Signal and Image Processing 2020/2021
Exam Questions

— Do not turn this page before the official start of the exam! —

First Name, Surname: ______________________________

Student ID: ______________________________

Program: Master Data Science for Decision Making

Course code: KEN4222

Examiner: Dr. P. Bonizzi and dr. J.M.H. Karel

Date/time: Thursday October 22$^{\text{th}}$, 2020, 9.00-11.00h

Format: Open book exam (for hardcopy materials; electronic materials as indicated below) / computer exam. The use of internet other then to access Matlab and the exam is strictly prohibited.

Allowed aids: Pens, Matlab (either installed or through student desktop anywhere, with the following set of toolboxes: Bioinformatics toolbox, Control Systems toolbox, Database toolbox, Deep Learning toolbox, Image processing toolbox, Optimization toolbox, Signal processing toolbox, Statistics and Machine learning toolbox, Symbolic Math toolbox), textbook - "Discrete Wavelet Transformations: An Elementary Approach with Applications" (hardcopy or electronic), the lecture slides (hardcopy or electronic), the Matlab files that you created yourself and those that were provided by us (electronic) and the visualizer notes (hardcopy or electronic) and your own notes (hard copy only)

Instructions to students:

- The exam consists of 5 questions on 2 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on each page, including the cover page.
- Answer every question on a separate sheet of paper.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- **Good luck!**

For on-site exams:

- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.

For proctored exams:

- The scan / photographs of papers with answers on the exam questions must be readable.
- Do not forget to place your ID card in the top-right corner while scanning a page.

The following table will be filled by the examiner:

Question:	1	2	3	4	5	Total
Points:	13	24	24	22	17	100
Score:						

1. Fourier series and Fourier transform

   (a) (5 points) Compute *analytically* (by hand!) the Fourier series coefficient c_0 (and ONLY c_0) of the following 2π periodic function that is given for $t \in [-\pi, \pi)$ as:

   $$f(t) = t^2$$

   True or False? For each of the statements below: if the claim is true briefly explain why; if it is false provide a counterargument (e.g., in the form of a counterexample).

   (b) (4 points) The Shannon sampling theorem applies to any continuous-time signal, even to a signal having a Fourier transform that is non-zero over an infinite region of frequencies.

   (c) (4 points) The spectrum of a constant signal is a single peak at 0 Hz.

2. Filtering, linear systems, and Fourier analysis

   (a) (5 points) Consider the high-pass filter with transfer function

   $$H(z) = \frac{\frac{1}{3}}{1 + \frac{2}{3}z^{-1}}$$

   Plot its frequency response by means of the function `freqz`. What do you think about the performance of this filter in removing the low-frequency content from a signal?

   Load the file `dataexam2020Question2.mat`. This file contains a discrete-time sequence `x` representing a signal which has been sampled with a sampling frequency of 7418 Hz.

   (b) (6 points) Filter the signal with the filter introduced at point (a). Plot the input and the output signal to the filter overlapped in the same plot and the amplitude spectra of the input and the output signal overlapped in a different plot. Make some considerations on the result of the filtering.

   (c) (6 points) Design a notch filter with two zeros to filter out the 11.12 Hz component. Plot its frequency response, and express some considerations on how you expect this filter to perform.

   (d) (7 points) Filter the filtered signal obtained at question 2(b) by means of the notch filter implemented at question 2(c). Display the input and the output signal to the notch filter in the same plot. Make some considerations on the performance of the notch filter. Where are the high frequencies mainly located in the time domain?

3. Principal component analysis (PCA)
   Load the file `dataexam2020Question3.mat` into Matlab. Matrix `data` contains 15 recordings of EEG measurements, sampled at 128 Hz.

   (a) (8 points) Assume you would like to decompose this dataset by means of PCA, such that the decomposition focuses on the variance of the original EEG recordings. Do you think it is relevant or negligible to center this dataset before applying PCA? Please, motivate your answer.

   (b) (8 points) Decompose this dataset by means of singular value decomposition (SVD), by focusing on the variance of the original signals. Look at the first principal component. What features of the original signals were mainly captured by the first principal component? Is this what you expected? Please, motivate your answer.

   (c) (8 points) Reconstruct `data` by exploiting only the second principal component. Plot the 1st signal in `data` overlapped with its reconstruction. Is the peak in the 1st signal in `data` properly captured by the first principal component? If yes, why? If not, what could you modify in your approach to be able to capture it with the first principal component?

4. Wavelet filters and filtering

   (a) (8 points) Filter design
   Someone wants to create an orthogonal wavelet filter bank. It is given that the low-pass filter $H(z)$ is of length 4 and that $h_2 = \frac{\sqrt{2}}{8}$. Write down (pen and paper) the conditions this filter should satisfy and state where they come from. If conditions are redundant, indicate this. Next, compute the other low-pass filter coefficients h_0, h_1 and h_3. Please be aware that the filter has been scaled such that $|H(1)| = \sqrt{2}$.

   (b) (14 points) Wavelet shrinkage
   Consider the picture in `pictureexam2020Question4.jpg`. Load this data. Since this is a grayscale image, we will only consider the first color channel, which we will call `im`.

   Perform wavelet shrinkage (Visushrink) using a three scale Daubechies 6 wavelet transform and soft-thresholding on `im`. You can use the following low-pass filter coefficients for the Daubechies 6 wavelet: 0.2352, 0.5706, 0.3252, -0.0955, -0.0604, 0.0249 (mind the scaling). Plot the original image, the wavelet decomposition and the resulting denoised image all in separate subplots of the same figure. Below you can indicate in which file you have solved this.

5. Time-frequency analysis

   (a) (12 points) This question concerns the signal `s` and time vector `t` in the file `dataexam2020Question5.mat`. This is a noisy signal. Plot both the signal and the spectrogram (ensure that the axis have proper units) and describe what the components in the signal are.

   (b) (5 points) Indicate whether the following proposition is true or false and provide argumentation to support that. “By making the window in the short-term Fourier transform shorter, one can increase the frequency resolution at the expense of the resolution in time, due to the Heisenberg uncertainty principle”