Master Project

Trace Abstraction with Accelerated Interpolants

Proposal

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1 Introduction

Assume we want to verify whether a program fulfills a given safety property. The program's control-flow graph represents every possible trace that can be taken by a program execution. Executions that violate the safety property end in a so called error location. Error traces are traces in the control-flow graph that start in the initial node and end in the error location. The goal is to check if there are feasible error traces. For this purpose one can apply an automata-based instance of the CEGAR scheme, called trace abstraction, on the program's control-flow graph.

Trace abstraction aims at constructing automata [6] from infeasible error traces. When the language recognized by the program's control-flow graph is a subset of these automata, it means, every possible error trace, and by that execution which ends in an error location, is infeasible. Proving that the program fulfills the safety property. If there is a single feasible error trace, then the program violates the safety property.

If an error trace is infeasible there is an infeasibility proof. Using this proof one can construct an automaton, which excludes the original error trace from the control-flow graph. However, this excludes only one error trace, making this approach not very efficient. To exclude more than one error trace, one can try to compute a generalization of the infeasibility proof.
A common strategy is [5] to calculate a generalization using Craig interpolation [3], where an SMT-solver computes a sequence of interpolants from an infeasibility proof.
But, it is not guaranteed that these interpolants are more general. This issue is most notably in program loops. Assume that the given program contains a loop with guard $x < 5000$, x being an

integer variable. It is possible that the computed interpolant sequence does not exclude every loop iteration, leading to the need of disproving every of the 5000 possible error traces individually.

A solution for this problem is accelerating the loop, meaning computing its transitive closure, and calculating interpolants on that.
This project aims at implementing exactly that. The goal is to combine interpolation and loop acceleration on the basis of the work of Hojjat et al [7] in the software analysis framework Ultimate [1].
The remainder of this proposal is structured as follows. Chapter 2 will give an overview of needed background information, like a more detailed look at trace abstraction, loop acceleration, and the combination of interpolation and acceleration. Chapter 3 will detail the approach this project will take to implement accelerated interpolation in Ultimate, and finally an outline of the project's deliverables and schedule.

2 Background

This project aims at combining loop acceleration and interpolant calculation based on the findings of Hojjat et al [7], however, instead of utilizing a CEGAR-scheme with predicate abstraction, it will be implemented as an automata-based CEGAR-scheme, called trace abstraction.
This section will introduce the basic ideas behind trace abstraction, loop acceleration, and finally accelerated interpolants.

2.1 Interpolating Trace Abstraction

Given a program P and its control-flow graph $A_P = (Loc, \Delta, \ell_0, \ell_E)$, where Loc is a set of program locations, Δ is a set of triples (ℓ, stm, ℓ') representing program transitions with $\ell, \ell' \in Loc$ and program statement stm, ℓ_0 being the initial location, and ℓ_E being the error location. One can interpret the control-flow graph as a nondeterministic automaton with ℓ_E as accepting state.

To check the reachability of ℓ_E, and with that correctness of P, use trace abstraction with interpolation according to the following paradigm [5]:

1. Search the program's control-flow graph for a trace that starts at ℓ_0 and ends in ℓ_E:

   $$\ell_0 \xrightarrow{stm_1} \ell_1 \xrightarrow{stm_2} \cdots \xrightarrow{stm_{n-1}} \ell_{n-1} \xrightarrow{stm_n} \ell_E$$

   With each $(\ell_i, stm_i, \ell_{i+1}) \in \Delta$.

2. Prove feasibility.
   In case that the trace is proven feasible, the program is incorrect, if the trace is infeasible construct an infeasibility proof.

3. Use the infeasibility proof to calculate interpolants.

4. Construct an automaton, $\mathcal{A}_i$, from the interpolants.

5. If the language $\mathcal{L}(\mathcal{A}_\mathcal{P})$, that is recognized by the program's control-flow graph, is a subset of the union of languages recognized by the constructed automata: $\mathcal{L}(\mathcal{A}_\mathcal{P}) \subseteq \mathcal{L}(\mathcal{A}_1) \cup ... \cup \mathcal{L}(\mathcal{A}_i)$ then the program is correct, else start again at step 1.

The interpolants generated in step 3 serve to generalize the infeasibility proof to exclude other possible error traces. However, the interpolants are not guaranteed to be general enough to exclude infinitely many error traces. Which poses a problem for loops. In the following we introduce a way to exclude an infinite number of traces going through a loop.

2.2 Trace Schemes

To make the exclusion of error traces through a loop possible, we need to preprocess error traces. Assume we are given the following error trace τ:

$$\tau : \ell_0 \xrightarrow{stm_1} \ell_1 \xrightarrow{stm_2} \ell_2 \xrightarrow{stm_3} \ell_3 \xrightarrow{stm_4} \ell_2 \xrightarrow{stm_5} \ell_E$$

We see there is a loop from ℓ_2 to ℓ_3 and back to ℓ_2.

To be able to efficiently accelerate the loop we firstly need to change its representation. We want to merge all loop statements into one transition. This is done by calculating the composition of the statements.
This composition then replaces the loop in the trace, resulting in a trace scheme:

$$\begin{tikzcd} \tau' : \ell_0 \arrow[r, "stm_1"] & \ell_1 \arrow[r, "stm_2"] & \ell_2 \arrow[loop above, "stm_3 \circ stm_4"] \arrow[r, "stm_5"] & \ell_E \end{tikzcd}$$

This trace scheme still includes the original trace. It is possible to check the trace scheme's feasibility however it is impossible to compute a sequence of interpolants yet. This is where acceleration is needed.

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2.4

2.3 Loop Acceleration

In program verification, loops introduce the problem of creating large or even infinite state spaces. Resulting in infinitely many error traces that have to be proven infeasible. However, there are two approaches changing the representation of loops to make them easier to deal with: Precise and approximate loop acceleration.

2.3.1 Precise Loop Acceleration

Precise loop acceleration means computing the transitive closure of a given loop:

$$R^* \equiv R^0 \ \vee \ R^1 \ \vee \ldots$$

Each relation R^i represents the state of a program after i loop iterations. This transitive closure represents each trace going through the loop.
In practice computing the transitive closure of a loop is not always possible, as it is a disjunction of infinitely many relations.

There is a class of relations whose transitive closure can be computed efficiently: ultimately periodic relations. Ultimately periodicity describes sequences that start to repeat after a prefix.
One class of such ultimately periodic relation is the octagon domain [8]. Octagons are characterized by a guard of the form:

$$\pm x_i \pm x_j \leq c_{i,j}$$

With integer variables $x_i, x, c_{i,j}$
This project will mainly focus on this class of ultimately periodic relations for its loop acceleration. Making use of the acceleration algorithm proposed by Bozga et al. [2].

2.3.2 Approximate Loop Acceleration

To deal with non-ultimately periodic relations, one can use an approximate loop acceleration method. Approximate loop acceleration, instead of modeling every trace through the loop, represents either an over- or underapproximation of the loop state space. On which one can compute the transitive closure more easily.
A possible future implementation of these loop acceleration techniques will be enabled by this project.

2.4 Meta-Traces

The transitive closure of a loop contains every trace going through it. To make use of loop acceleration, we need to pull apart the looping location ℓ_2 by introducing so called meta-transitions of the form:

$$\overset{stm_3 \,\circ\, stm_4}{\overset{\frown}{\ell_2}} \qquad \Rightarrow \qquad \ell_2' \xrightarrow{(stm_3 \circ stm_4)^*} \ell_2''$$

Where $(stm_3 \circ stm_4)^*$ symbolizes the calculated transitive closure of the loop.
Using meta-transitions, we can transform our trace scheme into a meta-trace:

$$\bar{\tau} : \ell_0 \xrightarrow{stm_1} \ell_1 \xrightarrow{stm_2} \ell_2' \xrightarrow{(stm_3 \circ stm_4)^*} \ell_2'' \xrightarrow{stm_5} \ell_E$$

The feasibility of the meta-trace is the same as the trace scheme before:

- If it is feasible then the original trace is feasible, making the program incorrect
- If it is infeasible, we can compute an interpolant sequence:

$$I_{\bar{\tau}} : \langle \top, I_1, I_2', I_2'', \bot \rangle$$

JW: todo: Why?

To guarantee inductiveness to the loop, we have to compute the strongest post of each interpolant, that coincide with a location in the loop, and its transitive closure:

$$I_{\bar{\tau}}^{post} : \langle \top, I_1, post(I_2', stm_3 \circ stm_4), \bot \rangle$$

This sequence is now general enough for the trace scheme to exclude the loop.

3 Approach

We want to implement accelerated interpolation into the software verification framework Ultimate [1]. Ultimate consists of multiple different libraries that can be executed in serial to form so called toolchains. There are already toolchains implementing trace abstraction as described above. This

project extends these toolchains with accelerated interpolation by creating a new interpolating trace checking library.
This library will be used for step 3 of the trace abstraction paradigm. For that it has to fulfill the following specifications:

- Computing loop accelerations of octagonal relations using ultimately periodic relations as shown by Bozga et al [2]. And being able to implement further acceleration techniques in the future.
- Construct trace schemes from given error traces and extend them using loop acceleration to meta schemes as presented by Hojjat et al [7].
- Check meta schemes' feasibility
- Construct interpolants for infeasible meta schemes to be used in trace abstraction.

This project is finished when this library has been implemented, tested, and evaluated by using it on several verification tasks.

Master projects award 16 ECTS points which translates to 16 weeks of work. Resulting in the following approach:

1. *Understanding the Matter:*
   Grasp how the combination of loop acceleration and interpolation works.
   Moreover, there have been previous projects that implemented the basis [9] and the ultimately periodic acceleration scheme itself [4] in Ultimate. It is important to understand those implementations before trying to adapt them for the usage of accelerated interpolants.

   Duration in weeks: 1

   Result: Being able to use the previous work and having an understanding on how to implement an accelerated interpolation library.

2. *Implement Trace Scheme Transformation:*
   The accelerated interpolation library should be able to transform a given error trace into a trace scheme.

   Duration in weeks: 4

   Result: Given an error trace, accelerated interpolation is able to transform it into a trace scheme.

3. *Implement Own Version of Loop Acceleration:*
   Implement a way of using Fast Acceleration of Ultimately Periodic Relations in the accelerated interpolation library.

   Duration in weeks: 4

   Result: Accelerated interpolation is able to construct the transitive closure of loops to use for interpolant generation. And is able to be extended to use other acceleration methods in the future.

4. *Combining Trace Schemes and Loop Acceleration:*
   Accelerated interpolation can construct meta-schemes from trace schemes by inserting the transitive closure of the loop, prove its feasibility, and furthermore, is able to generate interpolants from the meta-scheme.

   Duration in weeks: 3

   Result: Accelerated interpolation returns a sequence of interpolants for use in the trace abstraction paradigm, or returns that the program is incorrect, depending on the feasibility of an error trace.

5. *Evaluating the Library:*
   Search for possible bugs and compare it to other trace checkers, such as PDR.

   Duration in weeks: 2

   Result: A bugfree accelerated interpolation library and data comparing it to other trace checkers.

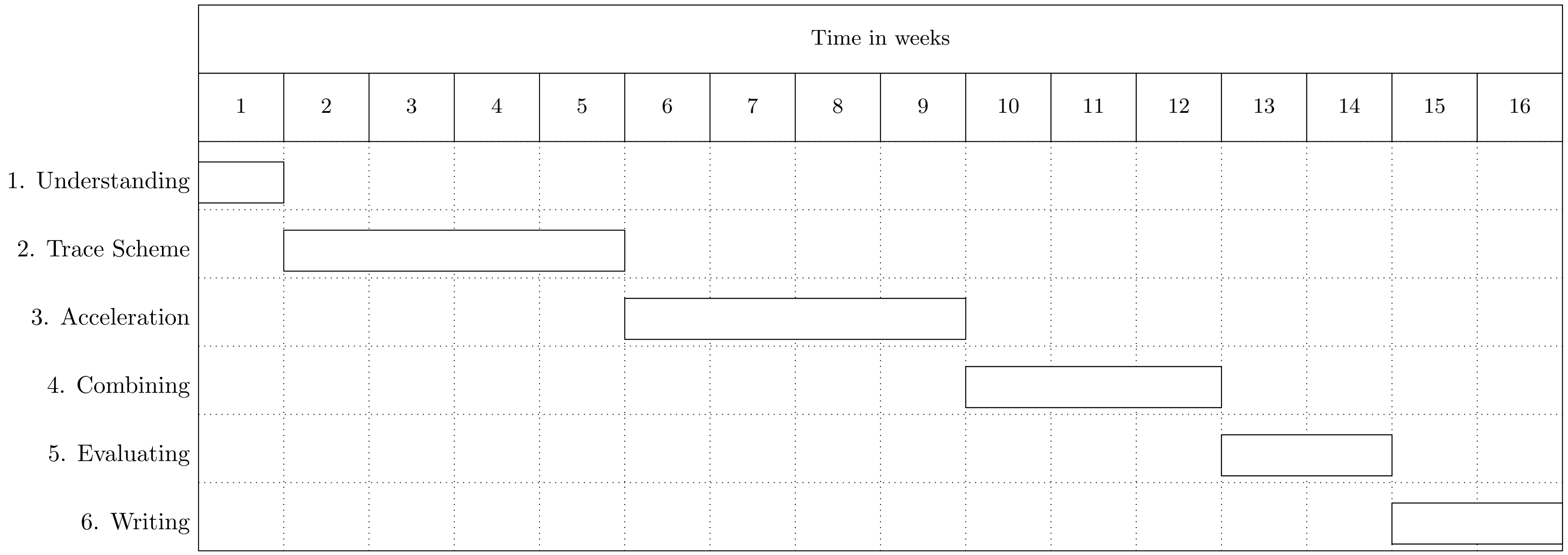
6. *Writing a Documentation about the Project:*
   Write in more detail than this proposal how accelerated interpolation works, and what new knowledge we discovered during this project

   Duration in weeks: 2

   Result: A written documentation about this project.

4 Schedule

This schedule illustrates the approach and shows the sequence of the individual tasks.



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