Bachelor Thesis

Property Directed Reachability

Proposal

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1 Introduction

SAT-based model-checking is a useful technique for both software and hardware verification. Most modern model-checkers are based on interpolation [?]. Recently a novel algorithm was devised by Aaron Bradley [?] called `IC3`. Because it was so new, it came as a surprise that it won third place in the hardware model-checking competition (HWMCC) at CAV 2010.
The model-checking method behind `IC3` is called *Property Directed Reachability*, *PDR* for short, which is not based on interpolation but on backward-search.
Ultimate [?] is a program analysis framework consisting of multiple plugins that perform steps of a program analysis, like parsing source code, transforming programs from one representation to another, or analyse programs. Ultimate already has analysis-plugins using different model-checking techniques like trace abstraction [?] or lazy interpolation [?]. The goal of this Bachelor's Thesis is to implement a new analysis-plugin that uses PDR in Ultimate and to compare it with the other techniques.

2 Bit-Level PDR

In the following I will describe the basic principle behind PDR as a hardware-checker as used in `IC3`, therefore we use only boolean variables. It is however possible to use PDR as a software-checker as shown later in chapter 3.

2.1 Preliminaries

First some preliminary definitions and notations:

A *literal* is a variable or its negation, e.g., x or $\bar{y}$
A *clause* is a disjunction of literals, e.g., $x \vee \bar{y}$
A *cube* is a conjunction of literals, e.g., $x \wedge \bar{y}$
Therefore, the negation of a cube is a clause. $\overline{(x \wedge \bar{y})} \equiv (\bar{x} \vee y)$

A *boolean transition system* is a tuple $S = (X, I, T)$ where X is a finite set of boolean variables, I is a cube representing the *initial state*, and T is a propositional formula over variables in X and $X' = \{x \in X \mid x' \in X'\}$, called transition relation, that describes updates to the variables.

For example, consider the transition system $U = (X, I, T)$ where
$X = \{x_1, x_2, x_3\}$
$I = \bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3$
$T = (x_1 \vee \neg x_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')$
With transition graph:

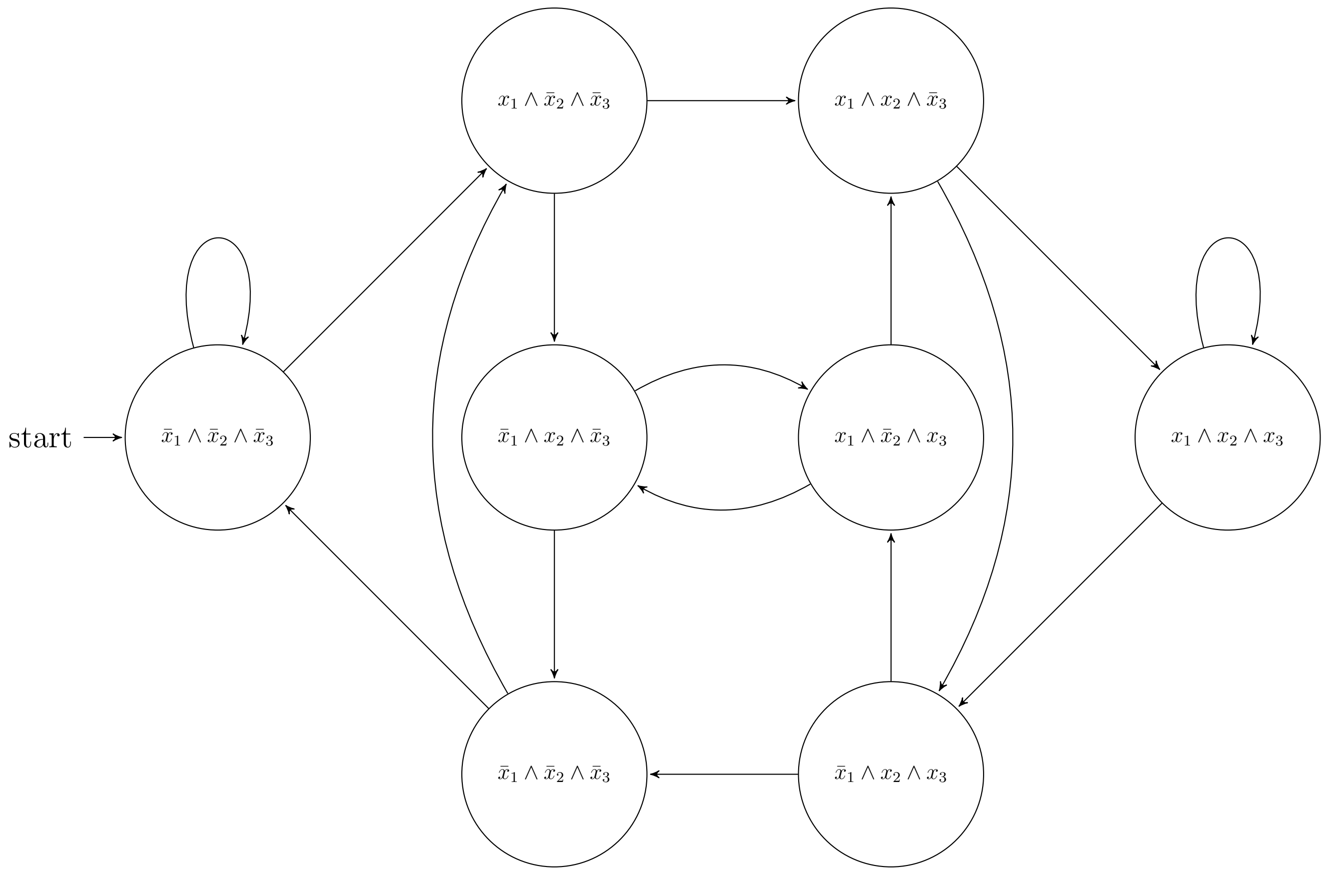


Figure 1: Transition Graph of U

Given a propositional formula ϕ over X we get a *primed formula* ϕ' by replacing each variable with its corresponding variable in X'.

A *state* in S is a cube containing each variable from X with a boolean valuation of it. For each possible valuation there is a corresponding state, resulting in $2^{|X|}$ states in S.
Like we see in the graph of U we have $2^{|X|} = 2^3 = 8$ states.

A *transition* from one state s to another state q exists if the conjunction of s, the transition relation, and q' is satisfiable.
For example in U the transition between the initial state $I = \bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3$ and state $r = x_1 \wedge \bar{x}_2 \wedge \bar{x}_3$ exists because

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{I} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T} \wedge \underbrace{x_1' \wedge \bar{x}_2' \wedge \bar{x}_3'}_{r'}$$

is satisfiable.

Given a propositional formula P over X, called *property*, we want to verify that every state in S that is reachable from I satisfies P such that, P describes a set of *good states*, conversely $\bar{P}$ represent a set of *bad* states.
Regarding U, let $P = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$ be given, making $\bar{P} = x_1 \wedge x_2 \wedge x_3$ a bad state.
We can use PDR to show that either $\bar{P}$ is unreachable from I or that there exists a sequence of transitions leading to $\bar{P}$ as counter-example.

2.2 Algorithm

A PDR-based algorithm tries to prove that a transition system $S = (X, I, T)$ satisfies a given property P by trying to find a formula F over X with the following qualities:

(1) $I \Rightarrow F$

(2) $F \wedge T \Rightarrow F'$

(3) $F \Rightarrow P$

F is called an *inductive invariant*.

To calculate an inductive invariant, PDR uses *frames* which are cubes of clauses representing an over-approximation of reachable states in at most i transitions from I.
PDR maintains a sequence of frames $[F_0, ..., F_k]$, called a *trace*, it is organized so that it fulfills the following characteristics:

(I) $F_0 = I$

(II) $F_{i+1} \subseteq F_i$, therefore $F_i \Rightarrow F_{i+1}$

(III) $F_i \wedge T \Rightarrow F'_{i+1}$

(IV) $F_i \Rightarrow P$

Now to the algorithm itself:
Start with checking for a *0-counter-example*, that means checking if $I \Rightarrow P$, by testing whether the formula $I \wedge \bar{P}$ is satisfiable. If it is, then I is a 0-counter-example, the algorithm terminates. If the formula is unsatisfiable, initialize the first frame $F_0 = I$, fulfilling (I), and moving on.

Let $[F_0, F_1, ..., F_k]$ be the current trace.
The algorithm repeats the following three phases until termination:

1. Next Transition
Check whether the next state is a good state meaning $F_k \wedge T \Rightarrow P'$ is valid, by testing the satisfiability of $F_k \wedge T \wedge \bar{P}'$

- If the formula is *satisfiable*, for each satisfying assignment $\vec{x} = (x_1, x_2, ..., x_{|X|}, x'_1, x'_2, ..., x'_{|X'|})$ get a new bad state $a = x_1 \wedge x_2 \wedge ... \wedge x_{|X|}$ and create tuple (a, k), this tuple is called a *proof-obligation*.
- If the formula is *unsatisfiable*, continue with the next phase.

2. Blocking-Phase
If there are proof-obligations:
Take proof-obligation (b, i) and try to block the bad state b by checking if frame F_{i-1} can reach b in one transition, i.e., test $F_{i-1} \wedge T \wedge b'$ for satisfiability.

- If the formula is *satisfiable*, it means that F_i is not strong enough to block b. For each satisfying assignment $\vec{x} = (x_1, x_2, ..., x_{|X|}, x'_1, x'_2, ..., x'_{|X'|})$ get a new bad state $c = x_1 \wedge x_2 \wedge ... \wedge x_{|X|}$ creating the new proof-obligation $(c, i-1)$.
- If the formula is *unsatisfiable*, strengthen frame F_i with $\bar{b}$ meaning $F_i = F_i \wedge \bar{b}$, blocking b at F_i

This continues recursively until either a proof-obligation $(d, 0)$ is created proving that there exists a counter-example terminating the algorithm, or there is no proof-obligation left.

3. Propagation-Phase
Add a new frame $F_{k+1} = P$ and propagate clauses from F_k forward, meaning for all clauses c in F_k check $F_k \wedge T \wedge \vec{c}'$ for satisfiability. If that formula is unsatisfiable, strengthen F_{k+1} with c: $F_{k+1} = F_{k+1} \wedge c$, else do nothing and

continue with the next clause. Because of this phase rule (II) is fulfilled.

After propagating all possible clauses, if $F_{k+1} \equiv F_k$ the algorithm found a fixpoint and terminates returning that P always holds with F_k being the inductive invariant.

To illustrate the procedure further consider the pseudo-code:

Algorithm 1 PDR-prove

procedure PDR-PROVE(I, T, P)
    check for 0-counter-example
    $trace.push(new\ frame(I))$

    loop
        while $\exists$ cube c, s.t. $trace.last() \wedge T \wedge c'$ is SAT and $c \Rightarrow \bar{P}$ **do**
            recursively block proof-obligation(c, trace.size() - 1)
            and strengthen the frames of the trace.
            if a proof-obligation(p, 0) is generated **then**
                return false ▷ counter-example found

        $F_{k+1} = new\ frame(P)$
        for all clause c $\in trace.last()$ **do**
            if $trace.last() \wedge T \wedge \bar{c}'$ is UNSAT **then**
                $F_{k+1} = F_{k+1} \wedge c$
        if $trace.last() == F_{k+1}$ **then**
            return true
        $trace.push(F_{k+1})$

2.3 Examples

2.3.1 With Failing Property

To show an application of the algorithm reconsider the example transition system $U = (X, I, T)$ with:
$X = \{x_1, x_2, x_3\}$,
$I = \bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3$,

$T = (x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')$

and the property:
$P = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$ with bad state $\bar{P} = x_1 \wedge x_2 \wedge x_3$
We now want to verify whether P holds or if there is a counter-example.

1. Step: Check for 0-Counter-Example
We need to make sure that $I \Rightarrow P$, we do that by testing if $I \wedge \bar{P}$ is satisfiable:

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{\text{I}} \wedge \underbrace{x_1 \wedge x_2 \wedge x_3}_{\bar{P}}$$

The formula is unsatisfiable meaning there is no 0-counter-example, we continue by initializing $F_0 = I$

2. Step: First Transition
Check if $F_0 \wedge T \Rightarrow P'$, by testing if $F_0 \wedge T \wedge \bar{P}'$ is satisfiable:

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_0} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T} \wedge \underbrace{x_1' \wedge x_2' \wedge x_3'}_{\bar{P}'}$$

Which it is not because $\bar{x}_1 \wedge (x_1 \vee \bar{x}_2') \wedge x_2'$ is unsatisfiable. We do not generate a proof-obligation so we can skip the blocking-phase and continue on with the propagation-phase.

3. Step: First Propagation-Phase
Initialize $F_1 = P$
Check each clause c in F_0 for $F_0 \wedge T \wedge \vec{c}'$ to strengthen F_1.

- $c = \bar{x}_1$

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_0} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T} \wedge \underbrace{x_1'}_{\vec{c}'}$$

Satisfiable with $(\bar{x}_1, \bar{x}_2, \bar{x}_3, x'_1, \bar{x}'_2, \bar{x}'_3)$
$\rightarrow$ Do not add $\bar{x}_1$ to F_1.

- $c = \bar{x}_2$

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_0} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (x_2 \vee \bar{x}'_3) \wedge (\bar{x}_2 \vee x'_3)}_{T} \wedge \underbrace{x'_2}_{\bar{c}'}$$

Unsatisfiable because $\bar{x}_1 \wedge (x_1 \vee \bar{x}'_2) \wedge x'_2$ is not satisfiable
$\rightarrow$ Add $\bar{x}_2$ to F_1
$\rightarrow F_1 = P \wedge \bar{x}_2$.

- $c = \bar{x}_3$

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_0} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (x_2 \vee \bar{x}'_3) \wedge (\bar{x}_2 \vee x'_3)}_{T} \wedge \underbrace{x'_3}_{\bar{c}'}$$

Unsatisfiable because $\bar{x}_2 \wedge (x_2 \vee \bar{x}'_3) \wedge x'_3$ is not satisfiable
$\rightarrow$ Add $\bar{x}_3$ to F_1
$\rightarrow F_1 = P \wedge \bar{x}_2 \wedge \bar{x}_3$

With that the first propagation-phase is done resulting in

$$F_1 = (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_2 \wedge \bar{x}_3$$

and because $F_1 \not\equiv F_0$ we continue.

4. Step: Second Transition
Check if $F_1 \wedge T \Rightarrow P'$ by testing $F_1 \wedge T \wedge \bar{P}'$ for satisfiability:

$$\underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_1} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (x_2 \vee \bar{x}'_3) \wedge (\bar{x}_2 \vee x'_3)}_{T} \wedge \underbrace{x'_1 \wedge x'_2 \wedge x'_3}_{\bar{P}'}$$

Which is unsatisfiable because $\bar{x}_2 \wedge (x_2 \vee \bar{x}'_3) \wedge x'_3$ is not satisfiable. We do not generate a proof-obligation so we continue with the second propagation-phase.

5. Step: Second Propagation-Phase
Initialize $F_2 = P$
Check each clause c in F_1 for $F_1 \wedge T \wedge \bar{c}'$ to strengthen F_2. We skip P, as it is already part of F_2.

This works exactly as in the 3. step:

- $c = \bar{x}_2$

$$\underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_1} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T} \wedge \underbrace{x_2'}_{\bar{c}'}$$

Satisfiable with $(x_1, \bar{x}_2, \bar{x}_3, x_1', x_2', \bar{x}_3')$
$\rightarrow$ Do not add $\bar{x}_2$ to F_2

- $c = \bar{x}_3$

$$\underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_1} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T} \wedge \underbrace{x_3'}_{\bar{c}'}$$

Unsatisfiable because $\bar{x}_2 \wedge (x_2 \vee \bar{x}_3') \wedge x_3'$ is not satisfiable.
$\rightarrow$ Add $\bar{x}_3$ to F_2
$\rightarrow F_2 = P \wedge \bar{x}_3$

That concludes the second propagation-phase resulting in

$$F_2 = (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_3$$

and because $F_2 \not\equiv F_1$ we continue.

6. Step: Third Transition Step
Check $F_2 \wedge T \Rightarrow \bar{P}'$ by testing $F_2 \wedge T \wedge \bar{P}'$ for satisfiability

$$\underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_3}_{F_2} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T} \wedge \underbrace{x_1' \wedge x_2' \wedge x_3'}_{\bar{P}'}$$

This time $F_2 \wedge T \wedge \bar{P}'$ is satisfiable with assignment $(\underbrace{x_1, x_2, \bar{x}_3}_{s}, x_1', x_2', x_3')$, we get the new bad state $s = x_1 \wedge x_2 \wedge \bar{x}_3$, and generate a proof-obligation $(s, 2)$, which we now try to block in the blocking-phase.

7. Step: First Blocking-Phase
Try to block proof-obligation $(s, 2)$ by checking if $F_1 \wedge T \wedge s'$ is satisfiable.

$$\underbrace{(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_1} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (x_2 \vee \bar{x}'_3) \wedge (\bar{x}_2 \vee x'_3)}_{T} \wedge \underbrace{x'_1 \wedge x'_2 \wedge \bar{x}'_3}_{s'}$$

This is again satisfiable with assignment $(\underbrace{x_1, \bar{x}_2, \bar{x}_3}_{\text{q}}, x'_1, x'_2, \bar{x}'_3)$, we get the bad state $q = x_1 \wedge \bar{x}_2 \wedge \bar{x}_3$ and generate a new proof-obligation $(q, 1)$.

Try to block proof-obligation $(q, 1)$ by checking if $F_0 \wedge T \wedge q'$ is satisfiable.

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{F_0} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (x_2 \vee \bar{x}'_3) \wedge (\bar{x}_2 \vee x'_3)}_{T} \wedge \underbrace{x'_1 \wedge \bar{x}'_2 \wedge \bar{x}'_3}_{q'}$$

This too is satisfiable with assignment $(\underbrace{\bar{x}_1, \bar{x}_2, \bar{x}_3}_{\text{I}}, x'_1, \bar{x}'_2, \bar{x}'_3)$, we get the bad state $I = x_1 \wedge \bar{x}_2 \wedge \bar{x}_3$ and generate a new proof-obligation $(I, 0)$.

With that we have found a counter-example, resulting in the termination of the algorithm returning the counter-example trace:

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{I} \rightarrow \underbrace{x_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{q} \rightarrow \underbrace{x_1 \wedge x_2 \wedge \bar{x}_3}_{s} \rightarrow \underbrace{x_1 \wedge x_2 \wedge x_3}_{\bar{P}}$$

Assume proof-obligation $(s, 2)$ would have been blocked, meaning $F_1 \wedge T \wedge s'$ was unsatisfiable, then we would have updated $F_2 = F_2 \wedge \bar{s}$ making absolutely sure that s is not reachable, every future proof-obligation containing s would have been blocked by F_2.

2.3.2 With Passing Property

To show a transition system with an inductive invariant consider $B = (X, I, T)$ with:
$X = \{x_1, x_2\}$,
$I = \bar{x}_1 \wedge \bar{x}_2$,
$T = (x_1 \vee \bar{x}_2 \vee x_2') \wedge (x_1 \vee x_2 \vee \bar{x}_1') \wedge (\bar{x}_1 \vee x_1') \wedge (\bar{x}_1 \vee \bar{x}_2') \wedge (x_2 \vee \bar{x}_2')$
and transition graph:

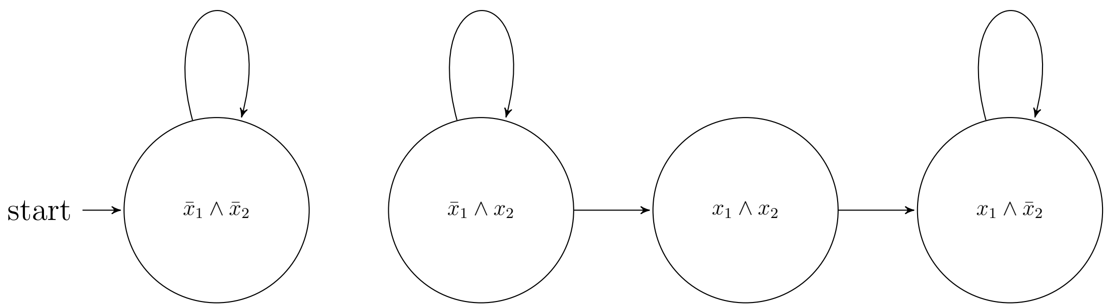


Figure 2: Transition Graph of B

Now given the property $P = \bar{x}_1 \vee x_2$, we want to check whether the bad state $\bar{P} = x_1 \wedge \bar{x}_2$ is reachable:

1. Step: Check for 0-Counter-Example
Check for 0-counter-example to make sure $I \Rightarrow P$ by testing $I \wedge \bar{P}$ for satisfiability:

$$\bar{x}_1 \wedge \bar{x}_2 \wedge x_1 \wedge \bar{x}_2$$

The formula is unsatisfiable because $\bar{x}_1 \wedge x_1$ that means there is no 0-counter-example.

2. Step: First Transition
Initialize $F_0 = I$ and check if $F_0 \wedge T \Rightarrow P'$ by testing $F_0 \wedge T \wedge \bar{P}'$ for

satisfiability:

$$\underbrace{\bar{x}_1 \wedge \bar{x}_2}_{F_0} \wedge \underbrace{(x_1 \vee \bar{x}_2 \vee x'_2) \wedge (x_1 \vee x_2 \vee \bar{x}'_1) \wedge (\bar{x}_1 \vee x'_1) \wedge (\bar{x}_1 \vee \bar{x}'_2) \wedge (x_2 \vee \bar{x}'_2)}_{T} \wedge \underbrace{x'_1 \wedge \bar{x}'_2}_{\bar{P}'}$$

Which is unsatisfiable because $\bar{x}_1 \wedge \bar{x}_2 \wedge (x_1 \vee x_2 \vee x'_1) \wedge \bar{x}'_1$ is not satisfiable. We generate no proof-obligation and continue with the propagation-phase.

3. Step: First Propagation-Phase
Initialize $F_1 = P$
For each clause c in F_0 check $F_0 \wedge T \wedge \bar{c}'$ for satisfiability to strengthen F_1.

- $c = \bar{x}_1$

  $$\bar{x}_1 \wedge \bar{x}_2 \wedge T \wedge x'_1$$

  Unsatisfiable because $\bar{x}_1 \wedge \bar{x}_2 \wedge (x_1 \vee x_2 \vee x'_1) \wedge \bar{x}'_1$ is not satisfiable.
  $\rightarrow$ Add $\bar{x}_1$ to F_1
  $\rightarrow F_1 = P \wedge \bar{x}_1$

- $c = \bar{x}_2$

  $$\bar{x}_1 \wedge \bar{x}_2 \wedge T \wedge x'_2$$

  Unsatisfiable because $\bar{x}_1 \wedge \bar{x}_2 \wedge (x_2 \vee \bar{x}'_2) \wedge x'_2$ is not satisfiable.
  $\rightarrow$ Add $\bar{x}_2$ to F_1
  $\rightarrow F_1 = P \wedge \bar{x}_1 \wedge \bar{x}_2$

That concludes the propagation-phase resulting in

$$F_1 = (\bar{x}_1 \vee x_2) \wedge \bar{x}_1 \wedge \bar{x}_2$$

and because $F_1 \not\equiv F_0$ we continue.

4. Step: Second Transition
Check if $F_1 \wedge T \Rightarrow P'$ by testing $F_1 \wedge T \wedge \bar{P}'$ for satisfiability:

$$(\bar{x}_1 \vee x_2) \wedge \bar{x}_1 \wedge \bar{x}_2 \wedge T \wedge x'_1 \wedge \bar{x}'_2$$

Which is unsatisfiable because $\bar{x}_1 \wedge \bar{x}_2 \wedge (x_1 \vee x_2 \vee \bar{x}_1') \wedge x_1'$ is not satisfiable. We again do not generate a proof-obligation, so that we continue with the second propagation-phase.

5. Step: Second Propagation-Phase
Initialize $F_2 = P$
For every clause c in F_1 check $F_1 \wedge T \wedge \bar{c}'$ for satisfiability, again skipping P.

- $c = \bar{x}_1$

  $$(\bar{x}_1 \vee x_2) \wedge \bar{x}_1 \wedge \bar{x}_2 \wedge T \wedge x_1'$$

  Unsatisfiable because $\bar{x}_1 \wedge \bar{x}_2 \wedge (x_1 \vee x_2 \vee \bar{x}_1') \wedge x_1'$ is not satisfiable
  $\rightarrow$ Add $\bar{x}_1$ to F_2
  $\rightarrow F_2 = P \wedge \bar{x}_1$

- $c = \bar{x}_2$

  $$(\bar{x}_1 \vee x_2) \wedge \bar{x}_1 \wedge \bar{x}_2 \wedge T \wedge x_2'$$

  Unsatisfiable because $\bar{x}_2 \wedge (x_2 \vee \bar{x}_2') \wedge x_2'$ is not satisfiable.
  $\rightarrow$ Add $\bar{x}_2$ to F_2
  $\rightarrow F_2 = P \wedge \bar{x}_1 \wedge \bar{x}_2$

With that the second propagation-phase ends, resulting in

$$F_2 = (\bar{x}_1 \vee x_2) \wedge \bar{x}_1 \wedge \bar{x}_2 \equiv F_1$$

The algorithm terminates returning that the property always holds and $(\bar{x}_1 \vee x_2) \wedge \bar{x}_1 \wedge \bar{x}_2$ being an inductive invariant.

2.4 Possible Improvements

The most time consuming part of the algorithm is the solving of SAT-queries, the larger the query the more time it takes. To improve this there are several ways to keep SAT-queries small:

- *Generalization of States*
  Blocking one state at a time is ineffective.

When blocking a state s do not add $\bar{s}$ but try to find and add a cube $c \subseteq \bar{s}$.
Most modern SAT-solver not only return unsatisfiable but also a reason for it, either by an UNSAT-core or through a final conflict-clause. Both of them deliver information about which clauses were actually used in the proof. To find a c just remove unused clauses of s.

- *Ternary Simulation*
  To reduce proof-obligations it is possible to eliminate not needed state variables by checking a satisfying assignment using ternary simulation. Ternary logic extends the binary logic by introducing a new valuation: X, called unknown, and new rules:

$$\begin{aligned}(X \wedge false) &= false,\\ (X \wedge true) &= X,\\ (X \wedge X) &= X,\\ \bar{X} &= X\end{aligned}$$

To remove state variables, set one variable at a time to X and try to transition to a next state using the transition relation, the variable is needed if X propagates into the next state, if it does not remove the variable from the proof-obligation.

Reconsider the prior example's first blocking phase resulting in the proof-obligation$(q, 1)$ with bad state $q = x_1 \wedge \bar{x}_2 \wedge \bar{x}_3$, we now want to reduce that proof-obligation using ternary simulation:

First of all, the transition formula:

$$\underbrace{x_1 \wedge \bar{x}_2 \wedge \bar{x}_3}_{q} \wedge \underbrace{(x_1 \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T}$$

Now set $x_1 = X$:

$$\underbrace{X \wedge \bar{x}_2 \wedge \bar{x}_3}_{q} \wedge \underbrace{(X \vee \bar{x}_2') \wedge (\bar{x}_1 \vee x_2') \wedge (x_2 \vee \bar{x}_3') \wedge (\bar{x}_2 \vee x_3')}_{T}$$

$(X \vee \bar{x}_2')$ is unknown meaning that x_1 is needed.

Now set $x_2 = X$:

$$\underbrace{x_1 \wedge X \wedge \bar{x}_3}_{q} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (X \vee \bar{x}'_3) \wedge (X \vee x'_3)}_{T}$$

$(X \vee \bar{x}'_3)$ is unknown meaning that x_2 is needed as well.

Now set $x_3 = X$:

$$\underbrace{x_1 \wedge \bar{x}_2 \wedge X}_{q} \wedge \underbrace{(x_1 \vee \bar{x}'_2) \wedge (\bar{x}_1 \vee x'_2) \wedge (x_2 \vee \bar{x}'_3) \wedge (\bar{x}_2 \vee x'_3)}_{T}$$

Because there is no clause being unknown, x_3 can be removed from the proof-obligation. We get the reduced proof-obligation$(x_1 \wedge \bar{x}_2, 1)$

3 Lifted PDR

We see that PDR is a useful hardware-model checking technique. If we want to use it on software, we need to *lift* the algorithm from bit-level propositional logic to first-order logic. There are multiple ways to do that, the following approach is based on the technique described in [**?**].
To use PDR on software we first need some new definitions and other preliminaries.

3.1 Preliminaries

A control flow graph (CFG) $\mathcal{A} = (X, L, G, \ell_0, \ell_E)$ is a tuple, consisting of a finite set of variables X, a finite set of locations L, a finite set of transitions $G \subseteq L \times FO \times L$, FO being a quantifier free first-order logic formula over variables in X and $X' = \{x \in X \mid x' \in X'\}$, an initial location $\ell_0 \in L$, and an error location $\ell_E \in L$.

For example consider the CFG $\mathcal{A} = (X, L, G, \ell_0, \ell_E)$ where
$X = \{x\}$,
$L = \{\ell_0, \ell_1, \ell_2, \ell_3, \ell_E\}$,
$G = \{(\ell_0, x' := 0, \ell_1), (\ell_1, x' := x + 1, \ell_2), (\ell_2, x = 1, \ell_E), (\ell_2, x \neq 1, \ell_3)\}$
with the graph:

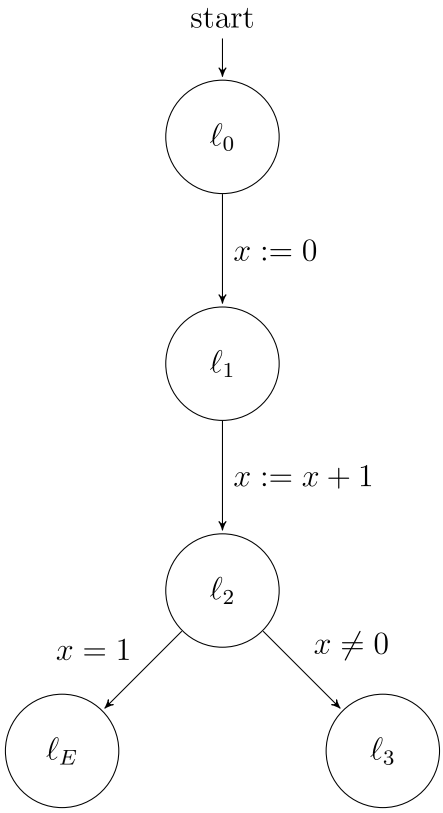


Figure 3: Graph of $\mathcal{A}$

The transition formula $T_{\ell_1 \to \ell_2}$ from one location ℓ_1 to another location ℓ_2 is defined as:

$$T_{\ell_1 \to \ell_2} = \begin{cases} (\ell_1, t, \ell_2), & (\ell_1, t, \ell_2) \in G \\ false, & otherwise \end{cases}$$

Resulting in the global transition formula:

$$T = \bigvee_{(\ell_1, t, \ell_2) \in G} T_{\ell_1 \to \ell_2}$$

The lifted algorithm no longer works on boolean transition systems but on CFGs. It tries to prove whether ℓ_E is reachable, by finding a feasible path from ℓ_0 to ℓ_E.

3.2 Lifted Algorithm

There are four main differences between bit-level PDR and lifted PDR:

- Instead of a global set of Frames $[F_0, ..., F_k]$ assign each program location $\ell \in L \backslash \{\ell_E\}$ a local set of frames $[F_{0,\ell}, ..., F_{k,\ell}]$. Each frame is now a cube of first-order formulas. As there are now multiple traces, proof-obligations get extended by another parameter, lifted proof-obligations are tuples (t, ℓ, i) where t is a first-order formula, ℓ describes the location where t has to be blocked, and i is a frame number, called level in the lifted algorithm.
- Because the states in a CFG are no formulas, the lifted algorithm no longer blocks states but transitions, there are no bad states only bad transitions.
- Because of the structure of the CFA, it is already known which states lead to the error location, as it is easy to extract the transitions in G that have ℓ_E as target, making the next transition phase, that was used to find proof-obligations before, obsolete.
  If there exists a transition to ℓ_E there will be an initial proof-obligation in each iteration of the algorithm, making the blocking-phase no longer optional.
- The propagation-phase is slimmed, it only checks for termination. In the phase the algorithm checks the frames to find a level i where all locations have a fixpoint, meaning $F_{i,\ell} = F_{i-1,\ell}$ for every location $\ell \in L \backslash \{l_E\}$, i is called a global fixpoint position. There is no more propagating formulas forward.

In more detail:
Given a CFG $\mathcal{A} = (X, L, G, \ell_0, \ell_E)$ we want to check if ℓ_E is reachable:

Again start with checking for a 0-counter-example, this is easily done by looking at ℓ_0. If $\ell_0 = \ell_E$ terminate and return that ℓ_E is indeed reachable, if $\ell_0 \neq \ell_E$ initialize level 0 frames for all locations $\ell \in L \backslash \{\ell_0, \ell_E\}$ as `false`, and for ℓ_0 as `true`.

Let k be the current level, so that each location $\ell \in L \backslash \{\ell_E\}$ has frames $[F_{0,\ell}, ..., F_{k,\ell}]$.
The algorithm repeats the following phases:

1. Next Level
Initialize for each $\ell \in L\backslash\{\ell_E\}$ a new frame $k+1$ as `true`.
For each location $\ell \in L$ where $(\ell, t, \ell_E) \in G$ generate an initial proof-obligation (t, ℓ, k).

2. Blocking-Phase
If there are proof-obligations:
Take proof-obligation (t, ℓ, i) with the lowest i and check for each predecessor location ℓ_{pre} if the formula:

$$F_{i-1,\ell_{pre}} \wedge T_{\ell_{pre} \to \ell} \wedge t'$$

is satisfiable.

- If it is *satisfiable*, it means that t could not be blocked at ℓ on level i, generate an new proof-obligation $(p, \ell_{pre}, i-1)$ where p is the weakest precondition of t.
- If the formula is *unsatisfiable*, strengthen each frame $F_{j,\ell}$, $j \leq i$ with $\bar{t}$, meaning $F_{j,\ell} = F_{j,\ell} \wedge \bar{t}$, blocking t at ℓ on level i.

This continues recursively until either a proof-obligation $(d, \ell, 0)$ is generated, proving that there exists a feasible path to ℓ_E terminating the algorithm, or there is no proof-obligation left.

3. Propagation-Phase
Check the frames if there exists a global fixpoint position i where

$$F_{i-1,\ell} = F_{i,\ell}$$

for every location $\ell \in L\backslash\{l_E\}$.
If there is such an i the algorithm terminates returning that ℓ_E is not reachable.

To illustrate the lifted algorithm further consider the updated pseudo-code:

Algorithm 2 lifted-PDR-prove

procedure LIFTED-PDR-PROVE(L, G)
    check for 0-counter-example
    $\ell_0.trace.push(new\ frame(true))$
    for all $\ell \in L \backslash \{\ell_0, \ell_E\}$ **do**
        $\ell.trace.push(new\ frame(false))$
    $level := 0$

    loop
        for all $\ell \in L \backslash \{\ell_E\}$ **do**
            $\ell.trace.push(new\ frame(true))$
        $level := level + 1$
        get initial proof-obligations

        while $\exists$ proof-obligation (t, ℓ, i), **do**
            Recursively block proof-obligation
            if a proof-obligation$(p, \ell, 0)$ is generated **then**
                return false

        for $i = 0;\ i \leq level;\ i := i + 1$ **do**
            for $\ell \in L \backslash \{l_E\}$ **do**
                if $\ell.trace[i] \neq \ell.trace[i-1]$ **then**
                    break
            return true

3.3 Example

3.3.1 Reachable Error State

To show an application of the lifted algorithm reconsider the example from earlier, we have CFA $\mathcal{A} = (X, L, G, \ell_0, \ell_E)$ with:
$X = \{x\}$
$L = \{\ell_0, \ell_1, \ell_2, \ell_3, \ell_E\}$,
$G = \{(\ell_0, x := 0, \ell_1), (\ell_1, x := x + 1, \ell_2), (\ell_2, x = 1, \ell_E), (\ell_2, x \neq 1, \ell_3)\}$

We now want to verify whether ℓ_E is reachable, using the lifted algorithm:

1. Step: Check for 0-Counter-Example
Is $\ell_0 = \ell_E$?
No, we continue with initializing level 0 by adding to each $\ell \in L \backslash \{\ell_0, \ell_E\}$ a new frame $F_{0,\ell} = false$, for ℓ_0 add $F_{0,\ell_0} = true$:

location \ level	0
ℓ_0	$true$
ℓ_1	$false$
ℓ_2	$false$
ℓ_3	$false$

2. Step: Next Level
Initialize new frames for level 1 as `true`:

location \ level	0	1
ℓ_0	$true$	$true$
ℓ_1	$false$	$true$
ℓ_2	$false$	$true$
ℓ_3	$false$	$true$

To generate the initial proof-obligations, check G and take the transitions where ℓ_E is the target.
There is one transition $(\ell_2, x = 1, \ell_E)$, that means we have to block $x = 1$ at ℓ_2 on level 1
$\rightarrow$ proof-obligation $(x = 1, \ell_2, 1)$

3. Step: First Blocking Phase
We need to block the initial proof-obligation $(x = 1, \ell_2, 1)$. Let ℓ_{pre} be a predecessor of ℓ_2, we need to check the formula $F_{0,l_{pre}} \wedge T_{\ell_{pre} \to \ell_2} \wedge x' = 1$ for satisfiability. As there is only one predecessor ℓ_1 we test:

$$\underbrace{false}_{F_{0,\ell_1}} \wedge \underbrace{x' := x + 1}_{T_{\ell_1 \to \ell_2}} \wedge x' = 1$$

Which is unsatisfiable
→ Add $\overline{(x = 1)} \equiv x \neq 1$ to F_{0,ℓ_2} and F_{1,ℓ_2}, blocking $x = 1$ at ℓ_2 on level 1.

location \ level	0	1
ℓ_0	$true$	$true$
ℓ_1	$false$	$true$
ℓ_2	$false \wedge x \neq 1$	$true \wedge x \neq 1$
ℓ_3	$false$	$true$

Because there are no proof-obligations left we continue with the propagation-phase.

4. Step: First Propagation-Phase
Check if there exists a global fixpoint position i where

$$F_{i-1,\ell} = F_{i,\ell}$$

for every location $\ell \in L \backslash \{l_E\}$.
→ There is no such i, we continue with the next level.

5. Step: Next Level
Initialize new frames for level 2 as `true`:

location \ level	0	1	2
ℓ_0	$true$	$true$	$true$
ℓ_1	$false$	$true$	$true$
ℓ_2	$false \wedge x \neq 1$	$true \wedge x \neq 1$	$true$
ℓ_3	$false$	$true$	$true$

Again generate the initial proof-obligation which is the same as before but on level 2 now:
→ proof-obligation $(x = 1, \ell_2, 2)$

6. Step: Second-Blocking Phase

We need to block the proof-obligation $(x = 1, \ell_2, 2)$ by testing

$$\underbrace{true}_{F_{1,\ell_1}} \wedge \underbrace{x' := x + 1}_{T_{\ell_1 \to \ell_2}} \wedge x' = 1$$

for satisfiability. Which is satisfiable with $p = (x = 0)$. Because p being also the weakest precondition, we generate a new proof-obligation $(p, \ell_1, 1)$, meaning we need to block p at location ℓ_1 on level 1.

Take the new proof-obligation $(x = 0, \ell_1, 1)$ and check

$$\underbrace{true}_{F_{0,\ell_0}} \wedge \underbrace{x' := 0}_{T_{\ell_0 \to \ell_1}} \wedge \underbrace{x' = 0}_{p'}$$

for satisfiability.

Which is valid, with `true` being the weakest precondition, we generate the new proof-obligation $(true, l_0, 0)$ and because this obligation is on level 0 we terminate, stating that ℓ_E is reachable by the counter-example trace:

$$\ell_0 \to \ell_1 \to \ell_2 \to \ell_E$$

3.3.2 Unreachable Error State

To show a CFA with an unreachable error state consider $\mathcal{B} = (X, L, G, \ell_0, \ell_E)$ with:
$X = \{x, y\}$
$L = \{\ell_0, \ell_1, \ell_2, \ell_E\}$
$G = \{(\ell_0, x' := 0 \wedge y' := x', \ell_1), (\ell_1, x' := x + 1 \wedge y' := y + 1, \ell_1),$
$(\ell_1, x = y, \ell_2), (\ell_1, x \neq y, \ell_E)\}$
with graph:

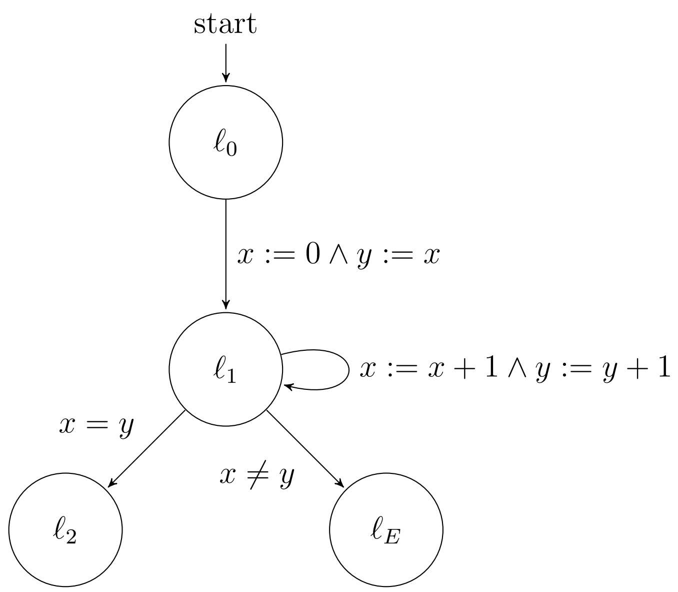


Figure 4: Graph of $\mathcal{B}$

We now want to check whether ℓ_E is reachable, using the lifted algorithm:

1. Step: Check for 0-Counter-Example
Is $\ell_0 = \ell_E$?
No, we continue with initializing level 0 by adding to each $\ell \in L \backslash \{\ell_0, \ell_E\}$ a new frame $F_{0,\ell} = false$, for ℓ_0 add $F_{0,\ell_0} = true$.

location \ level	0
ℓ_0	$true$
ℓ_1	$false$
ℓ_2	$false$

2. Step: Next Level
Initialize new frames for level 1 as `true`:

location \ level	0	1
ℓ_0	$true$	$true$
ℓ_1	$false$	$true$
ℓ_2	$false$	$true$

Generate the initial proof-obligations:
There is only one transition leading to ℓ_E, $(\ell_1, x \neq y, \ell_E)$
$\rightarrow$ proof-obligation $(x \neq y, \ell_1, 1)$.

3. Step: First Blocking Phase
To block the proof-obligation $(x \neq y, \ell_1, 1)$ check each predecessor of ℓ_1:

- predecessor: ℓ_0

  $$\underbrace{true}_{F_{0,\ell_0}} \wedge \underbrace{x' := 0 \wedge y' := x'}_{T_{\ell_0 \rightarrow \ell_1}} \wedge x' \neq y'$$

  Which is unsatisfiable
  $\rightarrow$ Add $\overline{(x \neq y)} \equiv x = y$ to F_{0,ℓ_1} and F_{1,ℓ_1}:

location \ level	0	1
ℓ_0	$true$	$true$
ℓ_1	$false \wedge x = y$	$true \wedge x = y$
ℓ_2	$false$	$true$

- predecessor: ℓ_1

  $$\underbrace{false \wedge x = y}_{F_{0,\ell_1}} \wedge \underbrace{x' := x + 1 \wedge y' := y + 1'}_{T_{\ell_1 \rightarrow \ell_1}} \wedge x' \neq y'$$

Which is unsatisfiable as well
$\rightarrow$ Because $x = y$ has already been added to F_{0,ℓ_1} and F_{1,ℓ_1} we move on.

As there are no proof-obligations left, we continue with the first propagation-phase.

4. Step: First Propagation-Phase
Check if there exists a global fixpoint position i where

$$F_{i-1,\ell} = F_{i,\ell}$$

for every location $\ell \in L \backslash \{l_E\}$.
$\rightarrow$ There is no such i, we continue with the next level.

5. Step: Next Level
Initialize new frames for level 2 as `true`:

location \ level	0	1	2
ℓ_0	$true$	$true$	$true$
ℓ_1	$false \wedge x = y$	$true \wedge x = y$	$true$
ℓ_2	$false$	$true$	$true$

Again generate the initial proof-obligation which is the same as before but on level 2 now:
$\rightarrow$ proof-obligation $(x \neq y, \ell_1, 2)$

6. Step: Second Blocking Phase
To block proof-obligation $(x \neq y, \ell_1, 2)$ we check the predecessors of ℓ_1:

- predecessor: ℓ_0

$$\underbrace{true}_{F_{1,\ell_0}} \wedge \underbrace{x' := 0 \wedge y' := x'}_{T_{\ell_0 \rightarrow \ell_1}} \wedge x' \neq y'$$

Which is unsatisfiable
→ Add $\overline{(x \neq y)} \equiv x = y$ to F_{0,ℓ_1}, F_{1,ℓ_1} and F_{2,ℓ_1}:

location \ level	0	1	2
ℓ_0	$true$	$true$	$true$
ℓ_1	$false \wedge x = y$	$true \wedge x = y$	$true \wedge x = y$
ℓ_2	$false$	$true$	$true$

- predecessor: ℓ_1

$$\underbrace{true \wedge x = y}_{F_{1,\ell_1}} \wedge \underbrace{x' := x + 1 \wedge y' := y + 1}_{T_{\ell_1 \to \ell_1}} \wedge x' \neq y'$$

Which is unsatisfiable as well
→ Because $x = y$ has already been added to F_{0,ℓ_1}, F_{1,ℓ_1}, and F_{2,ℓ_1} we move on.

As there are no proof-obligations left, we continue with the second propagation-phase

7. Step: Second Propagation-Phase

location \ level	0	1	2
ℓ_0	$true$	$true$	$true$
ℓ_1	$false \wedge x = y$	$true \wedge x = y$	$true \wedge x = y$
ℓ_2	$false$	$true$	$true$

We see that level 1 equals level 2 on all locations, with that we found global fixpoint position $i = 2$, the forumulas at that position are the inductive invariants proving that ℓ_E is not reachable.

3.4 Possible Improvements

As shown above, lifting PDR from bit-level to control flow graphs is possible. The problem of large, time consuming queries to the solver remain however.

Is it possible to lift the improvements of the bit-level algorithm too?

Ternary Simulation cannot be used on first-order formulas making it impossible to use it to reduce lifted proof-obligations.

Different generalization techniques are possible:

- *Syntactical Analysis*
  Given a cube c remove $a \subseteq c$, if no variable of a is assigned in T and

  1. a is already contained in a frame, or
  2. there exists an `assume a` in T

- *Weakest Precondition*
  The definition of the lifted algorithm above already contains an improvement, using weakest preconditions to find predecessors. Instead of generating multiple proof-obligations for each individual predecessor state, the weakest precondition covers all of them in a single one.

- *Using the Disjunctive Normal Form*
  After transforming the weakest precondition into its disjunctive normal form, each cube can be considered as a separate smaller proof-obligation, saving time on larger formulas.

- *Using Interpolation*
  Given a pair of formulas (A, B) where $A \wedge B$ is unsatisfiable, an interpolant I for (A, B) is a formula so that

  1. $A \Rightarrow I$
  2. $I \wedge B$ is unsatisfiable
  3. $vars(I) \subseteq vars(A) \cap vars(B)$

  After PDR has proven an error location unreachable, take the invariants of the locations that can lead to the error state and construct a

new interpolating trace. The invariants calculated by PDR are interpolants that are specific enough to prove one trace infeasible and at the same time are general enough to help proving other counter-example traces as infeasible.
Reconsider the unreachable example from before, a possible counter-example trace is:

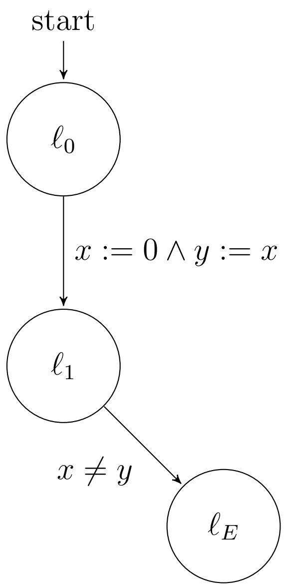


Figure 5: infeasible counter-example

location \ level	2
ℓ_0	*true*
ℓ_1	$true \wedge x = y$
ℓ_2	*true*

Table 1: invariants of the CFA

Using the calculated invariants generate a more general interpolating trace.

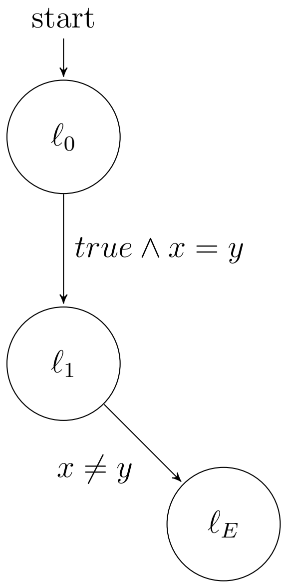


Figure 6: interpolating trace

location \ level	2
ℓ_0	$true$
ℓ_1	$true \wedge x = y$
ℓ_2	$true$

Table 2: invariants of the CFA

This generalized trace can help proving other counter-examples as infeasible.

The backwards-search nature of PDR does not get changed by using this form of interpolation as it is applied *after* PDR has proven an error location reachable or not.

4 Goals

At the moment Ultimate uses interpolation based model-checkers. This bachelor's thesis aims at implementing a new PDR-based approach and then comparing it with the existing ones. Furthermore should the correctness of this new approach be tested by unit-tests.

5 Approach

A bachelor's thesis takes 12 weeks of work:

1. Preplanning: Deciding what classes are needed, which parts of the papers are to be implemented, and so on.

   Duration: 1 week

   Outcome: Rough plan on how to implement PDR in Ultimate.

2. Implementing PDR: Implement the PDR-algorithm as described above with some changes to make it fit into the Ultimate framework.

   Duration: 4 weeks

   Outcome: A PDR based model-checking algorithm implemented in Ultimate.

3. Improving PDR: Adding some perfomance improving techniques as described in [?], [?], and [?] to the PDR-algorithm.

   Duration: 2 weeks

   Outcome: An improved PDR-algorithm, tuned for better performance.

4. Bugfixing: Finding and eliminating remaining bugs with help of unit tests.

   Duration: 1 week

   Outcome: A tested PDR-algorithm.

5. ANALYSIS: Comparing the implementation with the existing model-checkers performance-wise.

   Duration: 1 week

   Outcome: Data comparing model-checking methods.

6. WRITING THE THESIS: Writing down my results in a thesis. Proof-reading and printing it.

   Duration: 3 weeks

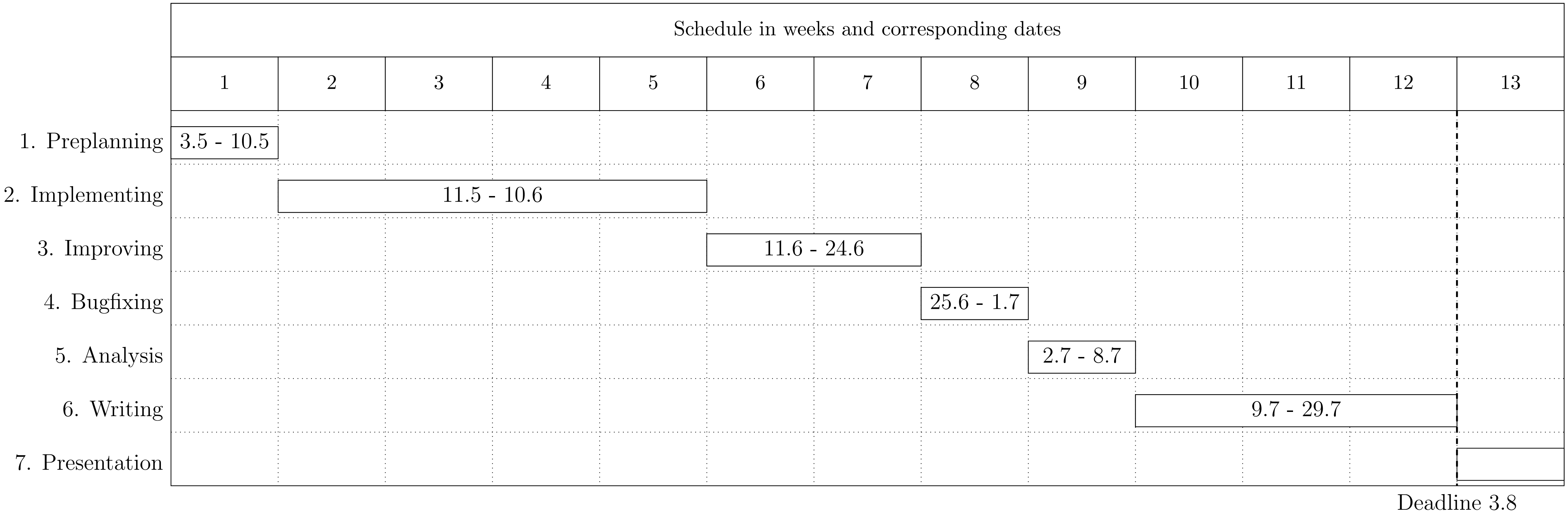
   Outcome: A Bachelor's thesis. Written and printed.

7. PREPARING A FINAL PRESENTATION: Preparing a presentation where I am able to show my results.

   Duration: 1 week   *Note: can be finished after deadline*

   Outcome: A presentation of my results of the previous points.

6 Schedule


Schedule in weeks and corresponding dates
1
2
3
4
5
6
7
8
9
10
11
12
13
1. Preplanning
3.5 - 10.5
2. Implementing
11.5 - 10.6
3. Improving
11.6 - 24.6
4. Bugfixing
25.6 - 1.7
5. Analysis
2.7 - 8.7
6. Writing
9.7 - 29.7
7. Presentation
Deadline 3.8

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