

Motivation



Overview

1. Introduction
2. Background: PDR on Hardware
3. PDR on Software
4. Implementation in Ultimate
5. Evaluation
6. Related Work
7. Future Work
8. Conclusion

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1. Introduction

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2.1 Preliminaries: Boolean Transition System

- A **Boolean Transition System** $S = (X, I, T)$ consists of
 - Set of **boolean variables** X
 - A conjunction representing the **initial state** I
 - A propositional formula T over variables in X and $X' = \{x' \in X \mid x' \in X'\}$, called **Transition Relation**
- **States** in S are cubes containing each variable from X with a boolean valuation of it
 - ➔ Finite number of states: $2^{|X|}$
- Transitions @Todo

2.1 Preliminaries: Formulas

- Given a formula φ over X , we get a primed formula φ' by replacing each variable with its corresponding variable in X'
- A literal is a variable or its negation
- A cube is a conjunction of literals
- A clause is a disjunction of literals
 - ➔ Negation of a cube is a clause and vice versa
- A Safety Property P is a formula over X that should be satisfiable by every state reachable from I
 - ➔ \bar{P} being a set of bad states

2.2 Algorithm: Overview

- PDR on hardware checks if states in \bar{P} are reachable from I
- For that it uses cubes of clauses, called Frames
 - Frame F_i represents an over-approximation of reachable states in at most i transitions from I
- PDR maintains sequence of frames $[F_0, F_1, \dots, F_k]$, called trace

2.2 Algorithm: Pseudo-Code

```
1: procedure PDR-PROVE( $I, T, P$ )
2:   check for 0-counter-example
3:   trace.push(new frame(I))

4:   loop
5:     while  $\exists$  cube  $c$ , s.t.  $\text{trace.last()} \wedge T \wedge c'$  is SAT and  $c \Rightarrow \bar{P}$  do
6:       recursively block proof-obligation( $c$ ,  $\text{trace.size()} - 1$ )
7:       and strengthen the frames of the trace.
8:       if a proof-obligation( $p$ , 0) is generated then
9:         return false

10:     $F_{k+1} = \text{new frame}(P)$ 
11:    for all clause  $c \in \text{trace.last()}$  do
12:      if  $\text{trace.last()} \wedge T \wedge \bar{c}'$  is UNSAT then
13:         $F_{k+1} = F_{k+1} \wedge c$ 
14:      if  $\text{trace.last()} == F_{k+1}$  then
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```

2.2 Algorithm: Checking for 0-Counter-Example

➤ Is $I \wedge \bar{P}$ satisfiable?

➔ If satisfiable:

- Algorithm terminates and returns that a bad state is reachable

➔ If unsatisfiable:

- Algorithm initializes the first frame in the trace: $F_0 = I$ and continues

2.2 Algorithm: Pseudo-Code

```
1: procedure PDR-PROVE( $I, T, P$ )
2:   check for 0-counter-example
3:   trace.push(new frame(I))

4:   loop                                     Next Transition Phase
5:     while  $\exists$  cube  $c$ , s.t.  $\text{trace.last()} \wedge T \wedge c'$  is SAT and  $c \Rightarrow \bar{P}$  do
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2.2 Algorithm: Next Transition Phase

- Checking if the next state is a good state:
 - Let $[F_0, F_1, \dots, F_k]$ be the current trace
 - Is $F_k \wedge T \wedge \overline{P'}$ satisfiable?
 - ➔ If unsatisfiable:
 - Continue with the next phase

2.2 Algorithm: Next Transition Phase

- Checking if the next state is a good state

➤ Let $[F_0, F_1, \dots, F_k]$ be the current trace

➤ Is $F_k \wedge T \wedge \overline{P'}$ satisfiable?

➔ If satisfiable:

- Take satisfying assignment $\vec{x} = \{x_1, x_2, \dots, x_{|X|}, x'_1, x'_2, \dots, x'_{|X'|}\}$
- The algorithm gets new bad state: $b = x_1 \wedge x_2 \wedge \dots \wedge x_{|X|}$
- Construct the tuple $t = (b, k)$, called proof-obligation

2.2 Algorithm: Pseudo-Code

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Blocking-Phase


```

4:   loop
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2.2 Algorithm: Blocking-Phase

Note: useful to have
Piece of pseudo-code?

- Proving that new bad states are not reachable

➤ If there are proof-obligations:

- Algorithm takes proof-obligation (b , i)
- Tries to block bad state b by checking $F_{i-1} \wedge T \wedge b'$ for satisfiability

➔ If satisfiable:

- Frame F_{i-1} is not strong enough to block b
- Take satisfying assignment $\vec{x} = \{x_1, x_2, \dots, x_{|X|}, x'_1, x'_2, \dots, x'_{|X'|}\}$
- The algorithm gets another new bad state: $c = x_1 \wedge x_2 \wedge \dots \wedge x_{|X|}$
- Construct new proof-obligation $u = (c, i - 1)$

2.2 Algorithm: Blocking-Phase

- Proving that new bad states are not reachable
- If there are proof-obligations:
 - Algorithm takes proof-obligation (b, i)
 - Tries to block bad state b by checking $F_{i-1} \wedge T \wedge b'$ for satisfiability
- ➔ If unsatisfiable:
 - Algorithm strengthens F_i with \bar{b}
 - ➔ $F_i = F_i \wedge \bar{b}$
 - Blocking bad state b at F_i

2.2 Algorithm: Blocking-Phase

➤ This continues recursively until:

- There are no proof-obligations left
 - ➔ Algorithm continues with the next phase
- A proof-obligation $(d, 0)$ is created
 - ➔ Proving that a bad state can be reached, terminating the algorithm

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Propagation-Phase

2.2 Algorithm: Propagation-Phase

- Propagating learned information
- After no proof-obligations are left, the algorithm initializes new frame $F_{k+1} = P$
- Algorithm passes on learned informations, e.g, which states are blocked:
 - For each clause c in F_k check: $F_k \wedge T \wedge \bar{c}'$ for satisfiability
 - ➔ If satisfiable:
 - Do nothing, continue with next clause

2.2 Algorithm: Propagation-Phase

- Propagating learned information
 - After no proof-obligations are left, the algorithm initializes new frame $F_{k+1} = P$
 - Algorithm passes on learned informations, e.g, which states are blocked:
 - For each clause c in F_k check: $F_k \wedge T \wedge \bar{c}'$ for satisfiability
 - ➔ If unsatisfiable:
 - Algorithm strengthens F_{k+1} with c
 - ➔ $F_{k+1} = F_{k+1} \wedge c$

2.2 Algorithm: Propagation-Phase

- Check for termination
- After all clauses have been tested, algorithm checks if $F_k \equiv F_{k+1}$
 - If so, the algorithm has found a fixpoint and terminates
 - ➔ No states of \bar{P} are reachable
 - If not, the algorithm continues with a new Next Transition Phase

2.2 Algorithm: Propagation-Phase

- Check for termination
- After all clauses have been tested, algorithm checks if $F_k \equiv F_{k+1}$
 - If so, the algorithm has found a fixpoint and terminates
 - ➔ No states of \bar{P} are reachable
 - If not, the algorithm continues with a new Next Transition Phase
- Algorithm repeats the three phases until a fixpoint is found, or a proof-obligation $(d, 0)$ is created

2.2 Algorithm: Pseudo-Code TEMPLATE

```
1: procedure PDR-PROVE( $I, T, P$ )
2:   check for 0-counter-example
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16:     $trace.push(F_{k+1})$ 
```

2.3 Example

2.4 Possible Improvements

- Blocking one state at a time is ineffective:
 - Generalize blocked states
 - ➔ Eliminate insignificant cubes from states, that are not used by UNSAT-cores

- Ternary Simulation to reduce proof-obligations:
 - Extend binary variables with a new value: unknown
 - Check state variables of proof-obligations for importance
 - ➔ Eliminate unimportant state variables

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3.1 Preliminaries

- To use PDR on software, we need to lift the algorithm from propositional-logic based systems to first-order logic based systems

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- To use PDR on software, we need to lift the algorithm from propositional-logic based systems to first-order logic based systems
- For that we first need new definitions

3.1 Preliminaries: Control Flow Graph

- A control flow graph (CFG) $A = (X, L, G, \ell_0, \ell_E)$ is a graph consisting of
- A finite set of variables X
 - A finite set of locations L
 - A finite set of transitions $G \subseteq L \times FO \times L$
 - ➔ FO being a quantifier free first-order logic formula over variables in X and $X' = \{x \in X \mid x' \in X'\}$
 - An initial location $\ell_0 \in L$
 - An error location $\ell_E \in L$

3.1 Preliminaries: Control Flow Graph

➤ The transition formula $T_{\ell_1 \rightarrow \ell_2}$ from location ℓ_1 to location ℓ_2 is defined as:

- $$T_{\ell_1 \rightarrow \ell_2} = \begin{cases} (\ell_1, t, \ell_2), & (\ell_1, t, \ell_2) \in G \\ \text{false}, & \text{otherwise} \end{cases}$$

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➔ Global Transition Formula $T = \bigvee_{(\ell_1, t, \ell_2) \in G} T_{\ell_1 \rightarrow \ell_2}$

3.2 Lifted Algorithm: Pseudo-Code

```
1: procedure PDR-PROVE( $I, T, P$ )
2:   check for 0-counter-example
3:   trace.push(new frame(I))

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```
1: procedure LIFTED-PDR-PROVE( $L, G$ )
2:   check for 0-counter-example
3:    $\ell_0.\text{trace.push}(\text{new frame}(\text{true}))$ 
4:   for all  $\ell \in L \setminus \{\ell_0, \ell_E\}$  do
5:      $\ell.\text{trace.push}(\text{new frame}(\text{false}))$ 
6:    $\text{level} := 0$ 

7:   loop
8:     for all  $\ell \in L \setminus \{\ell_E\}$  do
9:        $\ell.\text{trace.push}(\text{new frame}(\text{true}))$ 
10:     $\text{level} := \text{level} + 1$ 
11:    get initial proof-obligations

12:    while  $\exists$  proof-obligation  $(t, \ell, i)$ , do
13:      Recursively block proof-obligation
14:      if a proof-obligation( $p, \ell, 0$ ) is generated then
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16:    for  $i = 0; i \leq \text{level}; i := i + 1$  do
17:      for  $\ell \in L \setminus \{\ell_E\}$  do
18:        if  $\ell.\text{trace}[i] \neq \ell.\text{trace}[i - 1]$  then
19:          break
20:    return true
```

3.2 Lifted Algorithm: Pseudo-Code

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6:    $level := 0$ 

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18:        if  $\ell.trace[i] \neq \ell.trace[i - 1]$  then
19:          break
20:    return true
```

3.2 Lifted Algorithm: Checking for 0-Counter-Example

➤ Is $\ell_0 = \ell_E$?

➔ Yes:

- Algorithm terminates, returning that ℓ_E is reachable

➔ No:

- Algorithm continues

3.2 Lifted Algorithm: Pseudo-Code

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```

3.2 Lifted Algorithm: Local Traces

- There is no global trace $[F_0, F_1, \dots, F_k]$
 - ➔ Every location $\ell \in L \setminus \{\ell_E\}$ has its own local trace $[F_{0,\ell}, F_{1,\ell}, \dots, F_{k,\ell}]$
 - ➔ Lifted frames are cubes of first-order formulas
 - ➔ @ToDo, explain changes to proof obligations

3.2 Lifted Algorithm: Pseudo-Code

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12:    while  $\exists$  proof-obligation  $(t, \ell, i)$ , do
13:      Recursively block proof-obligation
14:      if a proof-obligation( $p, \ell, 0$ ) is generated then
15:        return false
16:    for  $i = 0; i \leq \text{level}; i := i + 1$  do
17:      for  $\ell \in L \setminus \{\ell_E\}$  do
18:        if  $\ell.\text{trace}[i] \neq \ell.\text{trace}[i - 1]$  then
19:          break
20:    return true
```

3.2 Lifted Algorithm: Initialization

➤ Initialize each local frames:

- $F_{0,\ell} = \begin{cases} true, & \ell = \ell_0 \\ false, & otherwise \end{cases}$

3.2 Lifted Algorithm: Pseudo-Code

```
1: procedure PDR-PROVE( $I, T, P$ )
2:   check for 0-counter-example
3:   trace.push(new frame(I))

4:   loop
5:     while  $\exists$  cube  $c$ , s.t. trace.last()  $\wedge T \wedge c'$  is SAT and  $c \Rightarrow \bar{P}$  do
6:       recursively block proof-obligation( $c$ , trace.size() - 1)
7:       and strengthen the frames of the trace.
8:       if a proof-obligation( $p$ , 0) is generated then
9:         return false

10:     $F_{k+1} = \text{new frame}(P)$ 
11:    for all clause  $c \in \text{trace.last()}$  do
12:      if trace.last()  $\wedge T \wedge \bar{c}'$  is UNSAT then
13:         $F_{k+1} = F_{k+1} \wedge c$ 
14:      if trace.last()  $== F_{k+1}$  then
15:        return true
16:    trace.push(Fk+1)
```

```
1: procedure LIFTED-PDR-PROVE( $L, G$ )
2:   check for 0-counter-example
3:    $\ell_0.\text{trace.push}(\text{new frame}(\text{true}))$ 
4:   for all  $\ell \in L \setminus \{\ell_0, \ell_E\}$  do
5:      $\ell.\text{trace.push}(\text{new frame}(\text{false}))$ 
6:    $\text{level} := 0$ 

7:   loop
8:     for all  $\ell \in L \setminus \{\ell_E\}$  do
9:        $\ell.\text{trace.push}(\text{new frame}(\text{true}))$ 
10:       $\text{level} := \text{level} + 1$ 
11:      get initial proof-obligations

12:      while  $\exists$  proof-obligation  $(t, \ell, i)$ , do
13:        Recursively block proof-obligation
14:        if a proof-obligation( $p, \ell, 0$ ) is generated then
15:          return false

16:      for  $i = 0; i \leq \text{level}; i := i + 1$  do
17:        for  $\ell \in L \setminus \{\ell_E\}$  do
18:          if  $\ell.\text{trace}[i] \neq \ell.\text{trace}[i - 1]$  then
19:            break
20:      return true
```

3.2 Lifted Algorithm: Next Level Phase

- Initializing the next level:
 - Let k be the current level
 - ➔ Every location $\ell \in L \setminus \{\ell_E\}$ has trace $[F_{0,\ell}, \dots, F_{k,\ell}]$
 - Algorithm initializes new level $k+1$ for all locations $\ell \in L \setminus \{\ell_E\}$
 - ➔ Adding new frame $F_{k+1,\ell} = \text{true}$

3.2 Lifted Algorithm: Next Level Phase

- Initializing the next level:
 - Let k be the current level
 - ➔ Every location $\ell \in L \setminus \{\ell_E\}$ has trace $[F_{0,\ell}, \dots, F_{k,\ell}]$
 - Additionally, algorithm computes initial proof-obligations:
 - Because of the structure of CFAs, we always know the transitions to ℓ_E
 - ➔ Check G for transitions of the form (ℓ, t, ℓ_E)
 - ➔ For each transition, get proof-obligation (t, ℓ, k)
 - ➔ @ToDo explain lifted proof-obligations

3.3 Example

3.4 Possible Improvements

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4.2 Implemented Improvements

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7.1 Further Improvements

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8. Conclusion
