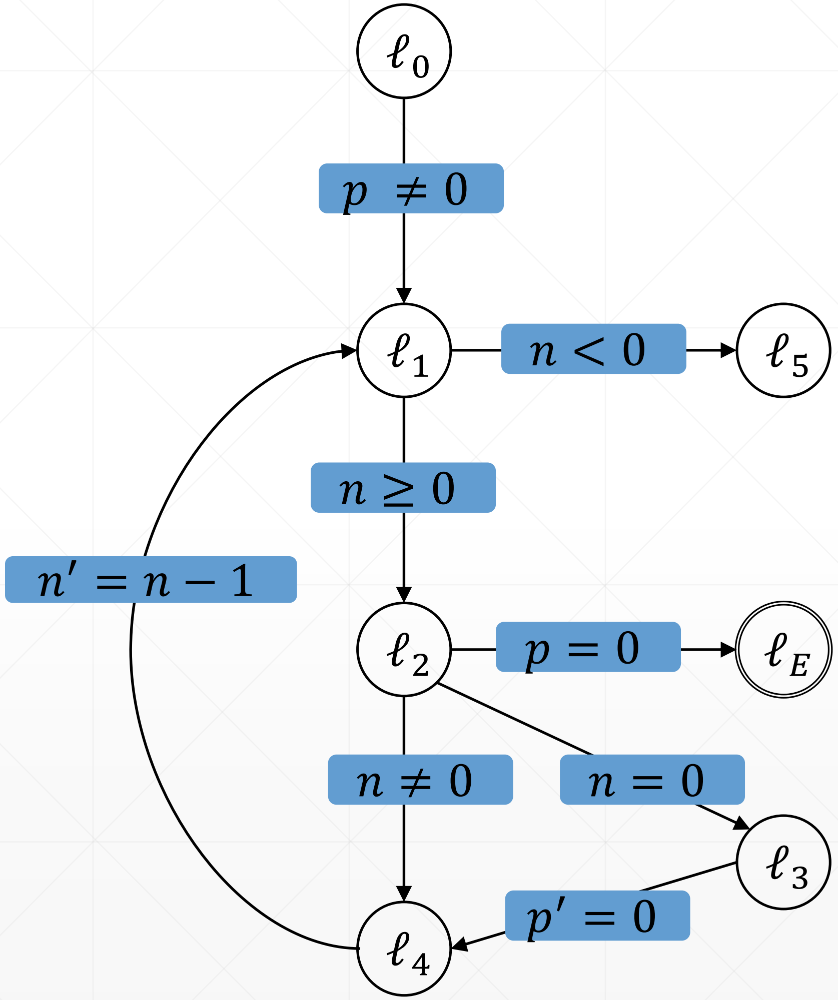
Example:



location	0	1	2	3	4	5
ℓ_0	t	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	$f \wedge f$	$t \wedge f$	$t \wedge f$	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t

15. Step: Iteration 5 Blocking-Phase

Example:

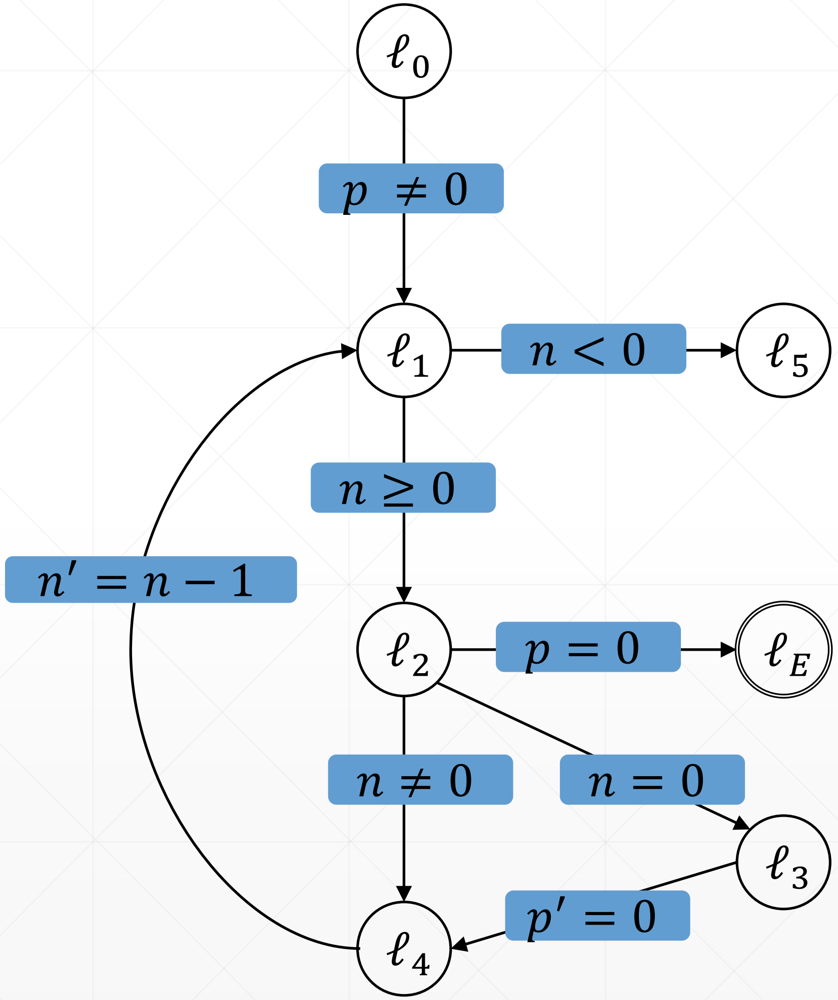


location	0	1	2	3	4	5
ℓ_0	t	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	$f \wedge f$	$t \wedge f$	$t \wedge f$	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t

16. Step: Iteration 5 Propagation-Phase

- Is there a global fixpoint?

Example:



location	0	1	2	3	4	5
ℓ_0	t	t	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
ℓ_3	$f \wedge f$	$t \wedge f$	$t \wedge f$	t	t	t
ℓ_4	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t	t

16. Step: Iteration 5 Propagation-Phase

- Is there a global fixpoint?

➔ Yes!
  - ➔ Algorithm termintes returning that error location is not reachable

PDR Algorithm: Termination

- Error location is reachable, if a proof-obligation $(p, \ell, 0)$ is generated

Related Work: Other Approaches

- Our Algorithm is based on the approach by Lange et al.[1]

- Other possible ways of using PDR on software:

Bit-Blasting[2]:

- Encode the variables as bitvectors with new variable pc representing the control-flow
- Use original bit-level PDR algorithm

➔ Not very competitive because tedious handling of pc variable

1: Tim Lange, Martin R. Neuhäußer, and Thomas Noll. IC3 software model checking on control flow automata. In *FMCAD*, pages 97–104. IEEE, 2015.

2: Tobias Welp and Andreas Kuehlmann. QF BV model checking with property directed reachability. In *DATE*, pages 791–796. EDA Consortium San Jose, CA, USA / ACM DL, 2013.

Related Work: Other Approaches

- Our Algorithm is based on the approach by Lange et al.[1]

- Other possible ways of using PDR on software:

Abstract Reachability Tree (ART) Unrolling[3]:

- Transform CFG into an ART
  - ➔ Attach program-counter variable pc and first-order formula φ to locations
- Block proof-obligations like in our approach

1: Tim Lange, Martin R. Neuhäußer, and Thomas Noll. IC3 software model checking on control flow automata. In *FMCAD*, pages 97–104. IEEE, 2015.

3:Alessandro Cimatti and Alberto Griggio. Software model checking via IC3. In *CAV*, volume 7358 of *Lecture Notes in Computer Science*, pages 277–293. Springer, 2012.

Implementation in Ultimate: Description Trace Abstraction with PDR

1. Calculate sequence of transitions from initial location to error location

   ➔ Possible error trace

2. Construct a path program of error trace, by projecting given program to the transitions found in trace

3. Use PDR to show if error is reachable or not

   ➔ If reachable:

   - Error trace is feasible, program is unsafe

Implementation in Ultimate: Description Trace Abstraction with PDR

1. Calculate sequence of transitions from initial location to error location

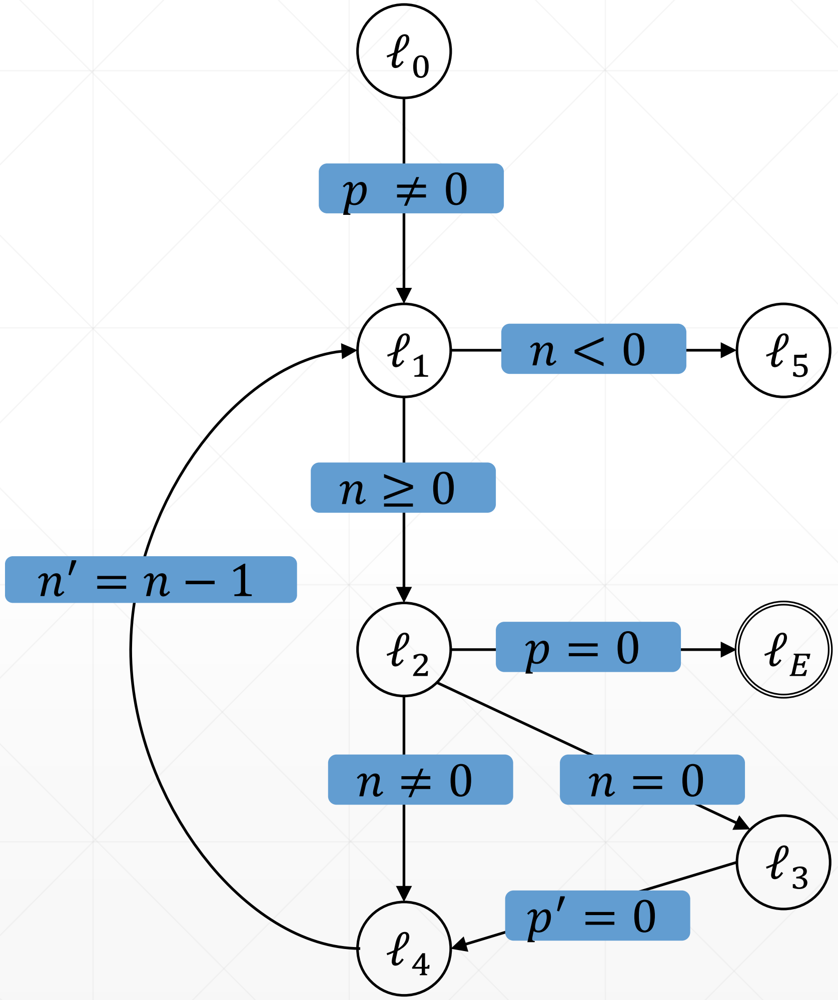
   ➔ Possible error trace

2. Construct a path program of error trace, by projecting given program to the transitions found in trace

3. Use PDR to show if error is reachable or not

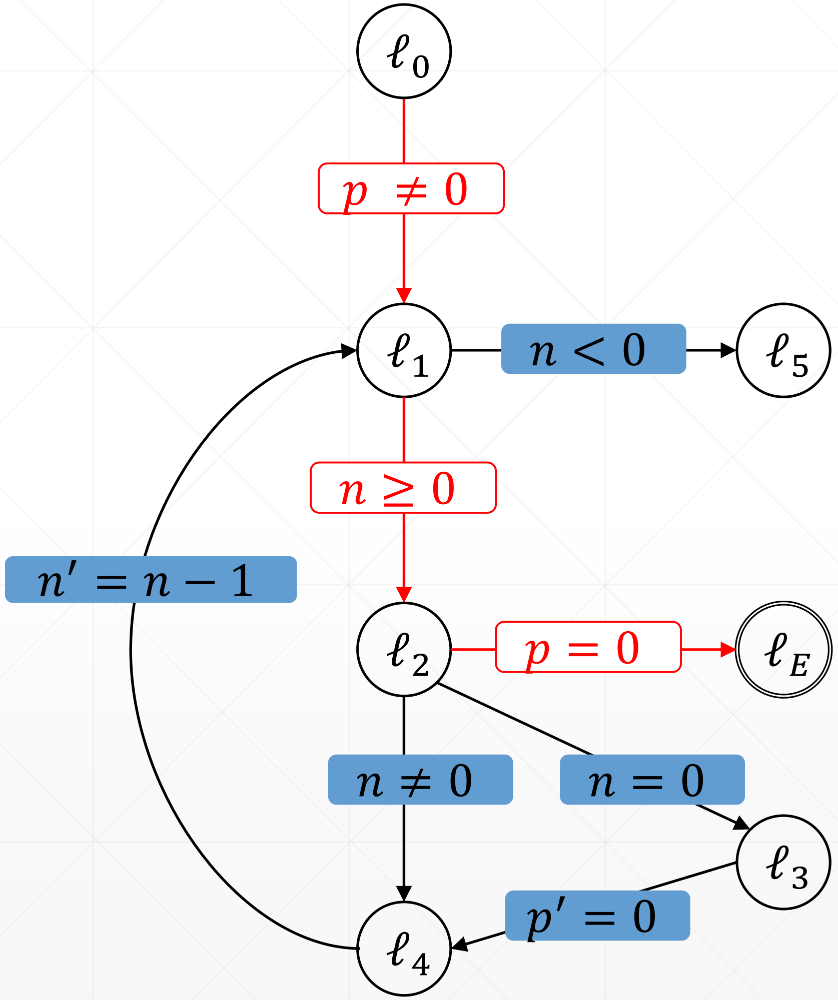
   ➔ If unreachable:
   - Use formulas at the fixpoint as interpolant sequence to refute other error traces

Implementation in Ultimate: Trace Abstraction with PDR



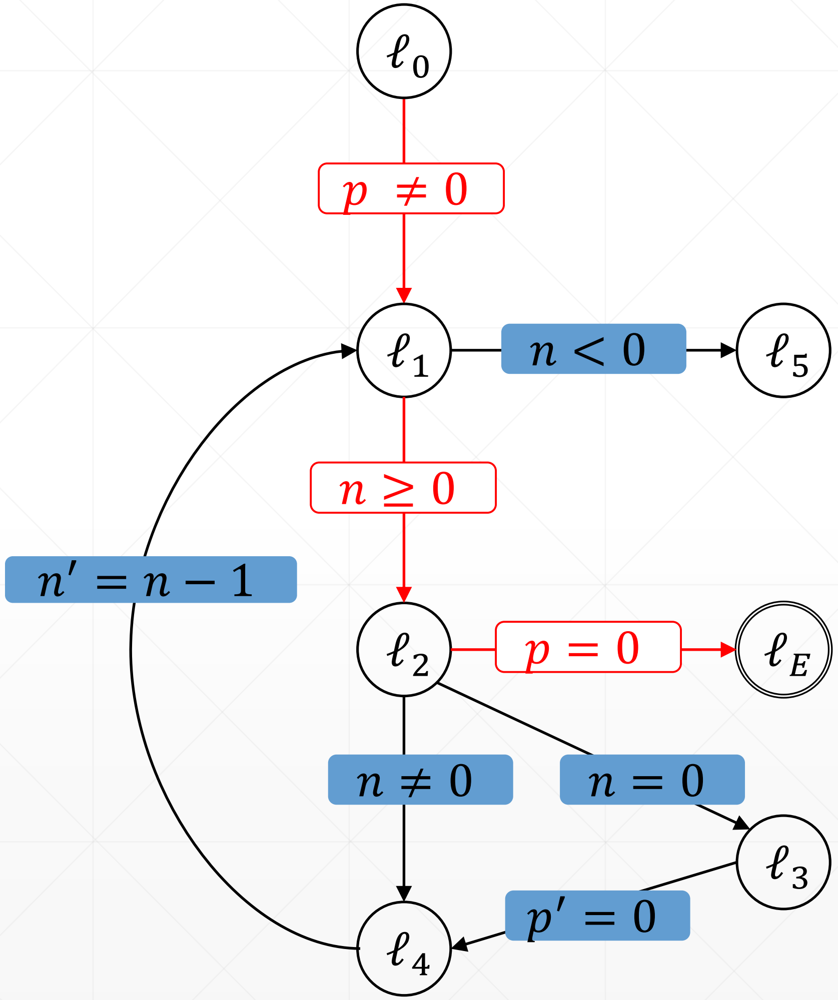
1. Step: Get possible error trace

Implementation in Ultimate: Trace Abstraction with PDR



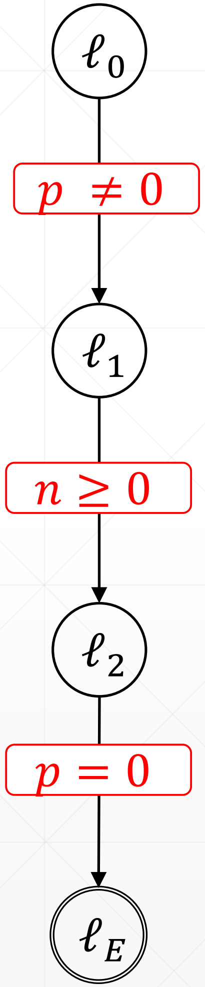
1. Step: Get possible error trace

Implementation in Ultimate: Trace Abstraction with PDR



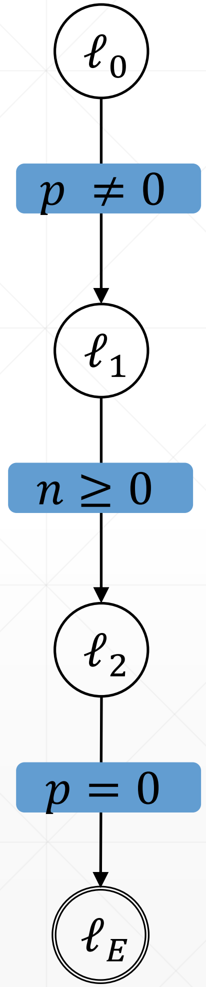
2. Step: Construct Path Program

Implementation in Ultimate: Trace Abstraction with PDR



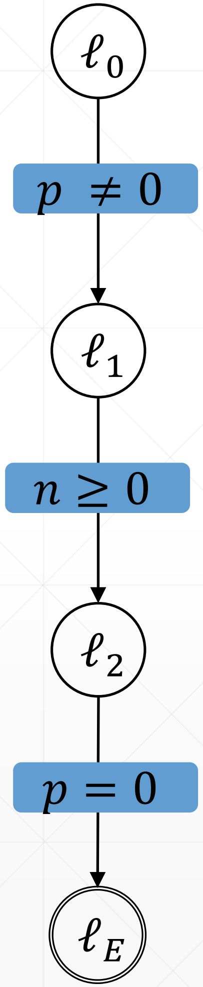
2. Step: Construct Path Program

Implementation in Ultimate: Trace Abstraction with PDR



3. Step: Use PDR

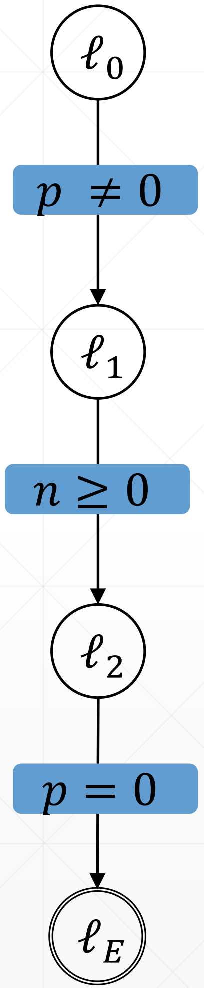
Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
ℓ_0				
ℓ_1				
ℓ_2				

3. Step: Use PDR

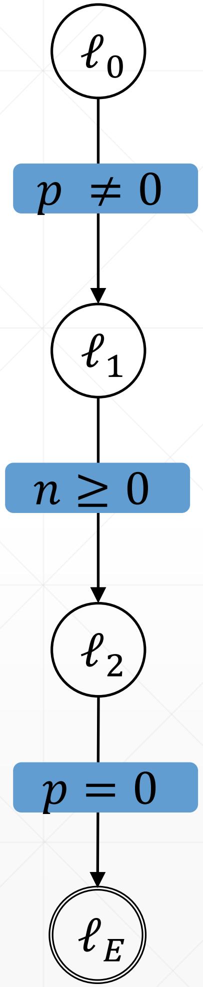
Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$

3. Step: Use PDR

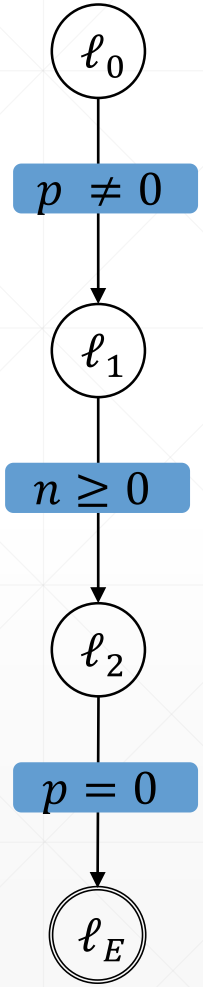
Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	t
ℓ_2	$f \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$	$t \land p \neq 0$

4. Step: Use fixpoint invariants as interpolant sequence

Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
ℓ_0	t	t	t	t
ℓ_1	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	t
ℓ_2	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$

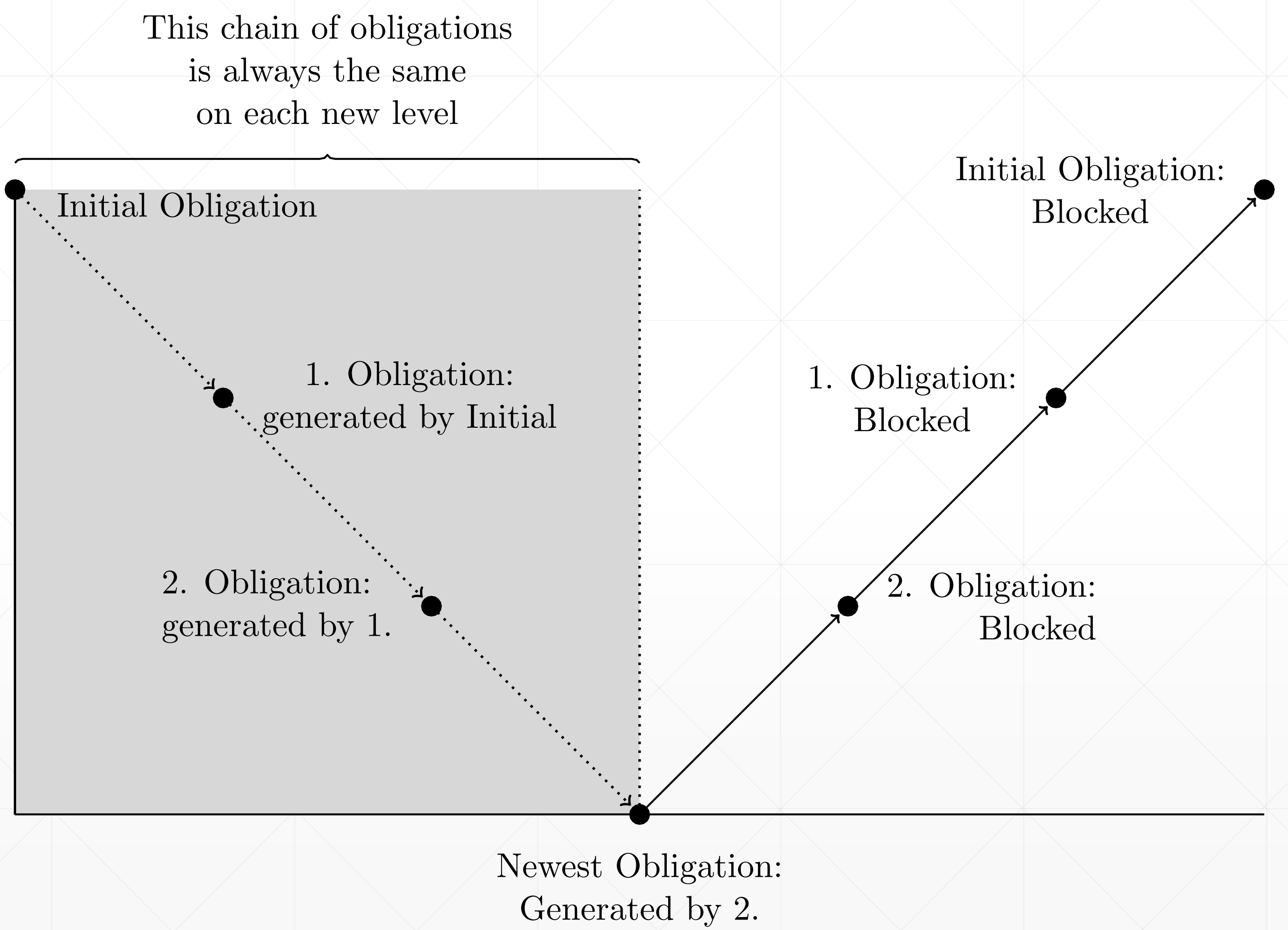
4. Step: Use fixpoint invariants as interpolant sequence

Implementation in Ultimate: Implemented Improvements

Caching proof-obligations:

- Cache the proof-obligation queue
- Start every new Iteration with the latest blocked proof-obligation

➔ Only proof-obligation that differs from Iteration before



Implementation in Ultimate: Implemented Improvements

Skipping already blocked proof-obligations:

- Cache unsatisfiable queries to SMT-solver
  - ➔ When a query to the SMT-solver is proven unsatisfiable, cache it
  - ➔ If a cached query is seen again, do not call SMT-solver again, strengthen frames right away

Evaluation: Introduction

- We compared Trace Abstraction using PDR with Trace Abstraction using Nested Interpolants

- Tested on Ultimate version 0.1.23-e6fd87c, time limit: 300s, memory limit: 8000MB

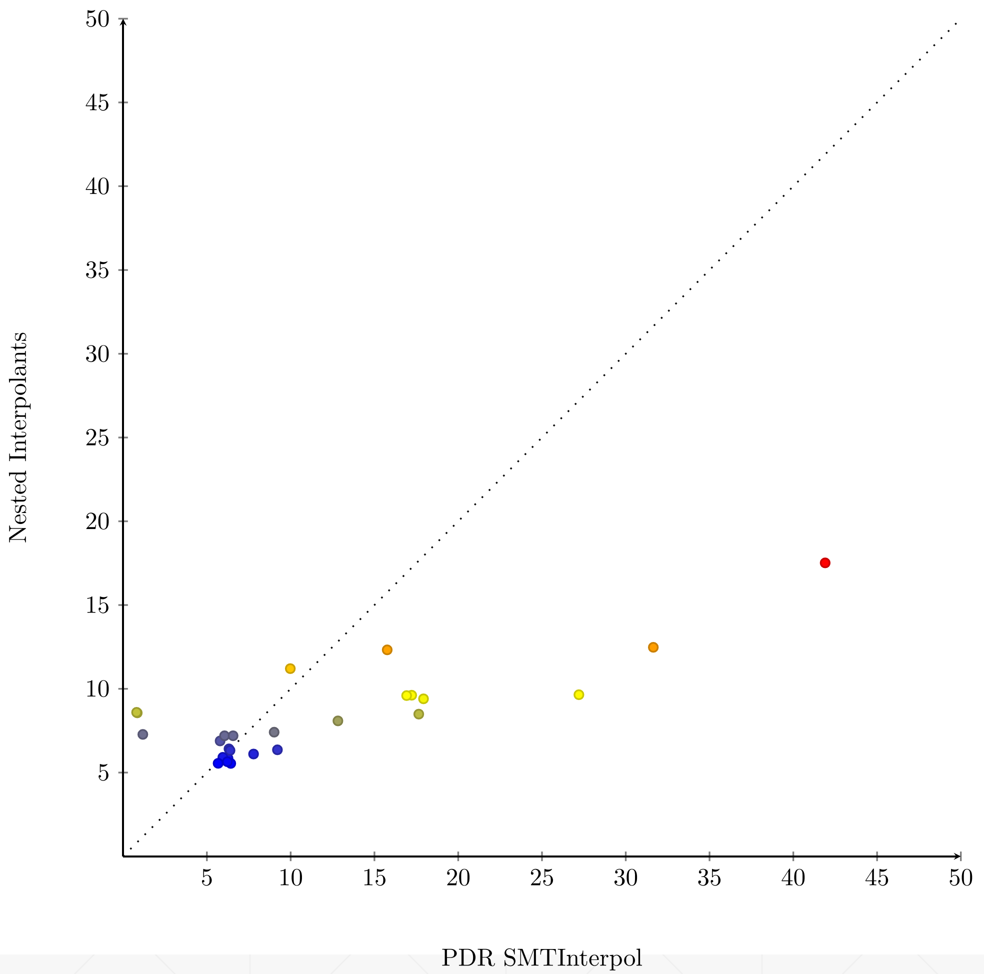
Evaluation: Introduction

- We compared Trace Abstraction using PDR with Trace Abstraction using Nested Interpolants
- Tested on Ultimate version 0.1.23-e6fd87c, time limit: 300s, memory limit: 8000MB
- Benchmarkset contained 250 Boogie[1] Programs
  - 31 real-life code
  - 40 programs without disjunctions
  - 134 difficult programs that could not be solved in three iterations
  - 37 programs with difficult loop invariants
  - 8 non-linear arithmetic

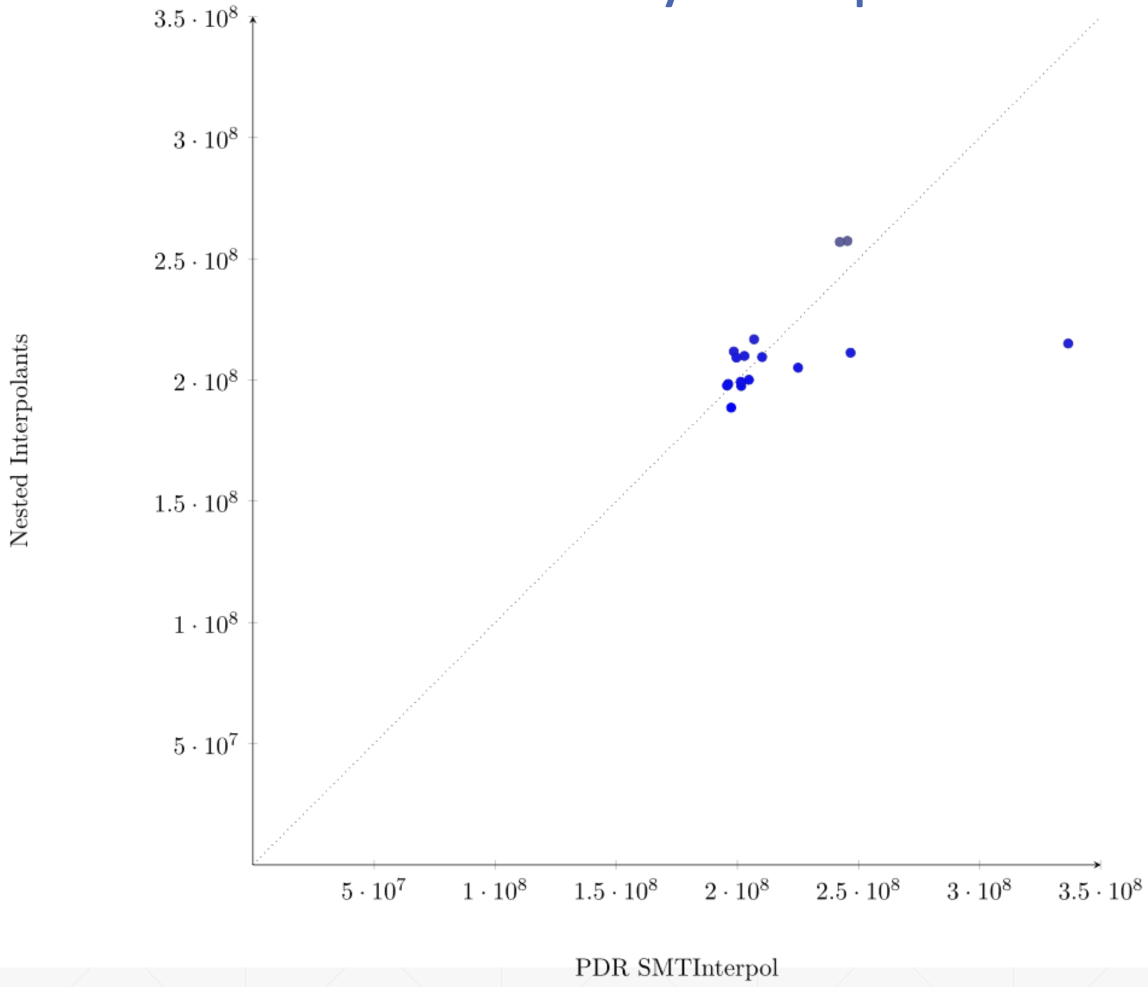
1: http://www.microsoft.com/en-us/research/project/boogie-an-intermediate-verification-language/

Evaluation: Data Comparison Successful Benchmarks

CPU Time Consumption in Seconds



Memory Consumption in Bit



	Nested Interpolants	PDR SMTInterpol	PDR Z3
Tests Solved	179/250	49/250	62/250
Solve Time	3543s	575s	1332s
Timeouts	65	90	133
Exceptions	6	111	55
`real-life`			
Tests Solved	20/31	3/31	9/31
Solve Time	598s	8s	76s
Timeouts	11	10	14
Exceptions	0	18	8
`20170319-ConjunctivePathPrograms`			
Tests Solved	29/40	6/40	16/40
Solve Time	531s	35s	191s
Timeouts	11	15	20
Exceptions	0	19	4
`20170304-DifficultPathPrograms`			
Tests Solved	105/134	24/134	24/134
Solve Time	1435s	449s	975s
Timeouts	24	44	74
Exceptions	5	66	36
`tooDifficultLoopInvariants`			
Tests Solved	17/37	8/37	8/37
Solve Time	944s	42s	57s
Timeouts	19	21	22
Exceptions	1	8	7
`nonlinear`			
Tests Solved	8/8	8/8	5/8
Solve Time	35s	41s	33s
Timeouts	0	0	3
Exceptions	0	0	0

Evaluation: Discussion

PDR with SMTInterpol:

- 90 Timeouts, mostly due to loops
- 111 Exceptions:
  - ➔ 16 Syntax Exceptions
  - ➔ 95 Exceptions due to exist quantifier

Evaluation: Discussion

PDR with z3:

- 131 Timeouts, mostly due to loops
- 55 Exceptions:
  - → 48 Solver returned unknown
  - → 2 overapproximation Exceptions
  - → 3 Unsupported Operation Exception
  - → 1 z3-Internal Exception

Evaluation: Discussion

	Nested Interpolants	PDR
Exclusively solved	116	13

Evaluation: Discussion

	Nested Interpolants	PDR
Exclusively solved	116	13

13 programs were exclusively solved by PDR

- ➔ Timed out with Nested Interpolants
- ➔ PDR solved them in 2 iterations each

Future Work: Implementing Further Improvements

➢ **Using Interpolation:**

- Our algorithm is inefficient when dealing with loops

Idea:

- Instead of strengthening frames with negated proof-obligation, calculate Interpolant for transition and proof-obligation and add that

Future Work: Implementing Further Improvements

➢ **Dealing with procedures:**

- C programs often contain procedures with which PDR cannot deal

Ideas:

1. Use a non-linear approach of PDR
2. Calculate a procedure summary, add that to the CFG, removing the procedure altogether

Conclusion

- **We have seen:**
  - How PDR works on software
  - How we combined Trace Abstraction and PDR
  - How the combination compared to Trace Abstraction with Nested Interpolants
  - What can be done to make it more efficient

Bibliography

- Aaron R. Bradley. Sat-based model checking without unrolling. In *VMCAI*, volume 6538 of *Lecture Notes in Computer Science*, pages 70–87. Springer, 2011.
- Hwmcc10 results. https://fmv.jku.at/hwmcc10/results.html. Accessed: 2018-07-20
- Niklas Een, Alan Mishchenko, and Robert Brayton. 2011. Efficient implementation of property directed reachability. In Proceedings of the International Conference on Formal Methods in Computer-Aided Design (FMCAD '11). FMCAD Inc, Austin, TX, 125-134.
- Tim Lange, Martin R. Neuhäußer, and Thomas Noll. IC3 software model checking on control flow automata. In *FMCAD*, pages 97–104. IEEE, 2015.
- Tobias Welp and Andreas Kuehlmann. QF BV model checking with property directed reachability. In *DATE*, pages 791–796. EDA Consortium San Jose, CA, USA / ACM DL, 2013.
- Alessandro Cimatti and Alberto Griggio. Software model checking via IC3. In *CAV*, volume 7358 of *Lecture Notes in Computer Science*, pages 277–293. Springer, 2012.
- Ultimate. https://ultimate.informatik.uni-freiburg.de. Accessed: 2018-07-20.
- https://www.microsoft.com/en-us/research/project/boogie-an-intermediate-verification-language/