



# Introduction: Motivation

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- PDR was first devised as hardware verification technique in 2010 by Aaron Bradley<sup>1</sup>
  - ➔ Surprisingly won 3<sup>rd</sup> place at CAV 2010 hardware checking competition<sup>2</sup>

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**“This new method appears to be the most important contribution to bit-level formal verification in almost a decade”<sup>3</sup>**

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**“This new method appears to be the most important contribution to bit-level formal verification in almost a decade”<sup>3</sup>**

- Using PDR on software may have similar performance!

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# Introduction: Motivation

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➤ Our goals:

- Use PDR on software in the verification framework Ultimate<sup>1</sup>
  - ➔ Combining Trace Abstraction and PDR
  - ➔ Comparison to existing techniques

# Overview

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## ➤ How does our PDR algorithm work?

- Preliminaries
- Running Example
- Related Work

## ➤ How do we use PDR in Ultimate?

- Combination of Trace Abstraction and our PDR algorithm
- Implemented Improvements

# Overview

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## ➤ **Evaluation:**

- Comparison of Trace Abstraction using PDR and Trace Abstraction using Nested Interpolants

## ➤ **What can be done in the future?**

- Implementing more Improvements

# PDR Algorithm: Preliminaries

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- A control flow graph (CFG)  $A = (X, L, E, \ell_0, \ell_E)$  is a graph consisting of
- A finite set of first-order variables  $X$
  - A finite set of locations  $L$
  - A finite set of transitions  $E \subseteq L \times FO \times L$ 
    - ➔  $FO$  is a quantifier free first-order logic formula over variables in  $X$  and  $X' = \{x \in X \mid x' \in X'\}$
  - An initial location  $\ell_0 \in L$
  - An error location  $\ell_E \in L$



# PDR Algorithm: Datastructures

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➤ Frame  $F_{i,\ell}$  :

- Represents a first-order formula
  - $\ell$  is the corresponding location
  - $i$  is the corresponding level
- ➔ Each location has multiple assigned frames

➤ Proof-Obligation  $(p, \ell, i)$  :

- $p$  is a first-order formula
  - $\ell$  is the corresponding location
  - $i$  is the corresponding level
- ➔ Need to be blocked

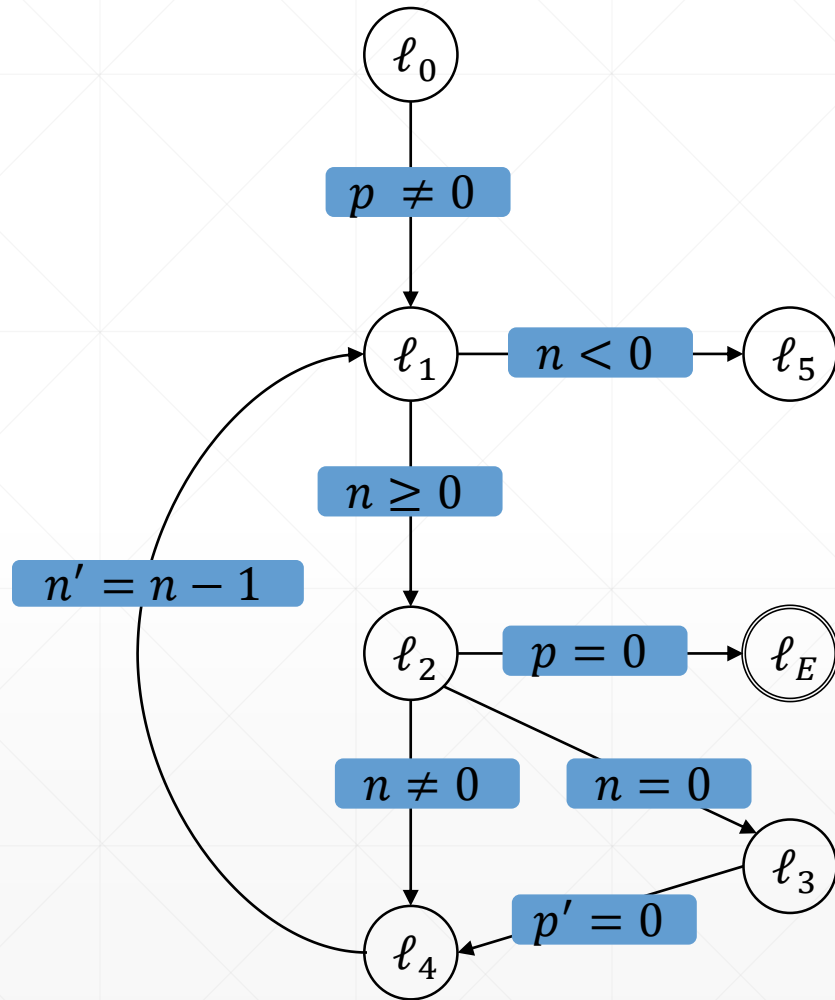
# PDR Algorithm: Description

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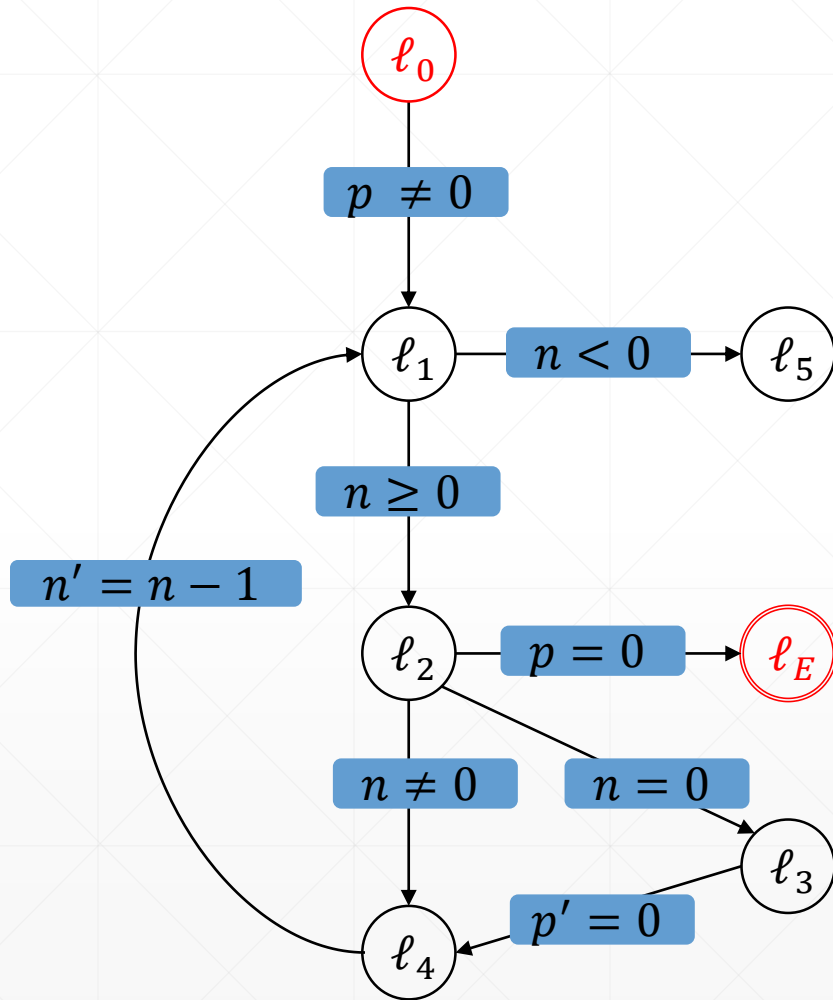
- Starts with checking for a 0-Counter-Example
- Repeats three phases until termination:
  1. Next Level Initialization Phase
  2. Blocking-Phase
  3. Propagation-Phase

## Example: Running Example

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## Example:

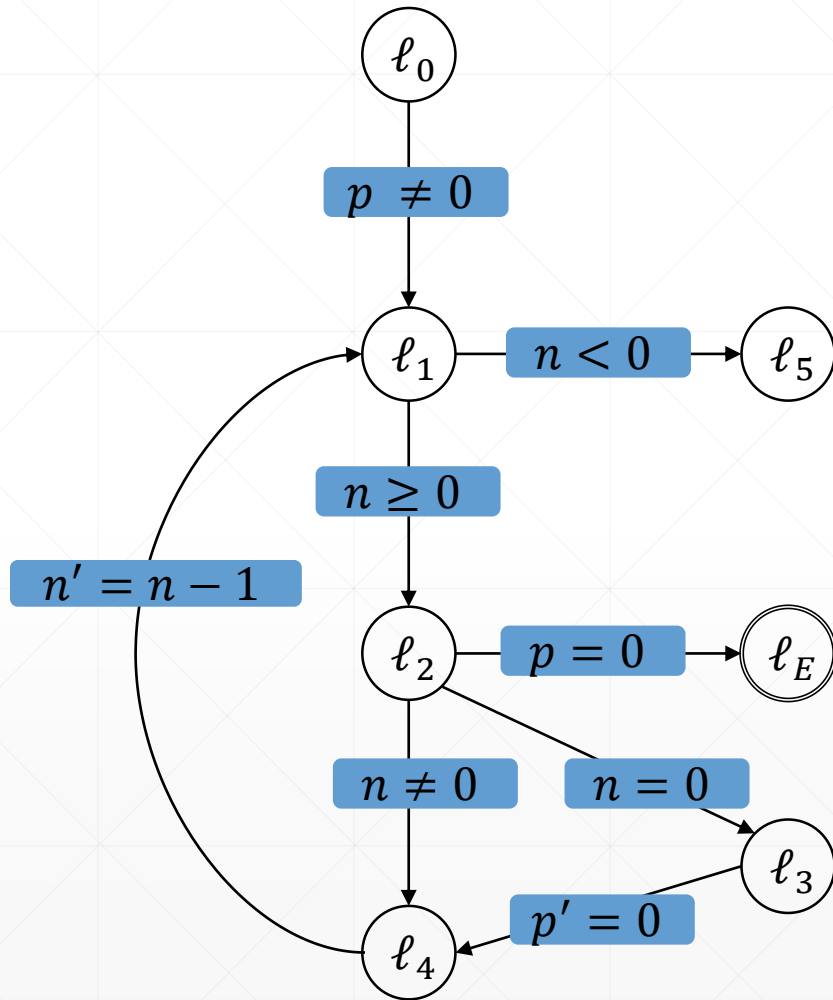


### 1. Step: Check for 0-Counter-Example

➤ Is  $\ell_0 = \ell_E$  ?

➔ No, continue with initialization

## Example:

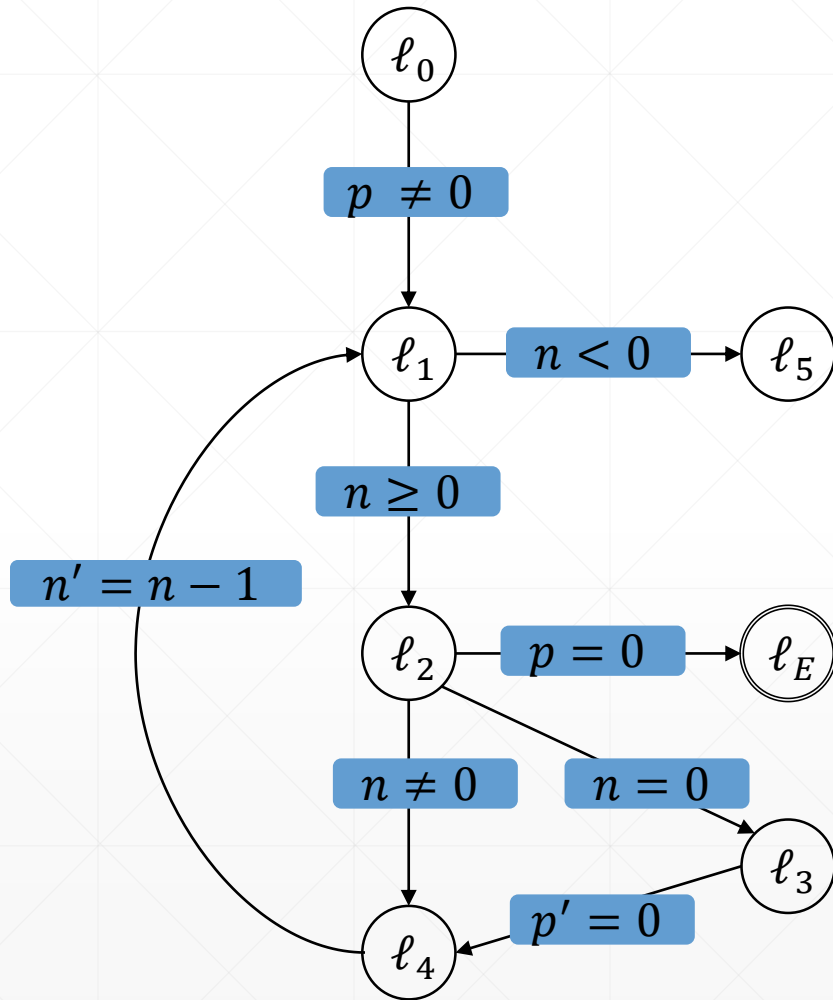


location	0
$\ell_0$	
$\ell_1$	
$\ell_2$	
$\ell_3$	
$\ell_4$	

### 2. Step: Initialization of level 0

$$\triangleright F_{0,\ell} = \begin{cases} \text{T}, & \ell = \ell_0 \\ \text{F}, & \text{otherwise} \end{cases}$$

## Example:

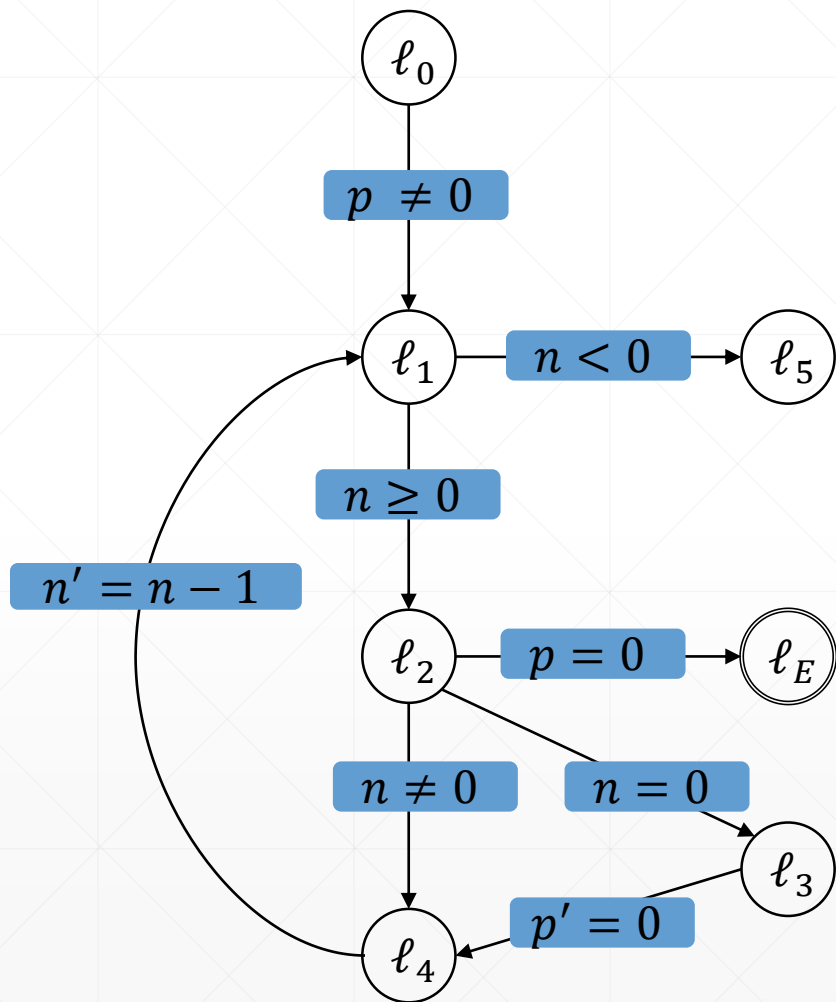


location	0
$\ell_0$	$t$
$\ell_1$	$f$
$\ell_2$	$f$
$\ell_3$	$f$
$\ell_4$	$f$

2. Step: Initialization of level 0

$$\triangleright F_{0,\ell} = \begin{cases} T, & \ell = \ell_0 \\ F, & \text{otherwise} \end{cases}$$

# Example:

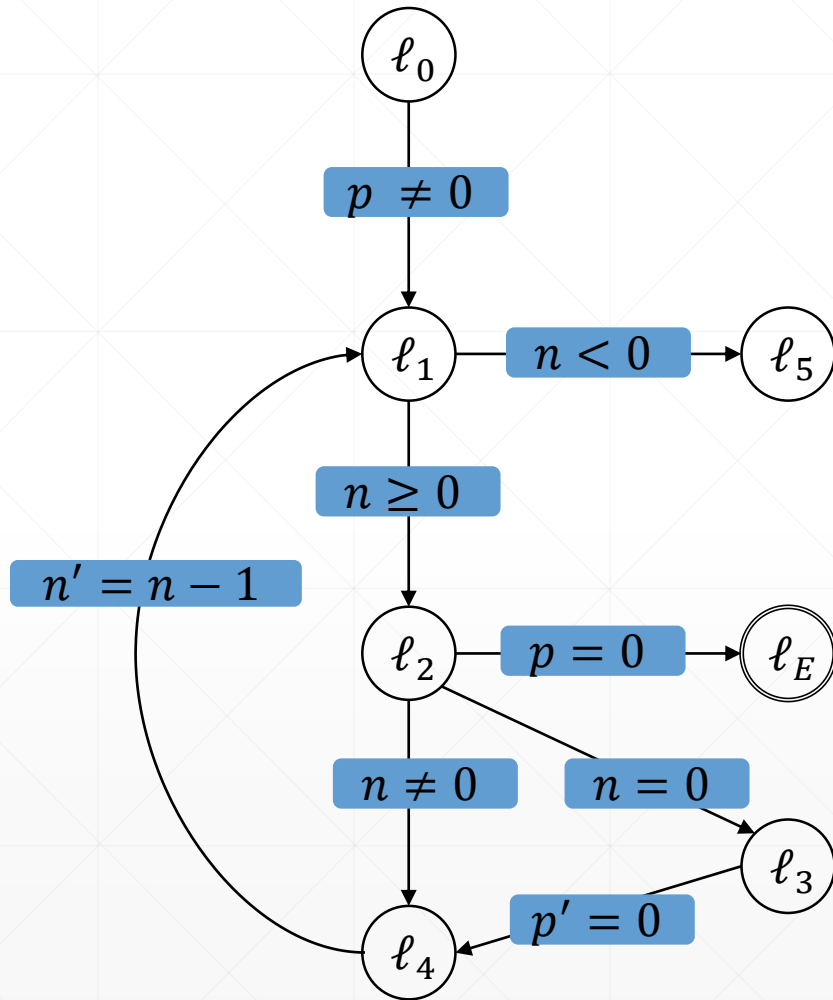


location	0	1
$\ell_0$	$t$	
$\ell_1$	$f$	
$\ell_2$	$f$	
$\ell_3$	$f$	
$\ell_4$	$f$	

## 3. Step: Level 1

➤ Initialize level 1 frames as true

## Example:



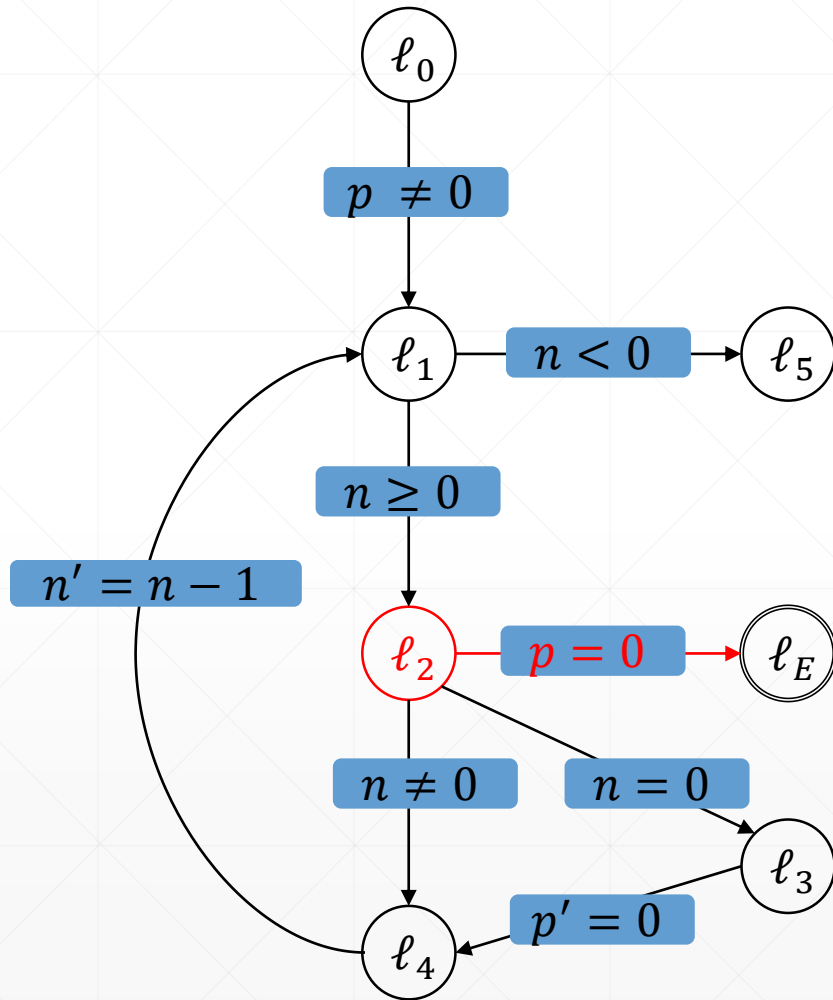
location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 3. Step: Level 1

- Initialize level 1 frames as true



## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

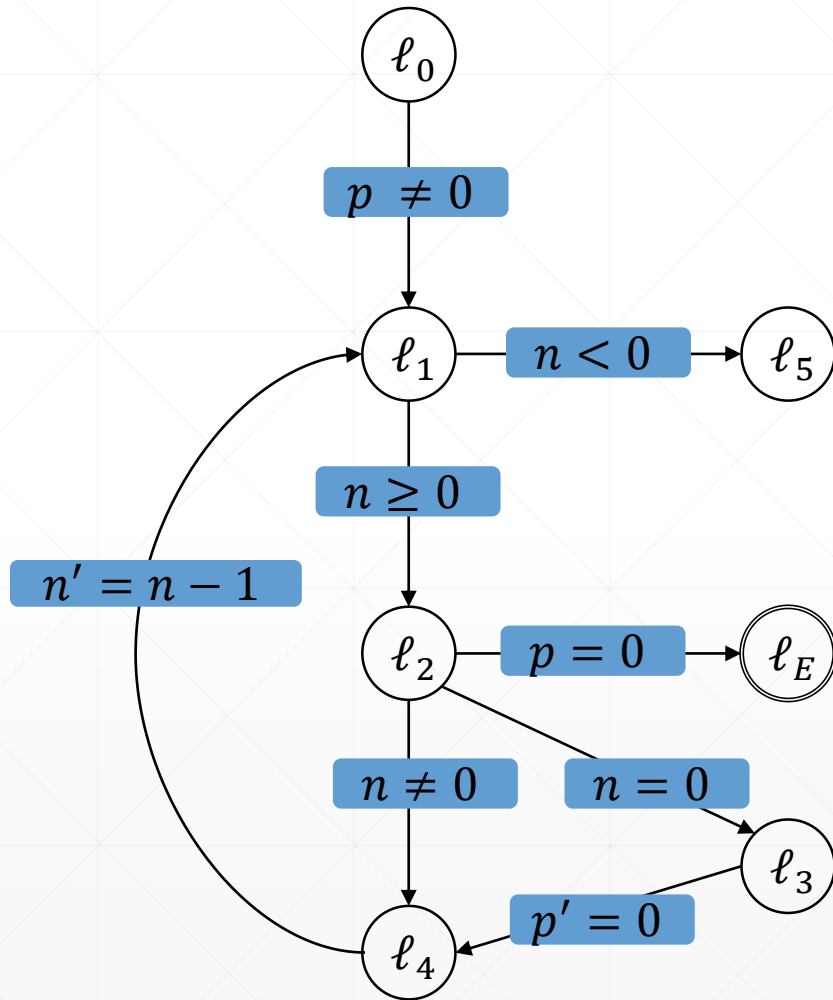
### 3. Step: Level 1

➤ Get initial proof-obligation

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

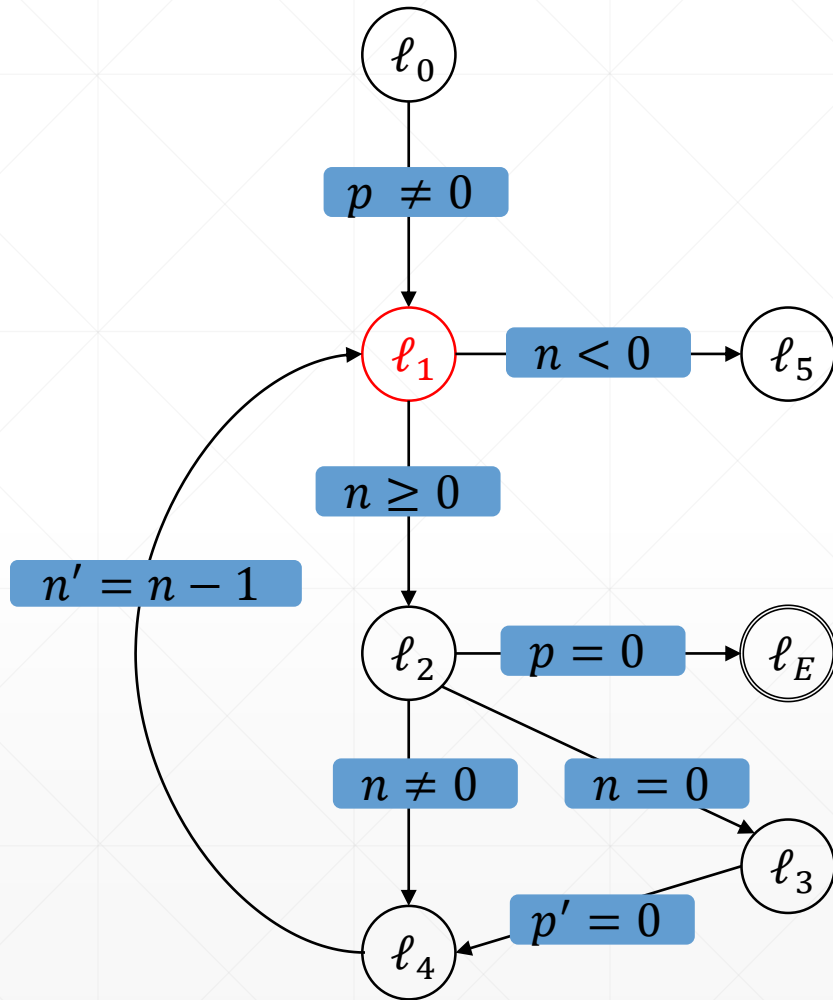
### 4. Step: Level 1 Blocking-Phase:

➤ Try to block  $(p = 0, \ell_2, 1)$

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

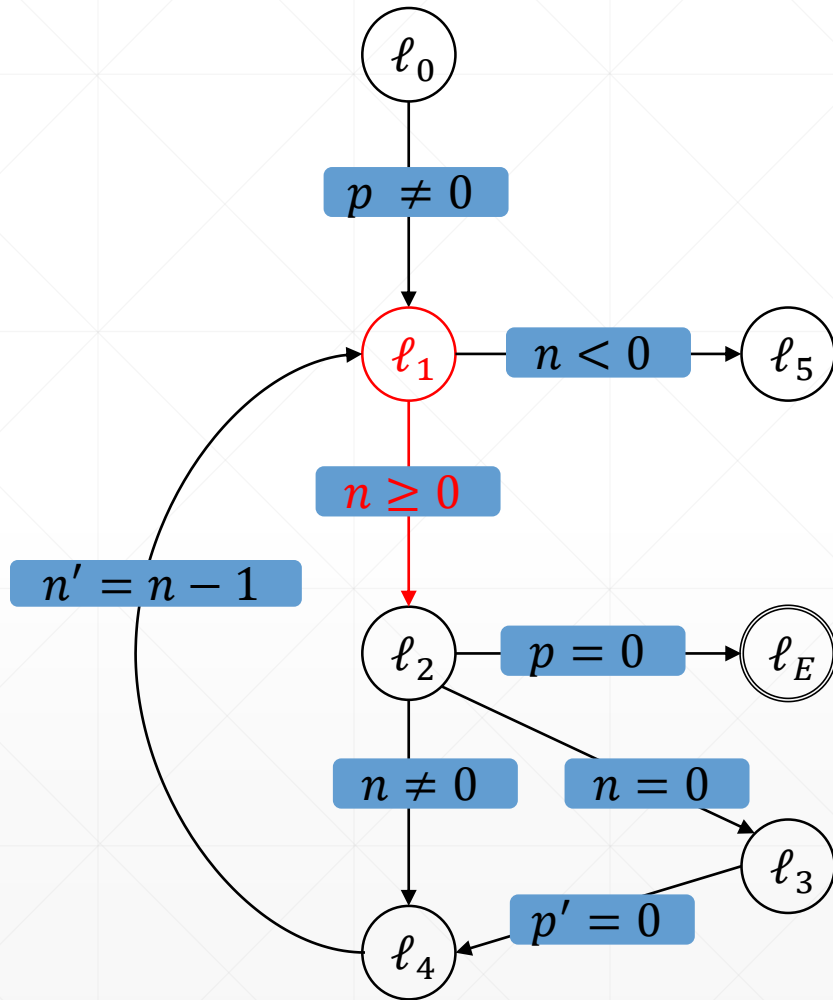
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $F_{0,\ell_1} \wedge T_{\ell_1 \rightarrow \ell_2} \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

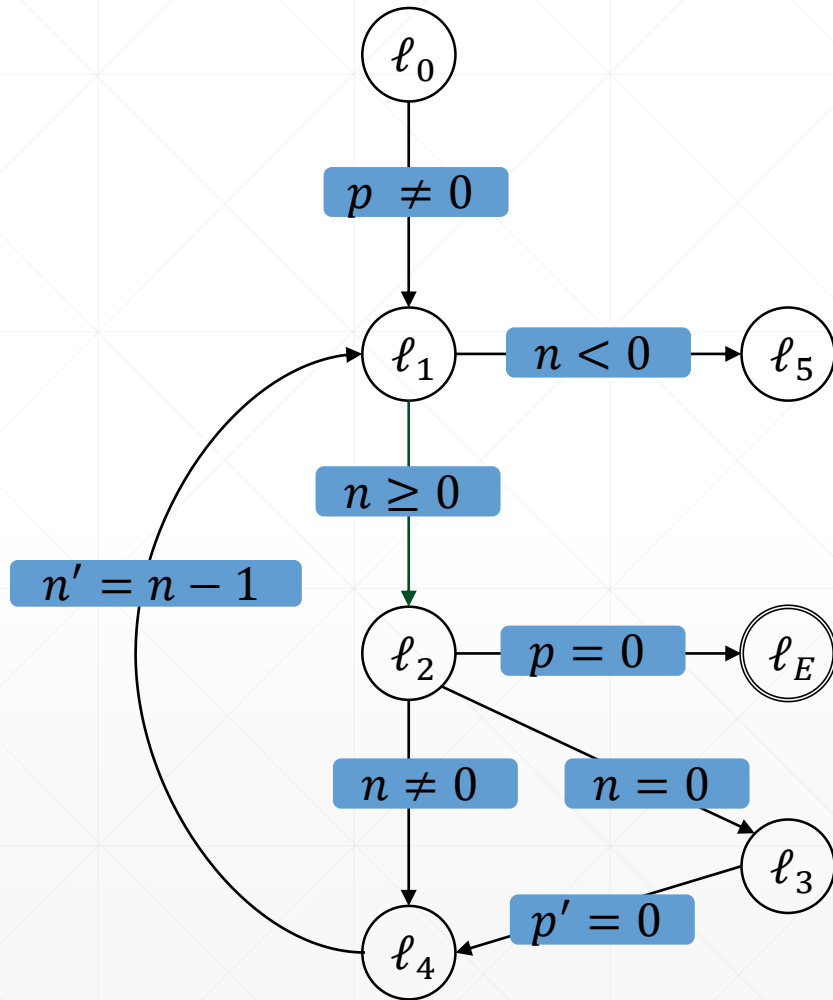
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $f \wedge n \geq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 1)$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f$	$t$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

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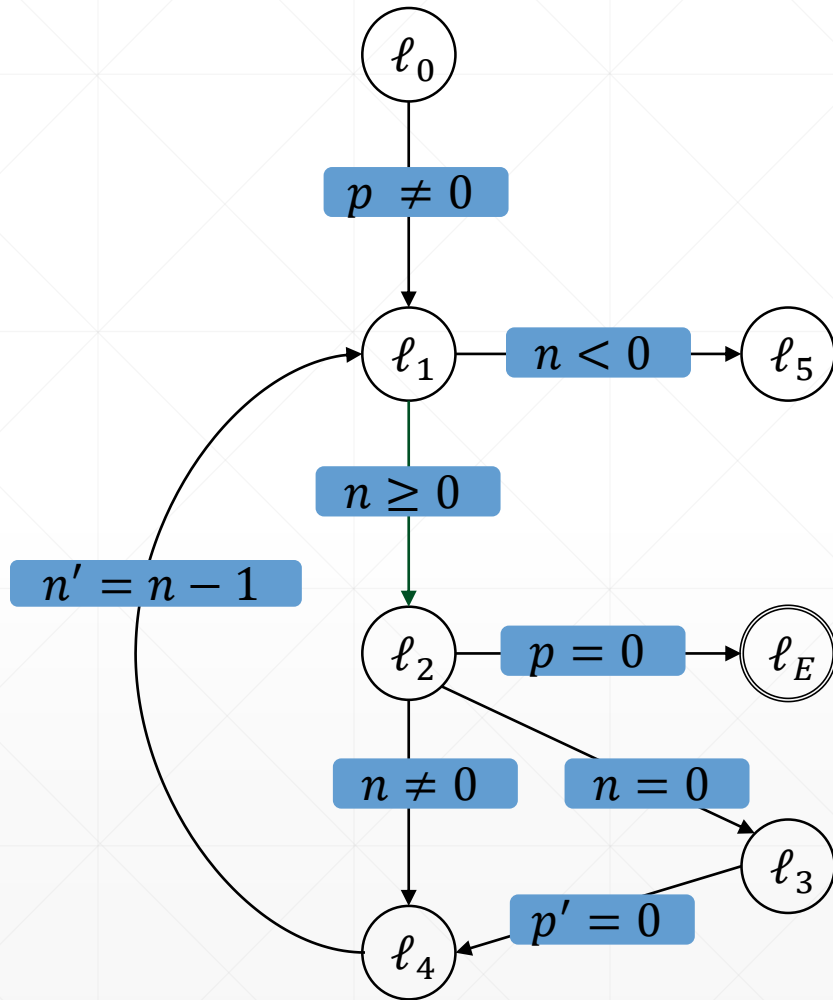
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $f \wedge n \geq 0 \wedge p' = 0$
  - ➔ **Unsatisfiable**
  - ➔ Strengthen frames  $F_{0,\ell_2}, F_{1,\ell_2}$

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 4. Step: Level 1 Blocking-Phase:

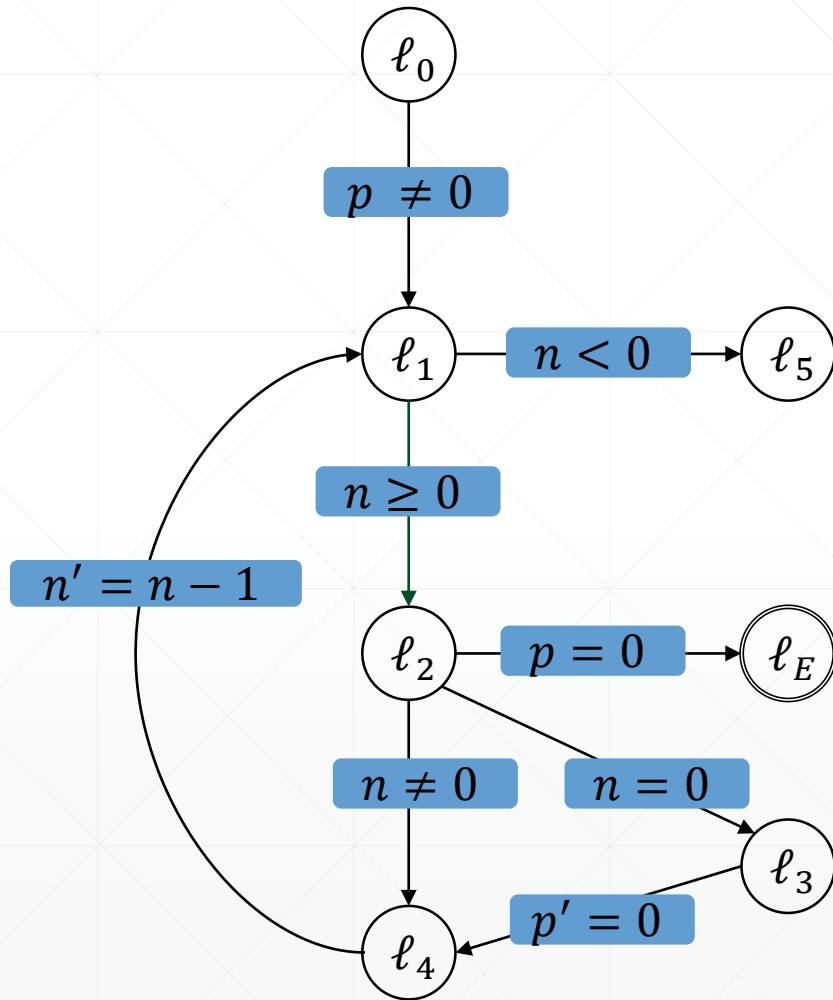
➤ Try to block  $(p = 0, \ell_2, 1)$

- Predecessor  $\ell_1$ :
  - $f \wedge n \geq 0 \wedge p' = 0$
  - ➔ Unsatisfiable
  - ➔ **Strengthen** frames  $F_{0,\ell_2}, F_{1,\ell_2}$

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

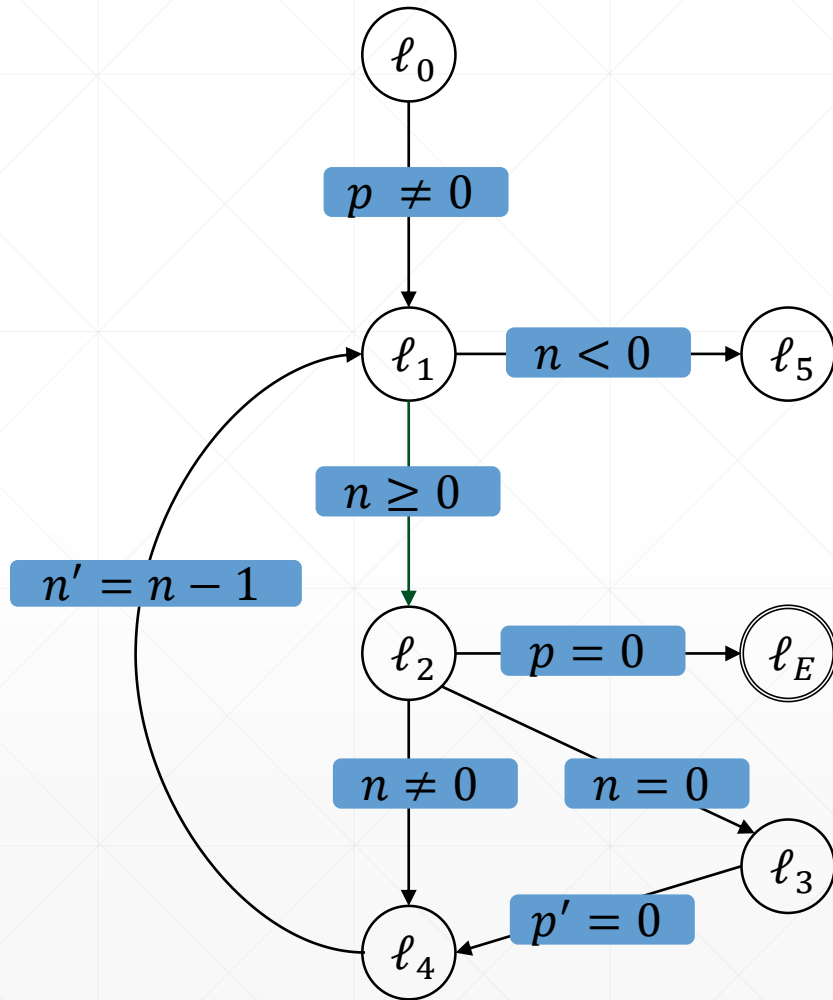
### 5. Step: Level 1 Propagation-Phase

➤ Is there a global fixpoint?

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 5. Step: Level 1 Propagation-Phase

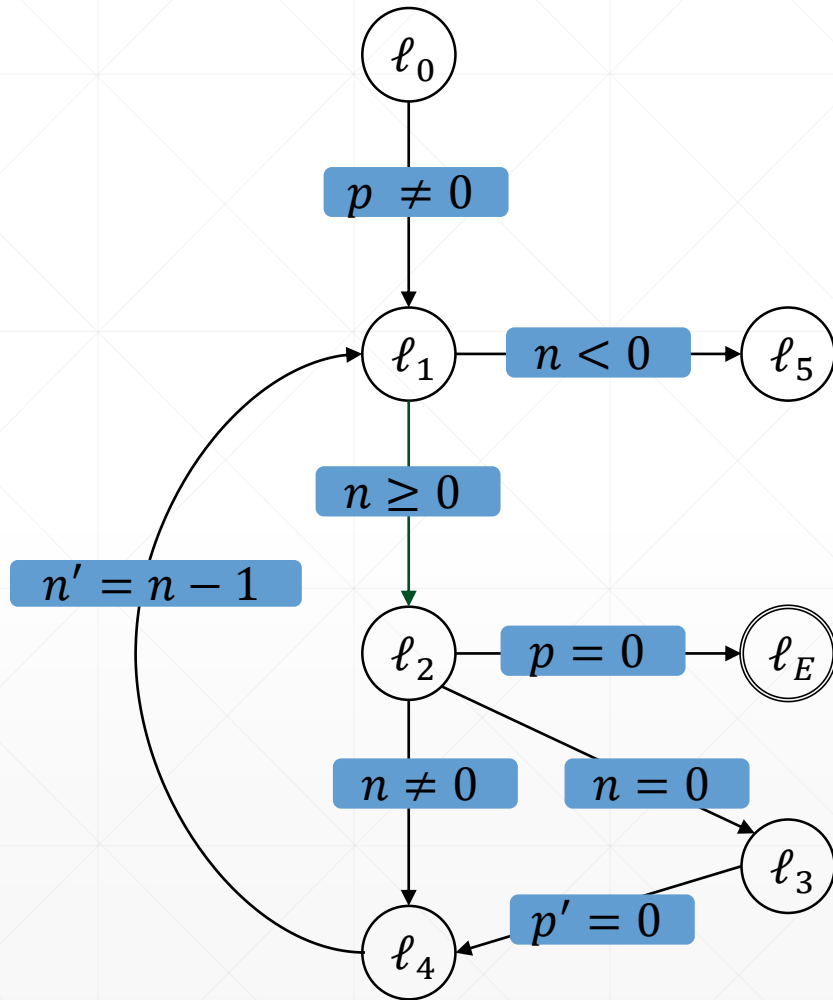
➤ Is there an  $i$  where  $F_{i-1,\ell} = F_{i,\ell}$  for  $\ell \in L \setminus \{\ell_E\}$ ?

### Proof-Obligations:

- $\emptyset$



## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

### 5. Step: Level 1 Propagation-Phase

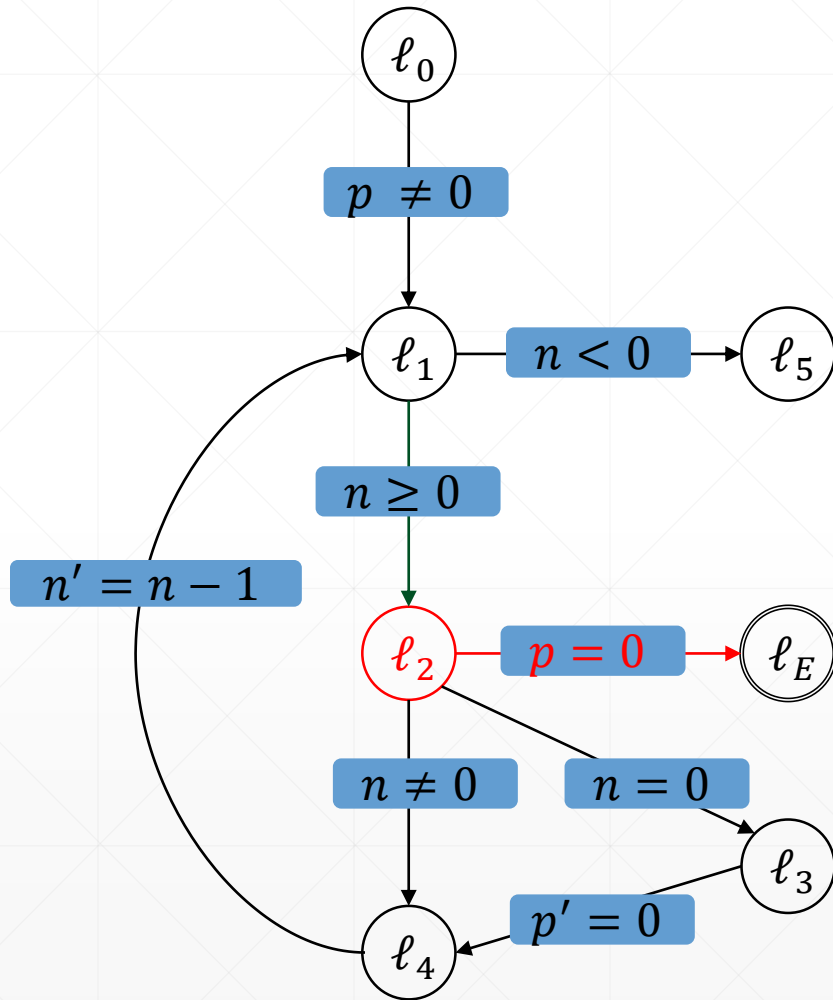
➤ Is there an  $i$  where  $F_{i-1,\ell} = F_{i,\ell}$  for  $\ell \in L \setminus \{\ell_E\}$ ?

➔ No. Continue with next level.

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1
$\ell_0$	$t$	$t$
$\ell_1$	$f$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$
$\ell_4$	$f$	$t$

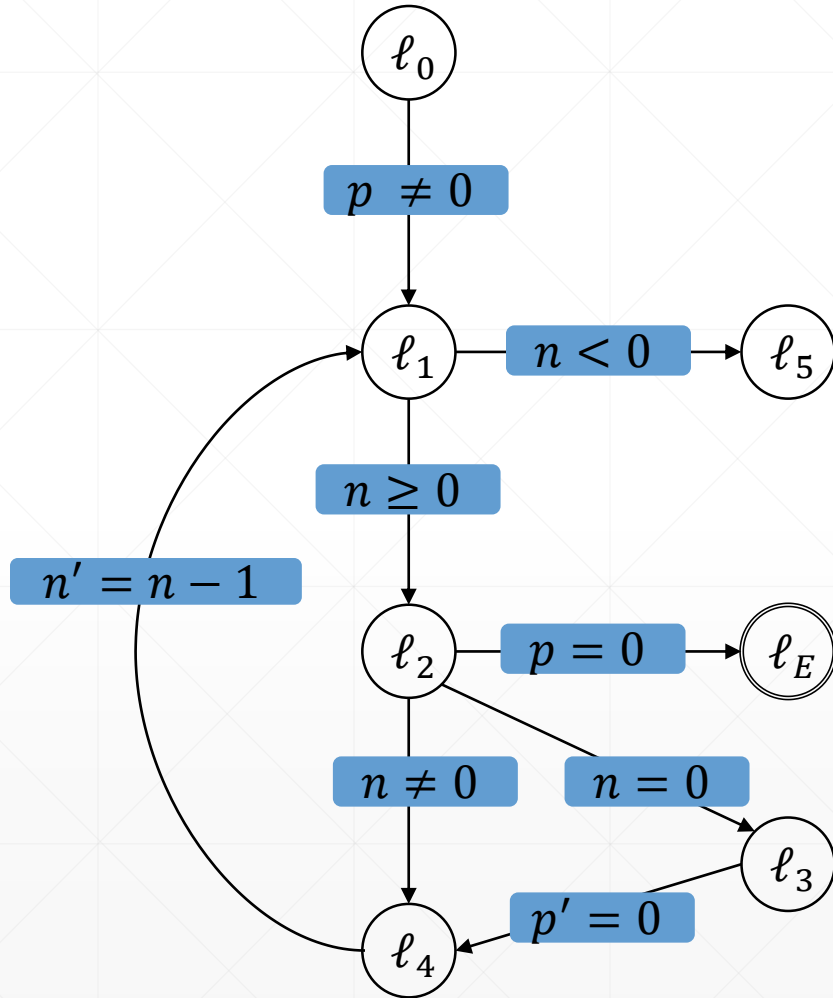
### 6. Step: Level 2

- Initialize new frames
- Add initial proof-obligation  
( $p = 0, \ell_2, 2$ )

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

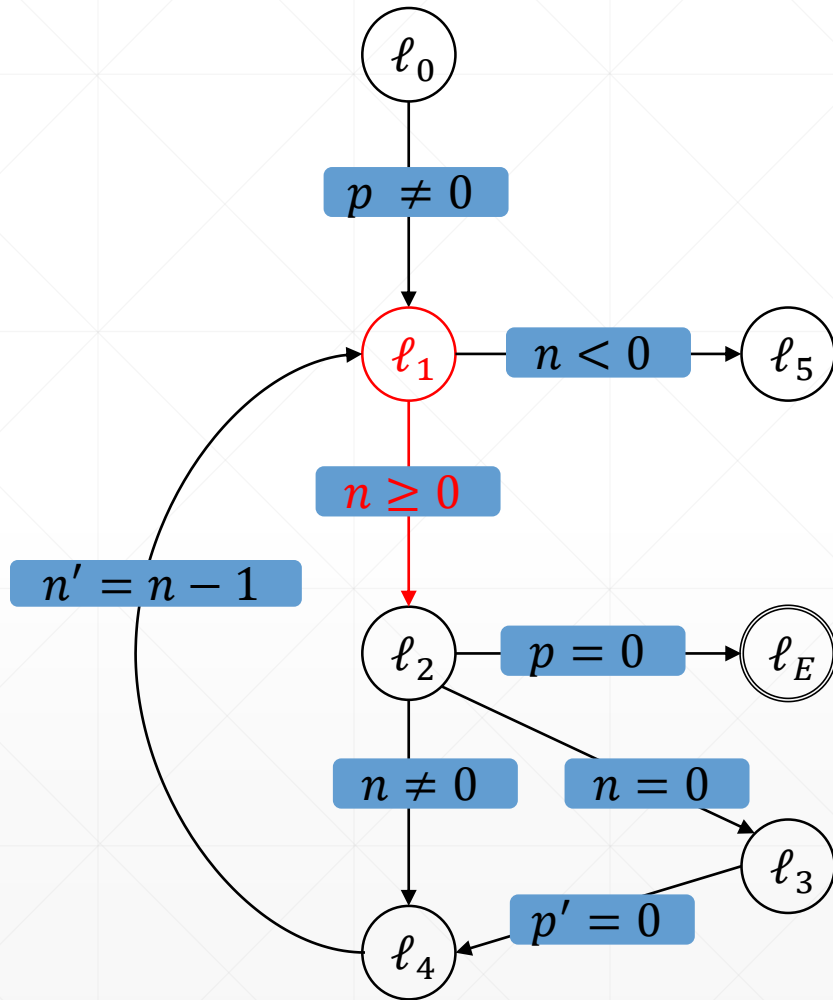
### 6. Step: Level 2

- Initialize new frames
- Add initial proof-obligation  $(p = 0, \ell_2, 2)$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

### 7. Step: Level 2 Blocking-Phase:

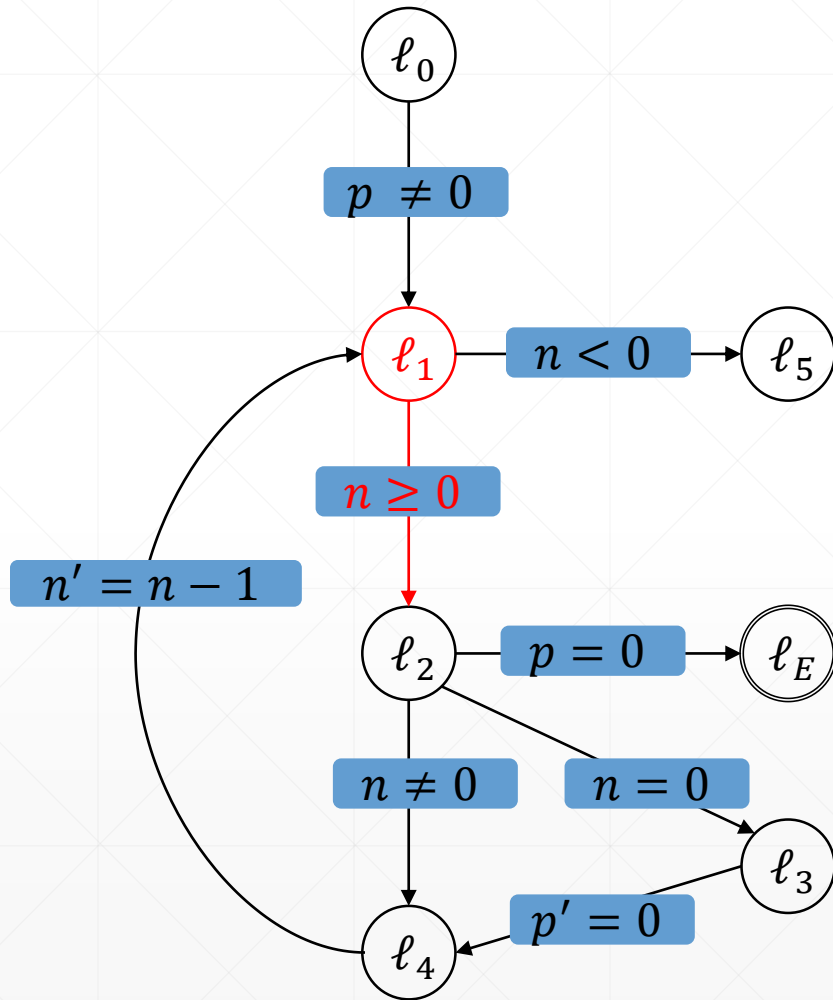
➤ Try to block  $(p = 0, \ell_2, 2)$

- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
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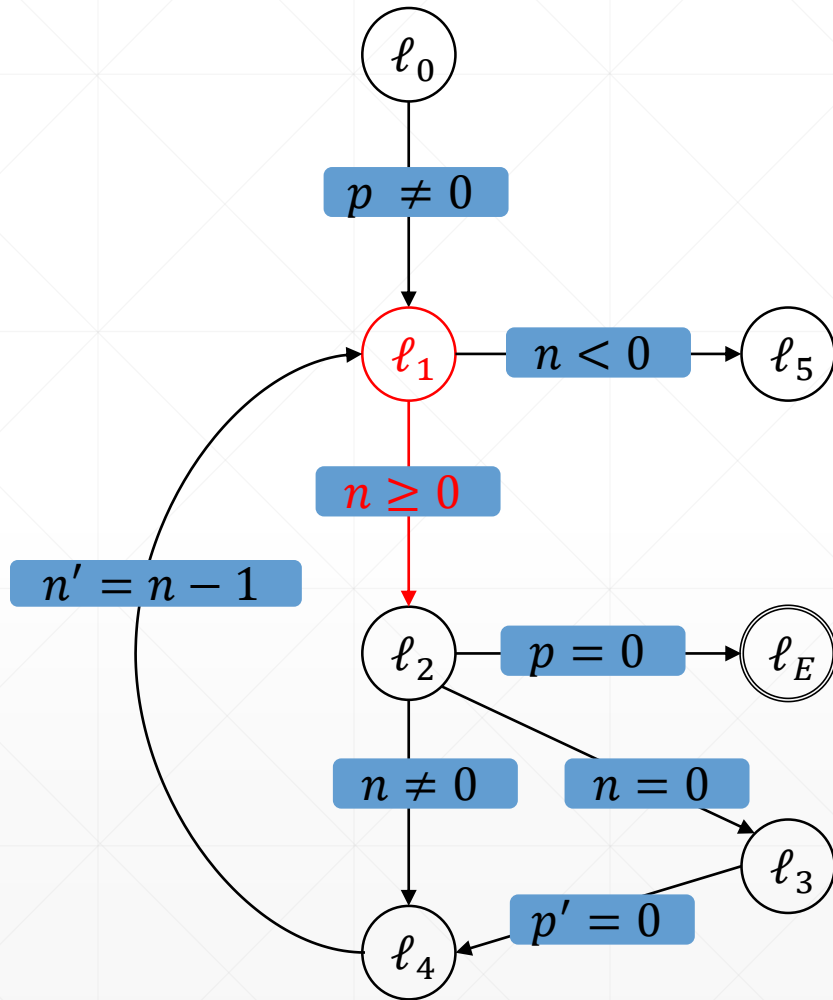
➤ Try to block  $(p = 0, \ell_2, 2)$

- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$ 
    - ➔ Satisfiable!
    - ➔  $wp(n \geq 0, p' = 0) = (p = 0)$
    - ➔ New proof-obligation  $(p = 0, \ell_1, 1)$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

### 7. Step: Level 2 Blocking-Phase:

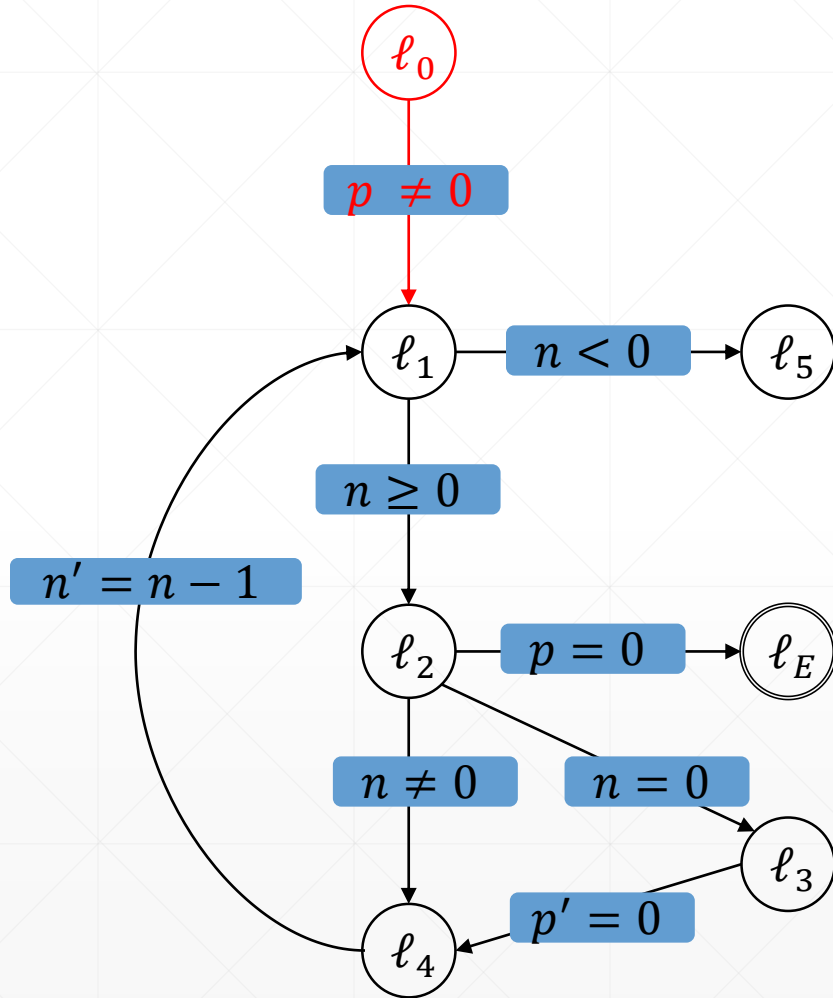
➤ Try to block  $(p = 0, \ell_2, 2)$

- Predecessor  $\ell_1$ :
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### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

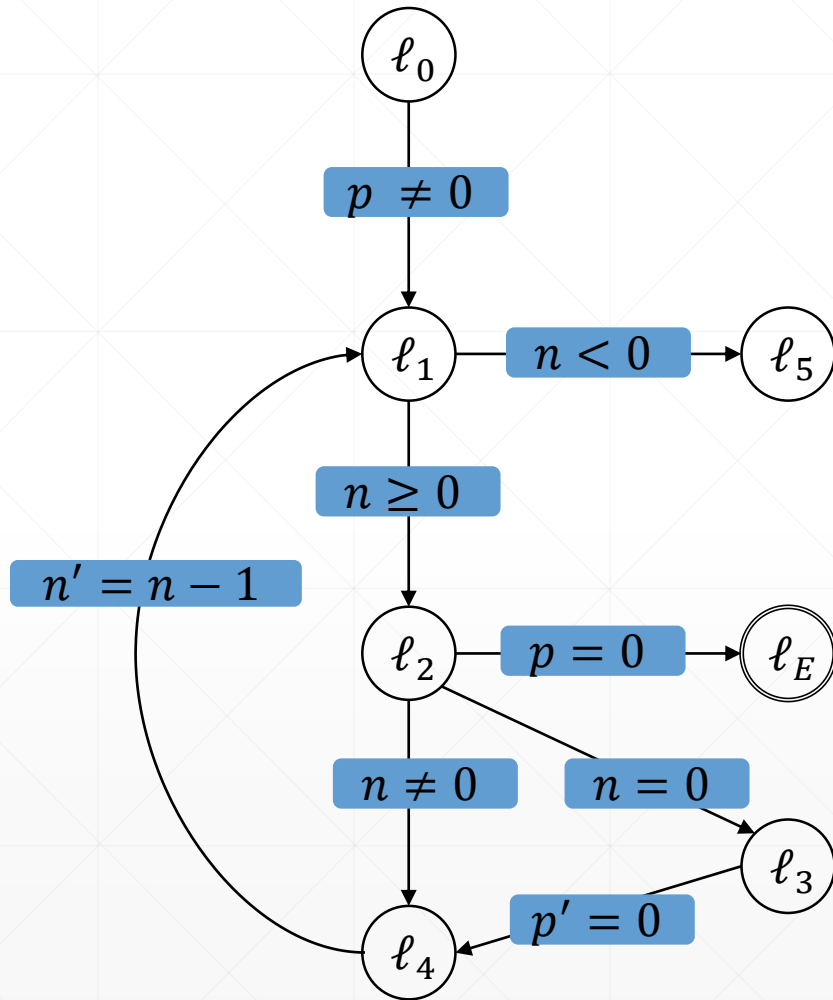
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_0$ :
  - $t \wedge p \neq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

### 7. Step: Level 2 Blocking-Phase:

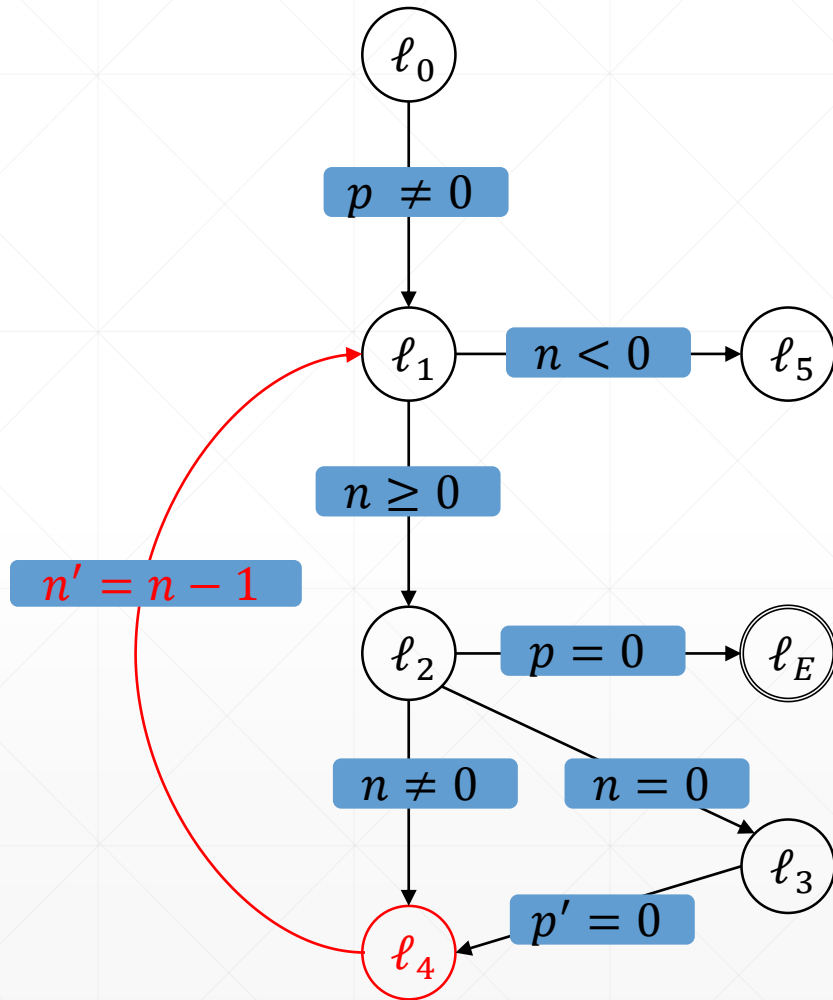
- Try to block  $(p = 0, \ell_1, 1)$ 
  - Predecessor  $\ell_0$ :
    - $t \wedge p \neq 0 \wedge p' = 0$ 
      - ➔ Unsatisfiable!
      - ➔ Strengthen frames  $F_{0,\ell_1}, F_{1,\ell_1}$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$



## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

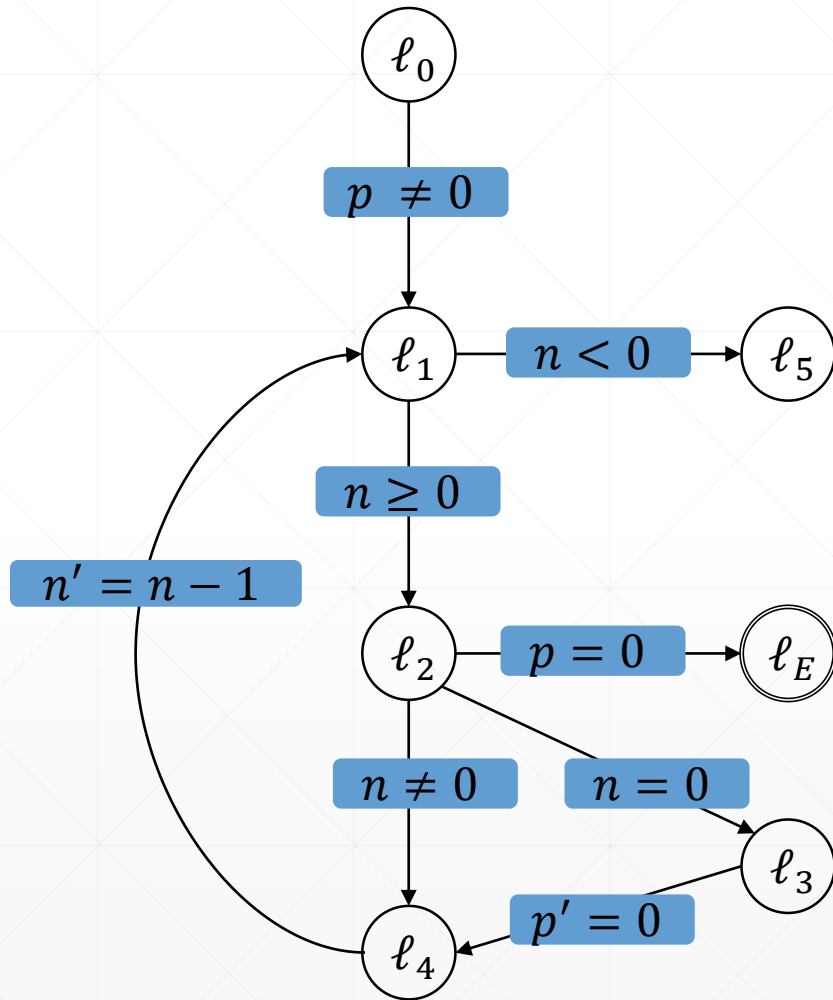
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_4$ :
  - $f \wedge n' = n - 1 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

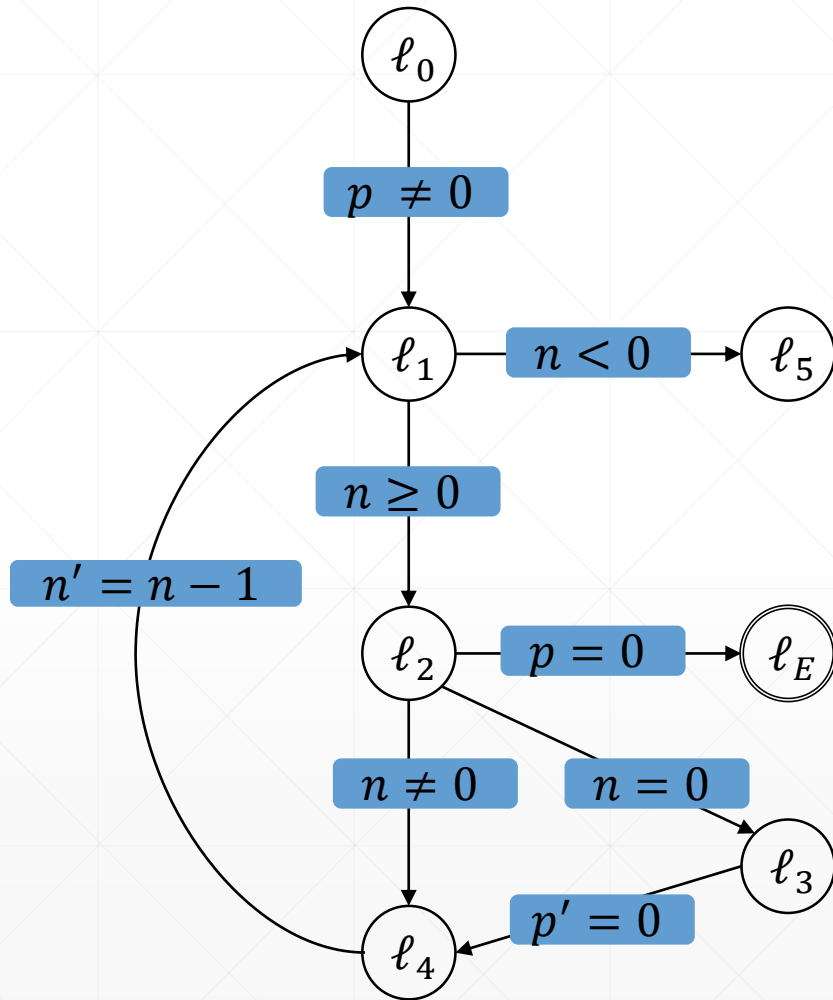
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_4$ :
  - $f \wedge n' = n - 1 \wedge p' = 0$
  - ➔ **Unsatisfiable!**

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$
- $(p = 0, \ell_1, 1)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

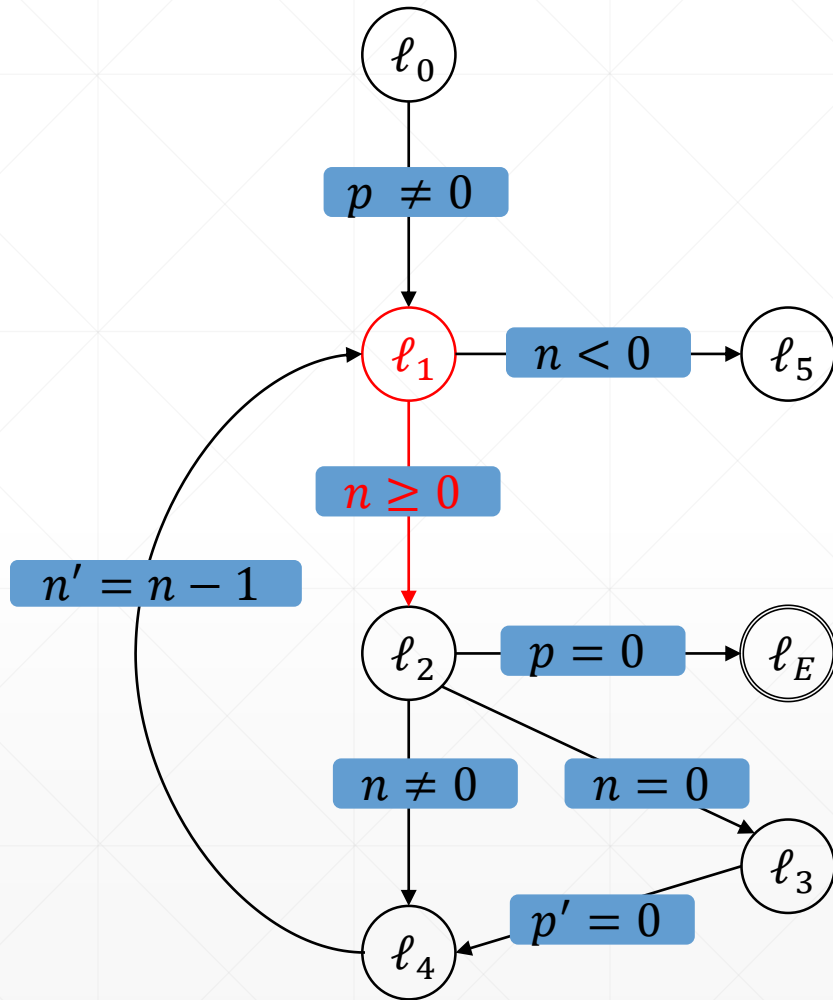
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_1, 1)$
- Predecessor  $\ell_4$ :
  - $f \wedge n' = n - 1 \wedge p' = 0$
  - ➔ **Unsatisfiable!**

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

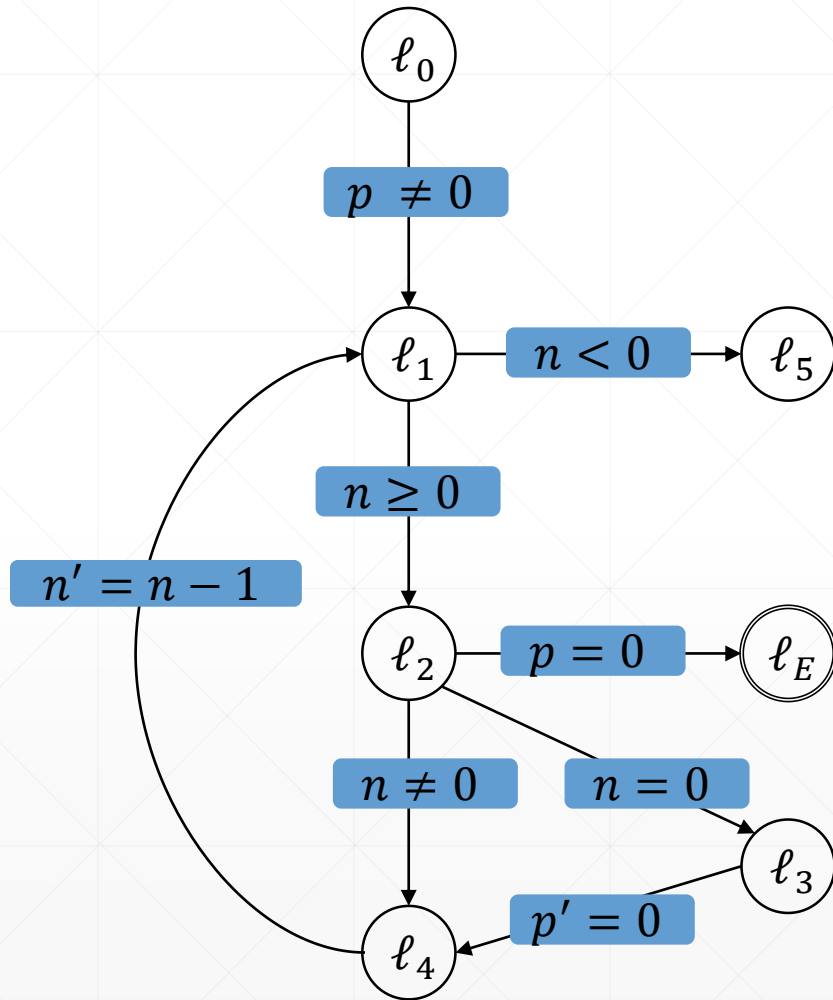
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_2, 2)$  again
  - Predecessor  $\ell_1$ :
    - $t \wedge p \neq 0 \wedge n \geq 0 \wedge p' = 0$

### Proof-Obligations:

- $(p = 0, \ell_2, 2)$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

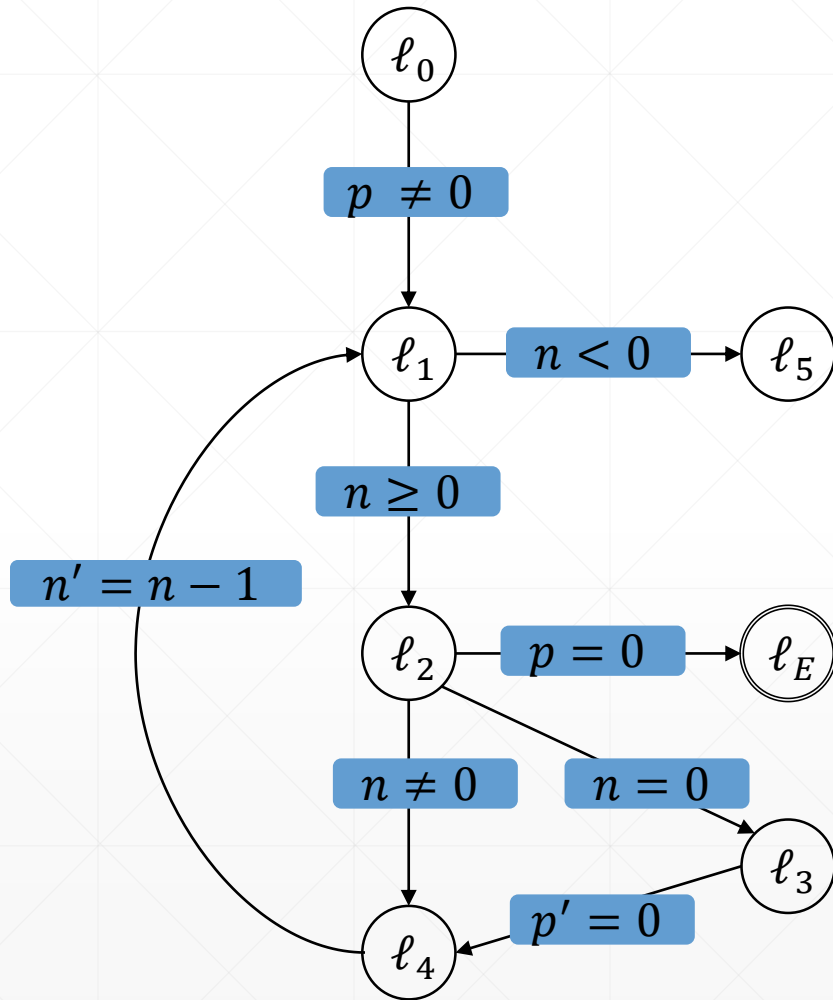
### 7. Step: Level 2 Blocking-Phase:

- Try to block  $(p = 0, \ell_2, 2)$  again
  - Predecessor  $\ell_1$ :
    - $t \wedge p \neq 0 \wedge n \geq 0 \wedge p' = 0$
    - ➔ **Unsatisfiable!**
    - ➔ Strengthen frames  $F_{2,\ell_2}$

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

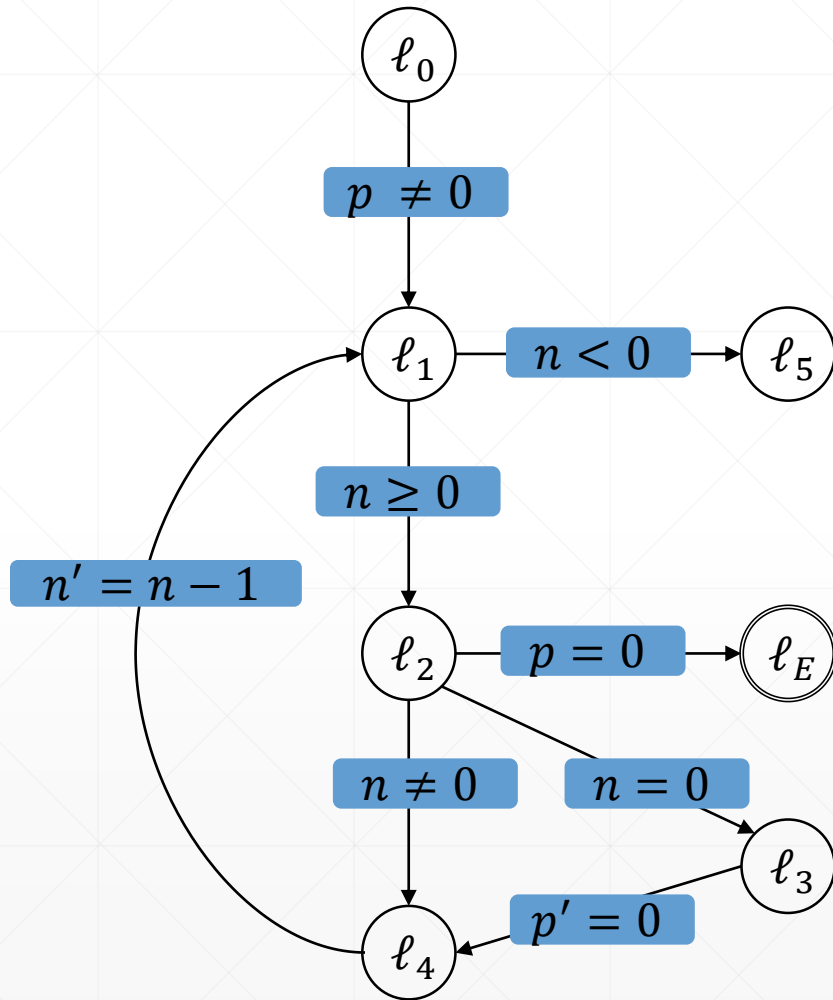
### 8. Step: Level 2 Propagation-Phase:

- Is there a global fixpoint?  
 ➔ No, continue with level 3

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2
$\ell_0$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$
$\ell_4$	$f$	$t$	$t$

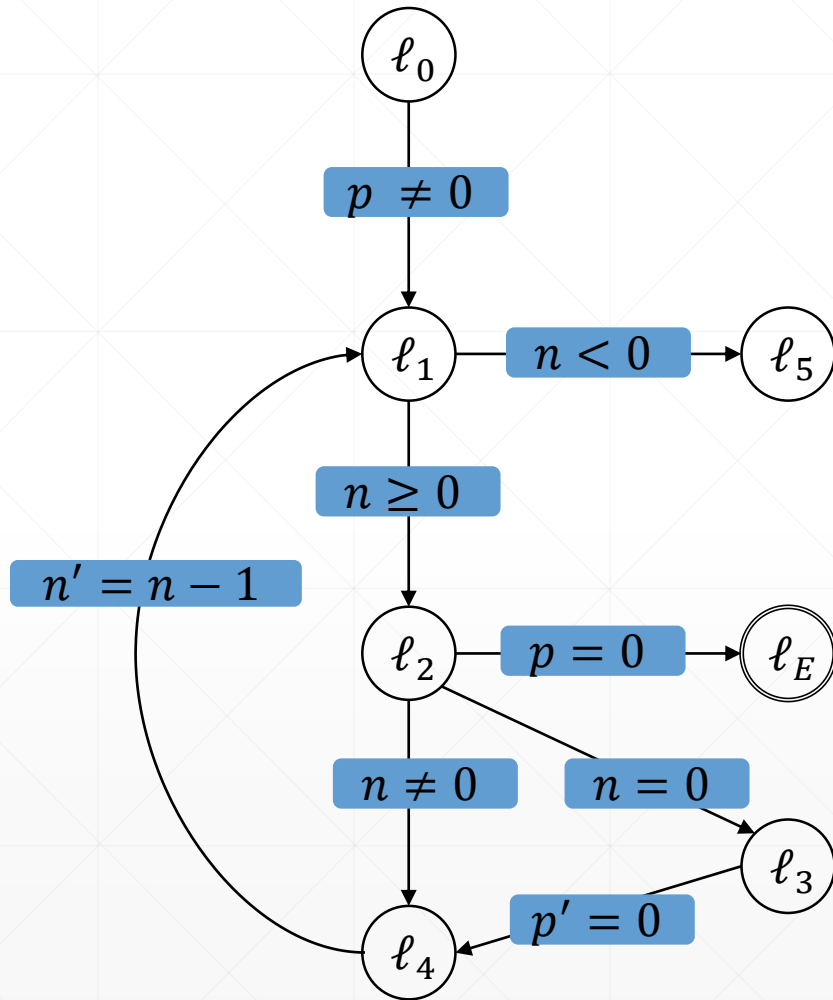
### 9. Step: Level 3

- Initialize new frames
- Get initial proof-obligations

### Proof-Obligations:

- $\emptyset$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

### 9. Step: Level 3

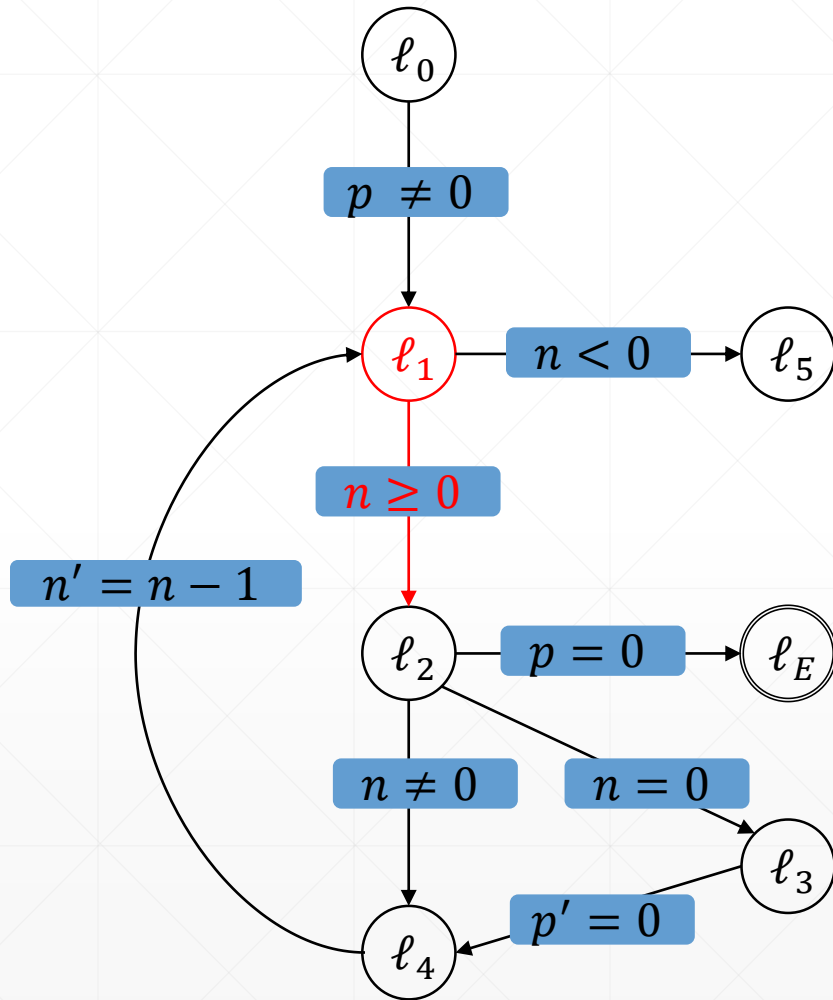
- Initialize **new frames**
- Get **initial proof-obligations**

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$



## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

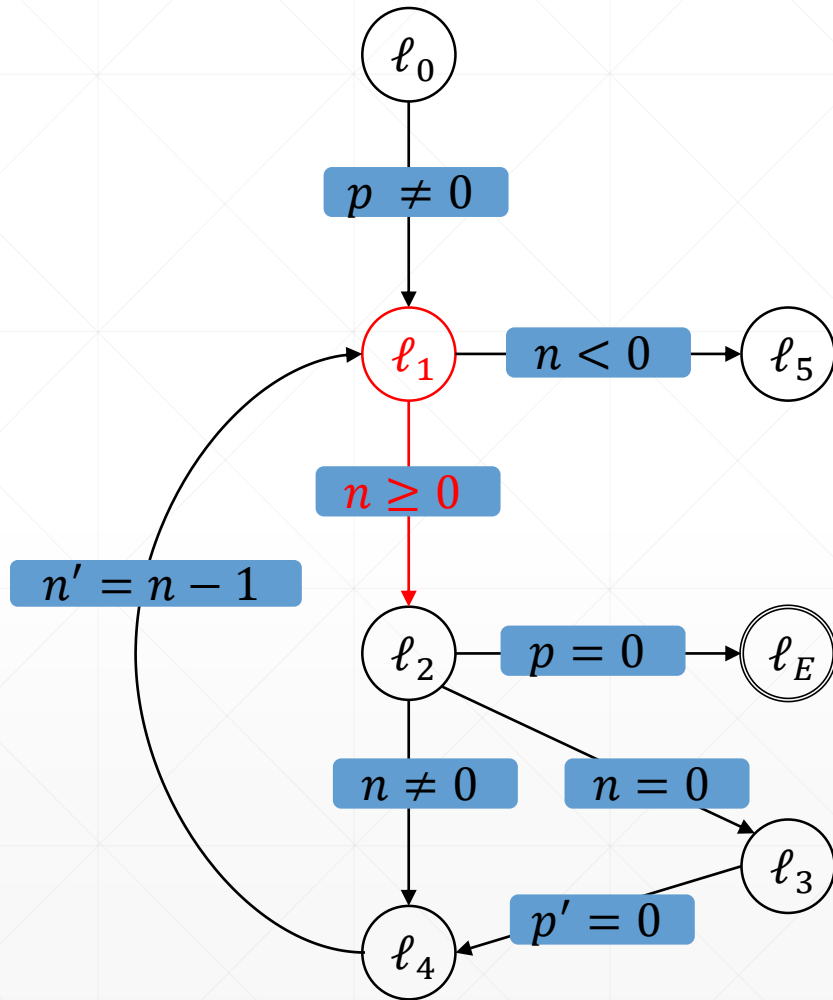
### 10. Step: Level 3 Blocking-Phase

- Try to block  $(p = 0, \ell_2, 3)$
- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$
  - Like the level before this is satisfiable

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

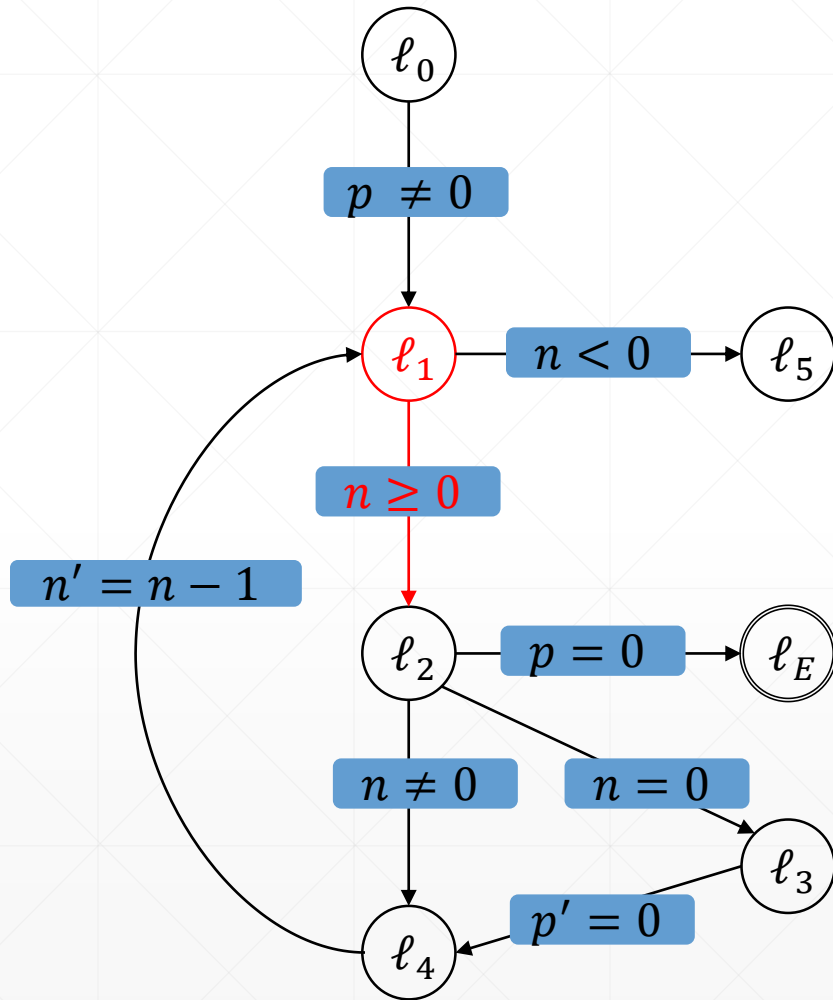
### 10. Step: Level 3 Blocking-Phase

- Try to block  $(p = 0, \ell_2, 3)$
- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$
  - Like the level before, get the same new proof-obligation but on level 2
  - $(p = 0, \ell_1, 2)$

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

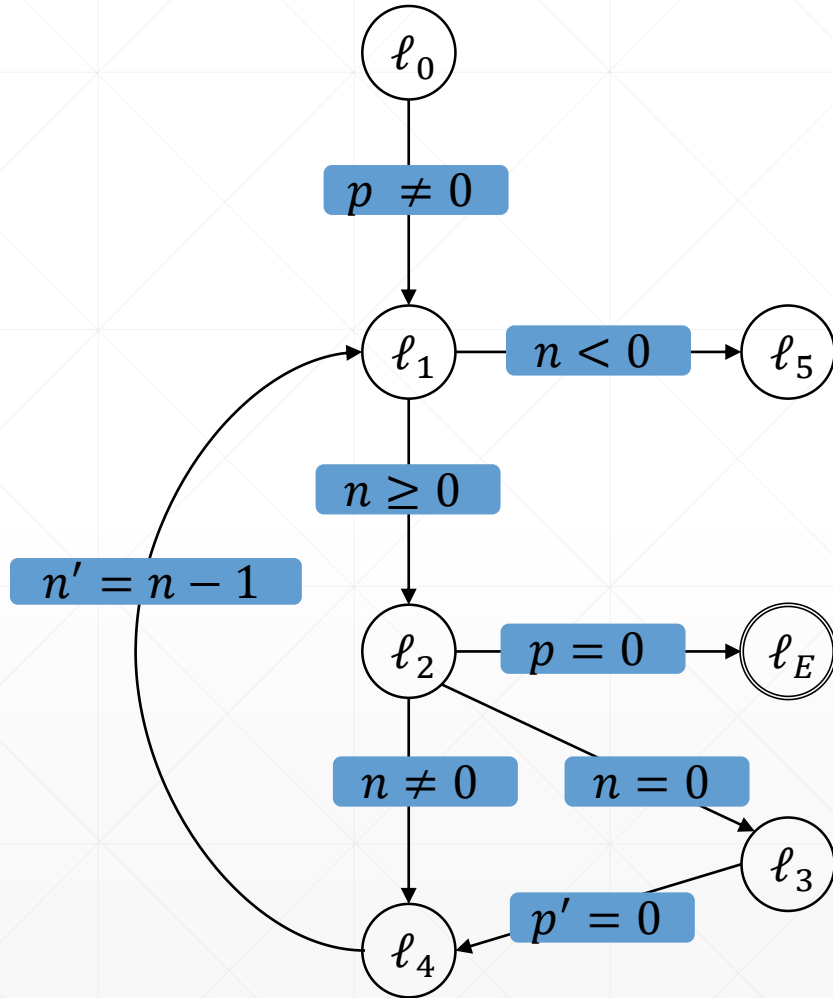
### 10. Step: Level 3 Blocking-Phase

- Try to block  $(p = 0, \ell_2, 3)$
- Predecessor  $\ell_1$ :
  - $t \wedge n \geq 0 \wedge p' = 0$ 
    - Like the level before, get the same new proof-obligation but on level 2
    - $(p = 0, \ell_1, 2)$

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$
- $(p = 0, \ell_1, 2)$

## Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f$	$t$	$t$	$t$

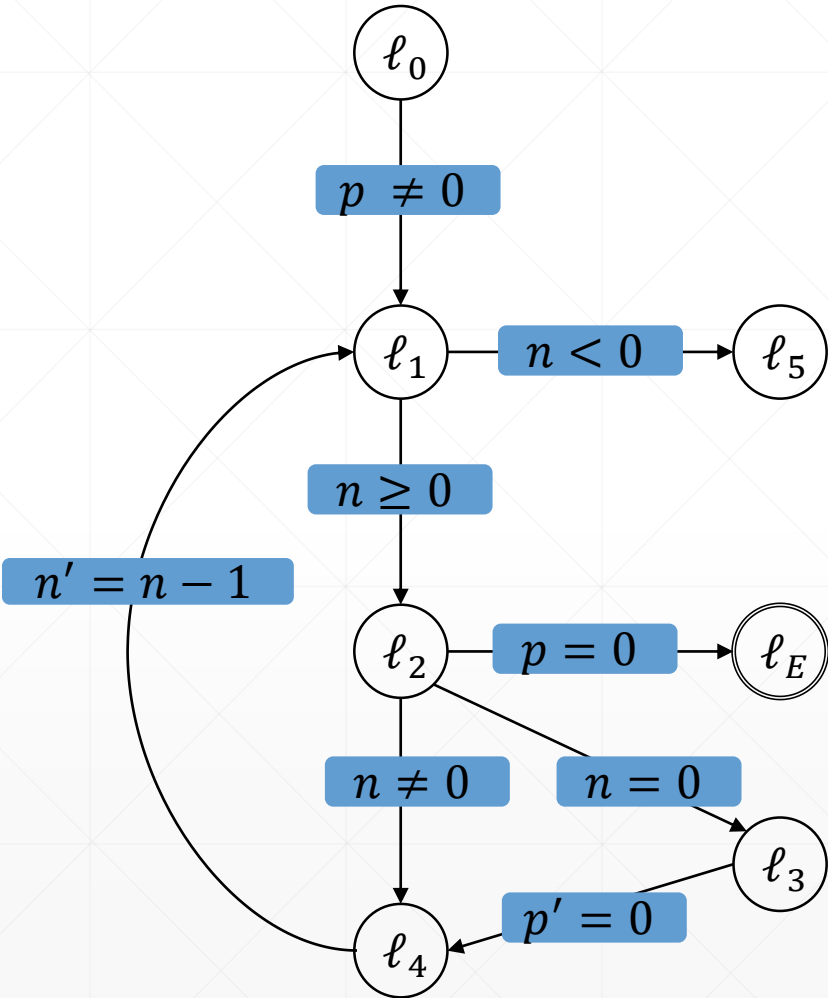
### 10. Step: Level 3 Blocking-Phase

- There are a lot of repetitions

### Proof-Obligations:

- $(p = 0, \ell_2, 3)$
- $(p = 0, \ell_1, 2)$

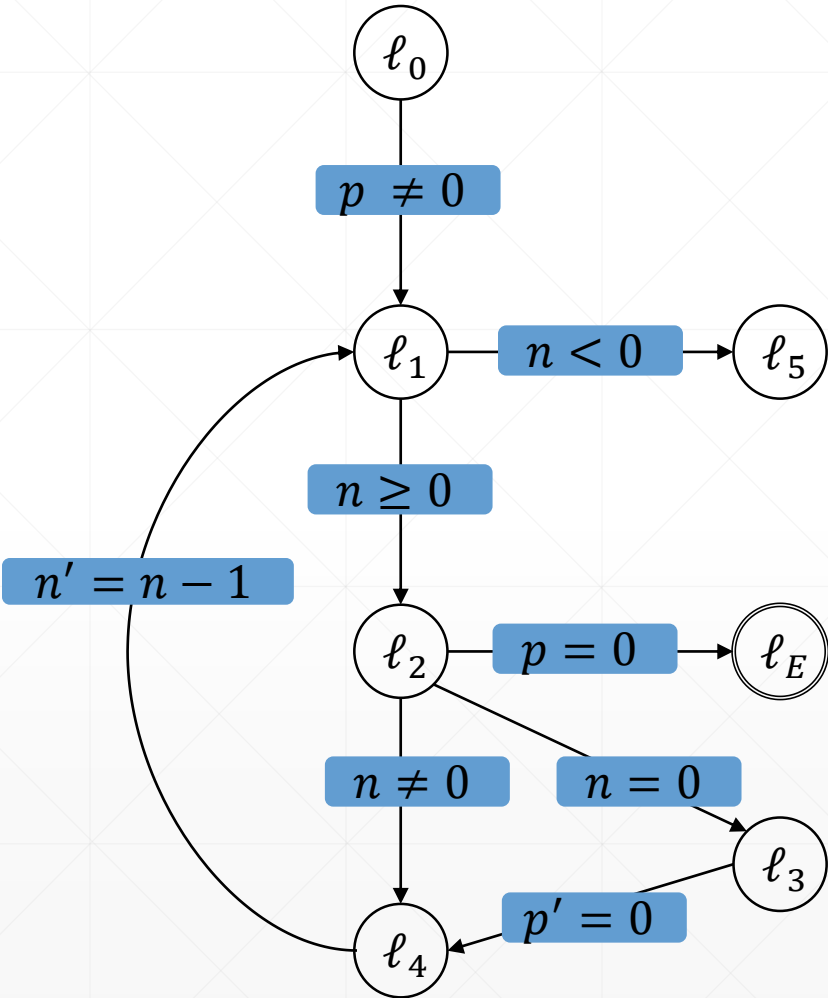
# Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$

11. Step: Level 3 Done

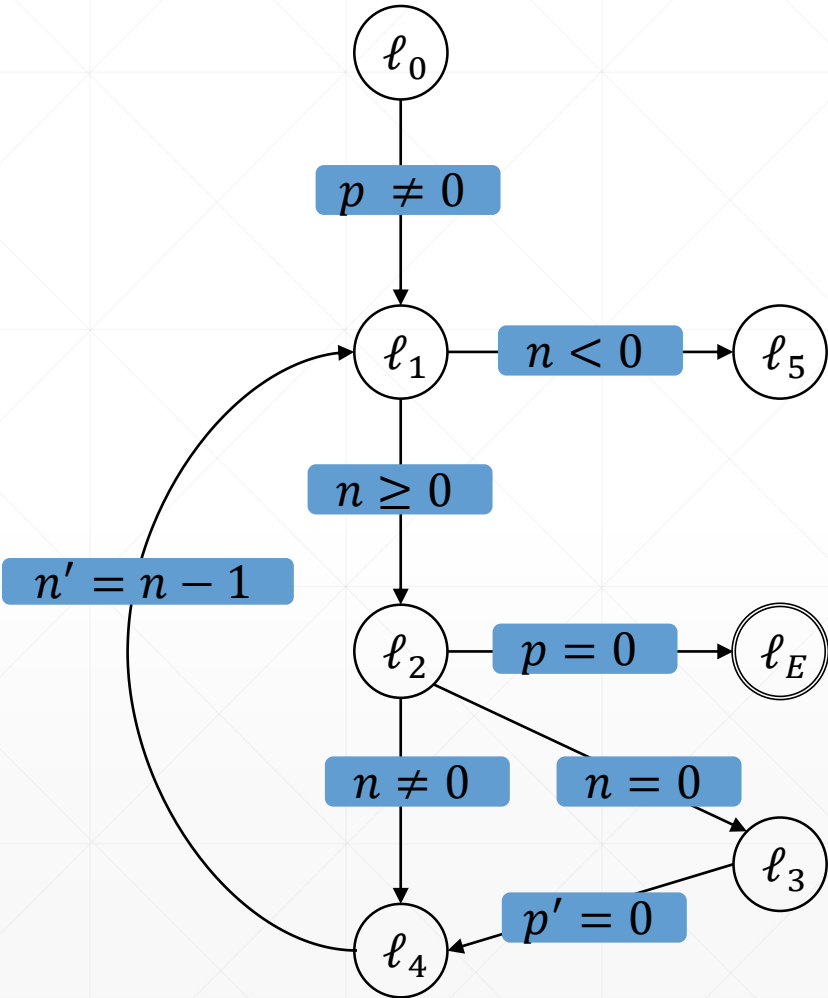
# Example:



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$

11. Step: Level 4

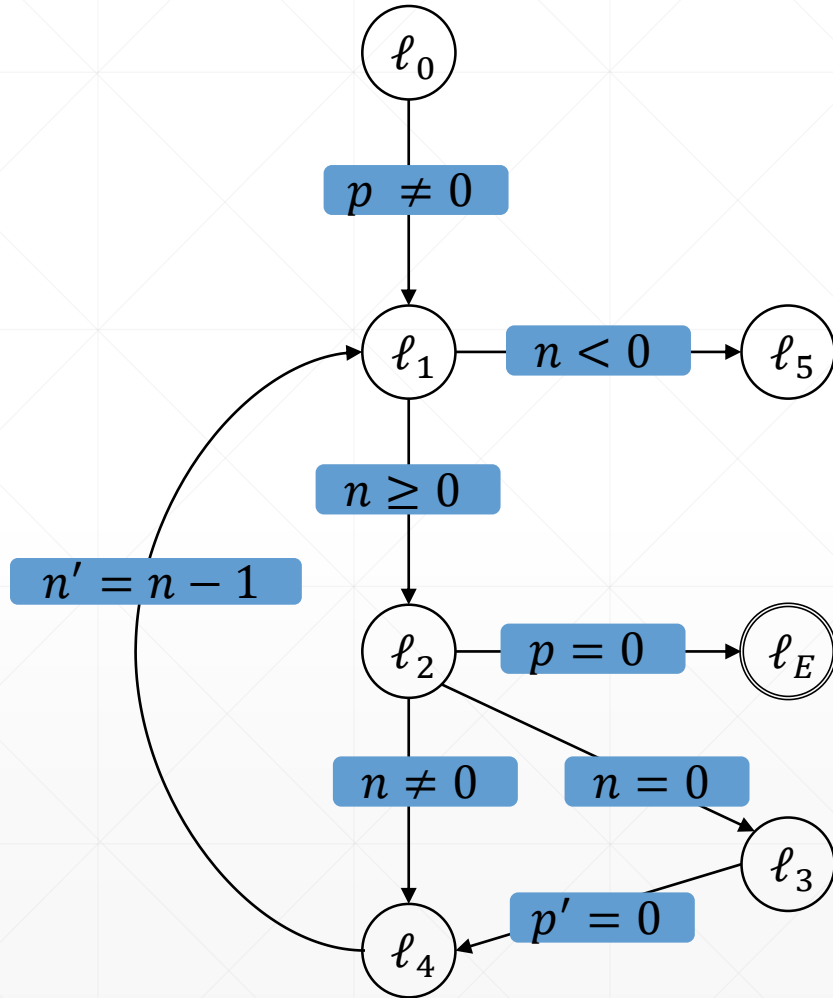
# Example:



location	0	1	2	3	4
$\ell_0$	$t$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$	$t$

11. Step: Level 4 Initialization

## Example:

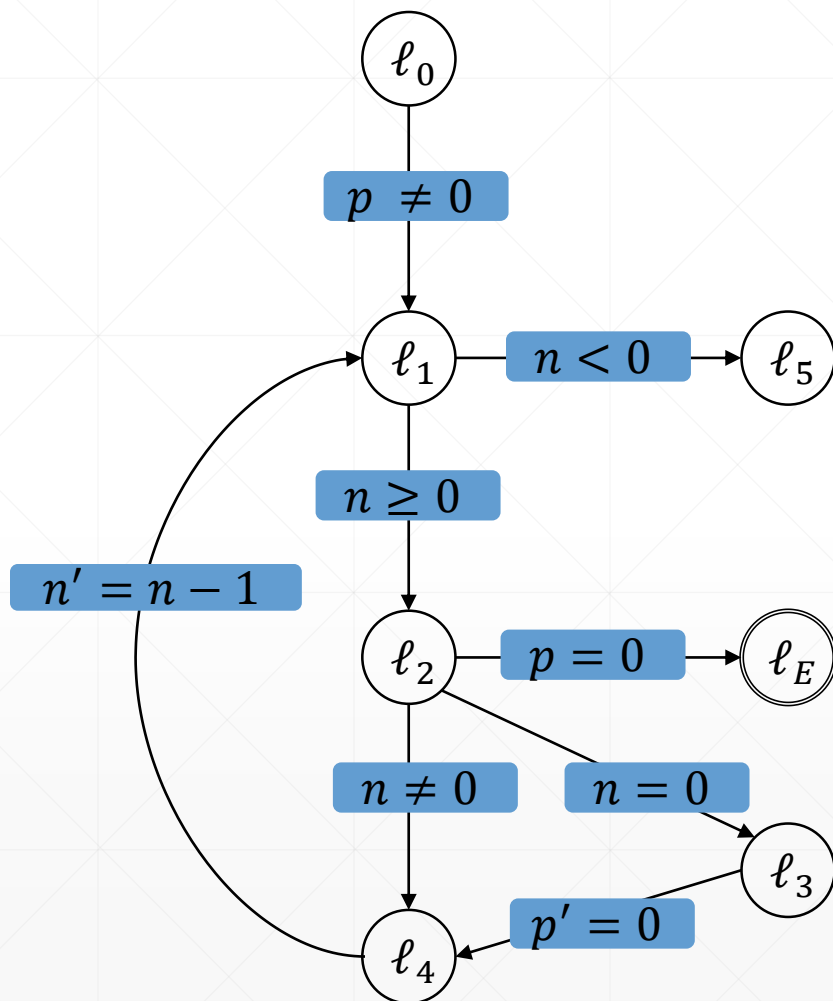


location	0	1	2	3	4
$\ell_0$	$t$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_3$	$f$	$t$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$	$t$

TODO The new interesting proof-obligation!



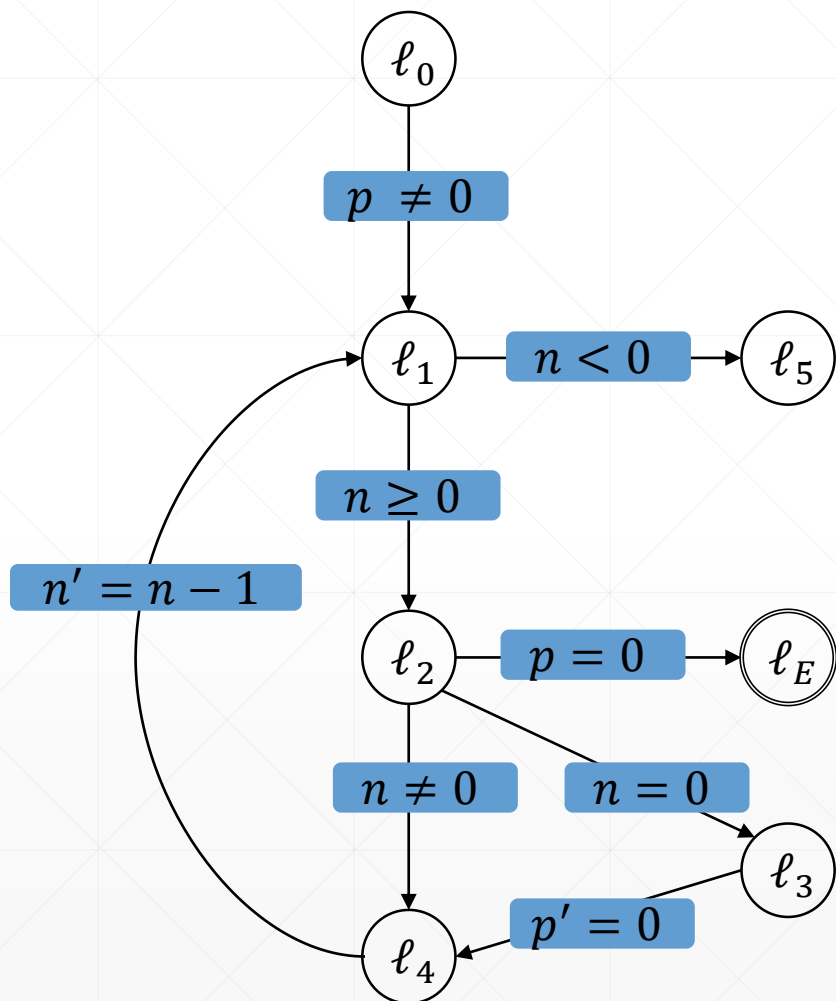
# Example:



location	0	1	2	3	4
$\ell_0$	$t$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$
$\ell_3$	$f \wedge f$	$t \wedge f$	$t$	$t$	$t$
$\ell_4$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$	$t$

TODO The Last step:  
 Spoiler: Error is unreachable

# Example:



location	0	1	2	3	4	5
$\ell_0$						
$\ell_1$						
$\ell_2$						
$\ell_3$						
$\ell_4$						

Text

Proof-Obligations:

## Related Work: Other Approaches

---

➤ Our Algorithm is based on the approach by Lange et al.<sup>1</sup>

➤ Other possible ways of using PDR on software:

- Bit-Blasting<sup>2</sup>:
  - Encode the variables as bitvectors with new variable  $pc$  representing the control-flow
  - Use the original bit-level PDR algorithm
  - ➔ Not very competitive because tedious handling of  $pc$  variable

---

1: Tim Lange, Martin R. Neuhäuser, and Thomas Noll. IC3 software model checking on control flow automata. In *FMCAD*, pages 97–104. IEEE, 2015.

2: Tobias Welp and Andreas Kuehlmann. QF BV model checking with property directed reachability. In *DATE*, pages 791–796. EDA Consortium San Jose, CA, USA / ACM DL, 2013.

## Related Work: Other Approaches

---

➤ Our Algorithm is based on the approach by Lange et al.<sup>1</sup>

➤ Other possible ways of using PDR on software:

- Abstract Reachability Tree (ART) Unrolling<sup>3</sup>:
  - Transform CFG into an ART
    - ➔ Attach program-counter variable  $pc$  and first-order formula  $\varphi$  to locations
  - Block proof-obligations like in our approach

---

1: Tim Lange, Martin R. Neuhäuser, and Thomas Noll. IC3 software model checking on control flow automata. In *FMCAD*, pages 97–104. IEEE, 2015.

3: Alessandro Cimatti and Alberto Griggio. Software model checking via IC3. In *CAV*, volume 7358 of *Lecture Notes in Computer Science*, pages 277–293. Springer, 2012.

# Implementation in Ultimate: Description Trace Abstraction with PDR

---

1. Calculate sequence of statements from initial location to error location

➔ Possible error trace

2. Construct a new CFG of error trace

3. Use PDR to show if error is reachable or not

➔ If reachable:

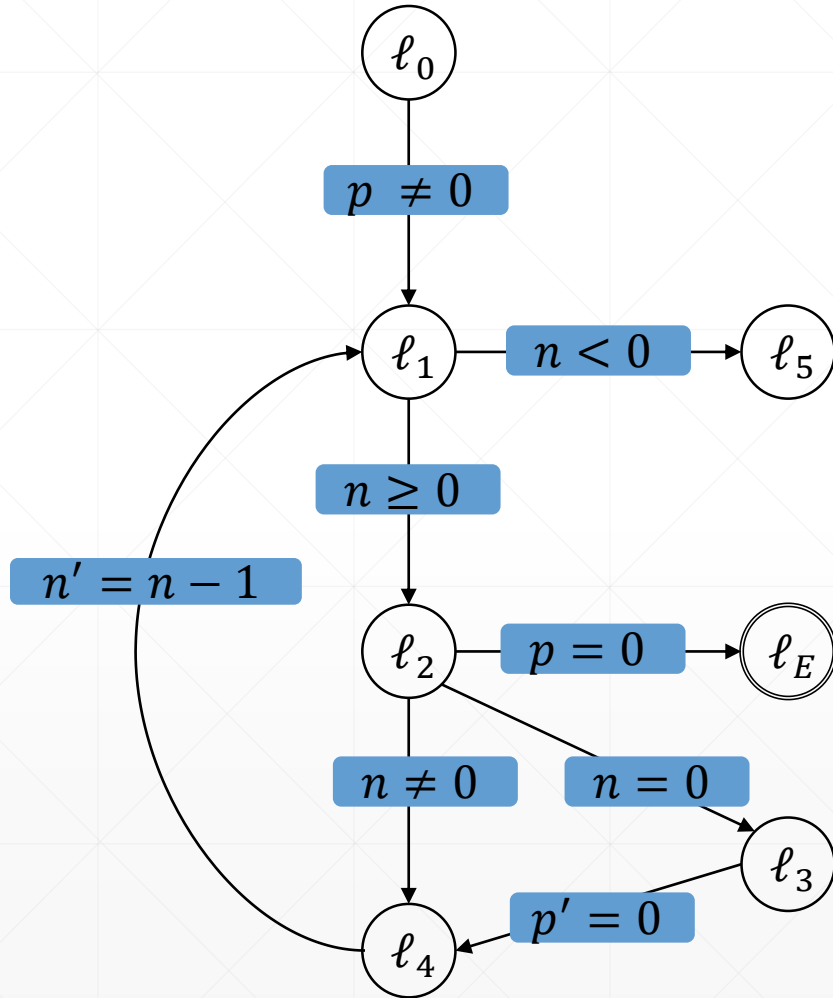
- Error trace is feasible, program is unsafe

# Implementation in Ultimate: Description Trace Abstraction with PDR

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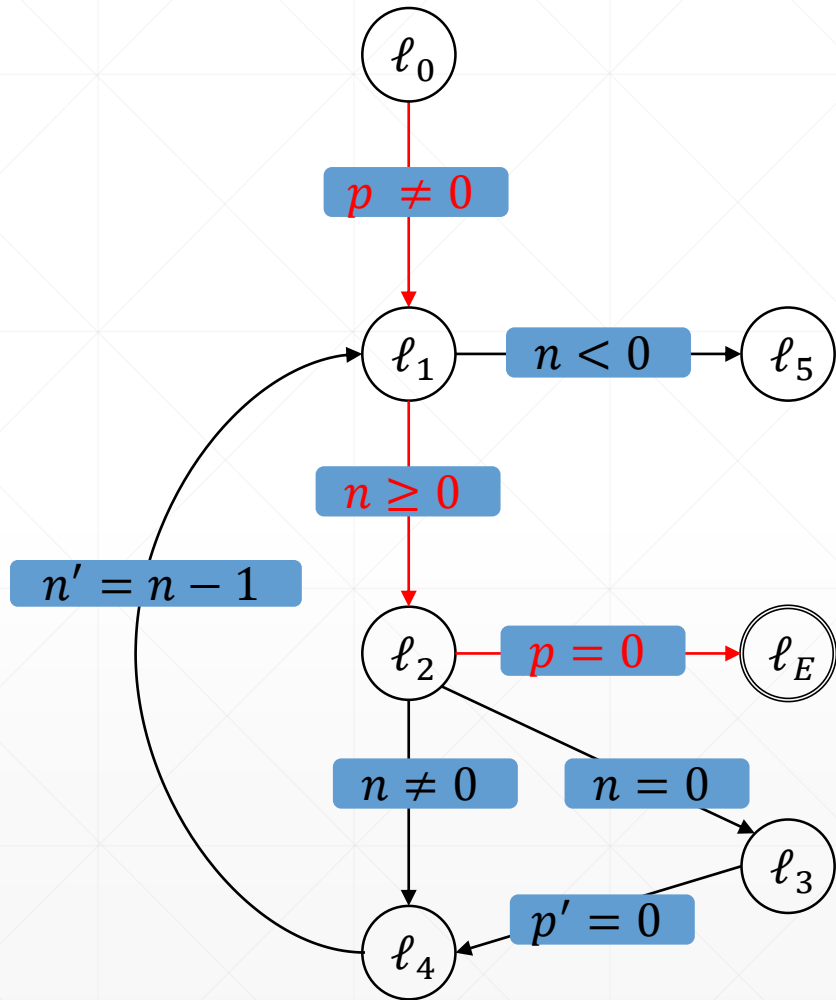
1. Calculate sequence of statements from initial location to error location
  - ➔ Possible error trace
2. Construct a new CFG of error trace
3. Use PDR to show if error is reachable or not
  - ➔ If unreachable:
    - Use formulas at the fixpoint as interpolant sequence to refute other error traces

# Implementation in Ultimate: Trace Abstraction with PDR



1. Step: Get possible error trace

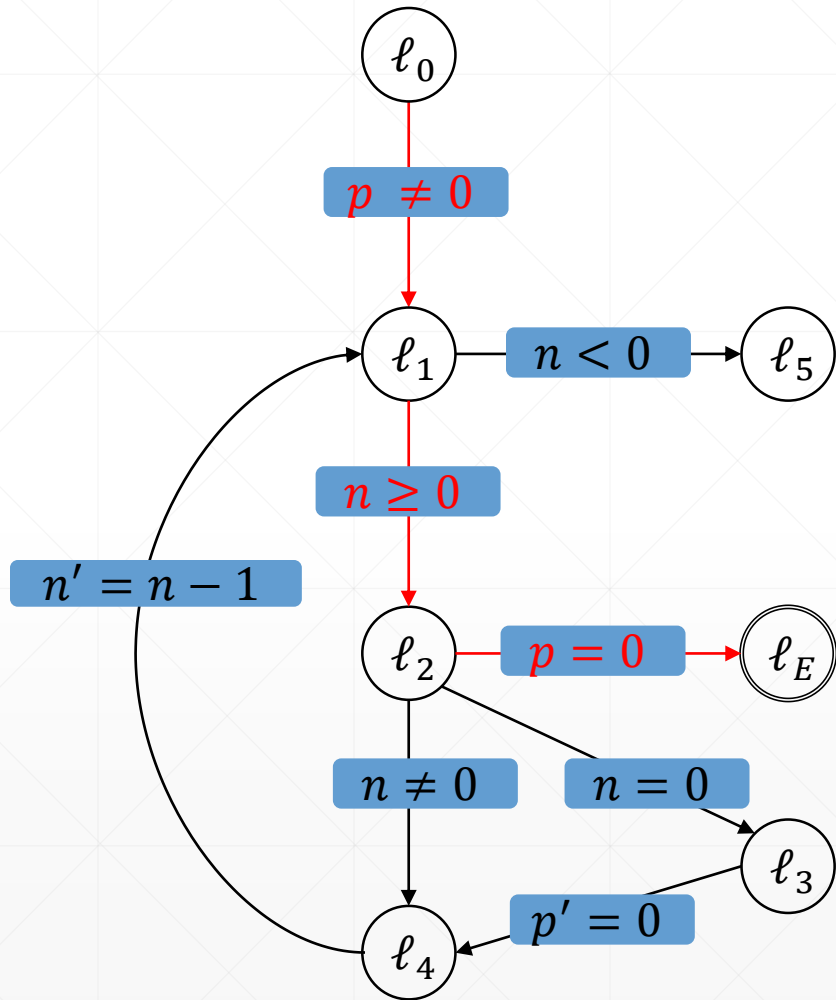
# Implementation in Ultimate: Trace Abstraction with PDR



1. Step: Get possible error trace



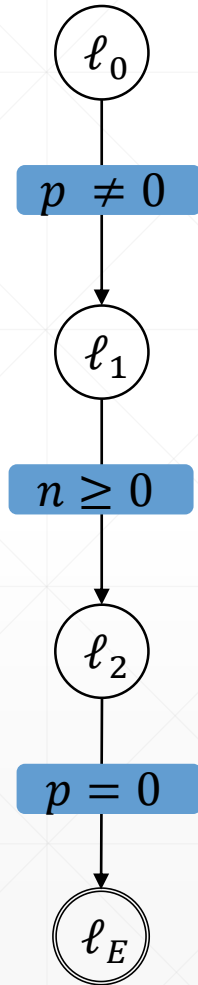
# Implementation in Ultimate: Trace Abstraction with PDR



2. Step: Construct new CFG

# Implementation in Ultimate: Trace Abstraction with PDR

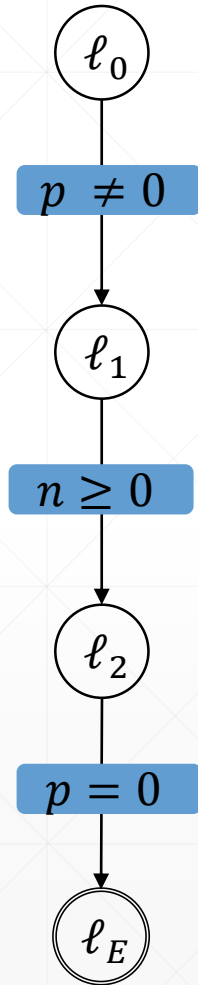
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2. Step: Construct new CFG

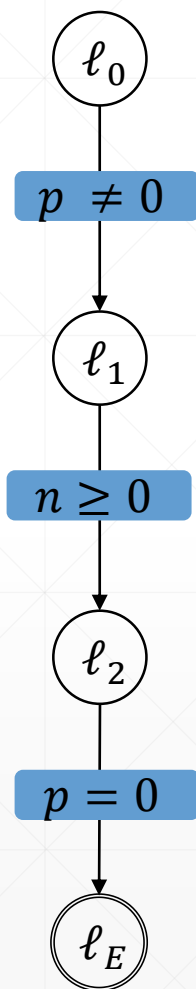
# Implementation in Ultimate: Trace Abstraction with PDR

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3. Step: Use PDR

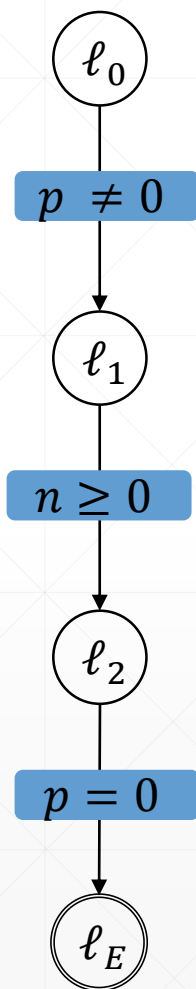
# Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
$\ell_0$				
$\ell_1$				
$\ell_2$				

3. Step: Use PDR

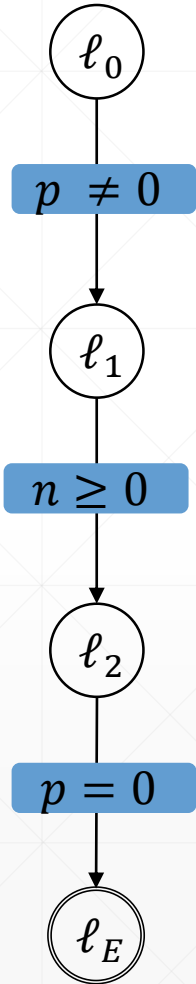
# Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$

3. Step: Use PDR

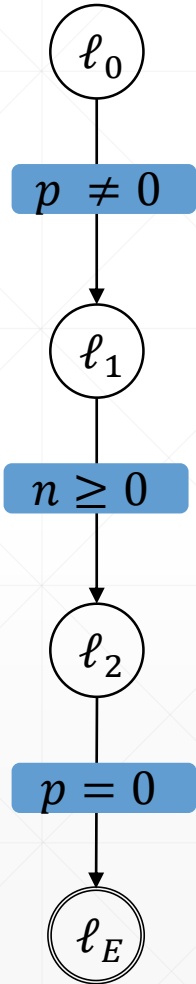
# Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$

4. Step: Use fixpoint invariants as interpolants

# Implementation in Ultimate: Trace Abstraction with PDR



location	0	1	2	3
$\ell_0$	$t$	$t$	$t$	$t$
$\ell_1$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t$
$\ell_2$	$f \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$	$t \wedge p \neq 0$

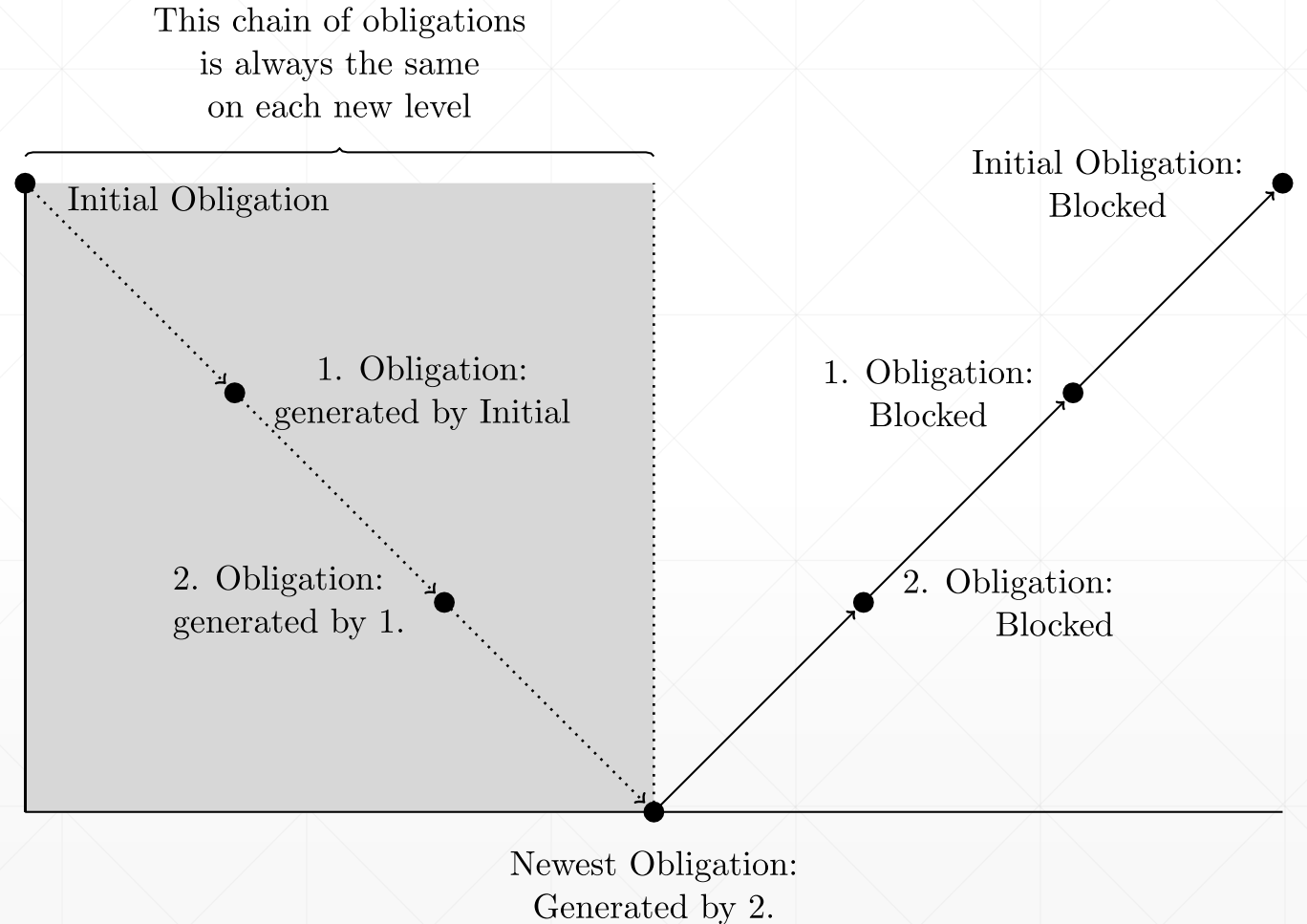
4. Step: Use fixpoint invariants as interpolant sequence

# Implementation in Ultimate: Implemented Improvements

## ➤ Caching proof-obligations:

- Save the proof-obligation queue
- Start every new level with the latest blocked proof-obligation

➔ Only proof-obligation that differs from level before





# Implementation in Ultimate: Implemented Improvements

---

- Skipping already blocked proof-obligations:
  - Save unsatisfiable queues to SMT-solver
    - ➔ If a saved queue is seen again, do not call SMT-solver again, strengthen frames right away

# Evaluation: Data Comparison

---

- We benchmarked PDR

# Evaluation: Discussion

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## Future Work: Implementing Further Improvements

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### ➤ Using Interpolation:

- Our algorithm is inefficient when dealing with loops
- Idea:
  - Instead of strengthening frames with negated proof-obligation, calculate Interpolant for transition and proof-obligation and add that

## Future Work: Implementing Further Improvements

---

### ➤ Dealing with procedures:

- C programs often contain procedures with which PDR cannot deal

- Idea:

- Use a non-linear approach of PDR
- Calculate a procedure summary and add that to the CFG, removing the procedure altogether

# Conclusion

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