



# Introduction: Motivation

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- PDR was first devised as hardware verification technique in 2010 by Aaron Bradley<sup>1</sup>
  - ➔ Surprisingly won 3<sup>rd</sup> place at CAV 2010 hardware checking competition<sup>2</sup>

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2: Hwmcc10 results. <http://fmv.jku.at/hwmcc10/results.html>. Accessed: 2018-07-20

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**“This new method appears to be the most important contribution to bit-level formal verification in almost a decade”<sup>3</sup>**

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**“This new method appears to be the most important contribution to bit-level formal verification in almost a decade”<sup>3</sup>**

- Using PDR on software may have similar performance!

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# Introduction: Motivation

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- Our goal:
  - Use PDR on software in the verification framework Ultimate<sup>1</sup>
    - ➔ Combining Trace Abstraction and PDR
    - ➔ Comparison to existing techniques

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# Overview

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- How does our PDR algorithm work?
  - Preliminaries
  - Running Example
  - Possible Improvements
  - Related Work
- PDR in Ultimate:
  - Combination of Trace Abstraction and our PDR algorithm
  - Implemented Improvements

# Overview

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## ➤ Evaluation:

- Comparison of Trace Abstraction using PDR and Trace Abstraction using Nested Interpolants

## ➤ Future Work:

- Implementing more Improvements

# PDR Algorithm: Preliminaries

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- Our algorithm uses CFGs



# PDR Algorithm: Datastructures

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➤ Frame  $F_{i,\ell}$  :

- Represents a first-order formula
  - $\ell$  is the corresponding location
  - $i$  is the corresponding level
- ➔ Each location has multiple assigned frames

➤ Proof-Obligation  $(p, \ell, i)$  :

- $p$  is a first-order formula
  - $\ell$  is the corresponding location
  - $i$  is the corresponding level
- ➔ Need to be blocked

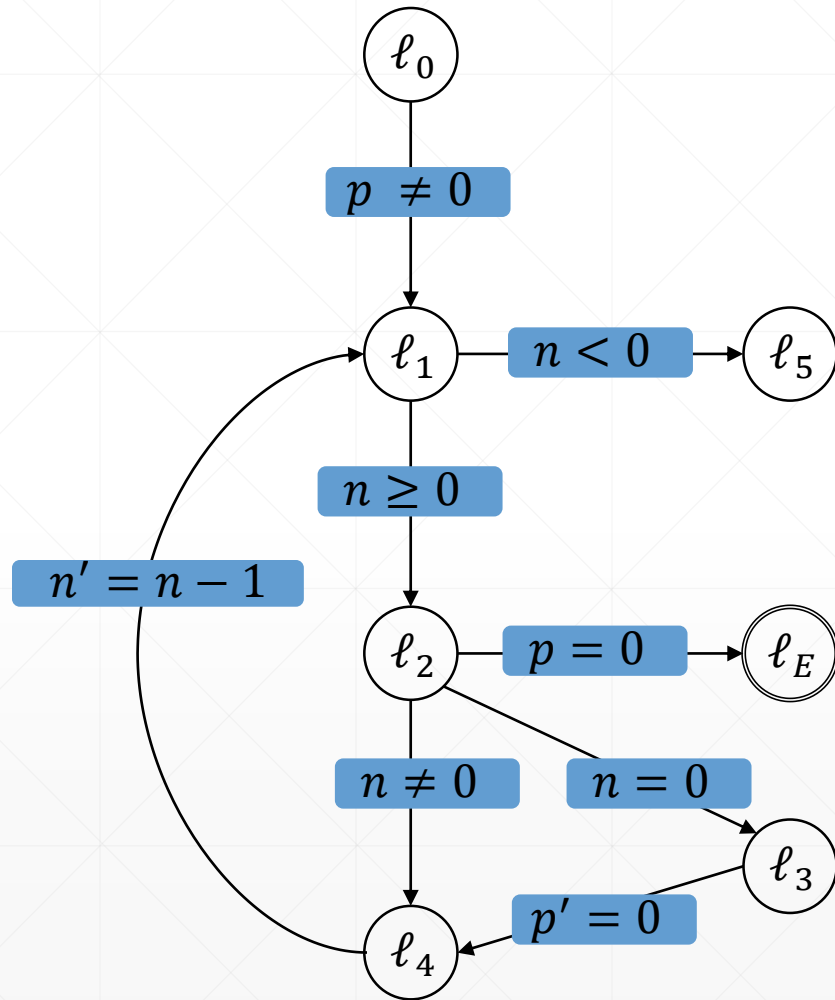
# PDR Algorithm: Description

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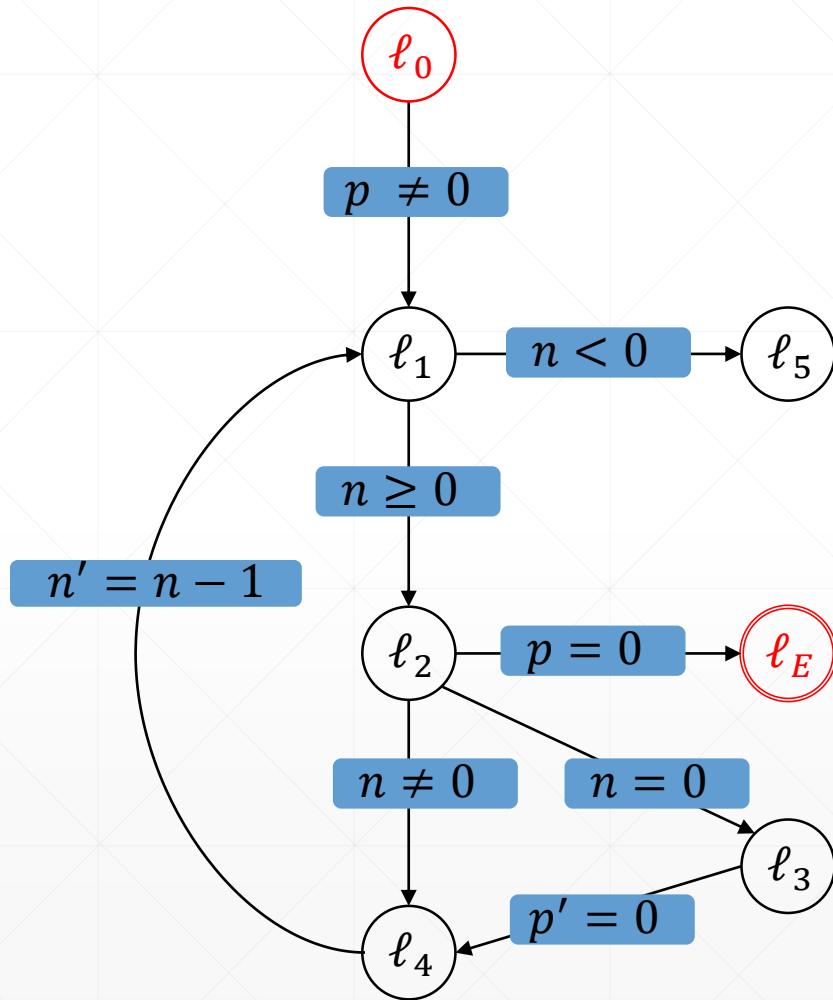
- Starts with checking for a 0-Counter-Example
  
- Repeats three phases until termination:
  1. Next Level Initialization Phase
  2. Blocking-Phase
  3. Propagation-Phase

# Example: Running Example

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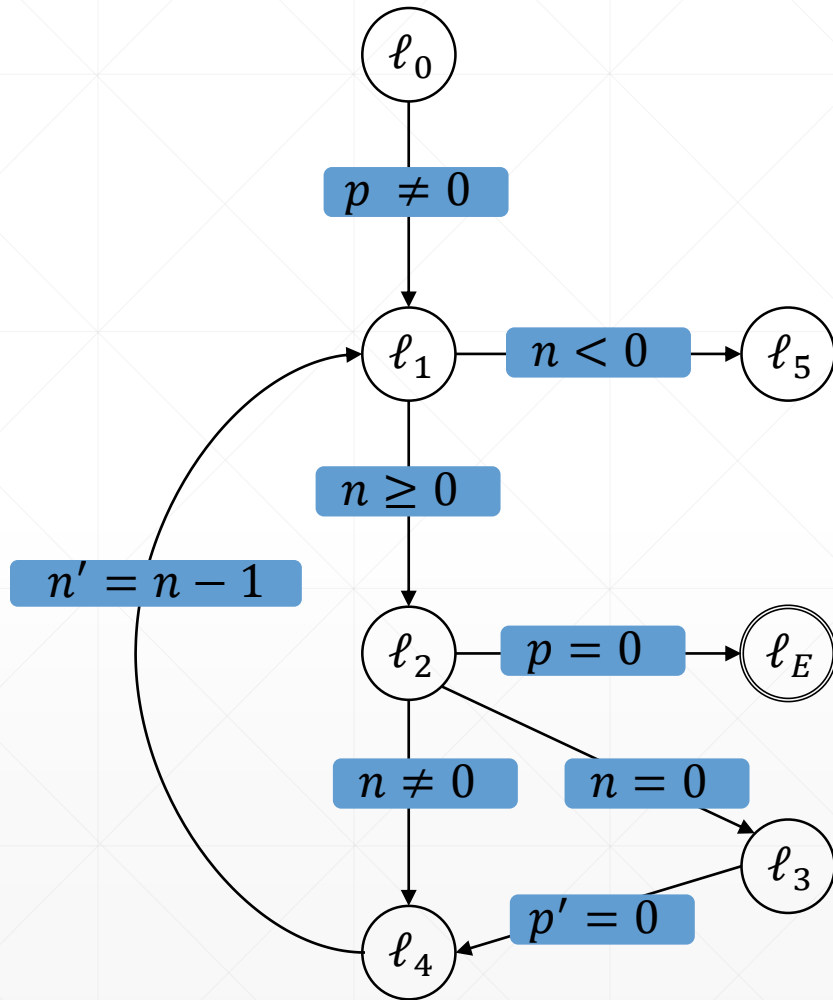
## Example:



### 1. Step: Check for 0-Counter-Example

- Is  $\ell_0 = \ell_E$  ?
  - ➔ No, continue with initialization

## Example:

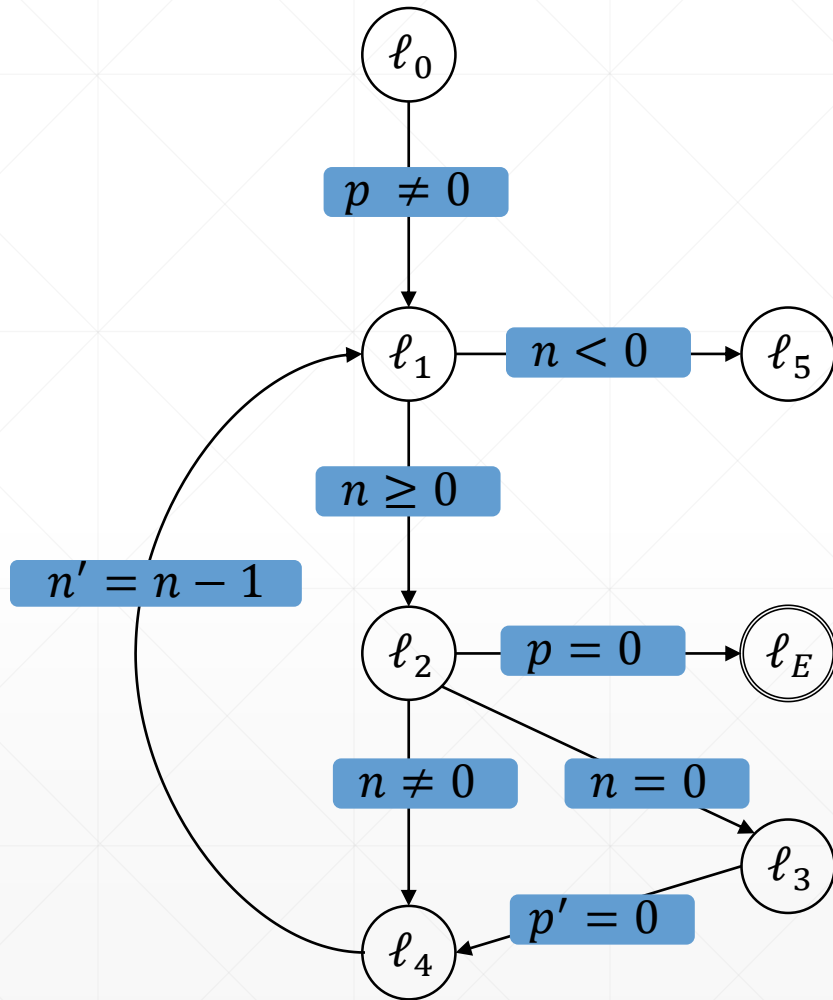


location	0
$\ell_0$	
$\ell_1$	
$\ell_2$	
$\ell_3$	
$\ell_4$	

2. Step: Initialization of level 0

$$\triangleright F_{0,\ell} = \begin{cases} \text{T}, & \ell = \ell_0 \\ \text{F}, & \text{otherwise} \end{cases}$$

## Example:

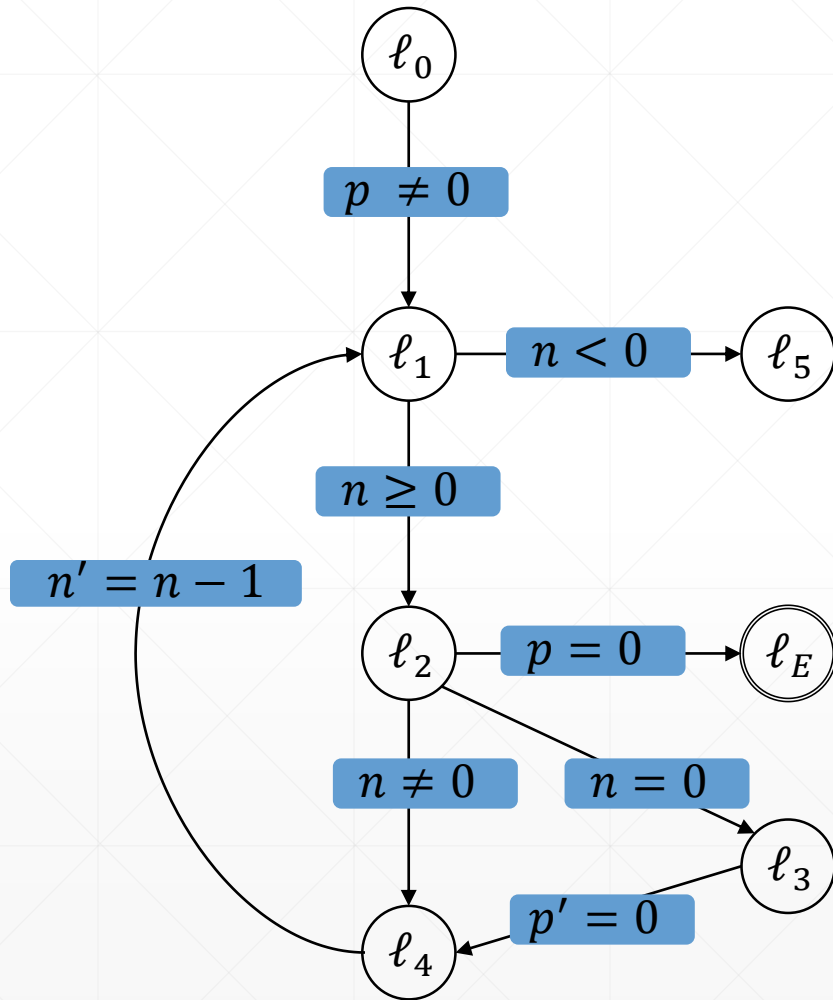


location	0
$\ell_0$	$t$
$\ell_1$	$f$
$\ell_2$	$f$
$\ell_3$	$f$
$\ell_4$	$f$

2. Step: Initialization of level 0

$$\triangleright F_{0,\ell} = \begin{cases} T, & \ell = \ell_0 \\ F, & \text{otherwise} \end{cases}$$

## Example:

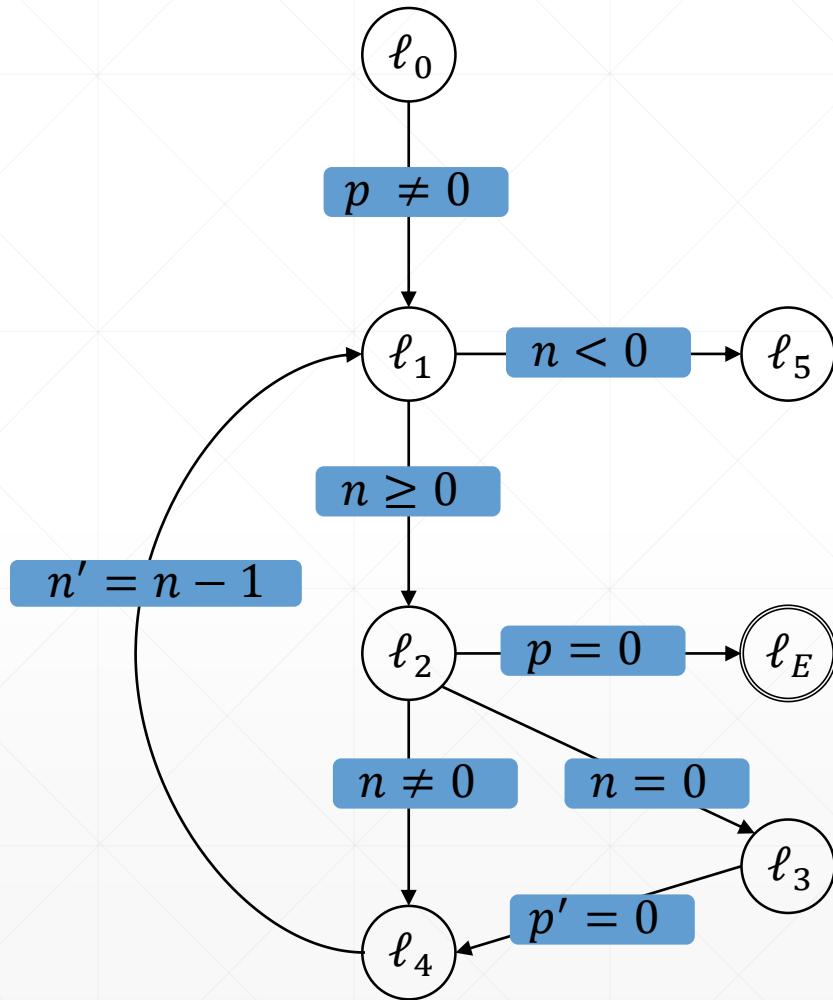


location	0	1
$\ell_0$	$t$	
$\ell_1$	$f$	
$\ell_2$	$f$	
$\ell_3$	$f$	
$\ell_4$	$f$	

### 3. Step: Level 1

- Initialize level 1 frames as true

## Example:



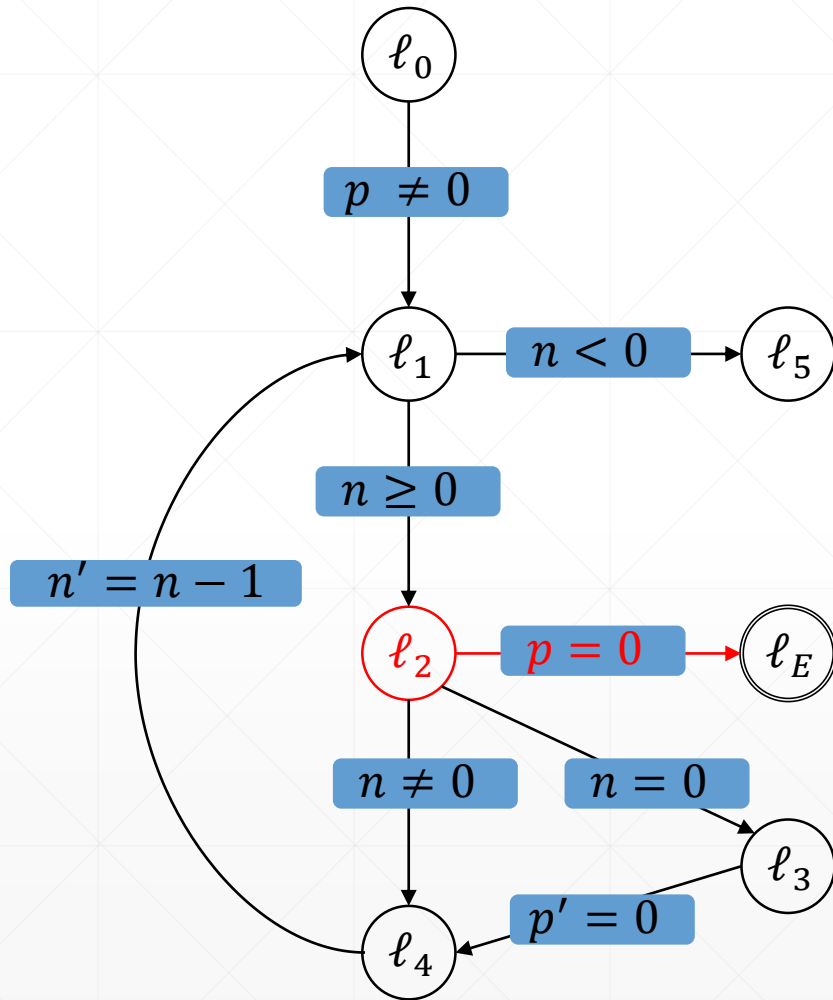
location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 3. Step: Level 1

- Initialize level 1 frames as true



## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

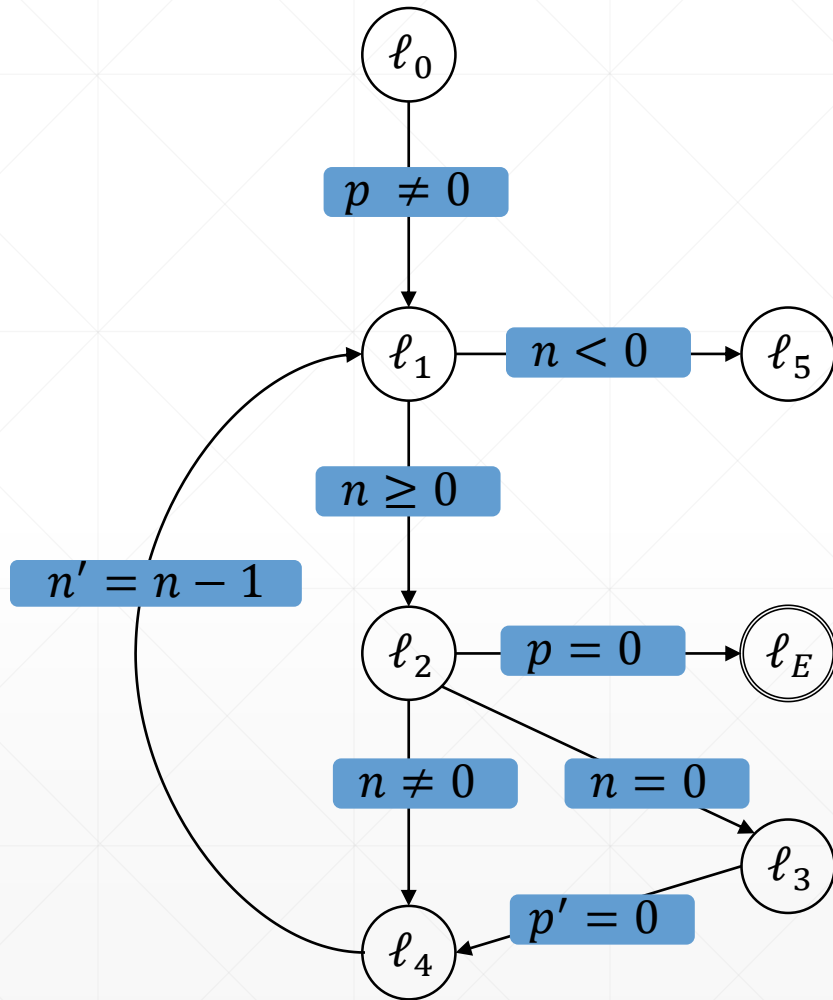
### 3. Step: Level 1

➤ Get initial proof-obligation

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

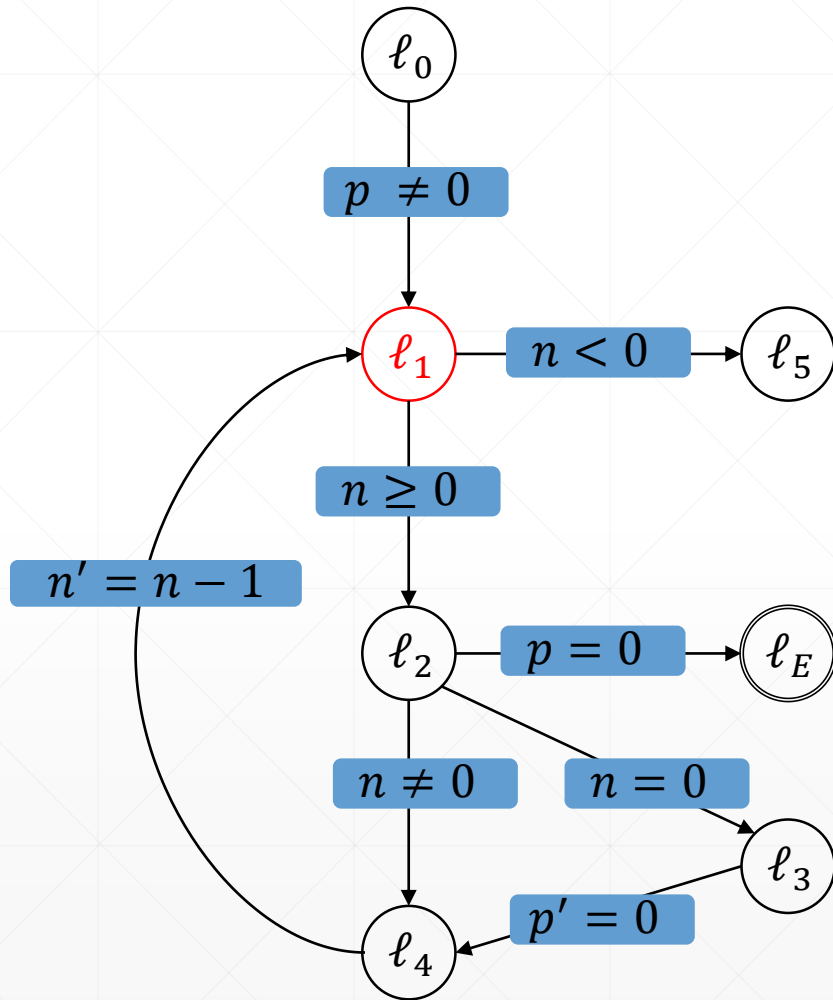
### 4. Step: Level 1 Blocking-Phase:

➤ Try to block  $(p = 0, \ell_2, 1)$

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

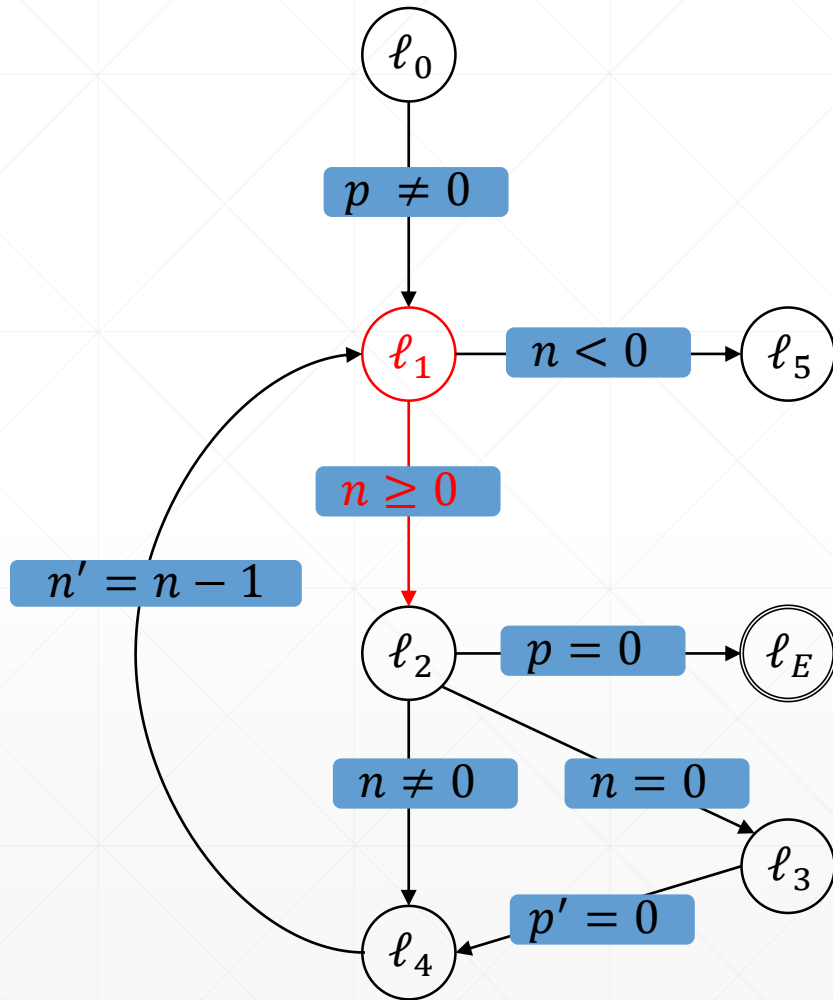
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $F_{0,\ell_1} \wedge T_{\ell_1 \rightarrow \ell_2} \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

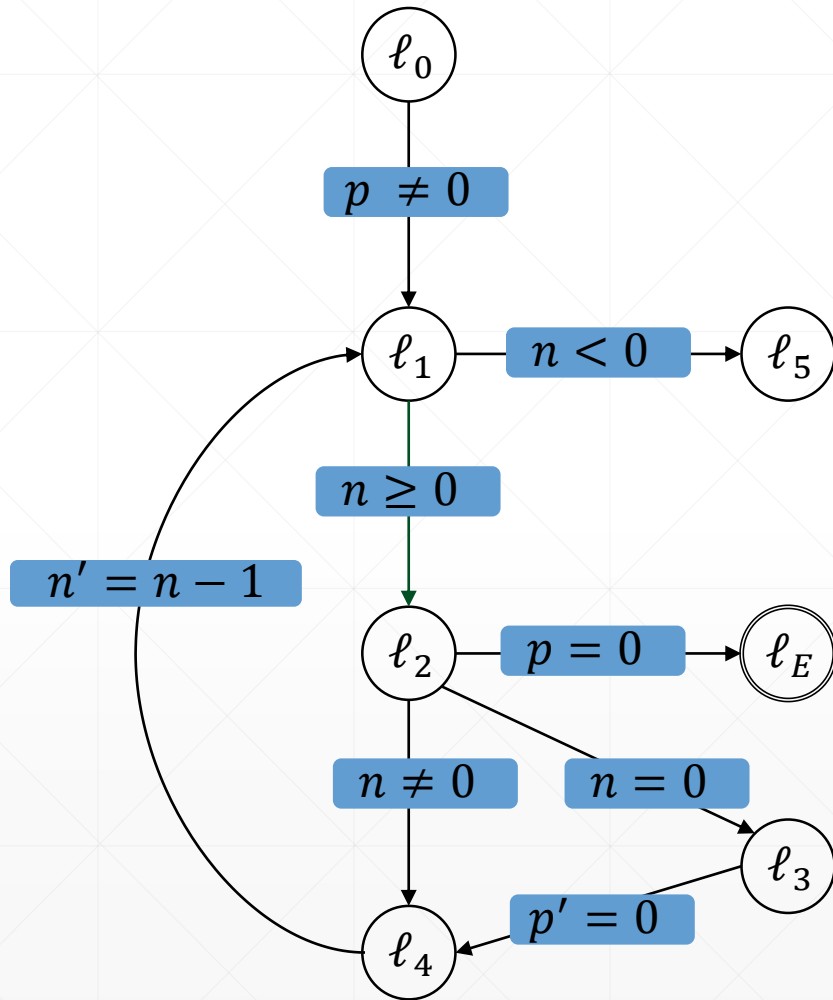
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $f \wedge n \geq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

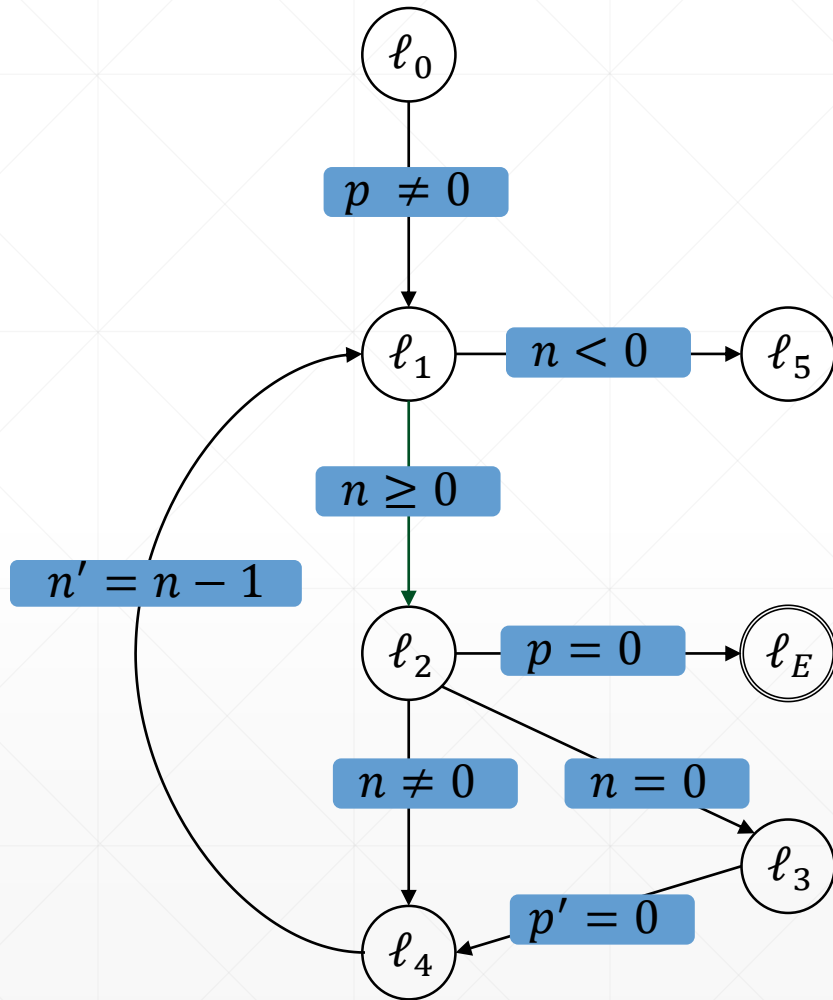
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $f \wedge n \geq 0 \wedge p' = 0$
  - ➔ **Unsatisfiable**
  - ➔ Strengthen frames  $F_{0,\ell_2}, F_{1,\ell_2}$

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

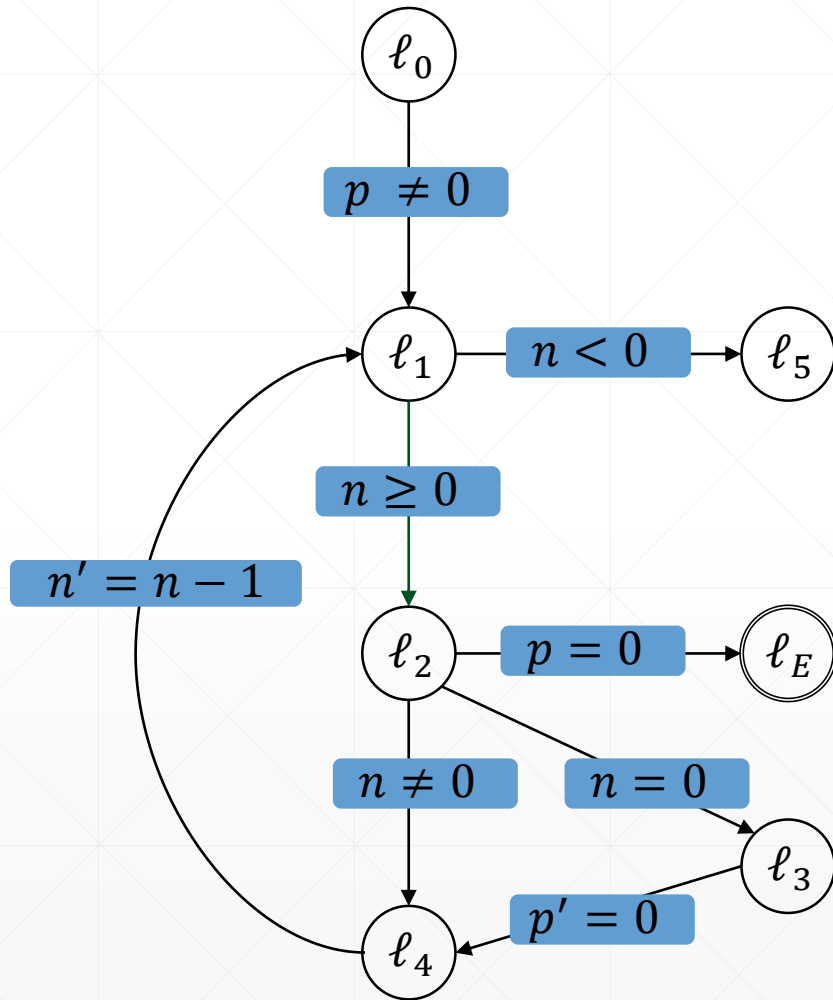
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $f \wedge n \geq 0 \wedge p' = 0$
  - ➔ Unsatisfiable
  - ➔ **Strengthen** frames  $F_{0,\ell_2}, F_{1,\ell_2}$

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

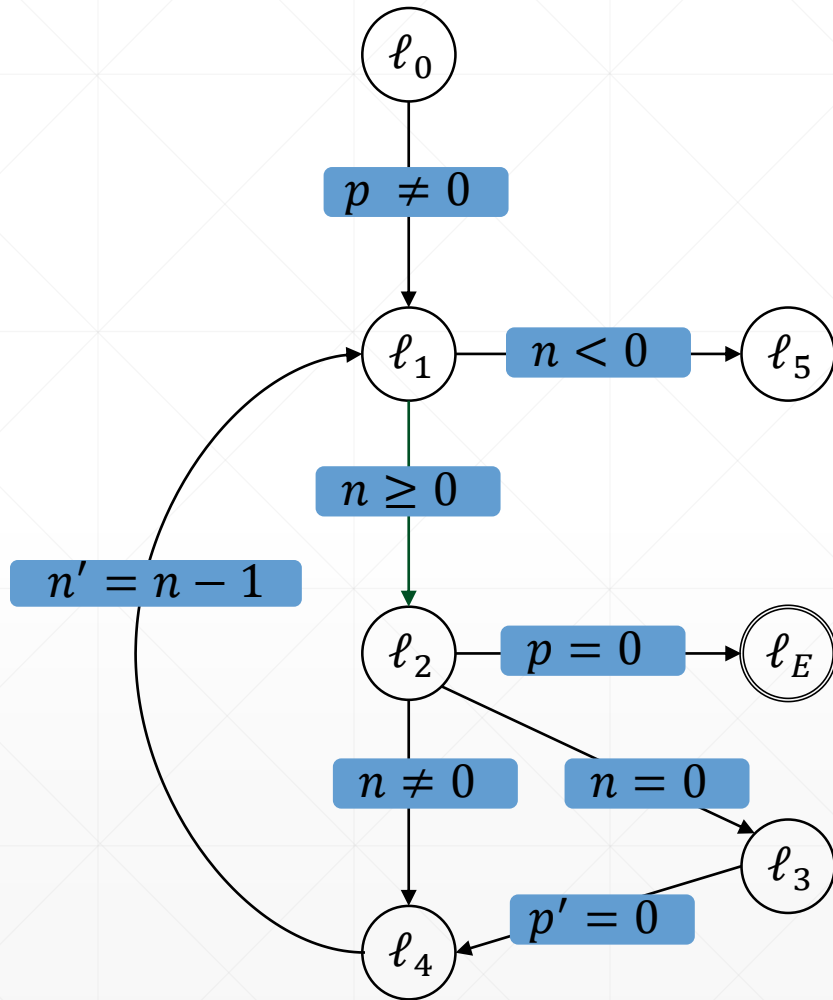
### 5. Step: Level 1 Propagation-Phase

➤ Is there a global fixpoint?

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 5. Step: Level 1 Propagation-Phase

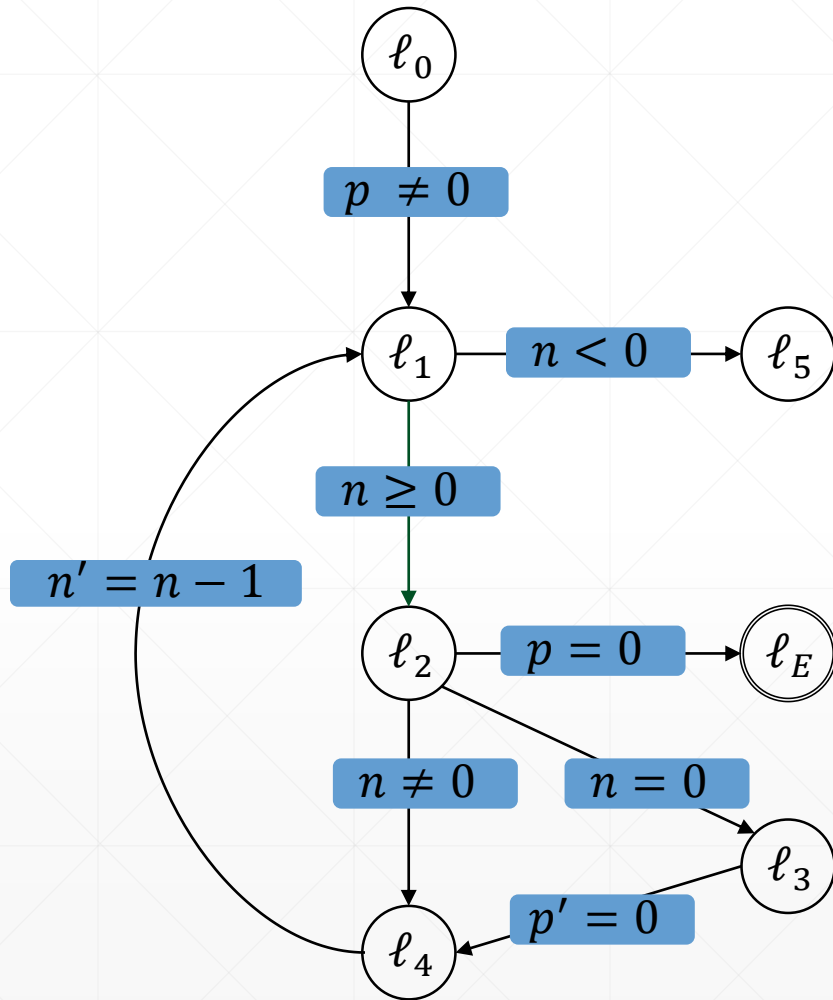
➤ Is there an  $i$  where  $F_{i-1,\ell} = F_{i,\ell}$  for  $\ell \in L \setminus \{\ell_E\}$  ?

### Proof-Obligations:

- $\emptyset$



## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
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$\ell_4$	$f$	$t$

### 5. Step: Level 1 Propagation-Phase

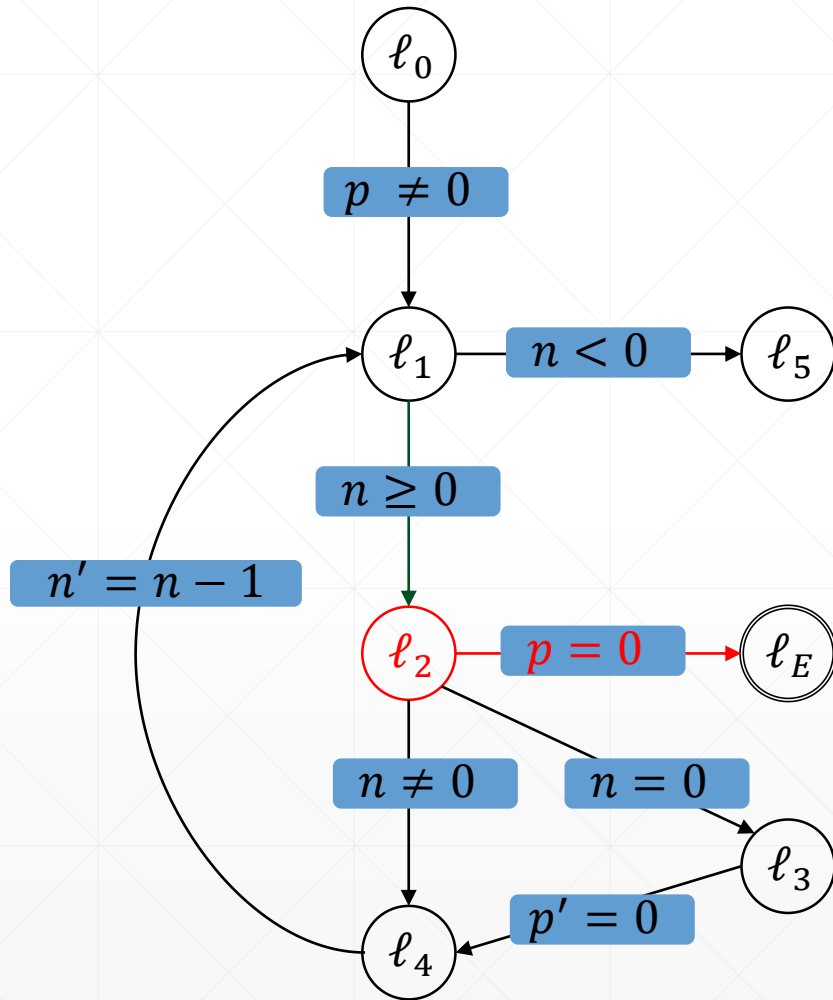
➤ Is there an  $i$  where  $F_{i-1,\ell} = F_{i,\ell}$  for  $\ell \in L \setminus \{\ell_E\}$ ?

➔ No. Continue with next level.

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

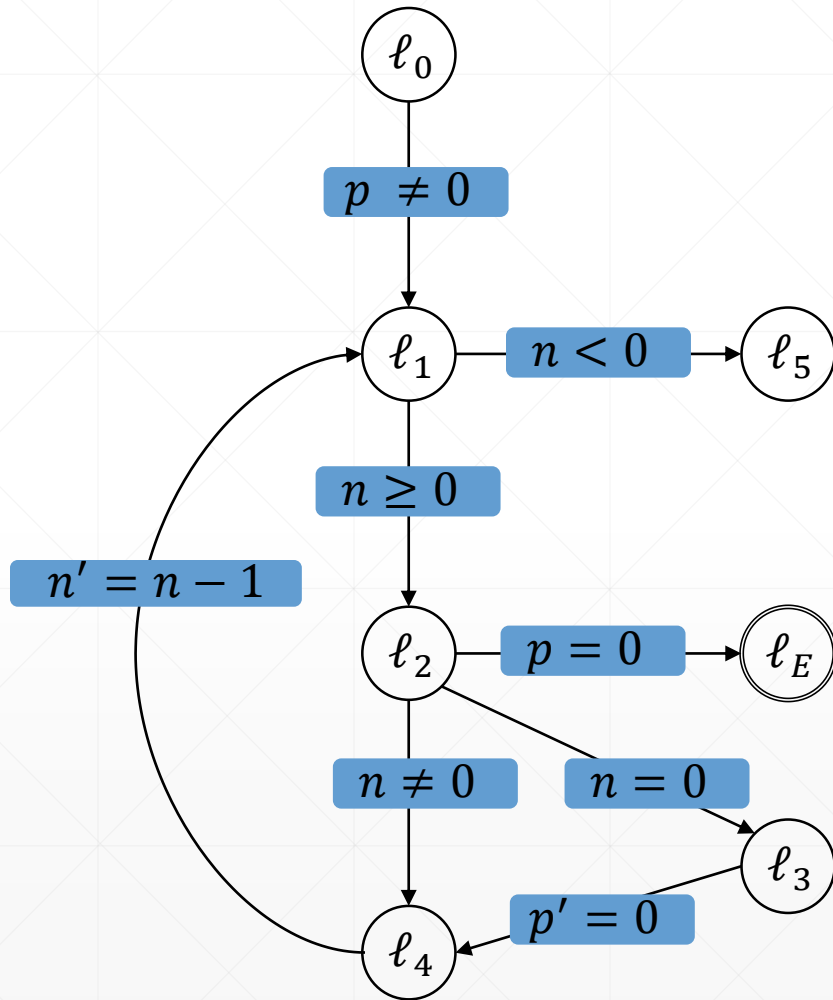
### 6. Step: Level 2

- Initialize new frames
- Add initial proof-obligation  
( $p = 0, \ell_2, 2$ )

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

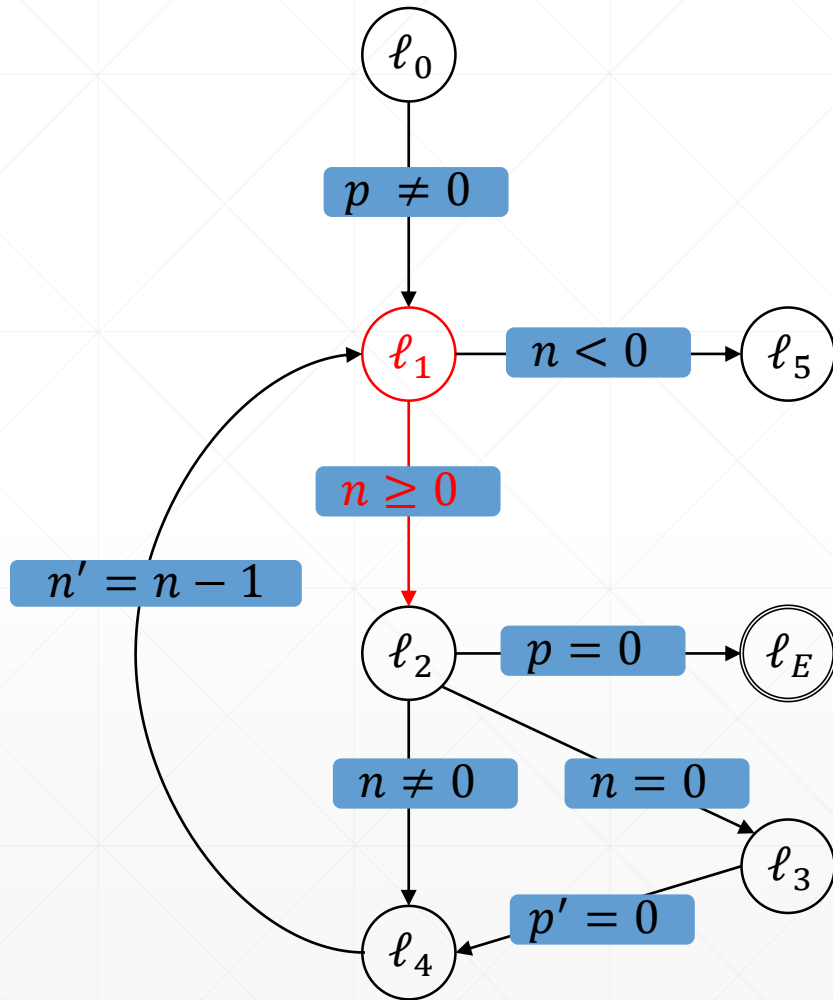
### 6. Step: Level 2

- Initialize new frames
- Add initial proof-obligation  $(p = 0, \ell_2, 2)$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

### 7. Step: Level 2 Blocking-Phase:

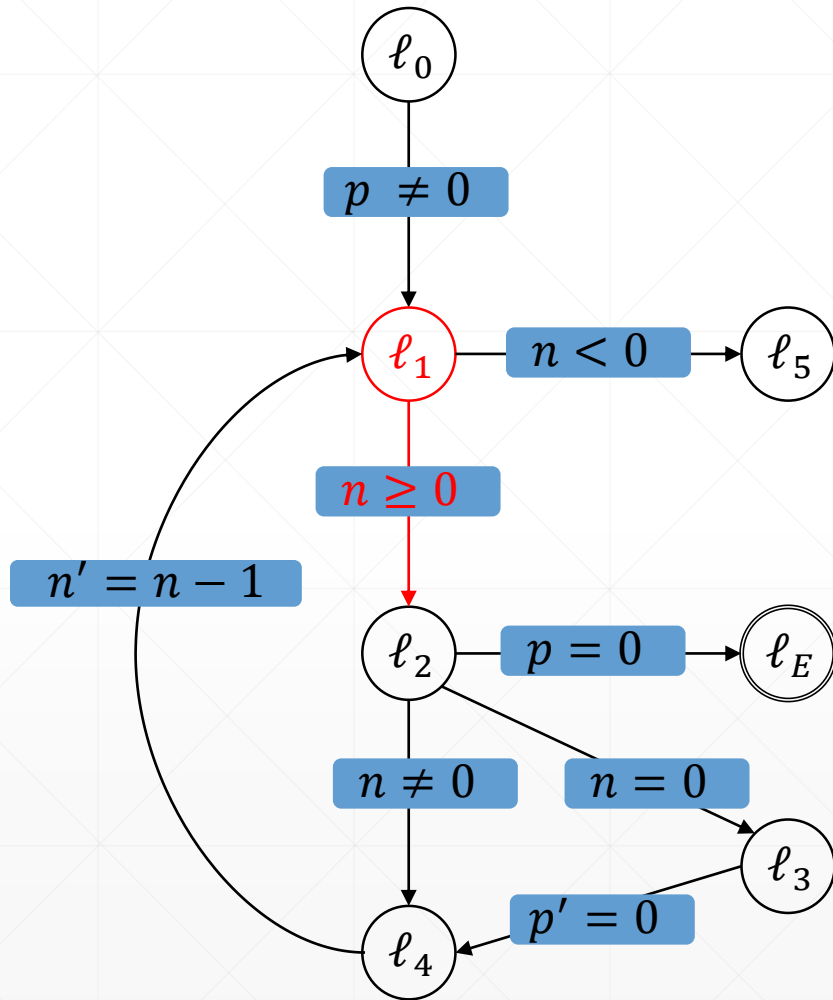
➤ Try to block  $(p = 0, \ell_2, 2)$

- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
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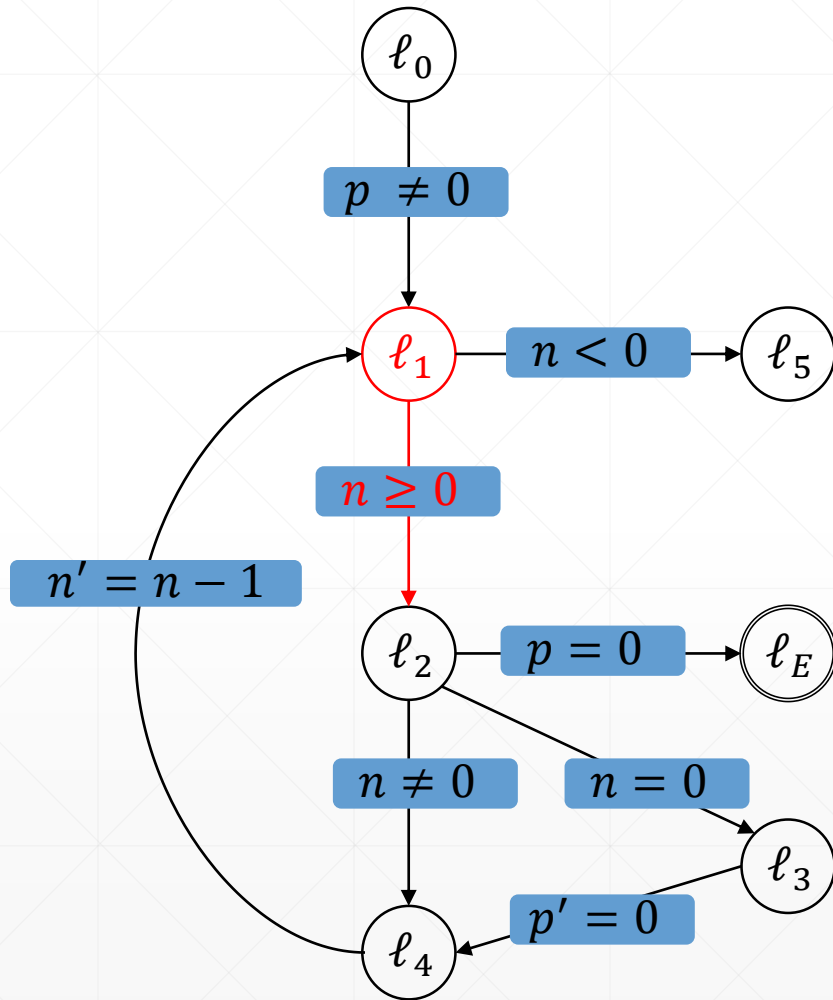
➤ Try to block  $(p = 0, \ell_2, 2)$

- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$ 
    - ➔ Satisfiable!
    - ➔  $wp(n \geq 0, p' = 0) = (p = 0)$
    - ➔ New proof-obligation  $(p = 0, \ell_1, 1)$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

### 7. Step: Level 2 Blocking-Phase:

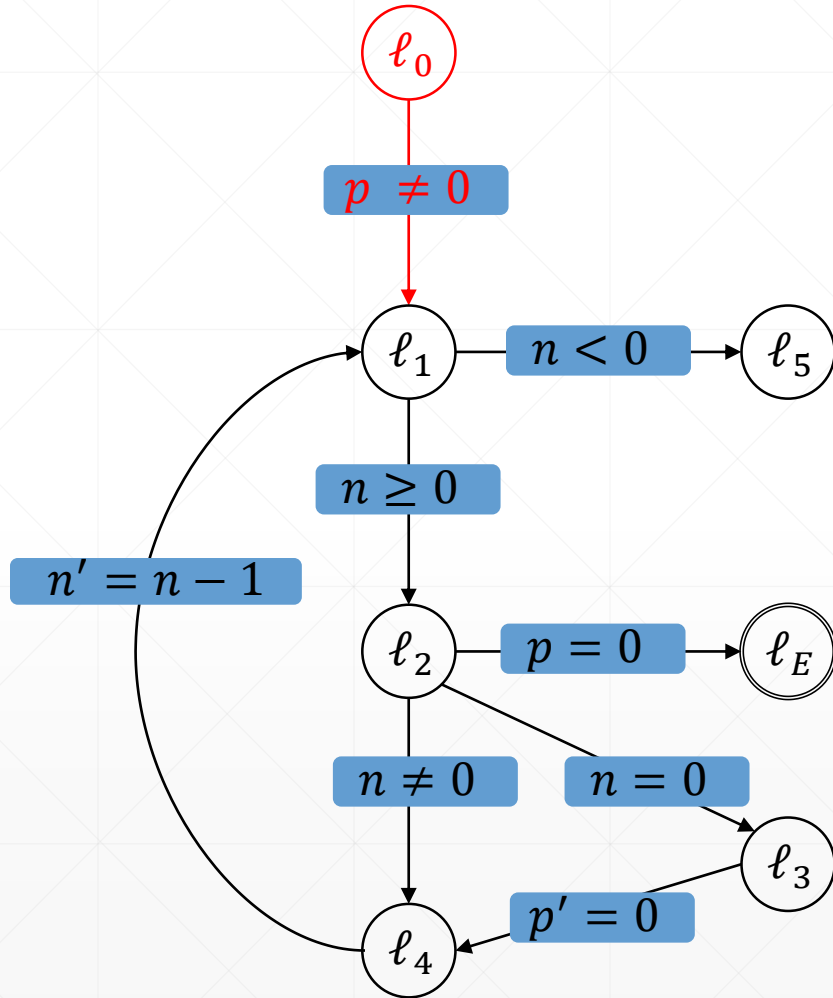
➤ Try to block  $(p = 0, \ell_2, 2)$

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### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

# Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

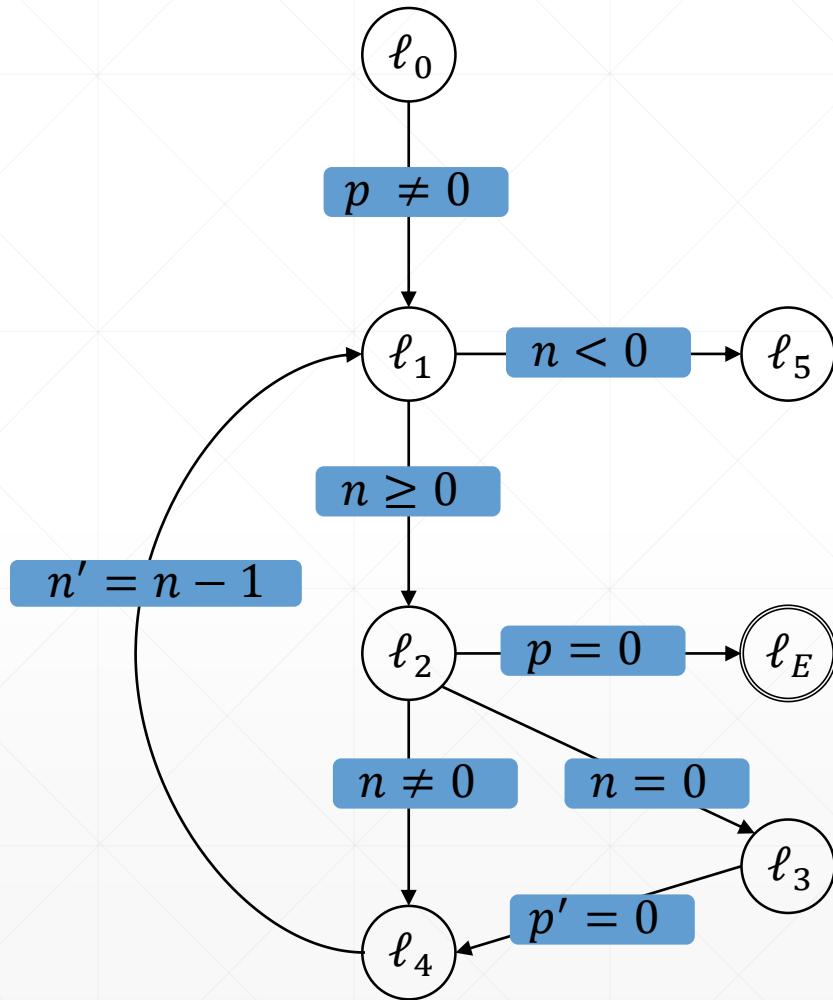
## 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_0$ :
  - $t \wedge p \neq 0 \wedge p' = 0$

## Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

### 7. Step: Level 2 Blocking-Phase:

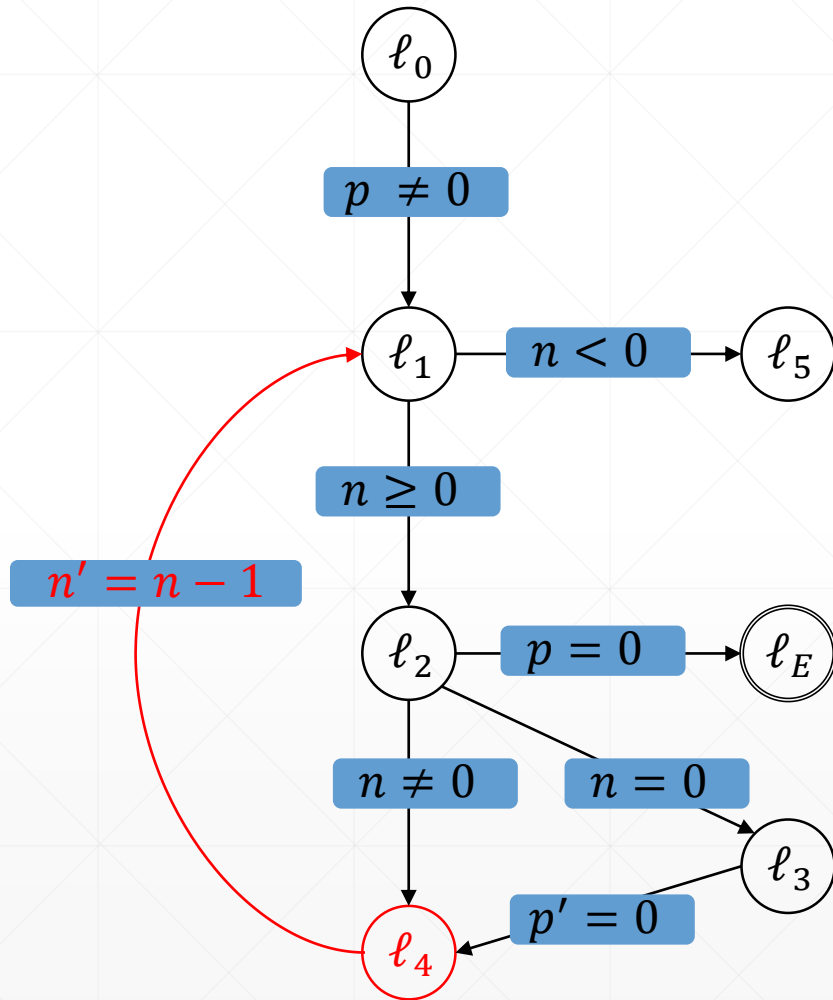
- Try to block  $(p = 0, \ell_1, 1)$ 
  - Predecessor  $\ell_0$ :
    - $t \wedge p \neq 0 \wedge p' = 0$ 
      - ➔ Unsatisfiable!
      - ➔ Strengthen frames  $F_{0,\ell_1}, F_{1,\ell_1}$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$



# Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

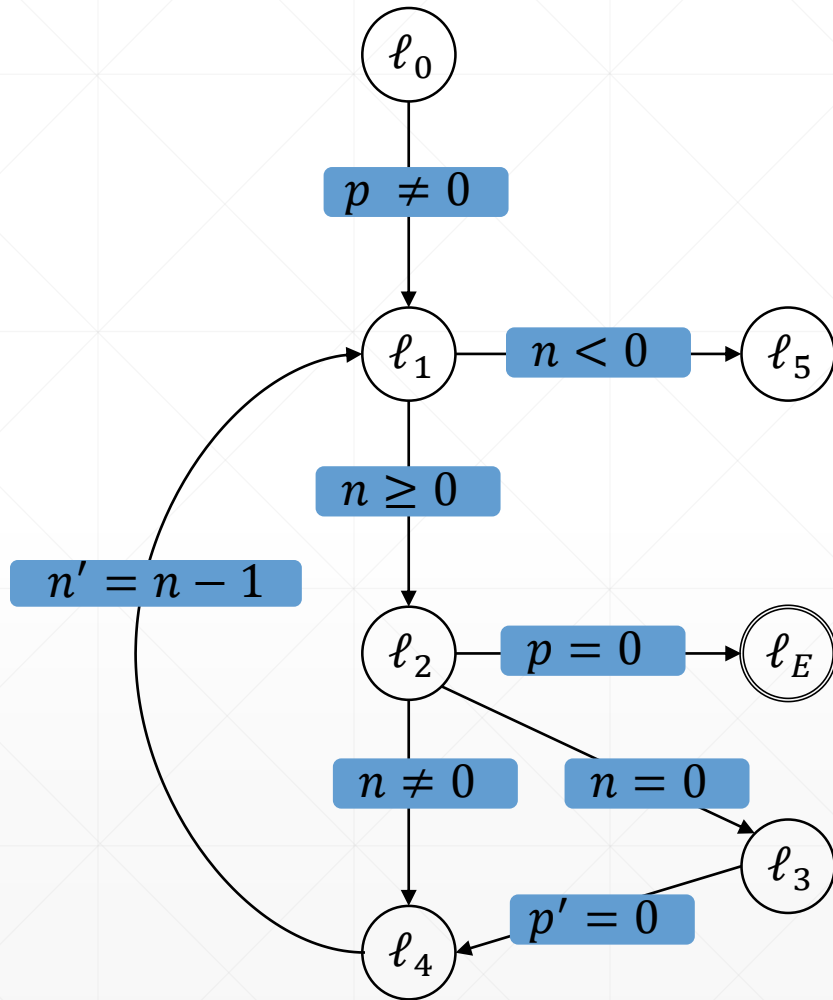
## 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_4$ :
  - $f \wedge n' = n - 1 \wedge p' = 0$

## Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

# Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

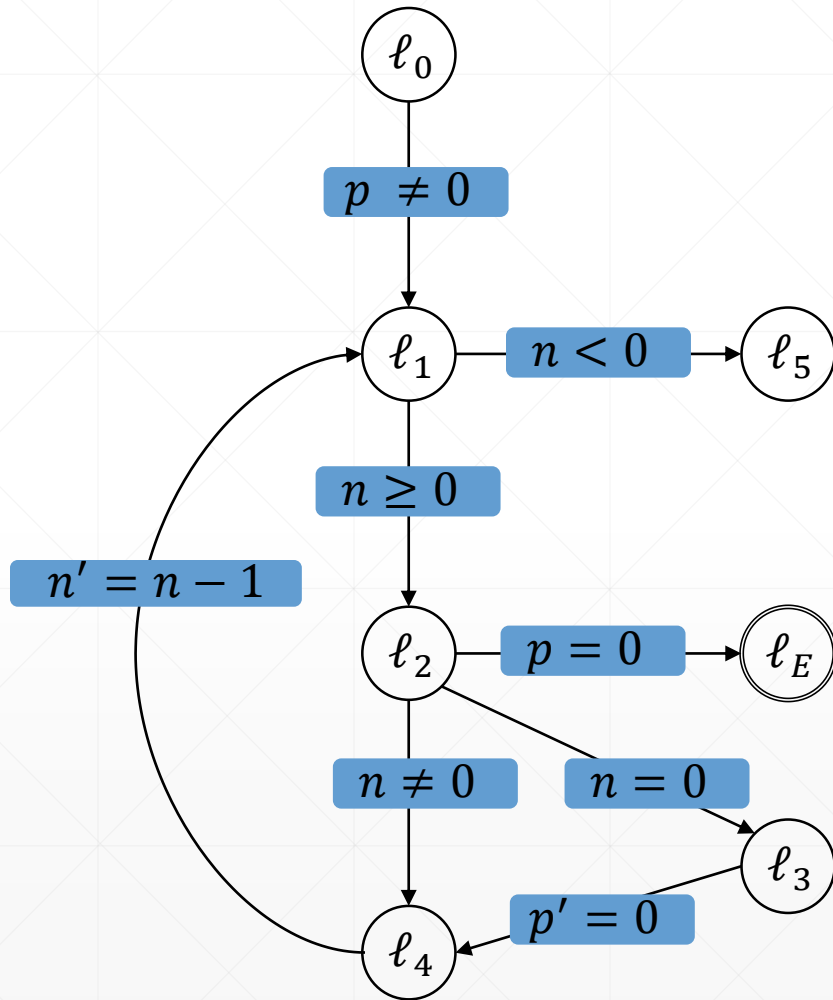
## 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_4$ :
  - $f \wedge n' = n - 1 \wedge p' = 0$
  - ➔ **Unsatisfiable!**

## Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

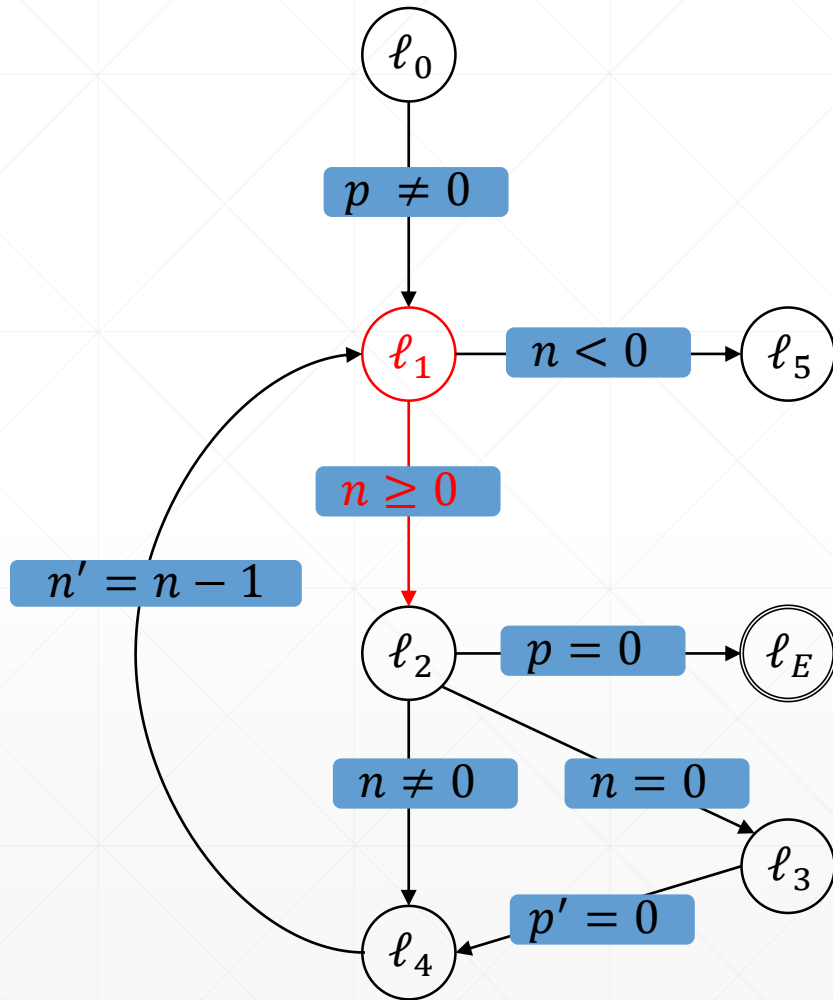
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$ 
  - Predecessor  $\ell_4$ :
    - $f \wedge n' = n - 1 \wedge p' = 0$
    - ➔ **Unsatisfiable!**

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

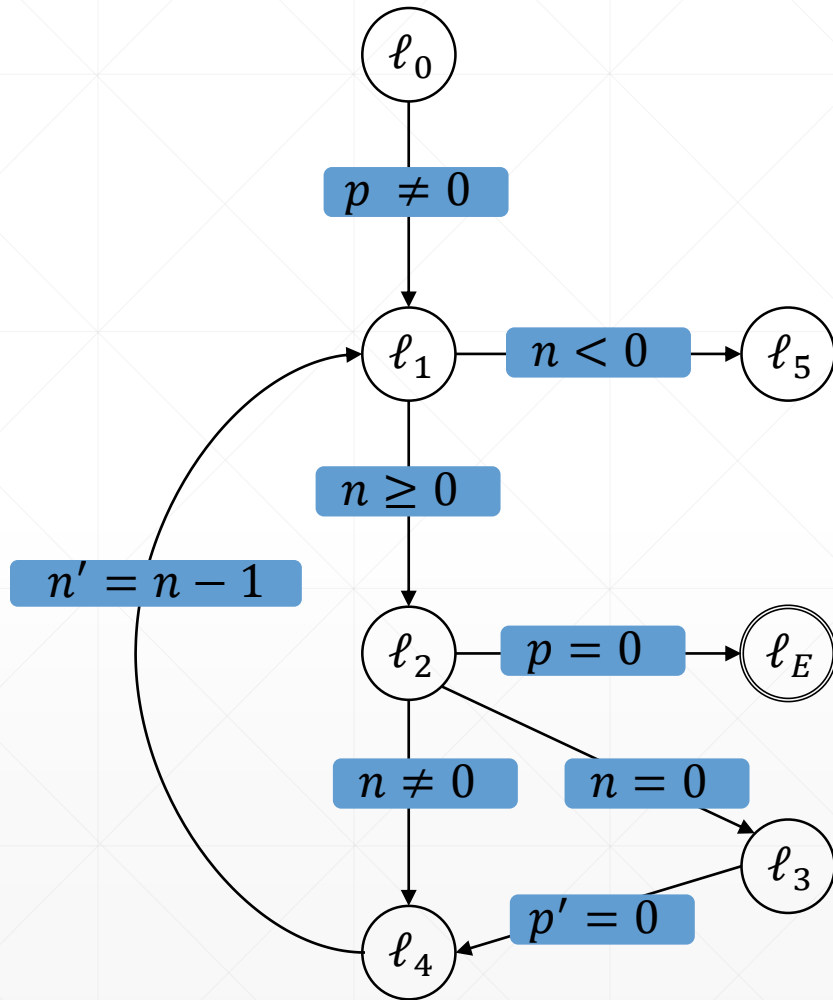
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_2, 2)$  again
  - Predecessor  $\ell_1$ :
    - $t \wedge p \neq 0 \wedge n \geq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

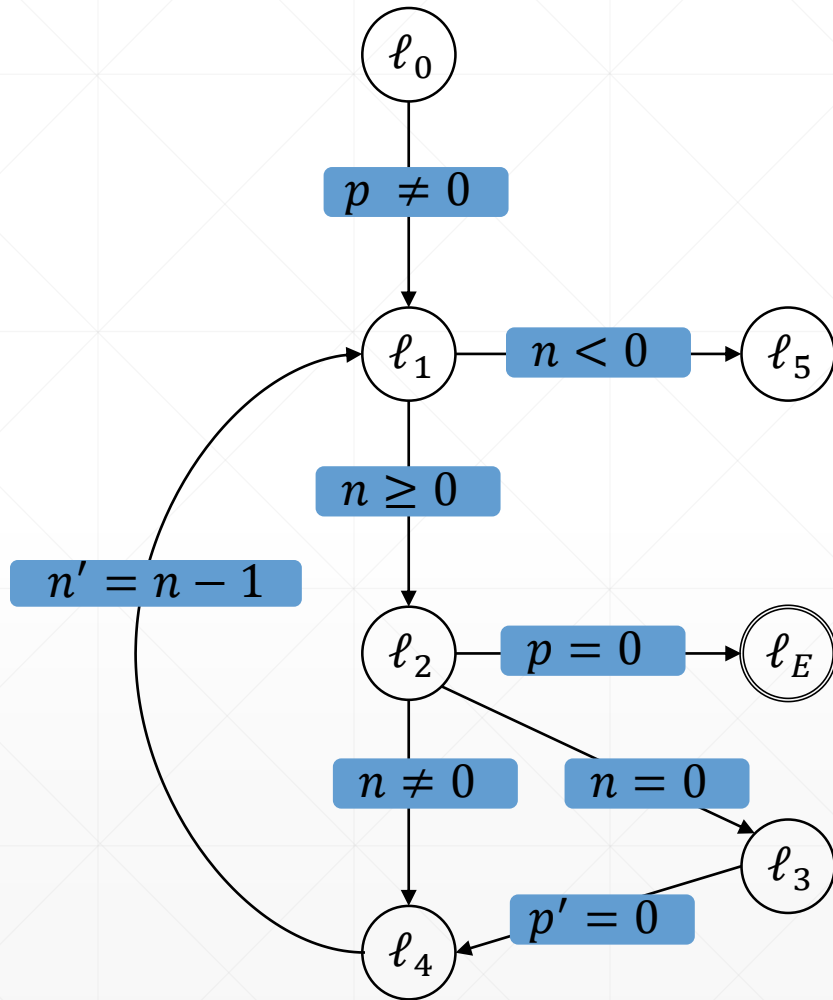
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_2, 2)$  again
  - Predecessor  $\ell_1$ :
    - $t \wedge p \neq 0 \wedge n \geq 0 \wedge p' = 0$
    - ➔ **Unsatisfiable!**
    - ➔ Strengthen frames  $F_{2,\ell_2}$

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

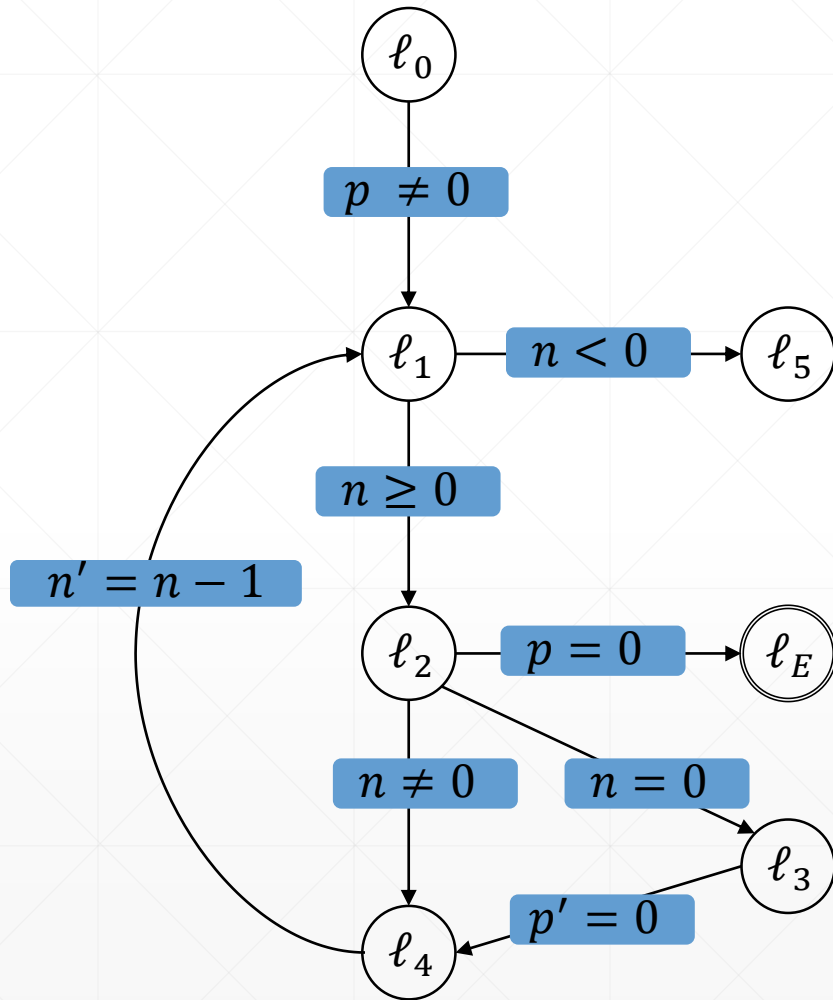
### 8. Step: Level 2 Propagation-Phase:

- Is there a global fixpoint?  
➔ No, continue with level 3

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

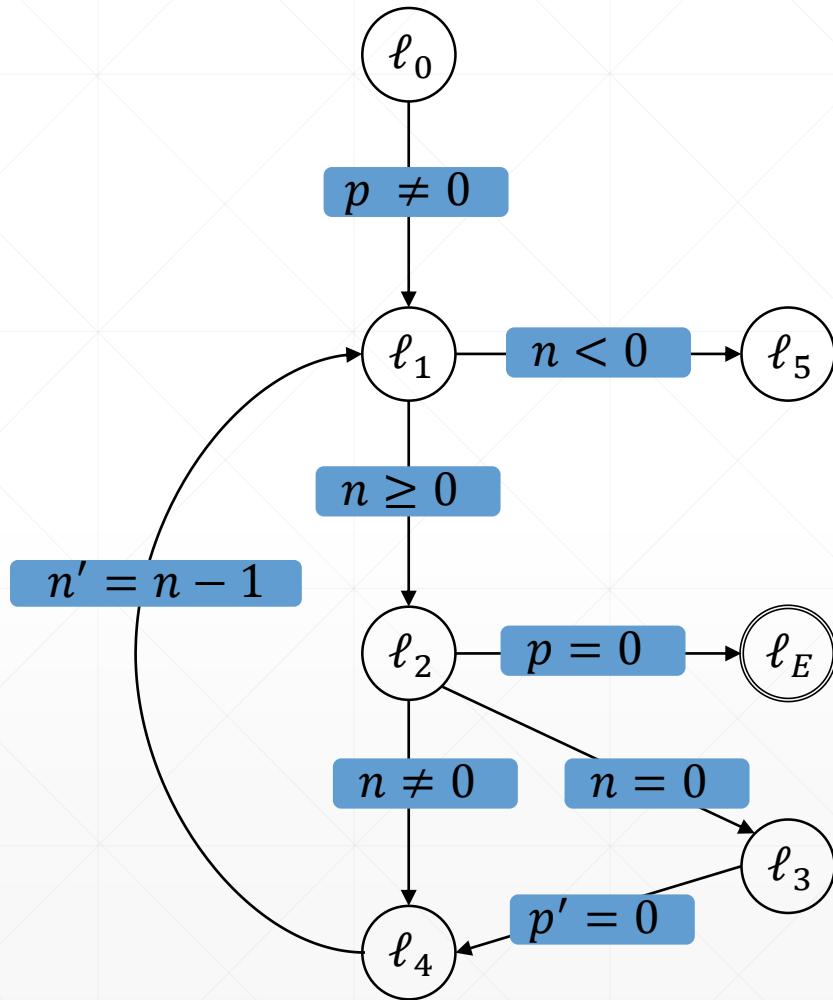
### 9. Step: Level 3

- Initialize new frames
- Get initial proof-obligations

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

### 9. Step: Level 3

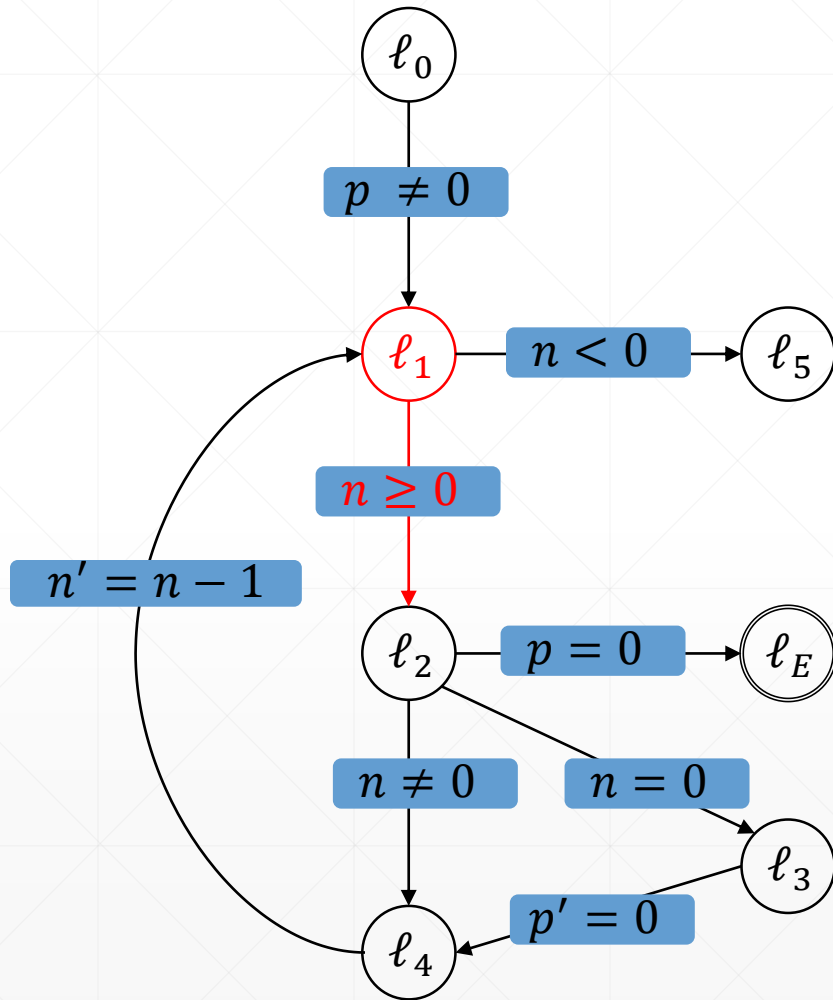
- Initialize **new frames**
- Get **initial proof-obligations**

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$



## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

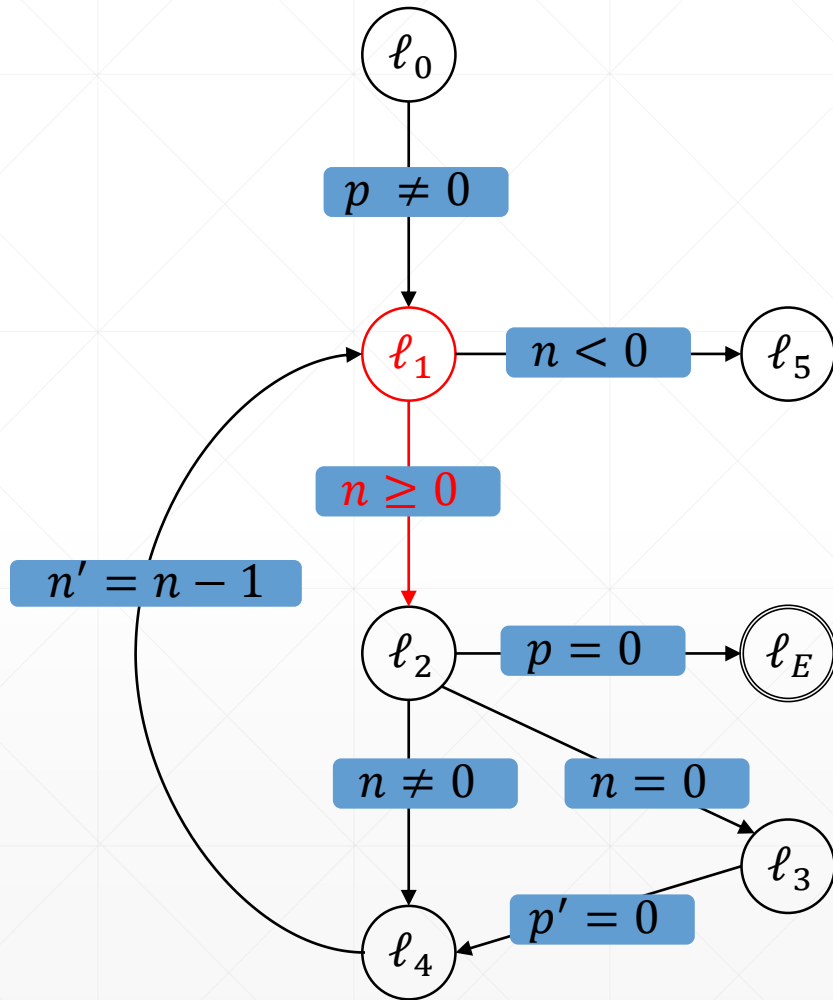
### 10. Step: Level 3 Blocking-Phase

- Try to block  $(p = 0, \ell_2, 3)$
- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$
  - Like the level before this is satisfiable

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$

# Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

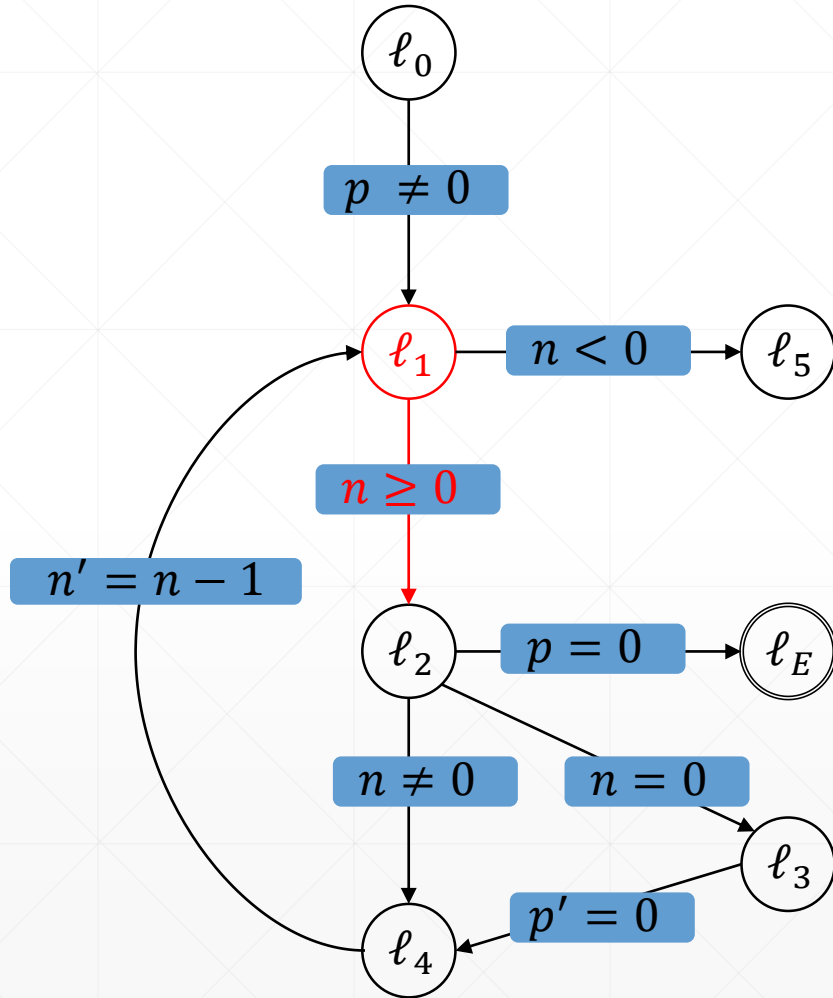
## 10. Step: Level 3 Blocking-Phase

- Try to block  $(p = 0, \ell_2, 3)$
- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$
  - Like the level before, get the same new proof-obligation but on level 2
  - $(p = 0, \ell_1, 2)$

## Proof-Obligations:

- $(p = 0, \ell_2, 3)$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

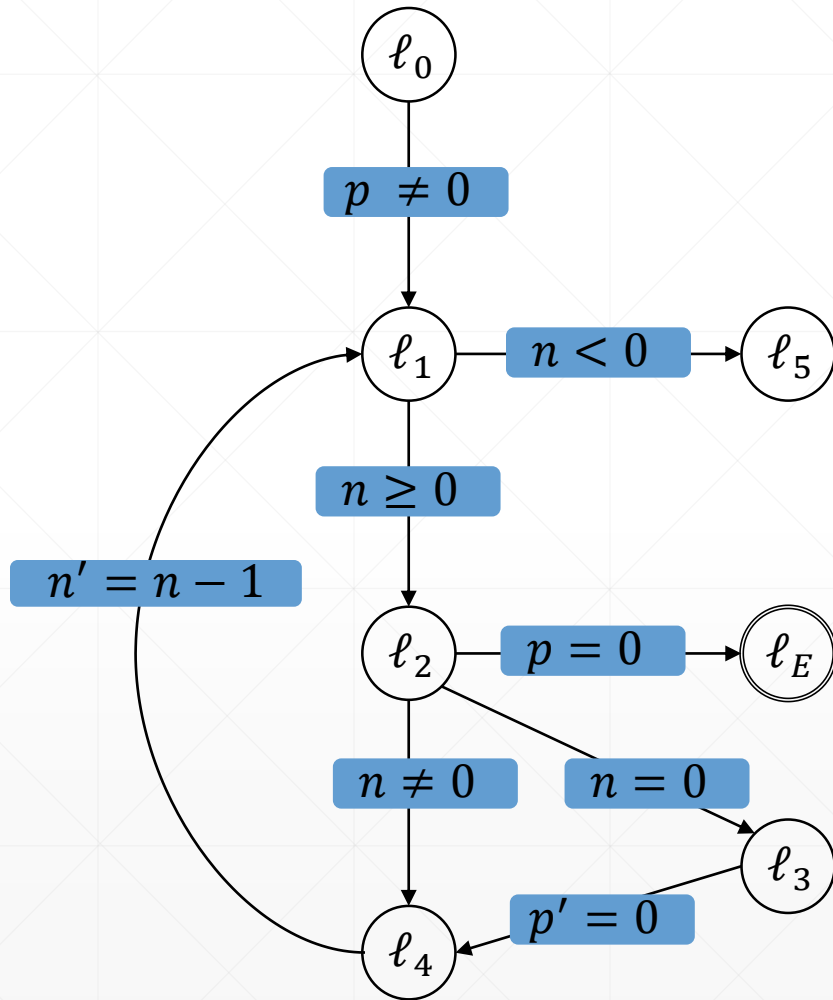
### 10. Step: Level 3 Blocking-Phase

- Try to block  $(p = 0, \ell_2, 3)$
- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$ 
    - Like the level before, get the same new proof-obligation but on level 2
    - $(p = 0, \ell_1, 2)$

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$
- $(p = 0, \ell_1, 2)$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

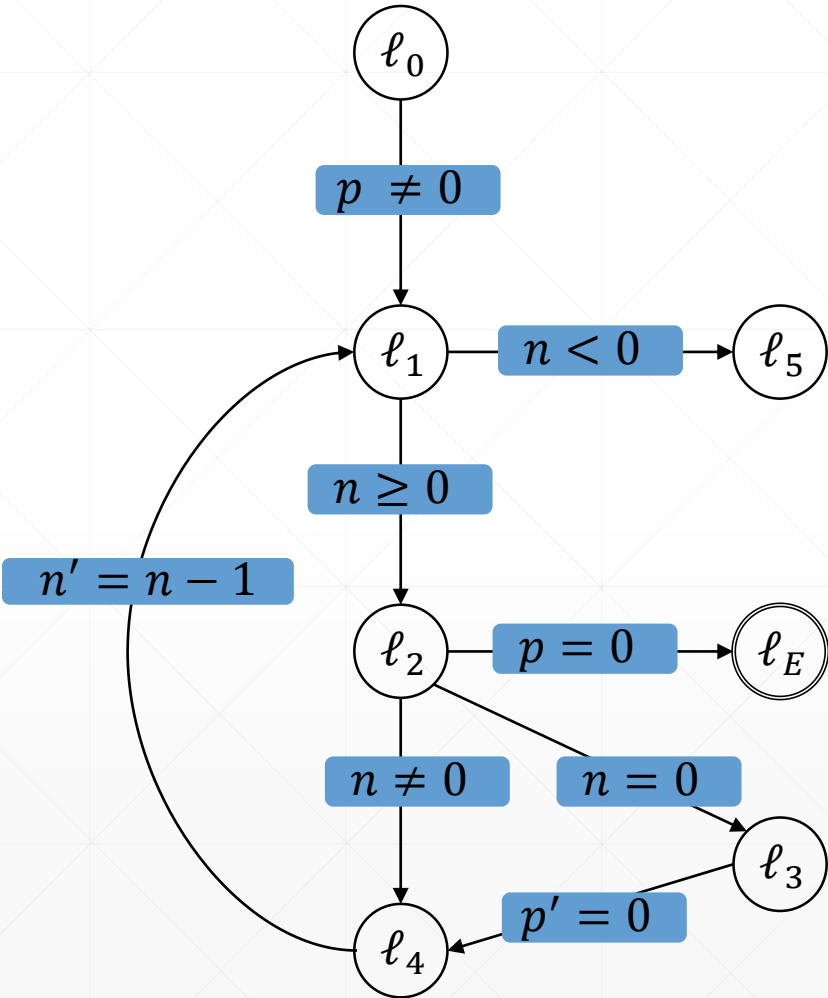
### 10. Step: Level 3 Blocking-Phase

- There are a lot of repetitions

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$
- $(p = 0, \ell_1, 2)$

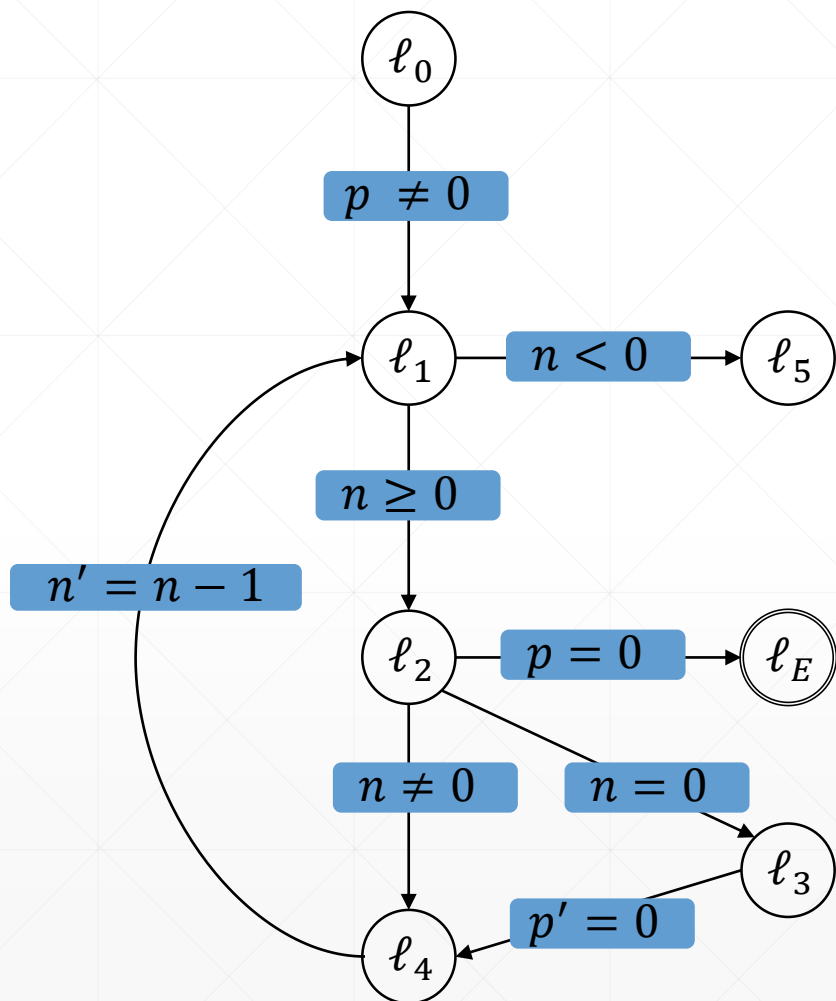
# Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$

11. Step: Level 3 Done

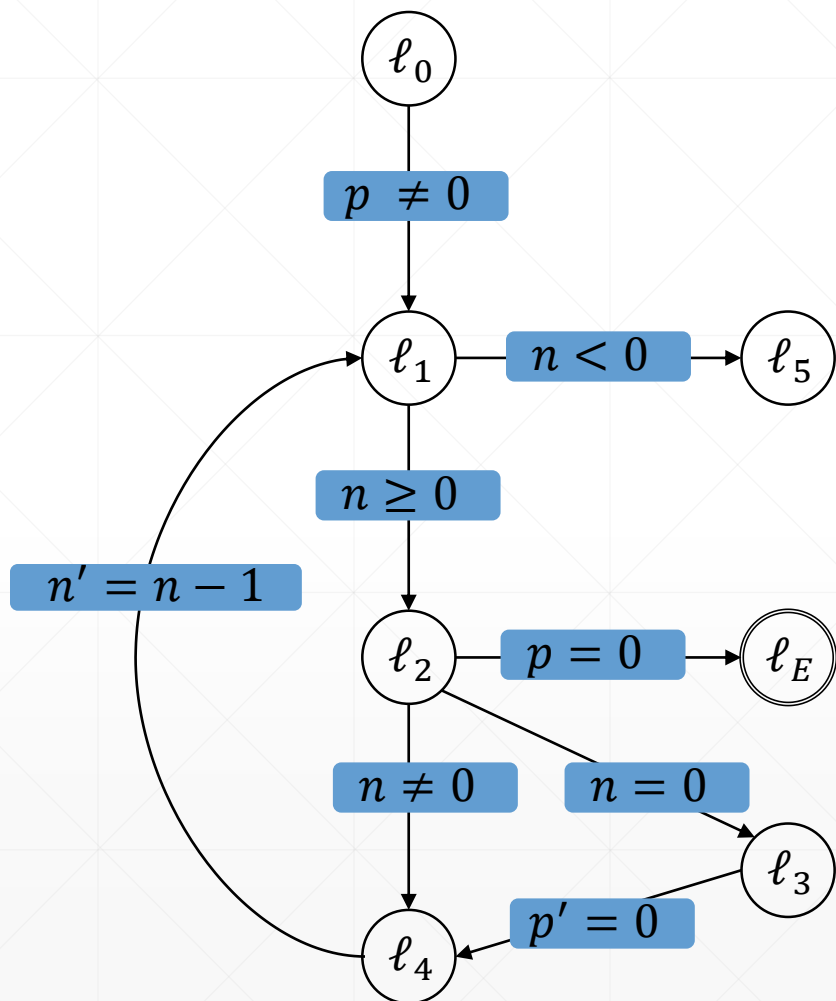
# Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$

11. Step: Level 4

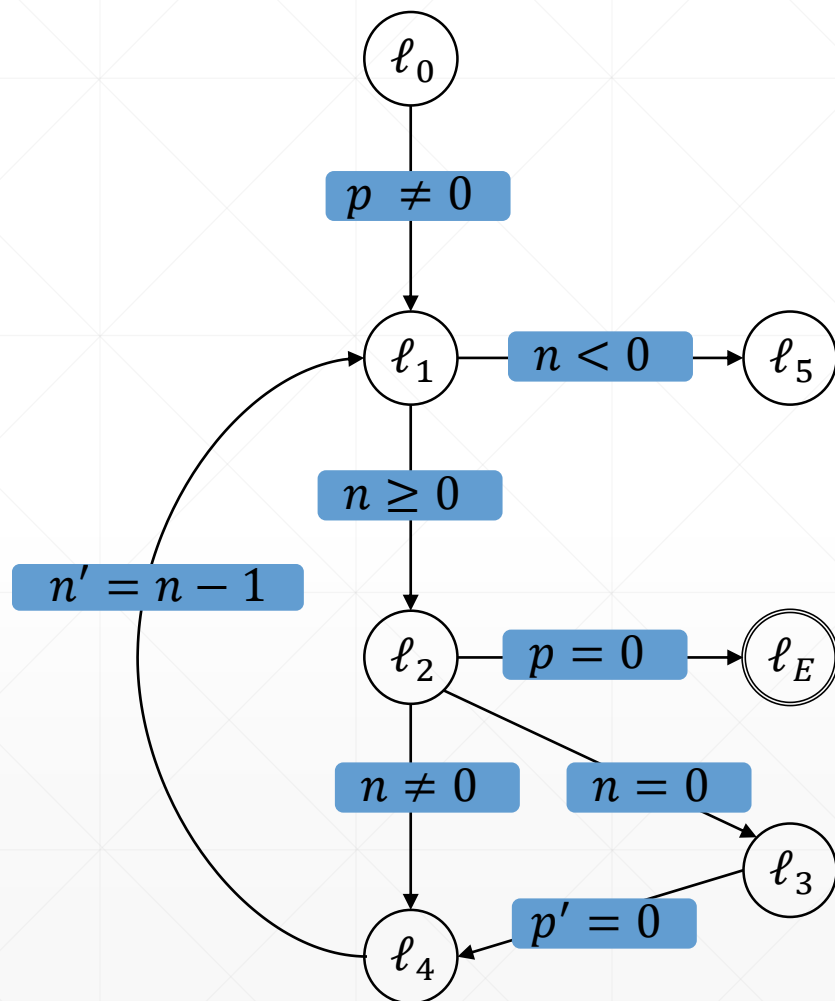
# Example:



location	0	1	2	3	4
$\ell_0$	$t$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$	$t$

11. Step: Level 4 Initialization

# Example:

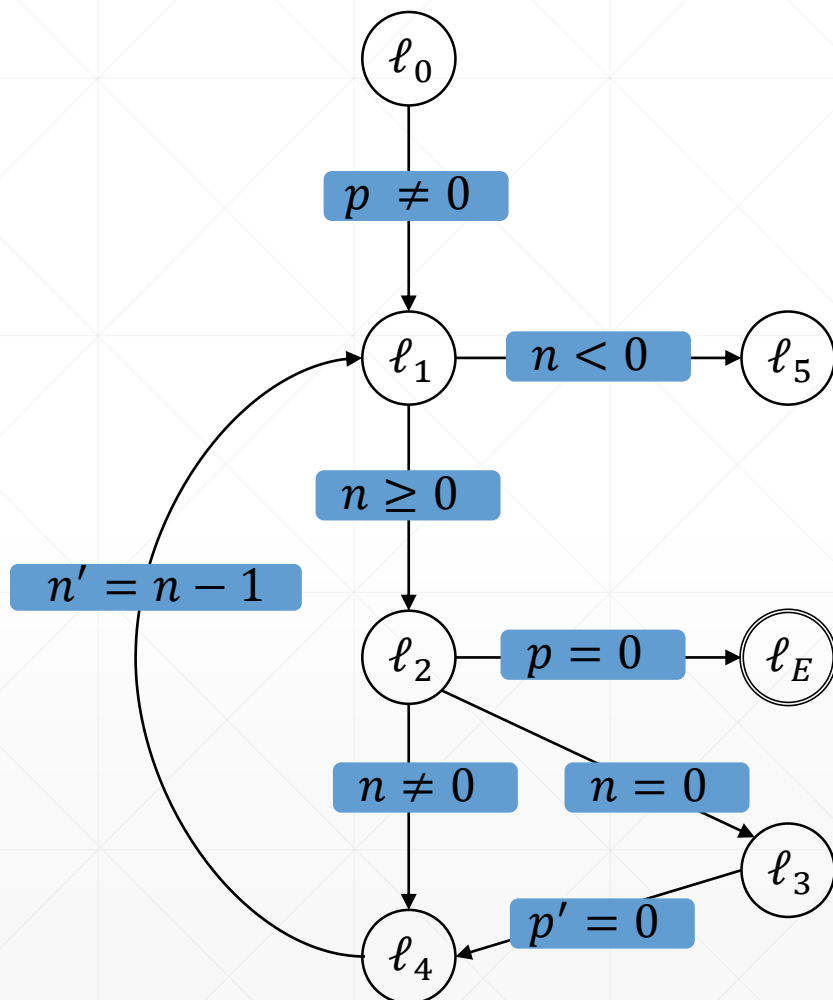


location	0	1	2	3	4
$\ell_0$	$t$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$	$t$

TODO The new interesting proof-obligation!

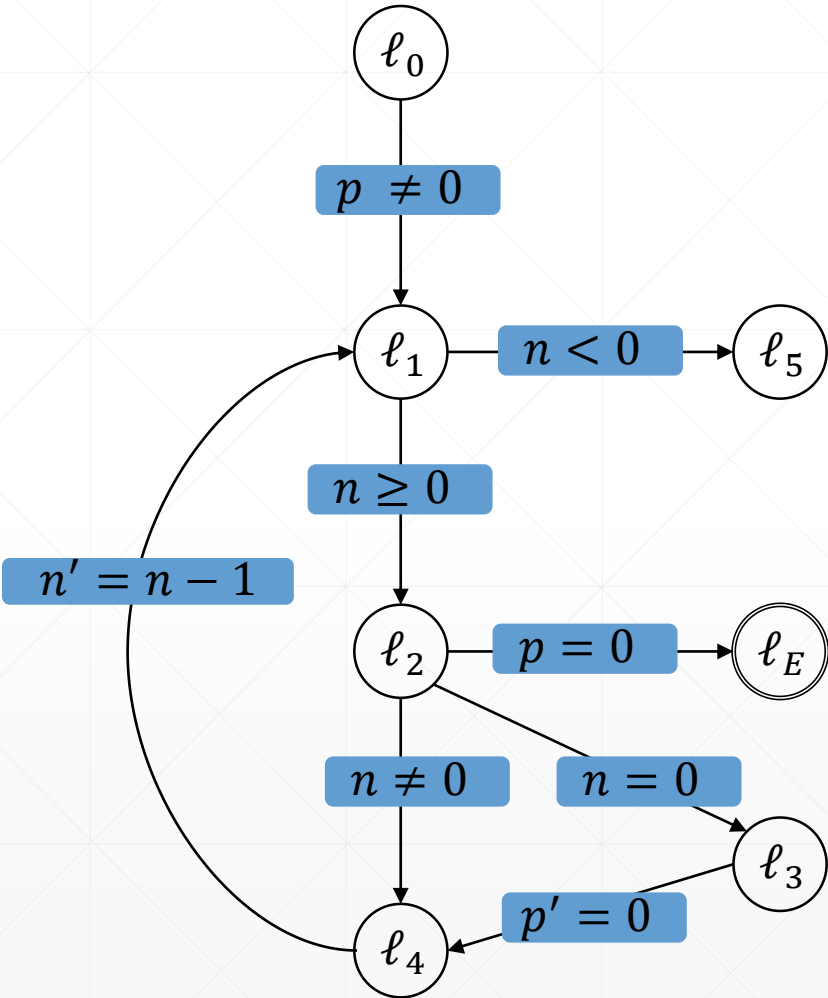


# Example:



location	0	1	2	3	4
$\ell_0$	$t$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f \wedge f$	$t \wedge f$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$

# Example:



location	0	1	2	3	4	5
$\ell_0$						
$\ell_1$						
$\ell_2$						
$\ell_3$						
$\ell_4$						

Text

Proof-Obligations:

# Related Work: Other Approaches

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- Bit-Blasting
- Using Abstract Reachability Trees

# PDR Algorithm: Possible Improvements

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## ➤ Generalization of Proof-Obligations:

- Using the disjunctive normal form (DNF):

@TODO EXPLANATION WITHOUT CUBES

➔ Split large proof-obligations into smaller ones by taking each conjunct of the DNF as a separate proof-obligation

- Using Interpolation:

- Instead of strengthening frames with the negated proof-obligation, compute an interpolant
- @ToDo MOAR

# Implementation in Ultimate: Trace Abstraction with PDR

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- Step by Step how trace abstraction works

# Implementation in Ultimate: Implemented Improvements

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# Evaluation: Data Comparison

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# Evaluation: Discussion

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# Future Work: Implementing Further Improvements

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# Conclusion

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